COSMOVIA Lectures
Virtual Institute of Astroparticle physics
Paris, March 13, 2020

Wormholes and Teleporters

Igor Nikitin

Fraunhofer Institute for Algorithms and Scientific Computing
Sankt Augustin, Germany

Content

- Static wormholes

  spherically symmetric wormholes (Morris, Thorne 1988)

  polyhedral wormholes (Visser 1989)

  stargates (dyhedral wormholes, Visser 1996)
- Dynamic wormholes
- Wormholes supported by Quantum Gravity

- Topology change

  Example 0: string theory

  GR: topological censorship theorems (Geroch 1967, Geroch, Horowitz 1979, Hawking, Ellis 1973, Borde 1994)

  GR: working them around (Sorkin 1986, Louko, Sorkin 1997, Horowitz 1991, Ionicioiu 1997, McCabe 2005)

  Example 1: dynamical opening of the wormhole

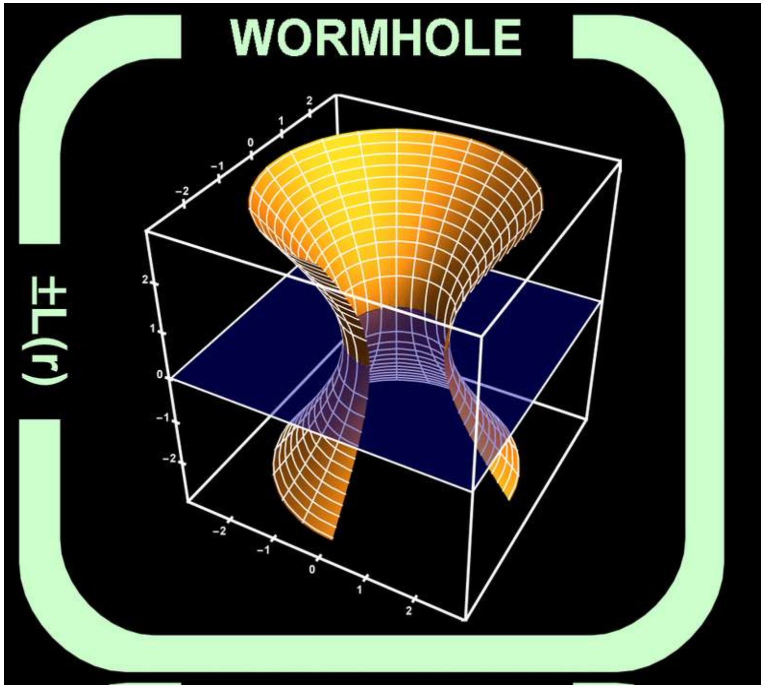
  Example 2: topological teleporter

arXiv:1909.08984

Static Wormoles: spherically symmetric case

- Morris-Thorne 1988 model
- no event horizon
- a tunnel connecting 2 universes or 2 sites of a single universe
- requires exotic matter ($\rho+p<0$)
- B→∞, A>0 is finite, L proper length integral is finite

$$L(r) = \int dr \sqrt{B(r)}$$



- metric, the square of the distance between points in spacetime:

$$ds^2 = -Adt^2 + Bdr^2 + Dr^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$A = 1 - r_0/r + \alpha/r^2,\ B = (1 - r_0/r)^{-1},\ D = 1$$

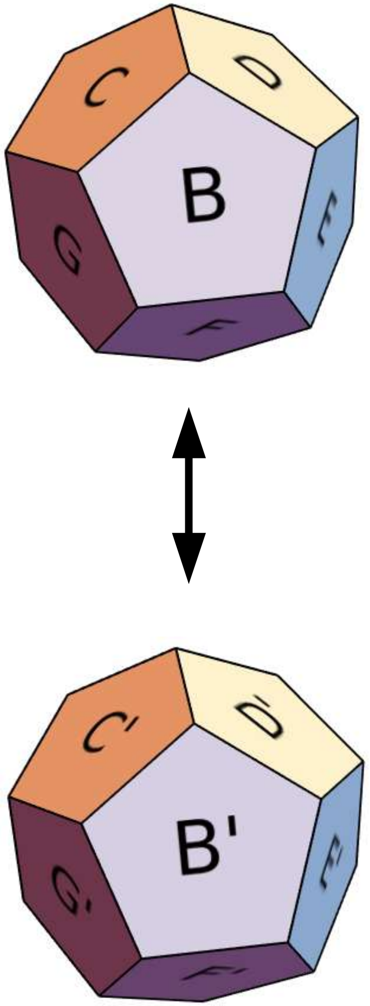
(a specific example, r_0,α >0, r→r_0+0 is the wormhole throat)

M.Visser, Lorentzian Wormholes: from Einstein to Hawking, Springer 1996

Static Wormoles: polyhedral case

- Visser 1989
- two copies of R^3 space, two congruent polyhedra cut out
- faces identified (AA', BB', ...)
- the space is locally flat near the faces

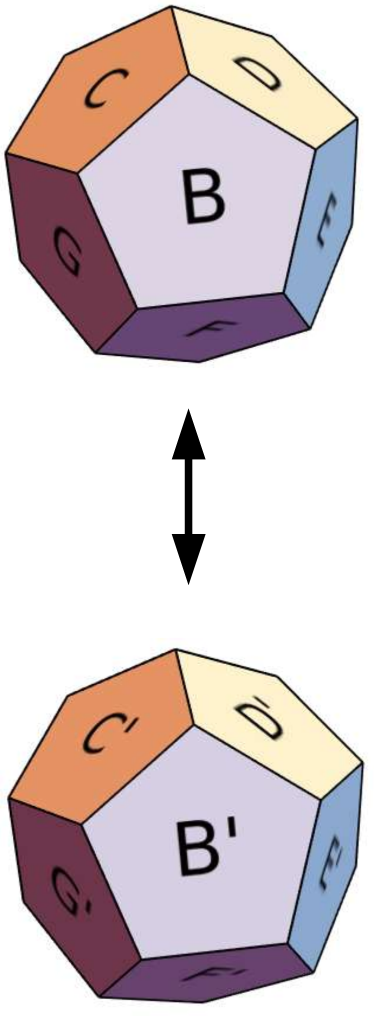
  => vacuum, no matter
- the space has a topological singularity near the edges, a conic type singularity with negative defect (positive excess) of angle
- angular arithmetics: $2\pi-\alpha$ in one universe plus $2\pi-\alpha$ in another universe, where $\alpha<\pi$ is the angle between the adjacent faces, result $=4\pi-2\alpha$, instead of 2π for flat space => $2(\pi-\alpha)$ excess, $-2(\pi-\alpha)$ defect



Static Wormoles: polyhedral case

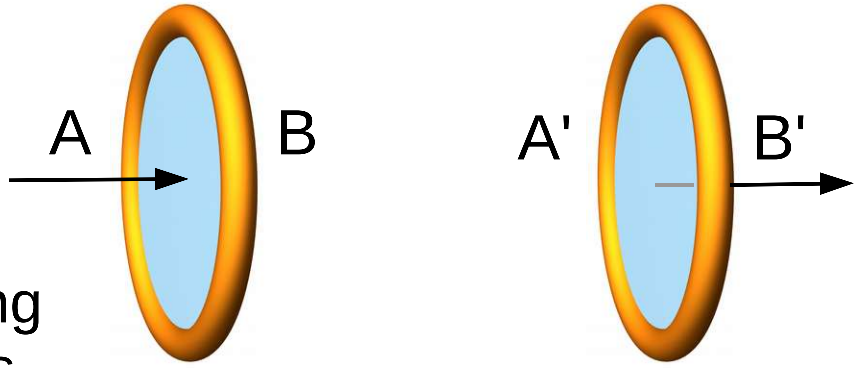
- Nambu-Goto string with linear mass density μ and tension τ has μ=τ=defect/(8π) in geometric units (G=c=1)
- on the edges: the pieces of exotic string with

  μ=τ=-(π-α)/(4π) <0
- exotic matter nevertheless, but the wormhole is traversable, the observer going through the face can avoid the regions of high curvature and exotic matter on the way from one uni to the other
- Remark: it's better to identify the faces with P-reflection to avoid P-reflection of the observer body (DNA composition etc) while going through the wormhole

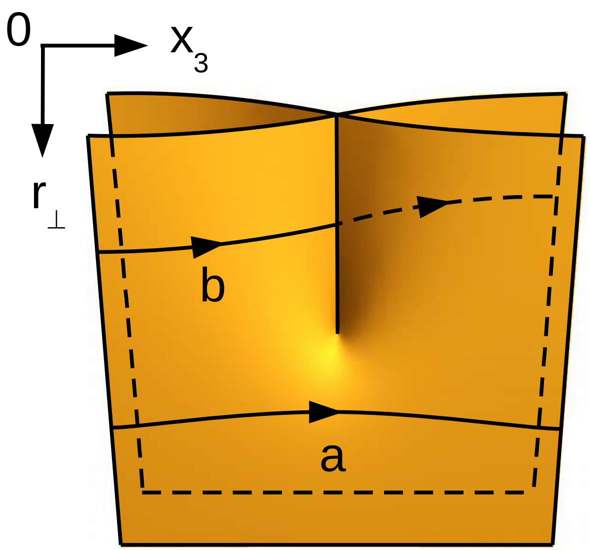


Static Wormoles: Stargate (dihedral case)

- Visser 1996, a particular case
- two-face degenerate polyhedron
- faces A,B with α=0
- any number of edges, N→∞ is a disk
- exotic string with μ=τ=-1/4 coiled in a ring
- in natural units approx -0.2 Jupiter mass per meter



- an embedding diagram in cylindric coords has a branching point
- (a) way not intersecting the face, remains in one uni
- (b) way intersects the face, goes to the another uni



arXiv:1909.08984

Static Wormoles: Einstein Field Equations (EFE)

spherically symmetric case, $Z=B^{-1}$

$$\rho = -(-1 + Z + rZ'_r)/(8\pi r^2),$$ density

$$p_r = (A(-1+Z) + rZA'_r)/(8\pi r^2 A),$$ radial pressure

$$p_t = (-rZ(A'_r)^2 + 2A^2 Z'_r$$ transversal pressure

$$+A(rA'_r Z'_r + 2Z(A'_r + rA''_{rr})))/(32\pi r A^2)$$

A(r),Z(r) profiles are freely selected

the matter terms are finite, if r,A in denominator are separated from zero (>Const>0) and r,A,Z and derivatives in the numerator are finite

arXiv:1909.08984

Static Wormoles: flare-out conditions

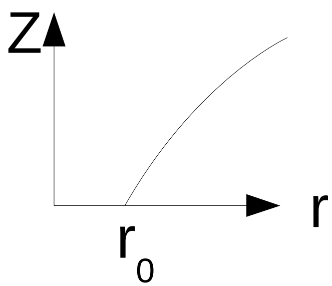
throat condition
($B^{1/2}$~$(r\text{-}r_0)^{-1/2}$→∞ in integrable way)

necessary and sufficient conditions
on density and radial pressure

$$\boxed{r = r_0,\ \ Z = 0,\ \ Z'_r > 0} \Leftrightarrow \boxed{p_r = -1/(8\pi r^2) < 0,\ \ \rho + p_r < 0}$$

$$\Rightarrow \boxed{p_t > 0,\ \ \rho + p_r + 2p_t > 0}$$

exotic matter



more necessary conditions on transversal pressure

(A'_r>0 as an additional condition imposed for the derivation)

Dynamic Wormoles

evolution of throat radius

$$r(t,L) = r_0(t) + c(t)L^2$$

shape function c>0
(c<0 for inverse wormhole, a bubble)

$$L(r,t) = ((r - r_0(t))/c(t))^{1/2},\ ds^2 = -Adt^2 + dL^2 + r^2 d\Omega^2$$
$$= (-A + (L_t')^2)dt^2 + (L_r')^2 dr^2 + 2L_t' L_r' dt dr + r^2 d\Omega^2$$

L'_t and L'_r contain a singular multiplier $|r-r_0|^{-1/2}$

non-diagonal term

$g_{tr} \sim -r'_0/(4c(r-r_0))$ and $g_{tt} \sim (r'_0)^2/(4c(r-r_0))$, the terms that remain active in a small vicinity $r \sim r_0$, even if r'_0 becomes arbitrarily small

(paper in preparation)

Dynamic Wormoles

The evaluation of Einstein tensor leads to a lengthy expression with the following properties:
(i) denominators of components are monomials of (r,A,c);
(ii) numerators are polynomials of (r,A,c,r_0) and their derivatives up to the second order;
(iii) all components are finite, provided that the denominator values are separated from zero and numerator values and derivatives are finite;
(iv) quasistatic property: when time derivatives vanish, the expressions coincide with those of static limit

in addition:

$$Z = (L'_r)^{-2} = 4c\,(r - r_0),\ \det g = -(A/Z)\, r^4 \sin^2\theta,$$

=> $(-\det g)^{1/2} \sim |r-r_0|^{-1/2}$, the densities $T^\mu{}_\nu (-\det g)^{1/2}$ are integrable

(paper in preparation)

Wormoles supported by QG

Energy conditions (Einstein, Hawking):

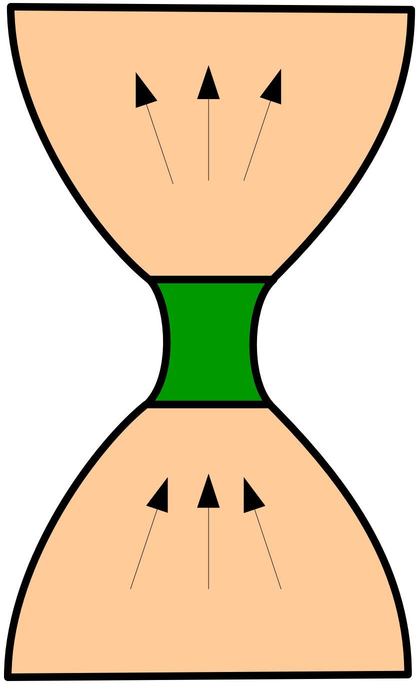
- there are *no* negative masses (weak energy condition)
- as well, there are *no* materials with ρ+p<0 (null energy condition)

Critics of energy conditions

- Barceló, Visser, Twilight for the energy conditions?, 2002

Models requiring exotic matter:

- wormholes (Morris-Thorne 1988)
- warp drives (Alcubierre 1994)
- Planck stars (Rovelli-Vidotto 2014, Barceló et al. 2015)
- RDM stars (arXiv:1701.01569)
- TOV stars with QG core (arXiv:1811.03368)



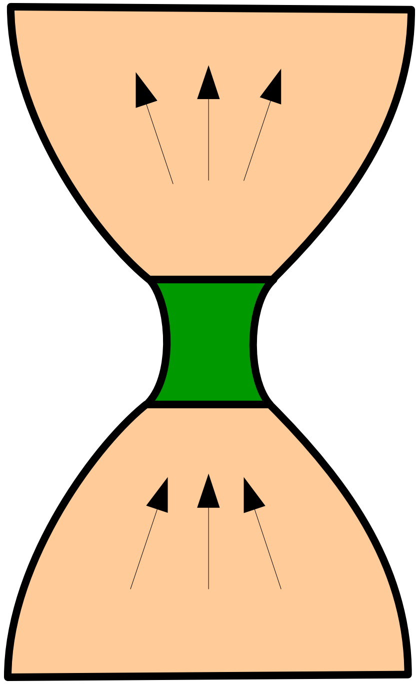
QG bounce of Planck star: collapse replaced by extension, black hole turns white

Wormoles supported by QG

QG able to generate effectively negative mass densities:

$\boldsymbol{\rho_X = \rho\,(1-\rho/\rho_P)}$ (Ashtekar et al. 2006)

- $\rho=\rho_P \Rightarrow \rho_X=0$ at Planck density the gravity is switched off
- $\rho>\rho_P \Rightarrow \rho_X<0$ in excess of Planck density the effective negative mass appears (**exotic matter**), gravitational repulsion (antigravity)



QG bounce of Planck star: collapse replaced by extension, black hole turns white

Wormoles supported by QG

Q: what if the effectively exotic matter terms created by quantum gravity do not lead to a quantum bounce, but to the formation of a wormhole?

The program:

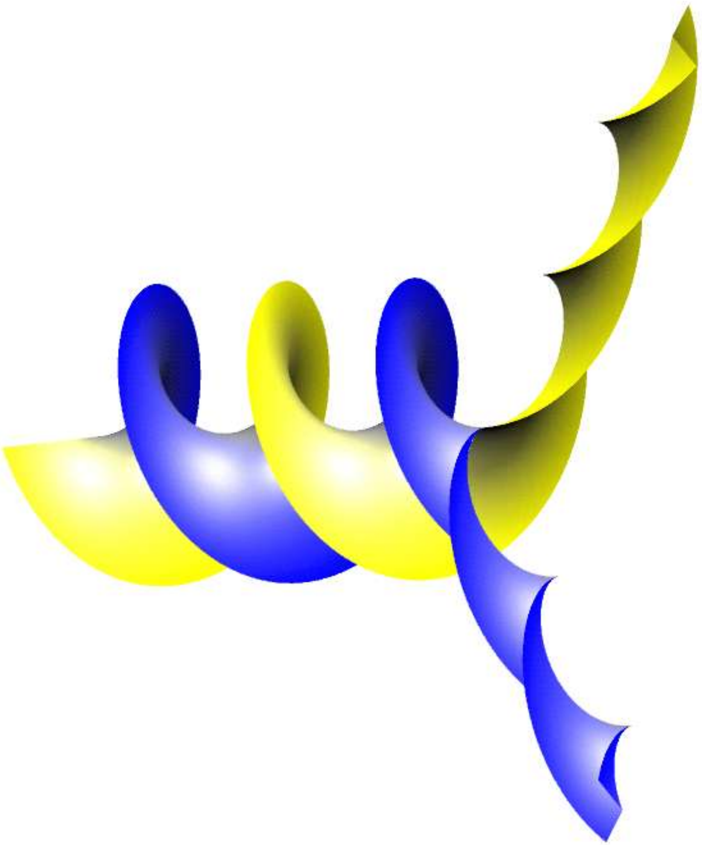
- take the environment model, where Planck density is reachable
  E.g., equation of state (EOS) $\rho = p_r$, $p_t = 0$, null radial dark matter (NRDM)
  solution: $\rho_{nom} = \varepsilon/(8\pi r^2 A(r))$, A(r) rapidly falling with decreasing r
  (mass inflation)
- in quantum region ($\rho_{nom} > \rho_P$) define a wormhole metric or a class of metrics, static or dynamic
- use EFE to convert it into QG modified EOS
  $\rho = \rho(\rho_{nom})$, $p_r = p_r(\rho_{nom})$, $p_t = p_t(\rho_{nom})$
- use flare-out conditions to make a qualitative prediction of what QG modified EOS should be to open the wormhole

(this program will be implemented below in frames of a more complex scenario, involving topology change)

arXiv:1909.08984

Topology change: string theory

- **Example:** Nambu-Goto classical string theory
- contains solutions of variable topology
- breaking and fusion of strings
- Artru 1983
- Klimenko, Nikitin 2001 (arXiv:hep-th/0110042)
- **Solution 1:** breaking in a regular point



Topology change: string theory

Details of construction (open string):

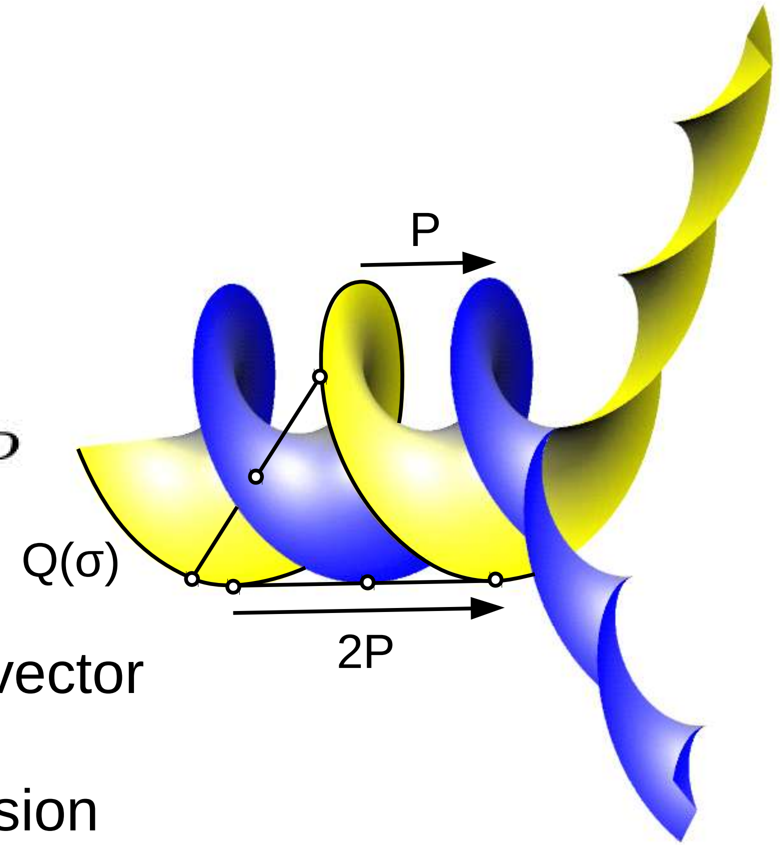
worldsheet (WS): a set of middles

$$x(\sigma_1,\sigma_2)=(Q(\sigma_1)+Q(\sigma_2))/2$$

supporting curve (1st edge of WS)

$$Q'(\sigma)^2=0,\ \ Q(\sigma+2\pi)=Q(\sigma)+2P$$

light-likeness periodicity



2nd edge is Q(σ)+P; P is energy-momentum vector

$Q'(\sigma)=\sum_n a_n e^{in\sigma}$ Fourier expansion

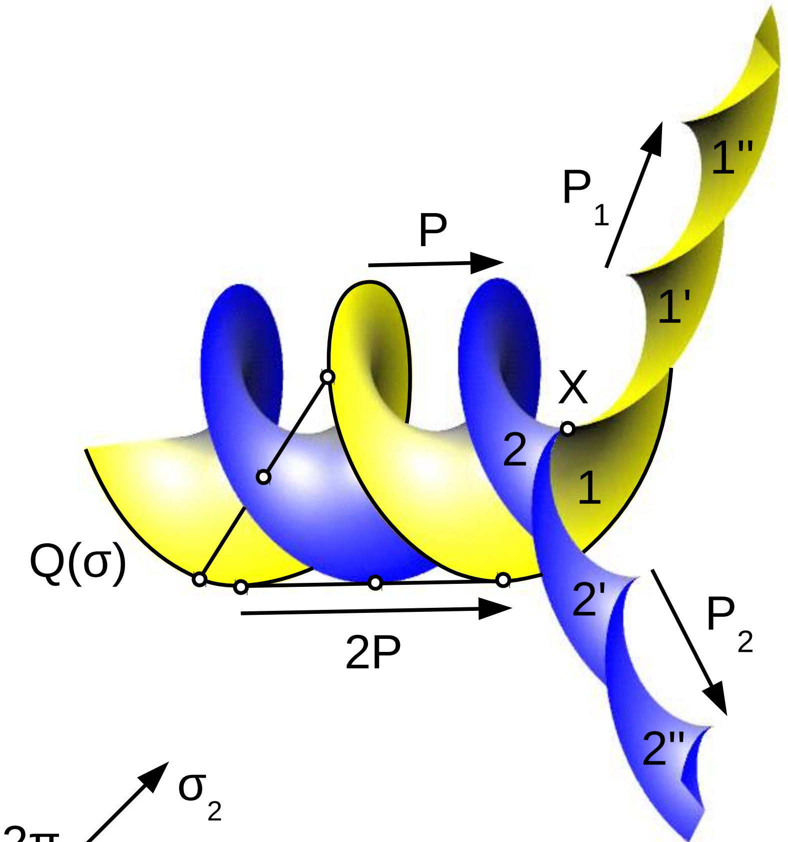
$L_n=\sum_k a_k a_{n-k}=0$ Virasoro constraints

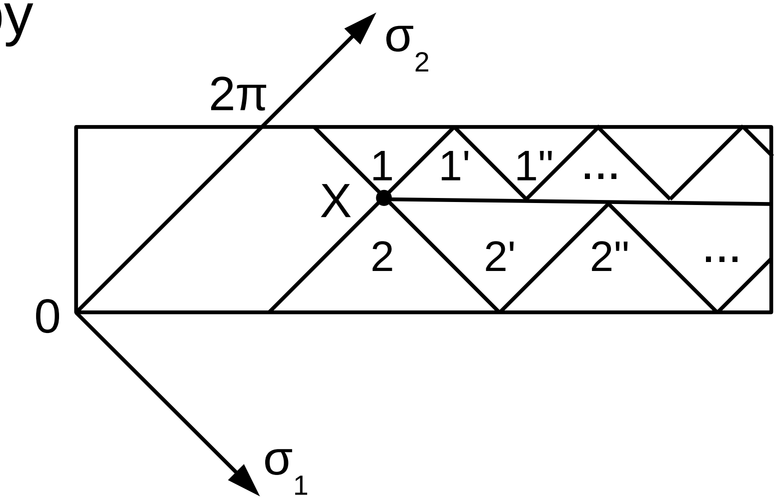
$\sigma_{1,2}=\tau\pm\sigma$ light-like parametrization of WS

Topology change: string theory

Details of construction (break of open string):

- for an arbitrarily selected point X, patches 1 and 2 are selected on the original WS
- separated by the lines $\sigma_{1,2}$=Const, characteristics, paths of light signal from X on WS
- continued periodically to the sequences {1',1'', ...} and {2',2'', ... }, the resulting 2WS after string break
- alt: periodic continuation of 1,2-segments of supporting curve and reconstruction by generic algorithm

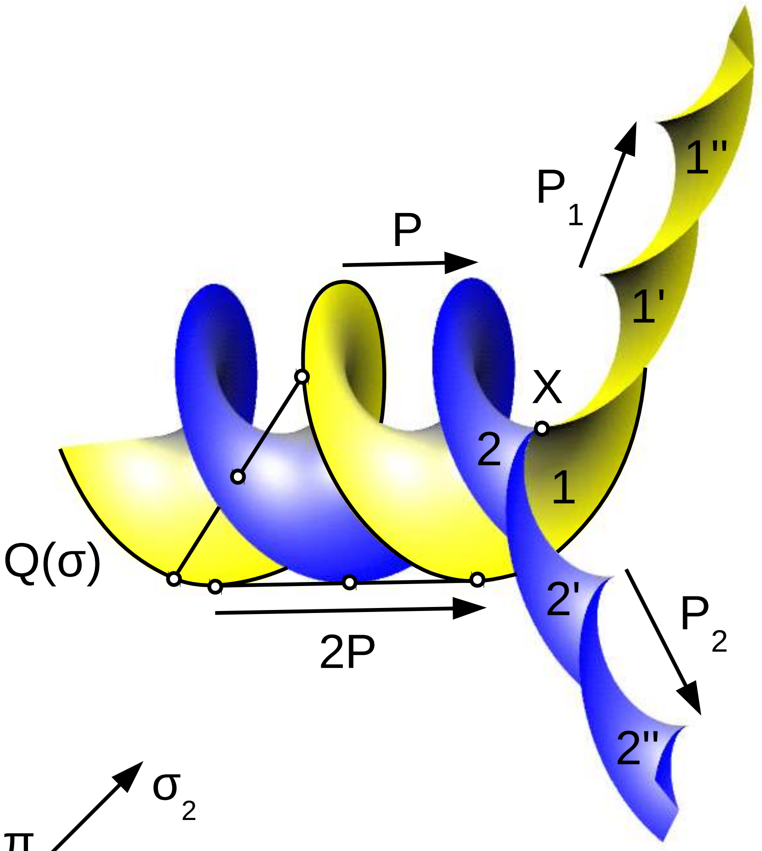


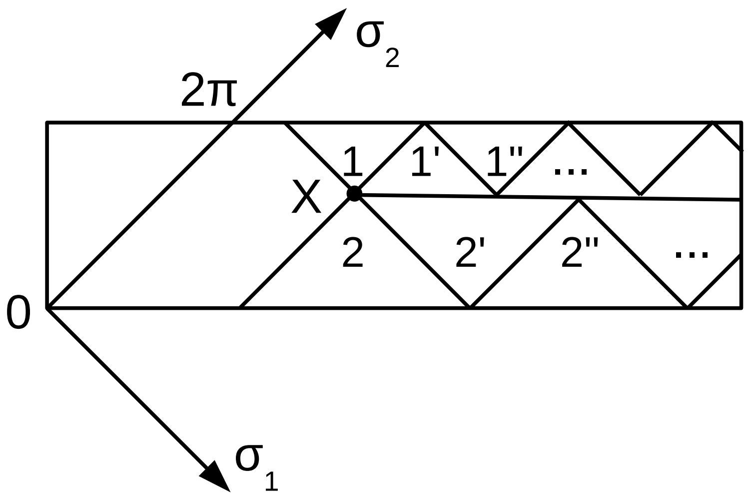


Topology change: string theory

Details of construction (break of open string):

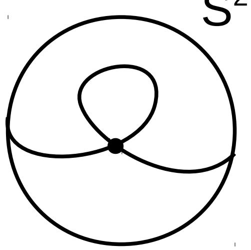
- the immediate consequence: $P = P_1 + P_2$, conservation of energy-momentum for string break
- C^0-fractures of WS appear on outgoing characteristics
- equations defining the shape of WS are satisfied in generalized functions
- (proof in arXiv:hep-th/0110042)





Topology change: string theory

Solution 2: break of open string in a singular point

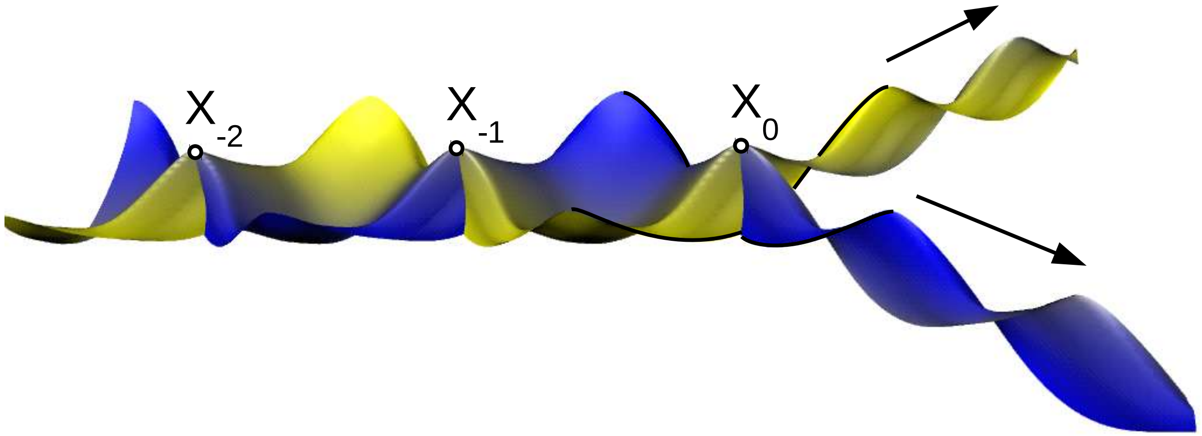


- let $q(\sigma)$ be a projection of $Q(\sigma)$ orthogonal to P (CMF) => a closed curve in R^3
- $N(\sigma) = q'(\sigma)/|q'(\sigma)|$ - unit tangent on S^2
- self-intersection point => lift to R^3, parallel tangents $q'_1 \sim q'_2$
- light-likeness $Q^{0\prime}=|q'|$ => lift to R^4, parallel tangents $Q'_1 \sim Q'_2$
- WS tangents $\partial_{1,2}x \sim Q'_{1,2}$ => collapse of tangent plane, singular point on WS
- stable to small variations of $Q(\sigma)$, or $q(\sigma)$, or $N(\sigma)$

Topology change: string theory

Solution 2: break of open string in a singular point

when the break occurs at a singular point, the continuation through parallel tangents is C^1-smooth



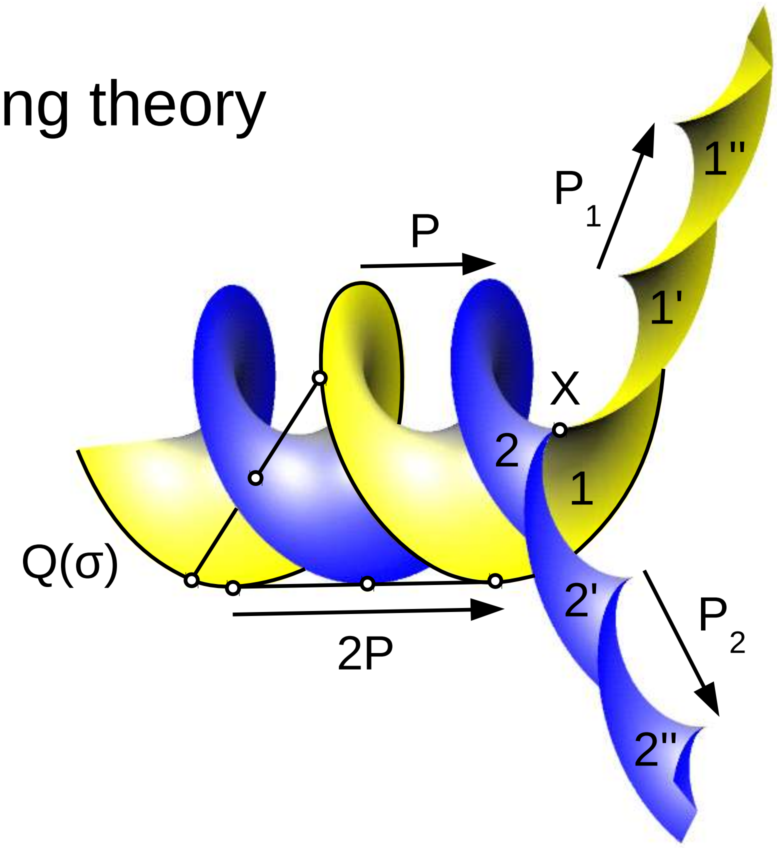
Topology change: string theory

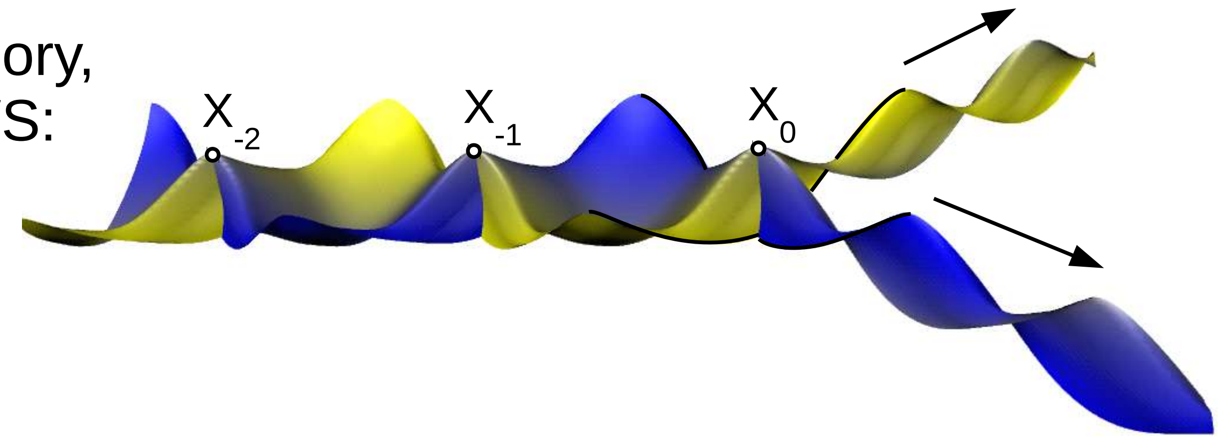
Resume:

string breaking processes are closely linked to singular points that either exist on the original WS or arise as a result of breaking

Remark:

- string breaking processes are available only in Lagrangian theory
- absent in the Hamiltonian theory, which fixes the topology of WS: the band IxR^1 open strings, the cylinder S^1xR^1 closed strings...





Topology change in General Relativity

Definition: Lorentzian time-orientable chronological manifold
- equipped with everywhere Lorentzian metric (- +++)
- with everywhere nonzero continuous vector field defining time flow
- there are no closed timelike curves (no time machines)

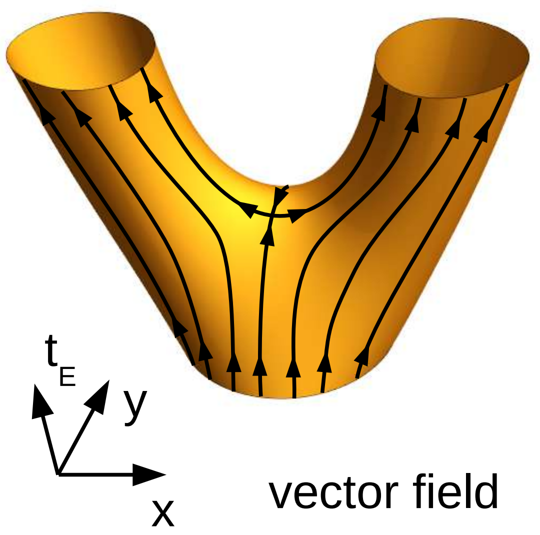
Theorem (Geroch 1967): change of topology not possible on such manifolds

Definition: almost everywhere Lorentzian manifold
- Lorentzian metric is introduced everywhere, except for a thin set of singular points

Theorem (Sorkin 1986): change of topology possible on such manifolds

Three methods for defining spacetime manifolds in GR

Method 1: Euclidean embedding diagrams
- define a surface in Euclidean space
- define a vector field V on it
- induce Euclidean metric on the surface
- redefine it to the Lorentzian metric



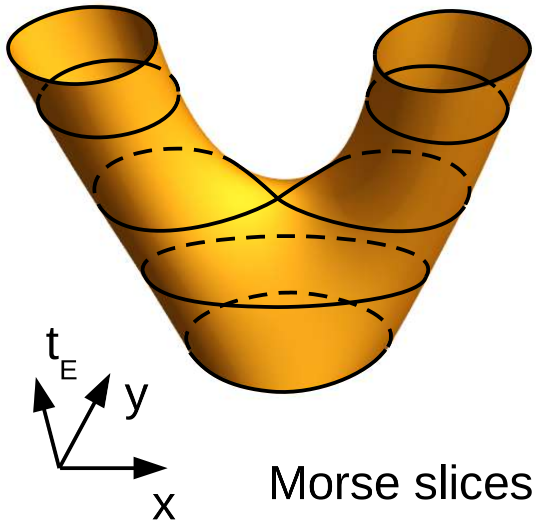
$$(g_L)^{\mu\nu} = (g_E)^{\mu\nu} - 2V^\mu V^\nu / ((g_E)_{\alpha\beta} V^\alpha V^\beta)$$

Properties:
- the metric is re-projected in the direction of V
- the components along V receive a Lorentzian signature
- with respect to the new metric, V is timelike
- can be used to specify the direction of time on the manifold

Three methods for defining spacetime manifolds in GR

A particular case: V ~ grad f

- For topology change $M_0 \rightarrow M_1$, an interpolating manifold, or *a cobordism* is a manifold M whose boundary is a disjoint union of M_0 and M_1, $\partial M = M_0 \underline{\cup} M_1$.
- *Morse function* is any smooth function, taking values $f(M_0) = a$, $f(M_1) = b$ on the boundaries and intermediate values in the interior $M \backslash \partial M$, whose critical points are non-degenerate and located in the interior $M \backslash \partial M$.
- Morse function can be taken as a global time coordinate t=f, interpolating between the initial and final states in the topological transition.



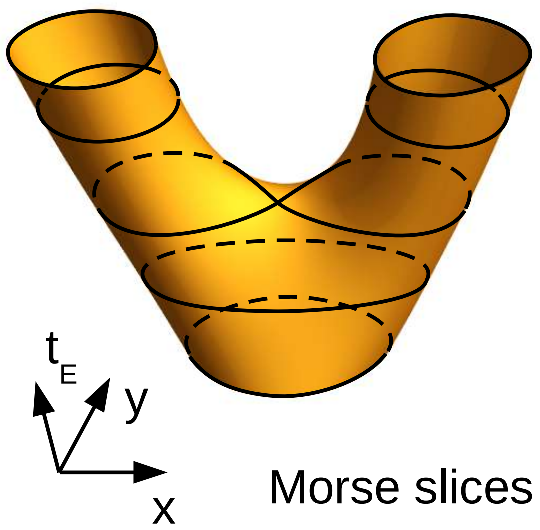
Morse slices

Three methods for defining spacetime manifolds in GR

A particular case: V ~ grad f

Equivalent to the previous definition with the replacement $V \rightarrow \partial f$, optionally:

- a trivial choice for the Euclidean metric $g_E \rightarrow \delta$
- an overall factor $\lambda \geq 0$
- an arbitrary reprojection factor $\zeta > 1$

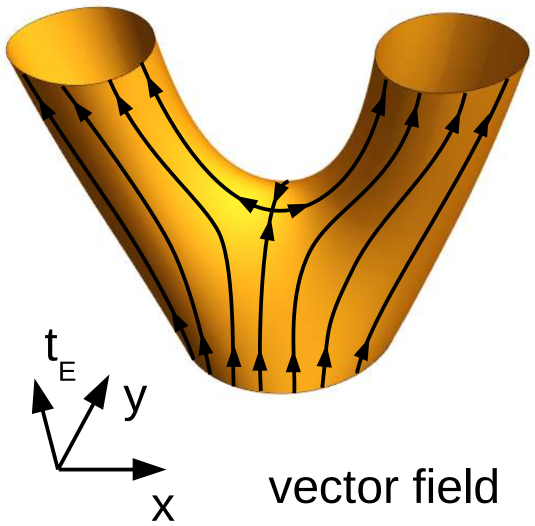


$$(g_L)_{\mu\nu} = \delta_{\mu\nu} \overset{\lambda}{\boxed{((\partial_\alpha f)(\partial_\beta f)\delta^{\alpha\beta})}} - \zeta(\partial_\mu f)(\partial_\nu f), \;\; \zeta > 1$$

Three methods for defining spacetime manifolds in GR

Example: $S^1 \to 2S^1$, closed uni splits to 2 closed unis. V necessarily has a singular point of saddle type.

Proof: fixing the topology of transition and direction of V at the boundaries, deform the surface to a sphere. Count indices: 3 foci (+1), sum must be $\chi=2$ (Poincaré-Hopf theorem) => should be at least one (-1) saddle.

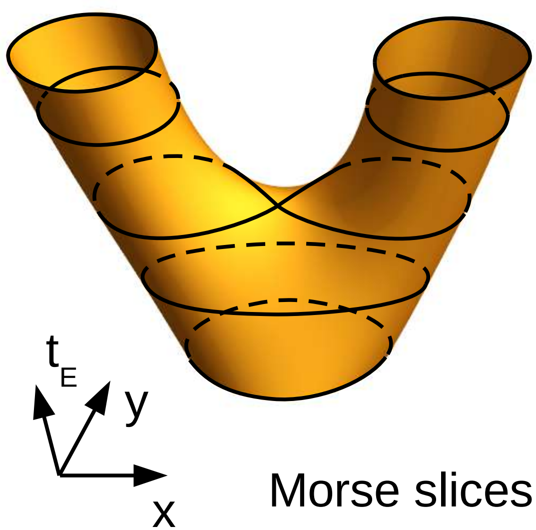


vector field

Check of chronological structure

- general V-def: closed integral trajectories of non-zero V possible => closed timelike paths
- Morse def: no closed timelike paths possible in general position

- *Proof:* consider closed timelike path not going through singular points; min f, max f reachable, where tangent to the path contained in f=Const section, which is spacelike (contradiction). If path comes through singular point, where metric is degenerate, difficulties arise. However, the path can be taken off singular points by a small variation (general position).



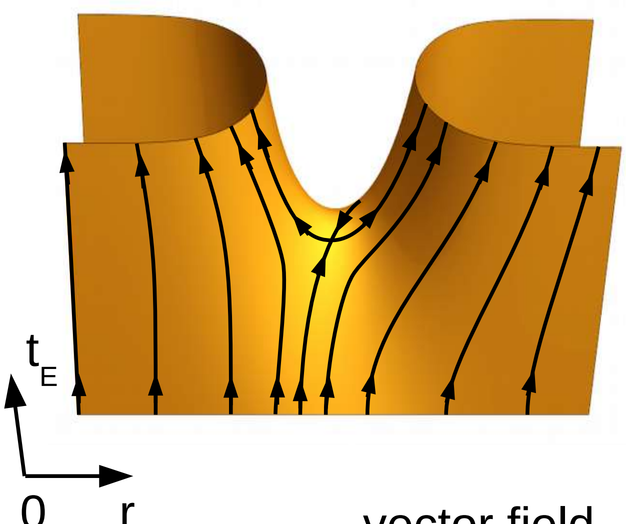
Morse slices

Three methods for defining spacetime manifolds in GR

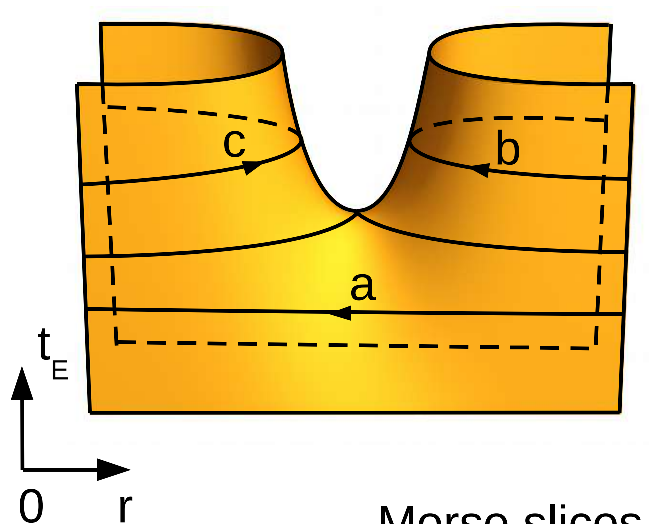
Example: $2R^3 \rightarrow R\times S^2 + S^3$

- dynamical opening of the wormhole, with the separation of a bubble (baby uni)
  (a) way from infinity to zero radius in one uni
  (b) way from one uni through wormhole throat (rmin) to another uni
  (c) way through rmax of the bubble

- Euclidean embedding diagram for (r,t) coordinates
- angular coordinates defined in a standard way



vector field



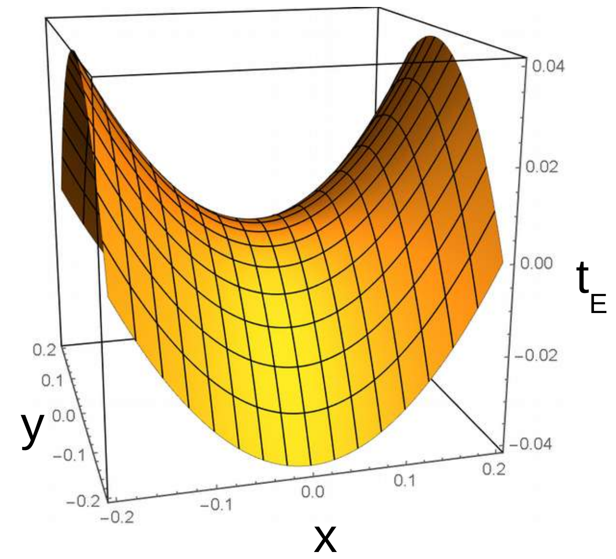
Morse slices

Three methods for defining spacetime manifolds in GR

Method 2: Lorentzian embedding diagrams

- Example: surface $t_E = x^2 - y^2$
- induction of Minkowski metric $ds^2 = dt^2 - dx^2 - dy^2$ onto it does not lead to the Lorentzian metric
- transformation $t_L = t_E^{1/3}$ leads to almost everywhere Lorentzian manifold

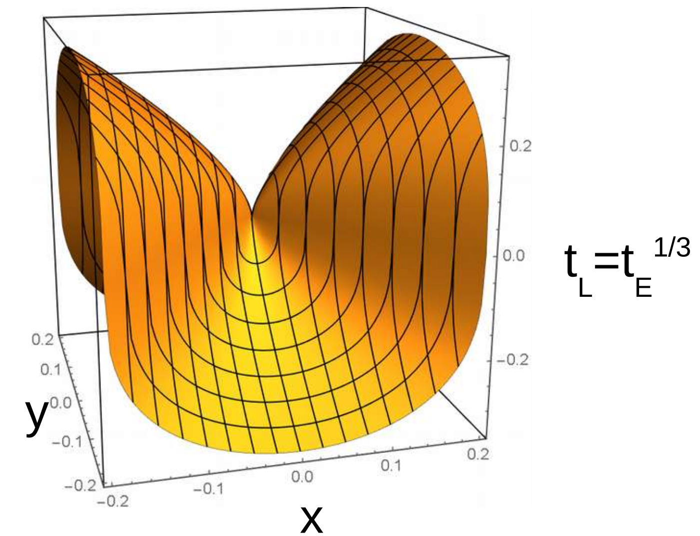
Proof: $(x, y) = r(\cos\alpha, \sin\alpha)$, $n_1^2 + n_2^2 - n_3^2 = -1 + (4/9)\, r^{-2/3} (\cos 2\alpha)^{-4/3} > 0$, $0 < r < 8/27$



Check of chronological structure:

- closed timelike paths not possible

Proof: otherwise they will be closed timelike in the ambient space, which is chronologically trivial (contradiction)



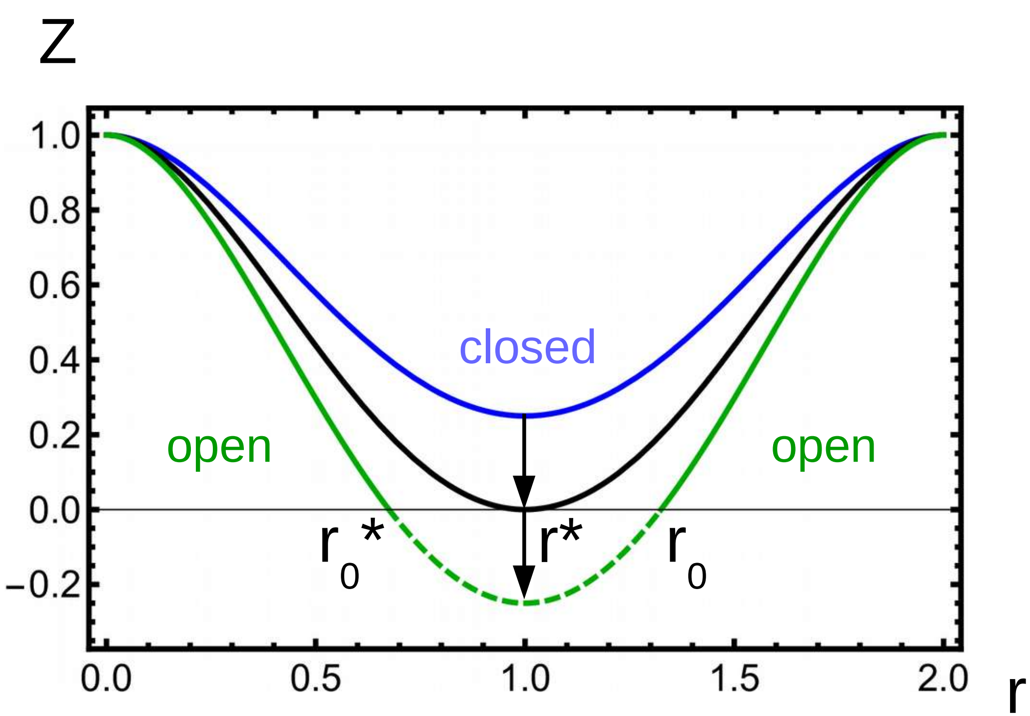
Three methods for defining spacetime manifolds in GR

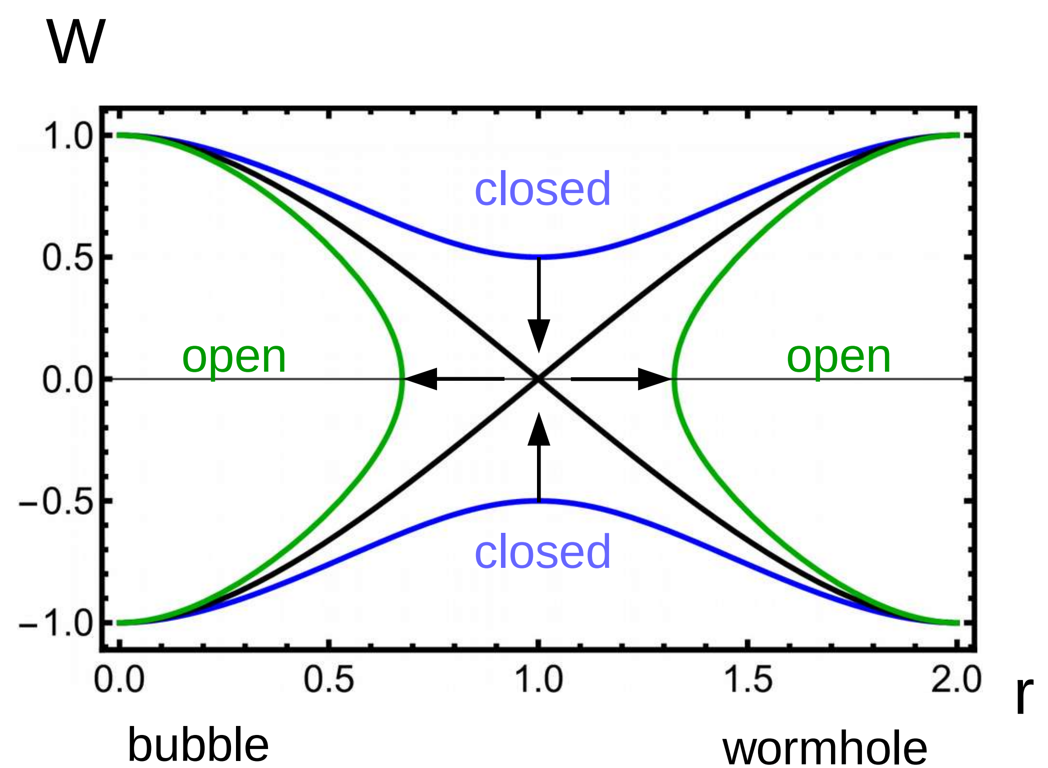
Method 3: direct definition of the metric components

$$ds^2 = -A(r,t)dt^2 + B(r,t)dr^2 + 2C(r,t)dtdr + D(r,t)r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

generic spherically symmetric; static: A(r)*, B(r), C=0, D=1;
auxiliary: $Z=B^{-1}$, $W=\pm\mathrm{sqrt}(Z)$

principal quasistatic scheme for opening the wormhole with bubble separation:

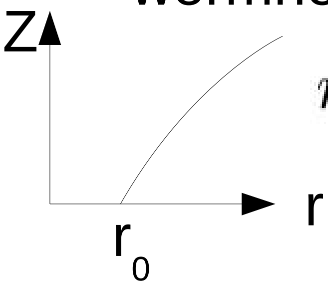




* A>0, A_r'>0 time-independent arbitrary

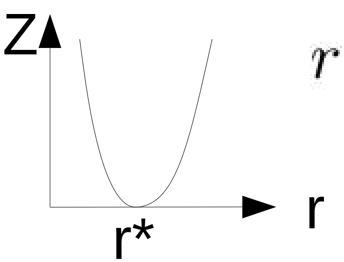
Generalized flare-out conditions

wormhole throat (as above)



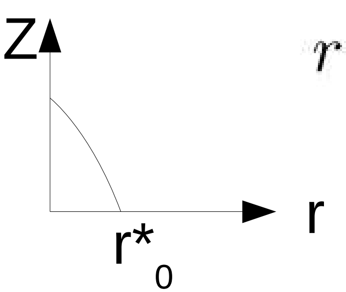
$$r = r_0,\ Z = 0,\ Z'_r > 0 \ \Leftrightarrow\ p_r = -1/(8\pi r^2) < 0,\ \rho + p_r < 0$$
$$\Rightarrow\ p_t > 0,\ \rho + p_r + 2p_t > 0$$

bifurcation point:



$$r = r^*,\ Z = 0,\ Z'_r = 0 \ \Leftrightarrow\ p_r = -1/(8\pi r^2) < 0,\ \rho + p_r = 0$$
$$\Rightarrow\ p_t = 0,\ \rho + p_r + 2p_t = 0$$

bubble rmax:



$$r = r^*_0,\ Z = 0,\ Z'_r < 0 \ \Leftrightarrow\ p_r = -1/(8\pi r^2) < 0,\ \rho + p_r > 0$$
$$\Rightarrow\ p_t < 0,\ \rho + p_r + 2p_t < 0$$

If A-profile monotonicity condition is not strict, $A'_r \geq 0$, the same conditions are met, except for $\rho + p_r + 2\ _t = 0$ at the points where $A'_r = 0$

Misner-Sharp mass (MSM)

$$M = r/2\ (1 - B^{-1})$$

equal to ∫dr $4\pi r^2 \rho$ for spherically symmetric problems (metric C=0, D=1)

$$A(r_0) > 0,\ \ B(r_0) \to +\infty,\ \ 2M(r_0) = r_0$$

in throat rmin, bifurcation r* and bubble rmax points

Q: should MSM (and radius) of the bubble be conserved?

- generally not (see Blau, Lecture Notes on General Relativity, Uni Bern 2018)
- MSM conservation should follow from EFE
- MSM is conserved for a spherically symmetric system surrounded by vacuum, where EFE => MSM conservation (Birkhoff's theorem)
- MSM is not conserved for FLRW with pressure, where MSM is changed by the work of pressure forces
- FLRW also does not have C=0, D=1 metric and has a different MSM definition

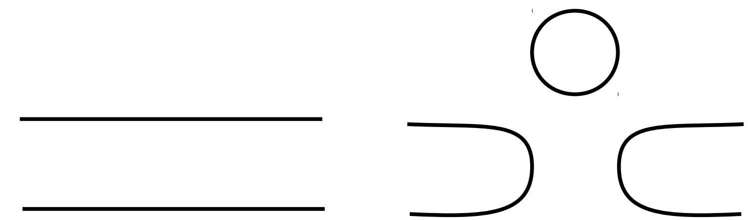
Misner-Sharp mass (MSM)

- generally, an arbitrary evolution of $Z(t,r)$ or $r(t,L)$ profiles is allowed, for which EFE will produce the corresponding matter terms
- in particular, the bubble can evolve according to closed scenarios of FLRW model
- nevertheless, we will consider a special scenario, when after the formation, the bubble comes to an equilibrium and preserves its external radius and MSM
- the dynamics of wormhole is not influenced by the behavior of the bubble

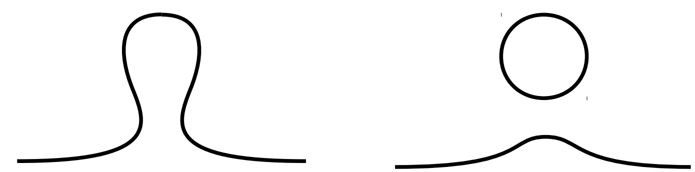
Other works on wormhole opening

- our scheme: $2R^3 \rightarrow R \times S^2 + S^3$

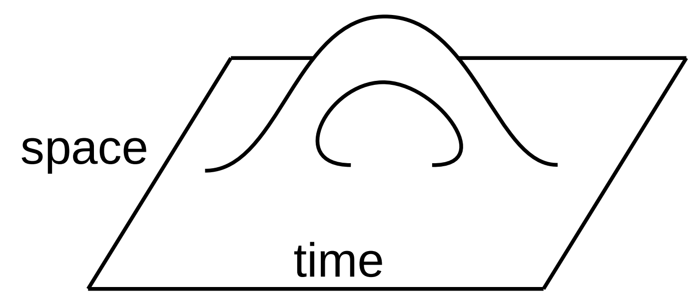
two copies of 3dim space converted to
two copies of 3dim space connected by
the wormhole plus the bubble

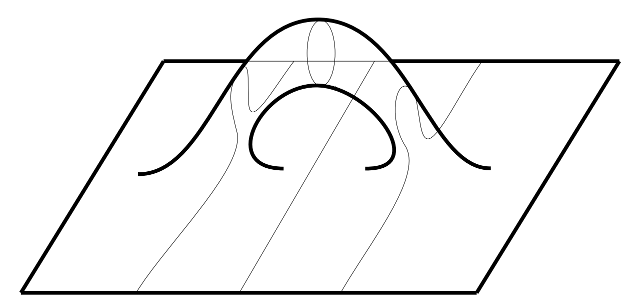


- Visser 1996, Waldrop 1987, Battarra et al. 2014: $R^3 \rightarrow R^3 + S^3$, bubble inflated through the wormhole, wormhole is torn*, leaving 3dim space and the bubble



- Hebecker et al. 2018: gravitational instantons, Euclidean metrics for quantum theory path integrals $R^3 \rightarrow R^3 + S^3 \rightarrow R^3$, a handle (also called a “wormhole”), the previous process combined with its inverse





* if $r_0 \rightarrow 0$ quasistatically, the radial pressure will be infinite: $p_r = -1/(8\pi r_0^2)$

Wormhole opening: the details

NRDM model taken as an environment (arXiv:1701.01569)
equation of state (EOS) $\rho = p_r$, $p_t = 0$,
solution: $\rho_{nom} = \varepsilon/(8\pi r^2 A(r))$, A(r) rapidly falling with decreasing r (mass inflation)
QG cutoff: $\rho_{nom} = \rho_P/N$, N=1-10 attenuation factor
logarithmic coordinate transformations, to display a large range of values of the graphs

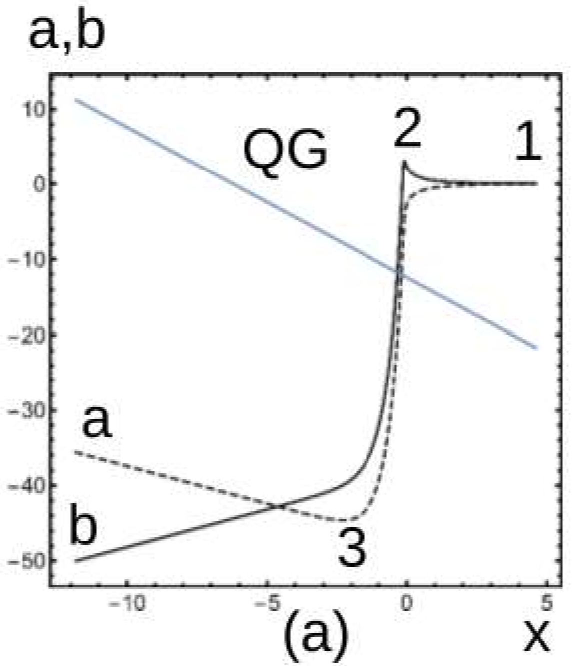
$$a = \log A,\ b = \log B,\ x = \log r$$

$$f(h) = \mathrm{arcsinh}(h/2),\ f^{-1}(h) = 2\sinh(h),$$
$$z = f(Z),\ w = f(W),\ y = f(r),\ l = f(L)$$

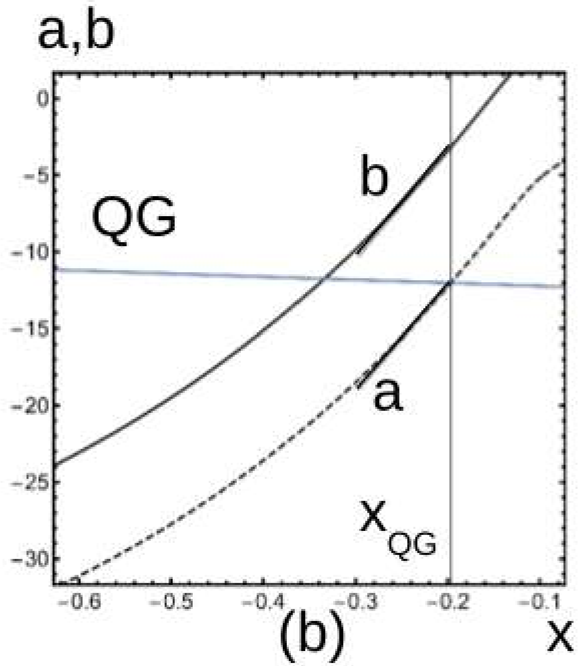
$f(v) \sim \log v$, $v \to \infty$

to model the dependencies, Bézier curves of order 3 are used with C^1-stitching between the segments

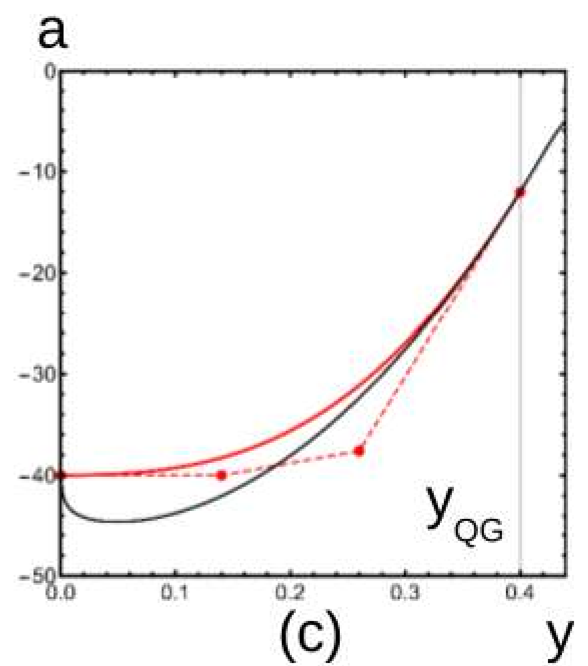
Wormhole opening: the details



NRDM solution with QG cutoff (a model example)



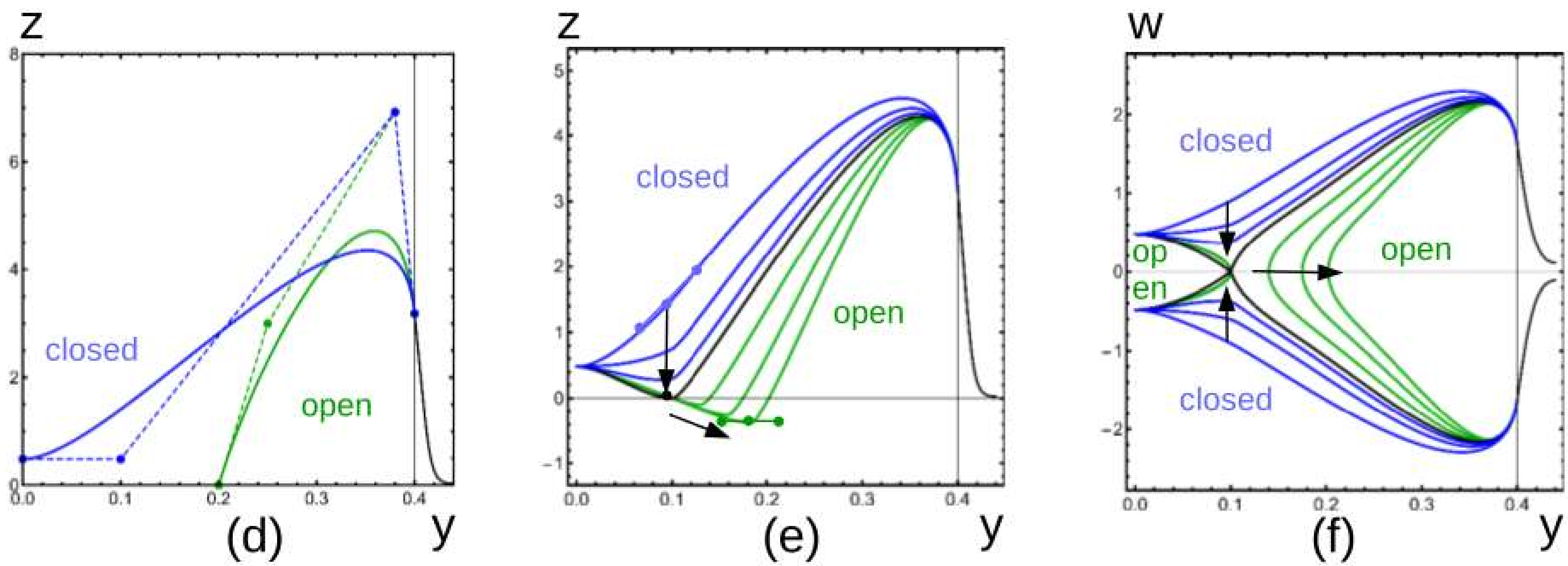
a closeup, tangents for C^1-stitching



a(y)-profile, monotone in y, constant in time

- since p_t contains 2nd derivatives of A, C^1-stitching produces a jump in p_t
- can be interpreted as sharp startup of the transverse interaction between radially converging flows of dark matter
- experiments with C^2-stitching give continuous but rapid increase of p_t, physically the same

Wormhole opening: the details



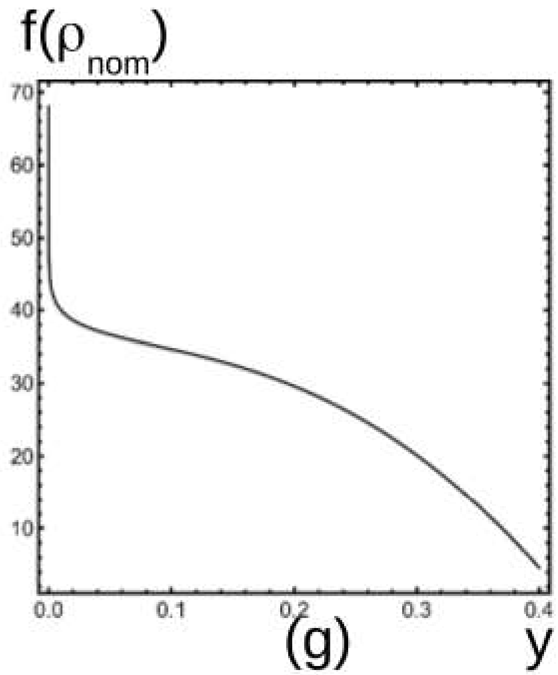
z(y)-profile, starting and ending positions

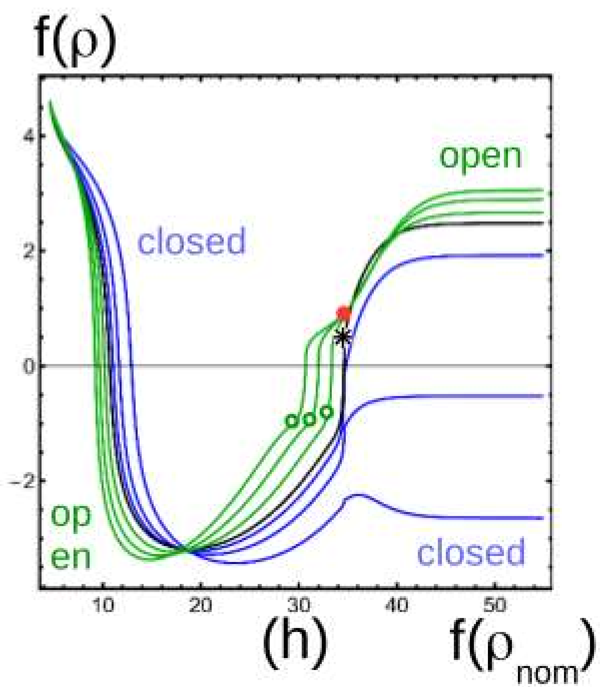
interpolation between them

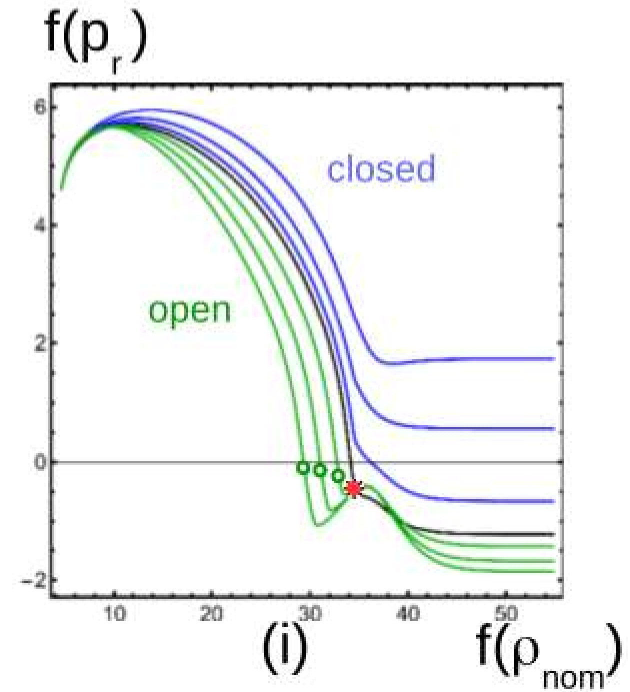
the same for w(y)-profile

- regular core, finite density in the center: $Z \sim 1 - 8/3\pi\rho(0)\, r^2$, $z(0) = f\,(1/2) \approx 0.481212$
- stationary r_{max} scenario: the left root of z(y) has fixed position
- only the parts $z \geq 0$ should be used
- in w(y) coordinates: the bubble profile after formation inflates a bit, then remains almost stationary

Wormhole opening: the details







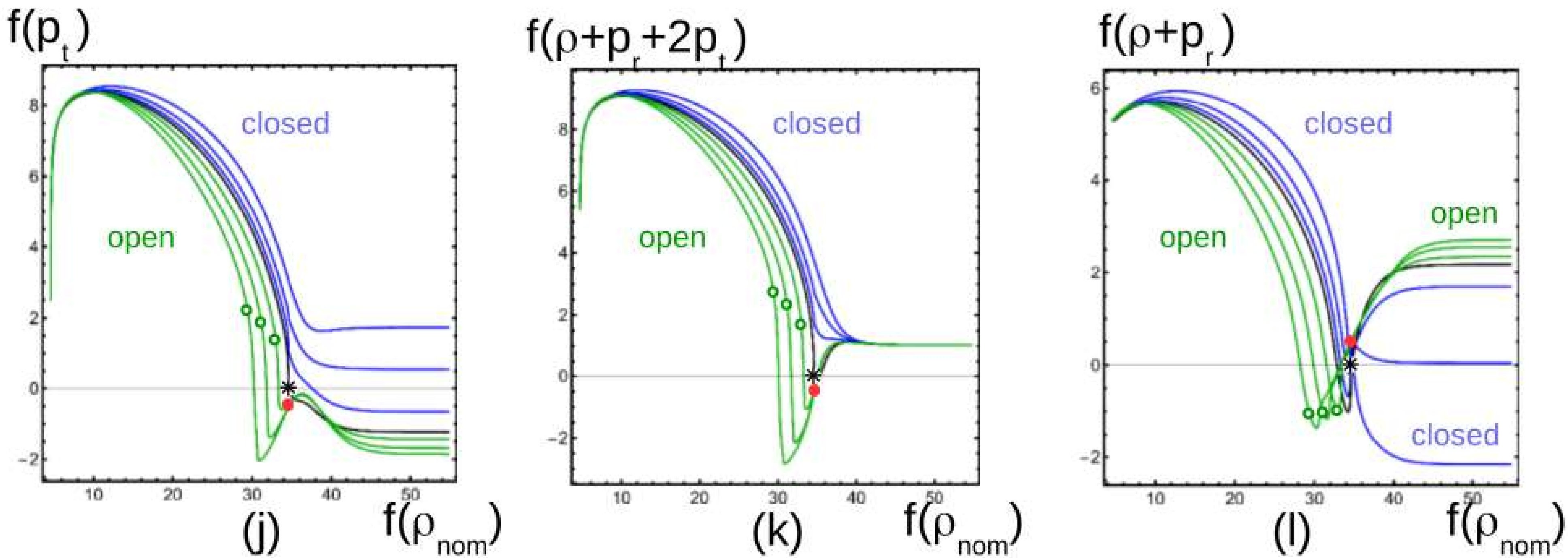
$y \rightarrow \rho_{nom}$ recalibration

EOS: $\rho(\rho_{nom})$

EOS: $p_r(\rho_{nom})$

- green circles – wormhole throat, red circle – buble r_{max}, black star – bif.point
- at first, density becomes negative, then radial pressure
- a window of opportunities opens for fulfilling the flare-out conditions $p_r < 0$, $\rho + p_r < 0$, necessary and sufficient to open the wormhole
- the wormhole is opened at the position r with $p_r = -1/(8\pi r^2)$

Wormhole opening: the details



EOS: $p_t(\rho_{nom})$

EOS: auxiliary components

- $p_t > 0$, $\rho + p_r + 2p_t > 0$ and all other flare-out conditions are satisfied at the wormhole throat, bubble and bifurcation point

The same in physical dimensions

QG wormhole in the center of Milky Way galaxy

Model parameters: $\boxed{\epsilon = 4 \cdot 10^{-7},\ r_{s,nom} = 1.32 \cdot 10^{10}\mathrm{m},}$ $r_2 = r_s = 1.1990455291886923 \cdot 10^{10}\mathrm{m}$	COSMOVIA 05-Jul-2019 18-Oct-2019
QG cutoff: $N = 10$, $\rho_P/N = 3.82807 \cdot 10^{68}\mathrm{m}^{-2}$, $\boxed{r_{QG} = r_s - 1.0003513617763519 \cdot 10^{6}\mathrm{m},}$ $y_{QG} = 23.207293345446693$, $a_{QG} = -222.2887073354903$, $z_{QG} = 192.8253039716802$, $\{y_i, a_i\} = \{\{23.207293345446693, -222.2887073354903\},$ $\{23.207283345446694, -247.28371742005763\}, \{1, -250\}, \{0, -250\}\}$	r_s-1 ths.km
Closed state: $\{y_i, z_i\} = \{\{23.207293345446693, 192.8253039716802\},$ $\{23.207193345446694, 442.77560481735327\}, \{15, 300\}, \{10, 200\}, \{5, 100\},$ $\{3, 0.4812118250596034 7\}, \{0, 0.48121182505960347\}\},$ $\boxed{\text{redshift factor in the center } a(0) = -250}$	similar model with static wormhole: Kardashev, Novikov, Shatskiy 2007
Open state: $\{y_i, z_i\} = \{\{23.207293345446693, 192.8253039716802\},$ $\{23.207193345446694, 442.77560481735327\}, \{21.237780648029343, -0.2\},$ $\{16.237780648029343, -0.2\}, \{11.237780648029343, -0.2\},$ $\{3, 0.48121182505960347\}, \{0, 0.48121182505960347\}\},$ redshift factor in the throat: $a(r_0) = -241.7284225293921$, radius of the throat: $r_0 = 1.3543177581795398 \cdot 10^{7}\mathrm{m}$, radius of the bubble: $r^* = 22026.465749406787\mathrm{m}$	14 ths.km 22km

arXiv:1909.08984

The bifurcation point

- Louko, Sorkin 1997
- the coordinates in the vicinity of Morse point of saddle type can be selected so that the Morse function will take a canonical form $t = x^2 - y^2$
- the metric will be

$$ds^2 = (x^2 + y^2 - \zeta x^2)dx^2 + (x^2 + y^2 + \zeta y^2)dy^2 + 2\zeta xydxdy$$

- using the coordinate transformation, the metric can be further reduced to

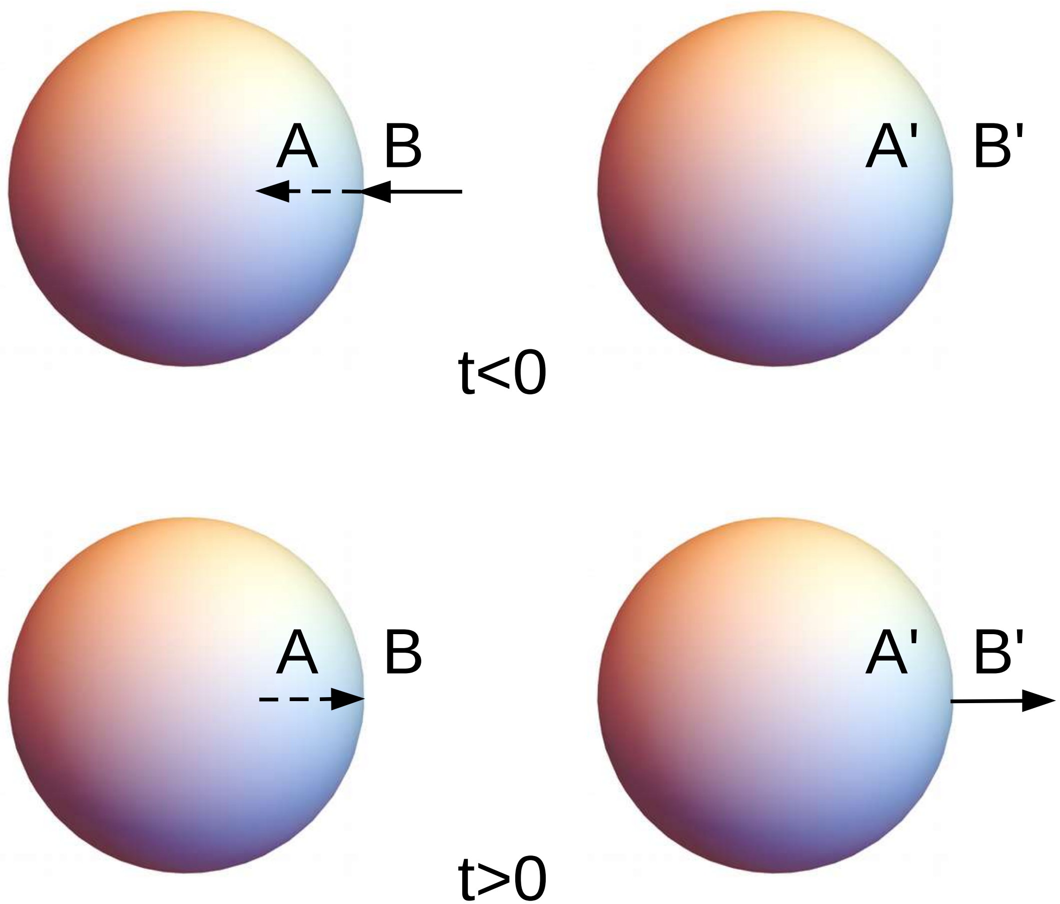
$$u = (x^2 - y^2)\sqrt{\zeta - 1}/2, \ v = xy, \boxed{ds^2 = -du^2 + dv^2}$$

- coincident with the *flat Minkowski plane*, covered twice by the transformation (x, y) → (u, v)
- This type of singularity will be considered in details below. The result is that the matter term vanishes everywhere, except of the origin, where a mild singularity is located, equivalent to zero in distributional sense.

(paper in preparation)

Topological teleporter

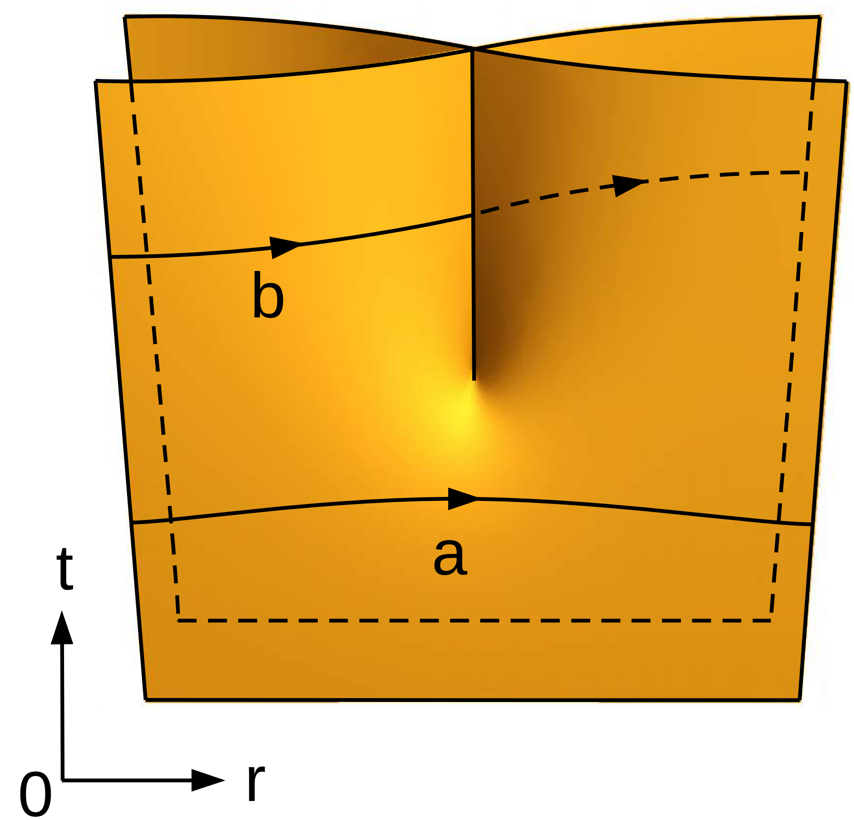
- two copies of R^3 space
- two spheres S^2 cut out
- connections:
  t<0 AB, A'B' (trivial)
  t>0 AB', A'B (crosslike)
- topologically dual to the opening wormhole, which is
  t>0 BB' (wormhole), AA' (bubble)
- instant swapping of two spherical volumes in space
- an event of teleportation
- At t<0, the observer crosses the sphere in one universe via the BA connection, then, at t>0, crosses the sphere in the AB' connection, in another universe, or in a remote part of the same universe.



arXiv:1909.08984

Topological teleporter

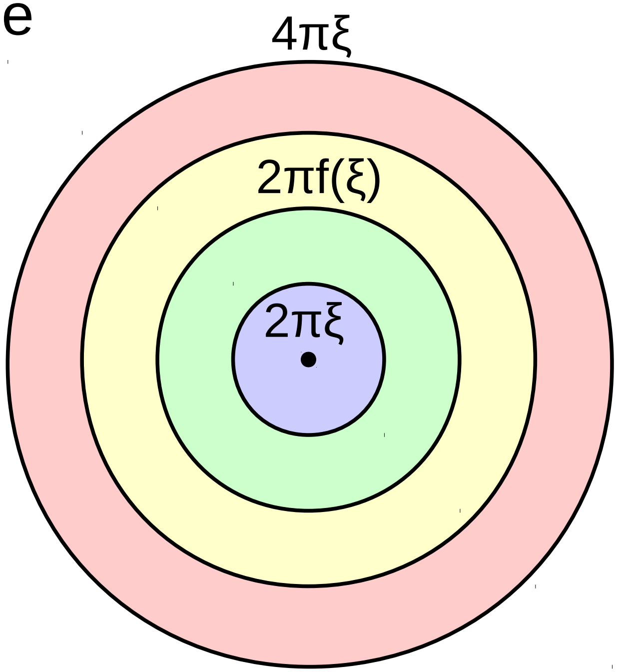
- embedding diagram the same as for the stargate
- in different coordinate system: sph-time (r,t) instead of cyl $(z,r_\perp)$
- these two cases are related by Wick rotation (will be shown below)
- (a) way corresponds to $t < 0$ and remains in one universe
- (b) way corresponds to $t > 0$ and goes to another universe
- the third coord is only for visualization; the metric is everywhere flat, except of the origin $t=0,r=a$, where it is singular
- the matter is concentrated in (immediate vicinity of) the origin: on stargate perimeter, on teleportation sphere



arXiv:1909.08984

Evaluation of matter term: stargate

- The problem: standard algebraic evaluation gives matter term =0 where the metric is flat and fails in the origin
- The method of solution: physical-based regularization
- double cover polar coordinate system (ξ, α)
- difference from flat Euclidean:
  the doubled circumference =4πξ
- let's interpolate the circumference:
  2πf(ξ), f(ξ)~2ξ at ξ>ε, f(ξ)~ξ at ξ~0
  => Euclidean flat near and including the origin
  => no concentrated matter term in the origin



- regularization displaces the matter concentrated at the origin to the ε-neighborhood, where its distribution can be calculated by the standard algebraic method
- localization of matter is controlled by ε-parameter, which can go to zero at the end of the calculation

(paper in preparation)

Evaluation of matter term: stargate

$$n=3,\ x=(t,\xi,\alpha),\ g=\mathrm{diag}(-1,1,f(\xi)^2),$$

$$G^t_t=-8\pi\rho=f''(\xi)/f(\xi),$$

a single non-zero component
of Einstein tensor

$$\int \rho f(\xi) d\xi d\alpha=(-1/4)\int_0^\epsilon f''(\xi)d\xi=(-1/4)(f'(\epsilon)-f'(0))=-1/4$$

$(-\det g)^{1/2} = f(\xi)$

$f'(\varepsilon) = 2$, $f'(0) = 1$

- f''/f plays the role of a regularized delta function (for the measure $f d\xi$)
- the result does not depend on the particular choice of the regularization

$$n=4,\ x=(t,z,\xi,\alpha),\ g=\mathrm{diag}(-1,1,1,f(\xi)^2)$$

$$G^t_t=G^z_z=f''/f$$

a straight string along z-axis,
negative linear mass and tension $\mu = \tau = -1/4$

(paper in preparation)

Evaluation of matter term: stargate

$$n=4,\ x=(t,\xi,\alpha,\phi),\ g=\mathrm{diag}(-1,1,f(\xi)^2,r_\perp^2),$$ a ring geometry

$$r_\perp=a+(2\xi-f(\xi))\cos\alpha+(f(\xi)-\xi)\cos 2\alpha,$$ interpolation of radial coord, flat space at ξ≥ε and at ξ~0

$$G_t^t=f''/f+\delta G,\ G_\phi^\phi=f''/f,$$

in addition, Gij in the 2x2 block (i, j) = (ξ, α) are nonzero.

Lemma 1: (i) all components of Gμν vanish at ξ≥ε and at ξ~0, where the metric is locally flat; (ii) the component δG after integration with the measure $(-\det g)^{1/2}\, d\xi$ gives an expression tending to zero with ε→0; (iii) the aforementioned (i, j)-components after going to the tensor densities $G^{\hat{i}\hat{j}}(-\det g)^{1/2}$ in an orthonormal basis are finite both before and after removing ε-regularization, the integrals of them with respect to the coordinate volume tend to zero at ε→0.

(paper in preparation)

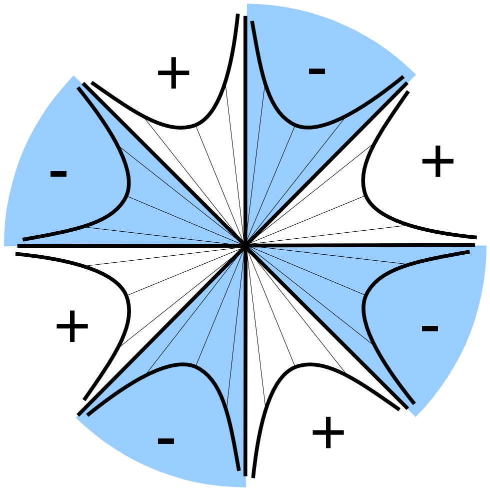
Evaluation of matter term: stargate

- Gij represent the internal stresses between the layers of the tube $0 < \xi < \varepsilon$, they depend on the chosen regularization
- physically important and independent of regularization are $G_t^{\ t}$ and $G_\phi^{\ \phi}$ density and tension components
- $\mu = \tau = -1/4$, as for the straight string; the same result obtained by other methods in Visser 1996, Louko, Sorkin 1997, Vickers, Wilson 1997, Balasin, Nachbagauer 1993

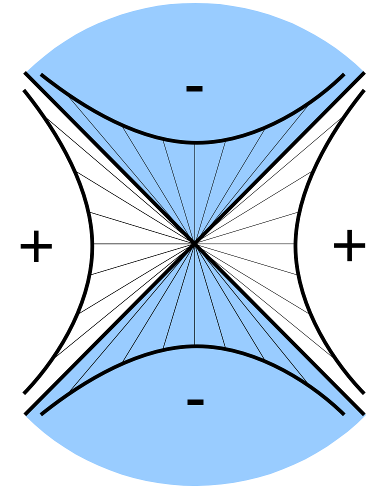
(paper in preparation)

Evaluation of matter term: teleporter

$$n = 3,\ x = (\xi, \alpha, z),\ g = \mathrm{diag}(\eta, -\eta\xi^2, 1),$$
$$\eta = +1,\ r = \xi \cosh\alpha,\ t = \xi \sinh\alpha,$$
$$\eta = -1,\ r = \xi \sinh\alpha,\ t = \xi \cosh\alpha$$

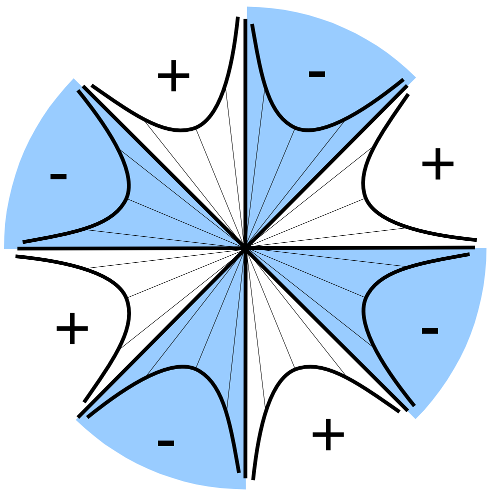


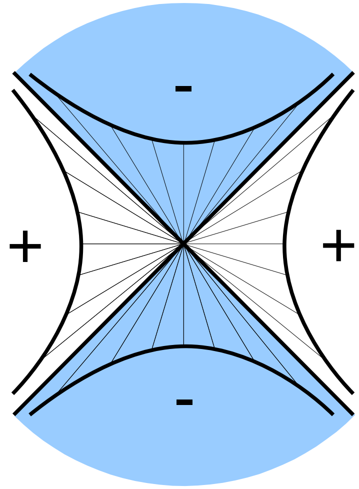
- double cover of flat Minkowski plane (r,t)
- eight hyperbolic maps
- rearrangement procedure:
  in standard calculation EFE are solved in inverse order: g→G; includes differentiations and algebraic operations
  => is local (compare with direct solution of EFE: G→g, which is non-local) => spacetime manifold can be subdivided into many submaps, the problem is solved in each of them individually
  => submaps can be rearranged in a different order to get a different problem, with an equivalent solution



(paper in preparation)

Evaluation of matter term: teleporter





- in our case, submaps are sectors with small dα
- at fixed η equivalent under Lorentz transformation
- submaps can be rearranged from eight to four hyperbolic maps
- the same as for flat Minkowski plane, with the defect for angular variable α

(paper in preparation)

Evaluation of matter term: teleporter

$$n = 3,\ x = (\xi, \alpha, z),\ g = \mathrm{diag}(\eta, -\eta f(\xi)^2, 1)$$

f(ξ)~2ξ at ξ>ε, f(ξ)~ξ at ξ~0 as earlier

$$G^z_z = f''/(\eta f),\ \eta = \pm 1$$

Note: g and G related to stargate case by Wick rotations, t ➔ iz, α ➔ iα, for η = −1 also ξ ➔ iξ

$$n = 4,\ x = (\xi, \alpha, y, z),\ g = \mathrm{diag}(\eta, -\eta f(\xi)^2, 1, 1),$$
$$G^y_y = G^z_z = f''/(\eta f),\ \eta = \pm 1$$

going to the next dimension

(paper in preparation)

Evaluation of matter term: teleporter

$$n = 4,\ x = (\xi, \alpha, \theta, \phi),\ g = \mathrm{diag}(\eta, -\eta f(\xi)^2, r^2, r^2 \sin^2\theta),$$
$$\eta = +1,\ r = a + (2\xi - f(\xi)) \cosh\alpha + (f(\xi) - \xi) \cosh 2\alpha,$$
$$\eta = -1,\ r = a + (2\xi - f(\xi)) \sinh\alpha + (f(\xi) - \xi) \sinh 2\alpha,$$

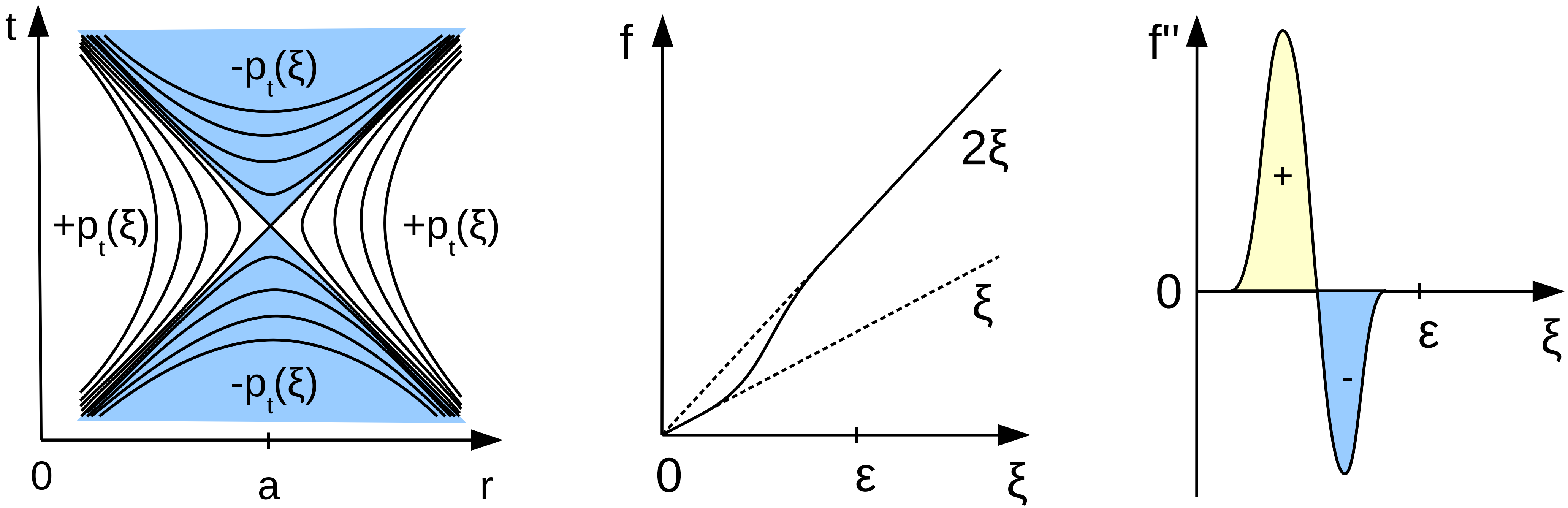
the last two dimensions rolled into the sphere

$$G^{\theta}_{\theta} = G^{\phi}_{\phi} = f''/(\eta f) + \delta G$$

Lemma 2: (i)-(iii) as above are satisfied
=> only transverse pressure components are physically important

(paper in preparation)

Evaluation of matter term: teleporter



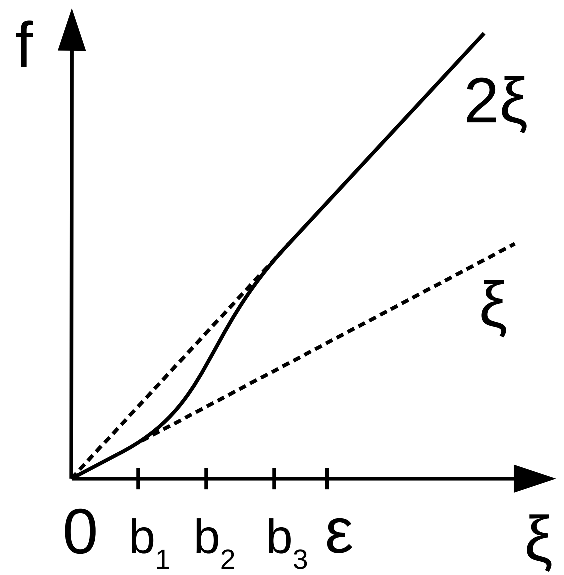
- only transverse pressure p_t is active
- concentrated in the vicinity of r=a t=0 sphere
- has alternating sign in 8 sectors (4 in each universe)

- the regularization function and its 2nd deriv.
- $p_t\,(-\det g)^{1/2} \sim f''(\xi)$
- the coefficient of proportionality: $1/(8\pi)$ in geometric units, 0.03 Jupiter's mass per meter in natural units

(paper in preparation)

Evaluation of matter term: teleporter

Definition:
(precise specification of regularization function)



let $f(\xi)$ be C^∞ smooth, equal to ξ at $\xi\leq b_1$, equal to 2ξ at $\xi\geq b_3$, with $f'(\xi)$ monotonously increasing from 1 to B_2 at $b_1\leq\xi\leq b_2$, $f'(\xi)$ monotonously decreasing from B_2 to 2 at $b_2\leq\xi\leq b_3$, with $0<b_1<b_2<b_3<\varepsilon$ and $B_2>2$. Let $f(\xi)$ be simply rescaled with ε, $f(\xi)\to\varepsilon f(\xi/\varepsilon)$, not changing the derivative $f'(\xi)$.

Lemma 3: $f''(\xi)\to\delta(\xi)$ and $f''(\xi)/\eta\to0$ at $\varepsilon\to0$
in the distributional sense

Proof: Consider a test function $g(\xi)$ of C^∞ class with finite support on R, write an estimation $|g(\xi)-g(0)|\leq C|\xi|$. Evaluate $\int_0^\varepsilon d\xi\, f''(\xi)g(0)=g(0)$ and $I=\int_0^\varepsilon d\xi\, f''(\xi)(g(\xi)-g(0))$, $|I|\leq C\int_0^\varepsilon d\xi\, |f''(\xi)|\xi \leq C\varepsilon\int_0^\varepsilon d\xi\, |f''(\xi)|=C\varepsilon(2B_2-3)\to0$ at $\varepsilon\to0$.
Consider a test function $g(x)$ of C^∞ class with finite support on R^2, write an estimation $|g(x_1)-g(x_2)|\leq C|x_1-x_2|$. Evaluate $I=\int_0^\varepsilon d\xi\, f''(\xi)/\eta \int_0^{+\infty} d\alpha\, g(x)$ over two adjacent maps. Using the symmetries, obtain $I=\int_0^\varepsilon d\xi\, f''(\xi) \int_0^{+\infty} d\alpha\, (g(x_1)-g(x_2))$, where $x_1=(\xi\cosh\alpha, \xi\sinh\alpha)$, $x_2=(\xi\sinh\alpha, \xi\cosh\alpha)$, $|x_1-x_2|=(\sqrt{2})\xi e^{-\alpha}$. Evaluate $|I|\leq C\int_0^\varepsilon d\xi\, |f''(\xi)| \int_0^{+\infty} d\alpha\, (\sqrt{2})\xi e^{-\alpha} \leq C(\sqrt{2})\varepsilon\int_0^\varepsilon d\xi\, |f''(\xi)| = C(\sqrt{2})\varepsilon\,(2B_2-3)\to0$ at $\varepsilon\to0$.

(paper in preparation)

Matter term for teleporter, physical interpretation

- the result is equivalent to zero as a generalized function
- not the same as the usual function tending to zero
- e.g., this function cannot be squared (infinite result at $\varepsilon \to 0$)
- comparison with toroidal compactification T^n, a flat space in a box, whose opposite sides are identified: the matter term vanishes identically as a function
- better to describe the regularized solution, a physical approximation to an idealized result at $\varepsilon \to 0$

- comparison with other works on topology change and degenerate metrics
- singularities of mild type obtained in Yodzis 1972, Horowitz 1991, Ellis et al. 1992, Bengtsson 1993, Ionicioiu 1997
- special opinion Louko, Sorkin 1997: complex regularization, adding $i\gamma$ to metric in Morse point, resulting to complex densitized scalar curvature $4\pi i \delta_2(x,y)$ at $\gamma \to 0$, having a profound meaning in quantum theory, for evaluation of path integrals
- we consider the classical theory and prefer to stay with real-valued expressions for the metric and the curvature tensor

(paper in preparation)

Matter term for teleporter, physical interpretation

Q: What are the stargate perimeter and teleportation sphere made of?

- Stargate: exotic string of negative mass and tension
- Teleporter: the matter has only transverse pressure, no radial pressure and no mass

- (i) A gas consisting of two components: (ρ, p_r, p_t) and $(-\rho, -p_r, 0)$, summing up to $(0,0, p_t)$.
- (ii) Tachyons. Consider a sphere existing for one instant of time. On the sphere, draw a system of great circles, each is a closed geodesic worldline of the tachyon. Thus, a two-dimensional tachyon gas is placed on the sphere, creating the necessary transverse pressure. The sign of this pressure is regulated by a common mass factor for tachyons.
- (iii) A string coiled into a ring and existing for one instant, having zero mass and non-zero (positive or negative) tension. A sphere with transverse pressure can be assembled from such strings, and from such spheres -- an alternating sign distribution necessary for the operation of teleporter.

(paper in preparation)

Conclusion 1/3

- a comparison of solutions of variable topology in string theory and general relativity is performed

- in both theories a change of topology is possible in the presence of singular points, in the class of almost everywhere Lorentzian manifolds

- string break: either occurs at a singular point, or singular points arise after the break

- general relativity: two examples

  (1) dynamic opening of a wormhole according to a scheme of new type
  (2) an instant swapping of two spherical volumes in space, an event of teleportation

  (1) and (2) are topologically dual, related by a reconnection of maps
  (2) is related to the stationary solution of stargate (dihedral wormhole) type by Wick rotation

Conclusion 2/3

- for both solutions, the corresponding matter distributions are calculated

  (1) the matter terms are finite, except of the immediate vicinity of the bifurcation point, where a mild singularity of Morse saddle type is located
  (2) the matter terms are concentrated near the teleportation sphere, similar to stargate, in which the matter terms are concentrated on the perimeter

- for both solutions, the bifurcation point of wormhole opening and the branching point of teleporter represent a sign alternating singularity, equivalent to zero in distributional sense

- the matter composition in all considered solutions is exotic, violating the energy conditions
- there is a principal possibility of creating such solutions via quantum effects

Conclusion 3/3

- similarity to Planck stars model, in which quantum gravity corrections led to effectively negative mass density, repulsive force and quantum bounce phenomenon
- a scenario is computed in which similar repulsive terms do not lead to a quantum bounce, but to the dynamic opening of a wormhole
- such scenario can be scaled to real astrophysical sizes corresponding to the central black hole in the Milky Way, describing the principal possibility of opening a wormhole as a result of natural astrophysical phenomena

- for solutions of stargate and teleporter types, several matter composition options were considered, in the form of an exotic string coiled into a ring, a string with zero-mass and non-zero tension, a two-dimensional tachyon gas, and a two-component gas of a normal and exotic type
- in particular, the combination of a normal matter with exotic matter from the core of Planck stars gives a principal possibility for engineering such solutions.

Thank you!