Laws of Galactic Rotation

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Galaxies

"Island Universes"

Gravitationally self-bound entities composed of stars, gas, dust, [& dark matter(?)]

Huge dynamic range in

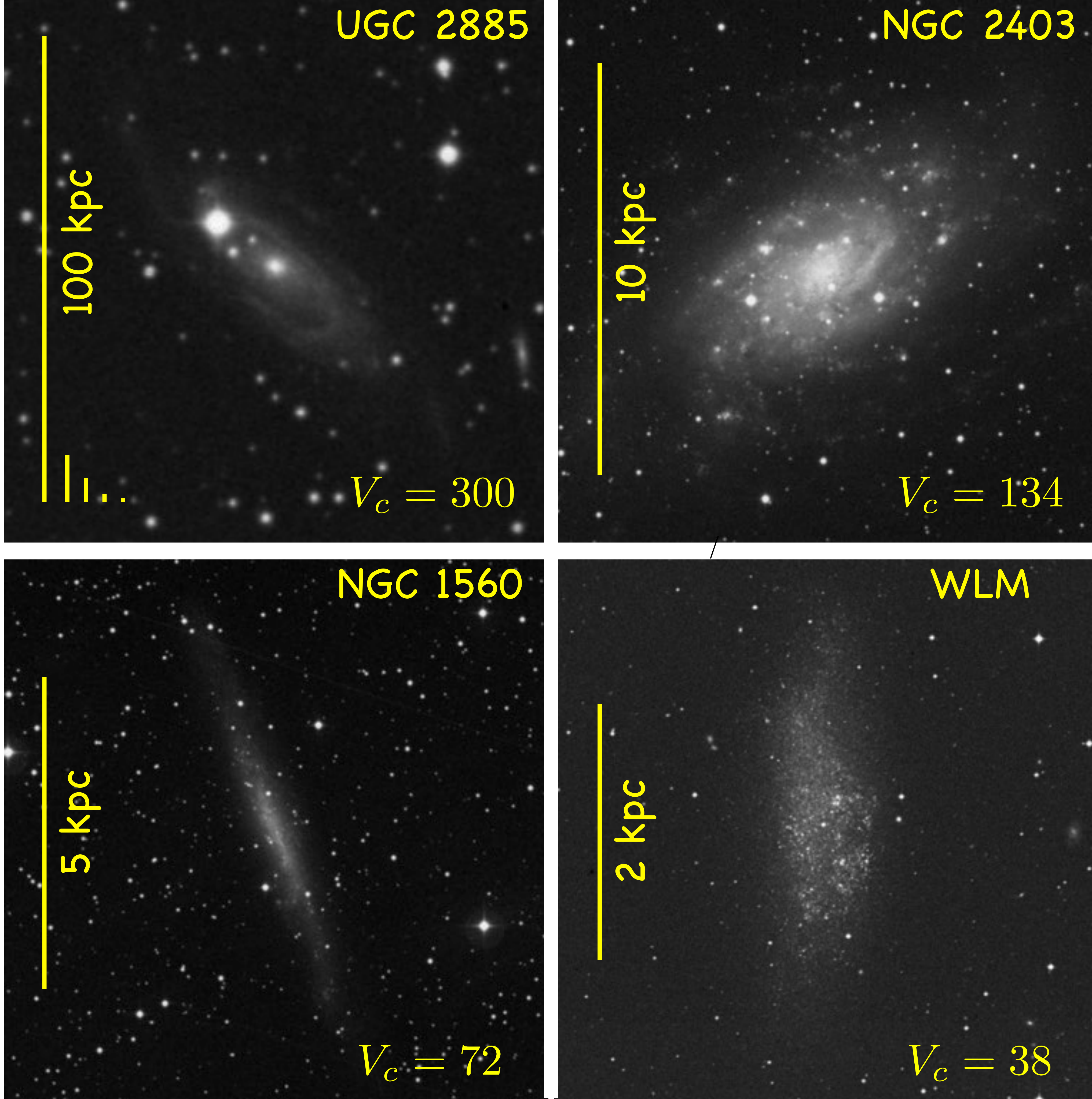
size
0.1 - 300 kpc

stellar mass
10^6 - 10^{12} $M_\odot$

gas fraction
0 - 99%

surface brightness
10 - 10^4 $L_\odot\,pc^{-2}$

dynamical mass
10^9 - 10^{13} $M_\odot$



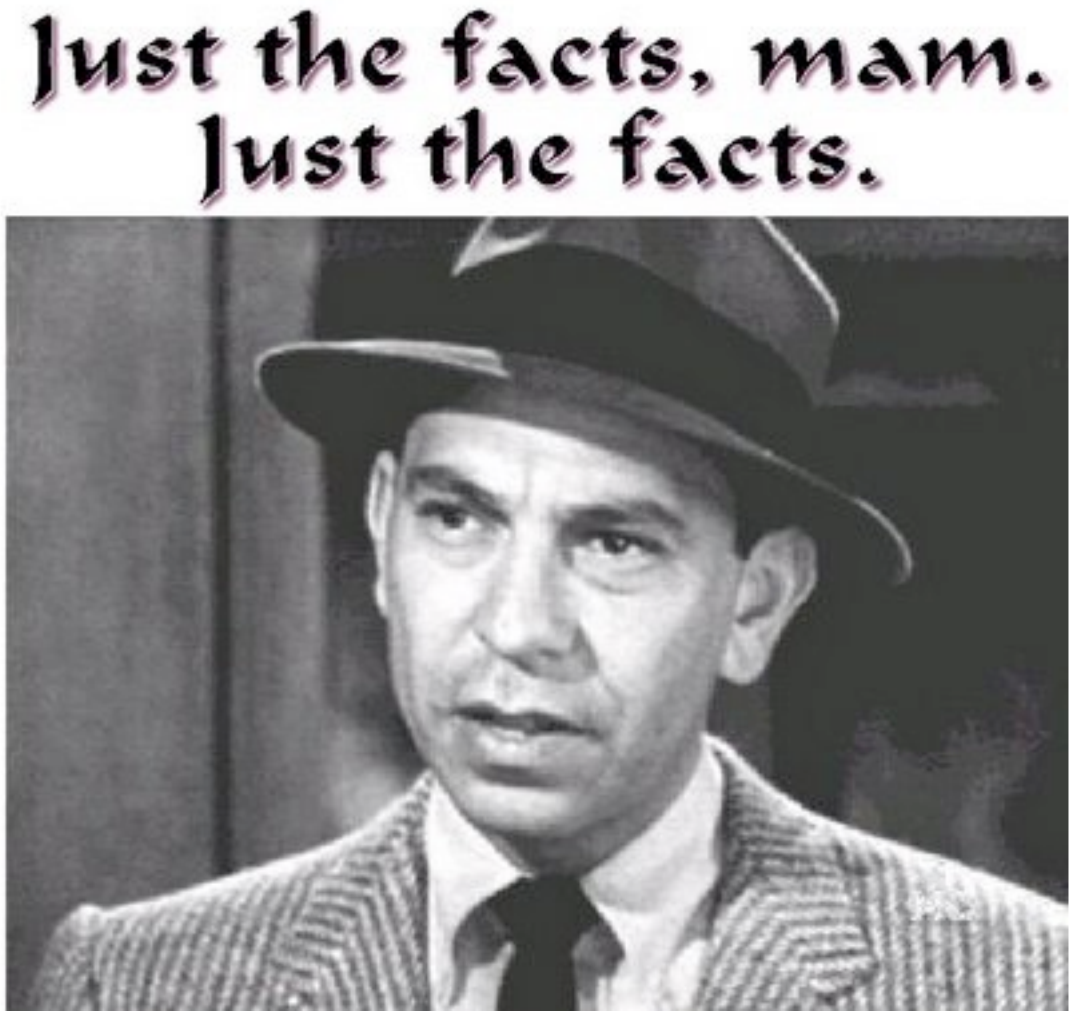
3 Laws of Galactic Rotation

1. Rotation curves tend towards asymptotic flatness $V_f \rightarrow \mathrm{constant}$

2. Baryonic mass scales as the fourth power of rotation velocity (Baryonic Tully-Fisher) $M_b \propto V_f^4$

3. Gravitational force correlates with baryonic surface density

$$-\frac{\partial \Phi}{\partial R} \propto \Sigma_b^{1/2}$$



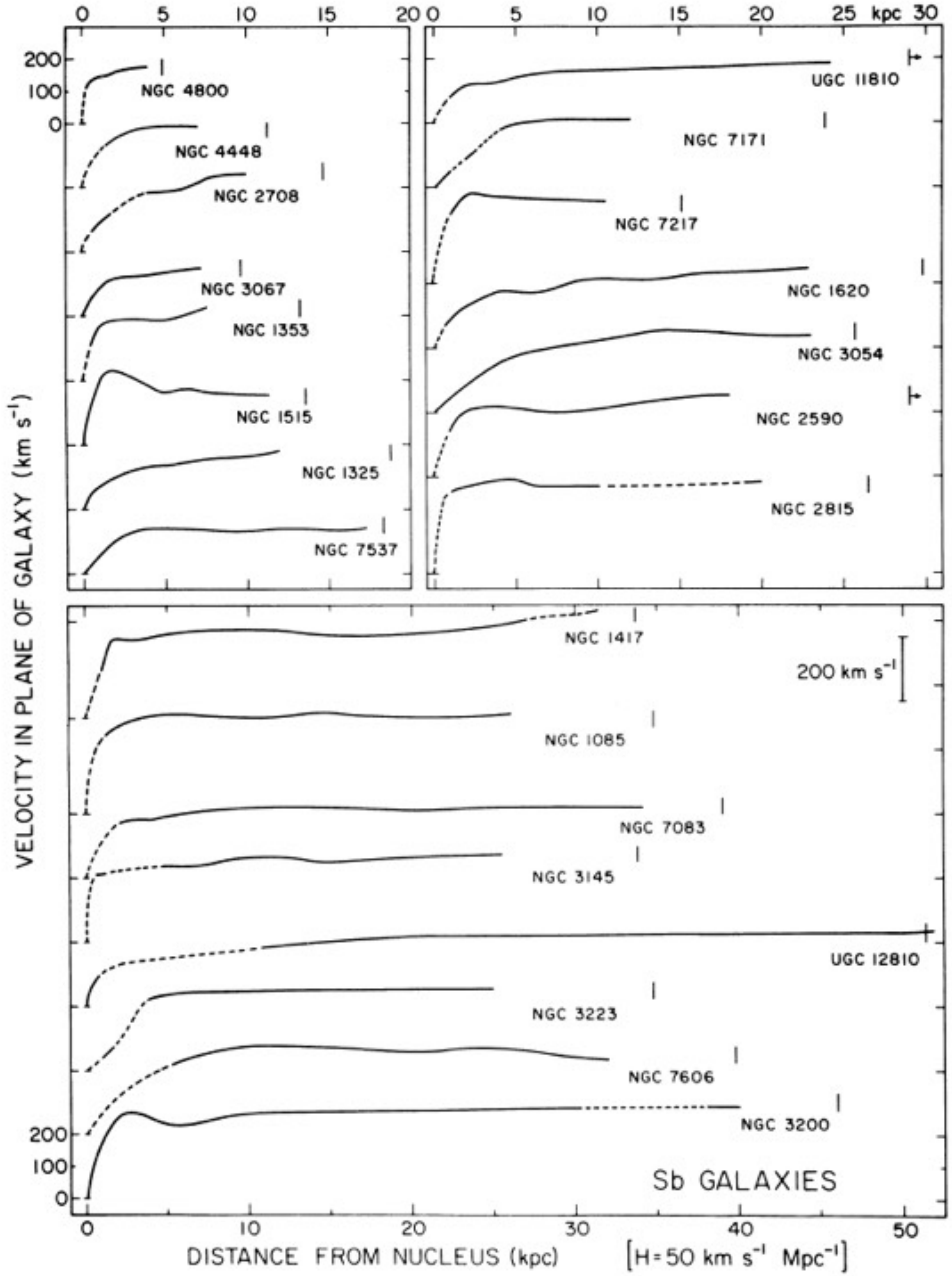


FIG. 3.—Mean velocities in the plane of the galaxy, as a function of linear radius for 23 Sb galaxies, arranged approximately according to increasing luminosity. Adopted curve is rotation curve formed from the mean of velocities on both sides of the major axis. Vertical bar marks the location of R_{25}, the isophote of 25 mag arcsec^{-2}, corrected for effects of internal extinction and inclination. Regions with no measured velocities are indicated by dashed lines.

1. Rotation curves tend to become flat at large radii

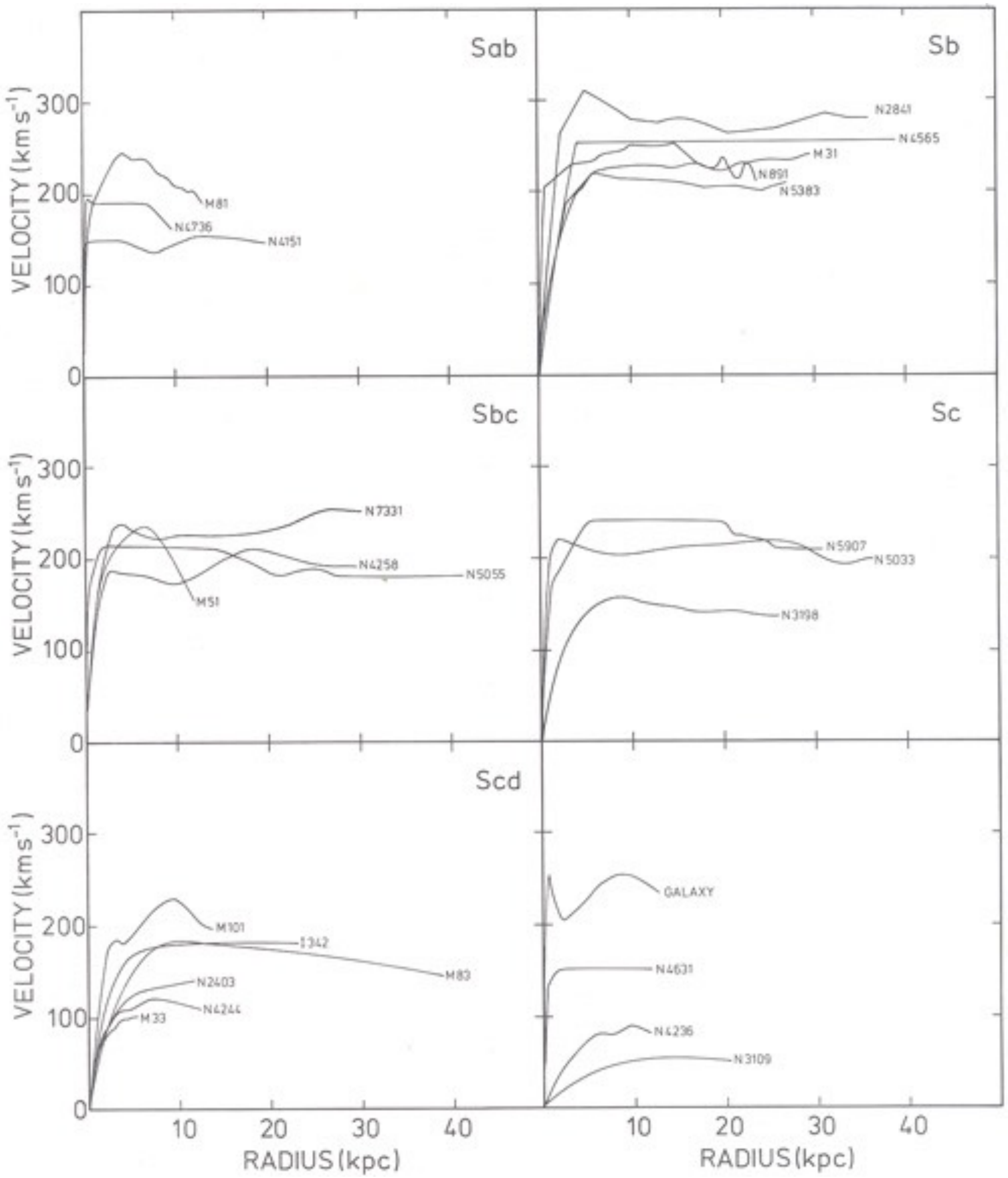
$$V \propto \text{const}$$

$$M \propto R$$

$$\rho \propto R^{-2}$$

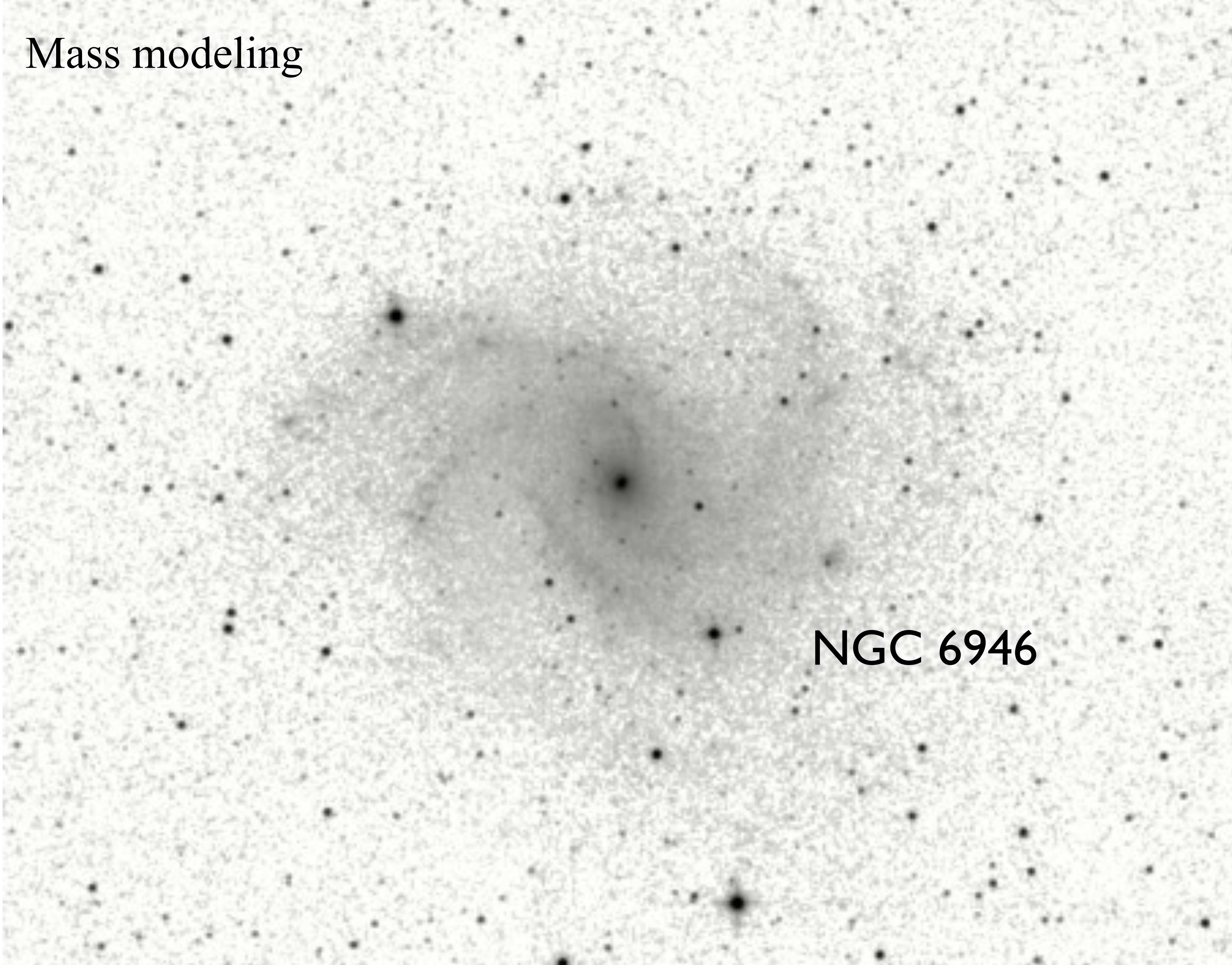
Optical data from
Rubin, Thonnard, & Ford 1978, *ApJ*, **225**, L107

...and stay flat to the largest radii probed

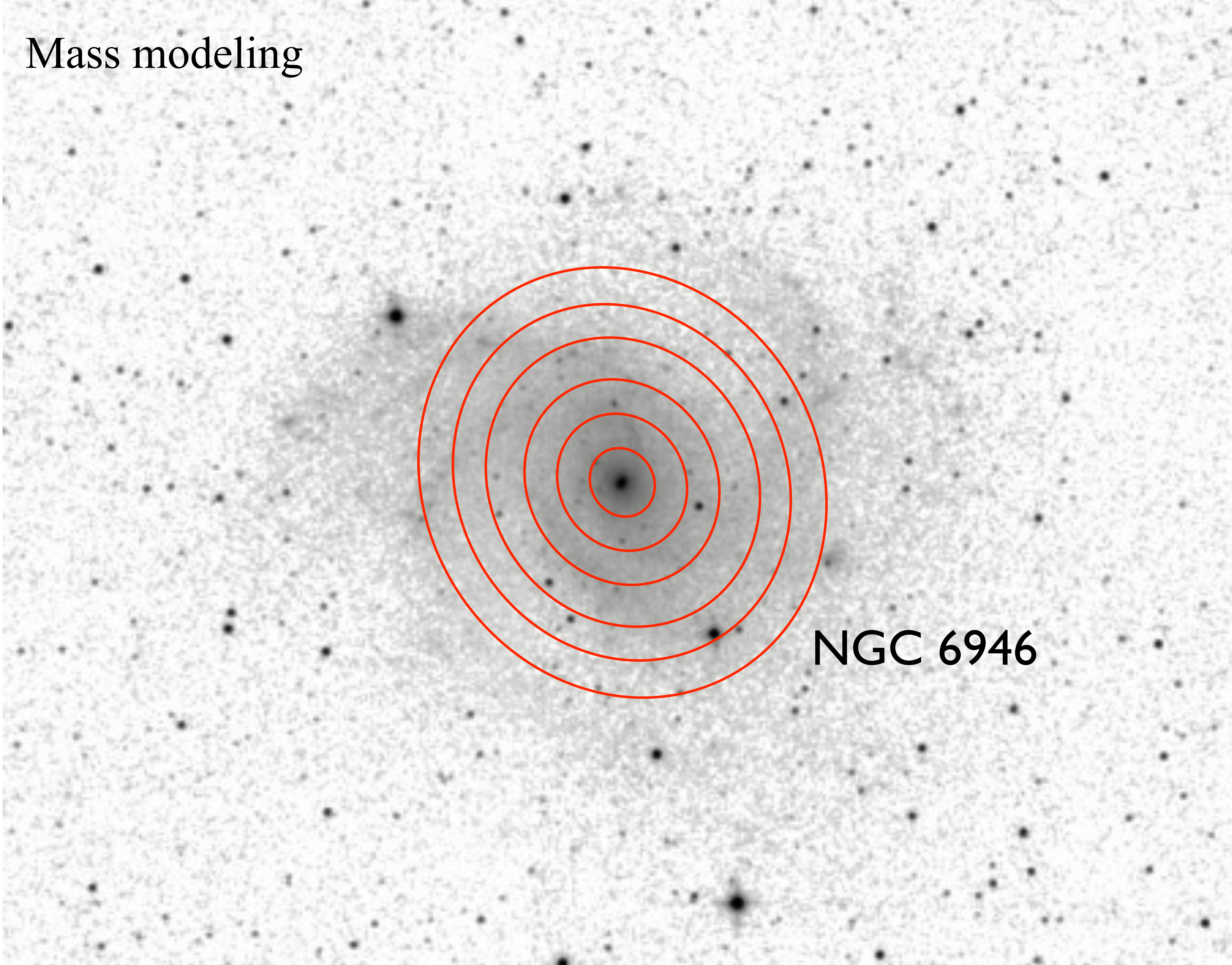


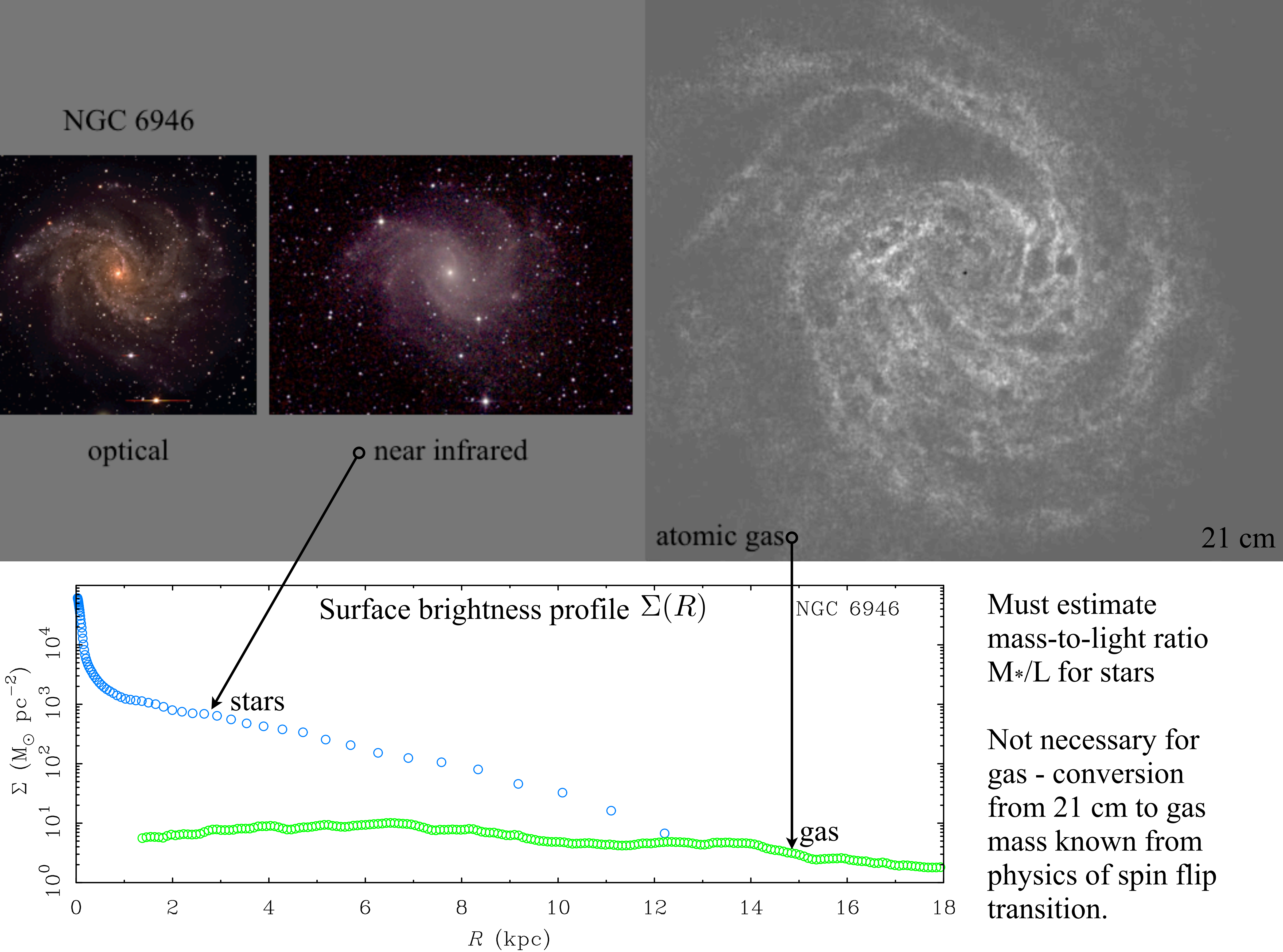
Radio data from
Bosma 1981, *AJ*, **86**, 1825

Mass modeling



Mass modeling

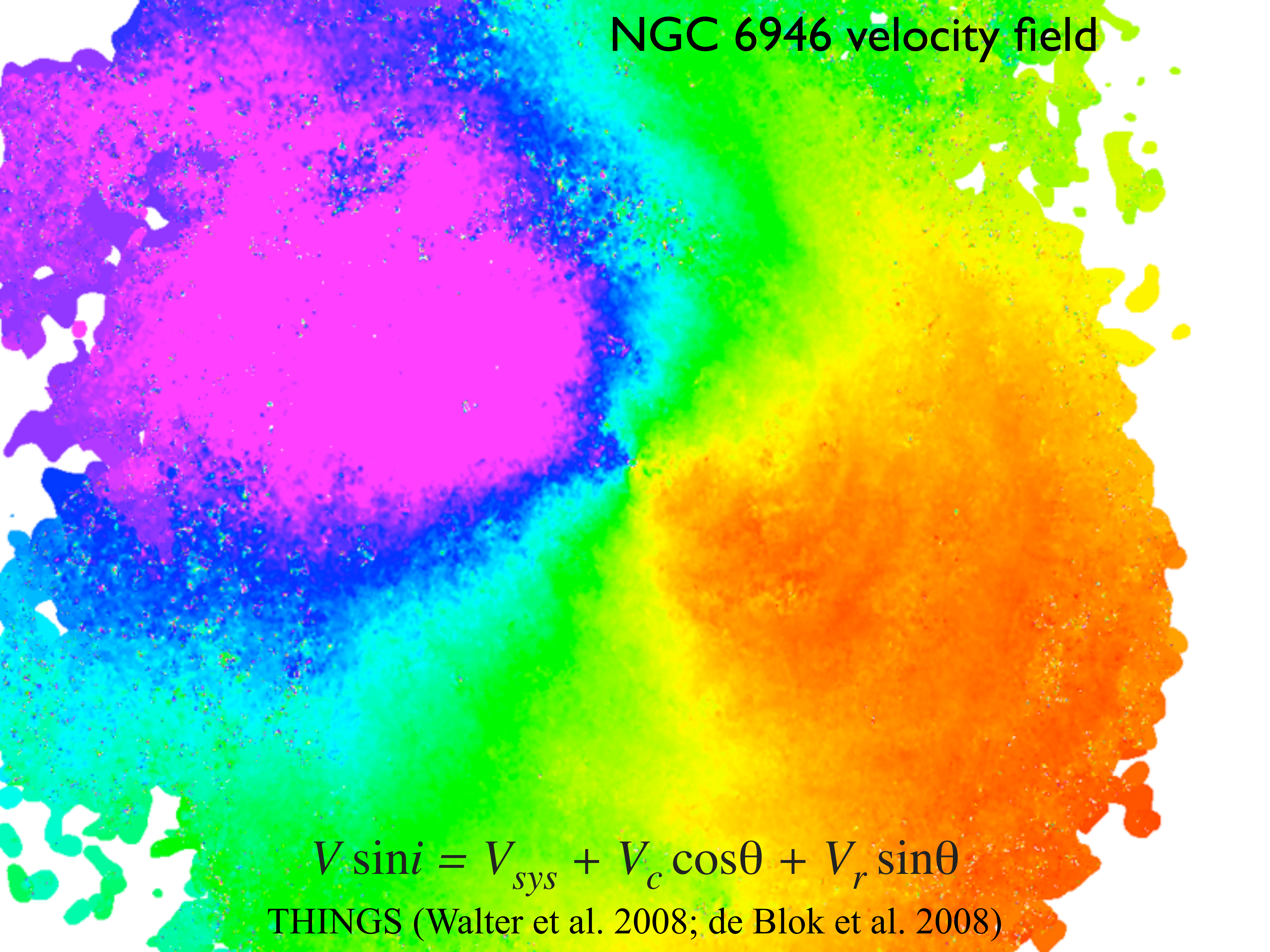




Must estimate mass-to-light ratio M*/L for stars

Not necessary for gas - conversion from 21 cm to gas mass known from physics of spin flip transition.

NGC 6946 velocity field



$$V \sin i = V_{sys} + V_c \cos\theta + V_r \sin\theta$$

THINGS (Walter et al. 2008; de Blok et al. 2008)

Mass model

Solve Possion's Eqn
for the observed
surface density
distribution

$$\nabla^2 \Phi = 4\pi G \rho$$

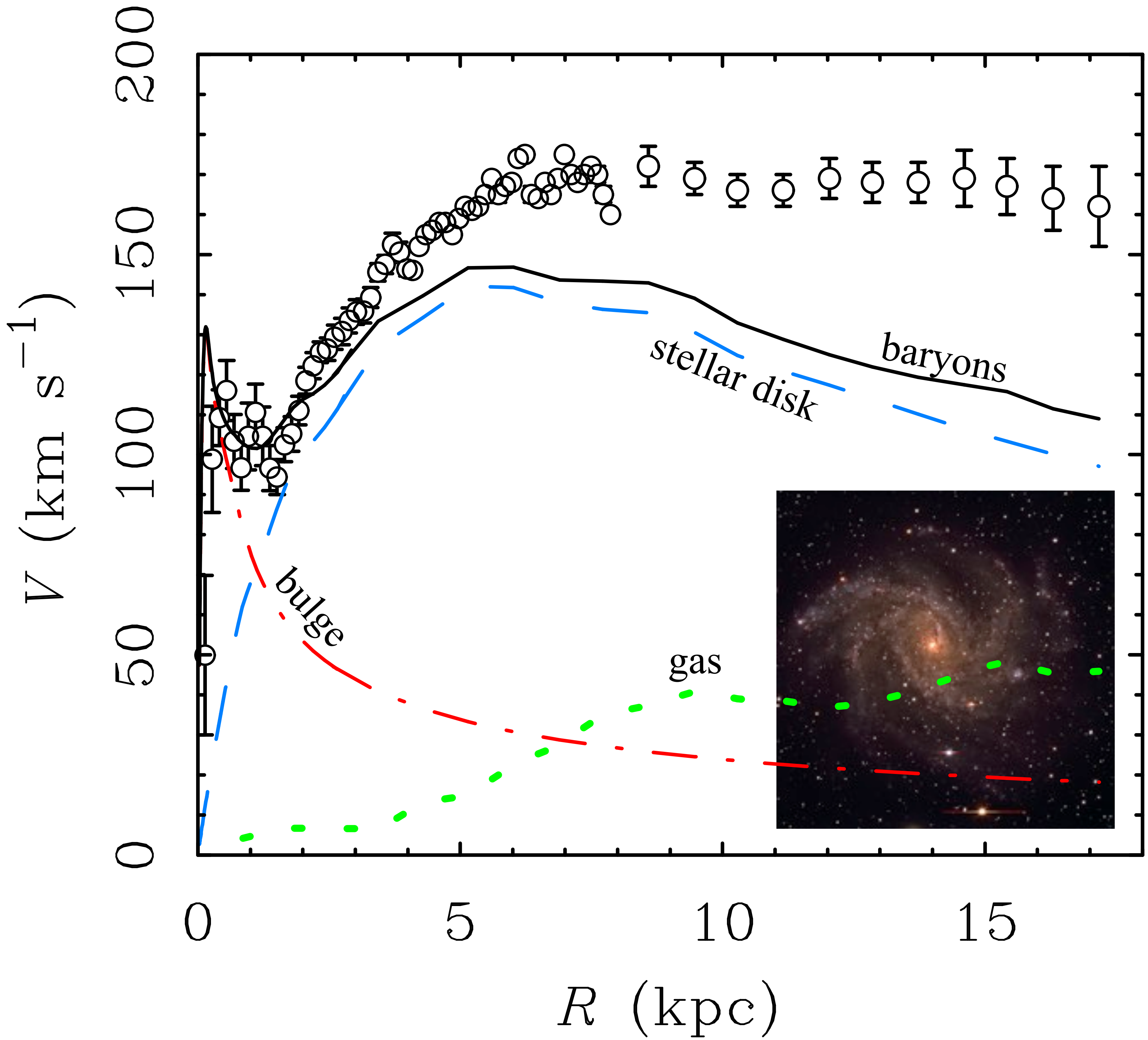
assuming a nominal
disk thickness

$$\rho(R, z) = \Sigma(R)\nu(z)$$

Equate
centripetal acceleration
with gravitational force

$$\frac{V^2}{R} = -\frac{\partial \Phi}{\partial R}$$

To find expected V(R)
for each baryonic component
- stars in bulge
- stars in disk
- gas



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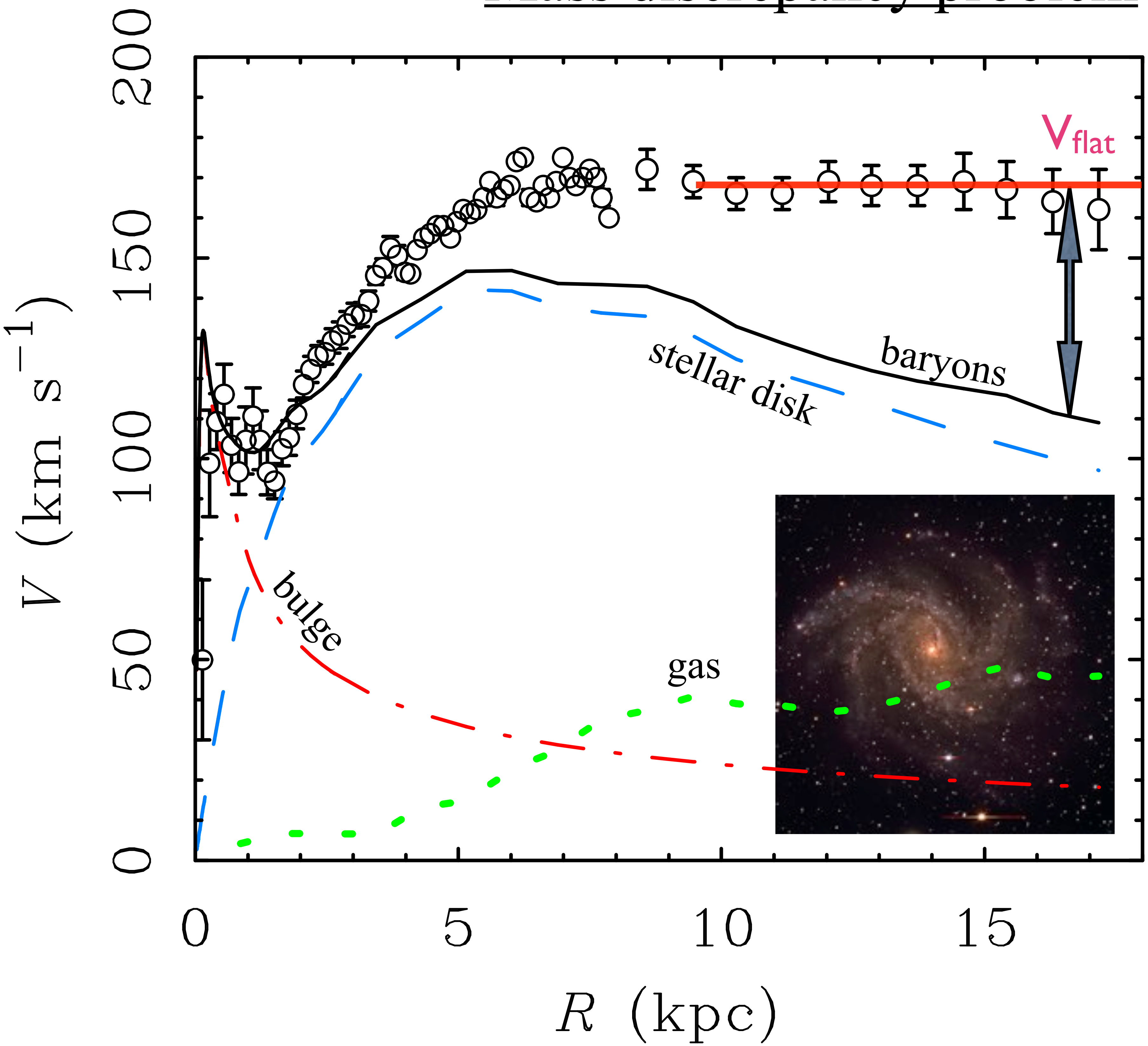
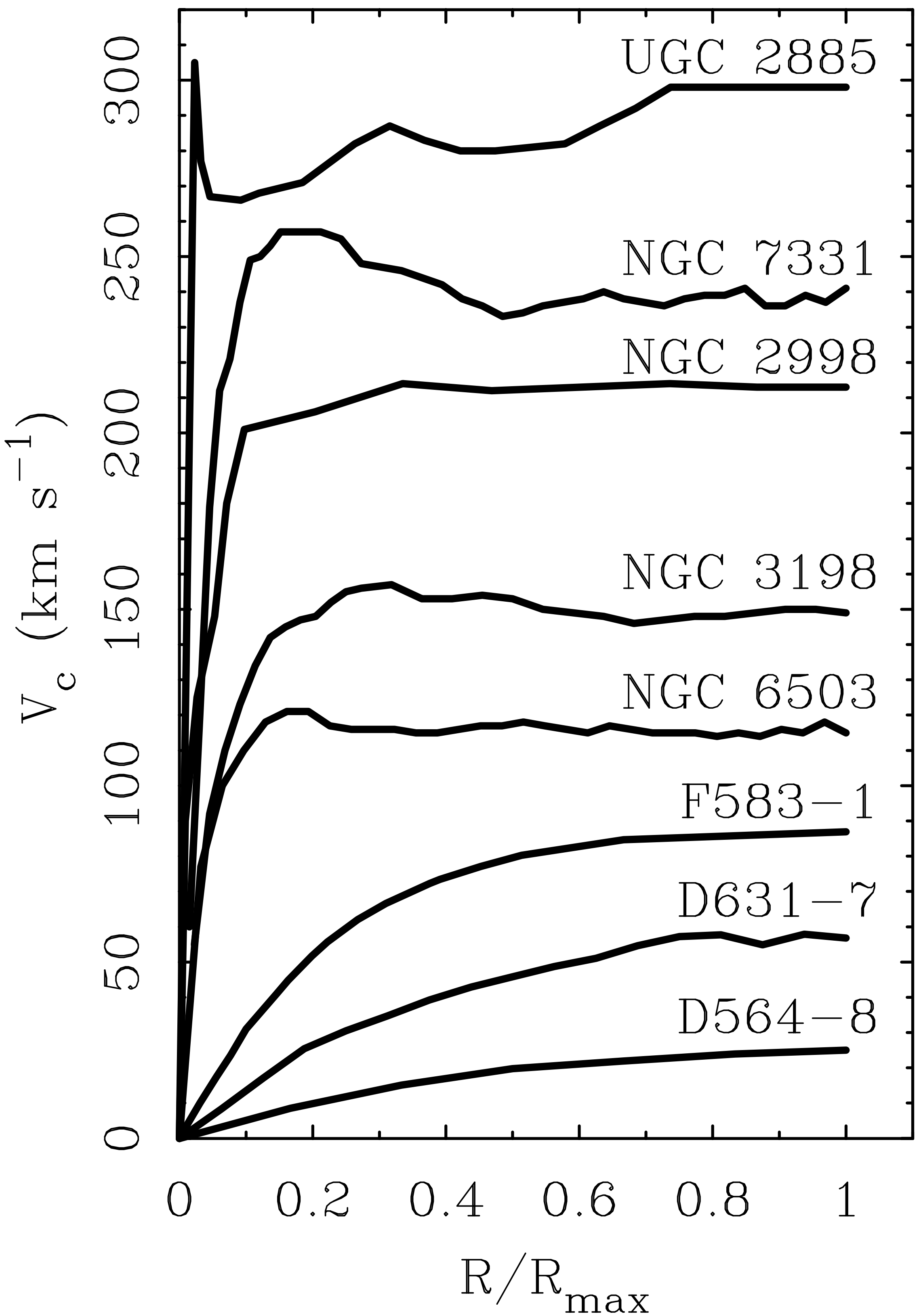
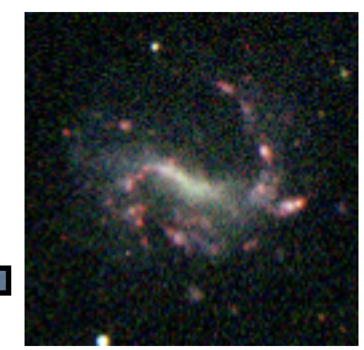
Mass discrepancy problem

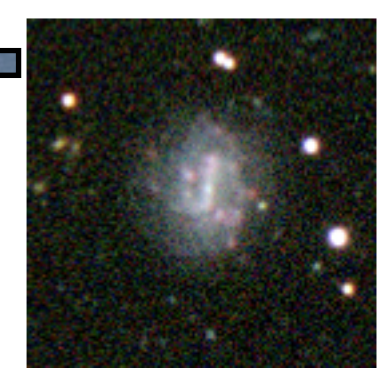


2. Rotation curves amplitude correlates with mass:



star dominated HSB

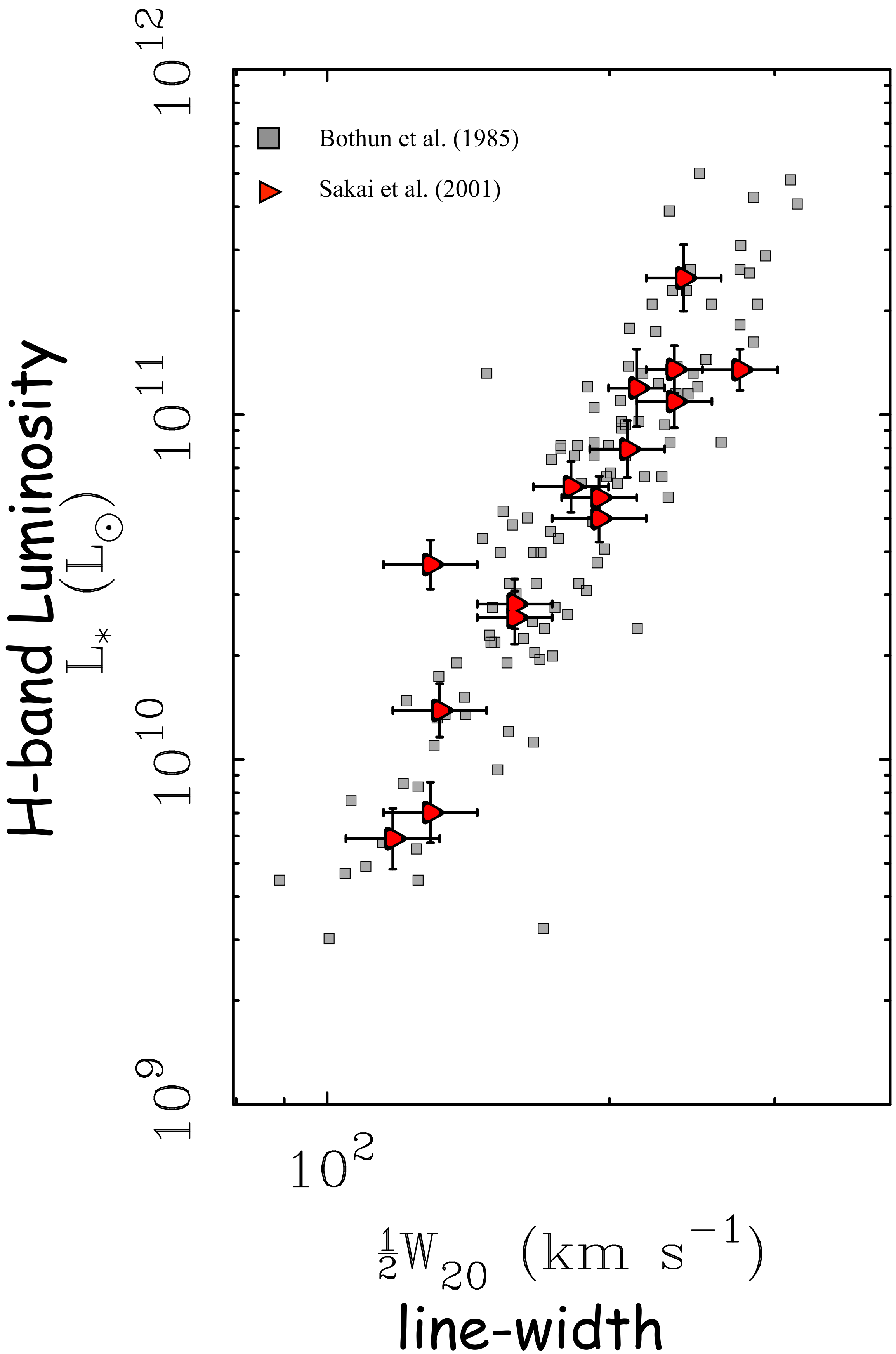




gas dominated LSBs

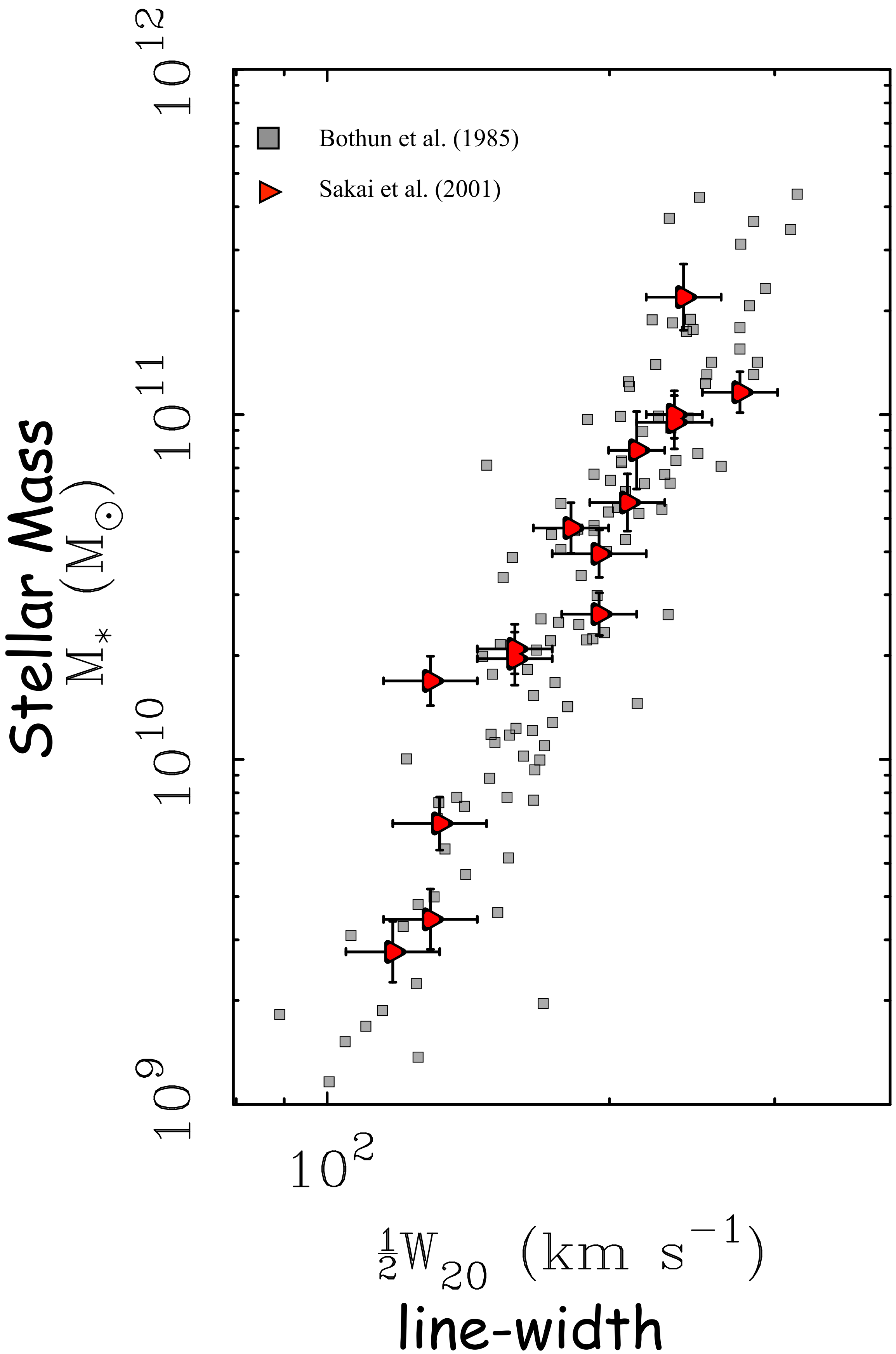
Flat rotation curves continue to occur in quite small systems (V_{flat} ~ 20 km/s)

Tully-Fisher relation



Luminosity and line-width are presumably proxies for stellar mass and rotation velocity.

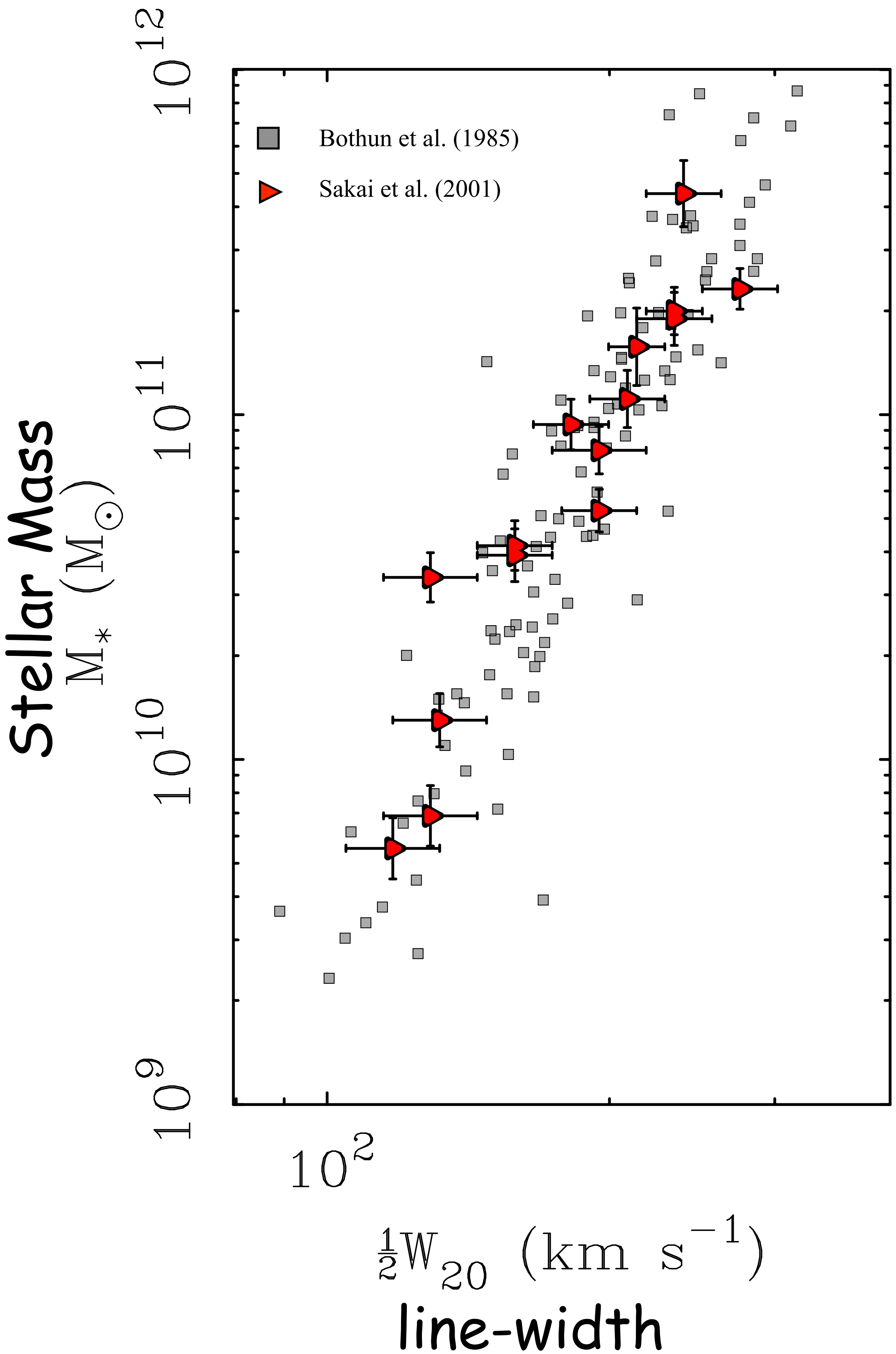
Stellar Mass Tully-Fisher relation



nominal M*/L (Kroupa IMF)

$$M_* = \left(\frac{M_*}{L}\right) L$$

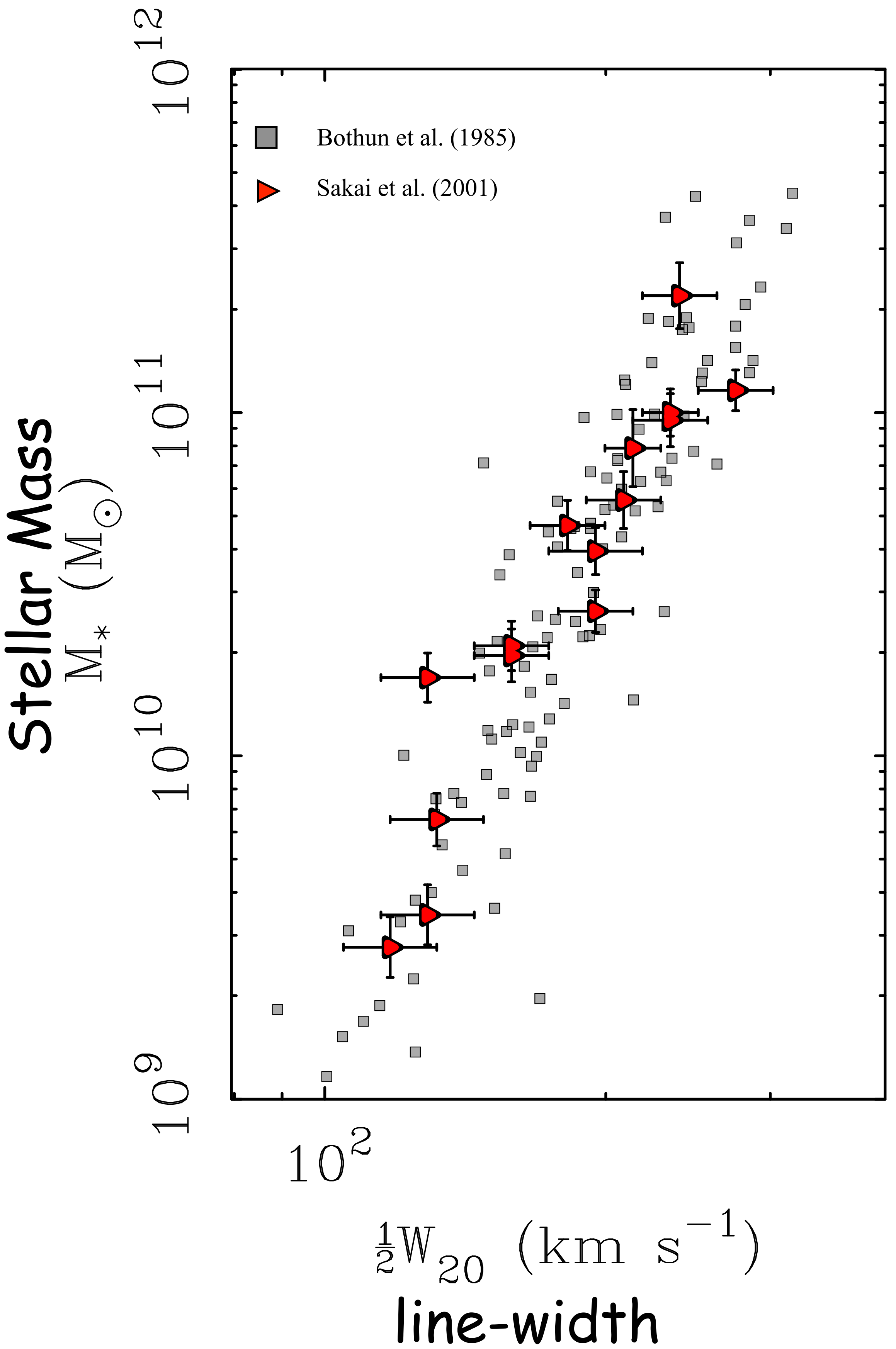
Stellar Mass Tully-Fisher relation



double M*/L

...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)

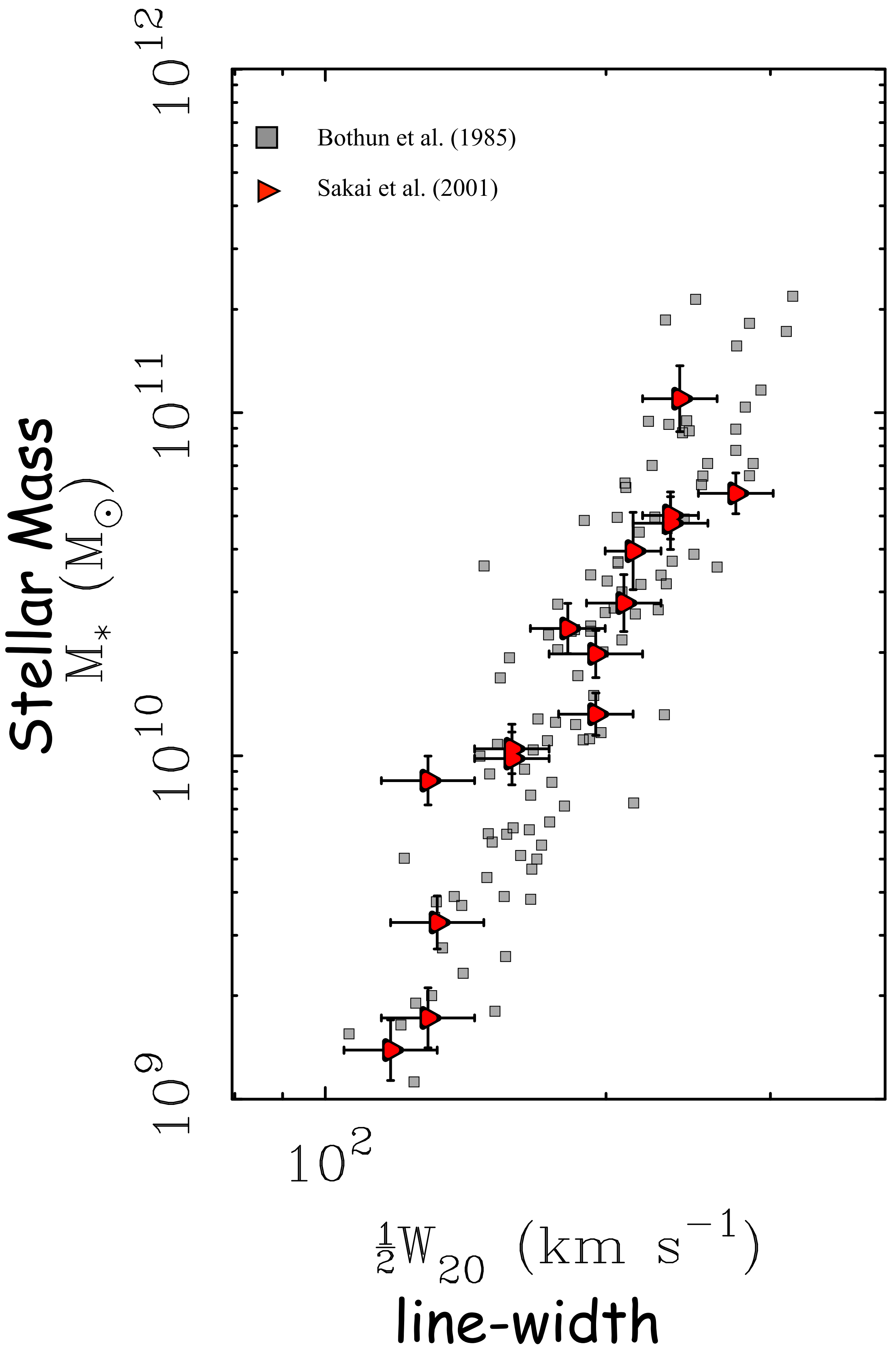
Stellar Mass Tully-Fisher relation



nominal M*/L

...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)

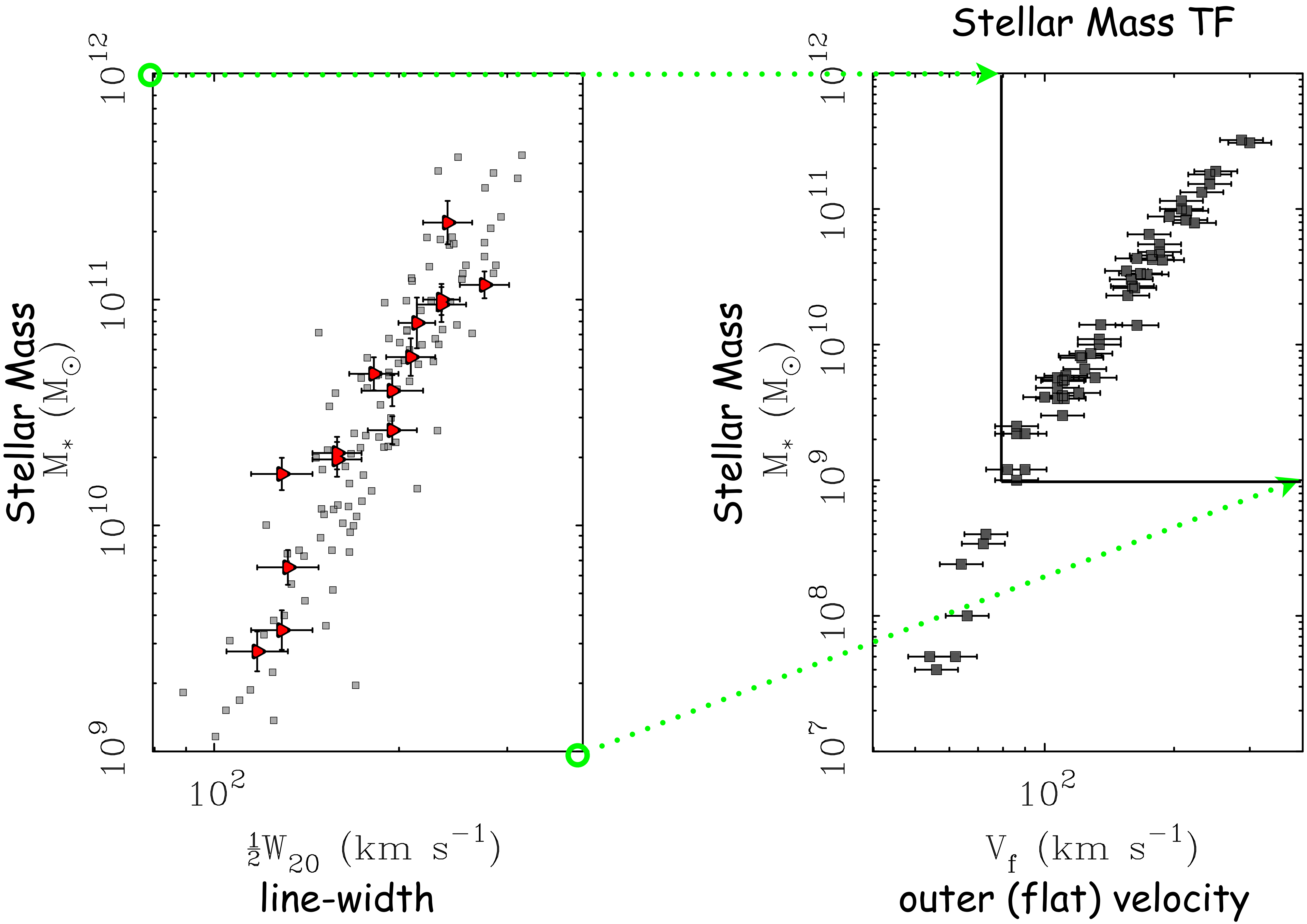
Stellar Mass Tully-Fisher relation



half M*/L

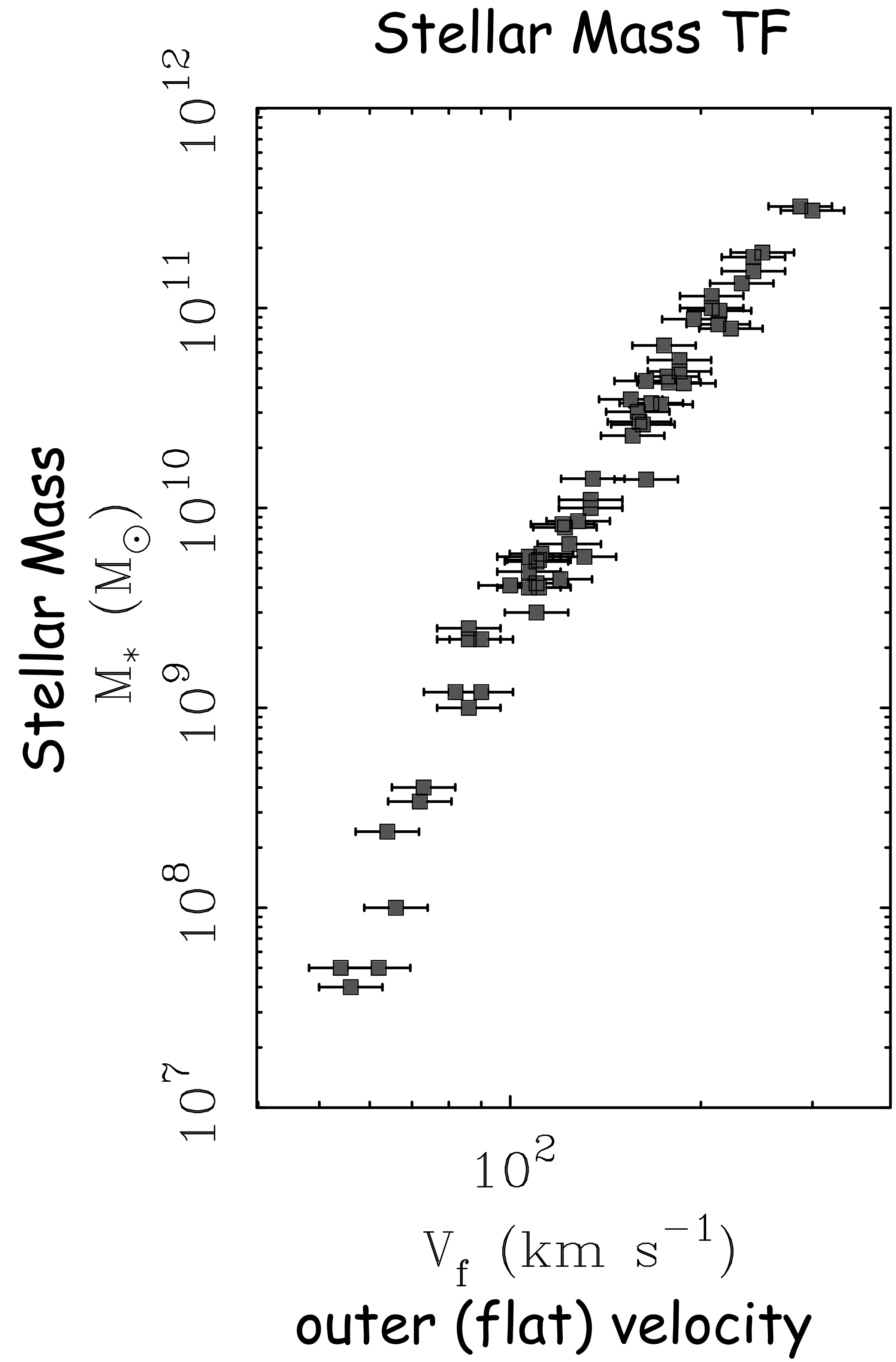
...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)

Scatter in TF relation reduced with resolved rotation curves (Verheijen 2001)



Low mass galaxies tend to fall below extrapolation of linear fit to fast rotators (Matthews, van Driel, & Gallagher 1998; Freeman 1999)

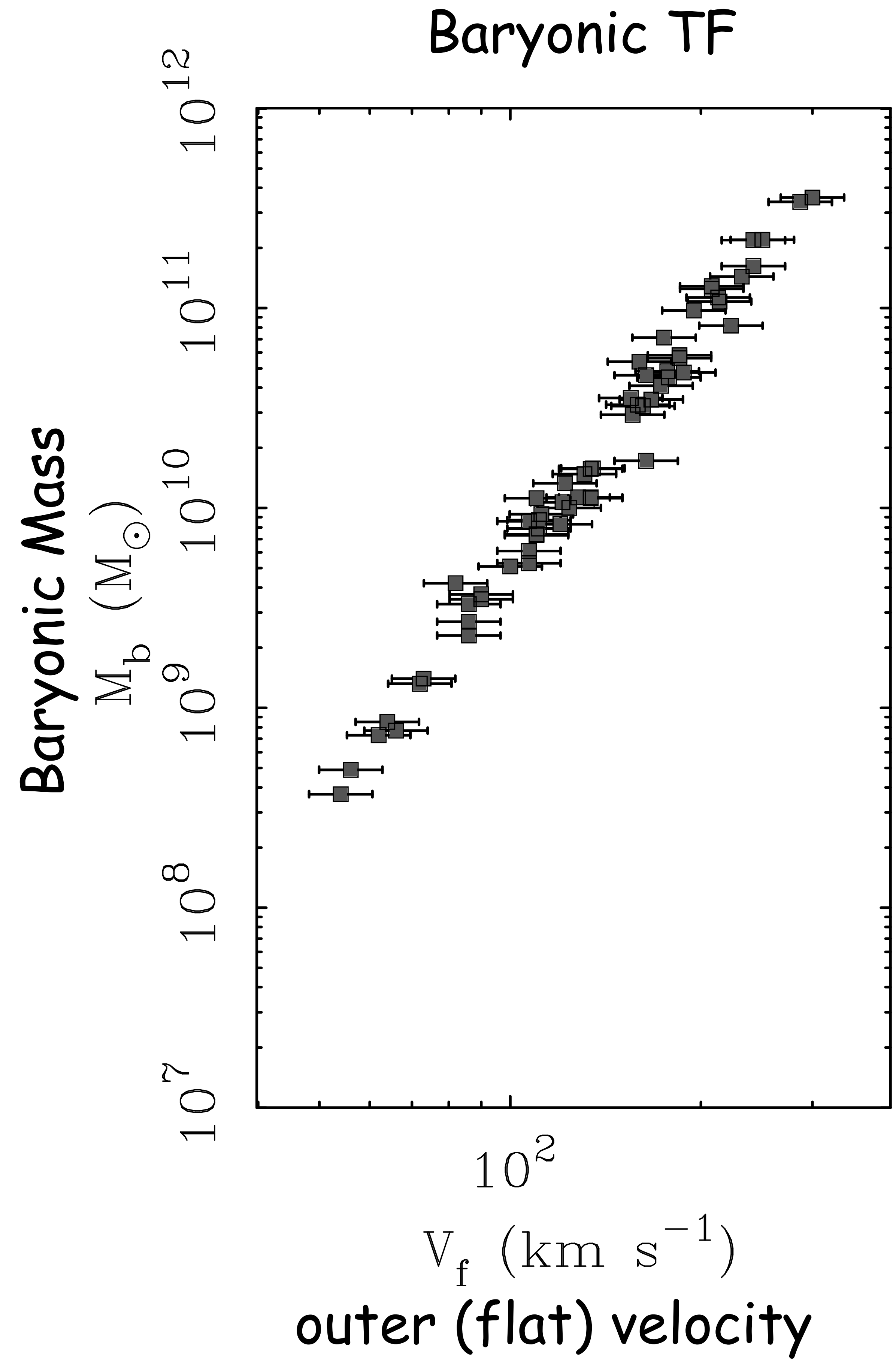
$$M_* = \left(\frac{M_*}{L}\right) L$$



Adding gas to stellar mass restores a single continuous relation for all rotators.

$$M_b = M_* + M_g$$

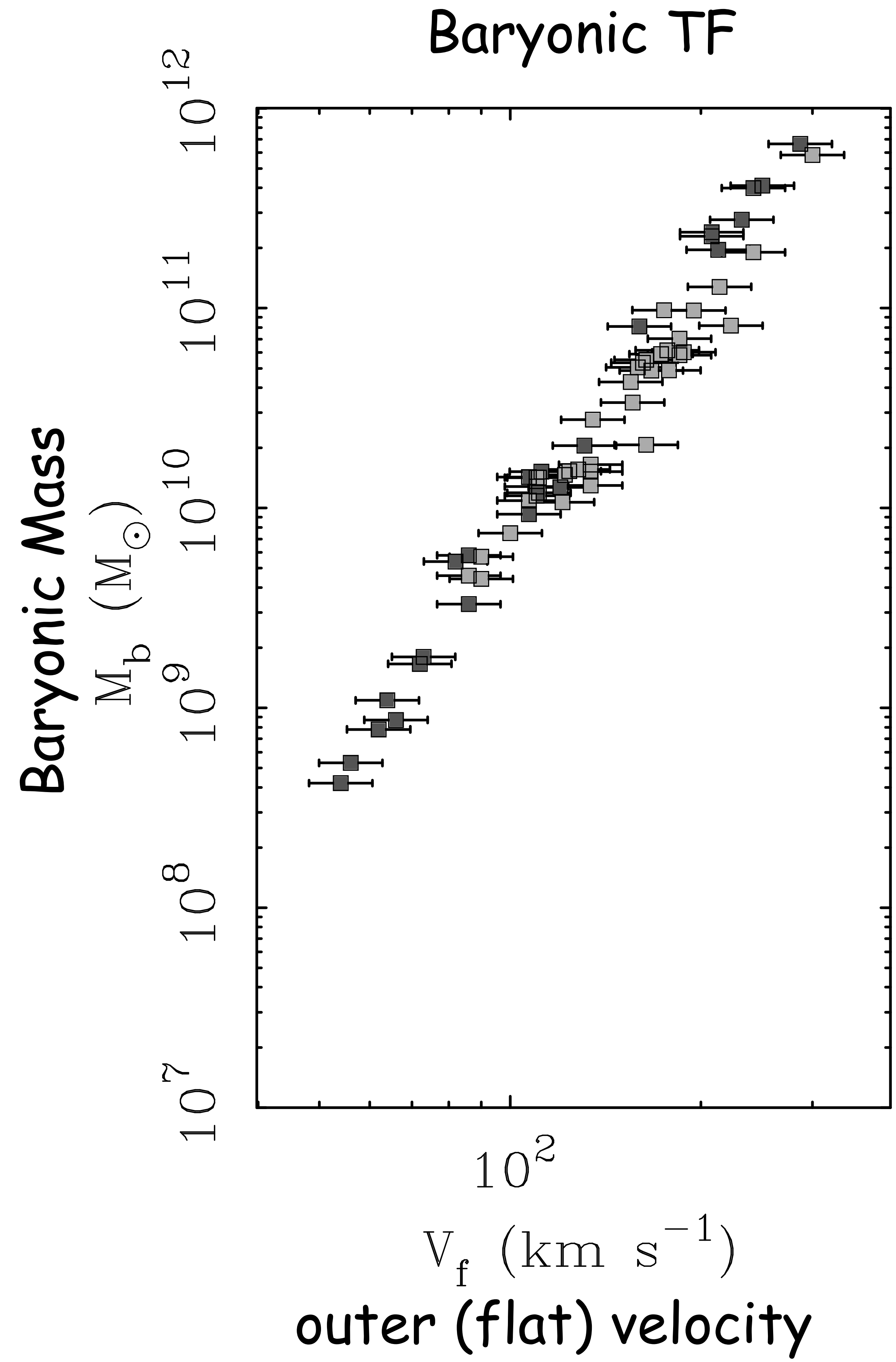
Baryonic mass is the important physical quantity. It doesn't matter whether the mass is in stars or in gas.



Twice Nominal M*/L

Now instead of a translation, the slope pivots as we vary M*/L.

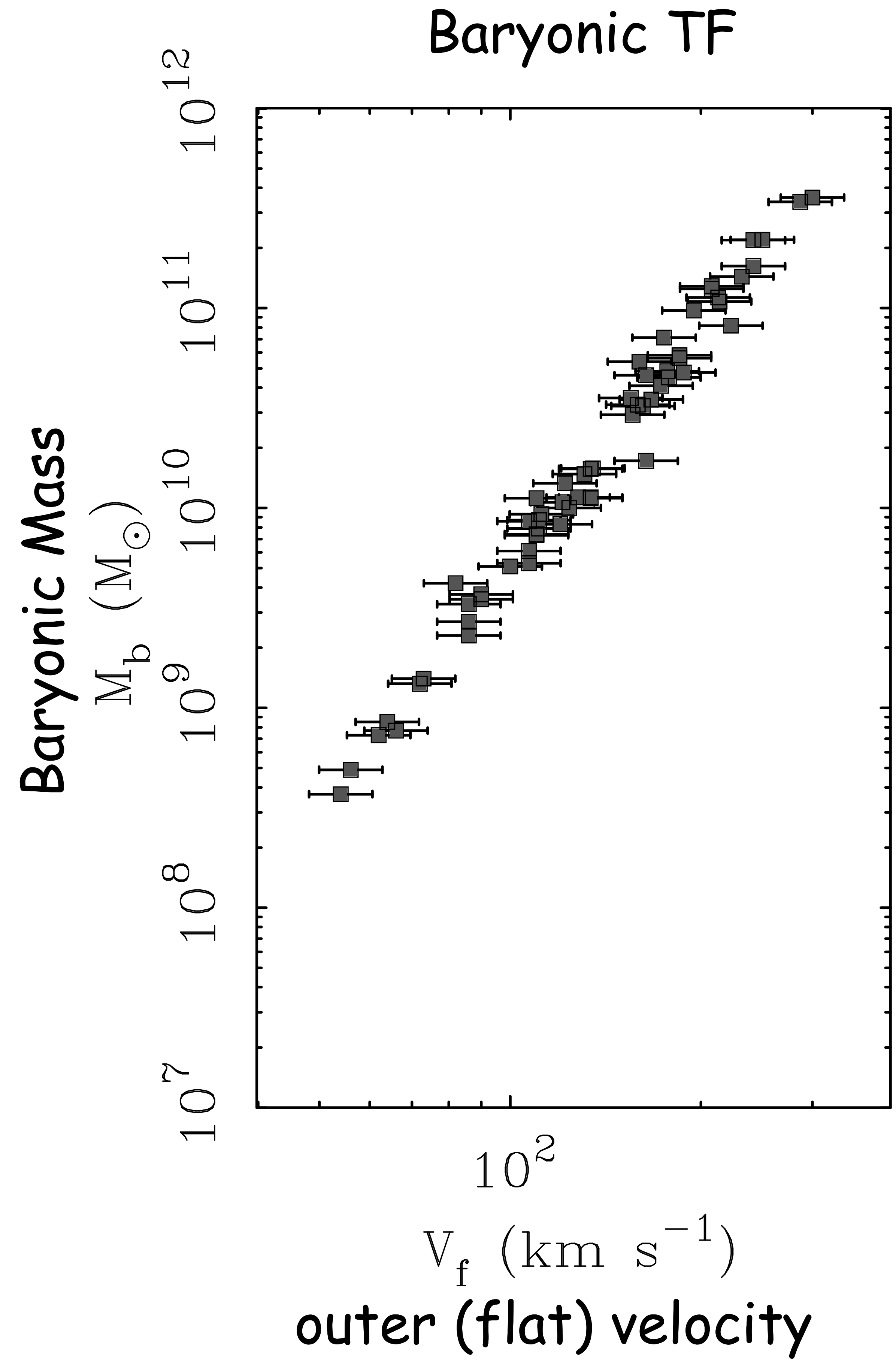
Scatter increases as we diverge from the nominal M*/L.



Nominal M*/L

Now instead of a translation, the slope pivots as we vary M*/L.

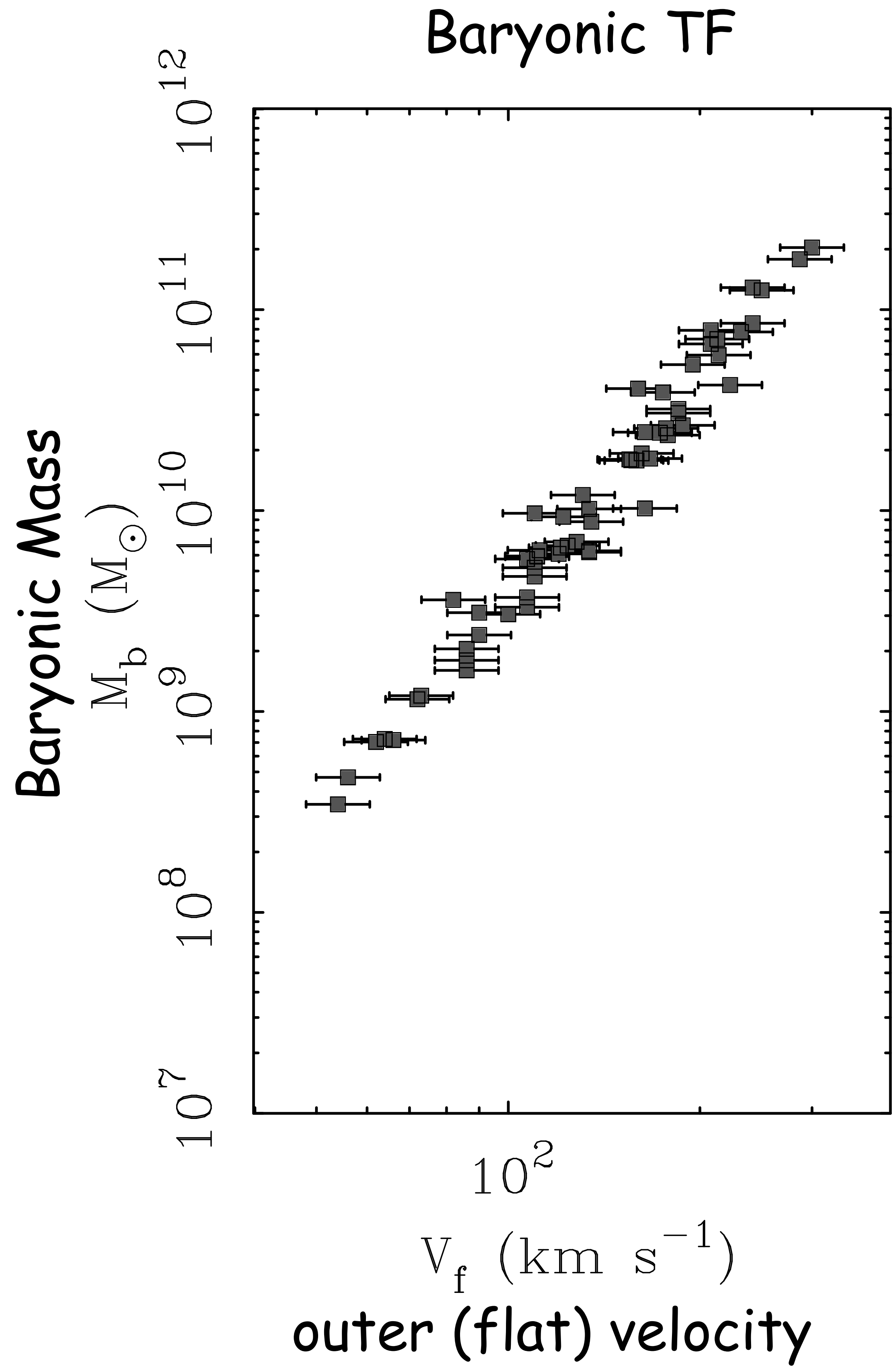
Scatter increases as we diverge from the nominal M*/L.



Half Nominal M*/L

Now instead of a translation, the slope pivots as we vary M*/L.

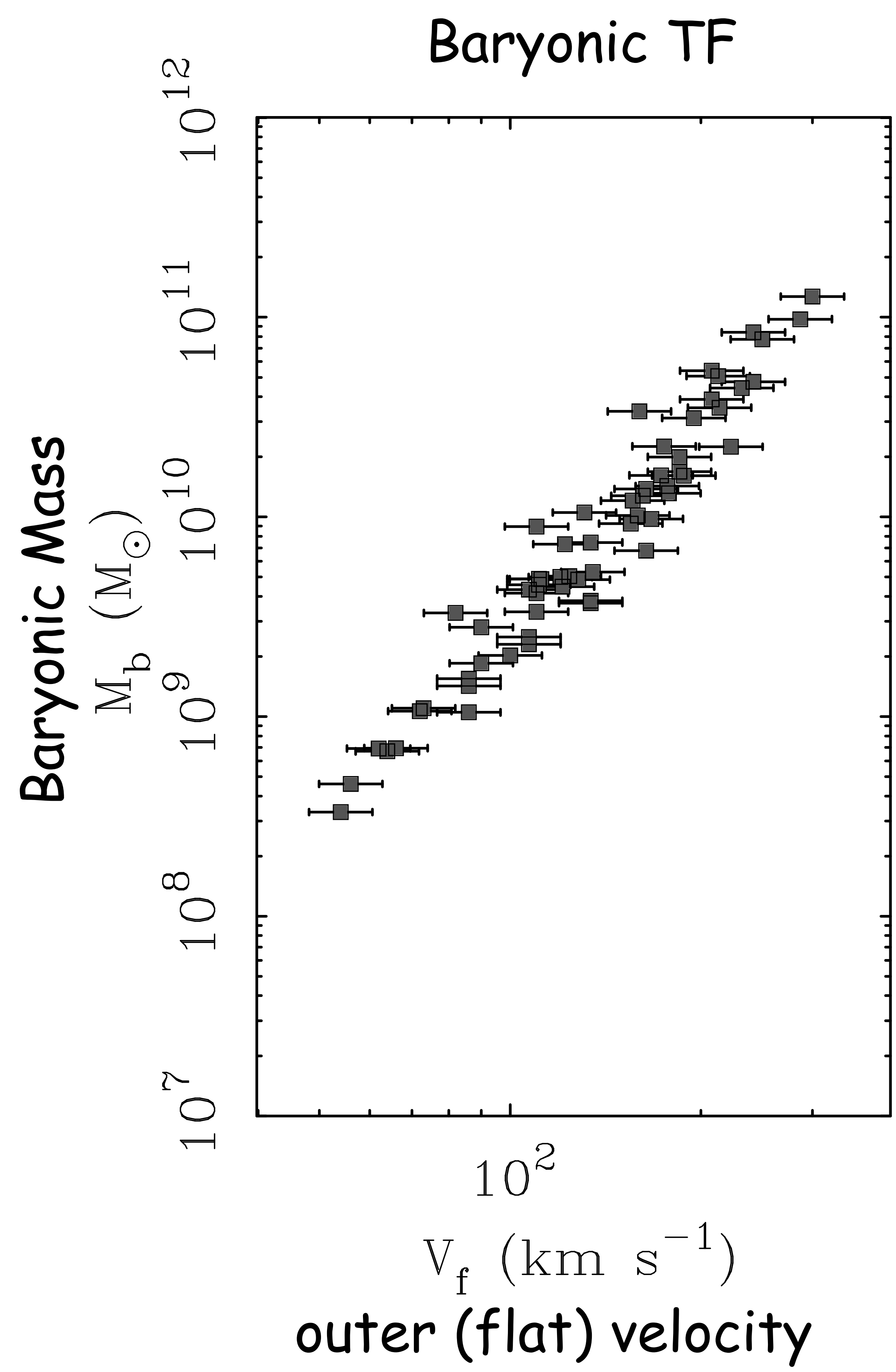
Scatter increases as we diverge from the nominal M*/L.



Quarter Nominal M*/L

Now instead of a translation, the slope pivots as we vary M*/L.

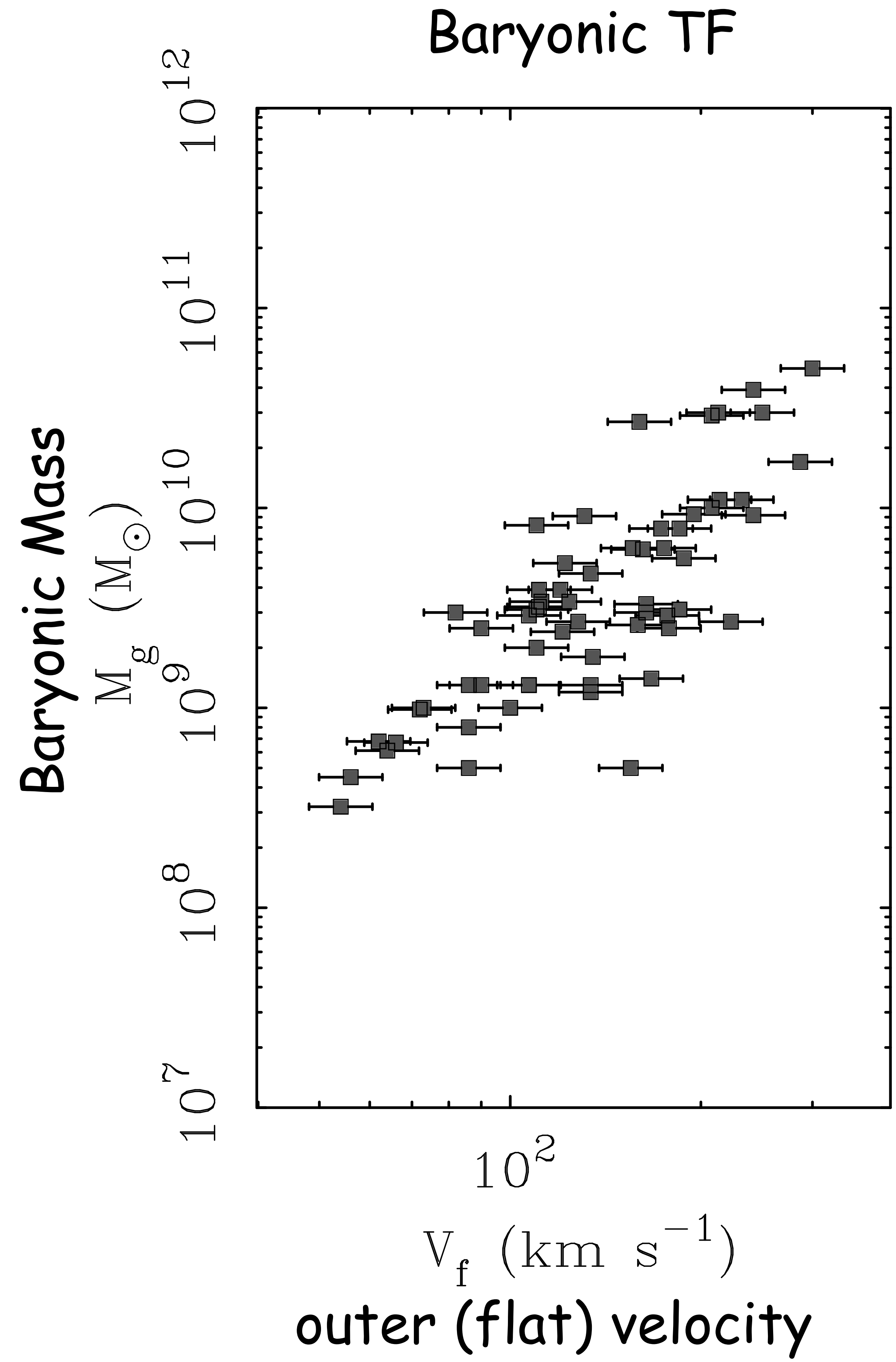
Scatter increases as we diverge from the nominal M*/L.



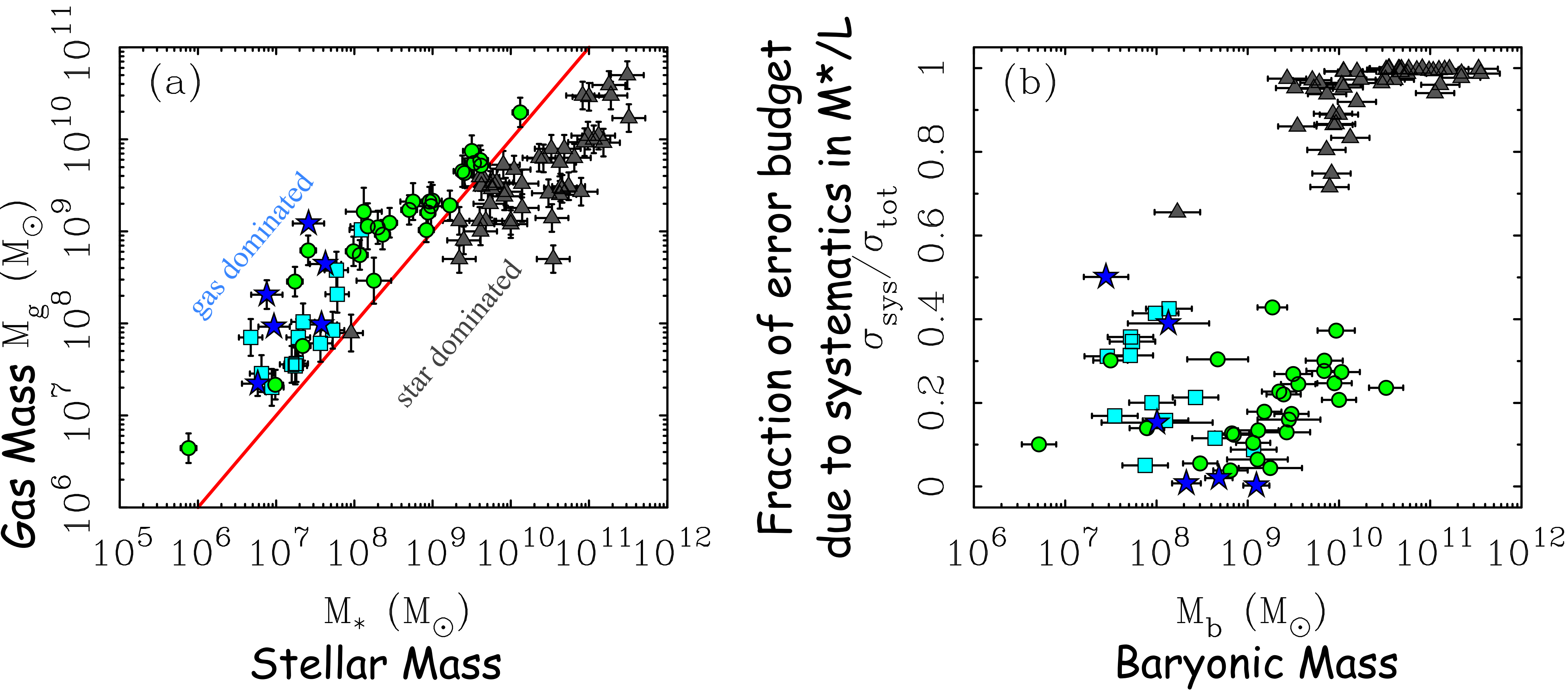
Zero M*/L

Now instead of a translation, the slope pivots as we vary M*/L.

Scatter increases as we diverge from the nominal M*/L.

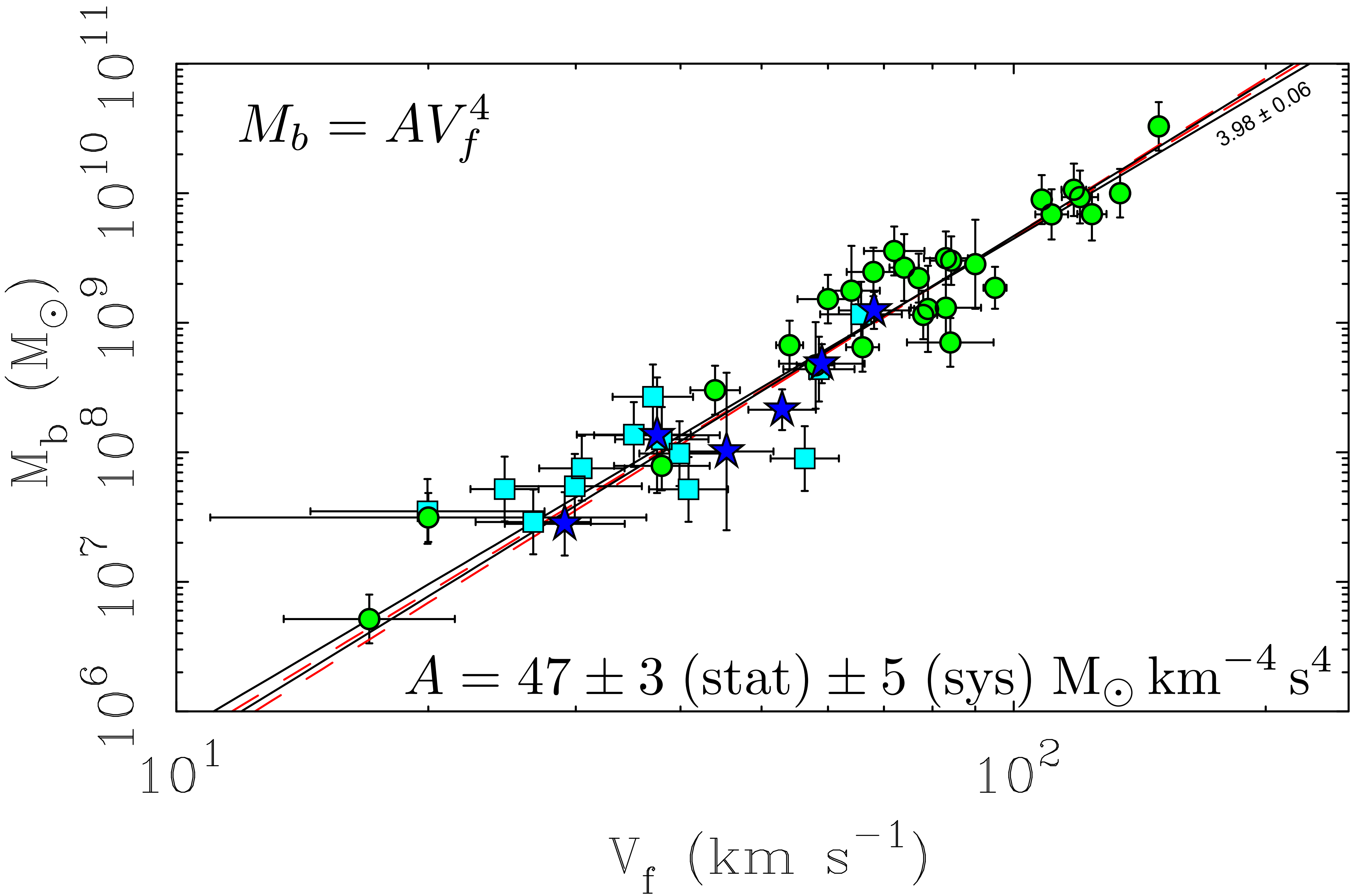


Gas dominated galaxies provide an absolute calibration of the mass scale.

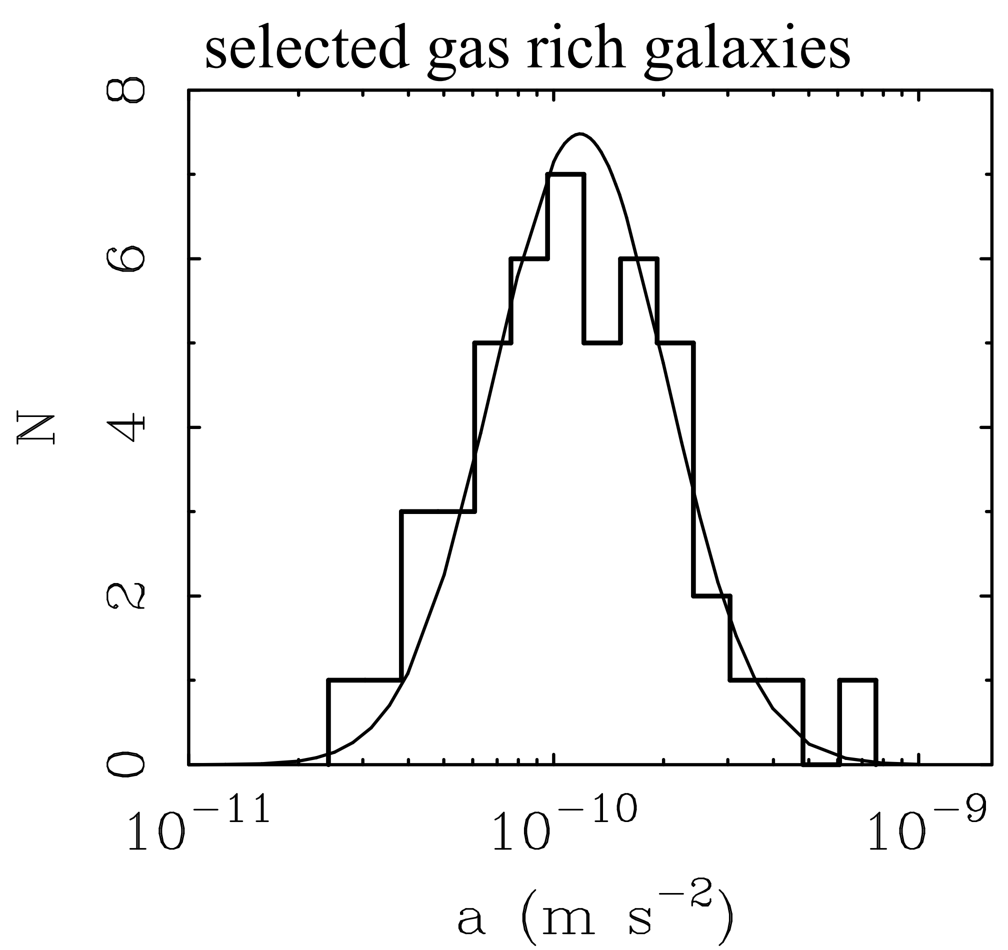


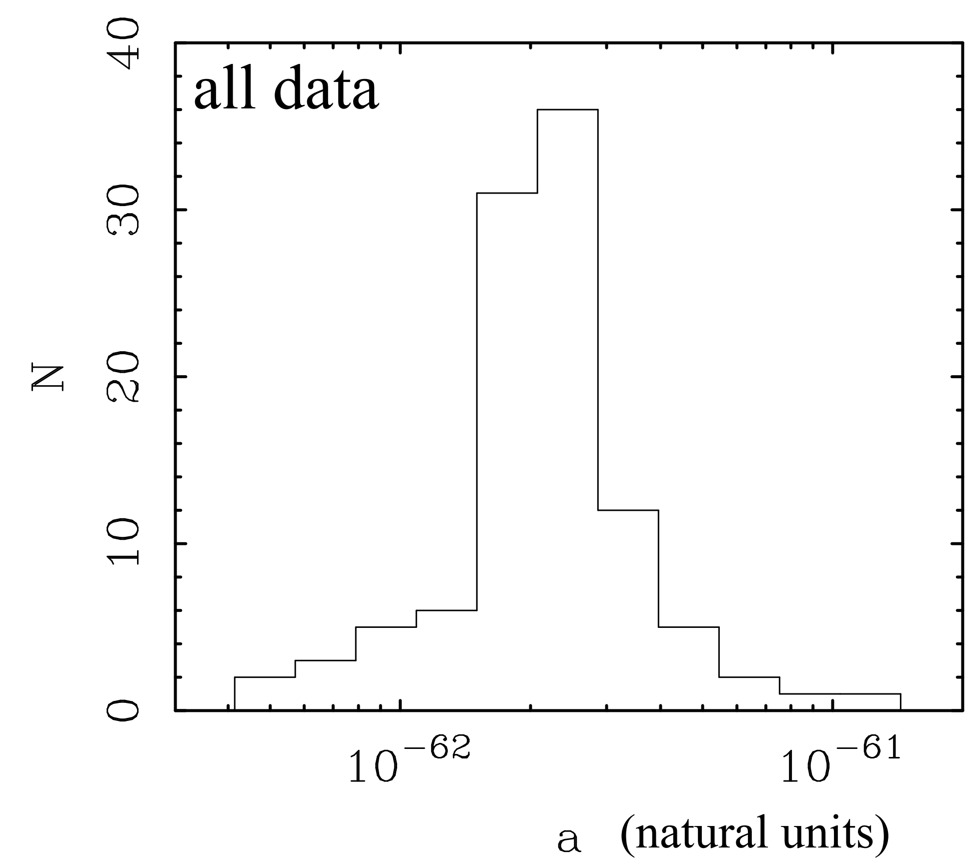
Systematic errors in M*/L no longer dominate the error budget for galaxies with Mg > M*.

Gas Rich Galaxy Baryonic Tully-Fisher relation
(Stark et al 2009; Trachternach et al 2009; McGaugh 2011, 2012)



The data specify a particular acceleration scale: $\mathrm{a} = \frac{V_f^4}{G M_b}$

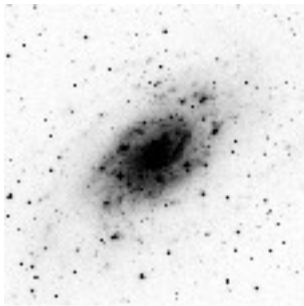




histogram: data
line: distribution expected from observational uncertainties.

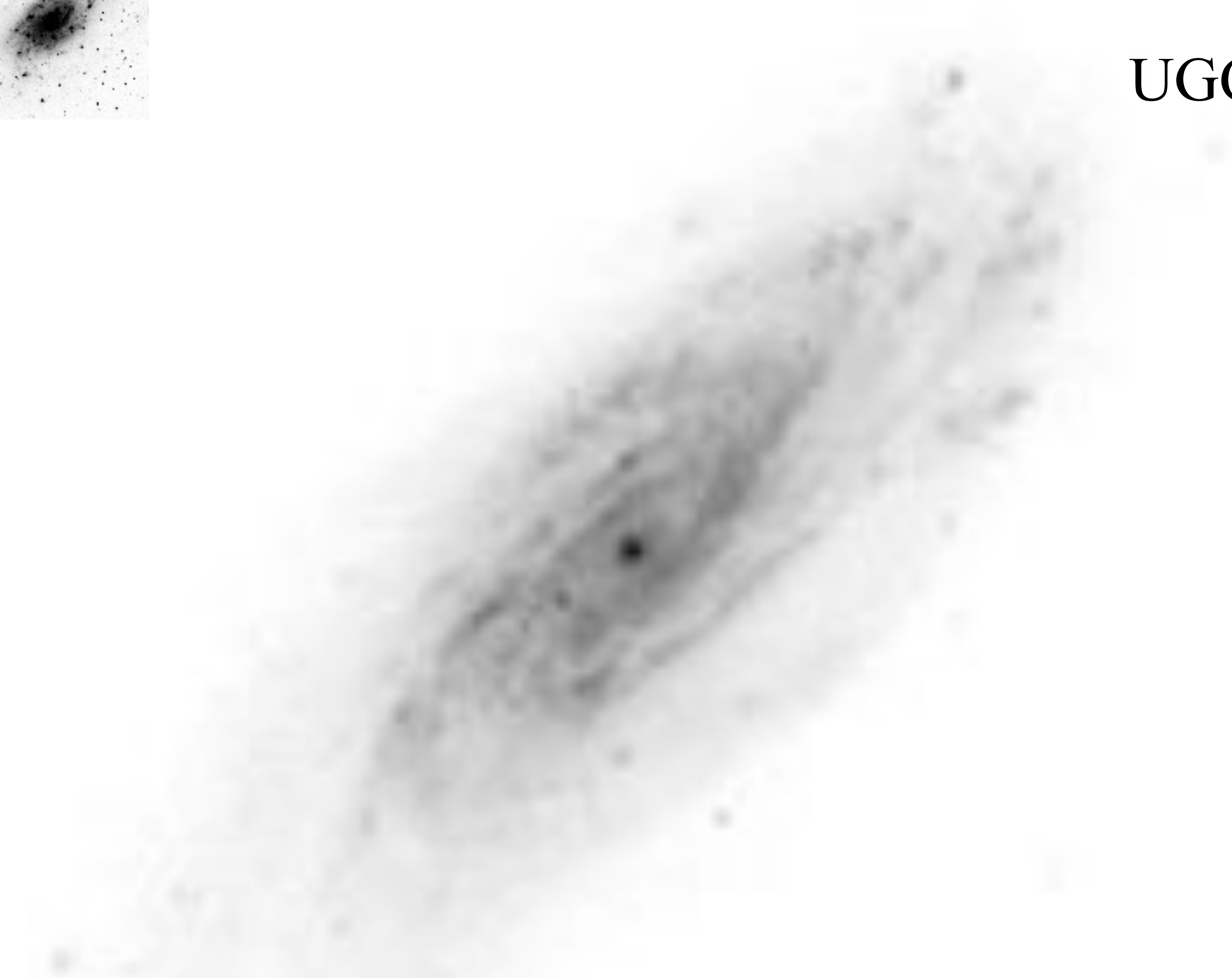
$$a_\dagger \approx \Lambda^{1/2}$$

The data are consistent with zero intrinsic scatter.



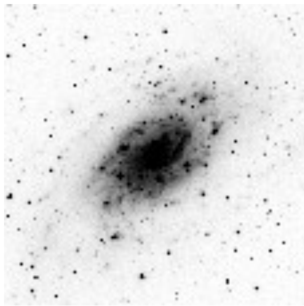
NGC 2403

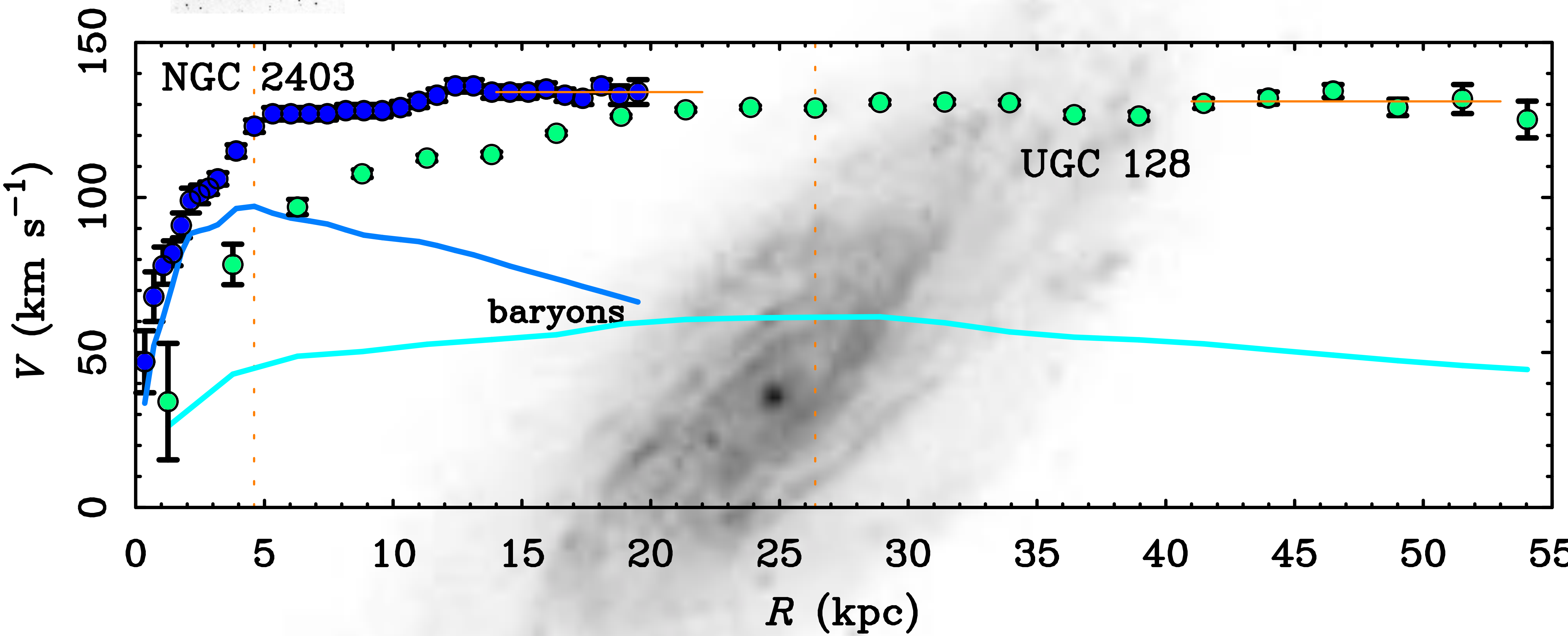
UGC 128



The BTFR is just the zeroth moment, as it were -
total baryonic mass vs. characteristic circular velocity.
There is more information in the distribution of mass.

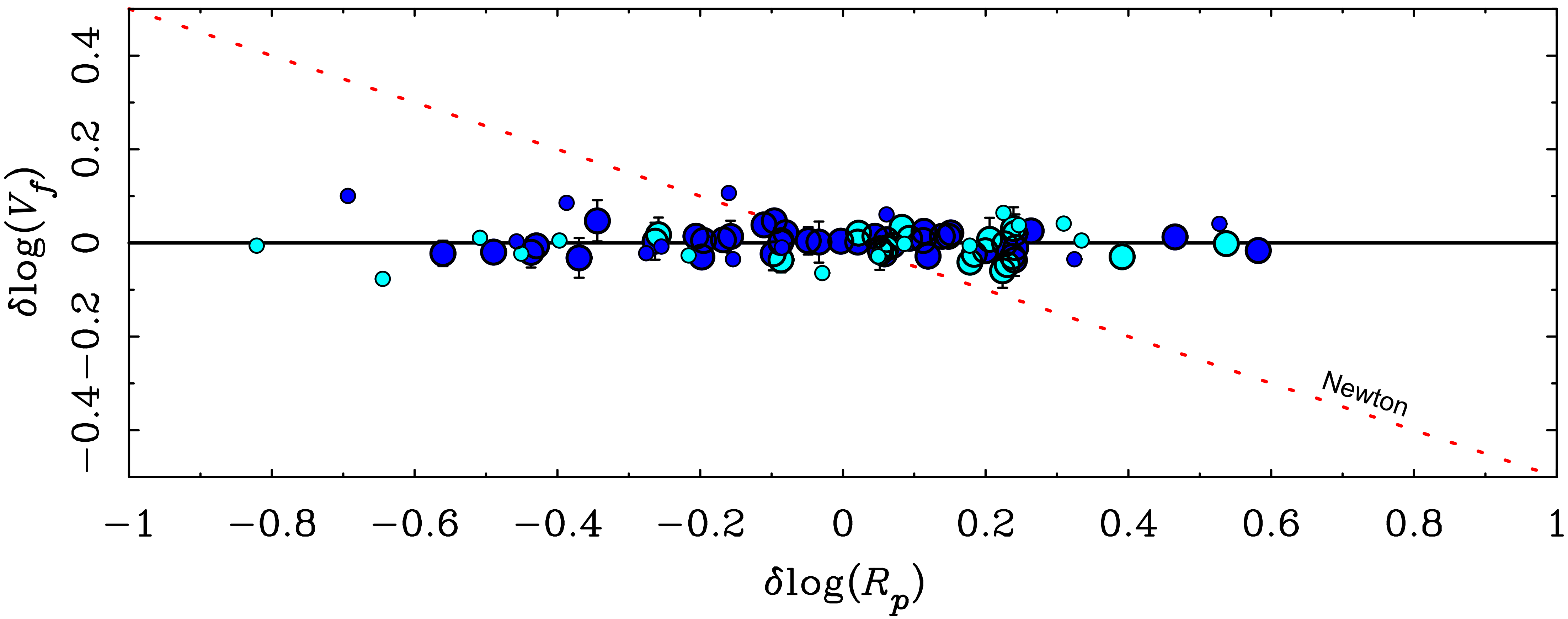
No residuals from TF with size or surface density





Same (M,V) but very different size and surface density

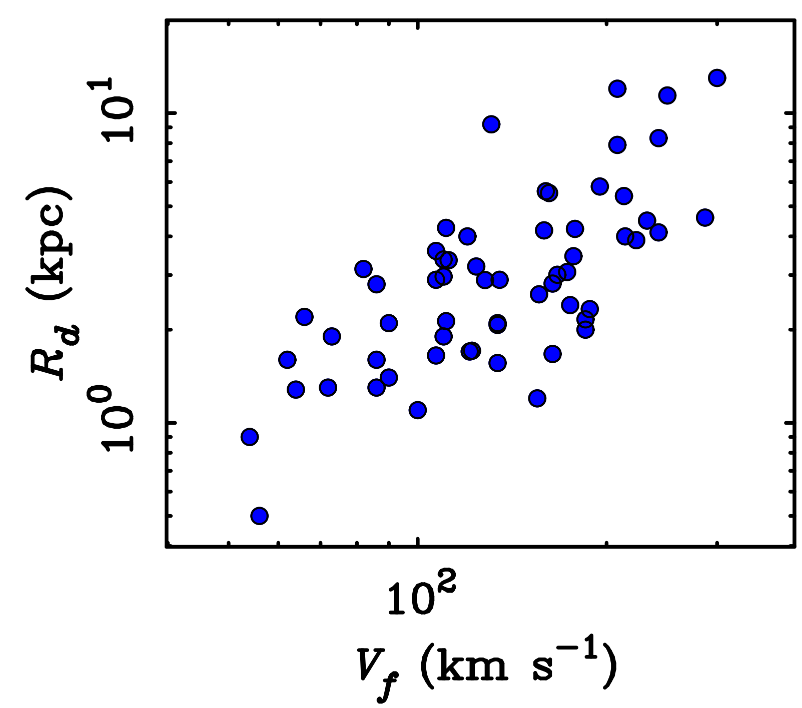
which is strange, since $V^2 = \frac{GM}{R}$



No residuals from TF with size or surface density for disks

$$V^2 = \frac{GM}{R} \rightarrow \frac{\delta\log(V)}{\delta\log(R)} = -\frac{1}{2}$$ expected slope (dotted line)

Note: large range in size at a given mass or velocity



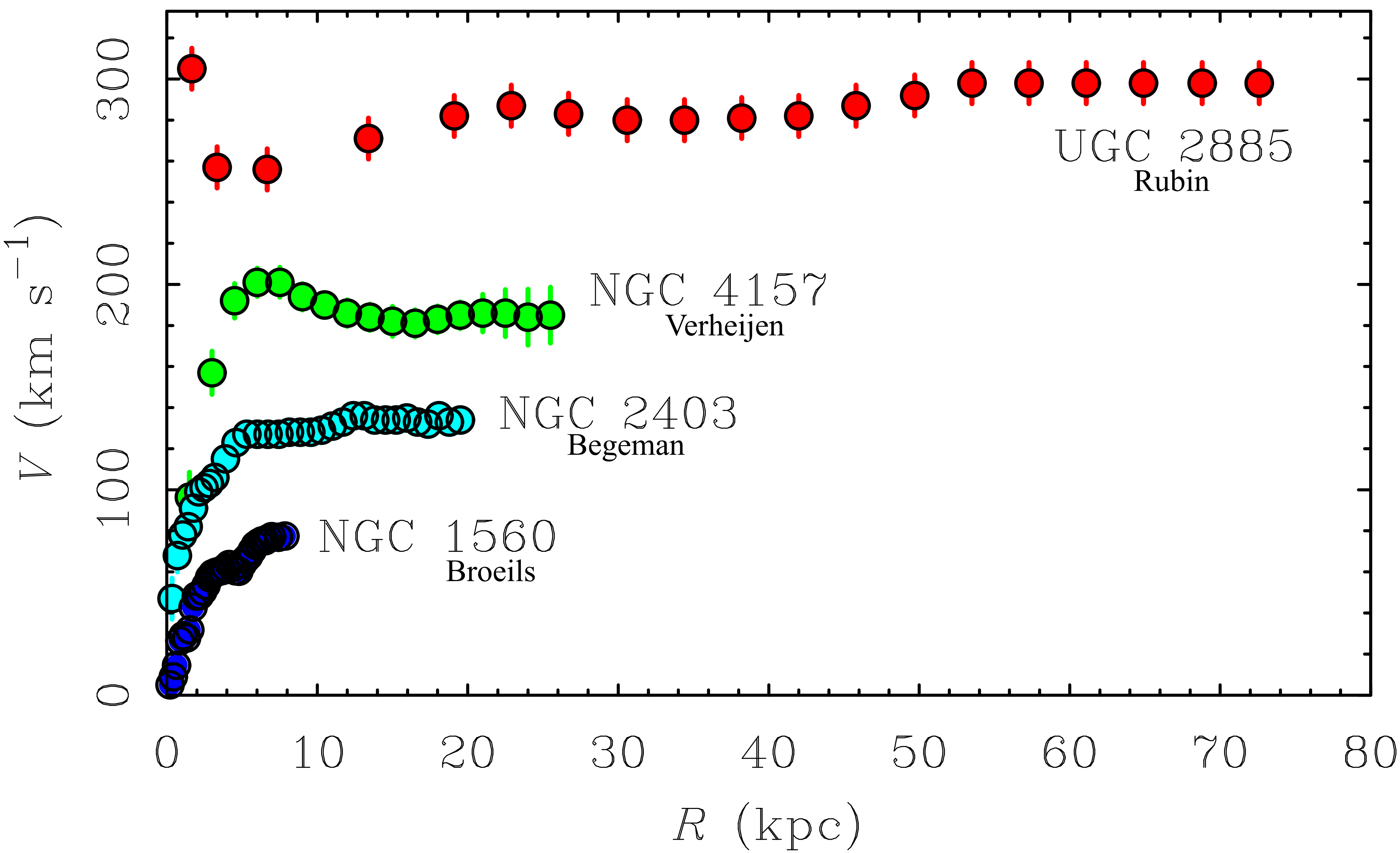
A contradiction to conventional dynamics?

2nd Law: the Baryonic Tully-Fisher Relation

- Fundamentally a relation between the baryonic mass of a galaxy and its rotation velocity
  - $M_b = M_* + M_g = 47 V_f^4$ (McGaugh 2012)
  - doesn't matter if it is stars or gas
- Intrinsic scatter negligibly small
  - Can mostly be accounted for by the expected variation in stellar M*/L
- Physical basis of the relation remains unclear

Relation has real physical units if slope has integer value - Slope appears to be 4 *if* Vflat is used.

Rotation curve amplitude and shape correlate with luminosity



Universal Rotation curve (Persic & Salucci 1996)

$V(R/R_{opt})$ correlates with Luminosity.

NOT just V(R) - must be normalized by optical size R_{opt}

Universal galaxy rotation and dark matter 33

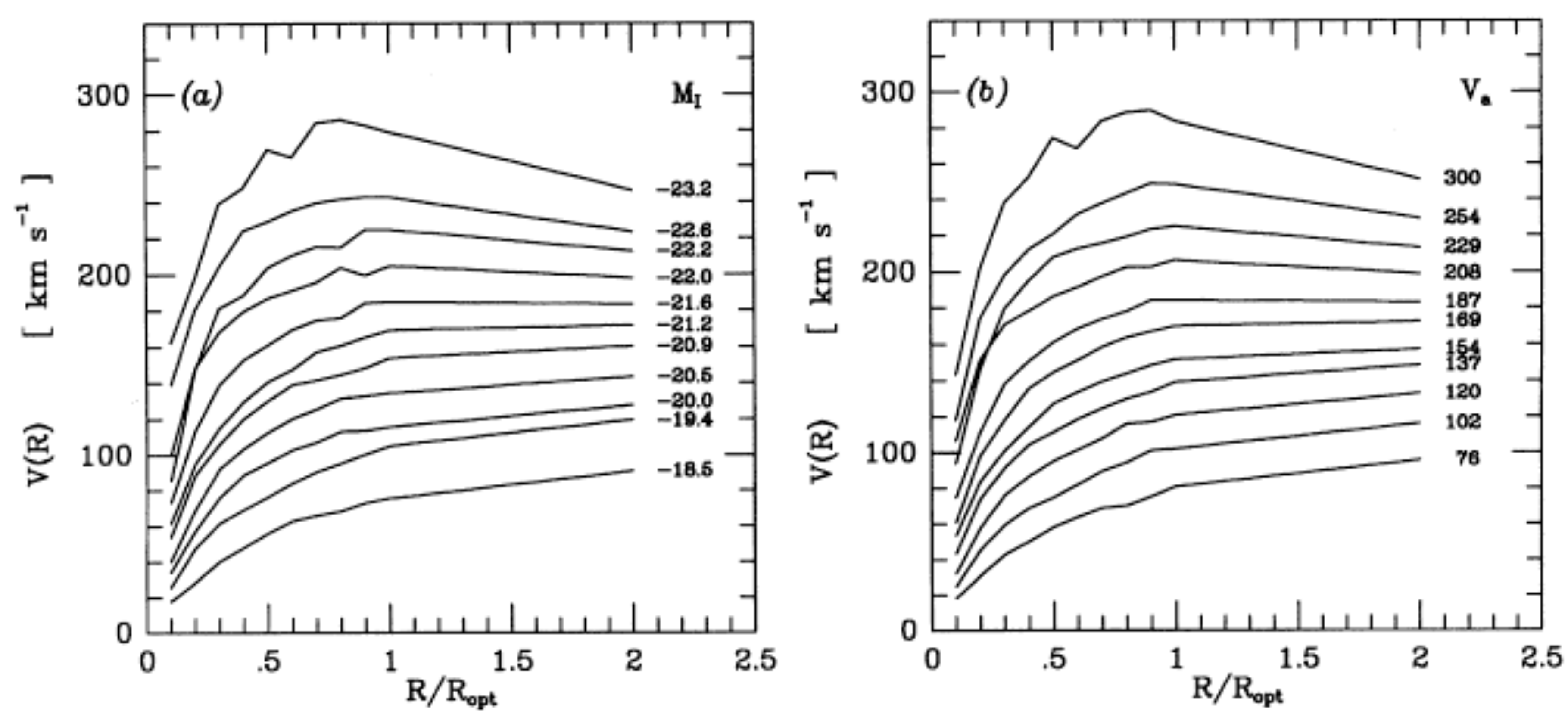
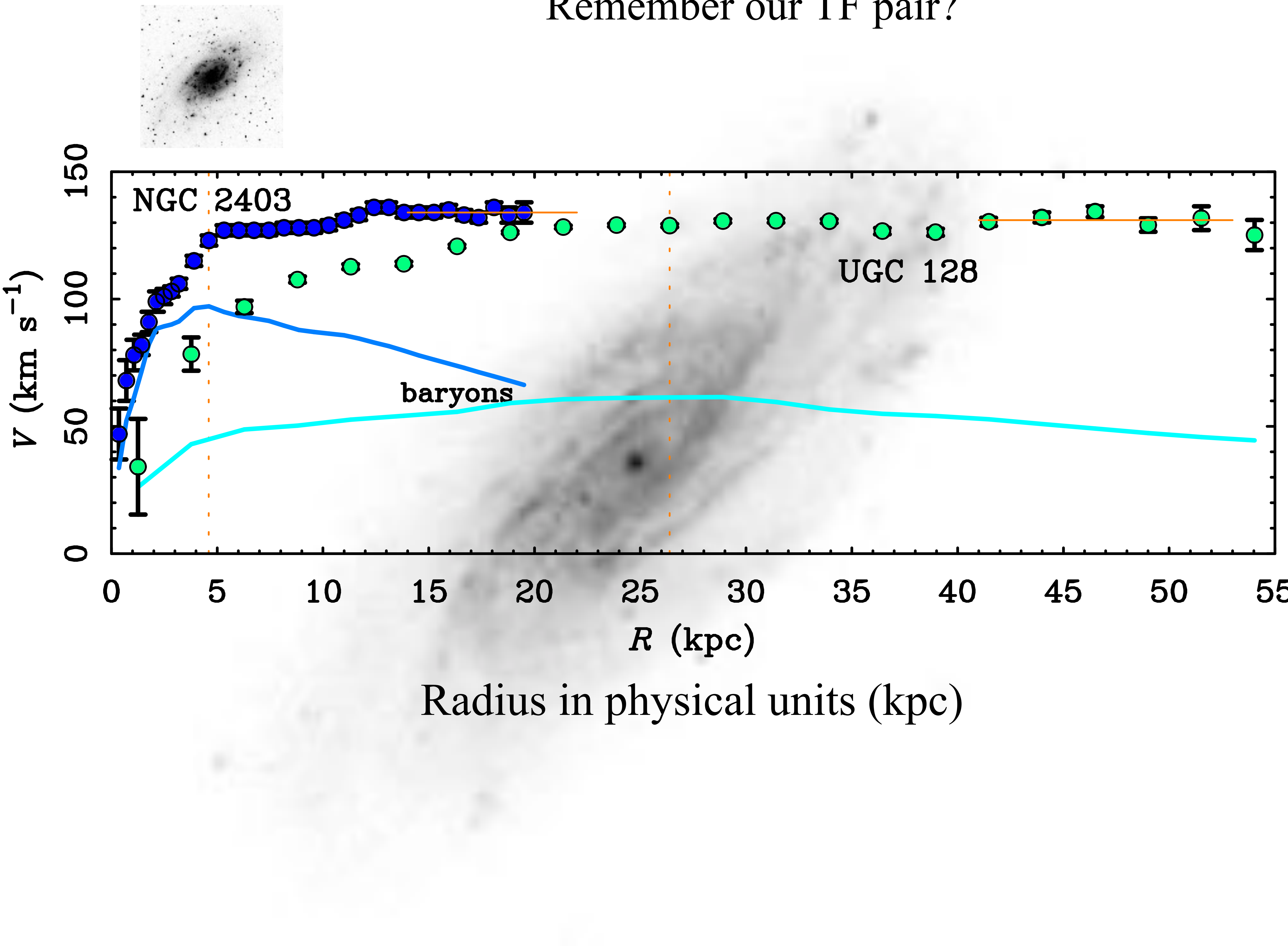
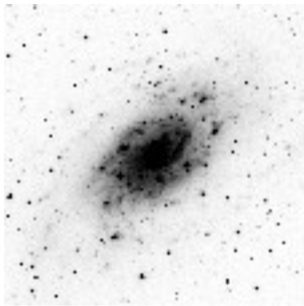


Figure 4. The universal rotation curve of spiral galaxies. Radii are in units of R_{opt}.

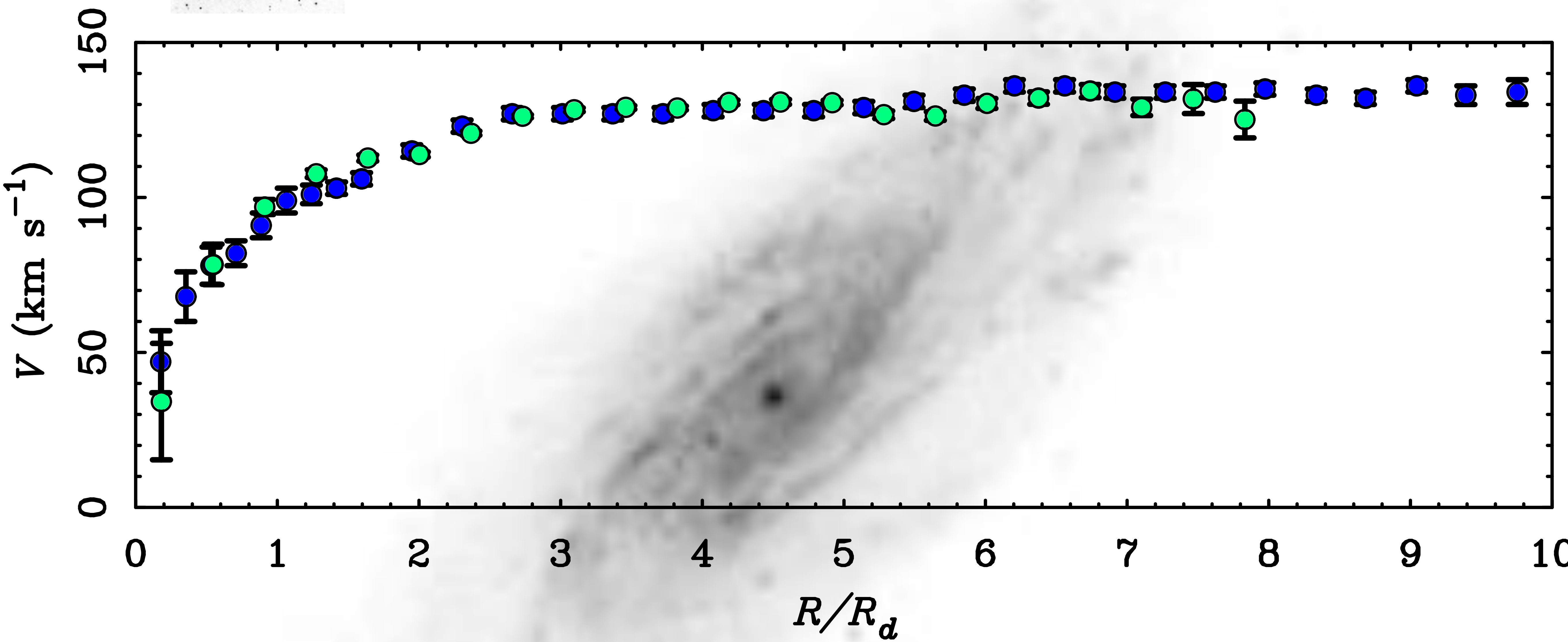
Remember our TF pair?



Radius in physical units (kpc)



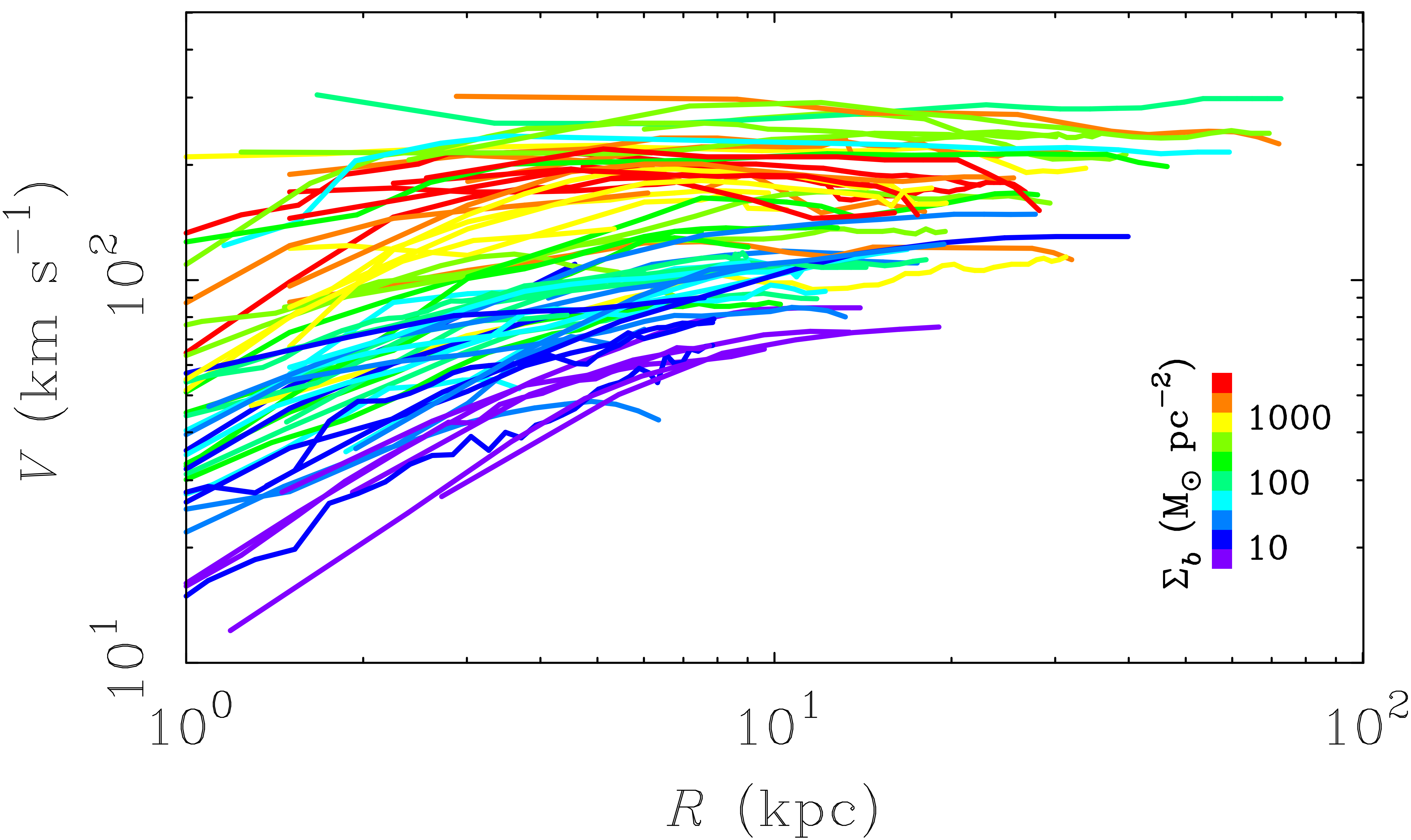
The dynamics knows about the distribution of baryons, not just their total mass



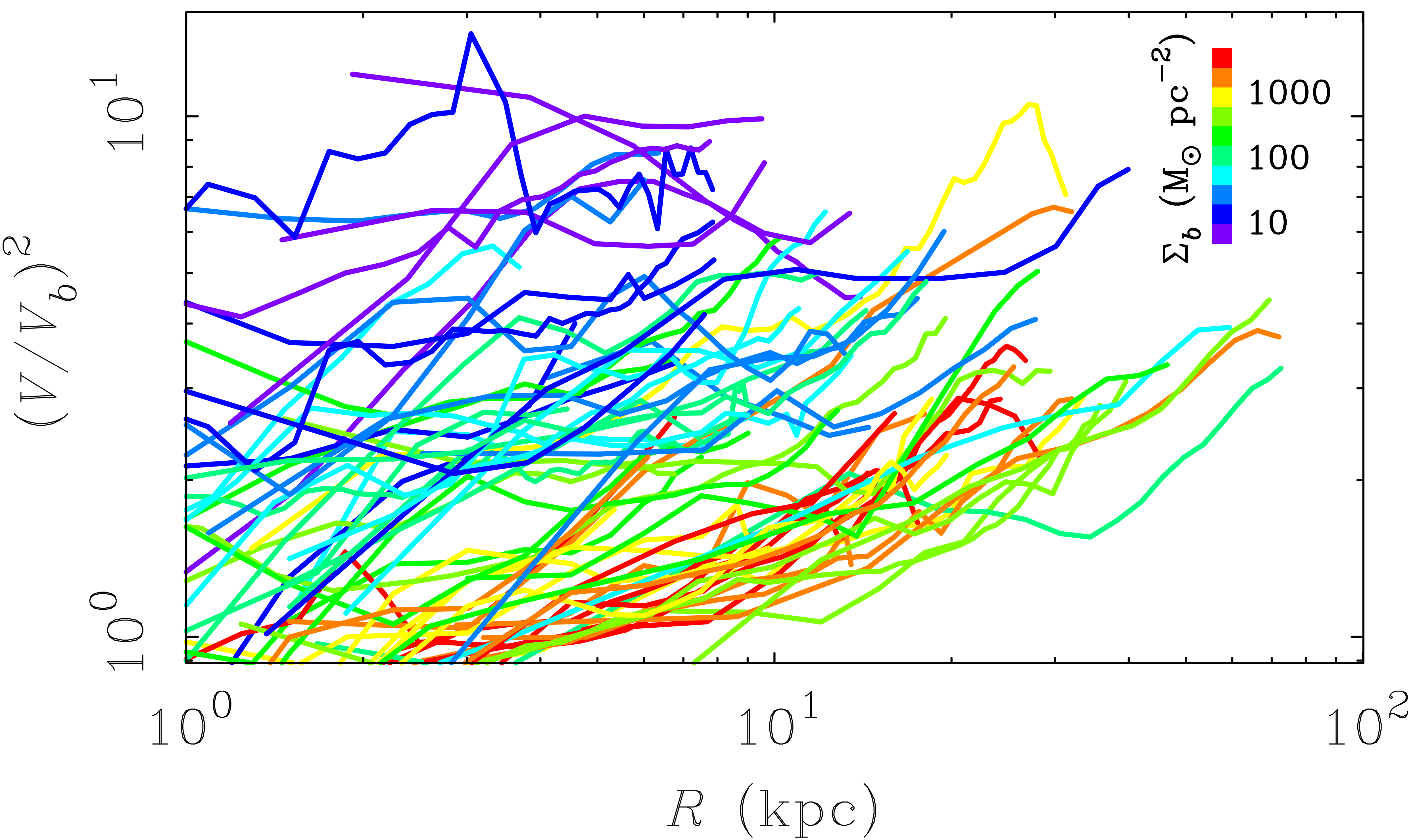
Radius normalized by size of disk.

Persic & Salucci 1996
de Blok & McGaugh 1996
Tully & Verheijen (1998)
Nordermeer & Verheijen (2007) [URC nor quite right formulation]
Swaters et al. (2009)

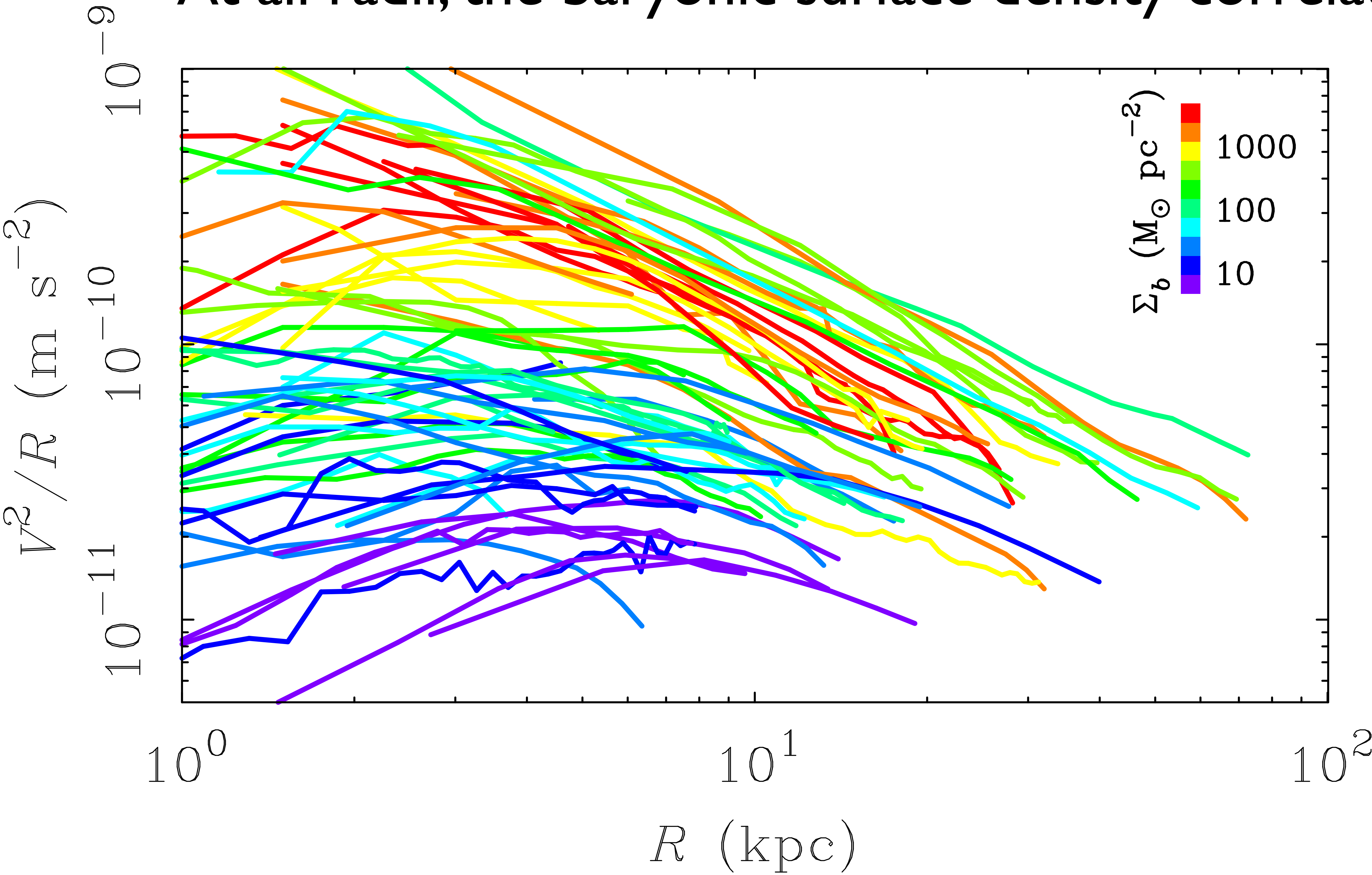
Rotation curve shapes depend on luminosity and surface density



The mass discrepancy sets in sooner and is more severe in LSB galaxies



At all radii, the baryonic surface density correlates

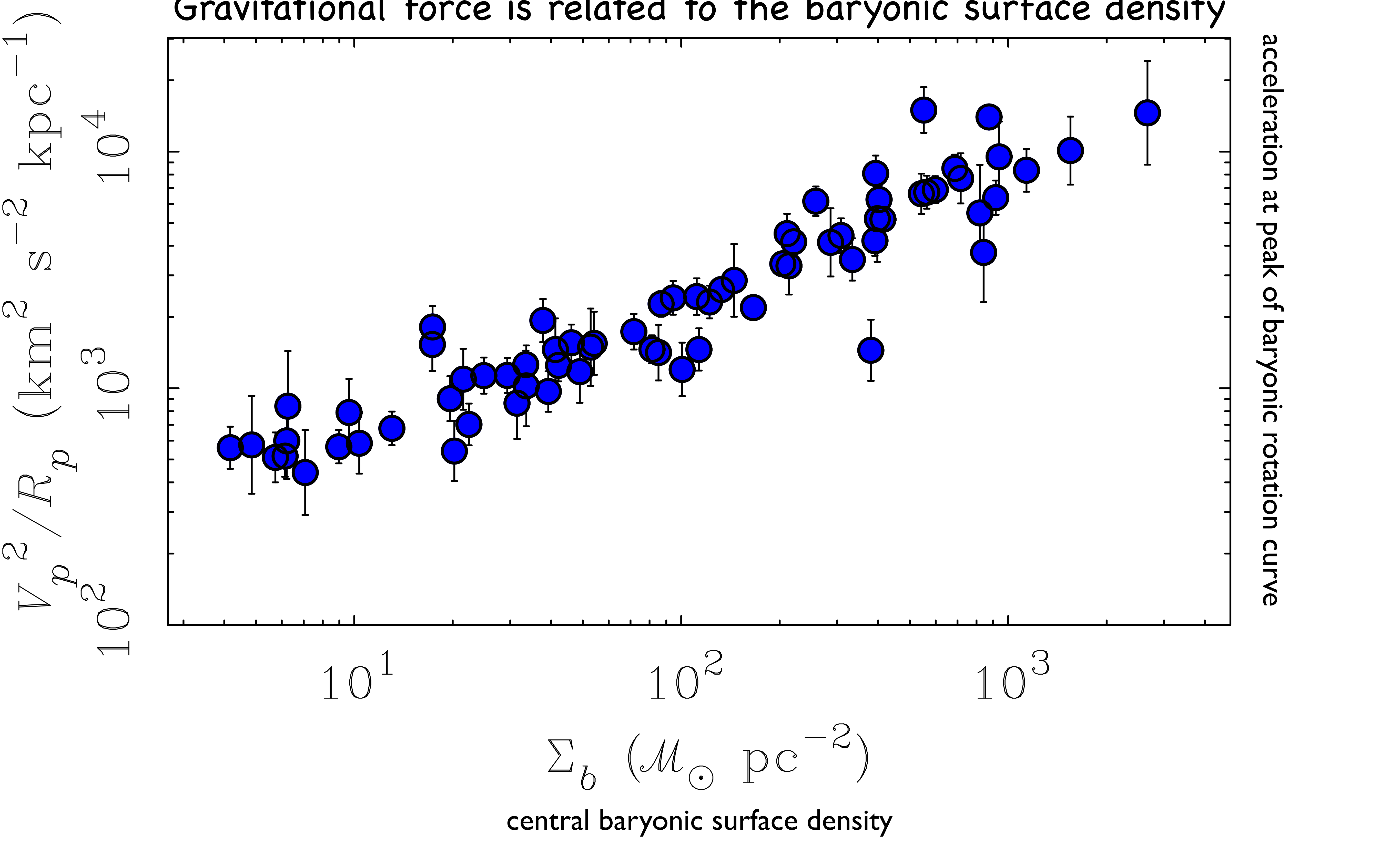


with the acceleration (gravitational force per unit mass)

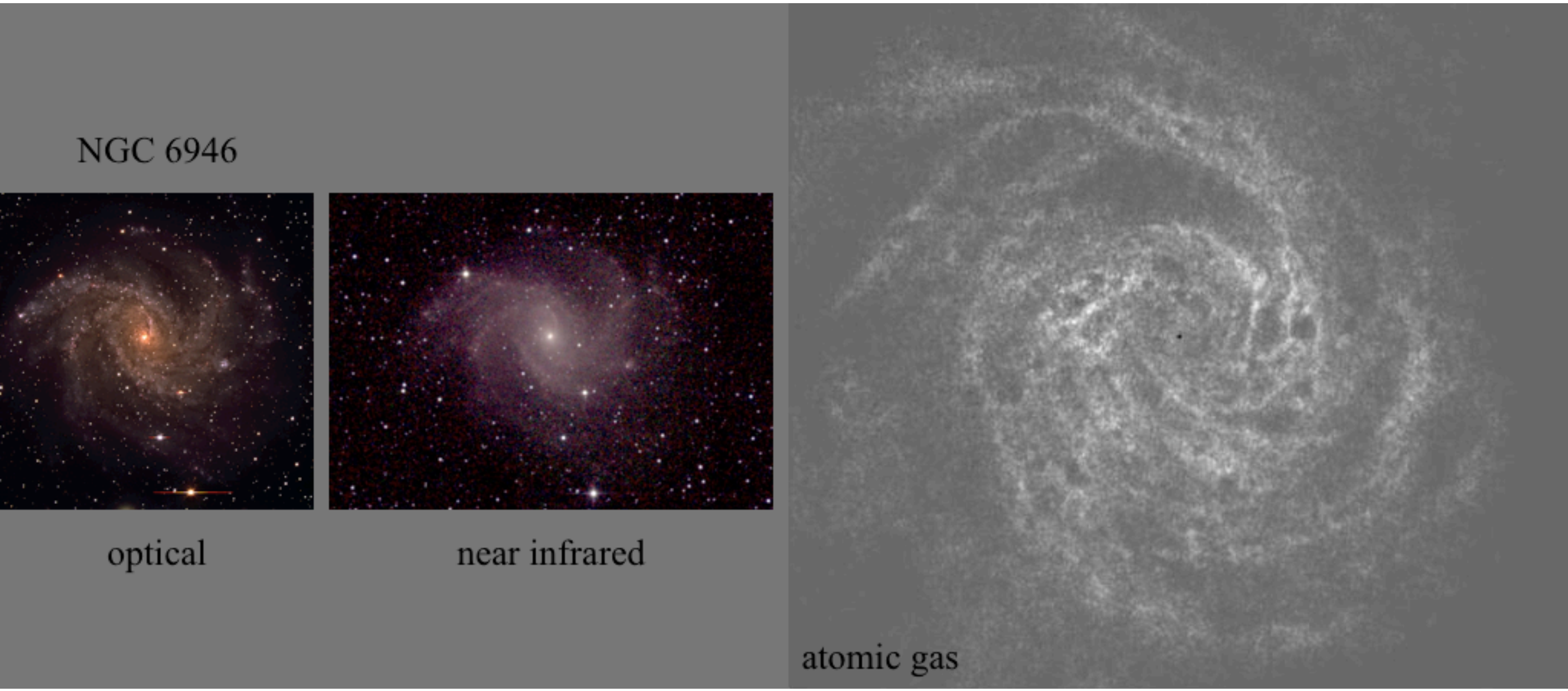
Just looking at the peak radius

$$a \sim \Sigma_b^{1/2}$$

Gravitational force is related to the baryonic surface density

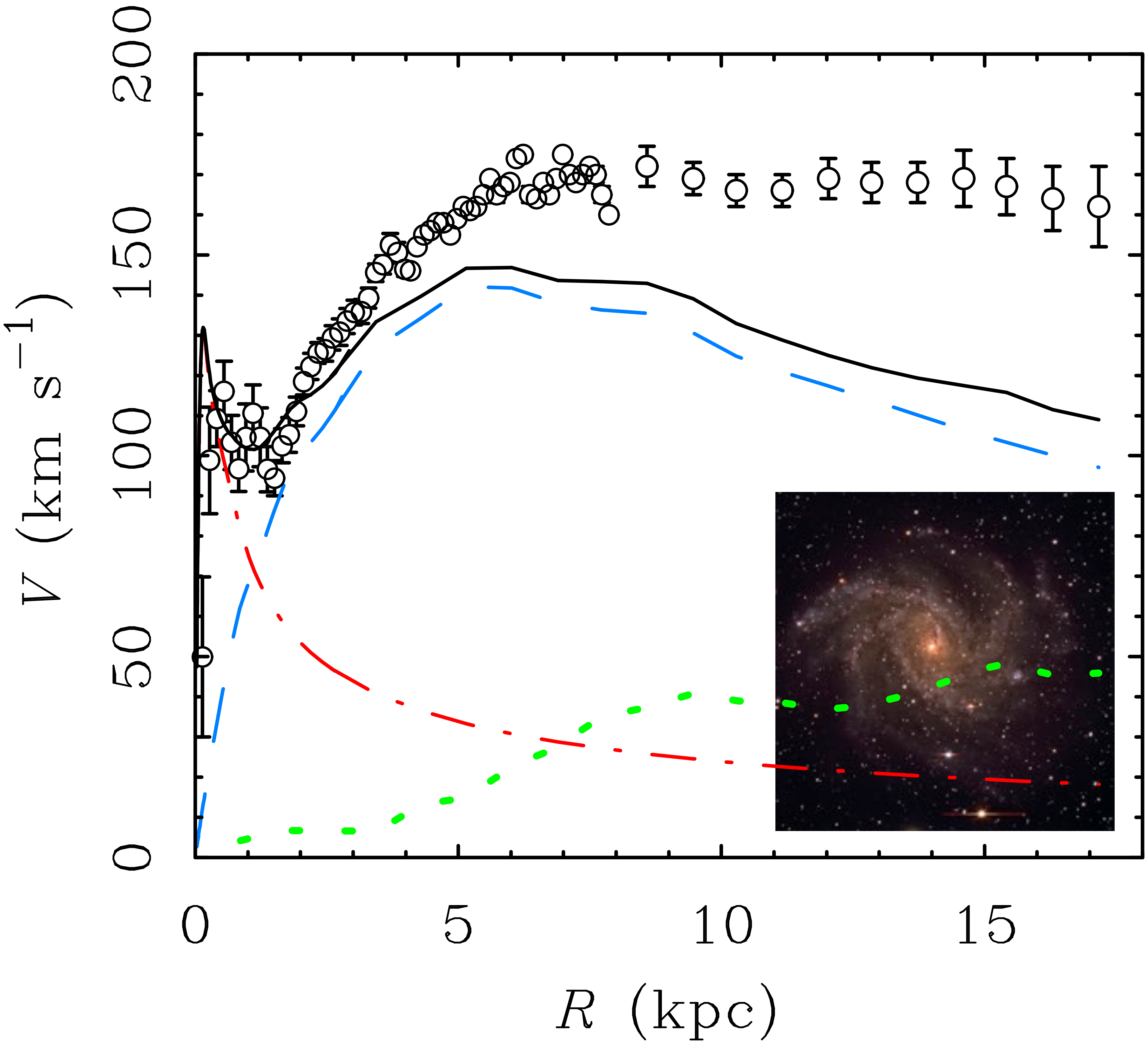


Devil in the details...



Renzo's Rule: (2004 IAU; 1995 private communication)
"When you see a feature in the light, you see a corresponding feature in the rotation curve."

In NGC 6946, a tiny bulge (just 4% of the total light) leaves a distinctive mark.



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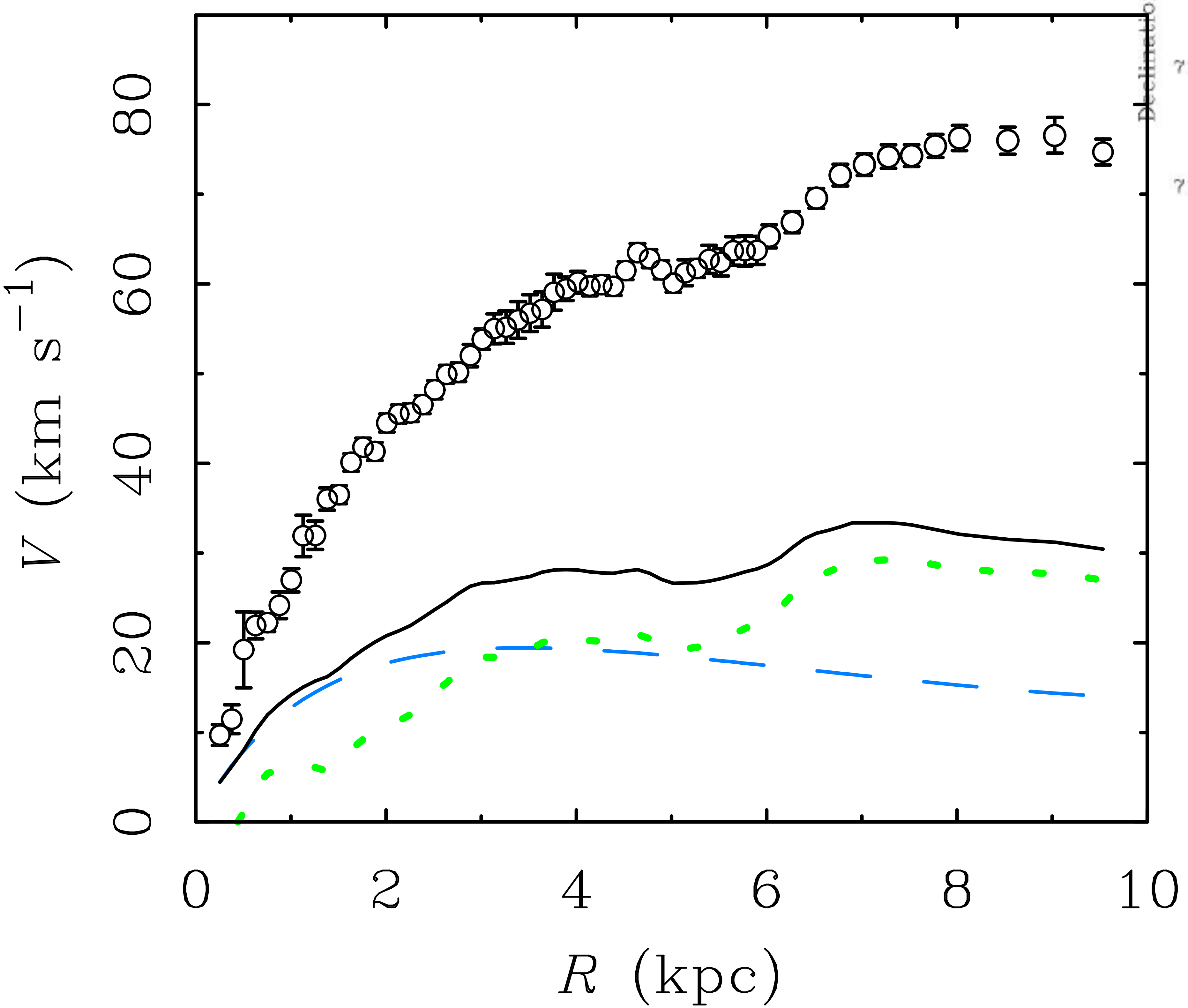
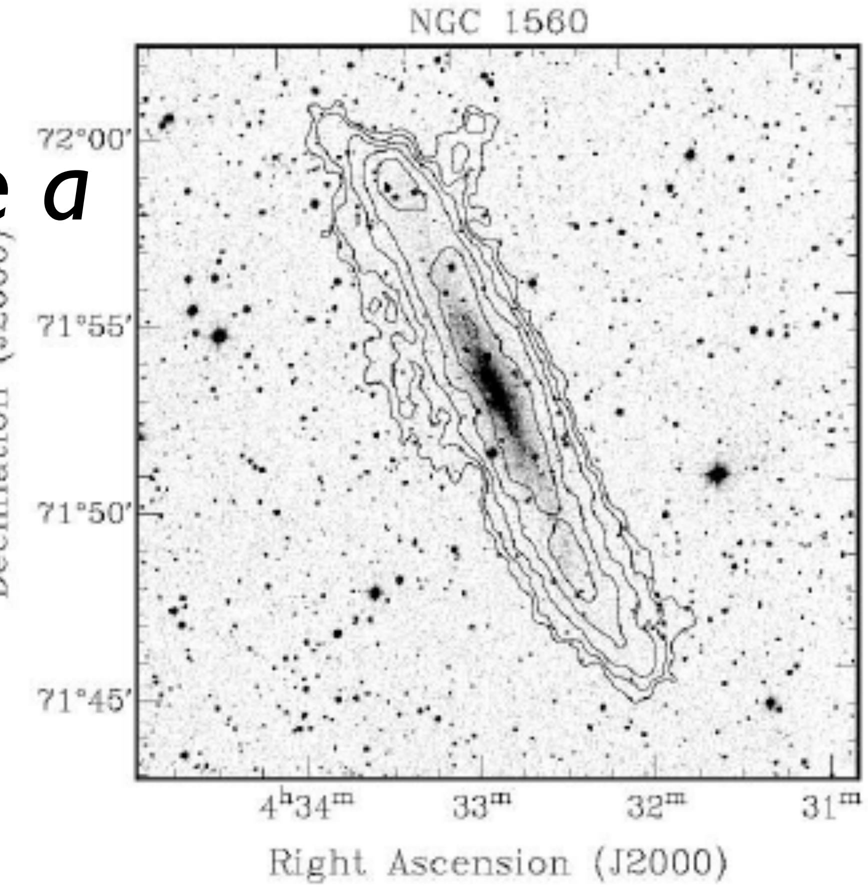




Gentile et al. (2010)

In NGC 1560, a marked feature in the gas is reflected in the kinematics, even though it accounts for little of the dynamical mass.

Renzo’s Rule:

“When you see a feature in the light, you see a corresponding feature in the rotation curve.”

The Mass Discrepancy correlates with acceleration and baryonic surface density

Renzo's Rule:

"When you see a feature in the light, you see a corresponding feature in the rotation curve."

The distribution of mass is coupled to the distribution of light.

The Mass Discrepancy correlates with acceleration and baryonic surface density

Renzo's Rule:

"When you see a feature in the light, you see a corresponding feature in the rotation curve."

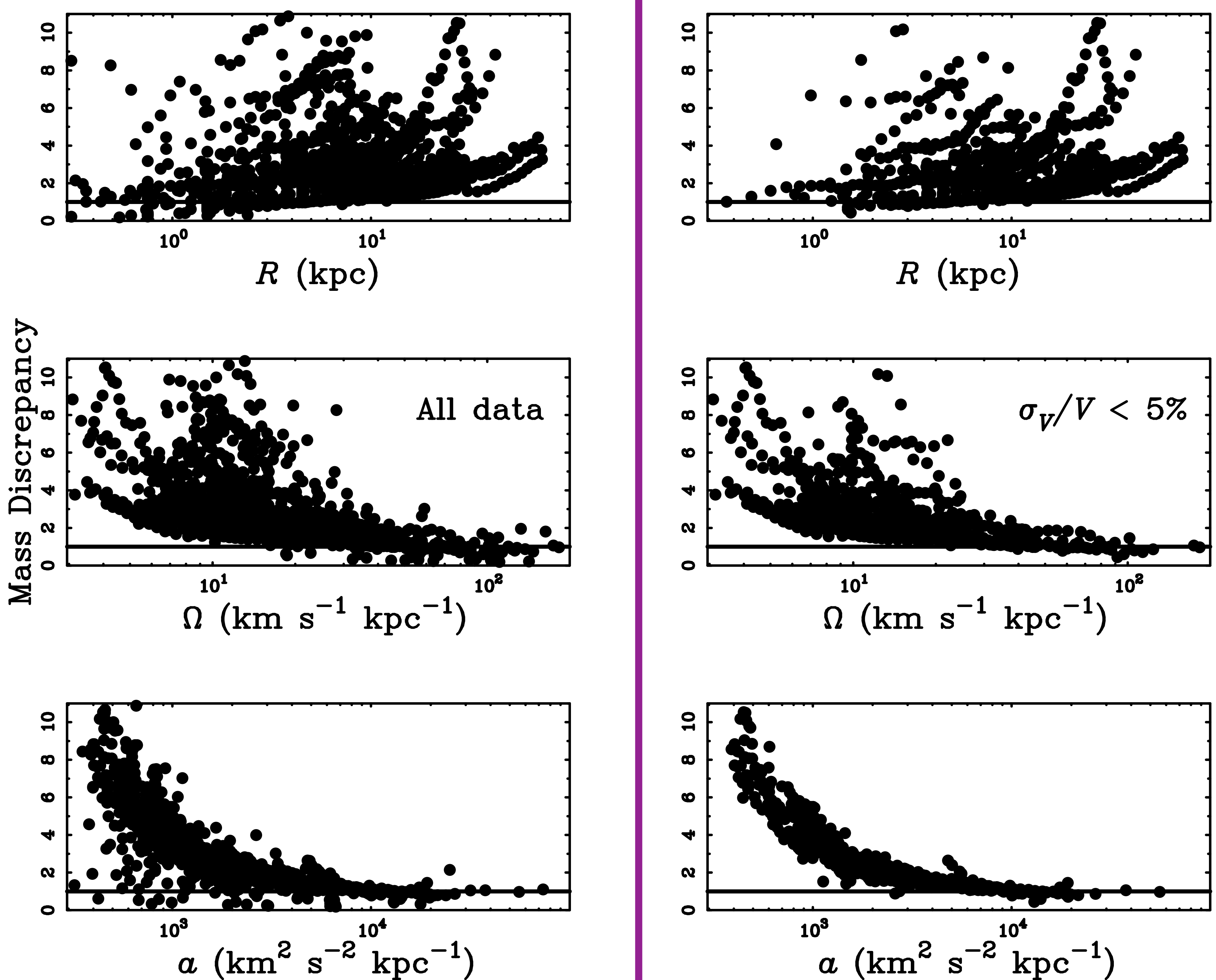
The distribution of mass is coupled to the distribution of light.

Quantify by defining the Mass Discrepancy:

$$\mathcal{D} = \frac{V^2}{V_b^2} = \frac{V^2}{\Upsilon_\star v_\star^2 + V_g^2}$$

The Mass Discrepancy correlates with acceleration and baryonic surface density

McGaugh (2004)



radius

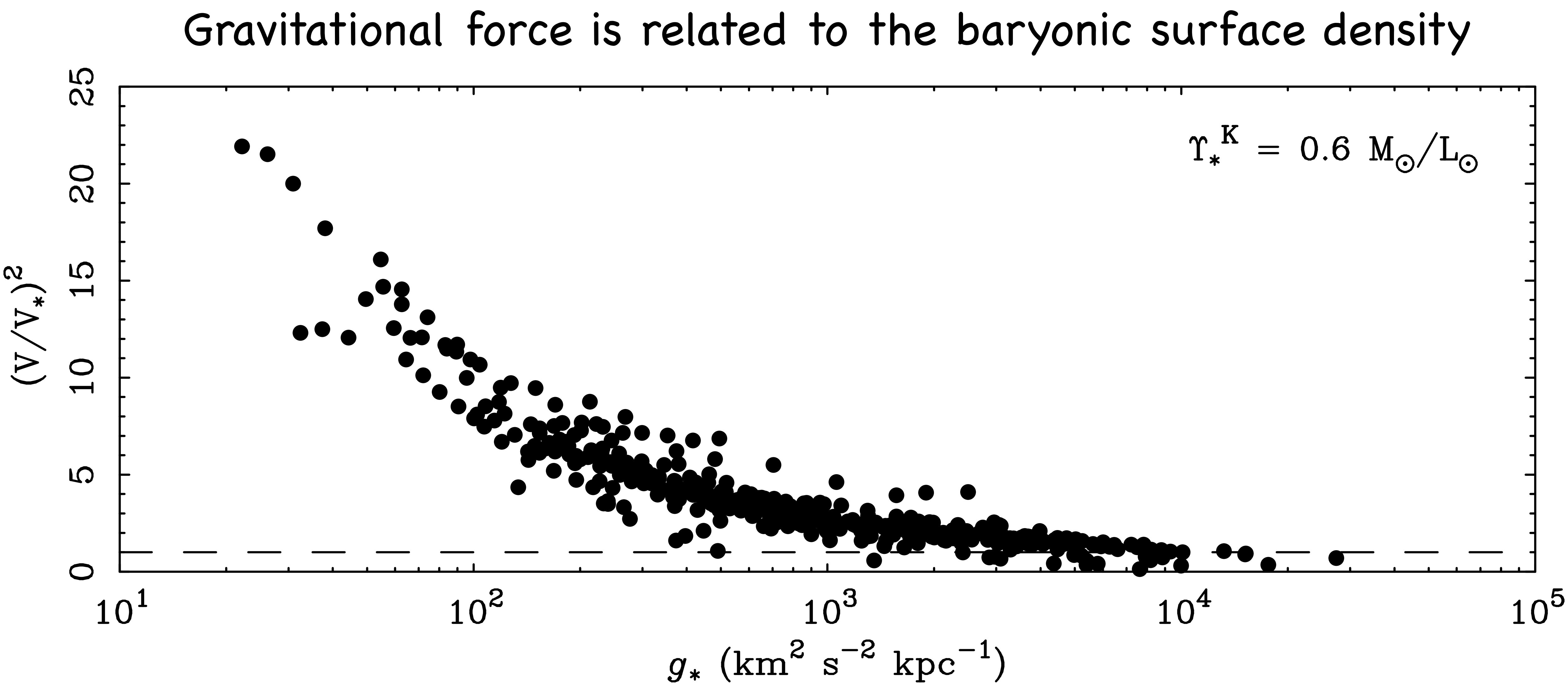
orbital frequency

acceleration

74 galaxies
> 1000 points
(all data)

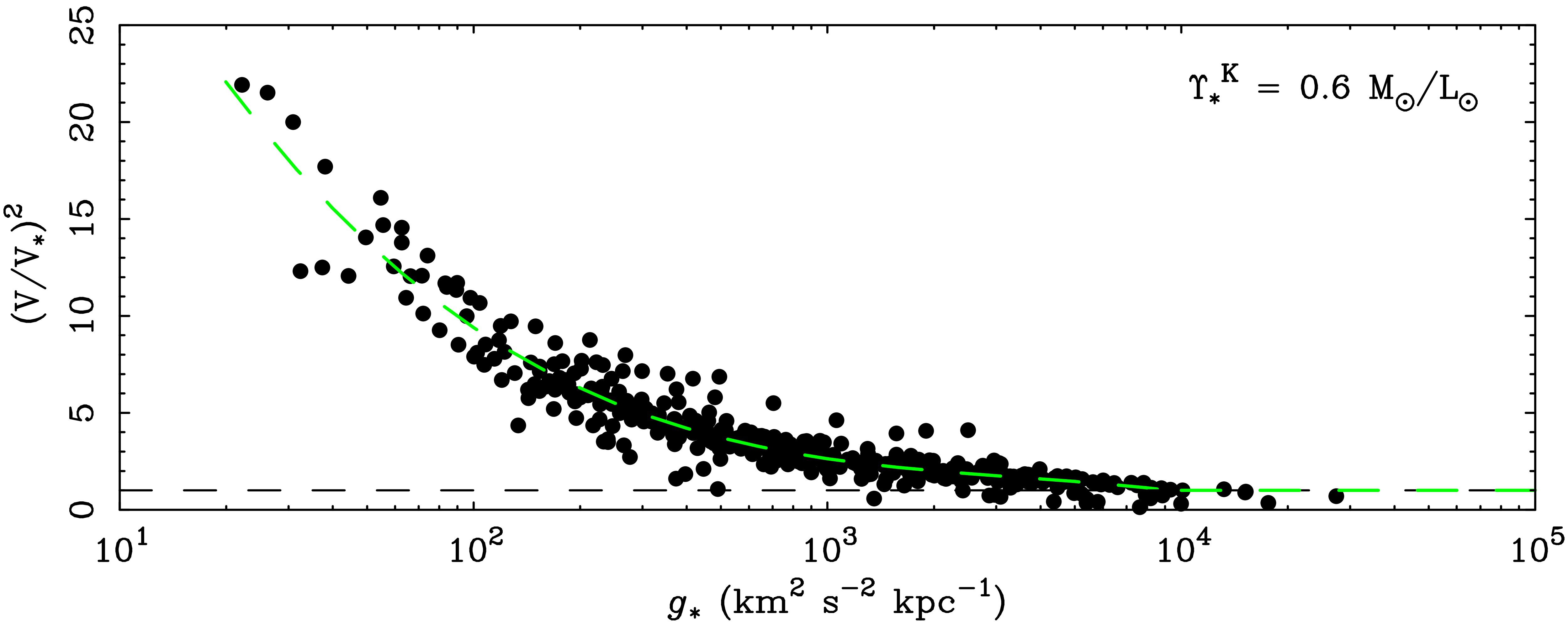
60 galaxies
> 600 points
(errors < 5%)

Empirical calibration of the mass discrepancy-acceleration relation



Empirical calibration of the mass discrepancy-acceleration relation

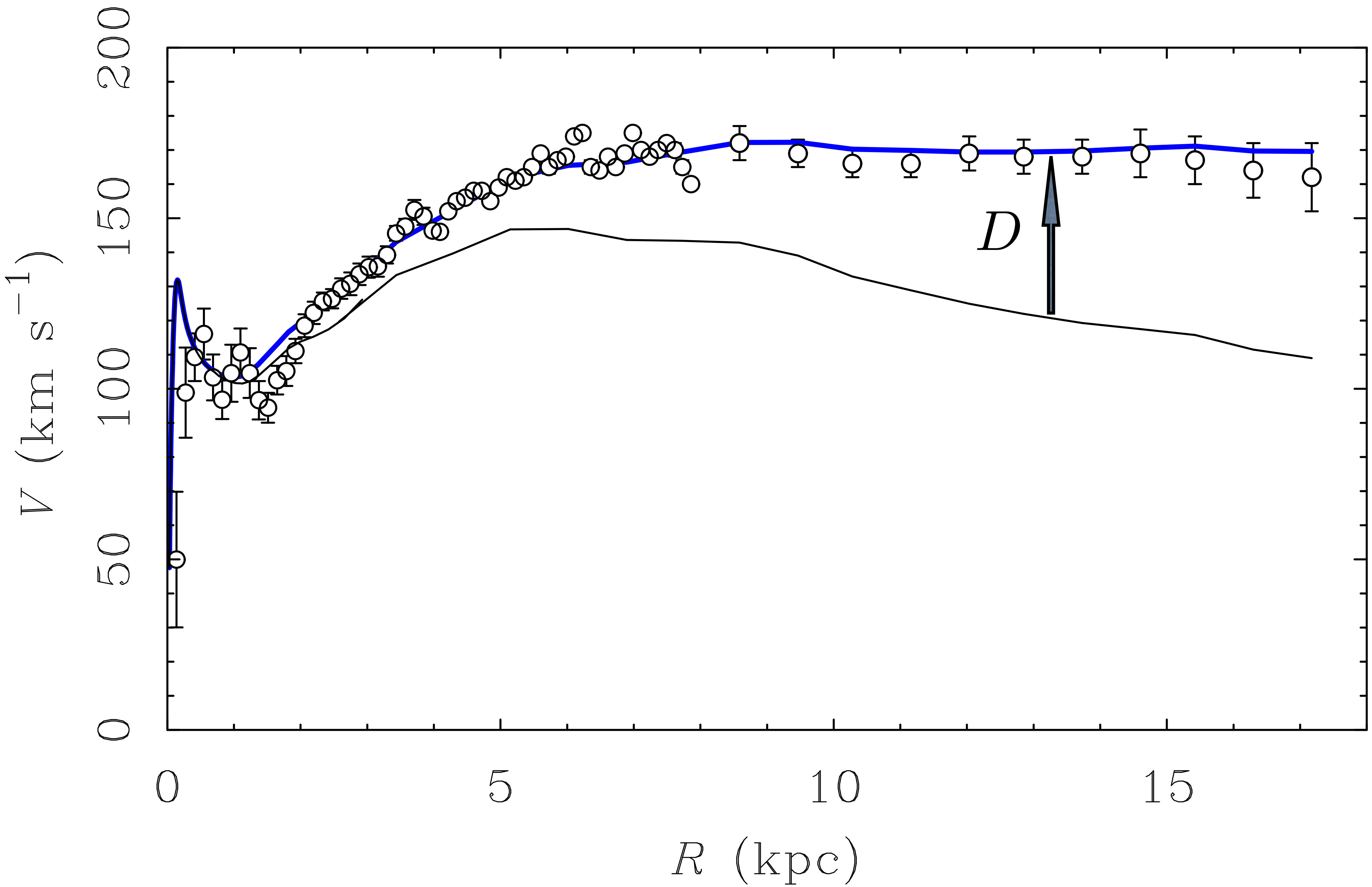
Can fit a fcn to the data $D(g_b/a_\dagger)$



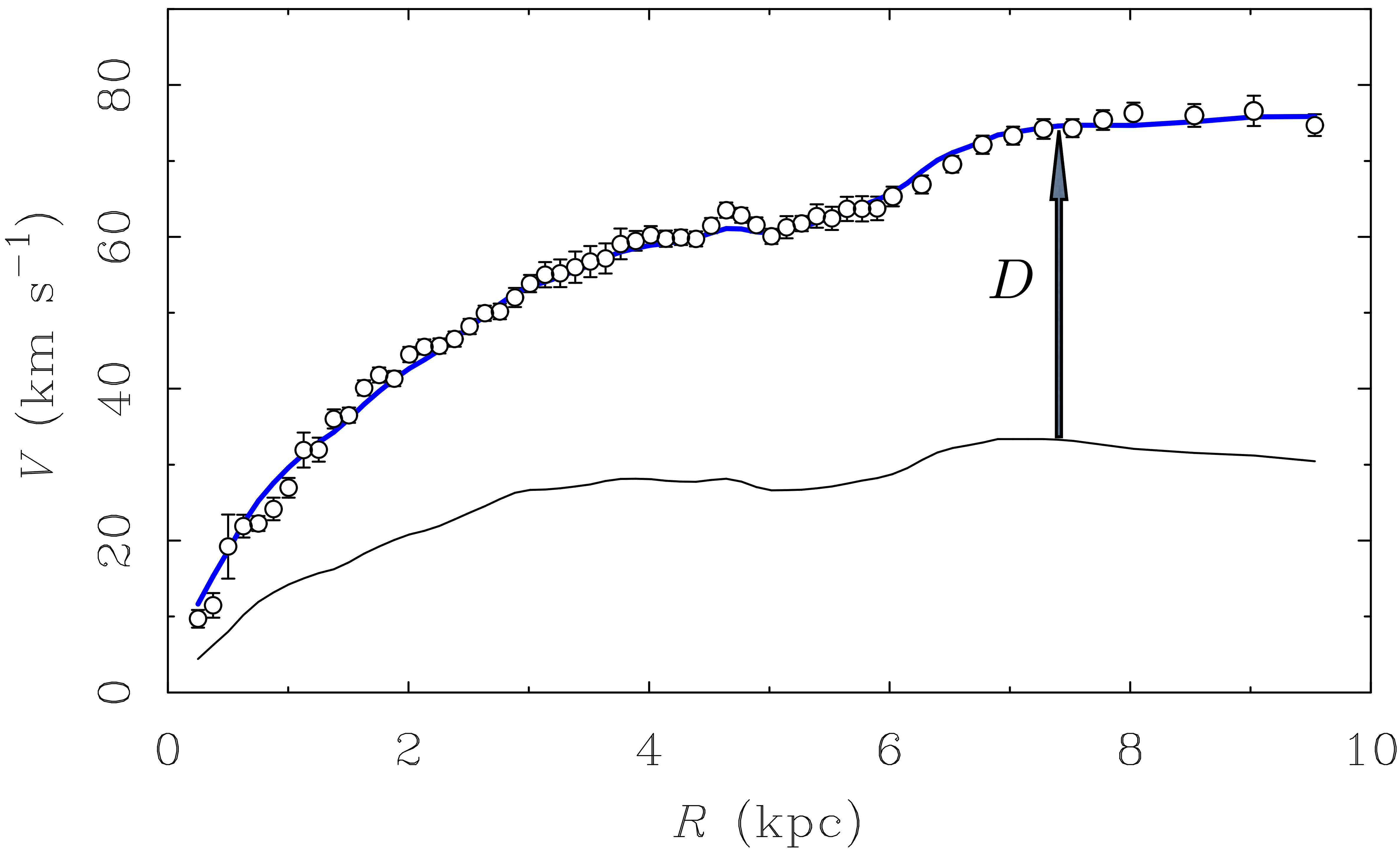
$$D = (1 - e^{-\sqrt{g_b/a_\dagger}})^{-1} \quad \text{with} \quad a_\dagger = 1.23 \pm 0.03 \text{ Å s}^{-2}$$

$$\text{or equivalently, } \Sigma_\dagger = a_\dagger/G = 880\ \mathrm{M}_\odot\,\mathrm{pc}^{-2} \approx 1.8\ \mathrm{kg\,m}^{-2}$$

- a stiff piece of construction paper

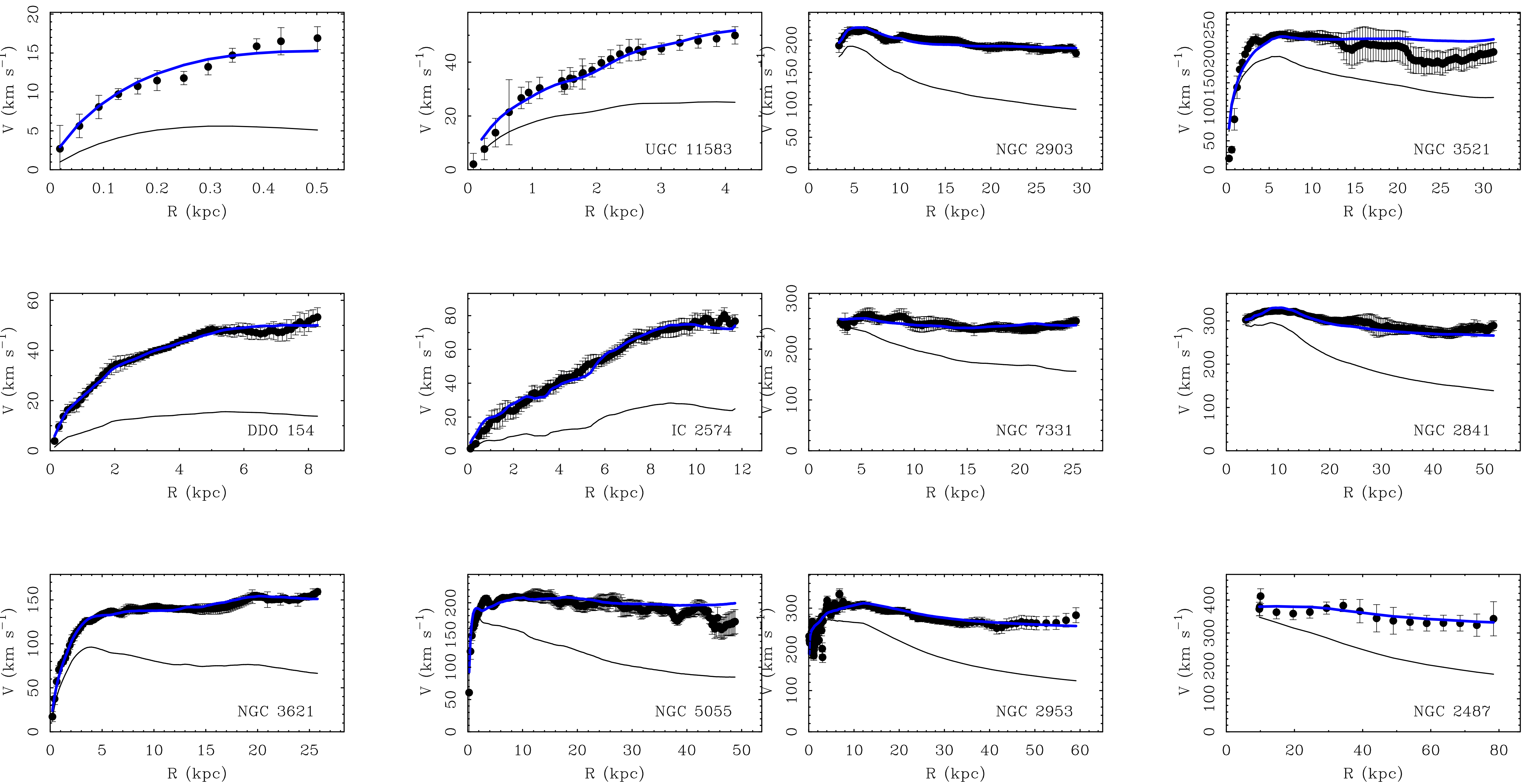


V (km s^{-1})
0
50
100
150
200
0
5
10
15
R (kpc)
D

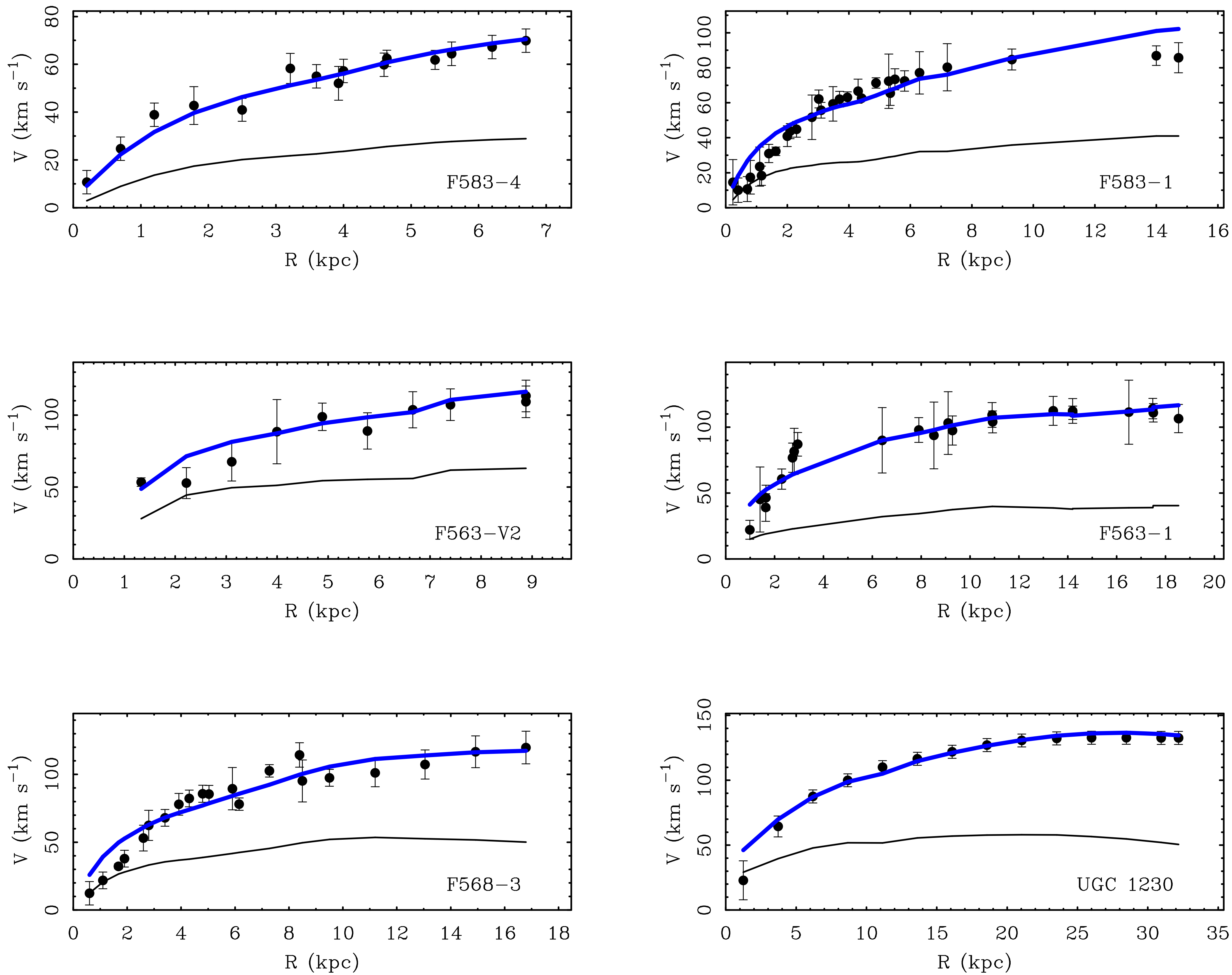


V (km s^{-1})
0
20
40
60
80
D
0
2
4
6
8
10
R (kpc)

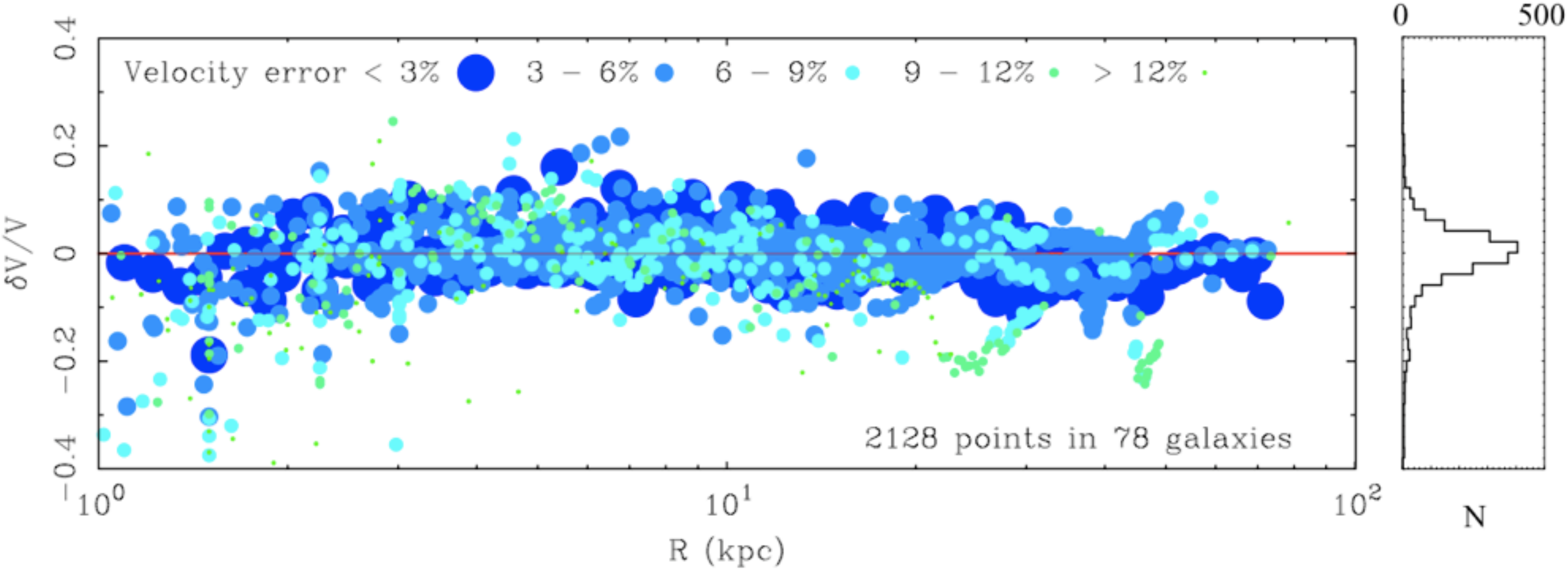
Can do it lots of times. Works for bright galaxies and faint,



and those of low surface brightness where baryons are everywhere sub-dominant.



Residuals from mapping baryonic rotation curve to the total rotation curve with mass-discrepancy acceleration relation



Light and Mass

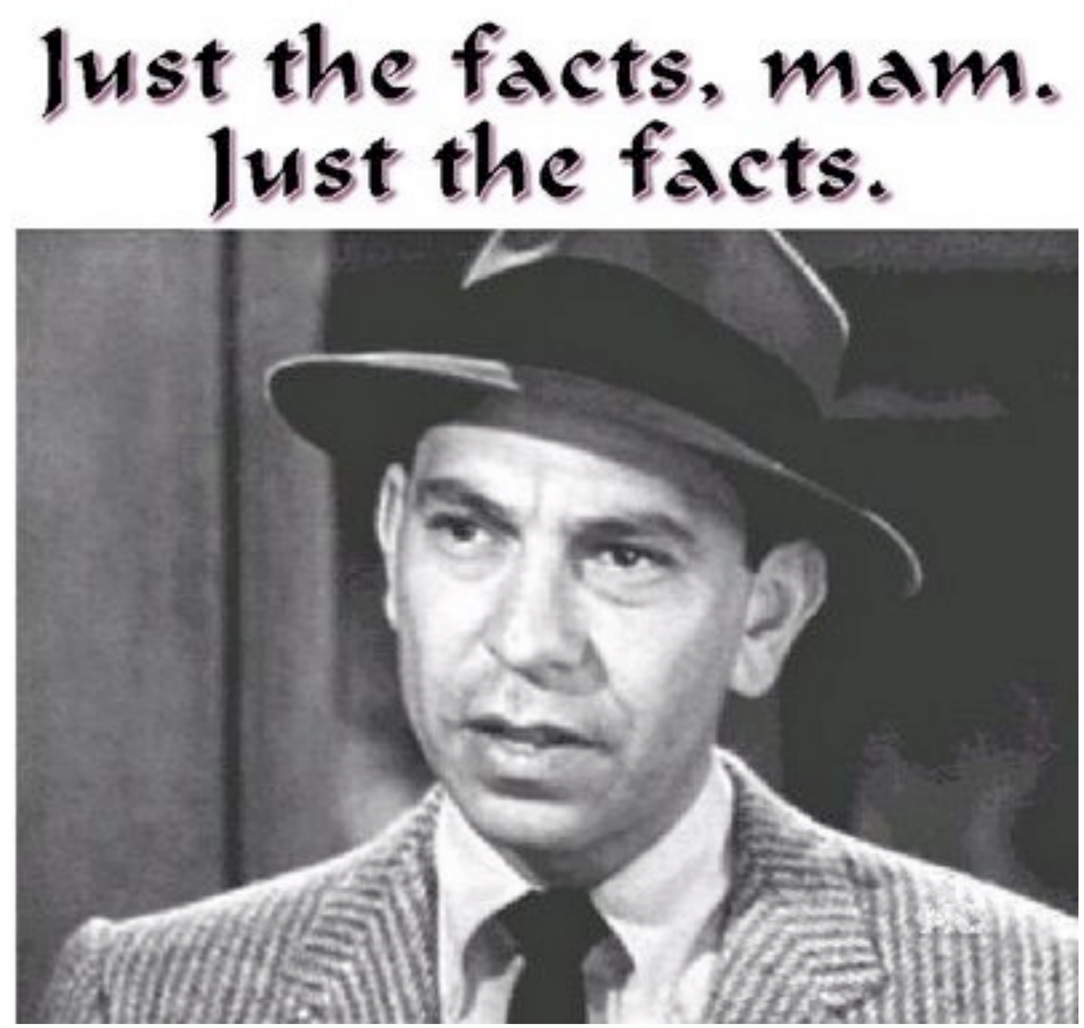
- Many indications of a strong connection between the distribution of baryons and the dynamics:
    - Rotation curve shape correlates with luminosity (Rubin et al. 1980)
    - Universal Rotation Curve (Persic & Salucci 1996)
    - Renzo's Rule (Sancisi 2004)
    - Mass Discrepancy-Acceleration Relation (McGaugh 2004)

3 Laws of Galactic Rotation

1. Rotation curves tend towards asymptotic flatness

2. Baryonic mass scales as the fourth power of rotation velocity (Baryonic Tully-Fisher)

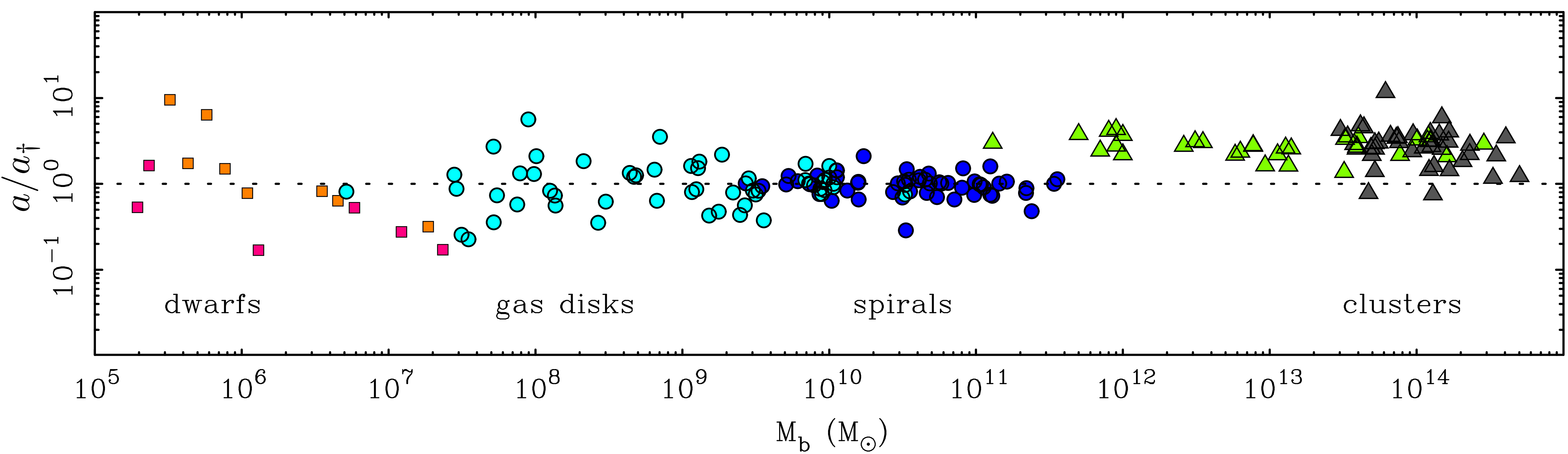
3. Gravitational force correlates with baryonic surface density - the dynamics knows about the baryons



- These appear to be Laws of Nature.
- There is something special about the scale $a_\dagger = 1.2\ \text{Å}\,\text{s}^{-2}$

$$\Sigma_\dagger = a_\dagger/G = 880\ \text{M}_\odot\,\text{pc}^{-2} \approx 1.8\ \text{kg}\,\text{m}^{-2}$$

$$a_\dagger \approx \frac{cH_0}{2\pi} \approx c\Lambda^{1/2}$$



Example application: our own Galaxy

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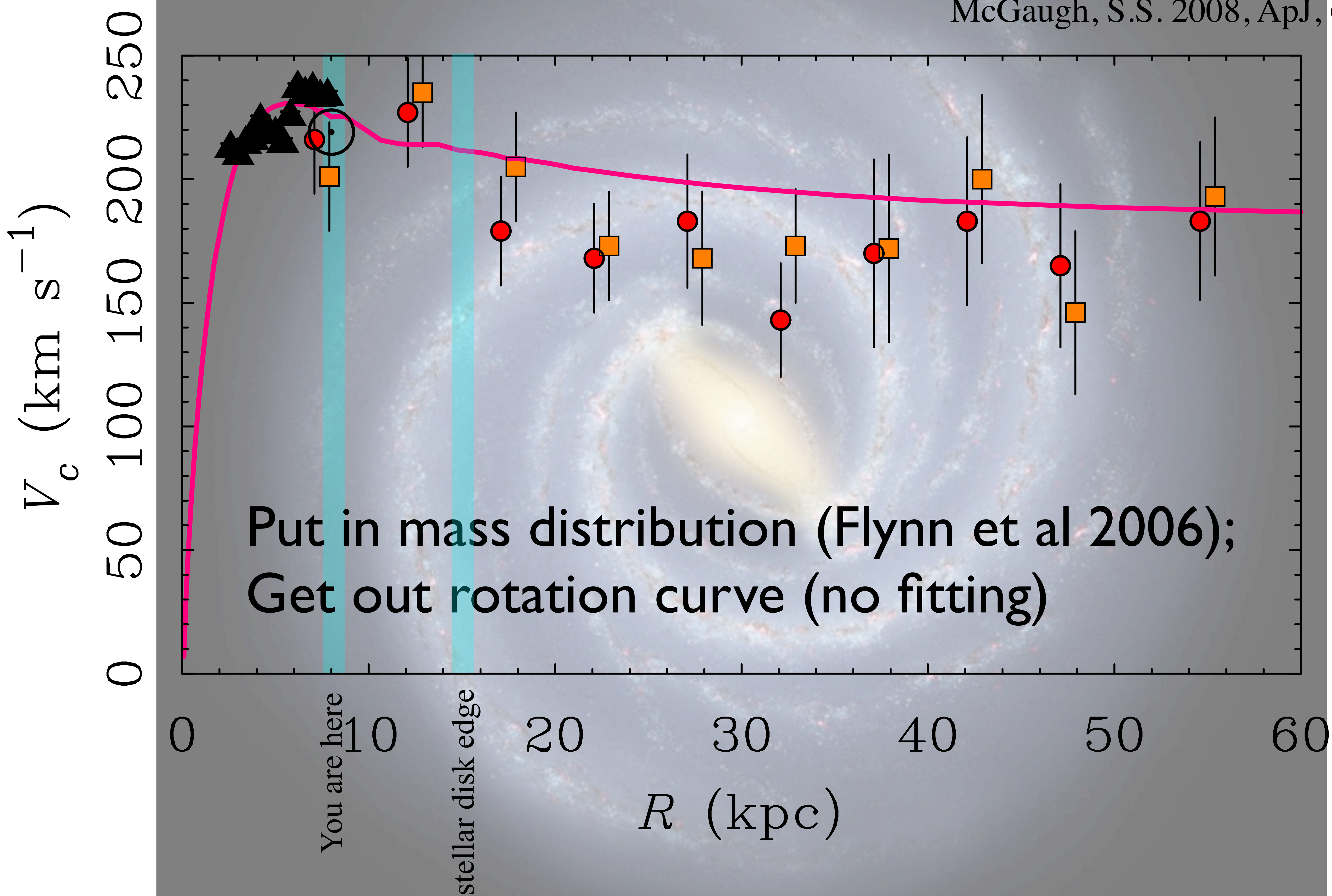
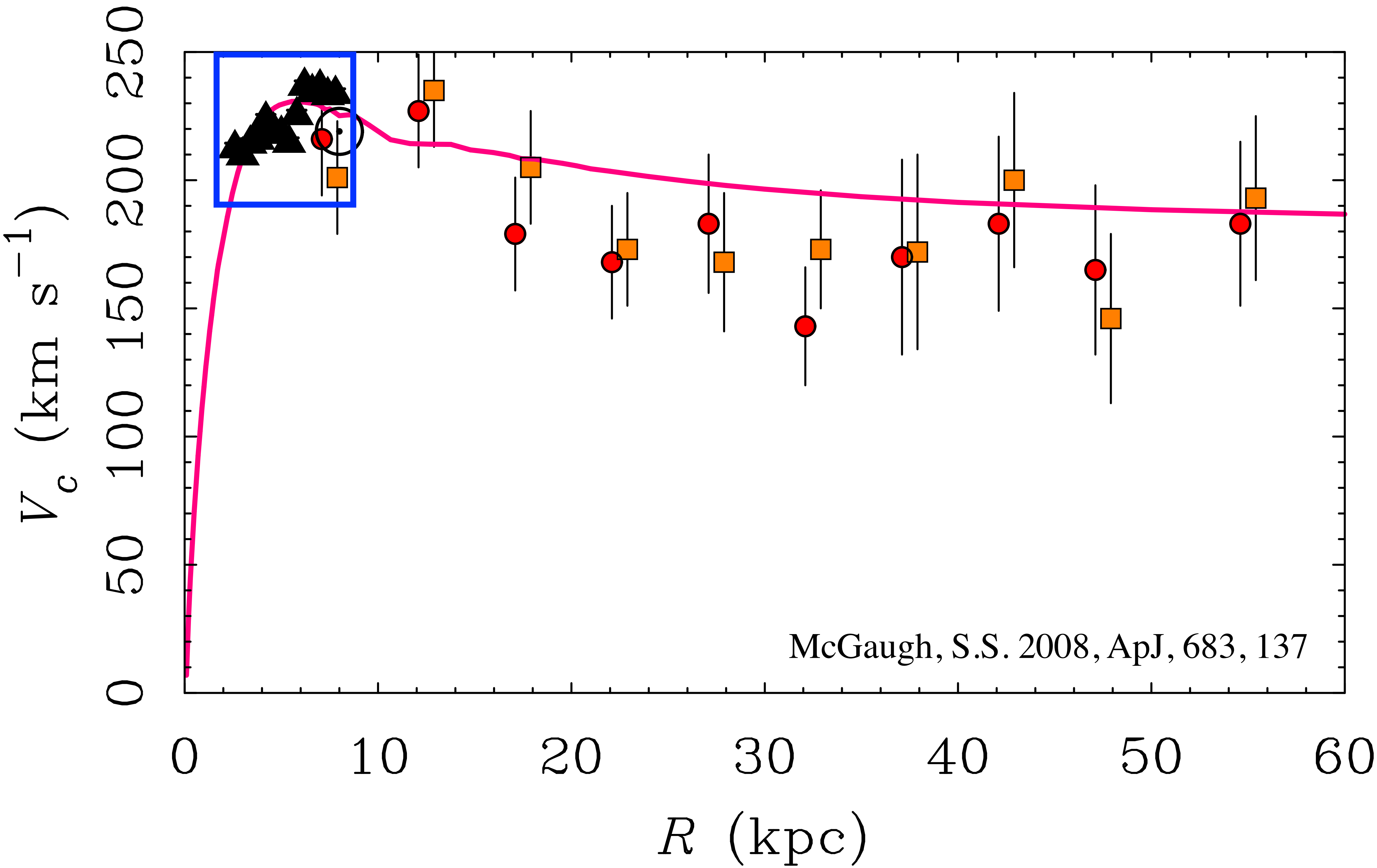
McGaugh, S.S. 2008, ApJ, 683, 137



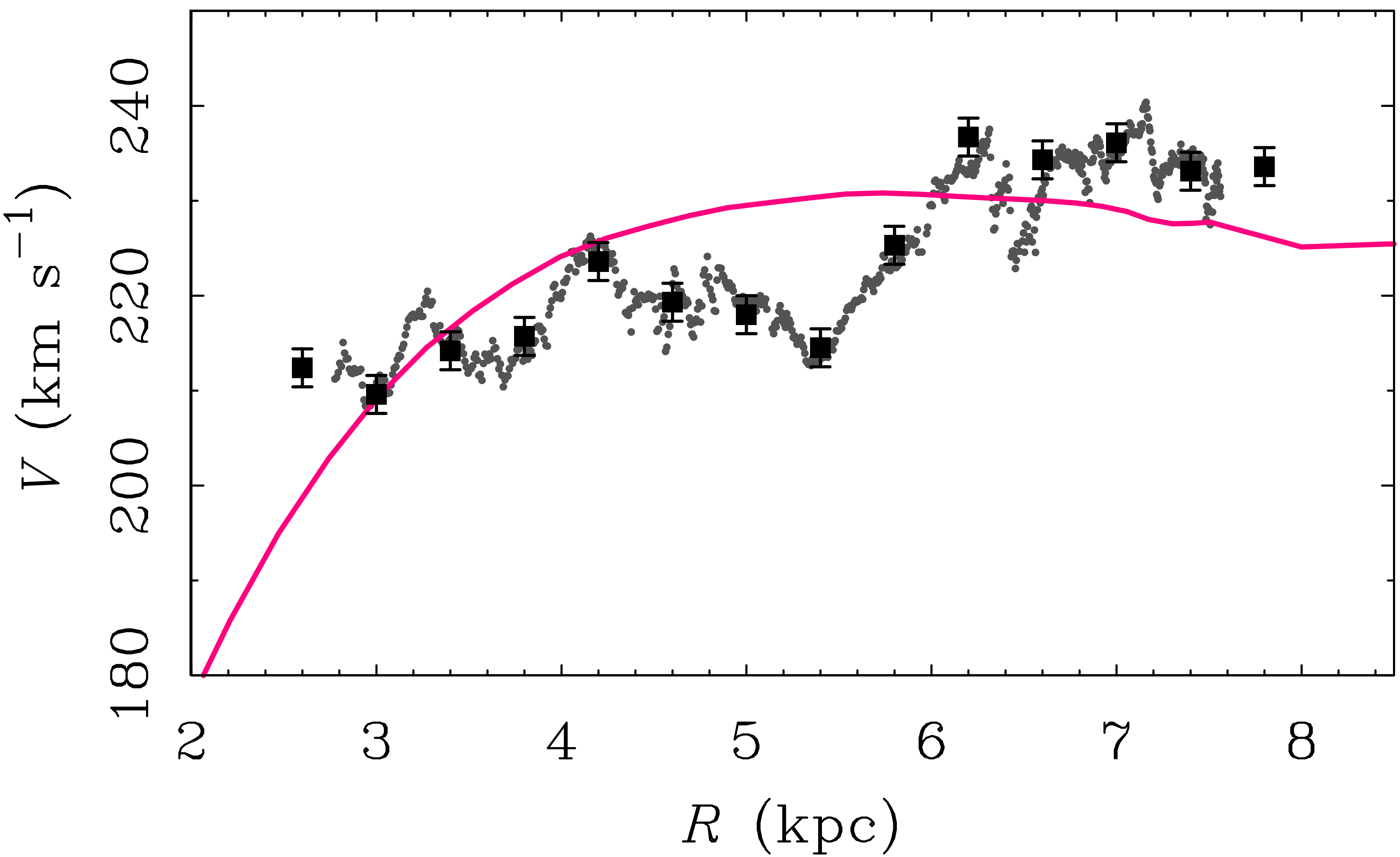
Luna et al. (2006: **CO**); McClure-Griffiths & Dickey (2007: **HI**); Xue et al. (2008: **BHB**)

Can we do better fitting the details of the terminal velocity curve?

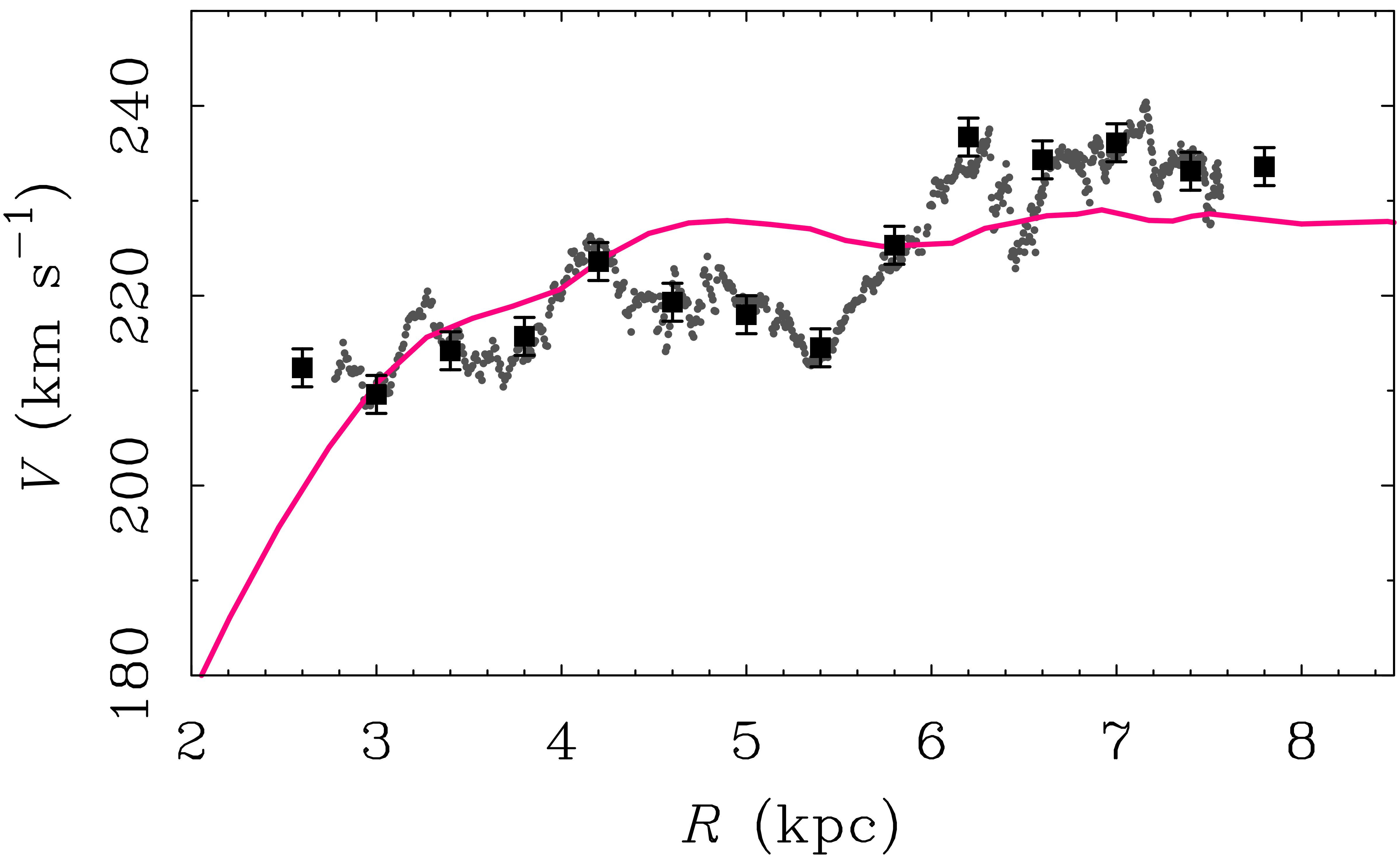


Luna et al. (2006: **CO**); McClure-Griffiths & Dickey (2007: **HI**); Xue et al. (2008: **BHB**)

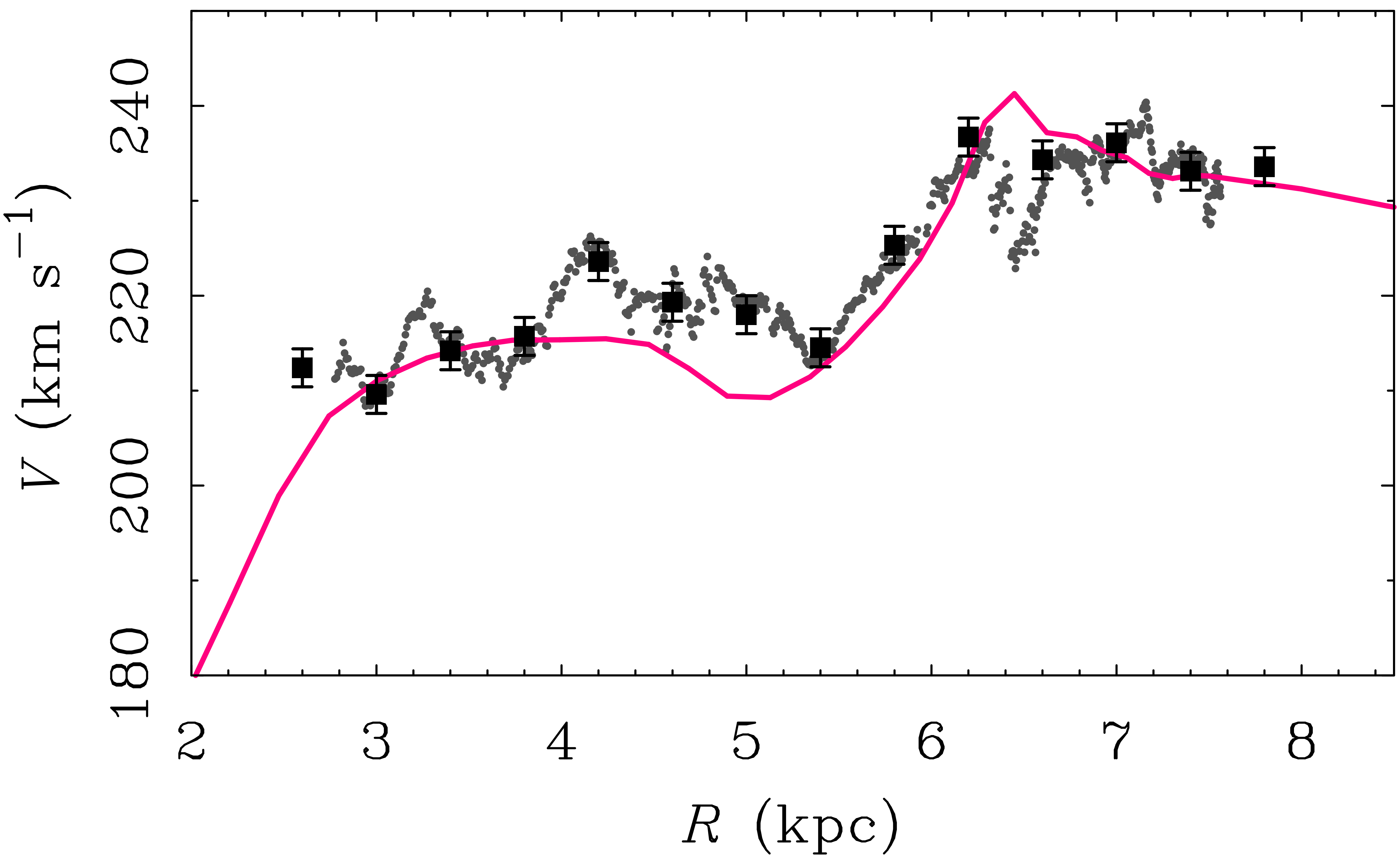
Fitting the details of the terminal velocity curve



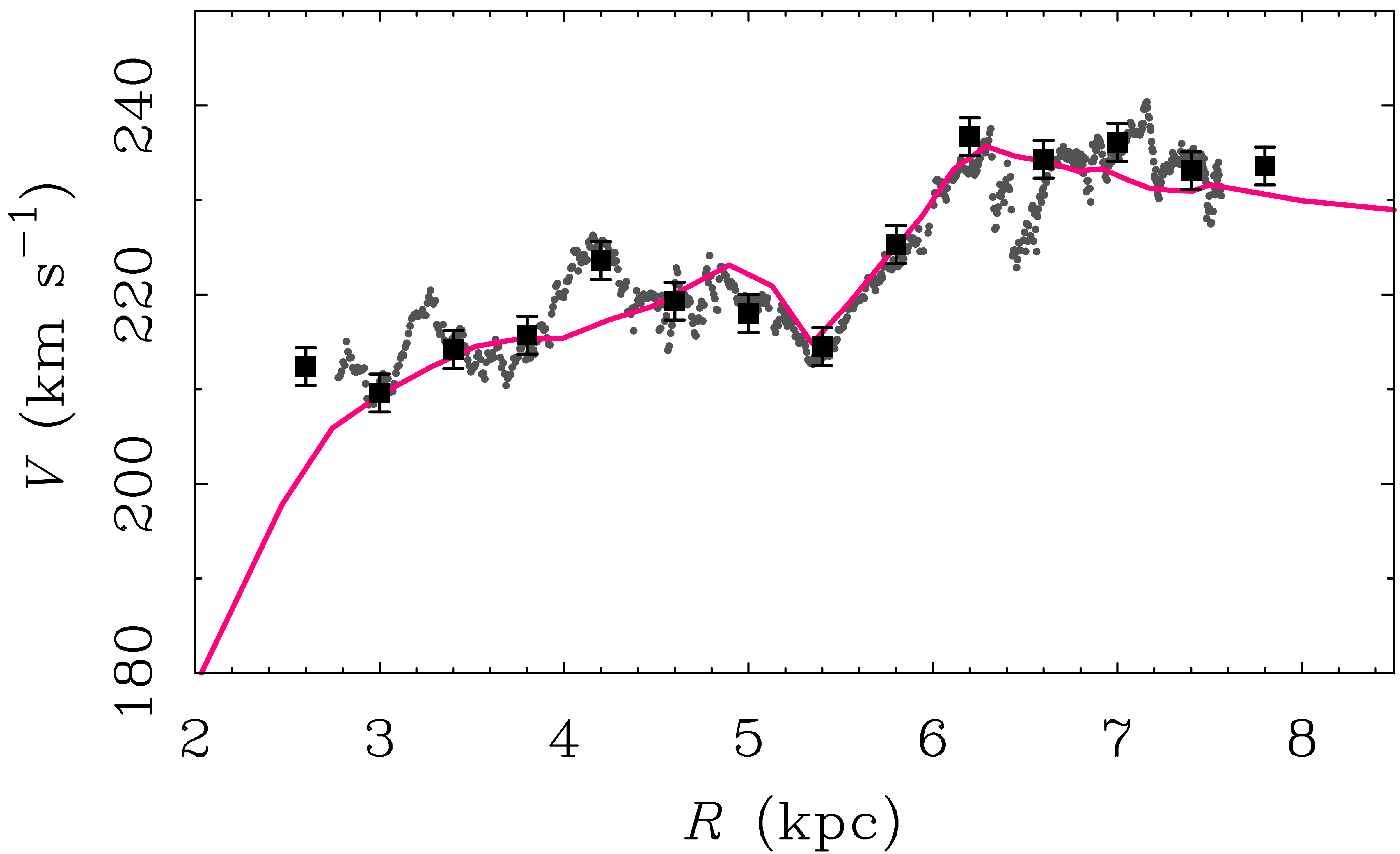
Fitting the details of the terminal velocity curve



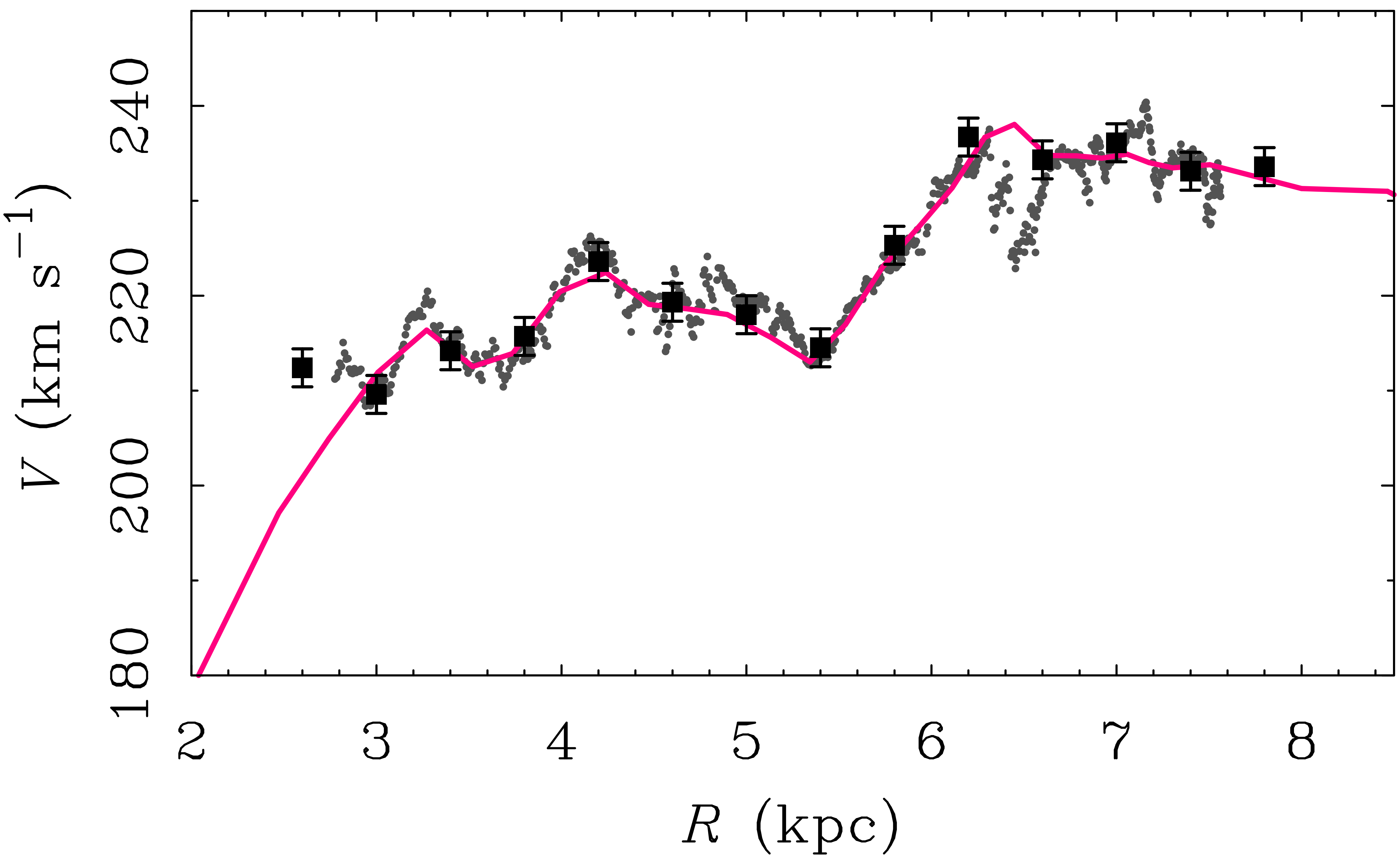
Fitting the details of the terminal velocity curve



Fitting the details of the terminal velocity curve



Fitting the details of the terminal velocity curve



Fitting the details of the terminal velocity curve uncovers details of Milky Way structure: the inferred density enhancement corresponds to the Centaurus spiral arm (McGaugh 2008).