A possible origin of masses, dark energy and inflation

Mikhail Shaposhnikov

VIA, 20 February 2009

Based on:

- F. Bezrukov, M.S., Phys. Lett. B **659** (2008) 703

- M.S., Daniel Zenhäusern, Phys. Lett. B **671** (2009) 162

- M.S., Daniel Zenhäusern, Phys. Lett. B **671** (2009) 187

- M.S., Igor Tkachev, arXiv:0811.1967 [hep-th]

- F. Bezrukov, A. Magnin, M.S., arXiv:0812.4950 [hep-ph]

Father of the God Particle

Photograph: Murdo McLeod, source: Guardian, Monday, June 30, 2008

Particle physicist Peter Higgs, who in 1964 proposed ...

... proposed the existence of a fundamental particle now known as the Higgs boson that gives all matter its mass.

But: who gives mass to the Higgs itself?

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But: who determines the Newton gravity constant?

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The Higgs boson is at most the deputy God particle...

Outline

- Classical scale invariance and its spontaneous breakdown

- Unimodular gravity

- Scale invariance, unimodular gravity, cosmological constant, inflation and dark energy

- Quantum scale invariance

- Key idea

- What to expect at LHC

- Conclusions

Scale invariance

Multiply all mass parameters in the theory

$$M_W, \quad \Lambda_{QCD}, \quad M_H, M_{Pl}, \ldots$$

by one and the same number : $M \to \sigma M$. Physics is not changed!

Indeed, this change, supplemented by a dilatation of space-time coordinates $x^\mu \to \sigma x^\mu$ and an appropriate redefinition of the fields does not change the complete quantum effective action of the theory.

First step: consider classical physics only (no parameters like Λ_{QCD}), just tree explicit mass parameters such as $M_H, \ M_W, M_{Pl}$.

Unique scale-invariant Lagrangian

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SM}[M \to 0]} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi, \chi)$$

Potential (χ - dilaton, φ - Higgs, $\varphi^\dagger \varphi = 2h^2$):

$$V(\varphi, \chi) = \lambda \left(\varphi^\dagger \varphi - \frac{\alpha}{2\lambda} \chi^2 \right)^2 + \beta \chi^4,$$

Gravity part

$$\mathcal{L}_G = - \left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi \right) \frac{R}{2},$$

Spontaneous breaking of scale invariance

Consider scalar potential

$$V(\varphi, \chi) = \lambda \left(\varphi^\dagger \varphi - \frac{\alpha}{2\lambda} \chi^2 \right)^2 + \beta \chi^4,$$

Requirements: vacuum state exists if $\lambda \geq 0, \quad \beta \geq 0$

For $\lambda > 0, \quad \beta > 0$ the vacuum state is unique: $\chi = 0, \quad \varphi = 0$ and scale invariance is exact.

Field propagators: scalar $1/p^2$, fermion $\not{p}/p^2$. Greenberg, 1961:

free quantum field theory!!

If not - theory does not describe particles !!.

In the presence of gravity the argument is weakened:

$\beta > 0$: there are time-dependent classical solutions with $\chi = \mathrm{const} \neq 0, \ h = \mathrm{const} \neq 0$ and De-Sitter space (positive cosmological constant $\propto \beta$)

$\beta < 0$: there are time-dependent classical solutions with $\chi = \mathrm{const} \neq 0, \ h = \mathrm{const} \neq 0$ and anti-De-Sitter space (negative cosmological constant $\propto \beta$)

Quantum sense of these solutions?

For $\lambda > 0$, $\beta = 0$ the scale invariance can be spontaneously broken.
The vacuum manifold:

$$h_0^2 = \frac{\alpha}{\lambda}\chi_0^2$$

Particles are massive, Planck constant is non-zero:

$$M_H^2 \sim M_W \sim M_t \sim M_N \propto \chi_0, \quad M_{Pl} \sim \chi_0$$

Phenomenological requirement:

$$\alpha \sim \frac{v^2}{M_{Pl}^2} \sim 10^{-38} \ll 1$$

Good news: cosmological constant is zero due to scale invariance

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Universe is in the state of accelerated expansion, $\Omega_{DE} \simeq 0.7$!

Unimodular gravity

Ordinary gravity:

the metric $g_{\mu\nu}$ is an arbitrary function of space-time coordinates.
Invariant under general coordinate transformations

Unimodular gravity:

the metric $g_{\mu\nu}$ is an arbitrary function of space-time coordinates with
$\det[g] = -1$. Invariant under general coordinate transformations
which conserve the 4-volume.

van der Bij, van Dam, Ng

Origin of UG: Field theory describing spin 2 massless particles is
either GR or UG

Number of physical degrees of freedom is the same.

Unimodular gravity and cosmological constant

Theories are equivalent everywhere except the way the cosmological constant appears

GR. Λ is the fundamental constant:

$$S = -M_P^2 \int d^4x \sqrt{-g}\,[R + \Lambda]$$

UG. Λ does not appear in the action:

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Cosmological constant problem is solved in UG??!!

Wilczek, Zee: NO!

UG is equivalent to

$$S = -M_P^2 \int d^4x \sqrt{-g} \left[R + \Lambda(x) \left(1 - \frac{1}{\sqrt{-g}} \right) \right]$$

Equations of motion ($G_{\mu\nu}$ - Einstein tensor):

$$G_{\mu\nu} = -\Lambda(x)\, g_{\mu\nu} \, , \sqrt{-g} = 1$$

Bianchi identity: $\Lambda(x)_; = 0 \longrightarrow \Lambda(x) = const.$

Solutions of UG are the same as solutions of GR with an arbitrary cosmological constant.

Conclusion: in UG cosmological constant reappears, but as an integral of motion, related to initial conditions

Scale invariance + unimodular gravity

Solutions of scale-invariant UG are the same as the solutions of scale-invariant GR with the action

$$S = -\int d^4x \sqrt{-g} \left[(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi) \frac{R}{2} + \Lambda + ... \right] ,$$

Physical interpretation: Einstein frame

$$g_{\mu\nu} = \Omega(x)^2 \tilde{g}_{\mu\nu} , \quad (\xi_\chi \chi^2 + \xi_h h^2)\Omega^2 = M_P^2$$

Λ is not a cosmological constant, it is the strength of a peculiar potential!

Relevant part of the Lagrangian (scalars + gravity) in Einstein frame:

$$\mathcal{L}_E = \sqrt{-\tilde{g}} \left(-M_P^2 \frac{\tilde{R}}{2} + K - U_E(h,\chi) \right) ,$$
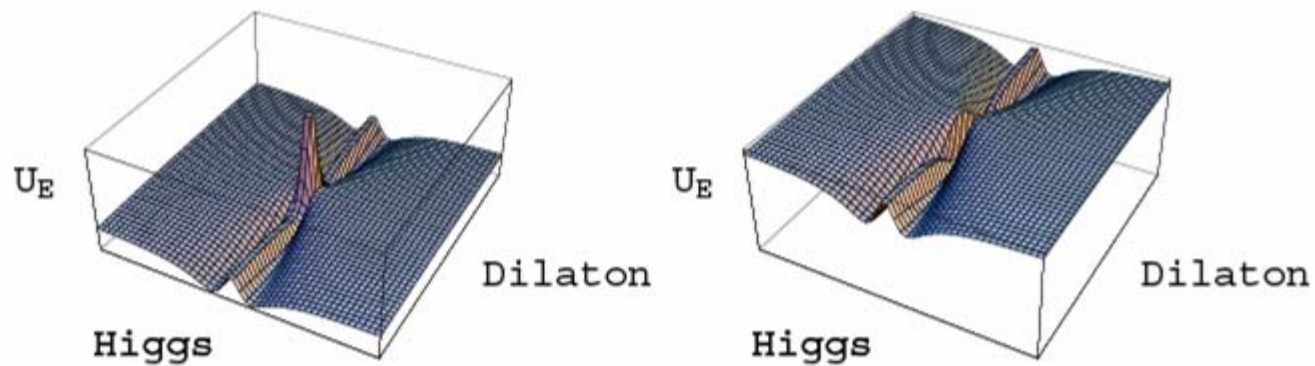
K - complicated non-linear kinetic term for the scalar fields,

$$K = \Omega^2 \left(\frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}(\partial_\mu h)^2) \right) - 3M_P^2 (\partial_\mu \Omega)^2 .$$

The Einstein-frame potential $U_E(h,\chi)$:

$$U_E(h,\chi) = M_P^4 \left[\frac{\lambda \left(h^2 - \frac{\alpha}{\lambda}\chi^2 \right)^2}{4(\xi_\chi \chi^2 + \xi_h h^2)^2} + \frac{\Lambda}{(\xi_\chi \chi^2 + \xi_h h^2)^2} \right] ,$$

Potential for the Higgs field and dilaton in the Einstein frame.

Left: $\Lambda > 0$, right $\Lambda < 0$.

50% chance ($\Lambda < 0$): inflation + late collapse

50% chance ($\Lambda > 0$): inflation + late acceleration

Chaotic initial condition: fields χ and h are away from their equilibrium values.

Choice of parameters: $\xi_h \gg 1, \quad \xi_\chi \ll 1$ (will be justified later)

Then - dynamics of the Higgs field is more essential, $\chi \simeq const$ and is frozen. Denote $\xi_\chi \chi^2 = M_P^2$.

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\tilde{h}}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2/M_P^2}{\Omega^4}} \implies \begin{cases} h \simeq \tilde{h} & \text{for } h < M_P/\xi \\[2ex] h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\tilde{h}}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\sqrt{\xi} \end{cases}$$
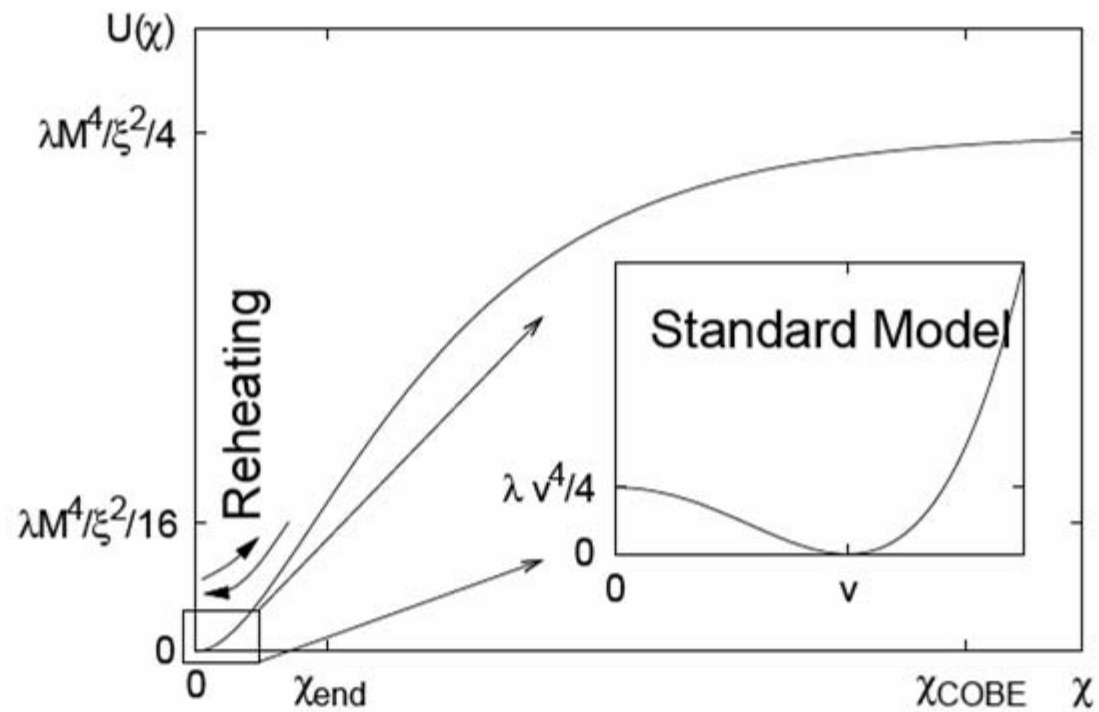
Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \tilde{h}\partial^\mu \tilde{h}}{2} - \frac{1}{\Omega(\tilde{h})^4}\frac{\lambda}{4}h(\tilde{h})^4 \right\}$$

Potential:

$$U(\tilde{h}) = \begin{cases} \frac{\lambda}{4}\tilde{h}^4 & \text{for } h < M_P/\xi \\[2ex] \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\tilde{h}}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases} \qquad .$$

Potential in Einstein frame

Slow roll stage

$$\epsilon = \frac{M_P^2}{2} \left(\frac{dU/d\chi}{U} \right)^2 \simeq \frac{4}{3} \exp\left(-\frac{4\chi}{\sqrt{6}M_P} \right)$$

$$\eta = M_P^2 \frac{d^2U/d\chi^2}{U} \simeq -\frac{4}{3} \exp\left(-\frac{2\chi}{\sqrt{6}M_P} \right)$$

Slow roll ends at $\chi_{\text{end}} \simeq M_P$

Number of e-folds of inflation at the moment h_N is $N \simeq \frac{6}{8} \frac{h_N^2 - h_{\text{end}}^2}{M_P^2/\xi}$
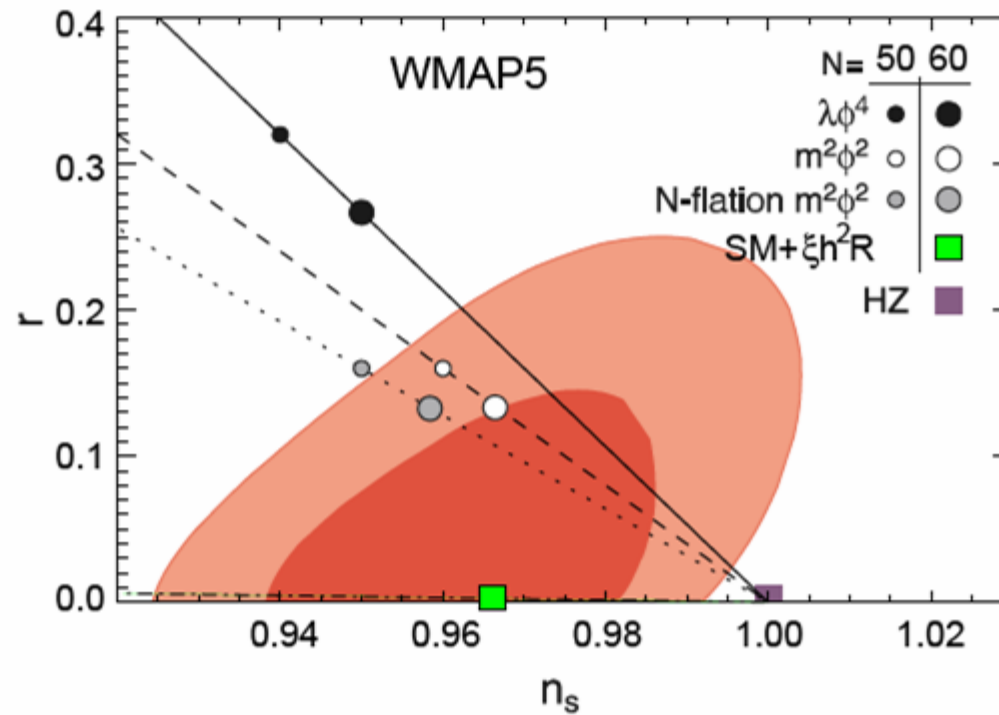
$$\chi_{60} \simeq 5M_P$$

COBE normalization $U/\epsilon = (0.027 M_P)^4$ gives

$$\xi \simeq \sqrt{\frac{\lambda}{3} \frac{N_{\text{COBE}}}{0.027^2}} \simeq 49000\sqrt{\lambda} = 49000 \frac{m_H}{\sqrt{2}v}$$

Connection of ξ and the Higgs mass!

CMB parameters—spectrum and tensor modes

Dark energy

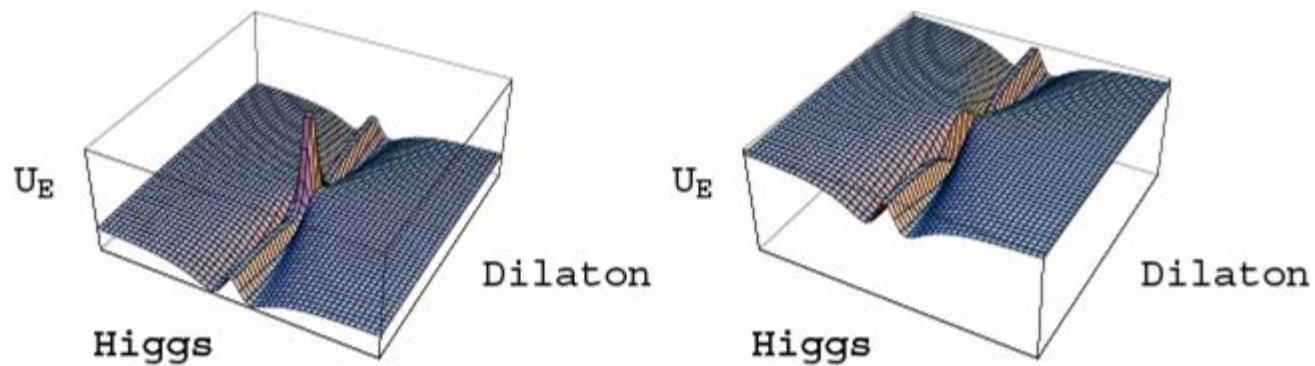Late time evolution of dilaton ρ along the valley, related to χ as

$$\chi = M_P \exp\left(\frac{\gamma\rho}{4M_P}\right), \quad \gamma = \frac{4}{\sqrt{6 + \frac{1}{\xi_\chi}}} \;.$$

Potential: Wetterich; Ratra, Peebles

$$U_\rho = \frac{\Lambda}{\xi_\chi^2} \exp\left(-\frac{\gamma\rho}{M_P}\right) \;.$$

From observed equation of state: $0 < \xi_\chi < 0.09$

Result: equation of state parameter $\omega = P/E$ for dark energy must be different from that of the cosmological constant, but $\omega < -1$ is not allowed.

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Quantum scale invariance

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Everything above does not make any sense???!!!

Standard reasoning

Dimensional regularisation $d = 4 - 2\epsilon$, $\overline{MS}$ subtraction scheme:

mass dimension of the scalar fields: $1 - \epsilon$,

mass dimension of the coupling constant: 2ϵ

Counter-terms:

$$\lambda = \mu^{2\epsilon}\left[\lambda_R + \sum_{k=1}^{\infty}\frac{a_n}{\epsilon^n}\right],$$

μ is a dimensionfull parameter!!

One-loop effective potential along the flat direction:

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2}\left[\log\frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2}\right],$$

Result: explicit breaking of the dilatational symmetry. Dilaton acquires a nonzero mass due to radiative corrections.

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Idea: Replace $\mu^{2\epsilon}$ by combinations of fields χ and h, which have the correct mass dimension:

$$\mu^{2\epsilon} \longrightarrow \chi^{\frac{2\epsilon}{1-\epsilon}} F_\epsilon(x) \ ,$$

where $x = h/\chi$. $F_\epsilon(x)$ is a function depending on the parameter ϵ with the property $F_0(x) = 1$.

Zenhäusern, M.S

Englert, Truffin, Gastmans, 1976

Natural choice:

$$\mu^{2\epsilon} \rightarrow \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \;,\; (\xi_\chi \chi^2 + \xi_h h^2) \equiv \omega^2$$

Potential:

$$U = \frac{\lambda_R}{4} \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[h^2 - \zeta_R^2 \chi^2\right]^2 \;,$$

Counter-terms

$$U_{cc} = \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[Ah^2\chi^2 \left(\frac{1}{\bar{\epsilon}} + a\right) + B\chi^4 \left(\frac{1}{\bar{\epsilon}} + b\right) + Ch^4 \left(\frac{1}{\bar{\epsilon}} + c\right)\right] \;,$$

To be fixed from conditions of absence of divergences and presence
of spontaneous breaking of scale-invariance

$$U_1 = \frac{m^4(h)}{64\pi^2} \left[\log \frac{m^2(h)}{v^2} + \mathcal{O}\left(\zeta_R^2\right) \right]$$

$$+ \frac{\lambda_R^2}{64\pi^2} \left[C_0 v^4 + C_2 v^2 h^2 + C_4 h^4 \right] + \mathcal{O}\left(\frac{h^6}{\chi^2}\right),$$

where $m^2(h) = \lambda_R(3h^2 - v^2)$ and

$$C_0 = \frac{3}{2} \left[2a - 1 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + \frac{4}{3}\log 2\lambda_R + O(\zeta_R^2) \right],$$

$$C_2 = -3 \left[2a - 3 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + O(\zeta_R^2) \right],$$

$$C_4 = \frac{3}{2} \left[2a - 5 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) - 4\log 2\lambda_R + O(\zeta_R^2) \right].$$

Consider the high energy ($\sqrt{s} \gg v$ but $\sqrt{s} \ll \chi_0$) behaviour of scattering amplitudes on the example of Higgs-Higgs scattering (assuming, that $\zeta_R \ll 1$). In one-loop approximation

$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2} \left[\log\left(\frac{s}{\xi_\chi \chi_0^2} \right) + \text{const} \right] + \mathcal{O}\left(\zeta_R^2 \right) .$$

This implies that at $v \ll \sqrt{s} \ll \chi_0$ the effective Higgs self-coupling runs in a way prescribed by the ordinary renormalization group! For QCD:

$$\Lambda_{QCD} = \chi_0 e^{-\frac{1}{2b_0\alpha_s}}, \quad \beta(\alpha_s) = b_0 \alpha_s^2$$

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- Unitarity: Do we need infinite counter-terms to remove divergences from higher-derivative operators? If no, then theory is unitary, if yes, this remains to be seen.

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- Cosmological constant is zero in all orders of perturbation theory

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The scalar self-coupling λ_R must be positive. Therefore, coupling at small energies must be within some interval. In other words, the SM must be valid up to the Planck scale.
Prediction for LHC: nothing but the Higgs in the mass interval
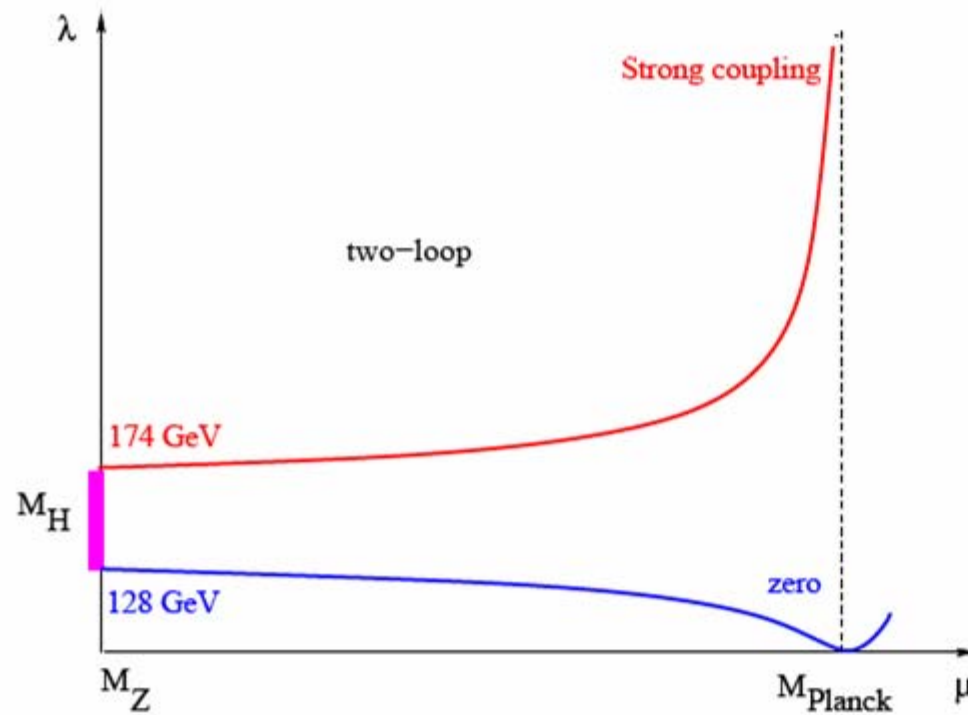
$$m_{\min} < m_H < m_{\max}$$

where

$$m_{\min} = \left[128.3 + \frac{m_t - 171.2}{2.1} \times 4.8 - \frac{\alpha_s - 0.118}{0.002} \times 1.5\right] \text{GeV}$$

$$m_{\max} = \left[173.6 + \frac{m_t - 171.2}{2.1} \times 1.5 - \frac{\alpha_s - 0.118}{0.002} \times 0.3\right] \text{GeV}$$

m_t is the mass of the top quark.

Behaviour of the scalar self-coupling

Cosmological constraint on the Higgs mass

The Standard Model Higgs boson can play the role of the inflaton
(Bezrukov, M.S.)
if the Higgs mass is between Bezrukov, Magnin, M.S.,
Dec 31, 2008 1 loop

$$m_{min} = 136.7 \text{ GeV}$$

$$m_{max} = 184.5 \text{ GeV}$$
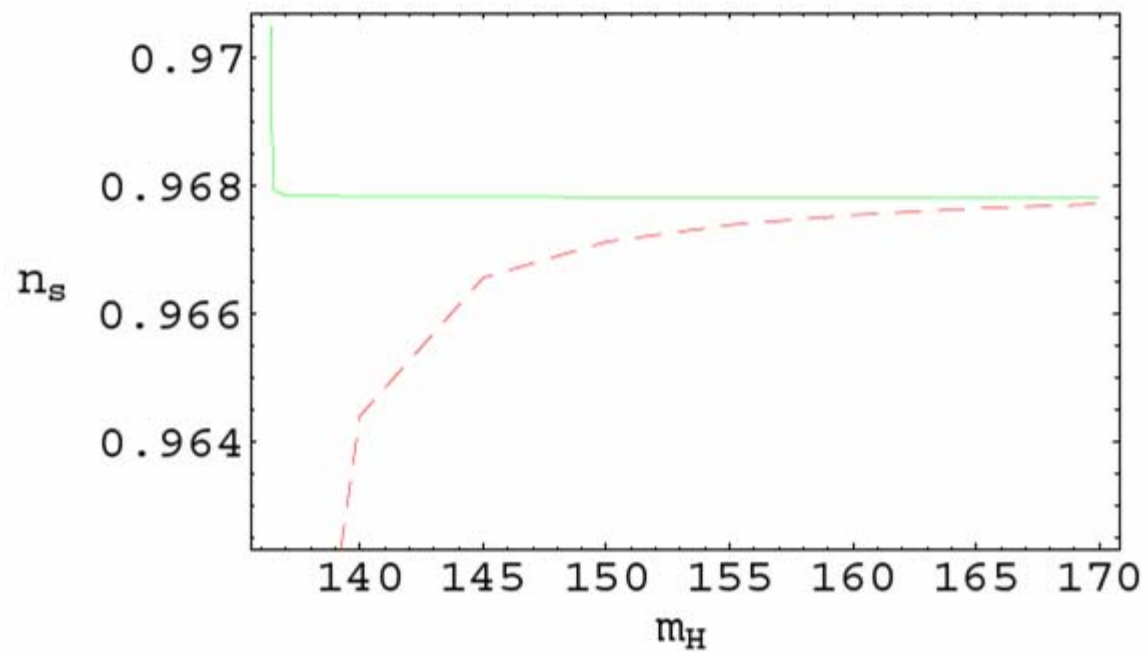
Another computation, 2 loop De Simone, Hertzberg, Wilczek,
Dec 31, 2008: $m_{min} = 126 \text{ GeV}$
Yet another computation, 2 loop Bezrukov, M.S.,
January 2009: $m_{min} = 128 \text{ GeV}, m_{max} = 181 \text{ GeV}$

Behaviour of the spectral index n_s

Physicists' Nightmare Scenario: The Higgs and Nothing Else

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If it has the right mass, the Higgs and nothing else **"would be the real five-star disaster, because that would mean there wouldn't need to be any new physics."**

—Jonathan Ellis, CERN

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All can be solved by adding three relatively light singlet leptons to the SM Lagrangian (νMSM).

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  - Dark energy is not a cosmological constant, $\omega > -1$

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