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# Time Machines

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# Content

Time machines: theoretical constructions, special solutions of General Relativity

Causality postulates and possible resolution of time paradoxes
- Hawking's chronology protection postulate
- Novikov's self-consistent chronology
- Wheeler-Feynman thought experiment

(one timeline)

- solution for mathematicians: non-Hausdorff topology (many timelines)

Models of time machines in GR
- van Stockum machine
- Godel's machine
- cosmological strings and Gott's machine
- spinning black holes and Kerr's machine

fast rotation

- wormholes leading to the past
- cyclic time universe
- general solution for mathematicians

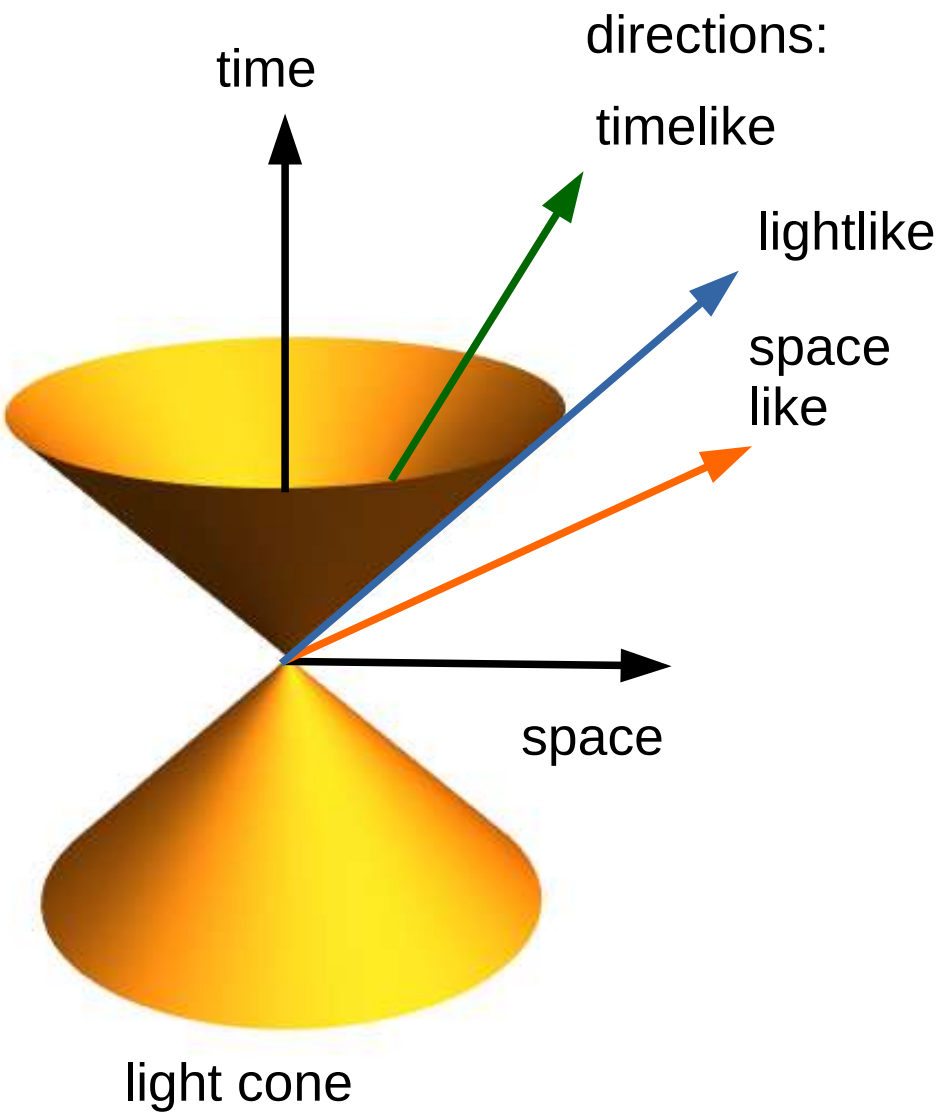
topology

Practical considerations
- required energy
- exotic matter
- quantum effects

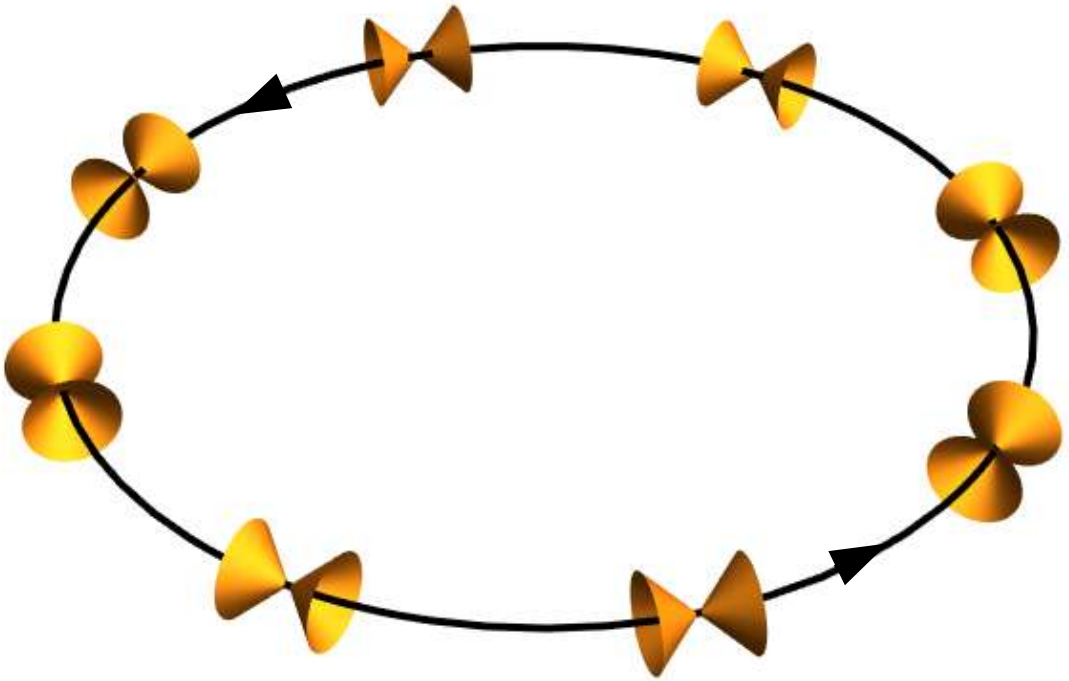
Overview of the book: M.Visser, Lorentzian Wormholes: from Einstein to Hawking, Springer 1996

Recent development: TARDIS, B.K.Tippett, D.Tsang 2017 Class. Quantum Grav. 34 095006, arXiv: 1310.7985, 1310.7983

# Time machines




**Definition:** spacetime represents ***time machine***, if it contains a closed timelike curve (the observer can return to the own past)



# Causality postulates

**Time paradoxes -** logical contradictions arising in the presence of time machine

**Progenitor paradox** (or grandfather paradox):
time traveler returns to the past and (in one way or another) hinders the *own birth,* as a result, time traveler disappears, cannot hinder the own birth, time traveler appears, hinders the own birth, etc.

**Time loop paradox** (or bootstrap paradox):
send a letter to yourself in the past with a drawing of a time machine, construct and use it to send the letter to yourself - who invented the time machine?

# Causality postulates

**Hawking's postulate** of the chronology protection:
time machines do not exist (therefore, no time paradoxes can arise)

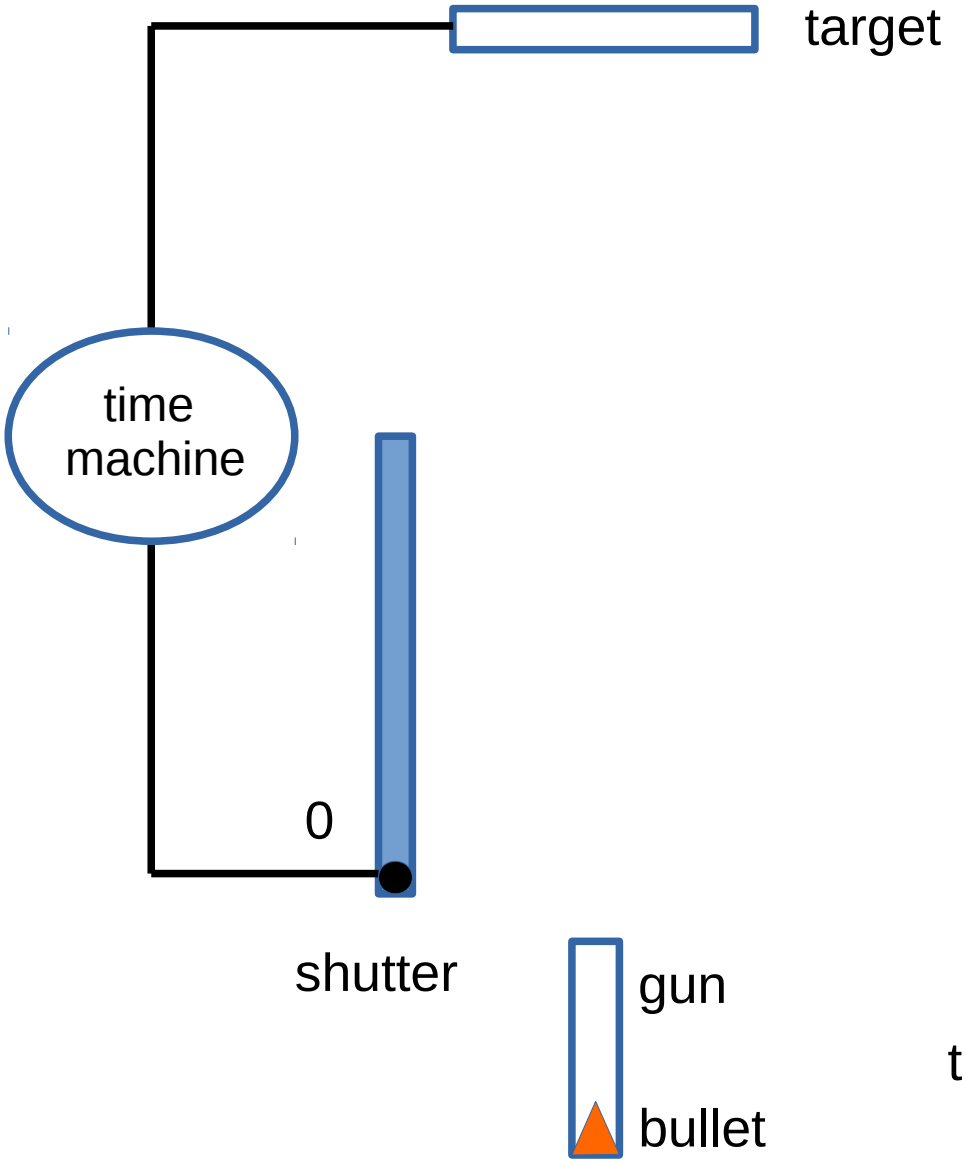
**Novikov's postulate** of self-consistent chronology:
time machines and timelike loops can exist if the entire chain of events along them is logically self-consistent

**Example:** grandfather paradox - an attempt returning to the past and (in one way or another) preventing the own birth (for one reason or another) *will fail*, so a logical contradiction will not arise

bootstrap paradox - no one invented the time machine, it just exists within the time loop it created (there is no logical contradiction)

# Causality postulates

Mechanistic example:
thought experiment
Wheeler-Feynman 1949



# Causality postulates

Mechanistic example:
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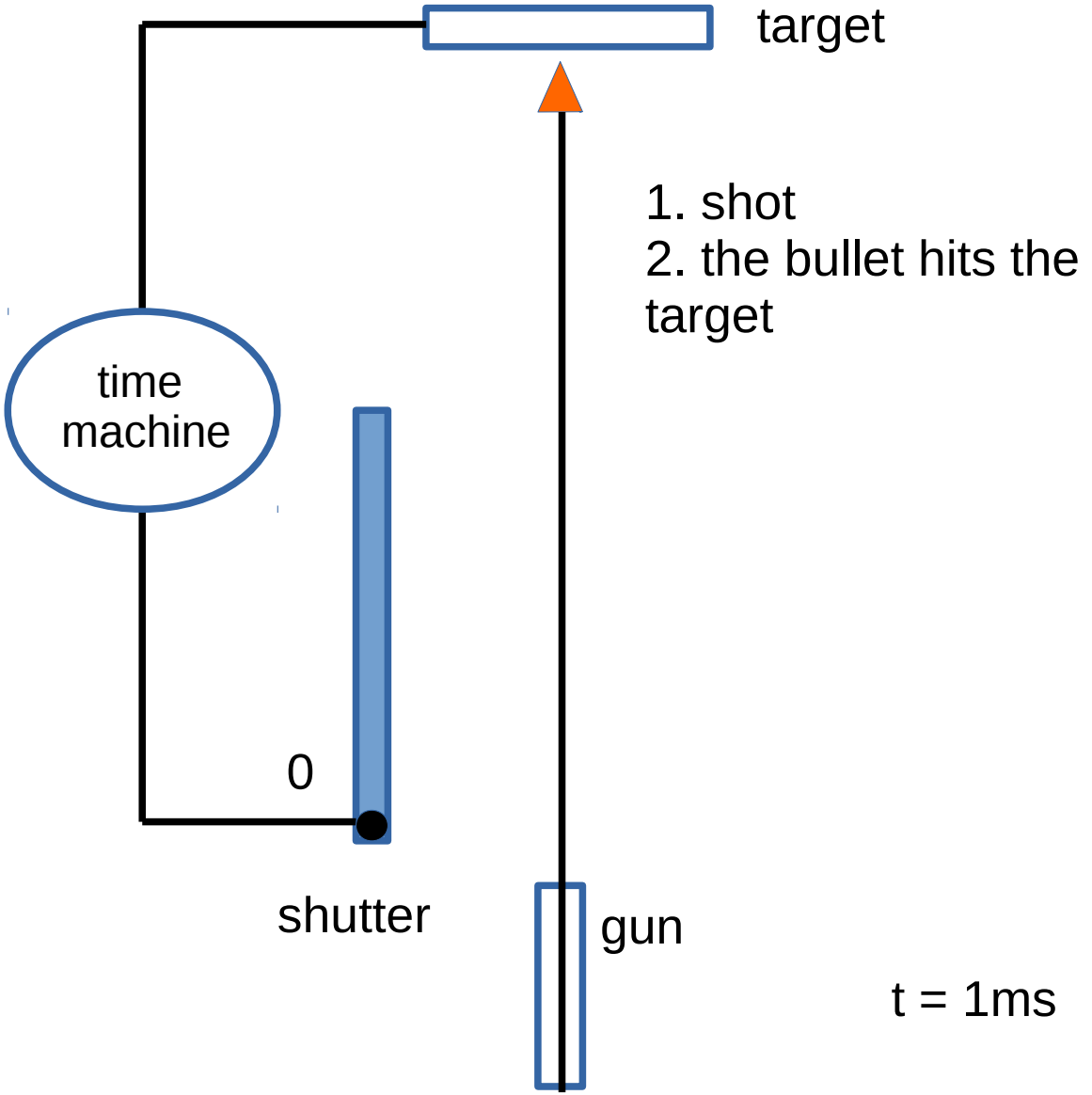
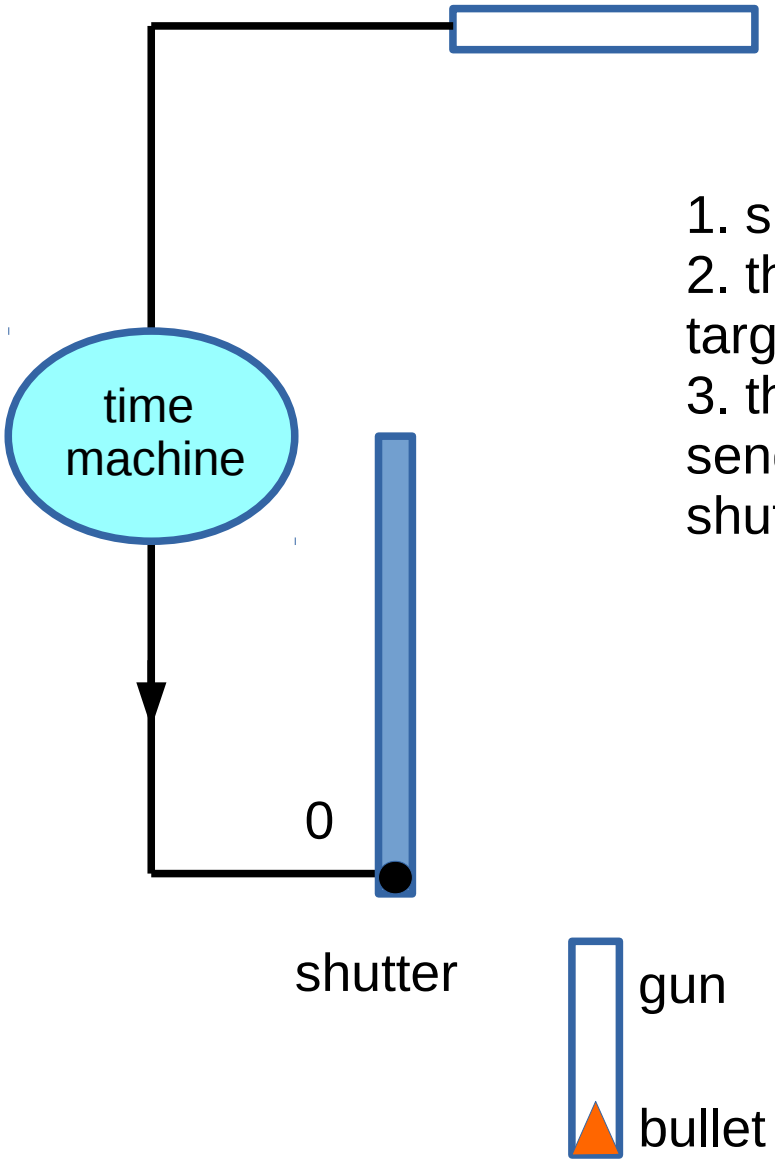


# Causality postulates

Mechanistic example:
thought experiment
Wheeler-Feynman 1949

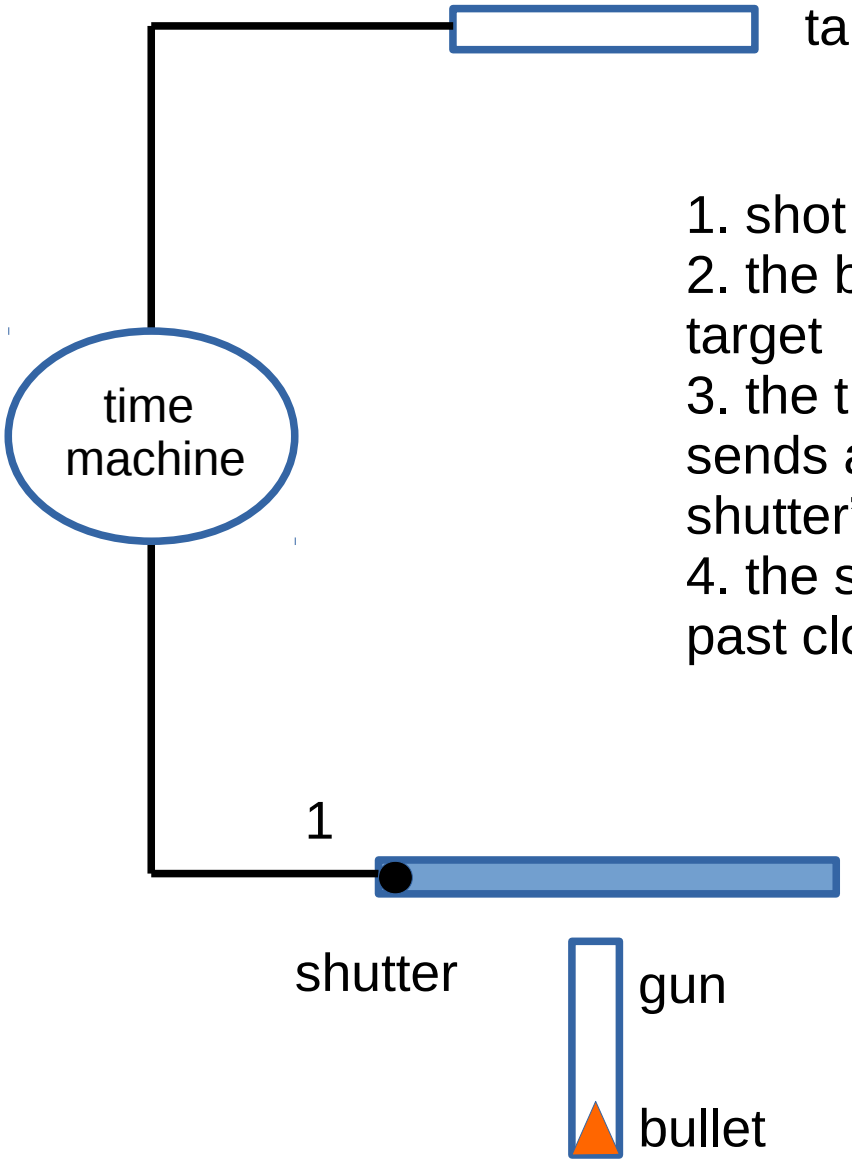



1. shot
2. the bullet hits the target
3. the time machine sends a signal “close shutter” to the past

t = 0ms

# Causality postulates

Mechanistic example:
thought experiment
Wheeler-Feynman 1949

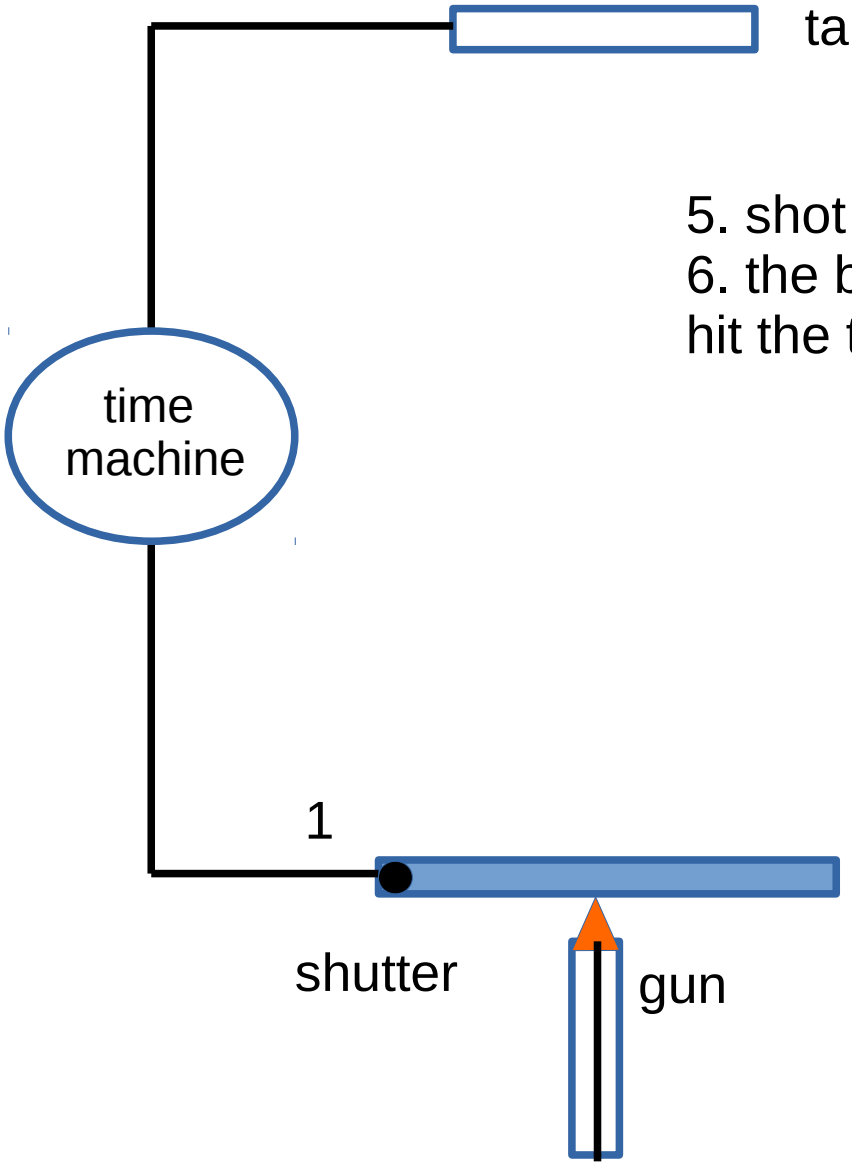



1. shot
2. the bullet hits the target
3. the time machine sends a signal “close shutter” to the past
4. the shutter in the past closes

t = 0.5ms

# Causality postulates

Mechanistic example:
thought experiment
Wheeler-Feynman 1949

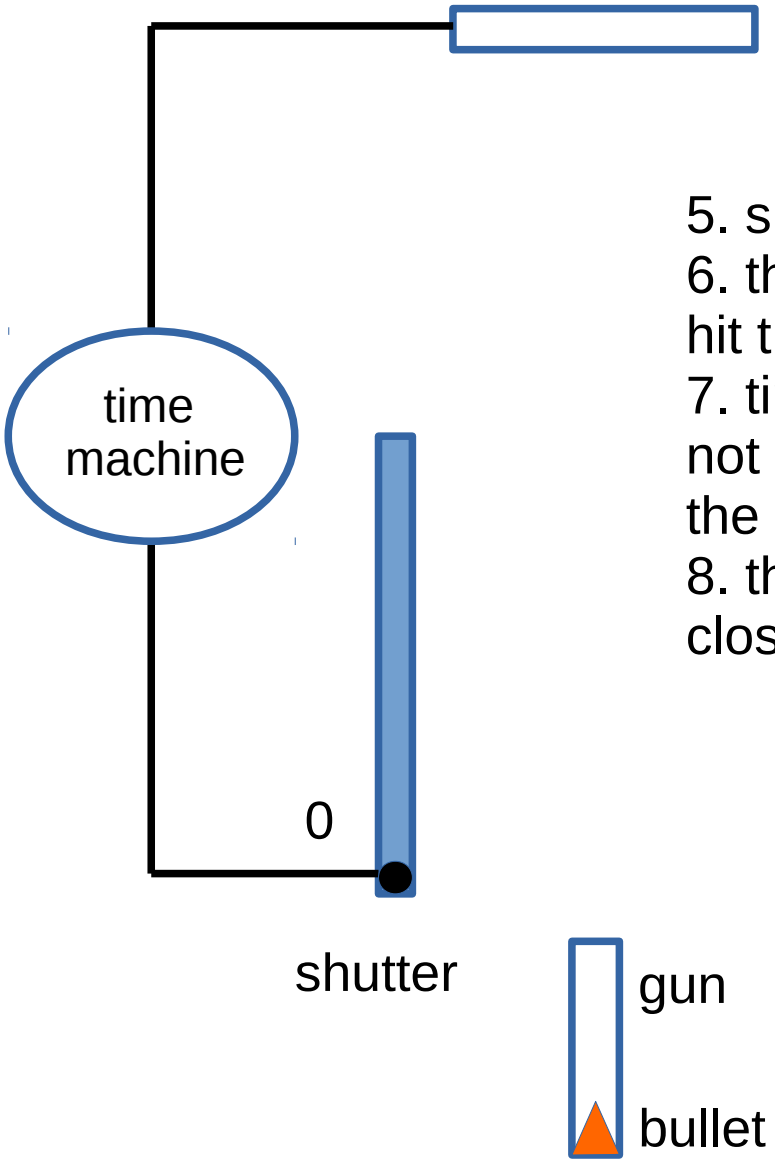



5. shot
6. the bullet does not hit the target

t = 1ms

# Causality postulates

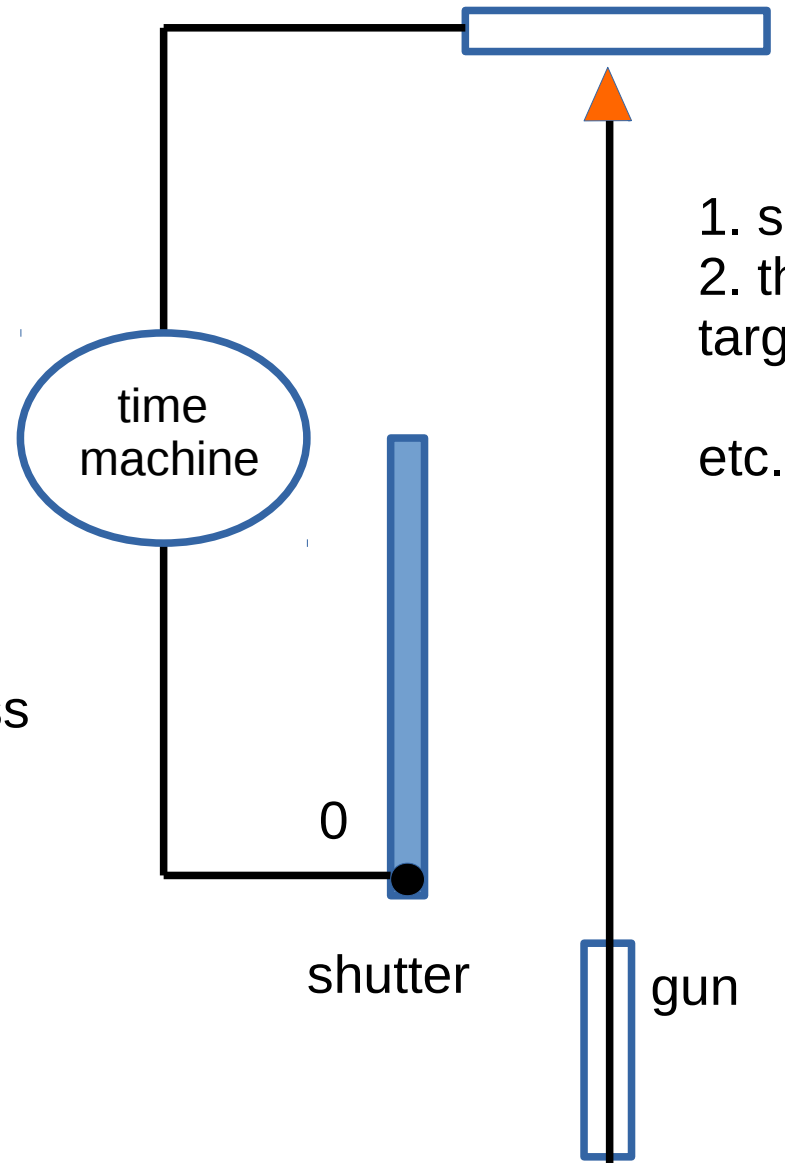
Mechanistic example:
thought experiment
Wheeler-Feynman 1949




5. shot
6. the bullet does not hit the target
7. time machine does not send a signal to the past
8. the shutter does not close in the past

# Causality postulates

Mechanistic example:
thought experiment
Wheeler-Feynman 1949




1. shot
2. the bullet hits the target

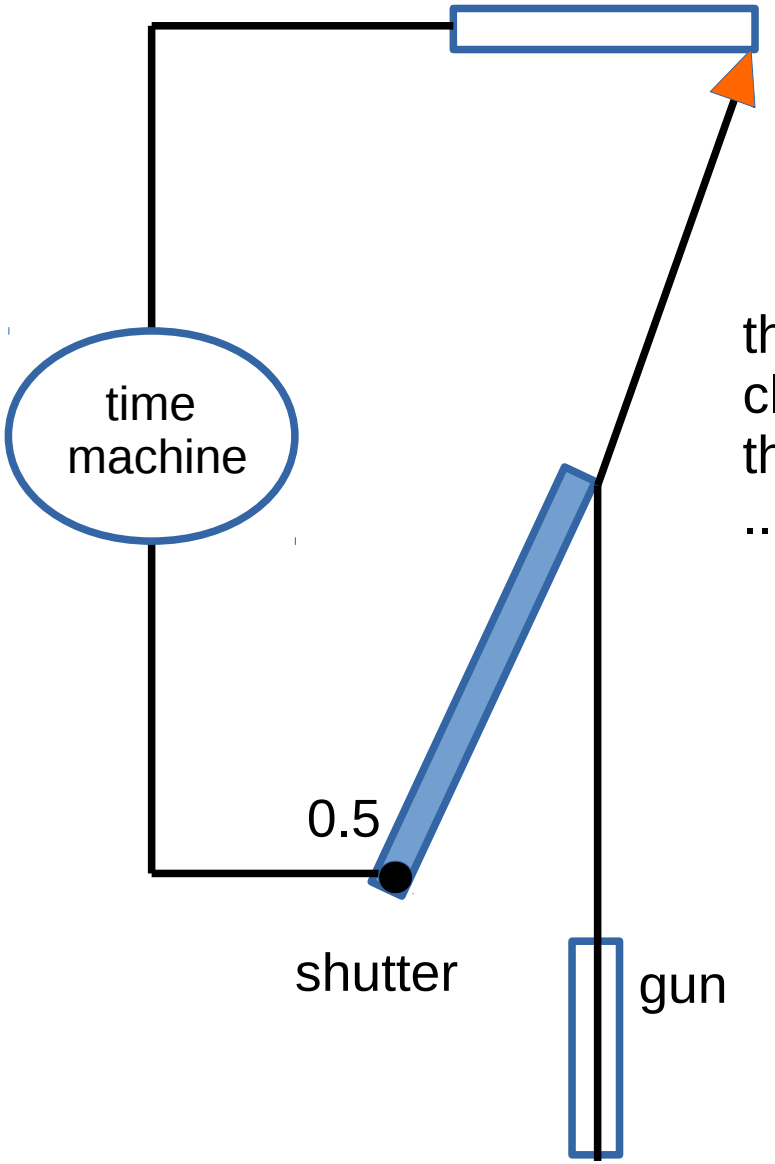
etc.

In the considered discrete class 0/1 there is no logically closed solution
(Novikov's postulate violated)

t = 1ms

# Causality postulates

Mechanistic example:
thought experiment
Wheeler-Feynman 1949



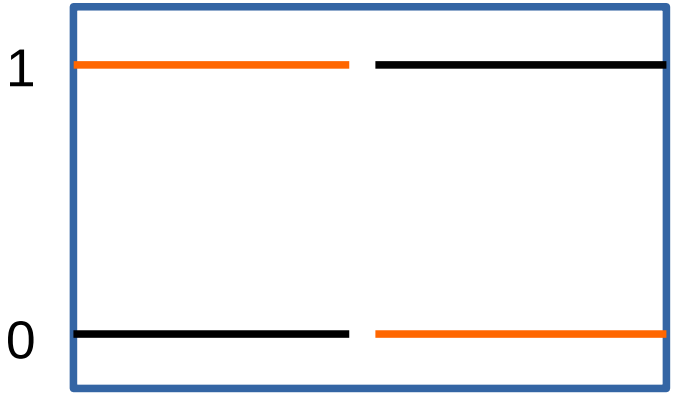

the shutter is not fully closed, the bullet hits the edge of the target ...

Self-consistent solution in a continuous class
(Novikov's postulate fulfilled)

t = 1ms

# Causality postulates

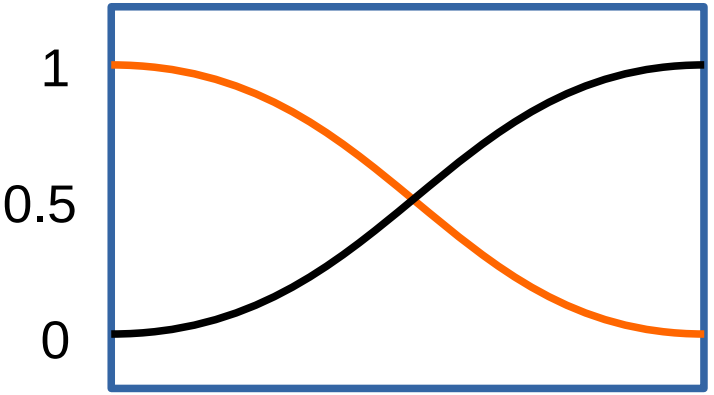
the question is in the existence of a solution
of the appropriate *mathematical model*




In discrete class 0/1 there is no logically closed solution
(Novikov's postulate violated)

A = .not.A  (liar paradox)

this model has no solution
and has no physical implementation




Self-consistent solution in the continuous class
(Novikov's postulate fulfilled)

A = 1-A

this model has a solution,
implementation possible

# Causality postulates

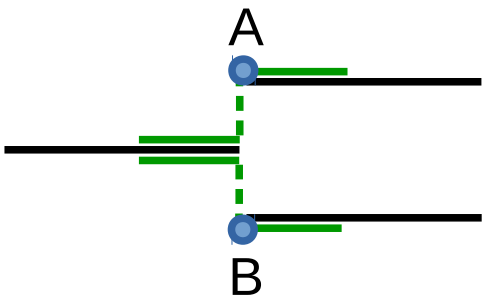
Paradoxes arise under the assumption that spacetime contains only *one timeline*

There are no paradoxes, if there are *multiple timelines*

The mathematical basis for such models is given by the following construction

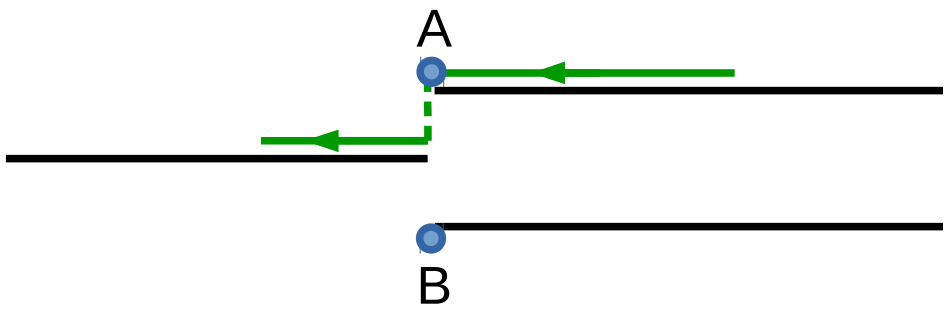
**Definition:** the topology is Hausdorff, if for any two different points there are disjoint neighborhoods surrounding them


**Example** of non-Hausdorff topology: neighborhoods of points A, B intersect (on the left side)



# Causality postulates

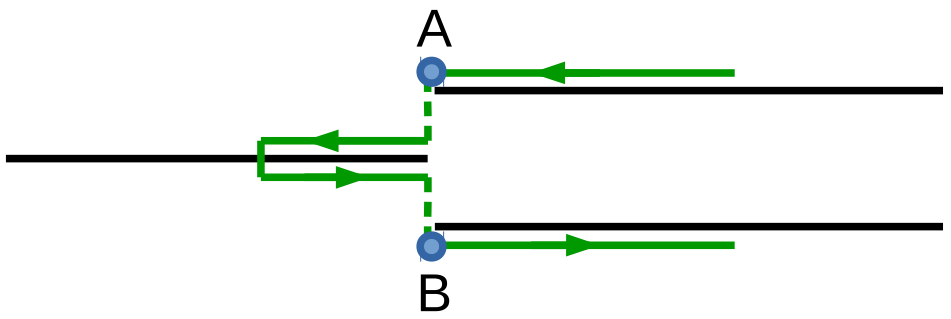
Resolving grandfather paradox using non-Hausdorff topology




1. a traveler moves into the past to the point before his birth A

# Causality postulates

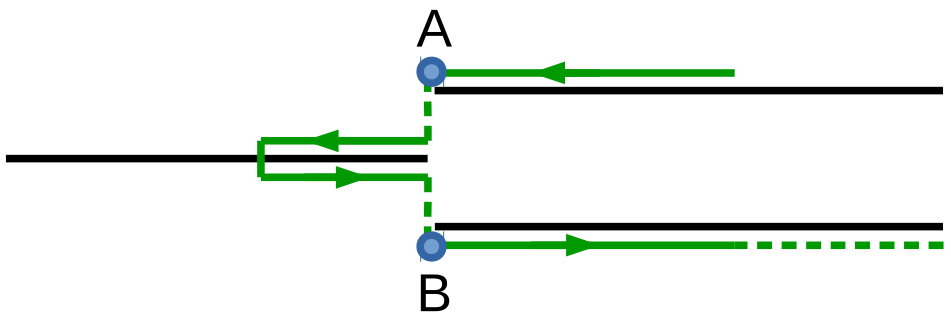
Resolving grandfather paradox using non-Hausdorff topology




1. a traveler moves into the past to the point before his birth A
2. prevents the birth at point B

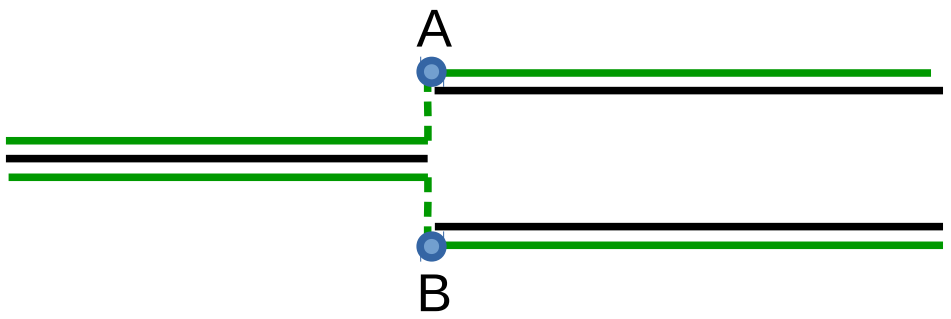
# Causality postulates

Resolving grandfather paradox using non-Hausdorff topology




3. continues to live in the alternative timeline that he created, where he was not born, but arrived there in a time machine

# Causality postulates




**Comment:** the general relativity operates on *maps* that cover spacetime, regarded as a manifold of a general form. These maps are equivalent to the neighborhoods of points considered here. Therefore, the general relativity is trivially transferred to the non-Hausdorff topology. In the example discussed here, spacetime is covered by two maps containing two alternative timelines.

# Models of time machines in astrophysics

The general theory of relativity considers spacetime as the manifold equipped with a *metric tensor*: 4x4 point-dependent symmetric matrix of a signature (+++-), three spatial coordinates, one time.

The structure of such manifolds is very rich, in particular, among them there are those containing closed timelike curves, time machines.

In general relativity, there is a large number of models of time machines, which can roughly be divided into two classes, where closed timelike curves are created by

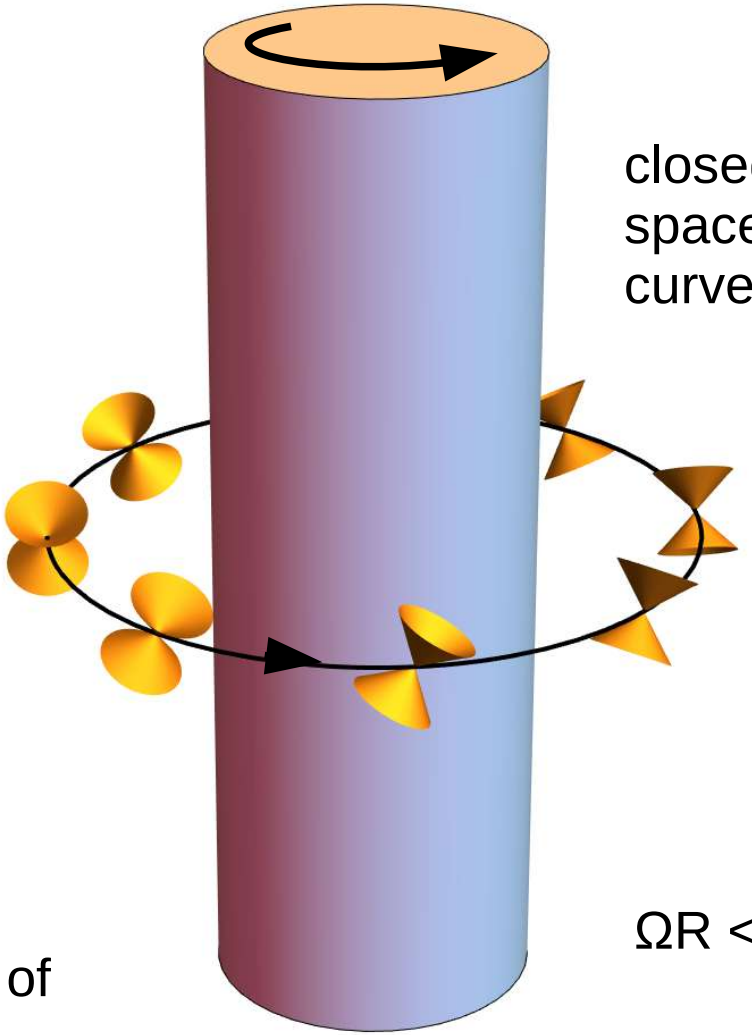
(1) fast rotation of massive objects or
(2) the nontrivial spacetime topology.

# Willem van Stockum Machine (1937)

a rotating cylinder filled by a cosmic dust (matter with positive density and zero pressure)

the gravitational forces acting on the dust are balanced by the centrifugal force

at high speed of rotation, the light cones begin to tilt (dragged by rotation)



closed spacelike curve

Ω is the angular velocity, R is the radius of the cylinder; hereinafter, geometric system of units is used: G = c = 1 (schematic image)

ΩR <1/2

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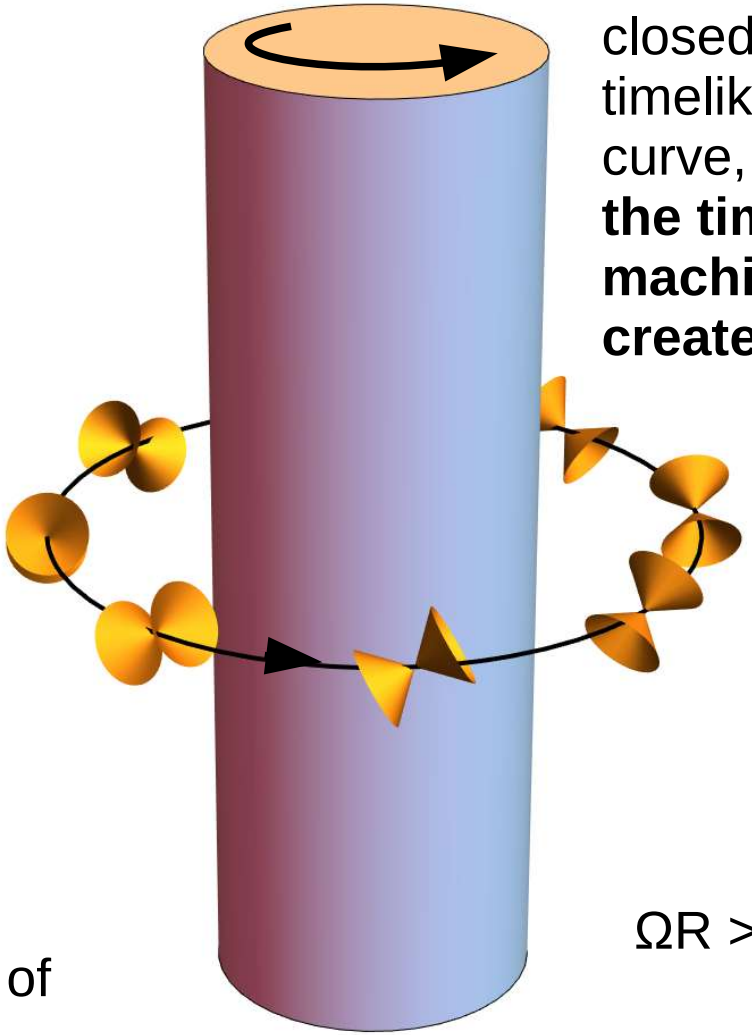


closed timelike curve, **the time machine created**

Ω is the angular velocity, R is the radius of the cylinder; hereinafter, geometric system of units is used: G = c = 1 (schematic image)

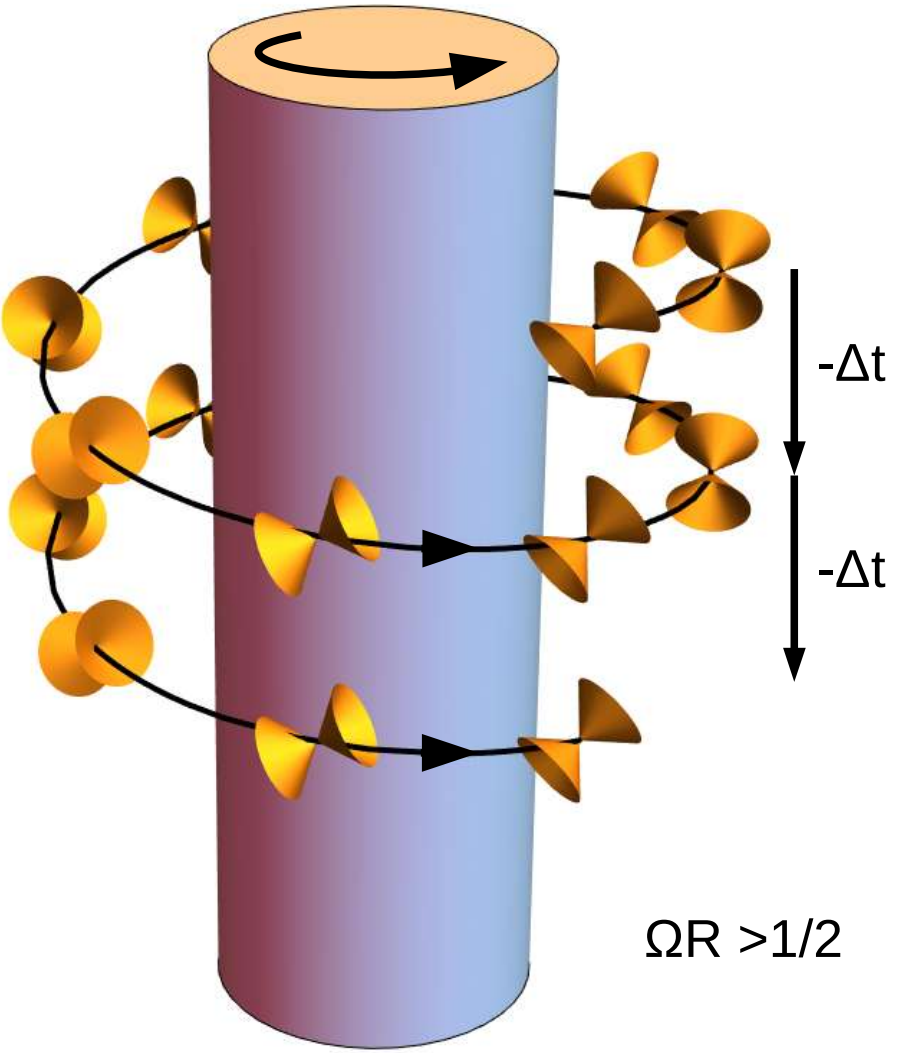
ΩR >1/2

# Willem van Stockum Machine (1937)

the practical use of time machine, not for walking in a loop, but for going backward in time along a spiral

although locally the world line of the observer is located inside the future light cone, as a whole, the trajectory goes to the past in global time

the observer can go back in time arbitrarily far, but not earlier than the date of constructing the time machine (all the models considered here possess this property)



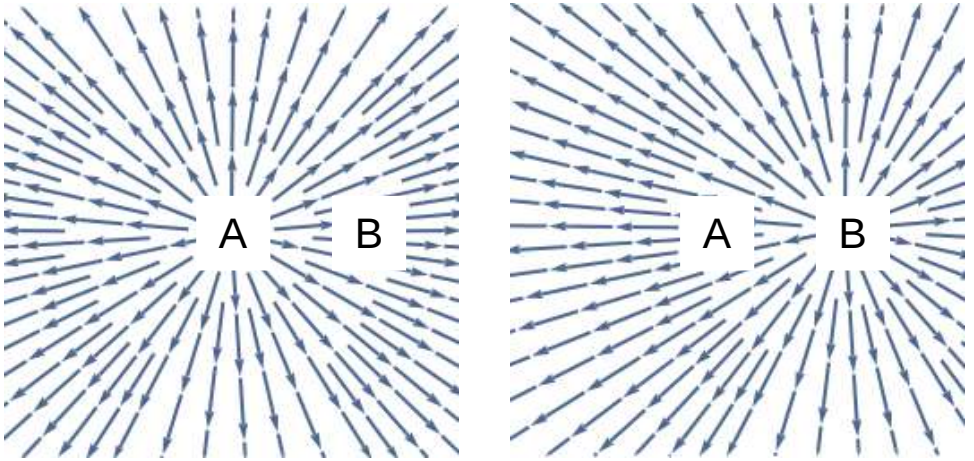
# Kurt Gödel's Machine / Universe (1949)

Gödel’s universe - a cosmological solution that describes an infinite rotating universe

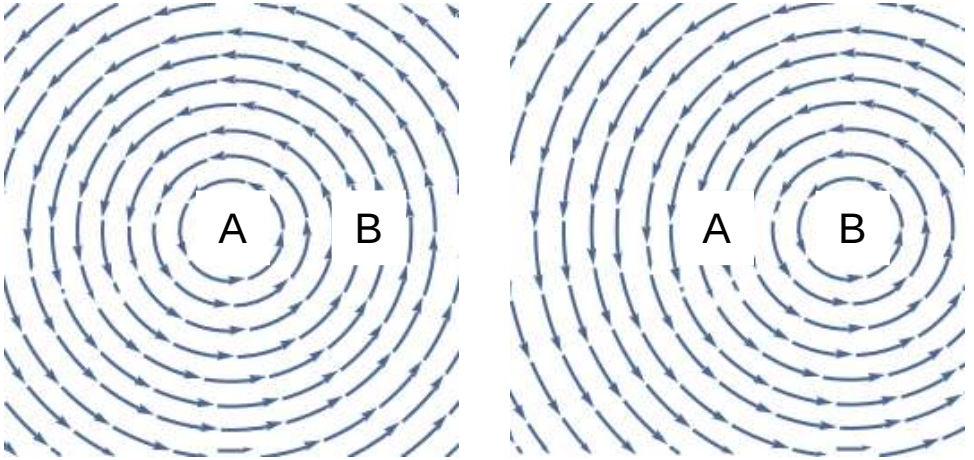
similar to the infinite expanding universe by Friedman

homogeneity principle: expansion or rotation with respect to any point looks equivalently (there is no selected center)

the rotation of the universe also produces the tilt of light cones and the formation of closed timelike curves




Friedman's universe




Gödel's universe

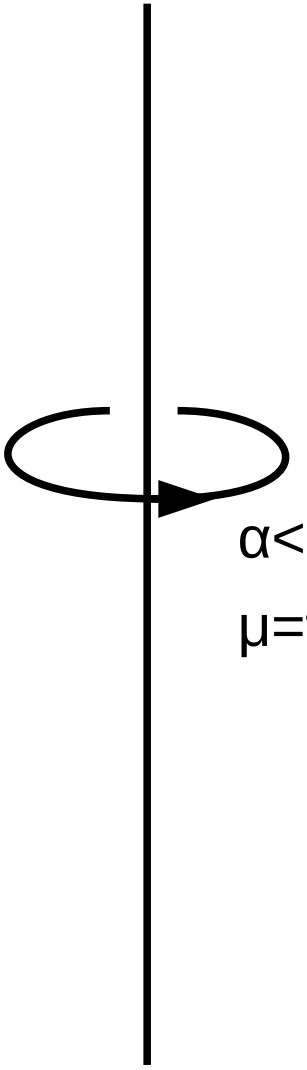
# Cosmological strings

massive extended infinitely thin objects

bend space in a specific way (angle deficit): when going around a string, the total angle α differs from 2π

$\mu = \tau = (2\pi - \alpha) / (8\pi)$,
where μ - linear mass density, τ - tension

deficit can be negative (excess of angle, negative density, exotic strings)



# Cosmological strings

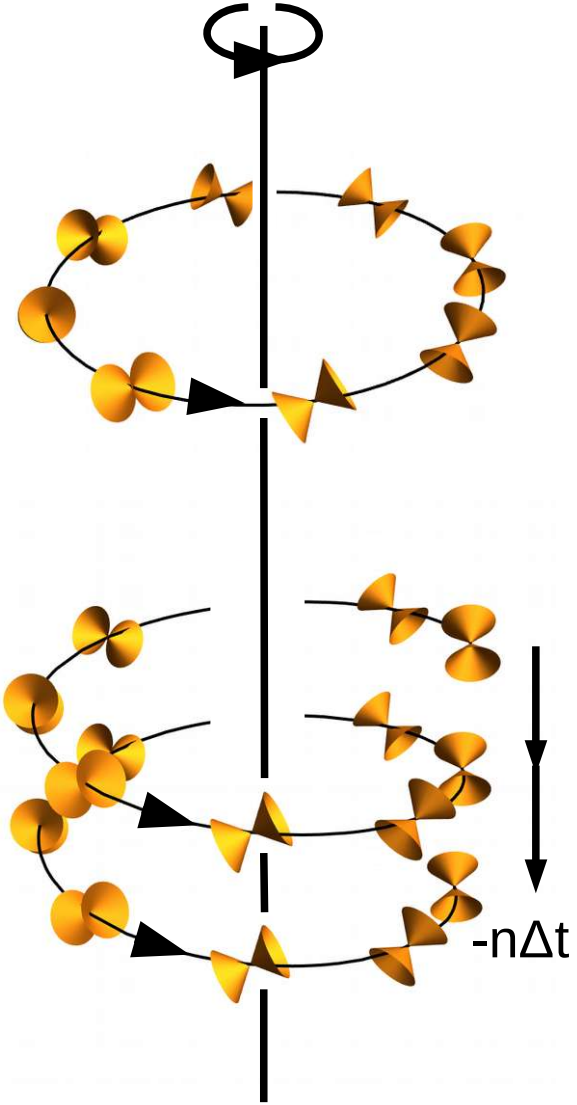
if the string rotates around its axis, this rotation tilts the light cones, as for van Stockum machine

the formation of a time machine becomes possible, for an orbit of radius r <4J / (1-4μ), where J is the angular momentum per unit length

Gott's machine 1991

arranged in a similar way, uses two parallel strings that move relative to each other and have an orbital momentum J per unit length

a time machine is formed if the strings are located at a distance r from each other, described by the same formula



# Kerr's solution 1963

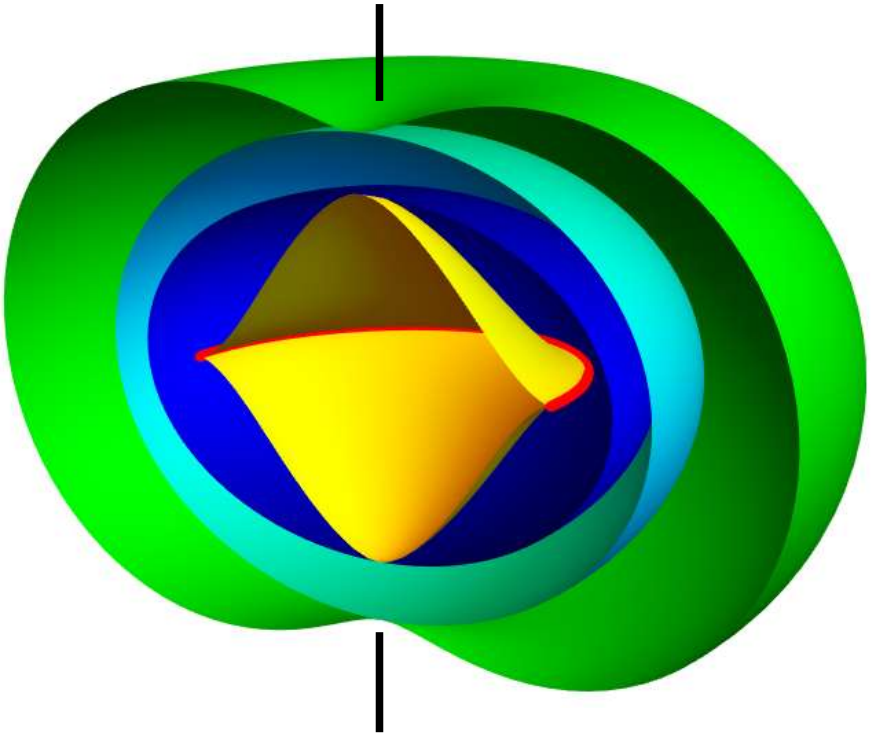
spinning black hole

maximally extended solution
(Penrose diagram) is very complex,
only one patch is shown here

case $J<M^2$, J is the moment of rotation,
M - mass

between two ergospheres the observer cannot be at rest, dragged by rotation of the hole

from under the outer horizon the observer cannot go outside ...



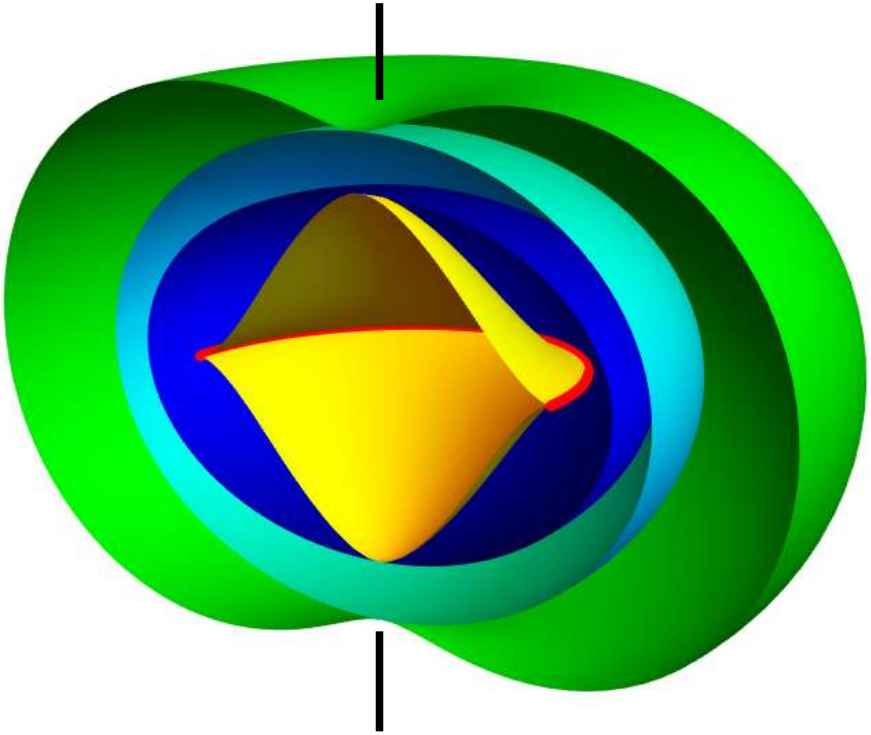
outside to inside: external ergosphere, outer horizon, inner horizon, inner ergosphere, ring singularity, axis of rotation;
$J = 0.99M^2$

# Kerr's solution 1963

on the inner horizon light rays entering from outside accumulate that cannot penetrate inside (however, if these rays change direction, they penetrate inside and go to the next patch of Penrose diagram...)

**Important:** for the observers under the inner horizon there are closed timelike curves

**Conclusion:** a time machine is formed, but hidden from the external observers by the event horizon

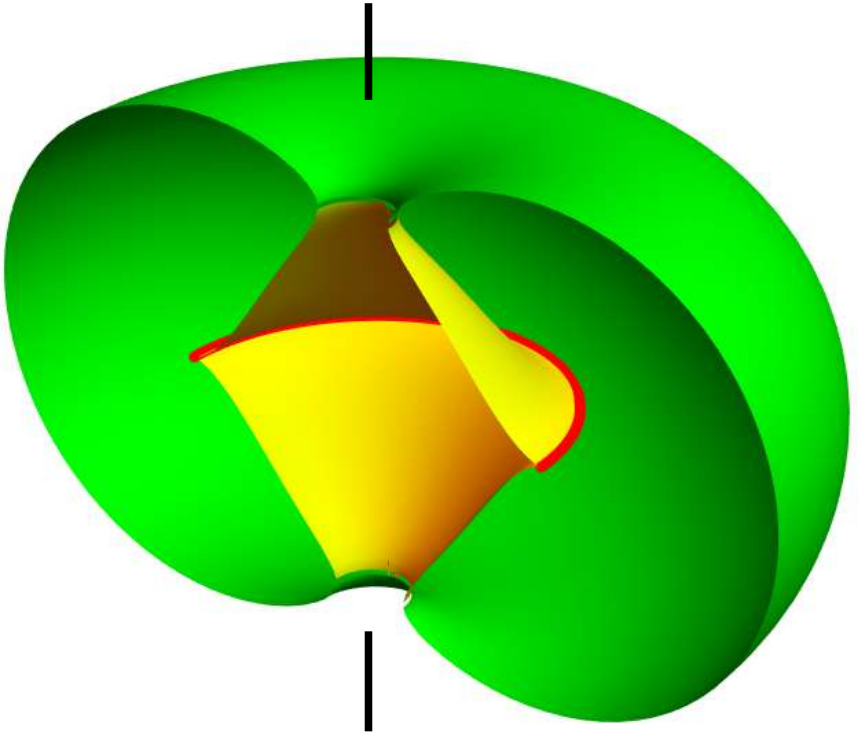


outside to inside: external ergosphere, outer horizon, inner horizon, inner ergosphere, ring singularity, axis of rotation; $J = 0.99M^2$

# Kerr's solution 1963

case $J>M^2$

the horizons disappear, two ergospheres and ring singularity remain



outside to inside: external ergosphere, inner ergosphere, ring singularity, axis of rotation; $J = 1.01M^2$

# Kerr's solution 1963

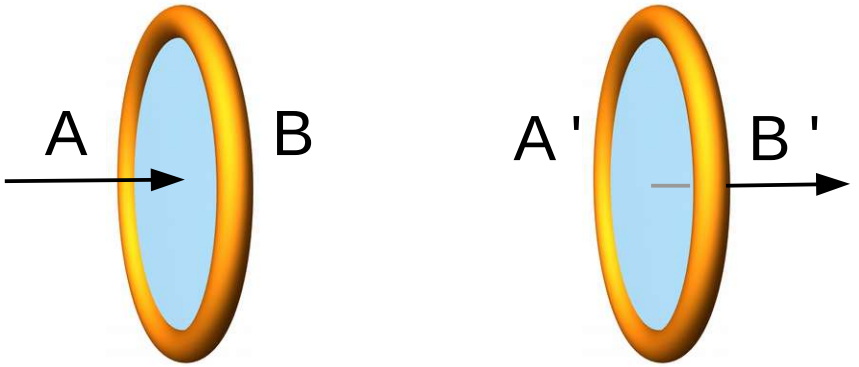
case $J>M^2$

naked ring singularity
(not covered by event horizon)

maximally extended solution:
2 universes connected by
a stargate wormhole

closed timelike curves, time machines
exist everywhere

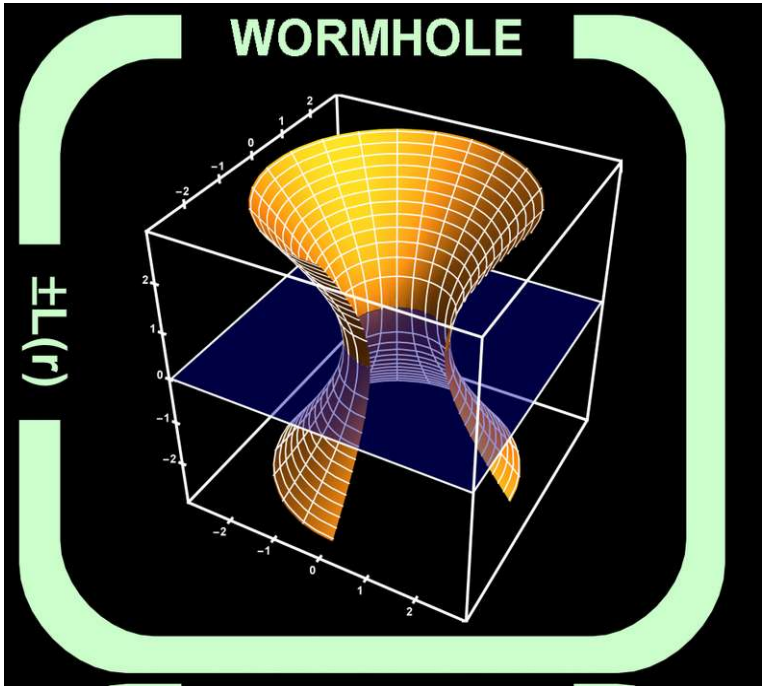
**Comment:** solutions with naked singularities are usually considered unphysical (the principle of cosmic censorship, which, however, has not been proved ...)




stargate: disk sides identified
crosslike, AB', BA'

# Wormhole of Morris-Thorne 1988

- stationary spherical symmetric solutions
- no event horizon
- tunnel connecting 2 universes or 2 sites of one
- exotic matter required ($\rho$+p<0)
- B→$\infty$, A>0 finite, L finite




$$L(r) = \int dr \sqrt{B(r)}$$

metric, the square of the distance between points in curved spacetime

$$ds^2 = -Adt^2 + Bdr^2 + Dr^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

particular example:

$$A = 1 - r_0/r + \alpha/r^2,\ B = (1 - r_0/r)^{-1},\ D = 1$$

# Wormhole “Stargate” Visser 1996

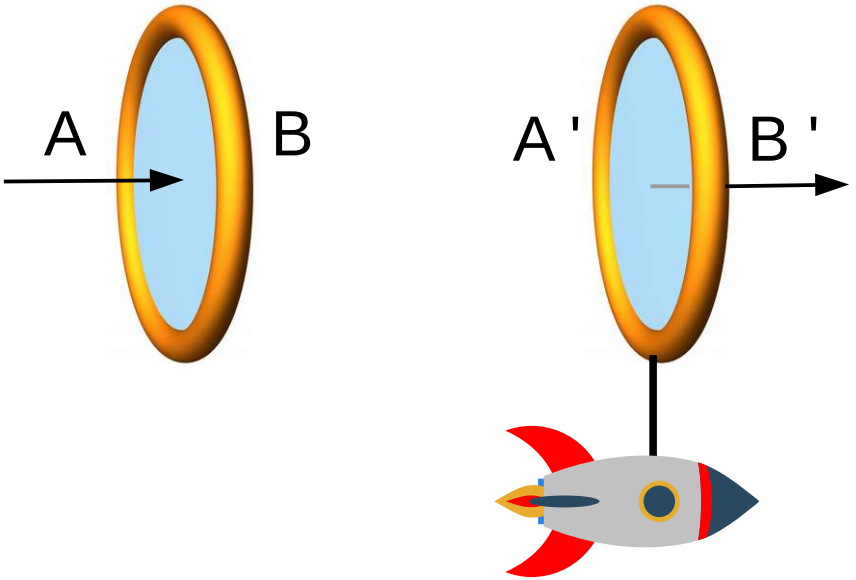
similar to Kerr's solution, differences:

spacetime non-curved (flat) everywhere except the disk perimeter

on the perimeter not a rotating ring singularity is located, but an exotic string of negative mass density and tension

total angle around the string across both universes $\alpha = 4\pi$

$\mu = \tau = (2\pi-\alpha) / (8\pi) = -1/4$




this type of wormhole is especially convenient for the transport, which will be required to build a time machine (it is enough to move the perimeter)

# Wormhole leading to the past

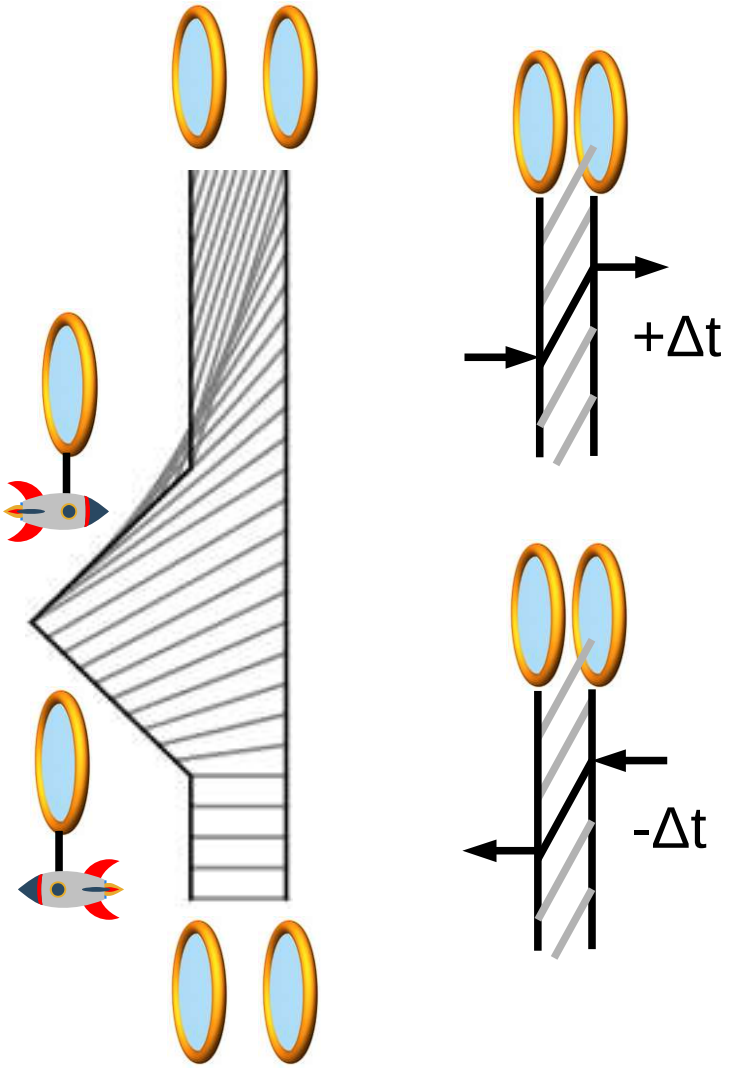
the effect of relativistic time dilation (the twins effect) is used

one wormhole opening moves at nearly light speed, first in one, then in opposite direction

in the final state, the openings are placed close to each other

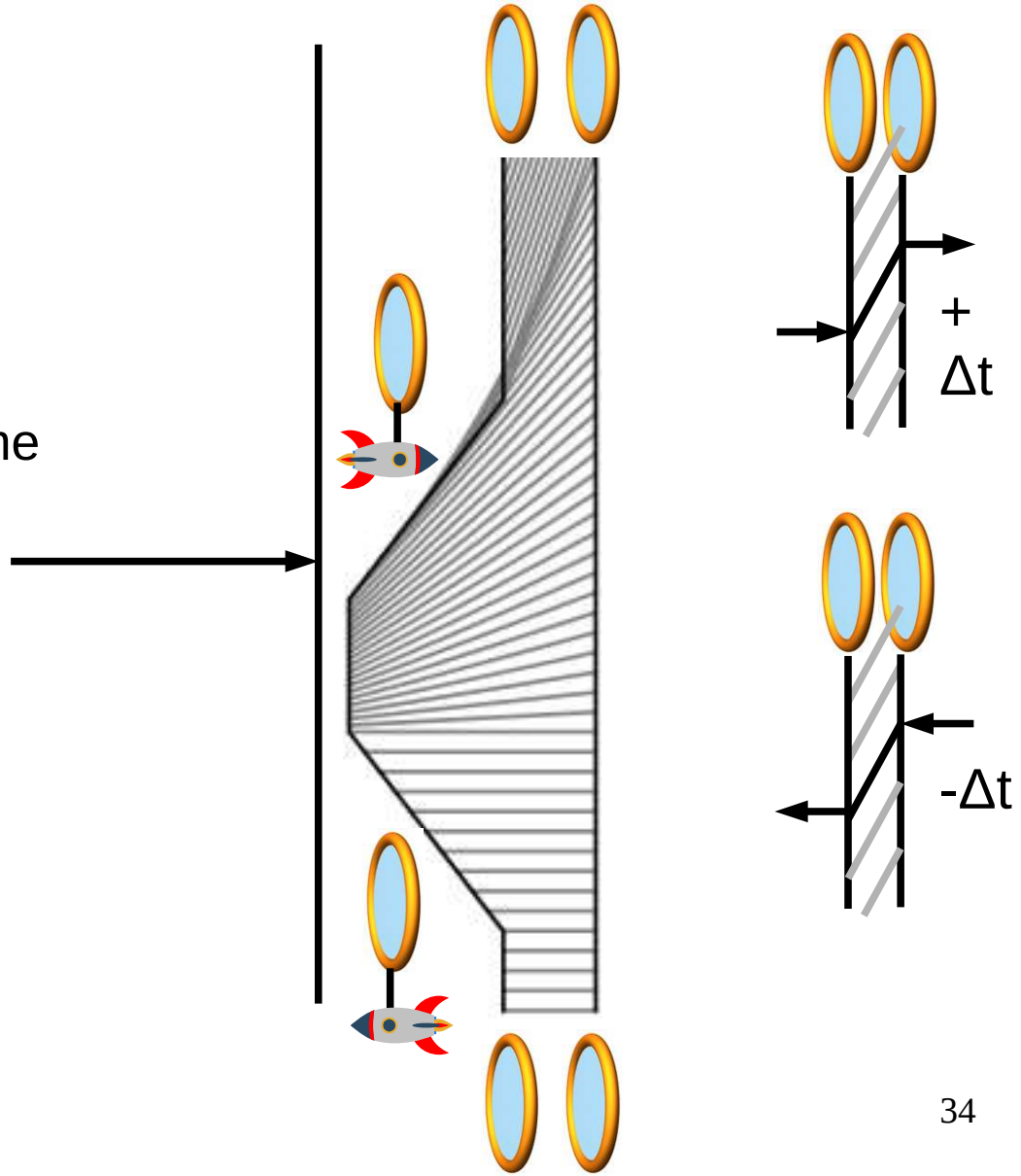
the accumulated time shift Δt can be used for travel to the future or to the past

the figure shows the slope of simultaneous cross sections associated with the relativistic time dilation



# Wormhole leading to the past

instead of fast motion, gravitational time dilation near the black hole event horizon can be used ...



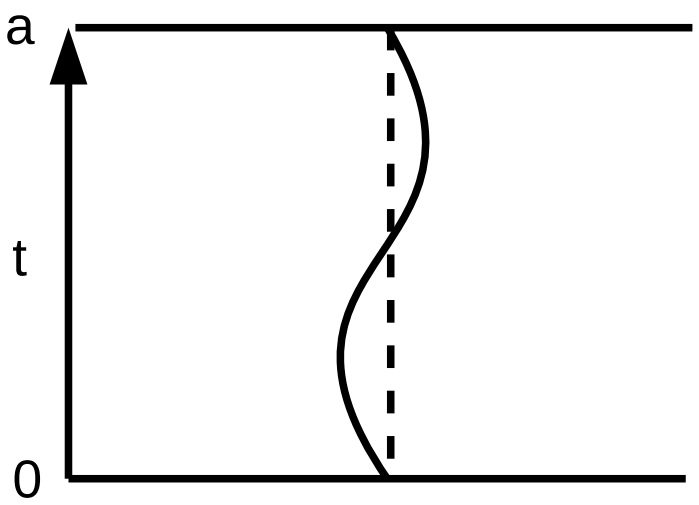
# Cyclic time universe

time changes on the interval [0,a], the end points are identified

spacetime with the topology of a cylinder $R^3 x S^1$

everywhere flat, vacuum solution (no matter)

obviously there are closed timelike curves (just sit and wait until the world line closes)

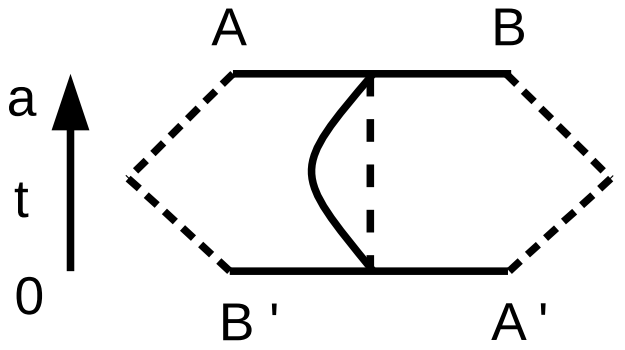


# Deutsch-Politzer Machine 1991

similar construction, 2-dimensional spacetime, where not infinite lines, but finite segments are identified

flat vacuum solution everywhere except the ends of the segments where the singularities are located

there is a variant of identification with reflection (twisted DP, TDP)




shows the region of existence of time machines, for TDP

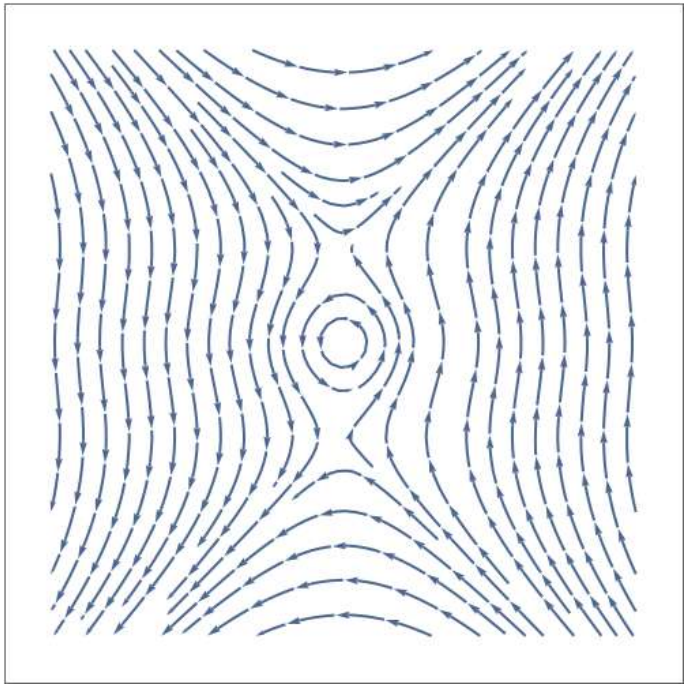
# General solution

**Algorithm** (of building time machines):

take an arbitrary manifold as spacetime

define: the Euclidean metric $g_E$ and
vector field V with closed integral lines

introduce the Lorentzian metric
according to the formula



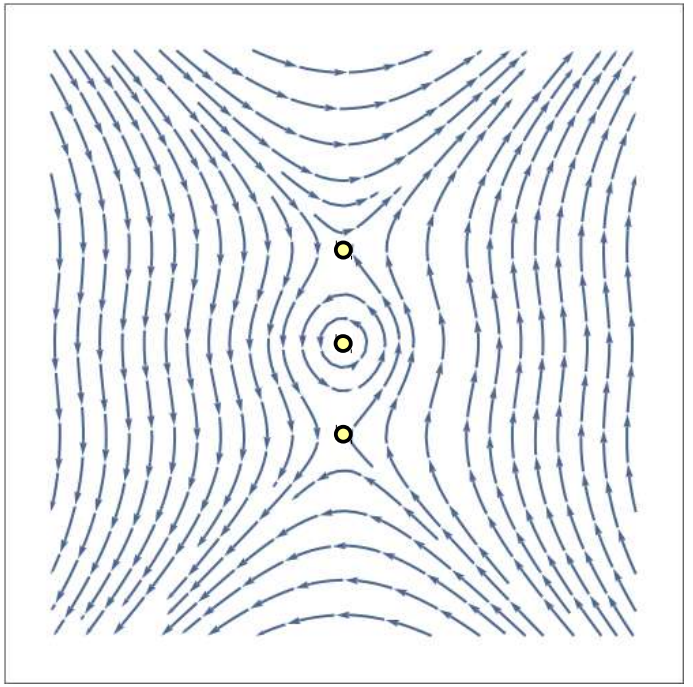
$$(g_L)^{\mu\nu} = (g_E)^{\mu\nu} - 2V^\mu V^\nu / ((g_E)_{\alpha\beta} V^\alpha V^\beta)$$

**Properties:**
the metric is reprojected in the V direction
V and -V give the same result (line field)
components along V get Lorentzian signature
with respect to the new metric, V is timelike
can be used to set time direction on the manifold

# General solution

Next, use the standard procedure $g \to G$ which defines from the metric g the Einstein tensor G, proportional to the distribution of matter T (the procedure contains differentiations and algebraic operations)

One can also impose additional restrictions on the localization and type of matter (sign of mass density, etc.)



**Comment:** singular points V = 0 can appear due to Poincaré-Hopf theorem (in particular, the impossibility of “combing the sphere”), leading to singularities in the metric tensor and (integrable) singularities in the distribution of matter.

**As a result:** we get an example of a time machine in which the position of closed timelike curves can be specified arbitrarily.

# Practical considerations

If we exclude from consideration non-standard cosmological models (Gödel's universe, cyclic time), as well as those requiring infinitely extended objects (van Stockum machine, cosmological strings)

If we also exclude the Kerr solution with a naked singularity (it corresponds to a large curvature of spacetime, which can lead to spontaneous production of particles from a vacuum and instability of the solution - Zel'dovich, Novikov, Starobinsky 1974)

Visser's stargate also has a naked singularity, but of a different type - the surrounding spacetime is flat

From the considered models, the wormholes leading to the past seem to be the most promising for the practical implementation of time machines, and we will consider more detailed estimations for them.

# Practical considerations

Theoretically, if a wormhole is created, then its relativistic movement in one and the other direction requires zero total energy (for acceleration, a supply of energy is needed, which can be returned back when braking)

Also, total zero energy is required for quasistatic sinking in a gravitational well of a black hole with subsequent elevation (when sinking, the energy is released, can then be used to raise)

In addition, for the stargate solution, surrounding one opening with a closed surface, the spacetime near the surface will be flat, the gravitational field and the total gravitational mass of the solution will be zero (here the compensation of negative mass density and positive string pressure happens: $\rho+p = 0$)

If we assume that the inertial mass bounded by the surface is equal to the gravitational mass (the principle of equivalence), then even the phases of acceleration and elevation will require zero energy (one only needs to take into account the kinetic and potential energy of the rocket and other auxiliary structures)

# Practical considerations

The main problem in creating a wormhole

For stargate solutions, linear mass density μ=-1/4 in geometric units corresponds μ= (-1/4) $c^2$/ G = -0.2 Jupiter masses per meter and corresponding pressure force, with plus sign

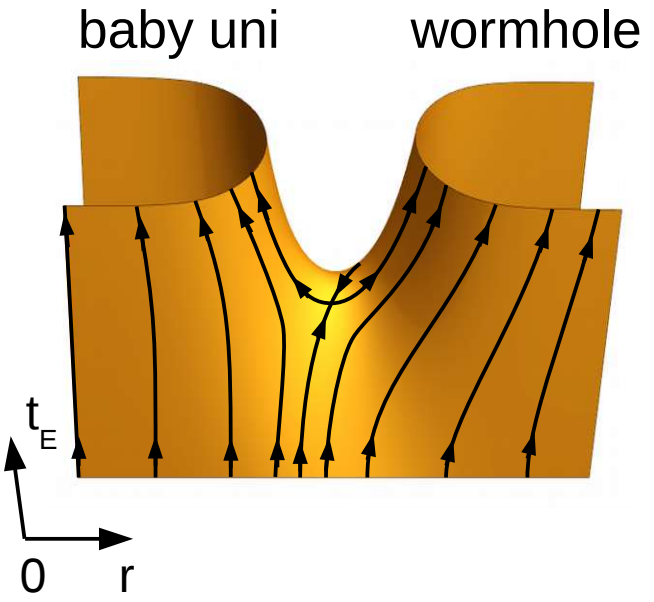
This estimation shows the very high energy characteristics of wormholes. In any case, their creation is beyond the capabilities of the mankind, with the modern level of development of science and technology.

This does not exclude the possibility of creating wormholes by advanced civilizations (assuming their existence) or a spontaneous emergence of wormholes as a result of natural astrophysical phenomena.

It is interesting to calculate the possible astrophysical manifestations of such systems (for example, Fast Radio Bursts are energetically equivalent to an annihilation of an asteroid of average size, which may be enough to open a microscopic wormhole about a nanometer in size).

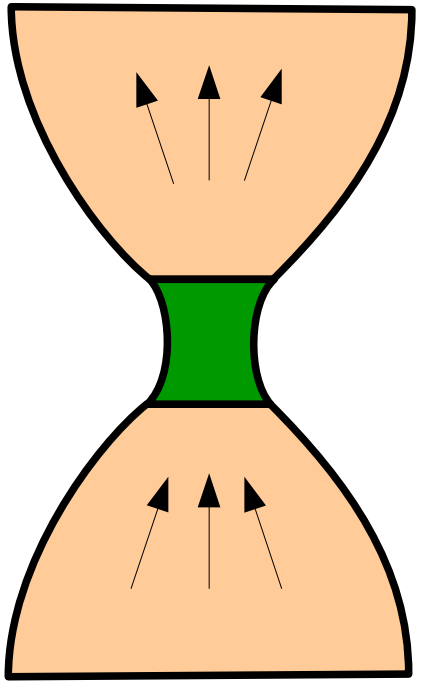
# Practical considerations

- opening a wormhole requires a change in the topology of space in time
- there are various schemes for such a change (Geroch 1967, Sorkin 1986, Borde 1994)
- Example: the opening of a wormhole with the detachment of the spatial bubble (baby universe), see COSMOVIA Lecture 13-Mar-20
- a vector field V defining the Lorentzian metric is shown
- the solution has Poincaré-Hopf singularity V = 0, which corresponds to a mild (integrable) singularity in the distribution of matter



# Practical considerations

- negative mass is required to create a wormhole of Visser's stargate type
- negative sum of mass density and pressure is required to create a Morris-Thorne wormhole
- computations in the 1st order of quantum gravity (QG) give a correction to the density: $\rho_X = \rho\,(1 - \rho / \rho_P)$ (Ashtekar et al. 2006)
- $\rho = \rho_P \Rightarrow \rho_X = 0$ at Plank's density the gravity is switched off
- $\rho > \rho_P \Rightarrow \rho_X < 0$ when the Plank's density is exceeded, the effective negative mass appears (**exotic matter**), the gravitational repulsion (quantum bounce)
- the model of Planck stars: Rovelli-Vidotto (2014), Barceló et al. (2016)
- this effect can be used for the practical creation of wormholes and time machines



Planck star: collapse changed to expansion, black hole turns white

# Practical considerations

- taking into account the quantum effects of a different kind, the vacuum self-energy in the lowest perturbation order (semiclassical limit) leads to a positive feedback loop
- as soon as the time machine is formed, vacuum fluctuations begin to amplify themselves
- this can lead to the destruction of the time machine

- the effect is reduced if the openings of the wormholes are far from each other and have smaller size
- the optimal configuration, described by Visser 1996, are two wormholes, together forming a closed timelike loop
- with openings at the distance **1 a.u.** and diameters **$10^{-20}$ m** a time machine is formed that allows one to send **8 minutes** to the past **1 bit** of information transmitted by a quantum of energy **20 TeV** (comp. LHC 13 TeV)

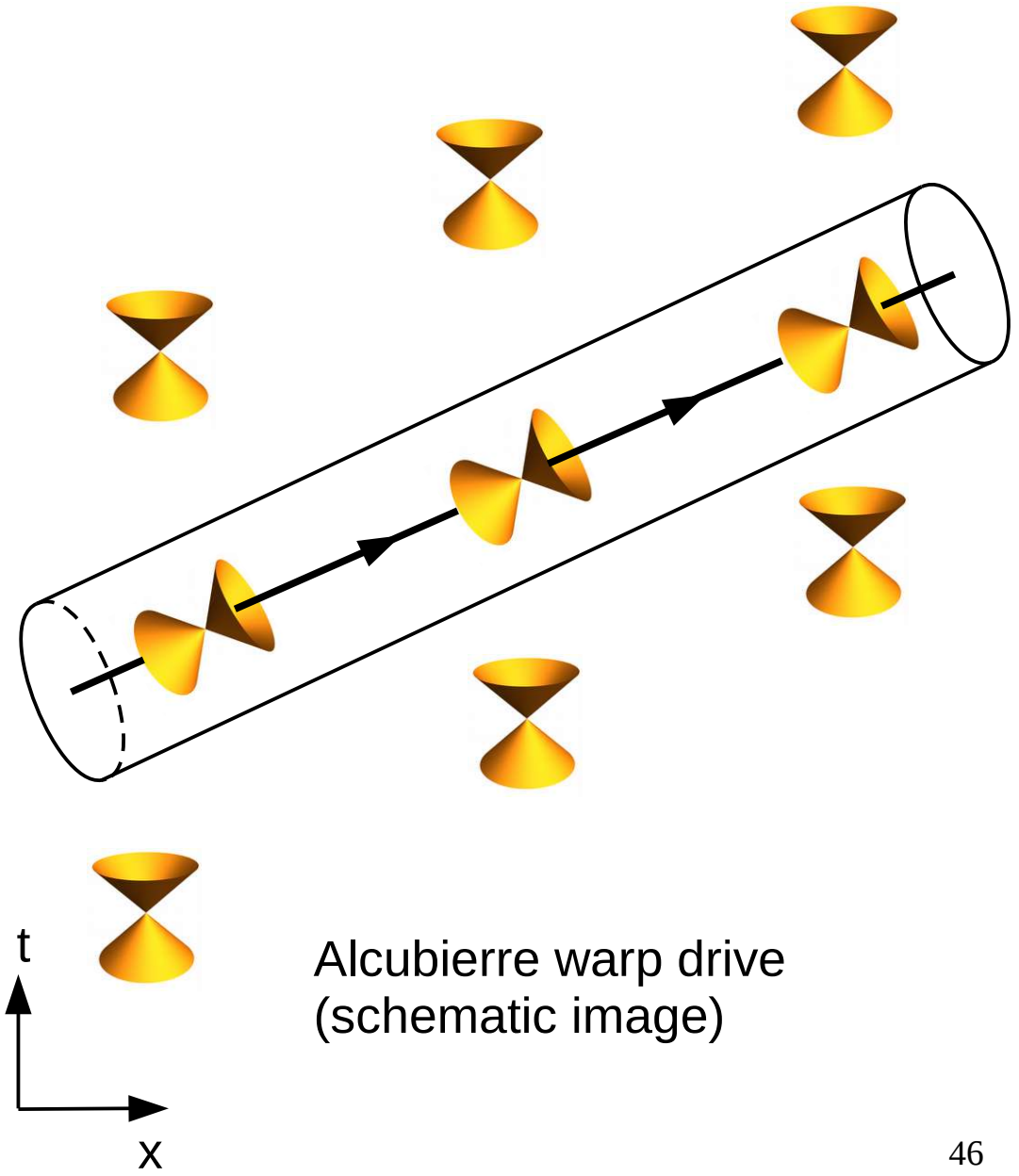
# Practical considerations

- can such divergences indicate the presence of nonperturbative effects (indecomposable to perturbation theory) ?
- can renormalization (redefinition) of the vacuum state be necessary in the presence of time machines ?
- can a compensation of positive vacuum energy be possible with an increase of negative energy of exotic matter filling the wormhole ?
- open questions for further research ...

# Addition: recent development

- Tippett, Tsang 2017
- TARDIS: Traversable Acausal Retrograde Domains In Spacetime
- (reminiscence on Dr. Who sci fi)
- solution closedly related to Alcubierre warp drive

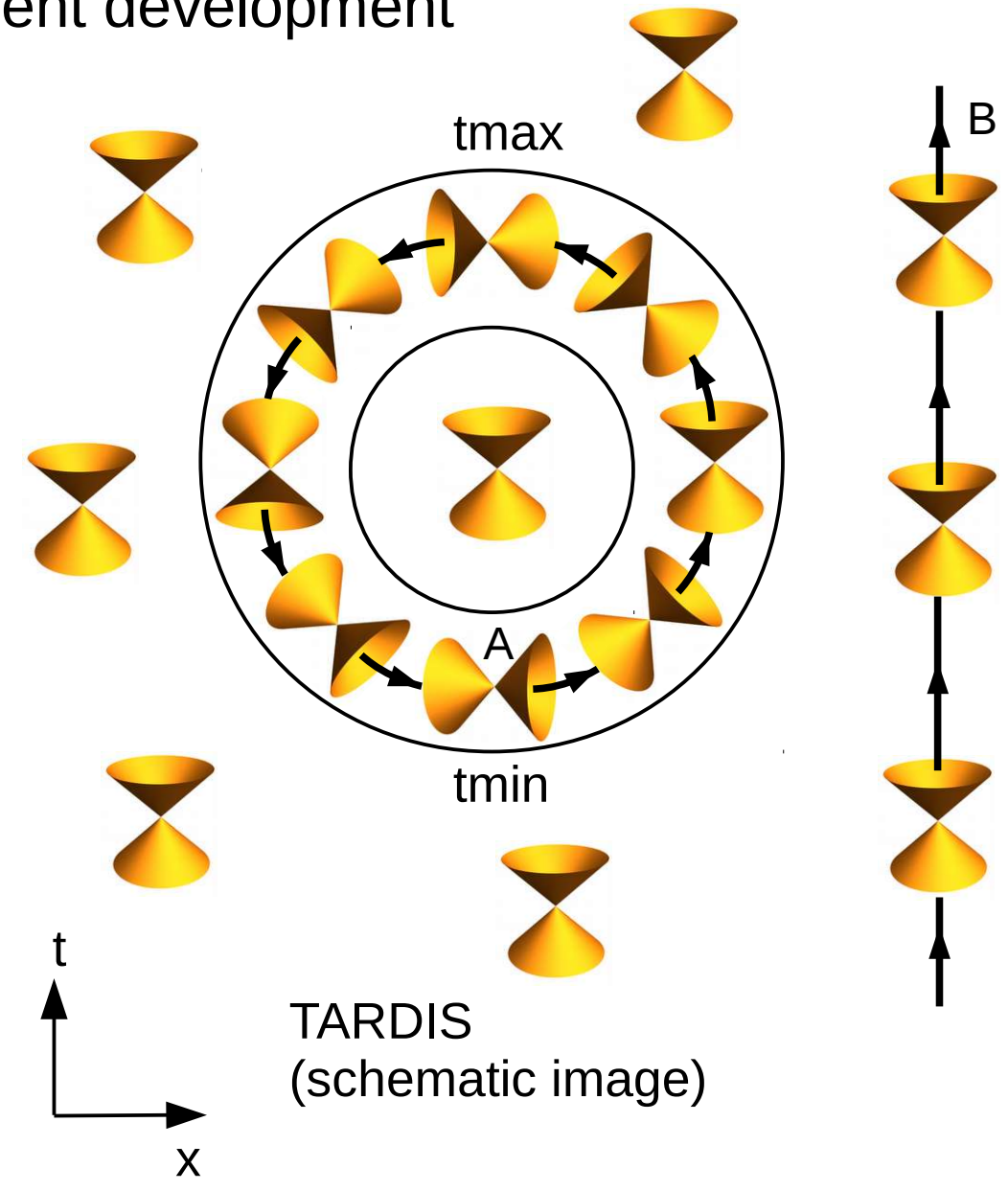
- here: warp drive geometry
- continuous transition between two flat (vacuum) domains, one has tilted light cones
- superluminal motion from the viewpoint of external observer
- locally all worldlines are inside light cones, the light speed never exceeded
- requires exotic matter




Alcubierre warp drive
(schematic image)

# Addition: recent development

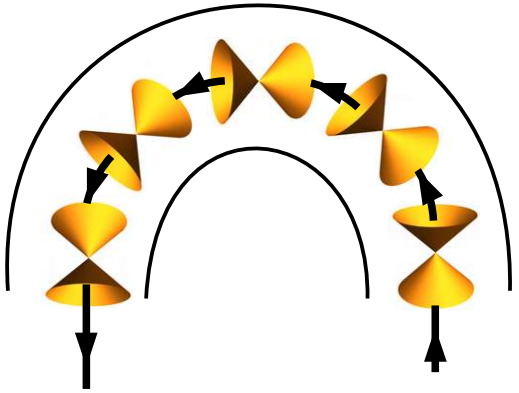
- TARDIS is a warp drive, moving in a loop in spacetime
- geometry: two flat (vacuum) domains, with continuous transition
- details: inner domain has Rindler metric, equivalent to Minkowski in constantly accelerated frame
- Rindler domain has cyclic time
- internal observer A performs cyclic evolution under Novikov's self-consistent chronology, perceives constant acceleration (controlled by radius of the loop a~1/R)
- for the external observer B, two copies of A appear at tmin, move apart (one evolves forward, other reverse in time), then come closer and annihilate each other at tmax

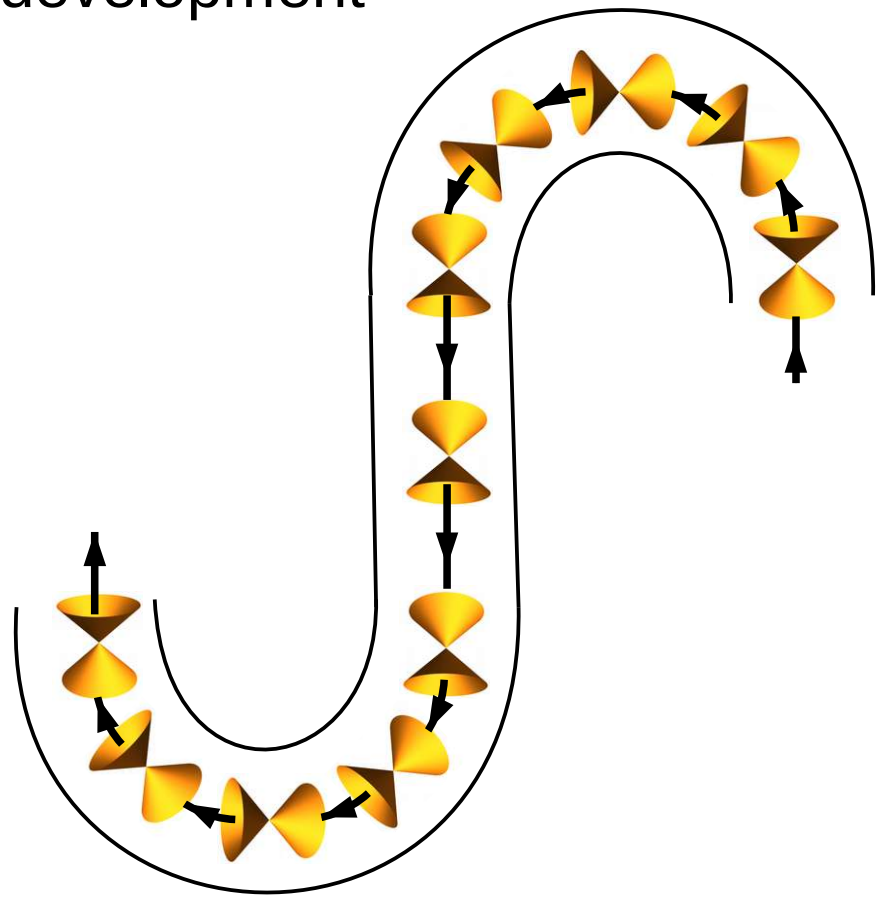



TARDIS
(schematic image)

# Addition: recent development

- S-geometry, modified not to go in cycle, but to travel back in time
- U-turn, “wrong” usage of TARDIS: on exit, makes the observer to propagate backward in time, be mirror reflected and (presumably) consist of antimatter – complete CPT conjugation is applied





TARDIS, modified geometries (schematic image)

# Addition: recent development

- Resume on TARDIS geometry: *„The spacetime geometry is geodesically incomplete, contains naked singularities, and requires exotic matter."*
- in detail: Tippett, Tsang 2017 have performed a deep analysis of TARDIS geometry, including ray-tracing by null geodesics, (pseudo) Cauchy horisons and other difficulties arising in Cauchy problem settings, singularities with metric degeneration (Poincaré-Hopf type), energy-momentum tensor...
- a deep discussion of selection rules, energy conditions, censorship theorems, in particular, an argument that exotic matter is obtained by application of *non-modified Einstein field eqs* to a proposed geometry
- while *extended gravity* inserts an additional degree of freedom f(R) between them and can generate non-exotic solutions, with examples known for wormholes and cosmology
- I also add that many solutions that were initially considered non-physical (black holes, Big Bang cosmology, dark matter, dark energy ...) subsequently became generally accepted, so it makes sense to keep track of exotic solutions of all kinds, considering them *not as forbidden, but as disputed*

# Conclusion

- the fundamental possibility of constructing time machines in the framework of the general theory of relativity is considered
- there are two ways to resolve time paradoxes:
  for one timeline, Novikov's postulate on the self-consistency of chronology,
  for many timelines, the use of non-Hausdorff topology
- there are two main ways to create time machines: systems with fast rotation of massive objects and nontrivial topology of spacetime
- one method - the creation of wormholes leading to the past, is considered in more detail
- the extremely high energy characteristics of wormholes currently do not allow the practical implementation of this scheme
- also, while implementing this scheme, it will be necessary to solve a number of fundamental problems: negative masses, Poincaré-Hopf singularities, instability of vacuum state
- recently developed TARDIS model also possesses most of these problems
- the possibility of solving these problems by advanced civilizations is not excluded, as well as the spontaneous creation of wormholes and time machines in natural astrophysical processes.