The Hubble flow:
an observer's perspective

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Figure 1:

B Examples of misconceptions or easily misinterpreted statements in the literature

In text books and works of popular science it is often standard practice to simplify arguments for the reader. Some of the quotes below fall into this category. We include them here to point out the difficulty encountered by someone starting in this field and trying to decipher what is really meant by 'the expansion of the Universe'.

[1] Feynman, R. P. 1995, *Feynman Lectures on Gravitation (1962/63)*, (Reading, Mass.: Addison-Wesley) p. 181, "It makes no sense to worry about the possibility of galaxies receding from us faster than light, whatever that means, since they would never be observable by hypothesis."

[2] Rindler, W. 1956, MNRAS, **6**, 662-667, *Visual Horizons in World-Models*, Rindler acknowledged that faster than c expansion is implicit in the mathematics, but expresses discomfort with the concept: "...certain physical difficulties seem to be inherent in models possessing a particle-horizon: if the model postulates point-creation we have material particles initially separating at speeds exceeding those of photons."

[3] McVittie, G. C. 1974, QJRAS, **15**, 246-263, *Distances and large redshifts*, Sect. 4, "These fallacious arguments would apparently show that many quasars had 'velocities of recession' greater than that of light, which contradicts one of the basic postulates of relativity theory."

[4] Weinberg, S. 1977, *The First Three Minutes*, (New York: Bantum Books), p. 27, "The conclusion generally drawn from this half century of observation is that the galaxies are receding from us, with speeds proportional to the distance (at least for speeds not too close to that of light).", see also p. 12 and p. 25. Weinberg makes a similar statement in his 1972 text *Gravitation and Cosmology* (New York: Wiley), p. 417, "a *relatively close* galaxy will move away from or toward the Milky Way, with a radial velocity [$v_{\rm rec} = \dot{R}(t_0)\chi$]." (emphasis ours). Shortly thereafter he adds a caution about SR and distant sources: "it is neither useful nor strictly correct to interpret the frequency shifts of light from very distant sources in terms of a special-relativistic Döppler shift alone. [The reader should be warned though, that astronomers conventionally report even large frequency shifts in terms of a recessional velocity, a "red shift" of v km/sec meaning that $z = v/(3 \times 10^5)$.]"

[5] Field, G. 1981, *This Special Galaxy*, in Section II of *Fire of life, the book of the Sun*, (Washington, DC: Smithsonian Books) "The entire universe is only a fraction of a kilometer across [after the first millionth of a second], but it expands at huge speeds — matter quite close to us being propelled at

How do we imagine the cosmic expansion? Usually this is a traditional image used in popular science, as well as in textbooks and even monographs. This is a "bird's-eye view" or a "god's view", when we find ourselves out of our space observing it from outside. For example, we imagine an inflating ball, or a stretching surface, which represent our expanding universe. More than that, it is convenient to imagine all points on this ball (or the surface) visible similtaniously, i.e. we see the whole picture "as it is now". Hence, we not only observe the universe "from outside", but also "see" all its points at the same time.

This is a useful image (maybe, it is necessary for understanding), however, a real observer never sees such a picture, it is impossible in principle. How the expansion would look like for an "internal" observer?

The flat Friedmann metric has the usual form:

$$ds^2 = c^2dt^2 - a(t)^2dl^2.$$

Here t is cosmic time. It is worth noting that cosmic time is not directly available to an "internal" observer who sees the universe as inhomogenious (the farther — the denser). The second term in the equation represents the Hubble flow — distant objects recede due to increasing scale factor a, while their comoving coordinates do not change.

As for comoving coordinates, it is natural to introduce a spherical system with an observer at the origin. Then distances and velocities defined below depend only on the radial comoving coordinate χ.

For the one-component FRW Universe with the perfect fluid in the form

$$p = w\rho$$

all formulae can be obtaned in a rather simple form.

The homogenious solution for the time evolution of the scale factor is $a \sim t^{1/\alpha}$, where $\alpha = 3(w+1)/2$.

Using the light propagation equation $ds^2 = 0$, the comoving coordinate of the object observable now at some redshift z can be obtained in a standard way for a given equation of state parameter α:

$$\chi = \frac{c}{a(t_0)H_0} \int_0^z \frac{dz}{H(z)} = \frac{c}{a(t_0)H_0} \frac{1}{1-\alpha} [(1+z)^{1-\alpha} - 1].$$

The proper distance is defined as $d = a\chi$. If we are interested in distances for $t = t_0$ (at the same moment of cosmic time for all sources) the scale factor in the equation should be equal to its present day value $a(t_0)$. To obtain the proper distance at the moment of light emission we need the scale factor at that time $a(t_{\rm em})$.

The general formula for the proper distance at the present moment, t_0, is the following:

$$d_{now} = \frac{c}{(1-\alpha)H_0}[(1+z)^{1-\alpha} - 1].$$

This distance is useful in order to imagine the general structure of the universe we live in. It grows monotonically with increasing redshift, tending to a finite value if $z \to \infty$ for $\alpha > 1$. This gives us an intuitively clear picture of a finite distance to the particle horizon.

The proper distance at the time of emission is

$$d_{em} = \frac{c}{H_0}\frac{1}{1-\alpha}[(1+z)^{1-\alpha} - 1]\frac{1}{(1+z)}.$$

In contrast to the present day proper distance (i.e., distance "now"), the dependence of the distance at emission upon the redshift is not monotonic for most realistic cosmological models. For example, for a dust-dominated universe ($p = 0$, so $\alpha = 3/2$) d_{em} reaches its maximum at $z = 5/4$ which is well within the currently observable universe.

Light trajectories in a flat universe are straight lines, so the angular size of distant objects grows with growing redshift has just this reason — objects with larger redshifts were at the time of emission closer to us than objects with lower redshifts (for z large enough).

From an observer's point of view, three different distances are often used:

- Angular distance d_Θ as a ratio of an object's physical transferse size to its angular size. It is easy to show that $d_\Theta = a(t_{\rm em})\chi$
- Proper motion distance $d_{\rm pm} = a(t_0)\chi$
- The photometric distance.It is defined as:

$$d_{\rm ph} = (L/4\pi f)^{1/2} = a^2(t_0)\frac{\chi}{a(t_{\rm em})},$$

  where L is the luminosity of an object, f is the observed flux. The photometric distance diverges at the event horizon. However, this is because of the energy dilution due to redshift encoded in the definition of this parameter.

It is clear that, formally, $d_{now} = d_{\rm pm}$ and $d_{em} = d_\theta$.

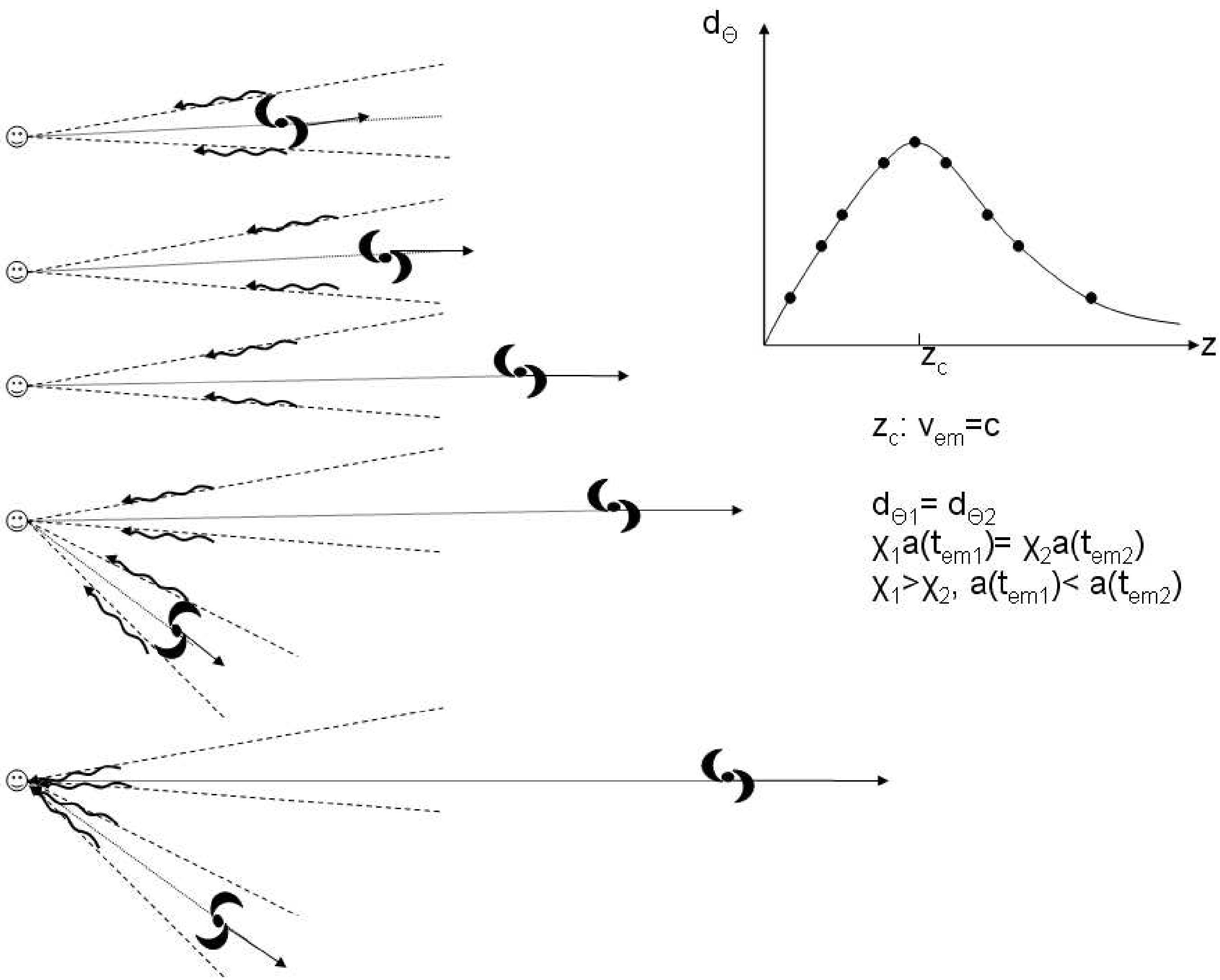

Figure 2:

Usually for the velocity of Hubble flow we have

$$\dot{d} = \dot{a}\chi,$$

because corresponding comoving coordinate does not change (we ignore pecular velocities, so $\dot{\chi} = 0$).

It is easy to see that for the Universe filled with a perfect fluid the velocity "now" is

$$v_{\mathrm{now}} = \frac{c}{1-\alpha}[(1+z)^{1-\alpha} - 1],$$

while the velocity at the moment of emission is

$$v_{\mathrm{em}} = \frac{c}{1-\alpha}[1 - (1+z)^{\alpha-1}].$$

Properties of the velocity at emission:

- Diverges for $z \to \infty$ for decelerating Universe
- Tends to final limit bigger than c for accelerating Universe (except de Sitter, where this limit is c

Properties of the velocity "now":

- Diverges for $z \to \infty$ for accelerating Universe
- Tends to a finite value bigger than c for $-1/3 < w < 1/3$
- Tends to a finite value smaller than c for $w > 1/3$

It is clear that the velocity now corresponds to the god's view, who can see all the Universe at the same cosmic time. What about the velocity at emission, is it corresponds to the observer's view? When we remember that cosmic time is used for derivation this velocity also, we understand that the answer is "no". The matter is that light signals emitted by the observed object during some time interval with respect to the cosmic time will reach the observer during bigger time measured by observer's clock, leading to smaller that $v_{\rm em}$ observed velocity. As a result, $v_{\rm em}$ also corresponds to god's view, who, being time traveller and observing the whole Universe at the time of emission, can see the observer and the observed object receeding with the relative velocity equal to $v_{\rm em}$. So, the question is: what the real observer can see?

Such kind of problem (when we totally neglect the existence of cosmic time and consider only observable values) begins to be practically important due to a possibility to detect time variation of redshifts (due to recession) in the near future. Corresponding formula for the redshift change measured by an observer's clock gives (Quercellini C, Amendola L, Balbi A, Cabella P, Quartin M *Phys. Rep.* **521** 95 (2012)):

$$\frac{dz}{dt} = H_0[1 + z - (1 + z)^\alpha].$$

If we also take into account that $\dot{H}/H^2 = -\alpha$, we obtain for the observable time derivative of the proper distance at the moment of emission the following estimate:

$$\frac{d(d_{\rm em})}{dt} \equiv \tilde{v}_{\rm em} = \frac{d(d_{\rm em})}{dH}\frac{dH}{dt} + \frac{d(d_{\rm em})}{dz}\frac{dz}{dt} = \frac{c}{1-\alpha}\frac{1-(1+z)^{\alpha-1}}{1+z}.$$

This velocity which is supposed to represent the velocity of the Hubble flow directly measured by an observer, does not generally coincide with any of velocities discussed above. The velocity at the moment of emission (defined with respect to cosmic time) differs from it by factor $(1+z)$ which represents the ratio of time intervals at the object at the moment of emission dt_1 and at the observer's location when he/she receives the signal $dt_2 = (1+z)dt_1$. As this velocity is, by definition, the rate of change of angular distance, we will denote it as $v_\Theta \equiv \tilde{v}_{\rm em}$.

Properties of the velocity v_Θ:

- It has a local maximum smaller than c for $w < 1/3$
- It reaches c at $z = \infty$ for $w = 1/3$
- it grows infinitly with z for $w > 1/3$

It is interesting that about 50 years ago such a velocity could pretend to be a measure of Hubble flow which is always subluminal, reaching c only at the particle horizon in the limit of ultrarelativistic equation of state of the matter filling the Universe. This equation of state have been considered as the stiffest for a physically reasonable matter in, for example, Landau-Lifshitz course of theoretical physics. Even now, the only "non-exotic" matter with stiffer equation of state is a massless scalar field – the object, strictly speaking, still existing only theoretically. However, keeping in mind that superluminal recession velocities are allowed, as well as the equation of state $p = \rho/3$ is not a limiting case in contemporary physics, we do not insist that abovemensioned asymptotics of the velocity v_Θ have some deep meaning.

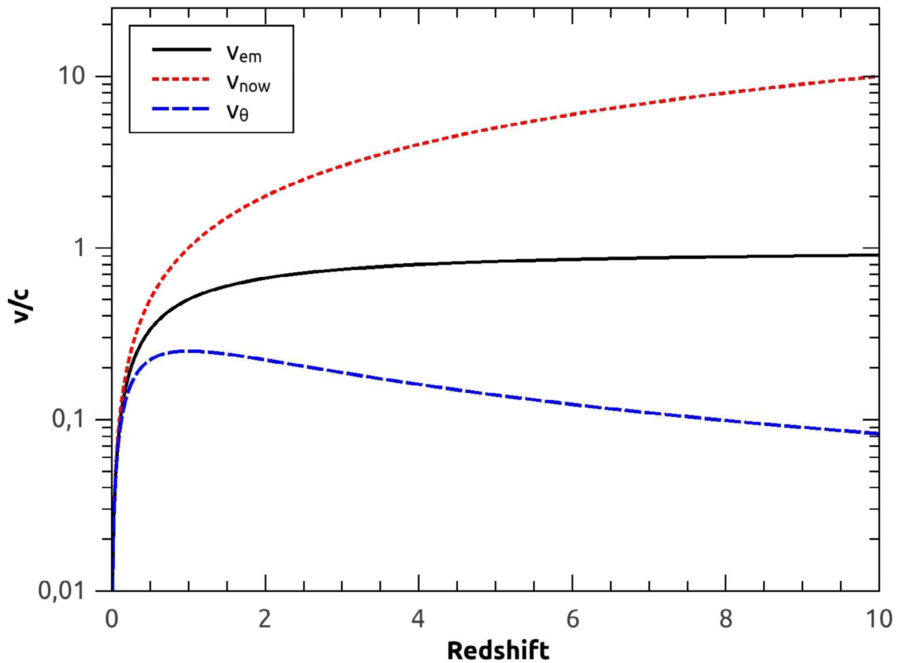

Figure 3:

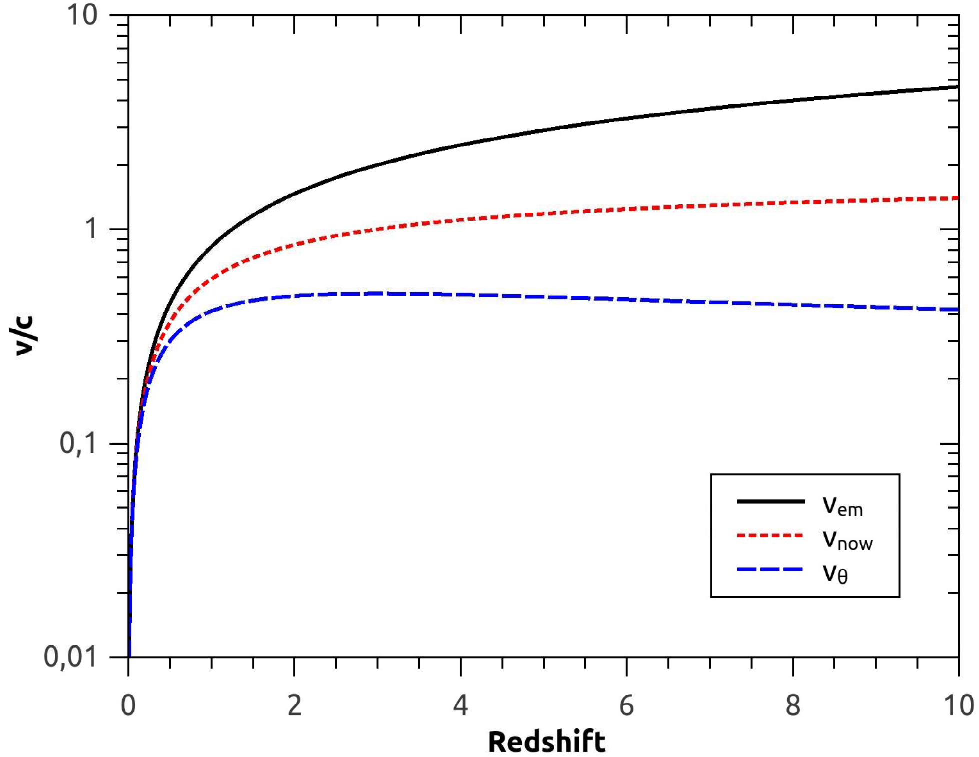

Figure 4:

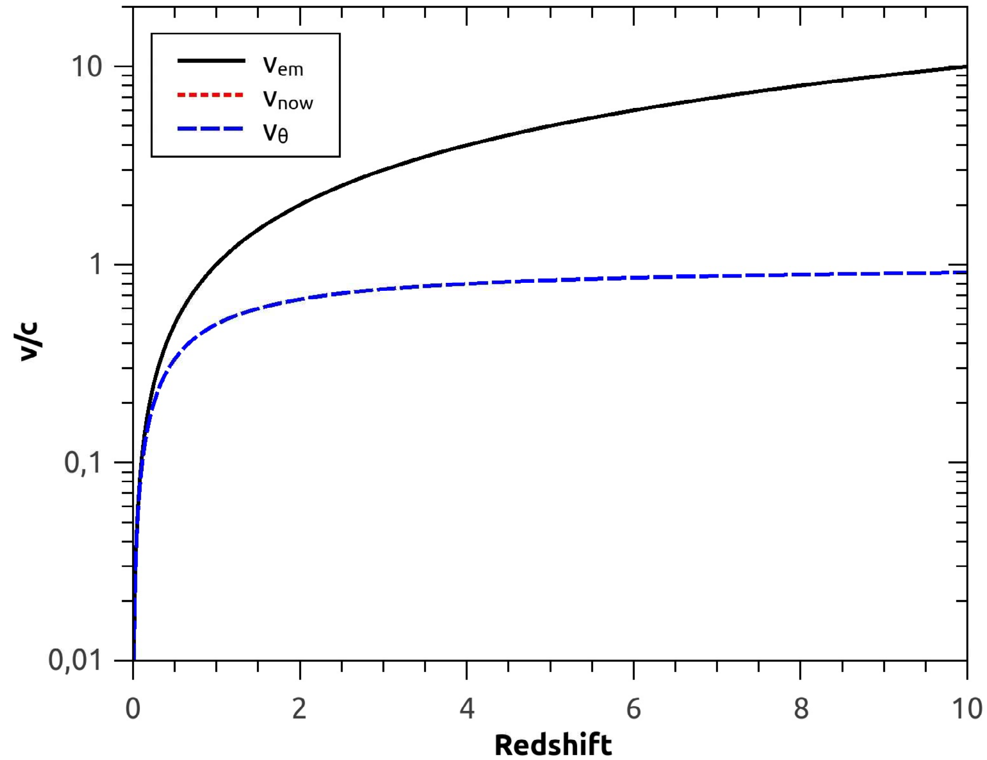

Figure 5:

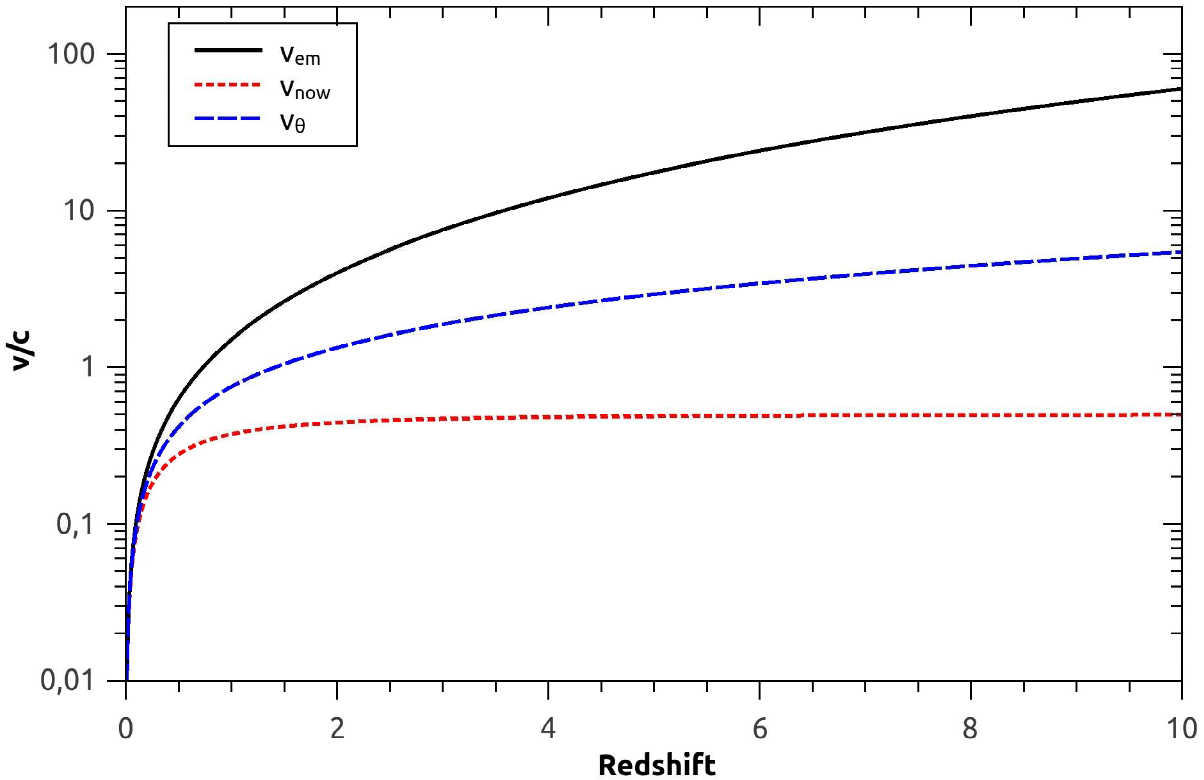

Figure 6:

CONCLUSION

In brief, our discussion can be summarized as follows:

- We want to define quantities which fit the best to our intuitive understanding of visible distance and velocity of the Hubble flow in the expanding universe.
- Proper distance is a fundamental quantity of the theory and does not depend on our observational abilities and current astrophysical knowledge.
- We see an object as it was at the moment of emission, so it is natural to consider the distance at the moment of emission as the characteristic of the source.

- Proper distance at the moment of emission can be calculated the same way as the angular distance. In addition, the angular distance and its derivative correspond to our psychological perception of receding and to intuitive expectations about the behavior of objects on the horizon. Therefore, just angular distance and its derivative (in time) are the most natural characteristics of the Hubble flow *from the observer's point of view*.