

THE CASIMIR EFFECT: FROM NANOTECHNOLOGY TO PHYSICS OF THE UNIVERSE

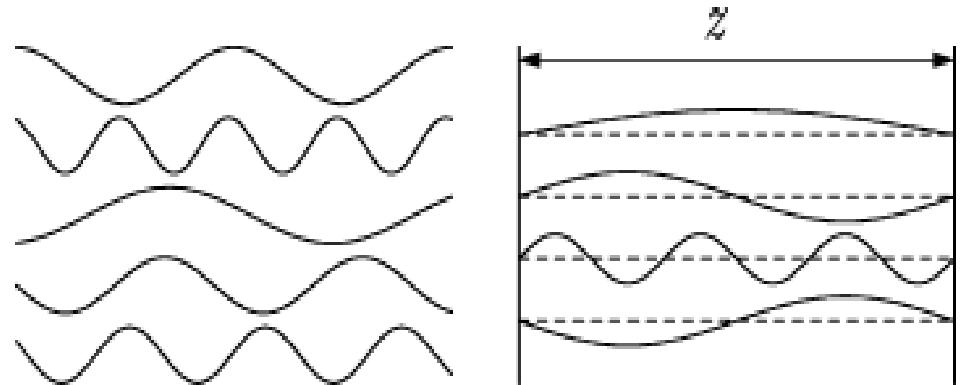
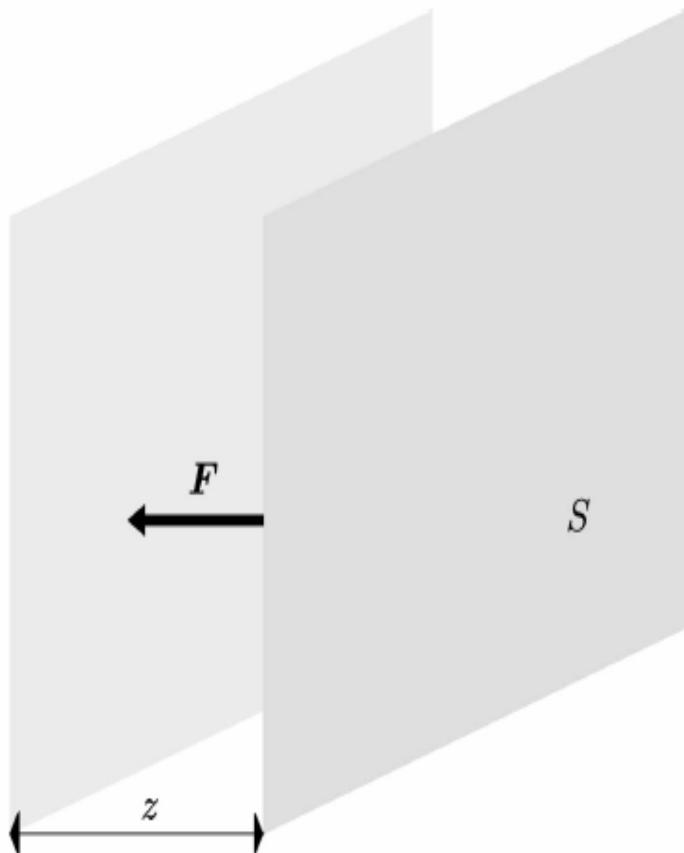
V. M. Mostepanenko

**Department of Astrophysics
Central Astronomical Observatory of the Russian
Academy of Sciences, St. Petersburg, Russia**

Content

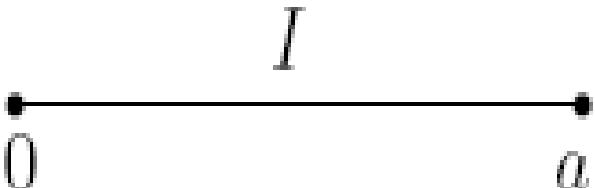
- 1. Introduction**
- 2. Two parallel ideal metal plates**
- 3. Cosmological models**
- 4. Lifshitz theory of the van der Waals and Casimir forces**
- 5. Comparison of the Lifshitz theory with the experimental data**
- 6. Constraints on corrections to Newton's gravitational law and parameters of dark matter from the Casimir effect**
- 7. Conclusions**

1. INTRODUCTION



The Casimir force arises due to the change of the spectrum of zero-point oscillations of the electromagnetic field by material boundaries.

Casimir, 1948

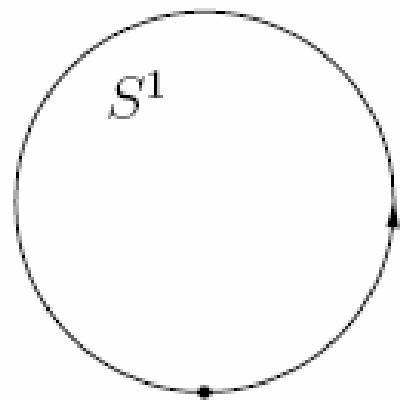


$$\varphi(t, 0) = \varphi(t, a) = 0$$

$$\omega_n=\frac{\pi n}{a},\quad n=1,\,2,\ldots$$

$$E=\frac{1}{2}\left(\sum_{n=1}^{\infty}\omega_n-a\int_{-\infty}^{\infty}\frac{dk}{2\pi}k\right)=-\frac{\pi}{24a}$$

$$F=-\frac{\partial E}{\partial a}=-\frac{\pi}{24a^2}$$



$$\varphi(t, 0) = \varphi(t, a)$$

$$\partial_x \varphi(t, x) \Big|_{x=0} = \partial_x \varphi(t, a) \Big|_{x=a}$$

$$\omega_n = \frac{2\pi n}{a}, \quad n = 0, \pm 1, \pm 2, \dots$$

$$E = \frac{1}{2} \left(\sum_{n=-\infty}^{\infty} \omega_n - a \int_{-\infty}^{\infty} \frac{dk}{2\pi} k \right) = -\frac{\pi}{6a}$$

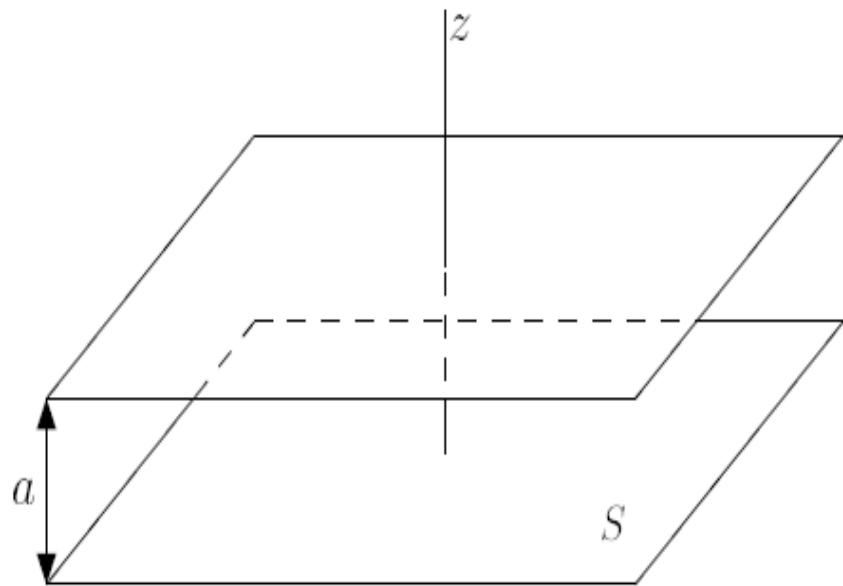
Nanotechnology

Micro- and nanoelectromechanical systems:

- **stiction of closely spaced elements;**
- **the Casimir force as a driving force at small distances.**

Chan, Aksyuk, Kleiman, Bishop, Capasso, Science, 2001;
Phys. Rev. Lett., 2001.

2. TWO PARALLEL IDEAL METAL PLATES



$$\mathbf{E}_t(t, \mathbf{r})|_S = \mathbf{B}_n(t, \mathbf{r})|_S = 0$$

2.1 The vacuum energy at zero temperature

$$E_0 = \int_V d\mathbf{r} \langle 0 | T_{00}^{(1)}(x) | 0 \rangle = \frac{1}{2} \sum_{\lambda=1}^2 \sum_J \omega_J = \sum_J \omega_J$$

$$\omega_{k_{\perp},n}^2 = k_{\perp}^2 + k_{zn}^2$$

$$\text{with } k_{zn} = \frac{\pi n}{a}, \quad n = 0, 1, 2, \dots$$

$$E_0(a) = \left(\frac{1}{2} \int_0^\infty \frac{k_\perp dk_\perp}{2\pi} \omega_{k_\perp,0} + \int_0^\infty \frac{k_\perp dk_\perp}{2\pi} \sum_{n=1}^\infty \omega_{k_\perp,n} \right) S$$

**After renormalization, i.e., subtraction
of the contribution of free space,**

$$E(a) \equiv E_{\text{IM}}(a) = -\frac{\pi^2}{720a^3}$$

$$P(a) = -\frac{\partial E(a)}{\partial a} = -\frac{\pi^2}{240a^4}$$

2.2 TWO PARALLEL IDEAL METAL PLATES AT NONZERO TEMPERATURE

$$\begin{aligned}\mathcal{F}_0(a, T) = & \int_0^\infty \frac{k_\perp dk_\perp}{2\pi} \left\{ \frac{1}{2}\omega_{k_\perp,0} + k_B T \ln \left(1 - e^{-\omega_{k_\perp,0}/k_B T} \right) \right. \\ & \left. + 2 \sum_{n=1}^{\infty} \left[\frac{1}{2}\omega_{k_\perp,n} + k_B T \ln \left(1 - e^{-\omega_{k_\perp,n}/k_B T} \right) \right] \right\} S\end{aligned}$$

After renormalization:

$$\mathcal{F}(a, T) = \frac{k_B T}{\pi} \sum_{l=0}^{\infty}' \int_0^{\infty} k_{\perp} dk_{\perp} \ln \left(1 - e^{-2a\sqrt{k_{\perp}^2 + \xi_l^2}} \right)$$

$$\xi_l = 2\pi k_B T l$$

$$P(a, T) = -\frac{\partial \mathcal{F}(a, T)}{\partial a} = -\frac{2k_B T}{\pi} \sum_{l=0}^{\infty}' \int_0^{\infty} k_{\perp} dk_{\perp} \frac{\sqrt{k_{\perp}^2 + \xi_l^2}}{e^{2a\sqrt{k_{\perp}^2 + \xi_l^2}} - 1}$$

$$S(a, T) = -\frac{\partial \mathcal{F}(a, T)}{\partial T} = -\frac{1}{T} \mathcal{F}(a, T) + \frac{k_B}{\pi} \sum_{l=1}^{\infty} \xi_l^2 \ln \left(1 - e^{-2a\xi_l} \right)$$

Limit of low temperature: $T \ll T_{\text{eff}} = \frac{1}{2ak_B}$

$$\mathcal{F}(a, T) = -\frac{\pi^2}{720a^3} \left[1 + \frac{45\zeta_R(3)}{\pi^3} \left(\frac{T}{T_{\text{eff}}} \right)^3 - \left(\frac{T}{T_{\text{eff}}} \right)^4 \right]$$

$$P(a, T) = -\frac{\pi^2}{240a^4} \left[1 + \frac{1}{3} \left(\frac{T}{T_{\text{eff}}} \right)^4 \right]$$

$$S(a, T) = \frac{3\zeta_R(3)k_B}{8\pi a^2} \left(\frac{T}{T_{\text{eff}}} \right)^2 \left[1 - \frac{4\pi^3}{135\zeta_R(3)} \frac{T}{T_{\text{eff}}} \right]$$

Limit of high temperature: $T \gg T_{\text{eff}}$

$$\mathcal{F}(a, T) = -\frac{k_{\text{B}}T}{8\pi a^2} \zeta_{\text{R}}(3)$$

$$P(a, T) = -\frac{k_{\text{B}}T}{4\pi a^3} \zeta_{\text{R}}(3)$$

$$S(a, T) = \frac{k_{\text{B}}}{8\pi a^2} \zeta_{\text{R}}(3)$$

3. COSMOLOGICAL MODELS

Einstein universe and closed Friedmann model

$$\begin{aligned}ds^2 &= dt^2 - a_0^2 [dr^2 + \sin^2 r (d\theta^2 + \sin^2 \theta d\varphi^2)] \\&= a_0^2 [d\eta^2 - dr^2 - \sin^2 r (d\theta^2 + \sin^2 \theta d\varphi^2)]\end{aligned}$$

$$\underline{T=0}$$

$$\langle 0 | T_0^{\ 0} | 0 \rangle = \frac{1}{4\pi^2 a_0^3} \sum_{k=1}^{\infty} k^2 \omega_k, \quad \quad \omega_k^2 = m^2 + \frac{k^2}{a_0^2}$$

$$\varepsilon = \frac{1}{2\pi^2 a_0^4} \int_{ma_0}^{\infty} \frac{\xi^2 d\xi}{\exp(2\pi\xi) - 1} \sqrt{\xi^2 - m^2 a_0^2}$$

Mamaev, Mostepanenko, Starobinsky, Sov. Phys. JETP, 1976

$$P=-\frac{\partial E}{\partial V}, \quad \text{where} \quad E=\varepsilon V, \quad V=2\pi^2a_0^3$$

$$\varepsilon = \frac{1}{2\pi^2a_0^4}\int_0^\infty \frac{\xi^3\,d\xi}{\exp(2\pi\xi)-1} = \frac{1}{480\pi^2a_0^4}$$

$$P=\frac{\varepsilon}{3}=\frac{1}{1440\pi^2a_0^4}$$

Inflation due to the Casimir effect

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2),$$

Topology of a 3-torus; the following points are identified:

$$(x + lL, y + nL, z + pL), \quad l, n, p = 0, \pm 1, \pm 2, \dots$$

$$\varepsilon(a) = -\frac{A}{a^4(t)L^4}, \quad A = 0.8375$$

From Einstein equations with the cosmological constant Λ

$$a(t) = \frac{1}{L} \left(\frac{8\pi G A}{\Lambda} \right)^{1/4} \left[\cosh \left(2\sqrt{\frac{\Lambda}{3}} t \right) \right]^{1/2}$$

Zel'dovich, Starobinsky, Sov. Astron. Lett., 1984

$$\underline{T\neq 0}$$

$$F=V\varepsilon+k_BT\sum_{k=1}^{\infty}k^2\ln\left(1-e^{-\frac{\omega_k}{k_BT}}\right)$$

Dowker, Critchley, Phys. Rev. D, 1977

$$\underline{a_0 k_B T \ll 1}$$

$$F \approx V\varepsilon - k_B T e^{-\frac{1}{a_0 k_B T}}$$

$$U = -T^2 \frac{\partial}{\partial T} \left(\frac{F}{T} \right) \approx V\varepsilon + \frac{1}{a_0} e^{-\frac{1}{a_0 k_B T}}$$

$$\underline{a_0 k_B T \gg 1}$$

$$F \approx -\frac{\pi^2}{90} V (k_B T)^4, \quad U \approx \frac{\pi^2}{30} V (k_B T)^4$$

Modified renormalization

$$F^{\text{ren}} = F - F_M$$

$$F_M = -\frac{\pi^2}{90}(k_B T)^4 V$$

**Bezerra, Klimchitskaya, Mostepanenko, Romero,
Phys. Rev. D, 2011**

Thermal Casimir effect for spinor and electromagnetic fields in closed Friedmann model

New aspects:

- spinor Casimir effect at high T does not possess the classical limit;**
- electromagnetic Casimir effect at high T does possess the classical limit.**

Bezerra, Mostepanenko, Mota, Romero, Phys. Rev. D, 2011.

4. LIFSHITZ THEORY OF THE VAN DER WAALS AND CASIMIR FORCES

Maxwell equations

$$\nabla \cdot \mathbf{D}(t, \mathbf{r}) = 0, \quad \nabla \times \mathbf{E}(t, \mathbf{r}) + \frac{1}{c} \frac{\partial \mathbf{B}(t, \mathbf{r})}{\partial t} = 0$$
$$\nabla \times \mathbf{B}(t, \mathbf{r}) - \frac{1}{c} \frac{\partial \mathbf{D}(t, \mathbf{r})}{\partial t} = 0, \quad \nabla \cdot \mathbf{B}(t, \mathbf{r}) = 0$$

Boundary conditions

$$E_{1t}(t, \mathbf{r}) = E_{2t}(t, \mathbf{r}), \quad D_{1n}(t, \mathbf{r}) = D_{2n}(t, \mathbf{r})$$
$$B_{1n}(t, \mathbf{r}) = B_{2n}(t, \mathbf{r}), \quad B_{1t}(t, \mathbf{r}) = B_{2t}(t, \mathbf{r})$$

$$\boldsymbol{D}(t,\boldsymbol{r})=\int_{-\infty}^t\varepsilon(t-t',\boldsymbol{r})\boldsymbol{E}(t',\boldsymbol{r})\,dt'$$

$$\boldsymbol{E}(t,\boldsymbol{r})=\!\int_{-\infty}^\infty\!\boldsymbol{E}(\omega,\boldsymbol{r}){\rm e}^{-{\rm i}\omega t}\,d\omega,\quad \boldsymbol{D}(t,\boldsymbol{r})=\!\int_{-\infty}^\infty\!\boldsymbol{D}(\omega,\boldsymbol{r}){\rm e}^{-{\rm i}\omega t}\,d\omega$$

$$\boldsymbol{D}(\omega,\boldsymbol{r})=\varepsilon(\omega,\boldsymbol{r})\boldsymbol{E}(\omega,\boldsymbol{r})$$

$$\mathcal{F}(a, T) = \frac{k_B T}{2\pi} \sum_{l=0}^{\infty}' \Phi_E(\xi_l), \quad P(a, T) = -\frac{\partial \mathcal{F}(a, T)}{\partial a},$$

$$\xi_l = 2\pi \frac{k_B T l}{\hbar} \quad \text{Matsubara frequencies,}$$

$$\Phi_E(x) = \int_0^\infty k_\perp dk_\perp \sum_\alpha \ln \left[1 - r_\alpha^{(1)}(ix, k_\perp) r_\alpha^{(2)}(ix, k_\perp) e^{-2aq} \right]$$

$$q \equiv q(ix, k_\perp) = \sqrt{k_\perp^2 + x^2/c^2}$$

Lifshitz, Sov. Phys. JETP, 1956

Reflection coefficients for two independent polarizations:

$$r_{\text{TM}}^{(n)}(ix, k_{\perp}) = \frac{\varepsilon^{(n)}(ix)q - k^{(n)}}{\varepsilon^{(n)}(ix)q + k^{(n)}}$$

$$r_{\text{TE}}^{(n)}(ix, k_{\perp}) = \frac{\mu^{(n)}(ix)q - k^{(n)}}{\mu^{(n)}(ix)q + k^{(n)}}$$

$$k^{(n)} \equiv k^{(n)}(ix, k_{\perp}) = \sqrt{k_{\perp}^2 + \varepsilon^{(n)}(ix)\mu^{(n)}(ix)\frac{x^2}{c^2}}$$

Models of the frequency-dependent dielectric permittivity

$$\epsilon_c(\omega) = 1 + \sum_{j=1}^K \frac{g_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}$$

**Permittivity of dielectric plates
as determined by core electrons**

$$\epsilon_d(\omega) = \epsilon_c(\omega) + i \frac{4\pi\sigma_0(T)}{\omega}$$

**Permittivity of dielectric plates
with dc conductivity included**

$$\epsilon_D(\omega) = \epsilon_c(\omega) - \frac{\omega_p^2}{\omega[\omega + i\gamma(T)]}$$

**The Drude model permittivity
for metallic plates**

$$\epsilon_p(\omega) = \epsilon_c(\omega) - \frac{\omega_p^2}{\omega^2}$$

**The plasma model permittivity
for metallic plates**

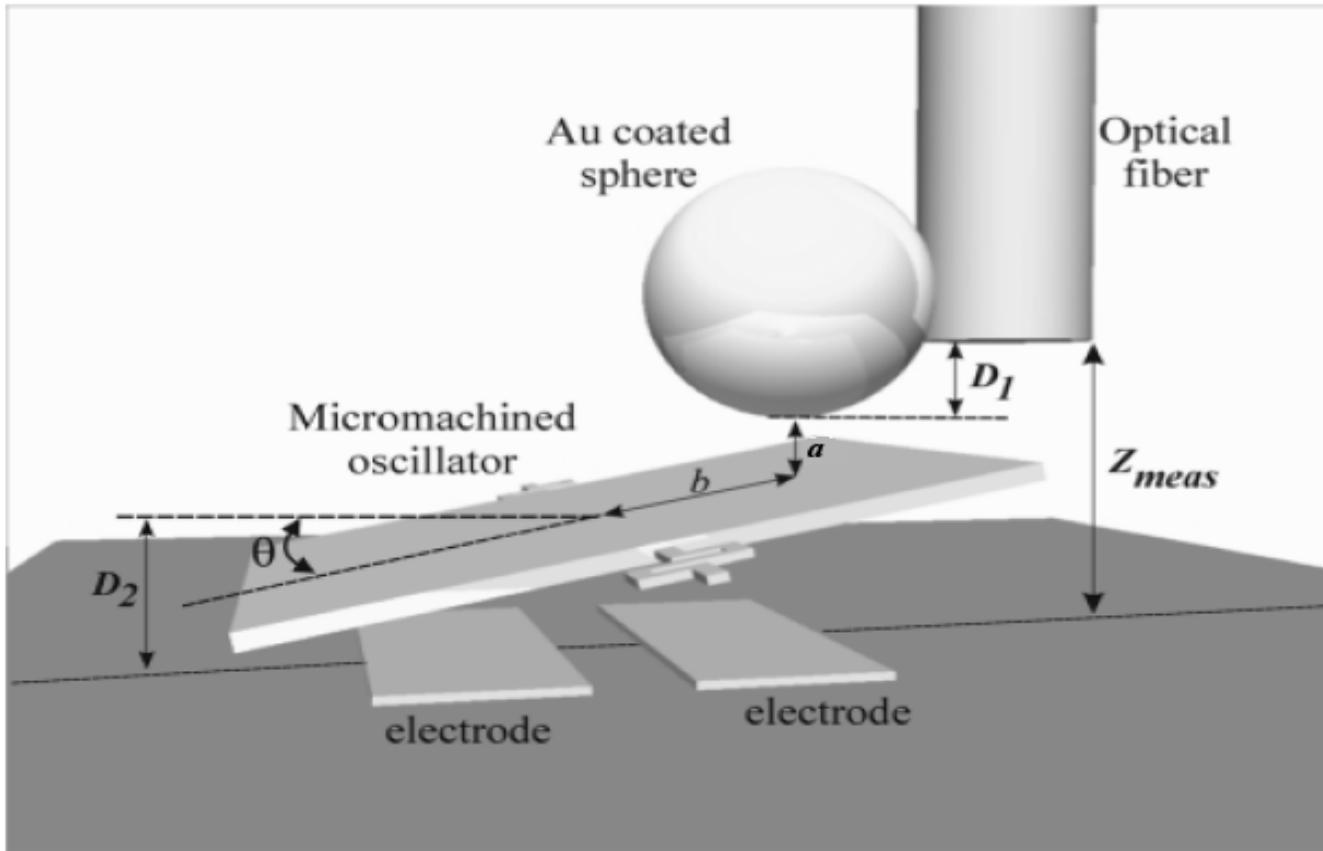
The Lifshitz theory with ϵ_d or ϵ_D violates the Nernst theorem

5. COMPARISON OF THE LIFSHITS THEORY WITH THE EXPERIMENTAL DATA

5.1 Indirect measurement of the Casimir pressure using a micromachined oscillator

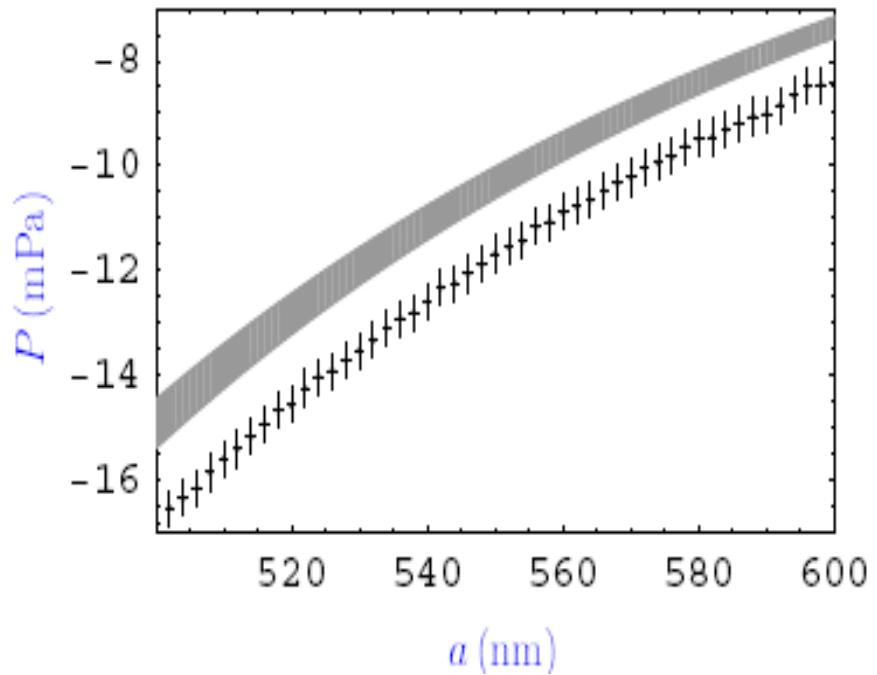
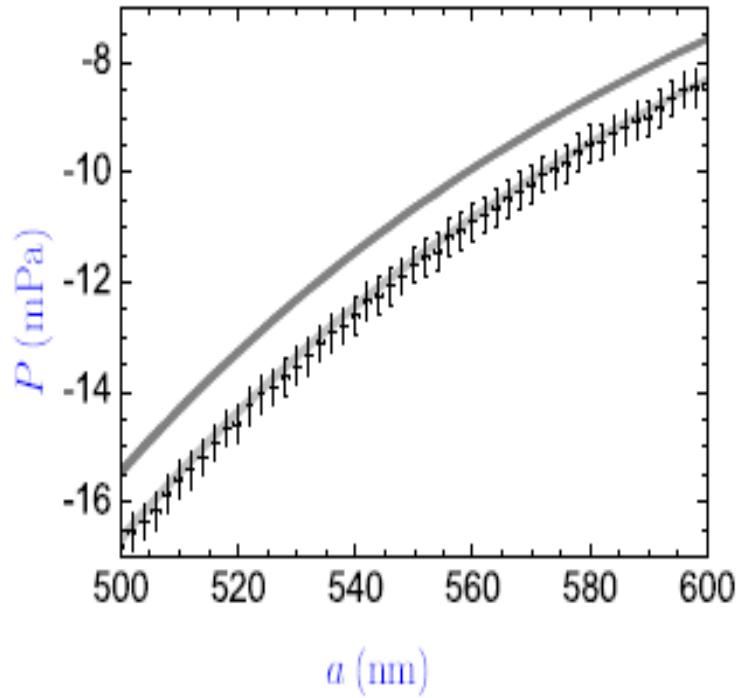
$$P(a, T) = -\frac{1}{2\pi R} \frac{\partial F_{sp}(a, T)}{\partial a}$$

Decca, Lopez, Fischbach, Klimchitskaya, Krause, Mostepanenko,
Phys. Rev. D (2003); Ann. Phys. (2005); Phys. Rev. D (2007);
Eur. Phys. J C (2007); Decca, Lopez, Osquigui, Int. J. Mod. Phys. A
(2010).



Schematic setup with a micromachined oscillator

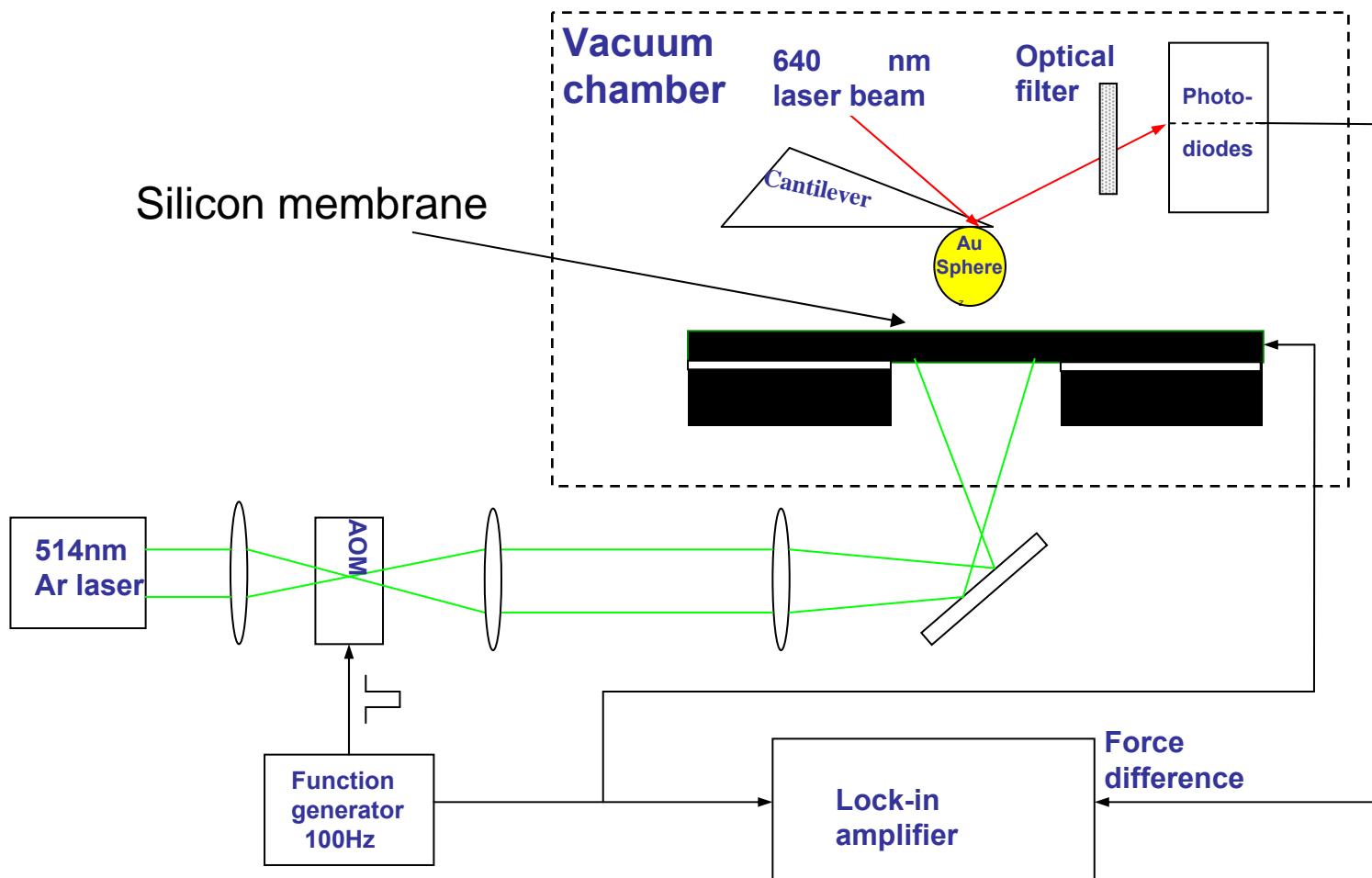
Comparison between experiment and theory



The relative experimental error (at a 95% confidence level) varies from 0.19% at 162 nm to 0.9% at 400 nm and 9% at 746 nm.

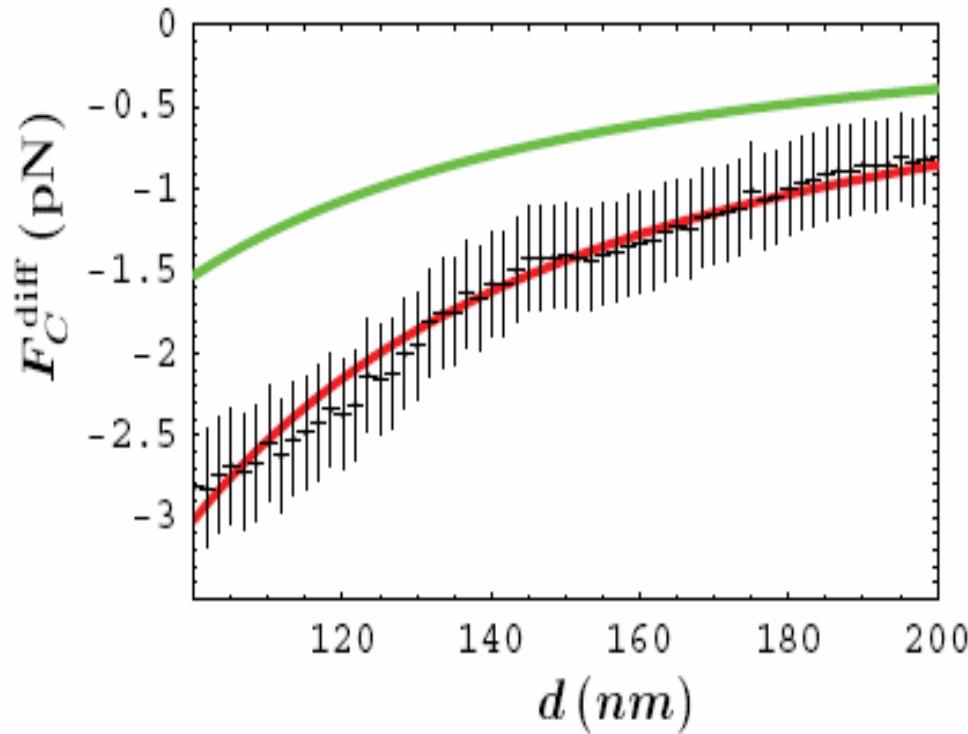
The Drude model is excluded by the data at a 95% confidence level.

5.2 Optical modulation of the Casimir force

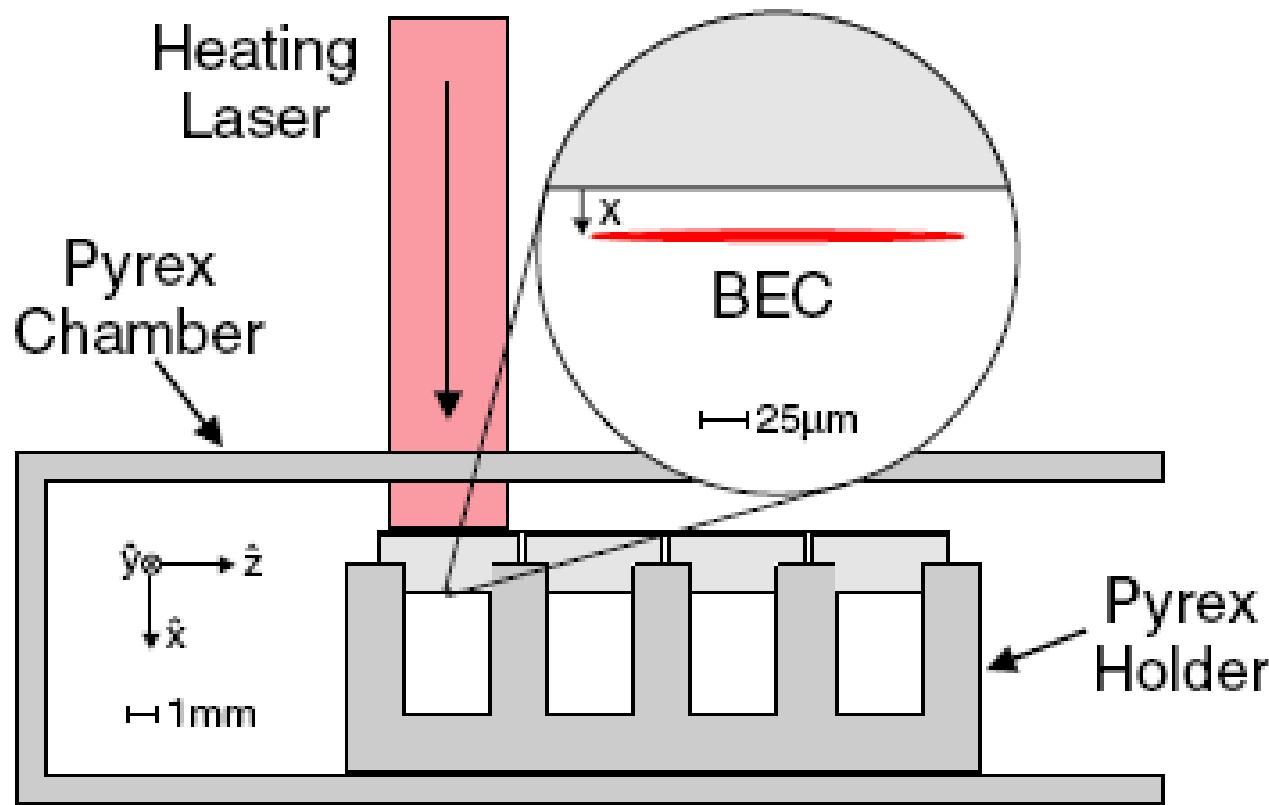


Chen, Klimchitskaya, Mostepanenko, Mohideen,
Optics Express (2007); Phys. Rev. B (2007).

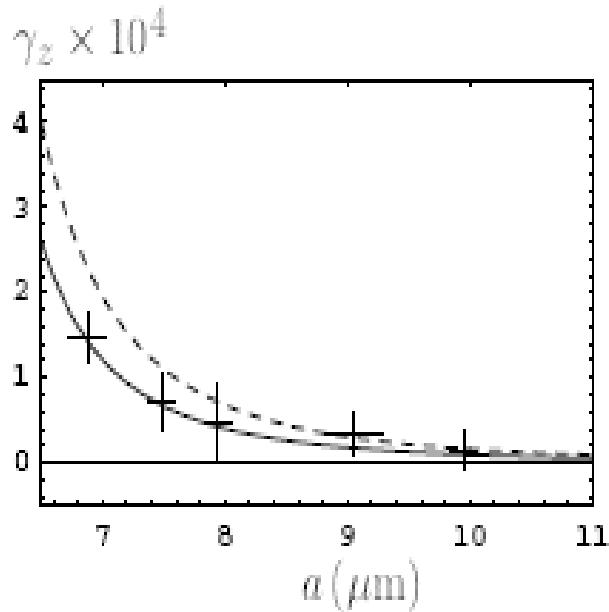
Difference of the Casimir force in the presence and absence of laser light



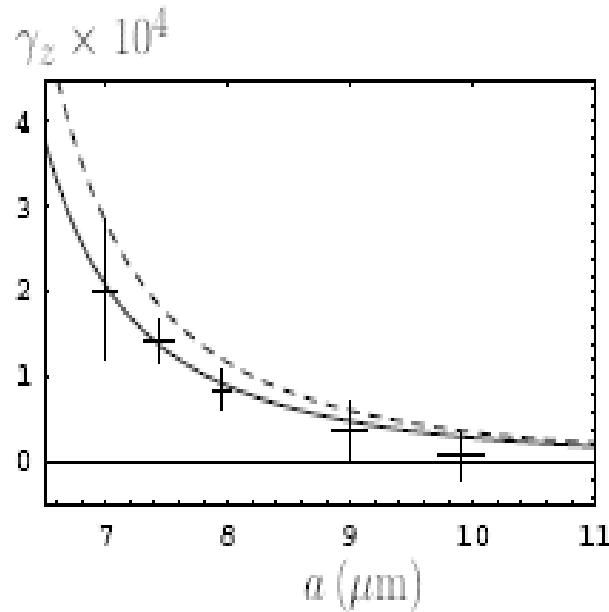
5.3. Measurement of the Casimir-Polder force



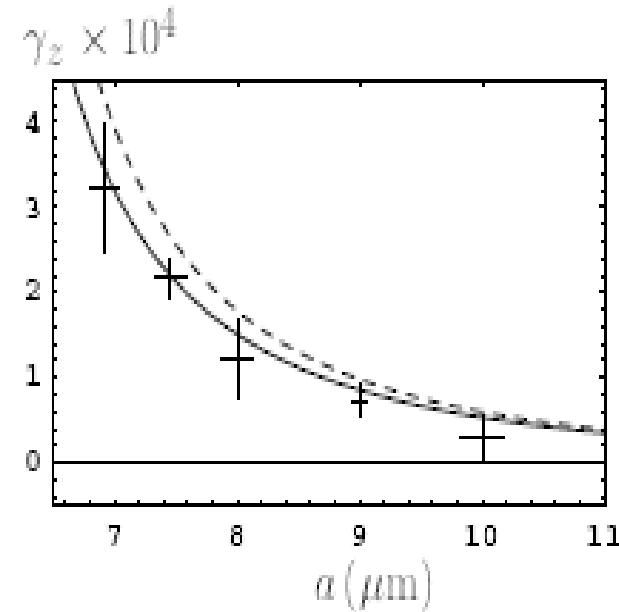
Frequency shift of center-of-mass oscillations of Bose-Einstein condensate of Rb atoms



$$T_p = T_e = 310 \text{ K}$$



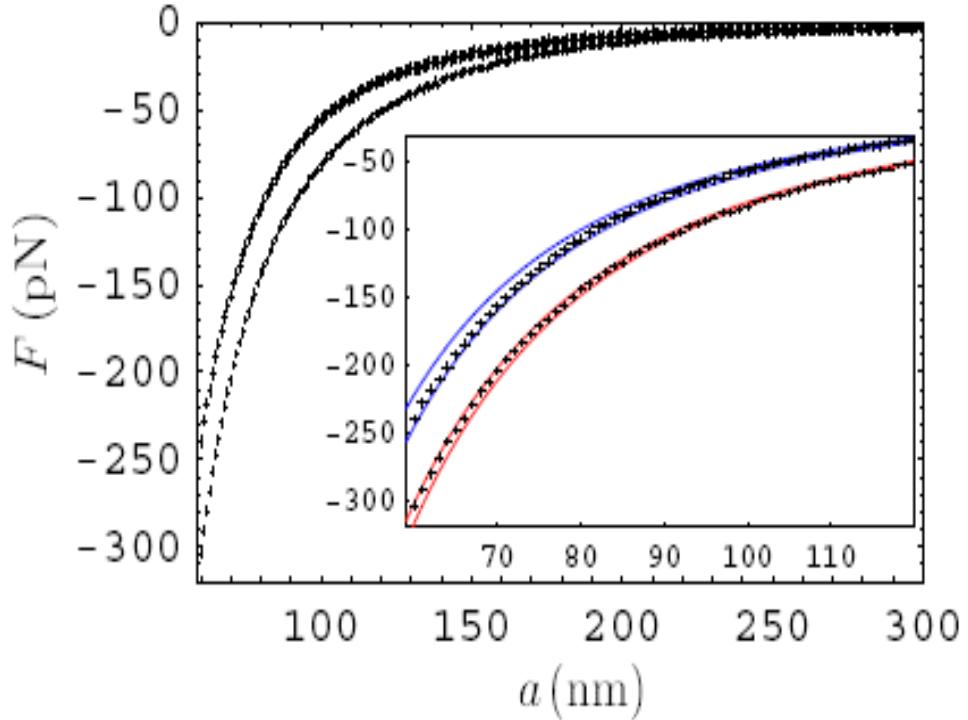
$$T_p = 479 \text{ K}$$



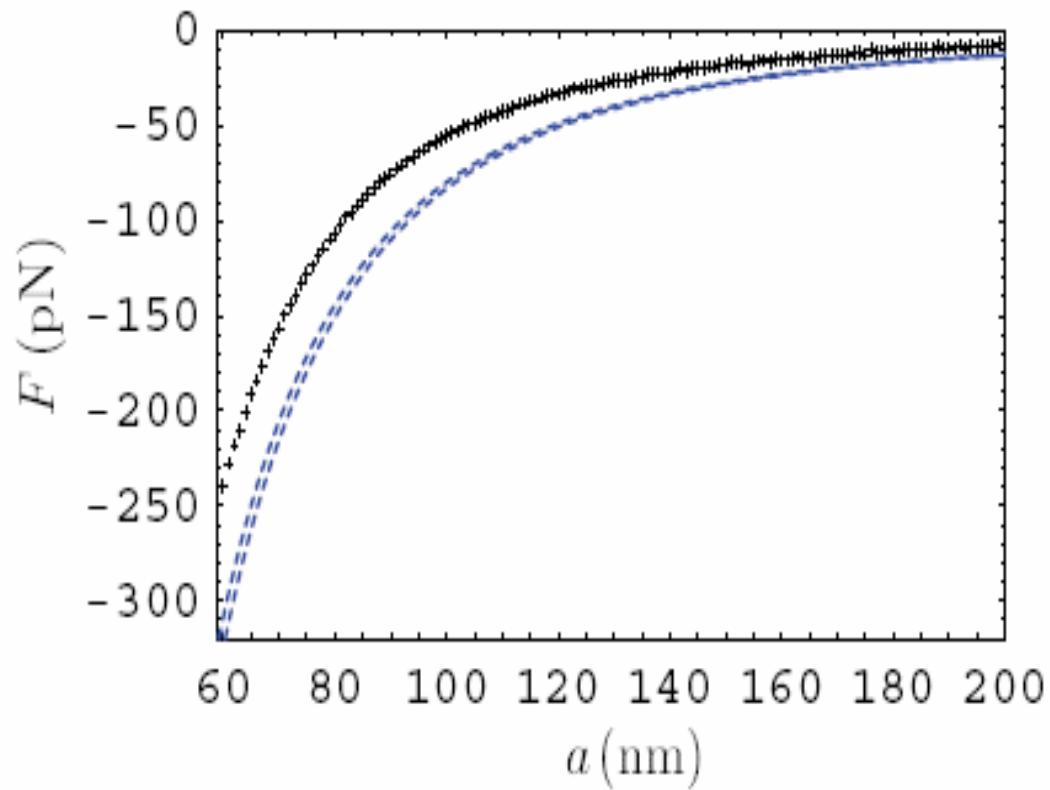
$$T_p = 605 \text{ K}$$

Obrecht, Wild, Antezza, Pitaevskii, Stringari, Cornell, Phys. Rev. Lett. (2007); Klimchitskaya, Mostepanenko, J. Phys. A (2008).

5.4 The Casimir force between Au sphere and ITO plate



**Chang, Banishev, Klimchitskaya, Mostepanenko,
Mohideen, Phys. Rev. Lett. (2011);
Banishev, Chang, Castillo-Garza, Klimchitskaya,
Mostepanenko, Mohideen, Phys. Rev. B (2012)**



6. Constraints on corrections to Newton's gravitational law and parameters of dark matter from the Casimir effect

Yukawa-type corrections to Newton's law:

$$V(r) = -\frac{Gm_1m_2}{r} \left(1 + \alpha e^{-r/\lambda}\right)$$

Power-type corrections to Newton's law:

$$V_l(r) = -\frac{Gm_1m_2}{r} \left[1 + \Lambda_l \left(\frac{r_0}{r}\right)^{l-1}\right]$$

Yukawa- and power-type potentials originate from:

1) Exchange of light and massless elementary particles,
such as:

- **arion;**
- **scalar axion;**
- **graviphoton;**
- **dilaton;**
- **goldstino;**
- **moduli.**

These particles may contribute
to the dark matter and dark energy.

2) Extra-dimensional theories with low-energy compactification scale

$$R_* \sim \frac{1}{E_{\text{Pl}}^{(D)}} \left(\frac{E_{\text{Pl}}}{E_{\text{Pl}}^{(D)}} \right)^{2/N} \sim 10^{(32-17N)/N} \text{ cm}$$

$$D \equiv 4 + N, \quad E_{\text{Pl}} = G^{-1/2} \sim 10^{19} \text{ GeV}$$

$$E_{\text{Pl}}^{(D)} = G_D^{-1/(2+N)}, \quad G_D = G\Omega_N \sim G R_*^N$$

For $N = 1$ $R_* \sim 10^{15} \text{ cm}$

$$N = 2 \quad R_* \sim 1 \text{ mm}$$

$$N = 3 \quad R_* \sim 5 \text{ nm}$$

Arkani-Hamed, Dimopoulos,
Dvali, Phys. Rev. D, (1999)

The Yukawa-type force between two macrobodies

$$E_{\text{Yu}}(a) = G\alpha\rho_1\rho_2 \int_{V_1} \int_{V_2} d\mathbf{r}_1 d\mathbf{r}_2 \frac{e^{-|\mathbf{r}_1 - \mathbf{r}_2|/\lambda}}{|\mathbf{r}_1 - \mathbf{r}_2|}$$

$$F_{\text{Yu}} = -\frac{\partial E_{\text{Yu}}(a)}{\partial a}$$

CONSTRAINTS FROM THE NORMAL CASIMIR FORCE BETWEEN TEST BODIES WITH SMOOTH SURFACES

Measured quantities are the Casimir force or its gradient:

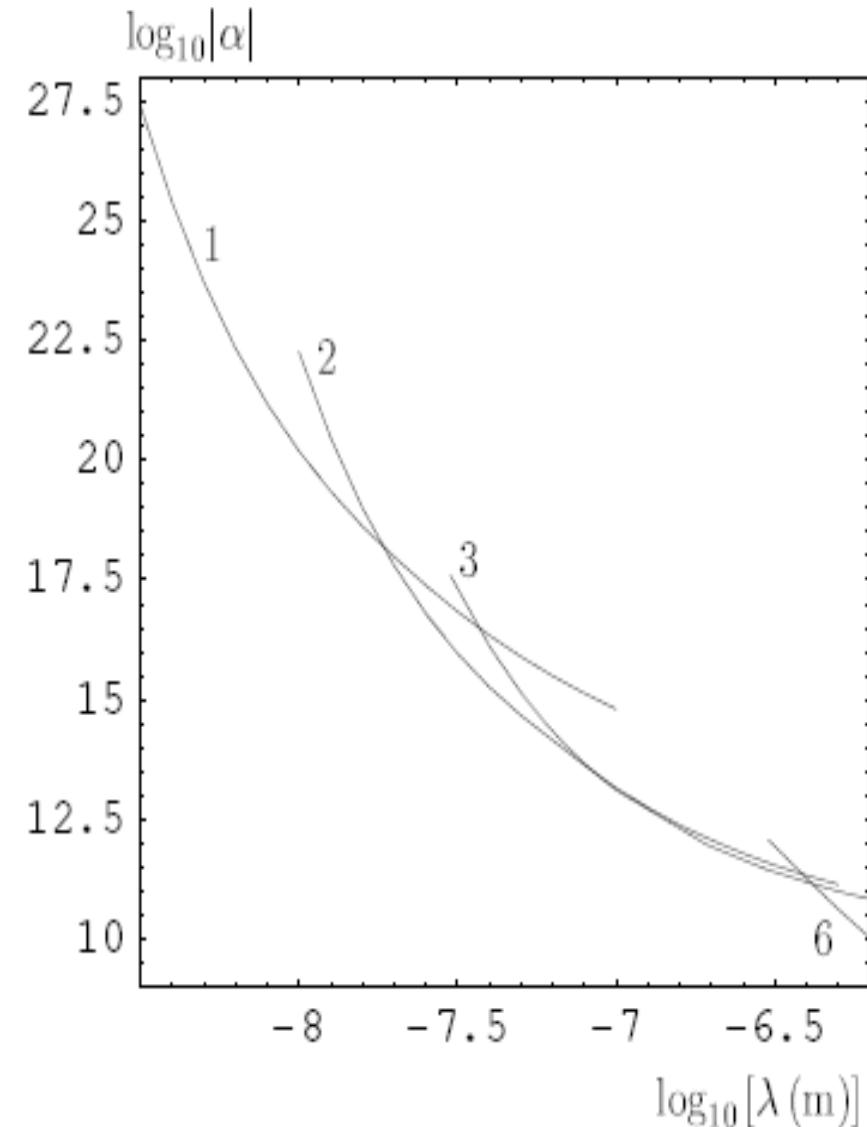
$$F_C(a, T) \quad \text{or} \quad F'_C(a, T) = \frac{\partial F_C(a, T)}{\partial a}$$

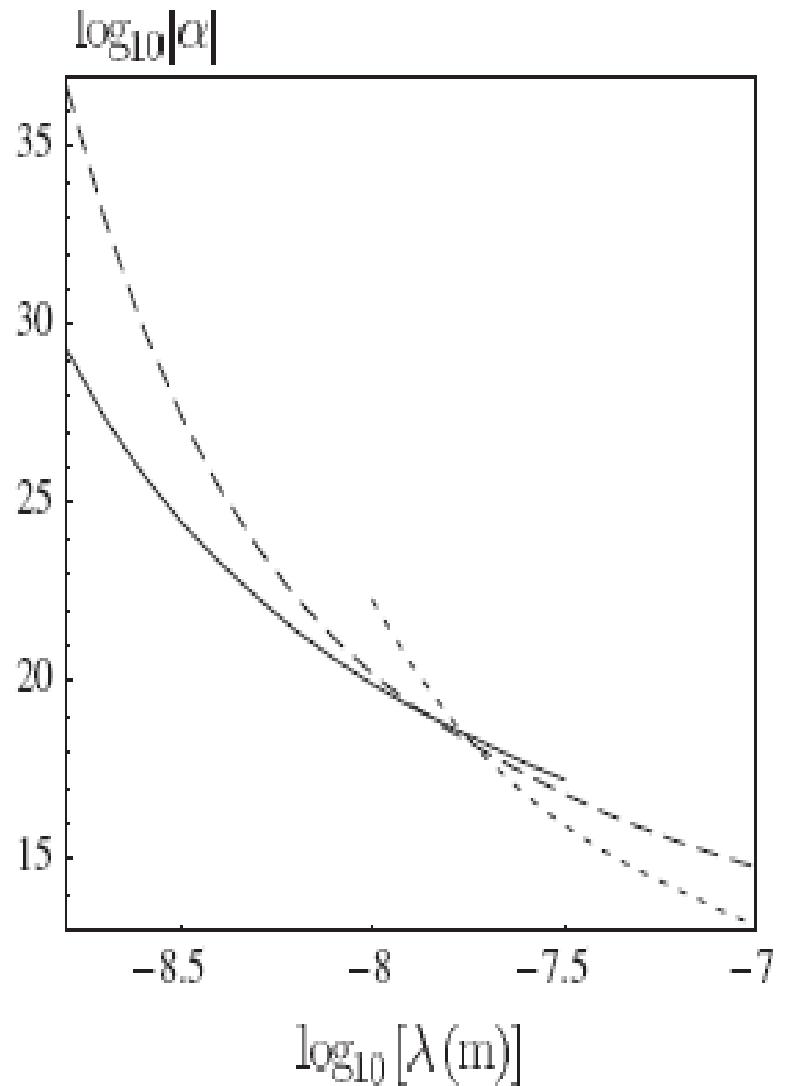
Obtaining constraints on Yukawa forces:

$$|F_{\text{Yu}}(a)| \leq \Delta^{\text{tot}} F_C(a, T) \quad \text{or} \quad |F'_{\text{Yu}}(a)| \leq \Delta^{\text{tot}} F'_C(a, T)$$

The strongest constraints on Yukawa-type corrections to Newton's gravitational law

obtained from the measurement
of the Casimir force using an atomic
force microscope (line 1),
from the measurement of the
Casimir pressure by means of a
micromachined oscillator (line 2),
and from the Casimir-less
experiment (line 3).
Line 6 indicates constraints obtained
from the torsion pendulum experiment





Constraints on the parameters of Yukawa-type interaction

from measurements of the lateral Casimir force between corrugated surfaces (the solid line),

and from measurements of the normal Casimir force by means of an atomic force microscope (the long-dashed line), and a micromachined oscillator (the short-dashed line).

Bezerra, Klimchitskaya, Mostepanenko, Romero,
Phys. Rev. D (2010); Phys. Rev. D (2011).

7. CONCLUSIONS

The Casimir effect is a multidisciplinary physical phenomenon . Besides applications to:

--- nanotechnology

--- condensed matter physics

--- cosmological models of the Universe

--- physics of dark matter,

which were touched above,

the Casimir effect plays an important role in

--- statistical physics

--- atomic spectroscopy

--- physics of elementary particles

--- surface science

--- mathematical physics.

**This promises new prospective results to
the Casimir physics in near future.**

INTERNATIONAL SERIES OF MONOGRAPHS ON PHYSICS • 145

Advances in the Casimir Effect

M. BORDAG
G. L. KLIMCHITSKAYA
U. MOHIDEEN
V. M. MOSTEFANENKO



OXFORD SCIENCE PUBLICATIONS