# STELLAR SUPERFLUIDS

## Probing the Neutron Star Interior from their Thermal Evolution

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unam

# STELLAR SUPERFLUIDS

## Probing the Neutron Star Interior from their Thermal Evolution


Collaborators:

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Charles de Gaule

Meudon

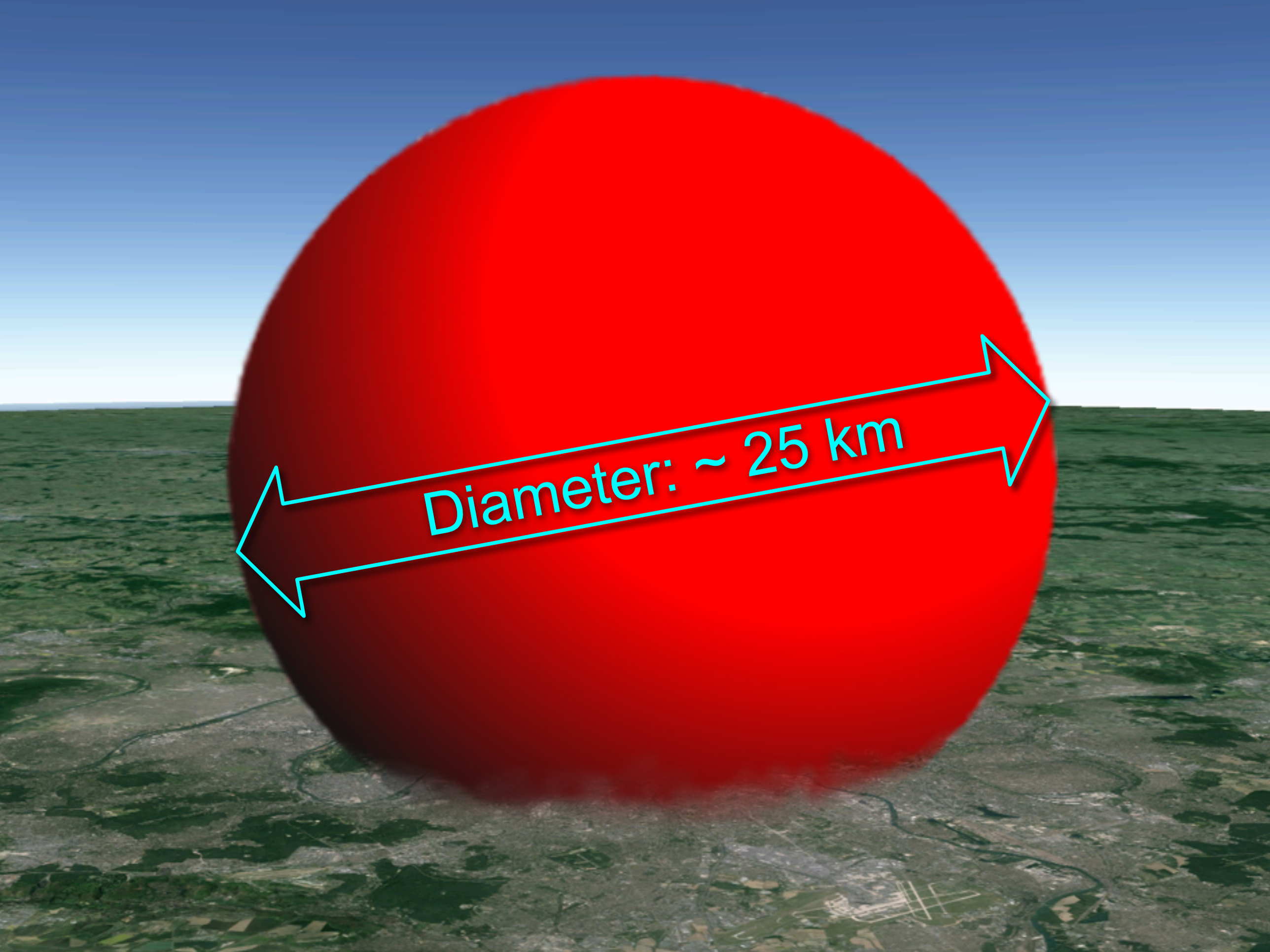
# This is Paris

Orly

This is a

Neutron Star

This is Paris


Diameter: ~ 25 km

Mass:
1-2 $M_\odot$

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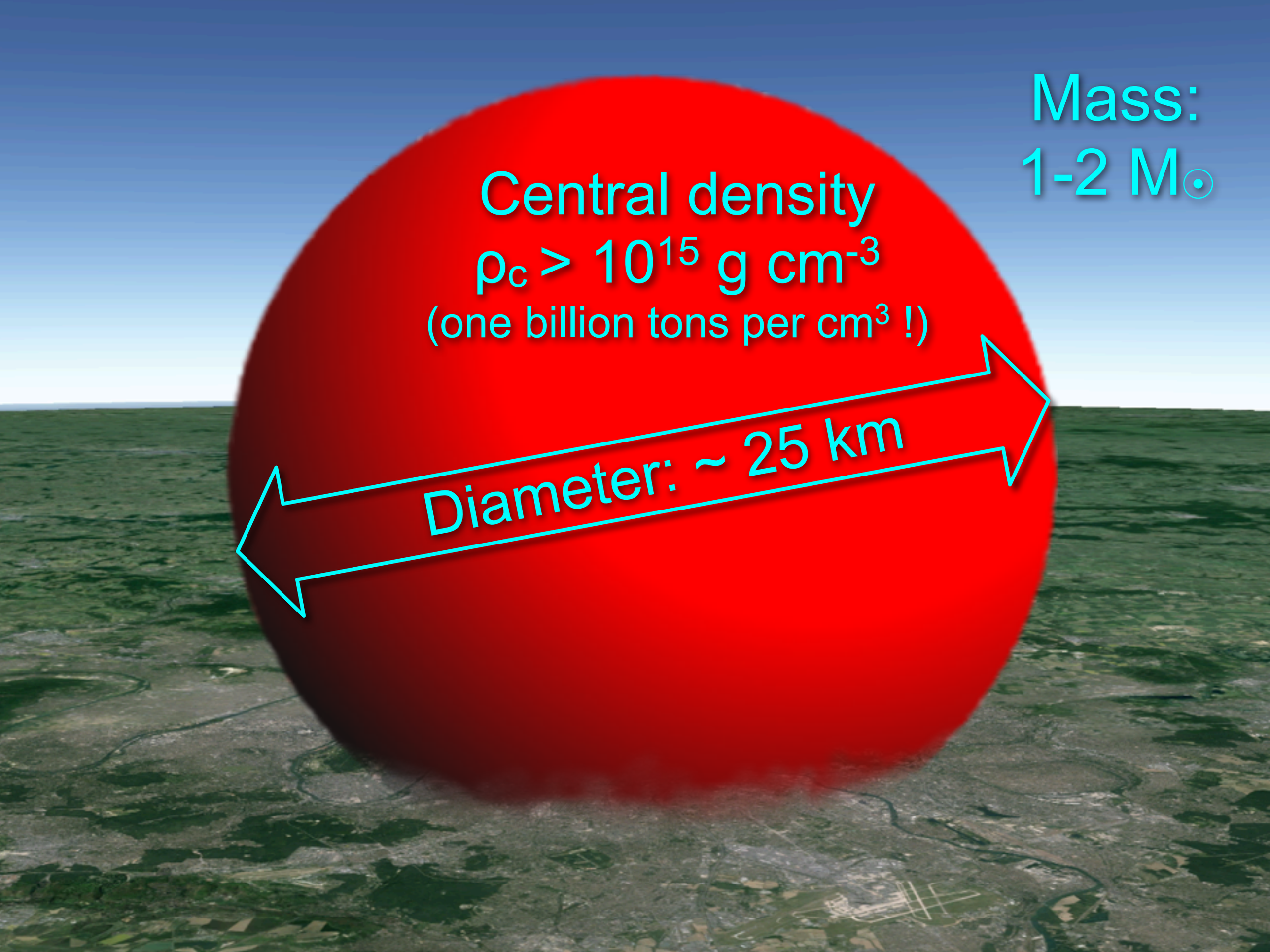
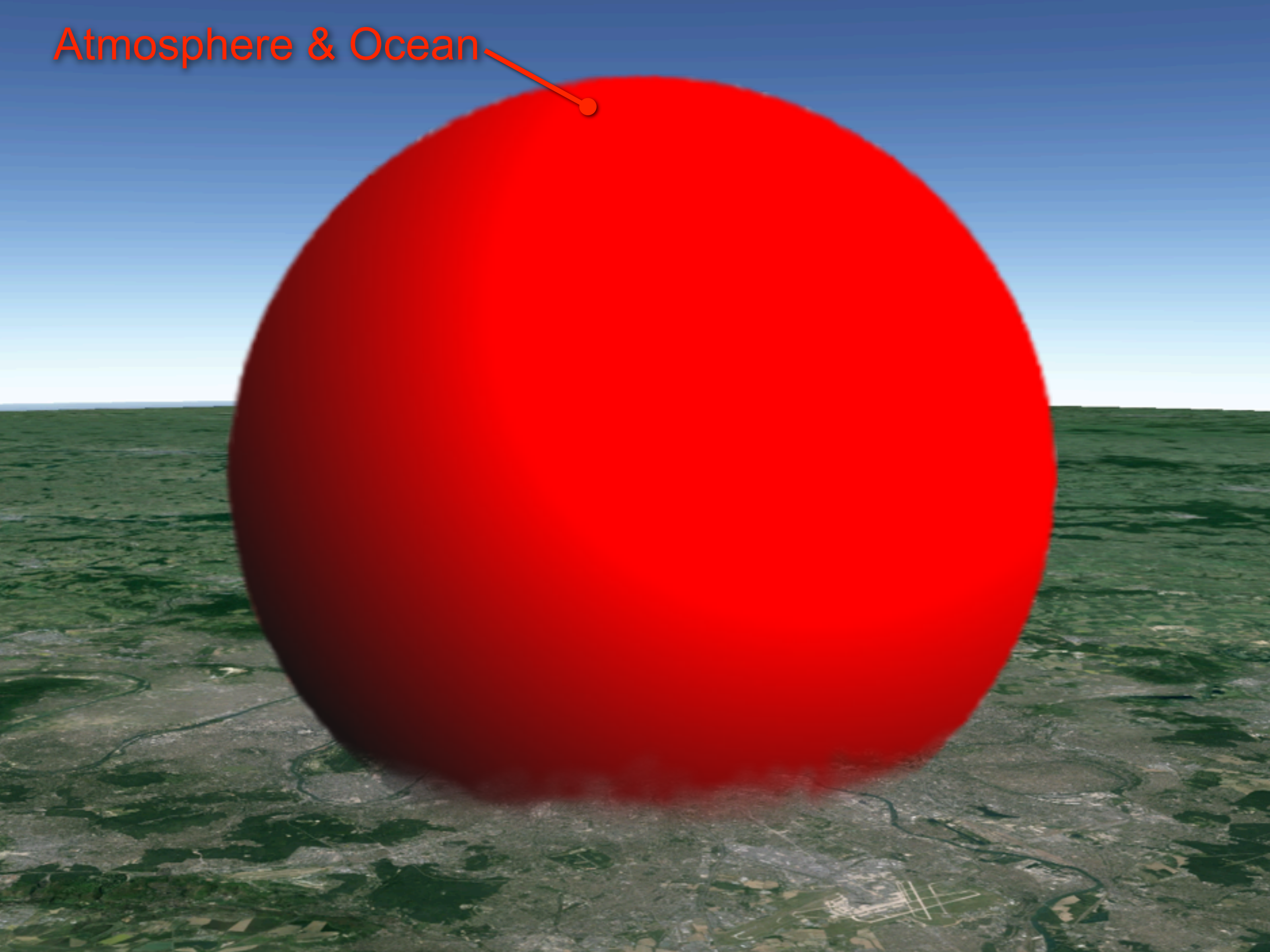

Mass:
1-2 M⊙
Central density
ρc > 10^15 g cm^-3
(one billion tons per cm^3 !)
Diameter: ~ 25 km

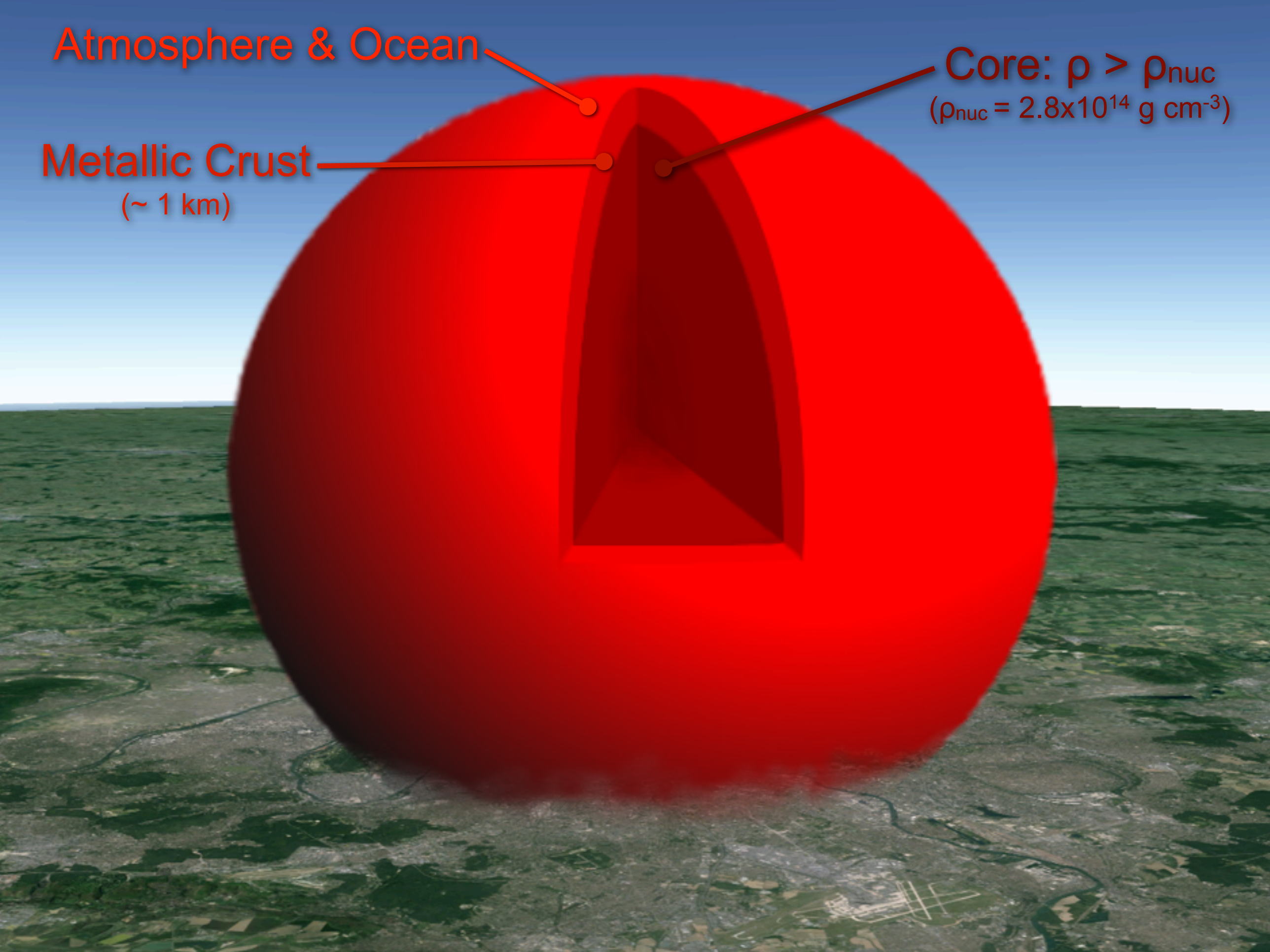
Compressed to neutron star matter density

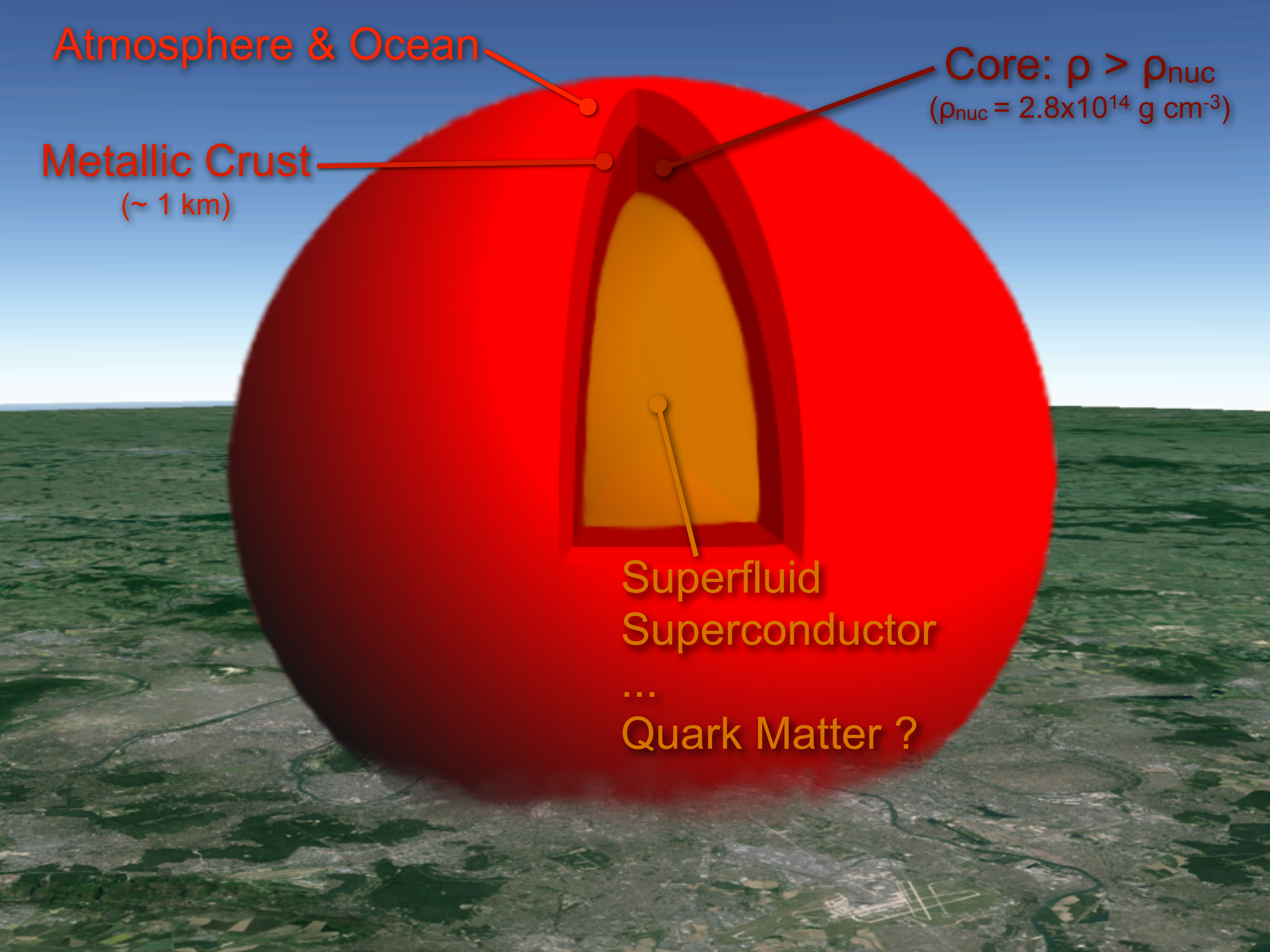
# Paris fits in a teaspoon


Atmosphere & Ocean

Atmosphere & Ocean

Metallic Crust
(~ 1 km)


Atmosphere & Ocean
Metallic Crust
(~ 1 km)
Core: ρ > ρnuc
(ρnuc = 2.8x10^14 g cm^-3)


Atmosphere & Ocean
Metallic Crust
(~ 1 km)
Core: ρ > ρnuc
(ρnuc = 2.8x10^14 g cm^-3)
Superfluid
Superconductor
...
Quark Matter ?

# “Neutron Stars” DO Exist

Consider the fastest known pulsar:
**PSR J1748.2448aD** in Terzan 5: rotational period **P=1.39 ms**

Velocity at equator < speed of light:

$$v_{\rm equator} = \Omega R = \frac{2\pi R}{P} < c \Longrightarrow R < 2\pi c P = 65\,{\rm km}$$

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---

Assume star is bound by gravity: $a_{\rm gravity} > a_{\rm centrifugal}$ at equator:

$$a_{\rm gravity} = \frac{GM}{R^2} > a_{\rm centrifugal} = \Omega^2 R = \frac{4\pi^2 R}{P^2} \quad \text{or} \quad \frac{M}{R^3} > \frac{4\pi^2}{GP^2}$$

$$\Longrightarrow \overline{\rho} = \frac{M}{\frac{4}{3}\pi R^3} > 8\times 10^{13}\ {\rm g\,cm^{-3}}$$

# “Neutron” Stars DO NOT Exist

“Neutron” stars as originally conceived by Landau, Baade & Zwicky and Oppenheimer & Volkoff DO NOT exist:

A ball of neutrons will immediately undergo decay into protons:

$$n \longrightarrow p + e^- + \overline{\nu}_e$$

since $M_n > M_p + m_e + m_\nu$

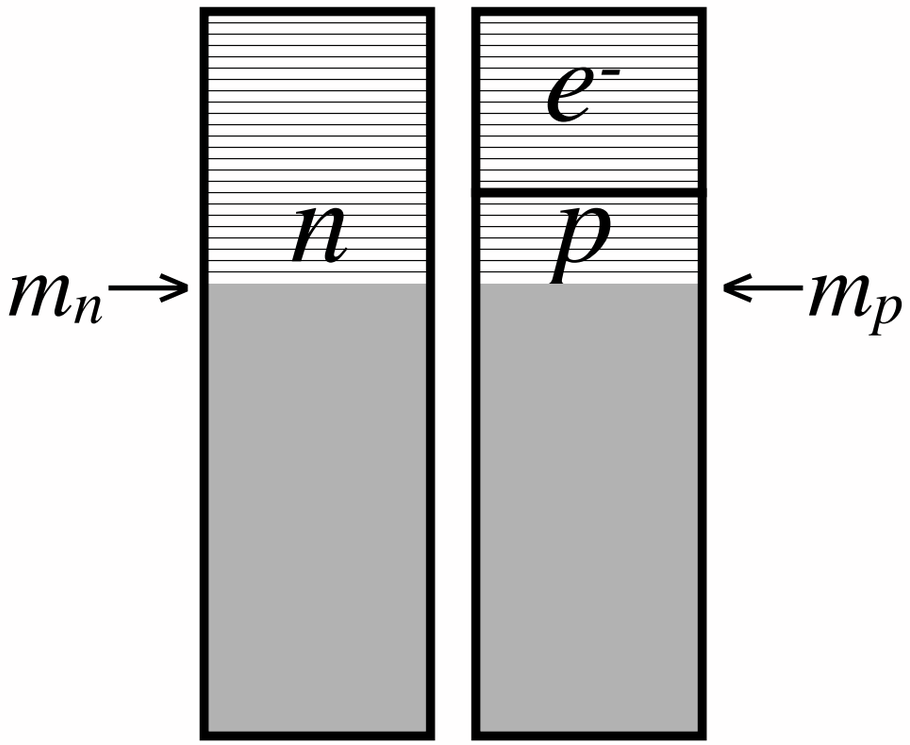
# Neutron Decay in Dense Matter

$$n \longrightarrow p + e^- + \overline{\nu}_e$$

Energies involved are the Fermi energies, not only the masses:
(assuming star is cold enough to be transparent to neutrinos: $E_{F\nu} = 0$)

Energy conservation:

$$E_{Fn} = E_{Fp} + E_{Fe}$$




For electrons: $E_F = p_F c$

For neutrons and protons:

$$E_F = mc^2 + p_F^2/2m + V$$

$V$ = potential energy ⇒

Need a model for strong interactions.

⇒ Increasing uncertainty with increasing density

# Neutron Decay in Dense Matter

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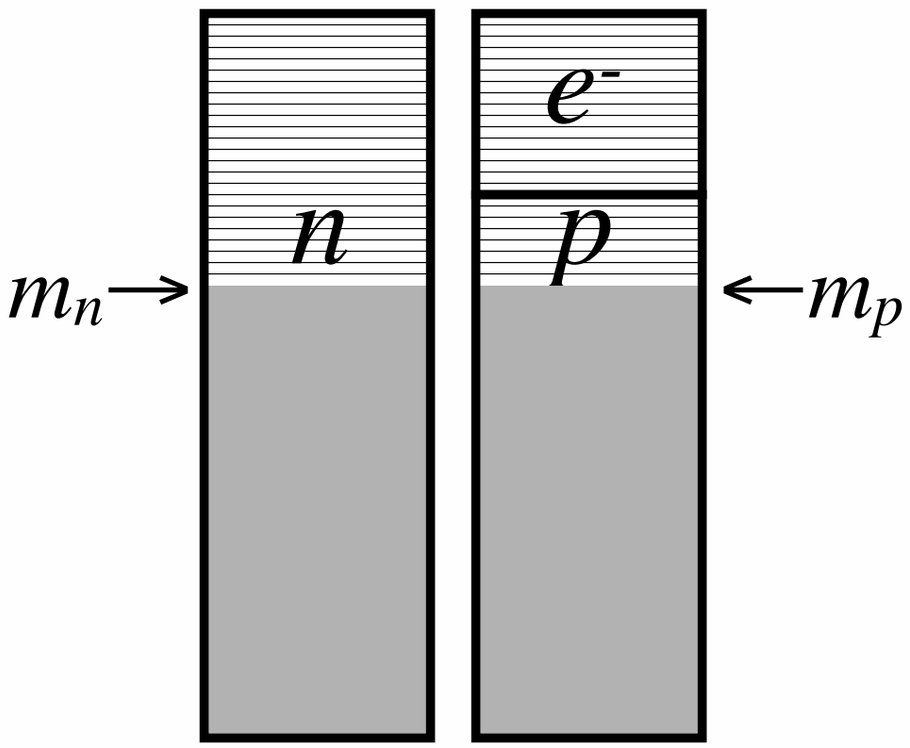



Then

$$p + e^- \longrightarrow n + \nu_e$$

is also possible, giving the DUrca cycle:

$$\begin{cases} n \longrightarrow p + e^- + \overline{\nu}_e \\ p + e^- \longrightarrow n + \nu_e \end{cases}$$

# “Chemical”, or Beta, Equilibrium

Creation (or destruction) of a particle X involves an energy $\mu_X$ (= chemical potential).

For fermions at $T \sim 0$ :

$$\mu_X = E_F \text{ (Fermi energy)}$$

$$n \longrightarrow p + e^- + \overline{\nu}_e$$

$$p + e^- \longrightarrow n + \nu_e$$

Such beta equilibrium between n & p implies:

$$\mu_n = \mu_p + \mu_e$$

(assuming neutrinos leave the star: $\mu_\nu = 0$)

# Hyperons in Neutron Stars ?

$$\mu_{B^0} = \mu_n$$

$$\mu_{B^-} = \mu_n + \mu_e$$

$$\mu_{B^+} = \mu_n - \mu_e$$

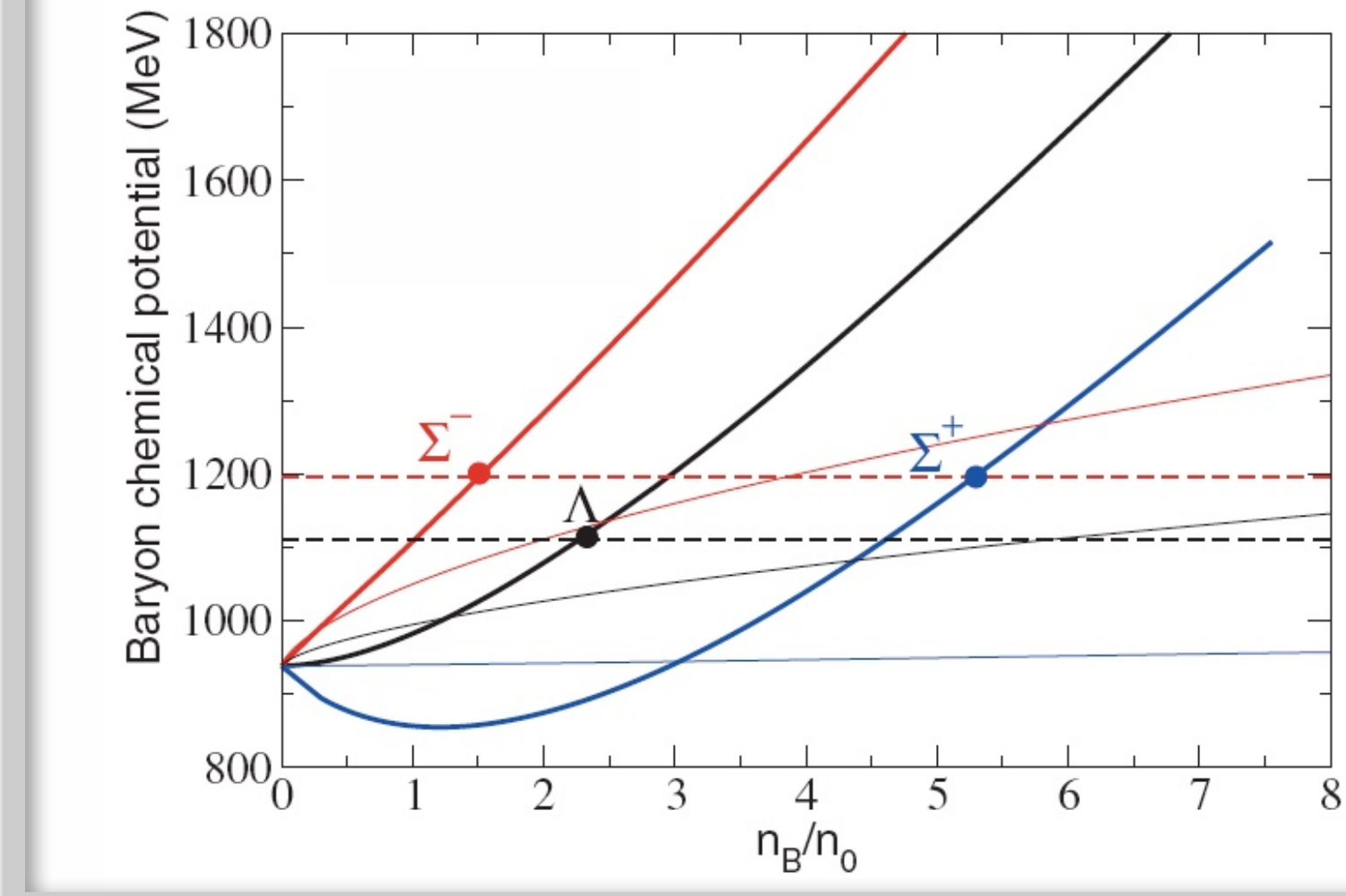


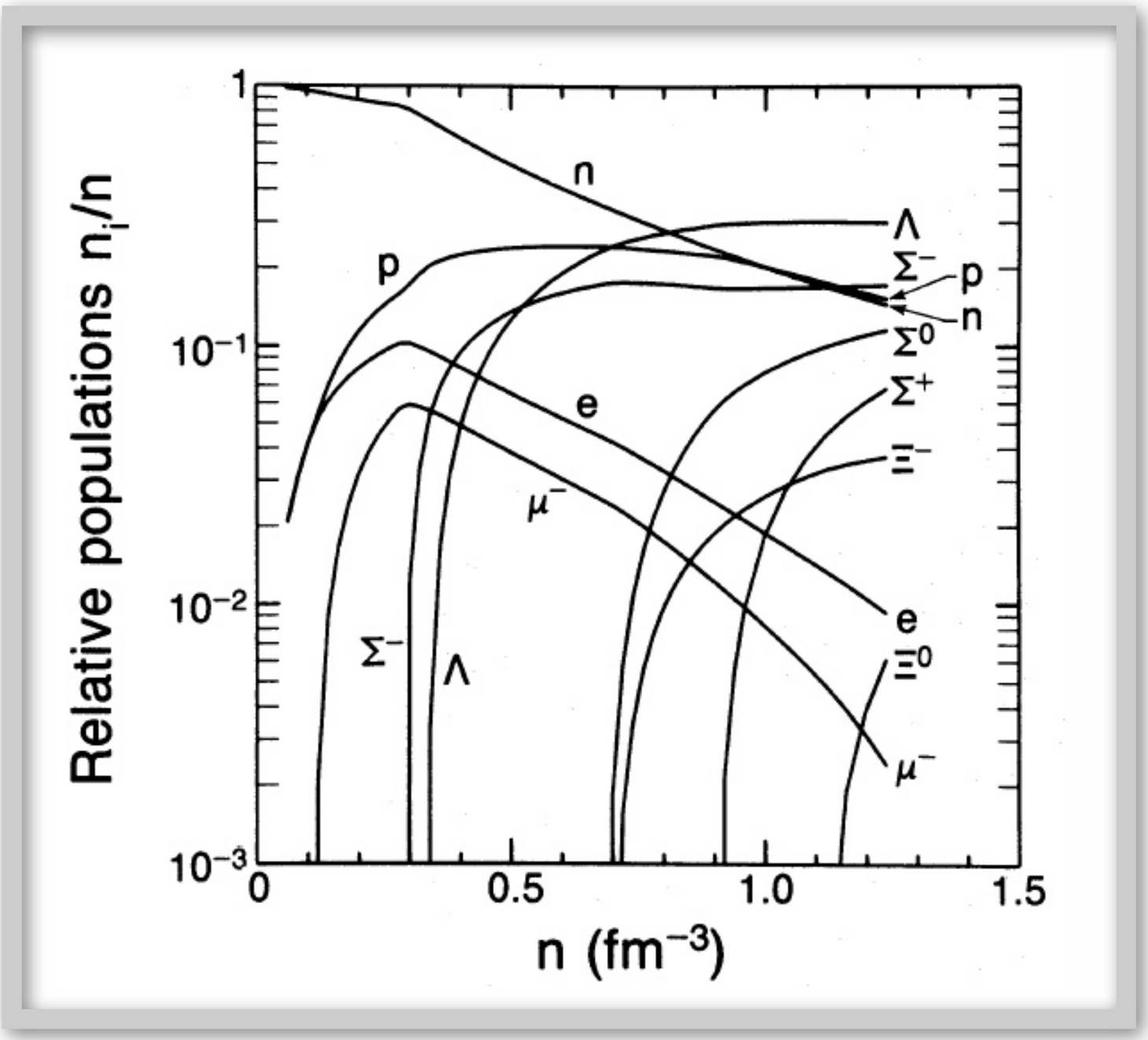

**Figure 10** Baryon chemical potentials in dense stellar matter.

Thick lines: Walecka type model

Thin lines: free gases

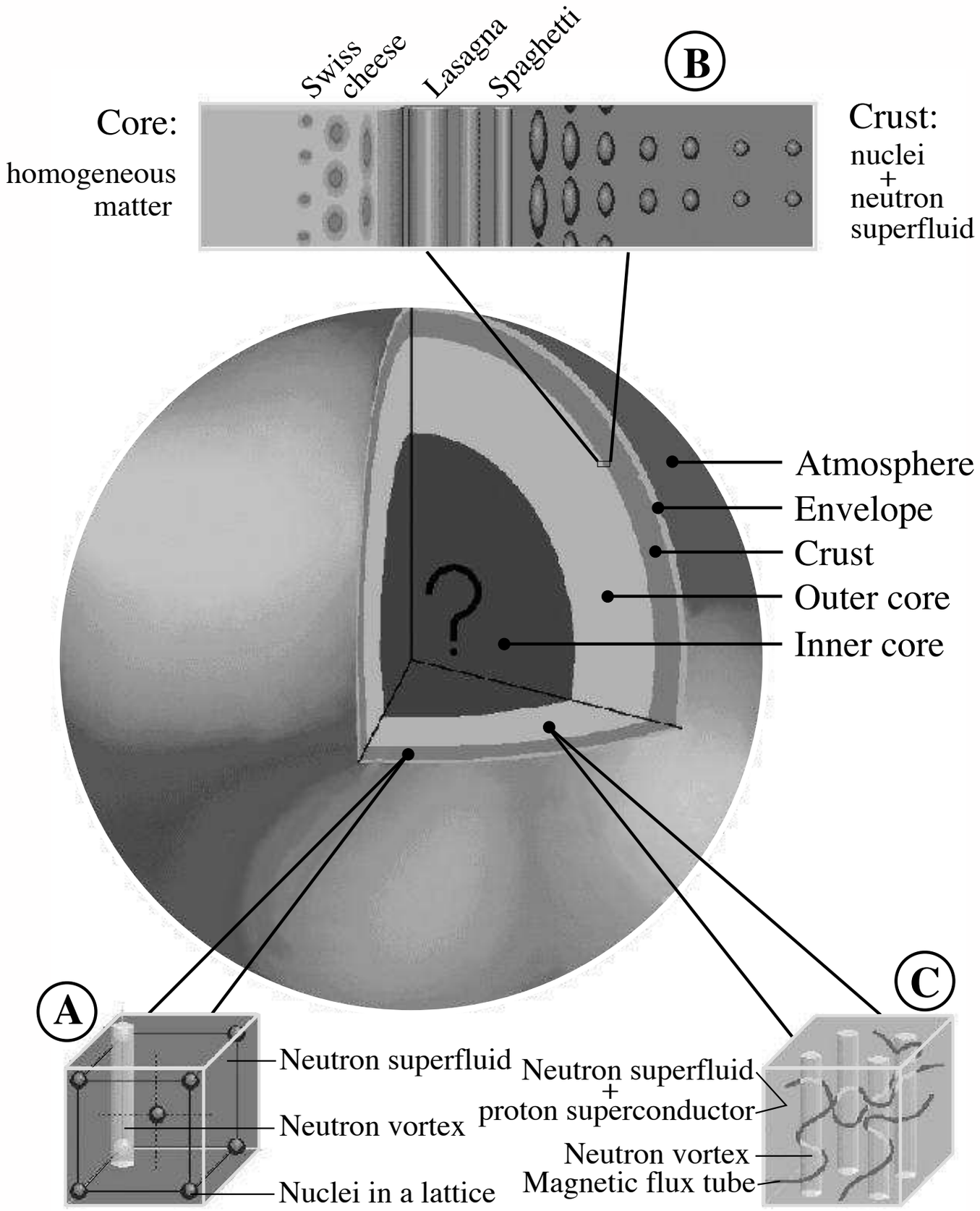
*Dense Matter in Compact Stars: Theoretical Developments and Observational Constraints,* Page D. & Reddy S., 2006, Annu. Rev. Nucl. Part. Sci. 56, 327

# Hyperons in Neutron Stars ?




*Neutron stars are giant hypernuclei ?*

N.K. Glendenning, ApJ 293, 470 (1985)




*Dense Matter in Compact Stars: Theoretical Developments and Observational Constraints,* Page D. & Reddy S., 2006, Annu. Rev. Nucl. Part. Sci. 56, 327

# SUMMARY

The interior of a neutron star is amazingly rich in physical structures and phases, from traditional solid state physics to quark-gluon plasma, high-energy physics (quark, neutrinos), low-temperature physics (superfluids/superconductors), complex systems (“pasta phase” = liquid crystal), ...

Magnetospheres are described by extreme QED (super-critical fields).

Gravitational field is the strongest observable in the Universe (modified gravity ?) and ...

it can accumulate Dark Matter !

# SUMMARY

The interior of a neutron star is ... al
structures and phases, f... s to
quark-gluon pla... s),
low-temp...
c...

Ma... D (super-
critic...

Gravit... strongest observable in the Universe
(modifi... y ?) and ...
it can accumulate Dark Matter !

JUST NAME IT ...

... WE HAVE IT !

# So What ?

- Many possibilities for ultra-dense matter.
- Poor understanding of interactions (e.g., hyperons, quarks) precludes any definitive answer.
- Many-body effects are often handled in simplified ways, or with toy models.

**The answer is an experimental/observational issue.**

# Neutrino Emission

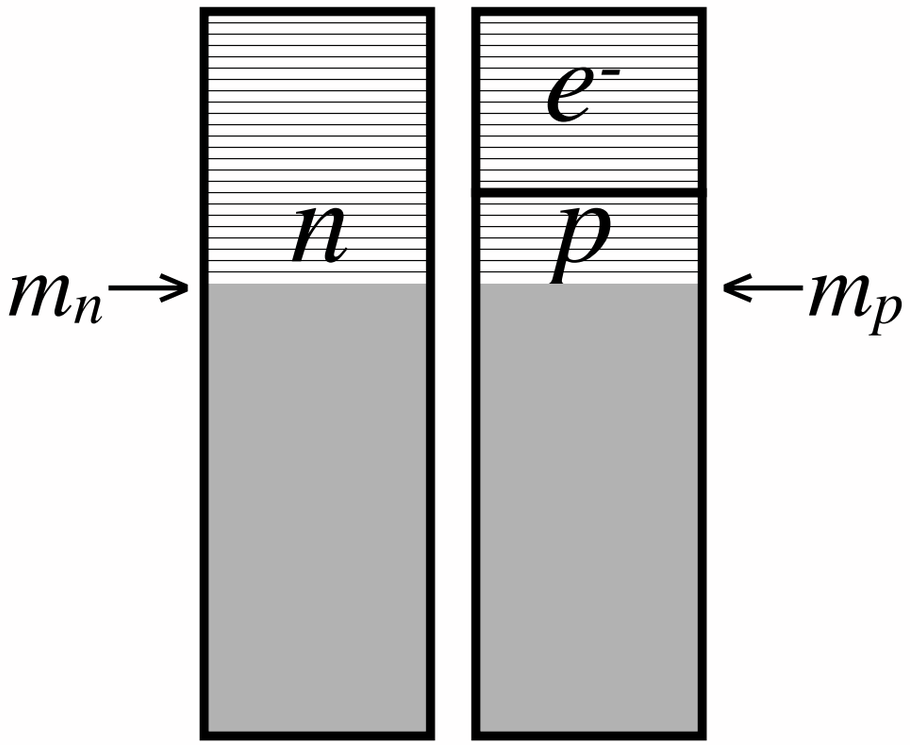
# The Direct Urca Process

Basic mechanism: β and inverse β decays:

$$n \longrightarrow p + e^- + \overline{\nu}_e \quad \text{and} \quad p + e^- \longrightarrow n + \nu_e$$

Energy conservation:

$E_{Fn} = E_{Fp} + E_{Fe}$




Momentum conservation:

"Triangle rule": $p_{Fn} < p_{Fp} + p_{Fe}$

$$n_i = \frac{k_{F\,i}^3}{3\pi^2} \;\Rightarrow\; n_n^{1/3} \leq n_p^{1/3} + n_e^{1/3} = 2n_p^{1/3}$$

$$x_p \equiv \frac{n_p}{n_n + n_p} \geq \frac{1}{9} \approx 11\%$$

# The Modified Urca Process

Direct Urca process:

$$\begin{cases} n & \longrightarrow & p + e^- + \overline{\nu}_e \\ p + e^- & \longrightarrow & n + \nu_e \end{cases}$$

DUrca needs $x_p$ > 11%: possible only at high densities

Modified Urca process:

$$\begin{cases} n + n' & \longrightarrow & p + n' + e^- + \overline{\nu}_e \\ p + n' + e^- & \longrightarrow & n + n' + \nu_e \end{cases}$$

The “spectator” neutron n´ gives or take the needed extra momentum: possible at any density

---

$$\epsilon_{\mathrm{DUrca}} \simeq 10^{27} \cdot T_9^6 \ \mathrm{erg\ cm^{-3}\ s^{-1}}$$

$$\epsilon_{\mathrm{MUrca}} \simeq 10^{21} \cdot T_9^8 \ \mathrm{erg\ cm^{-3}\ s^{-1}}$$

← One cubic meter of matter at $T$=$10^9$ K emits $10^{33}$ erg $s^{-1}$ ~ $L_\odot$

# Neutrino Emission from Hyperons

Hyperons, as $\Lambda$ and $\Sigma^-$, can be produced through reactions as, e.g.

$$\begin{cases} p + e^- & \longrightarrow & \Lambda + \nu_e \\ \Lambda & \longrightarrow & p + e^- + \overline{\nu}_e \end{cases}$$

$$\begin{cases} n + e^- & \longrightarrow & \Sigma^- + \nu_e \\ \Sigma^- & \longrightarrow & n + e^- + \overline{\nu}_e \end{cases}$$

Energy conservation requires: $\mu_\Lambda = \mu_n$ and $\mu_{\Sigma^-} = \mu_n + \mu_e$

Momentum conservation: very easily satisfied for $\Lambda$
and not very difficult to satisfy for $\Sigma^-$

**Hyperons will result in DUrca processes if they can be present**

# Neutrino Emission from Meson Condensates

Mesons, as $\pi^-$ and $K^-$ can be produced through reactions as, e.g.

$$\begin{cases} n + e^- \longrightarrow n + \pi^- + \nu_e \\ n + \pi^- \longrightarrow n + e^- + \overline{\nu}_e \end{cases}$$

$$\begin{cases} n + e^- \longrightarrow n + K^- + \nu_e \\ n + K^- \longrightarrow n + e^- + \overline{\nu}_e \end{cases}$$

Energy conservation requires: $m_\pi^* = \mu_e$ or $m_K^* = \mu_e$

Momentum conservation: trivially satisfied because mesons condense (they are bosons) and the condensate can absorb *any* extra needed momentum

**Charged mesons will result in DUrca processes if they can be present**

# Neutrino Emission from Deconfined Quarks

2 DUrca processes are possibles (energy and momentum are easily conserved):

$$\begin{cases} u + e^- & \longrightarrow & d + \nu_e \\ d & \longrightarrow & u + e^- + \overline{\nu}_e \end{cases} \qquad \begin{cases} u + e^- & \longrightarrow & s + \nu_e \\ s & \longrightarrow & u + e^- + \overline{\nu}_e \end{cases}$$

In $d \longrightarrow u + e^- + \overline{\nu}_e$ the matrix element (squared)

$$|M_{fi}|^2 = 32 G_F^2 \cos^2\theta_C (p_1 \cdot p_2)(p_3 \cdot p_4)$$

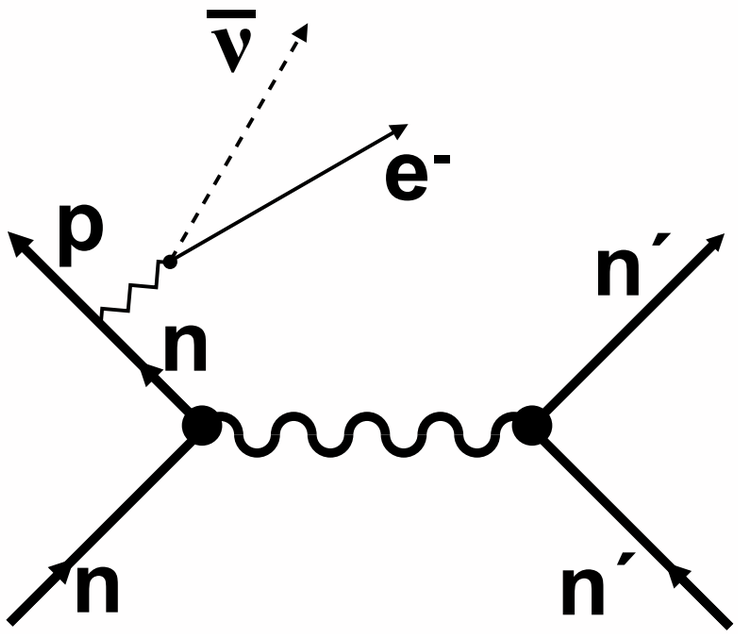
vanishes if $\mu_q = c p_{Fq}$ $\Rightarrow$ need $\alpha_c$ correction: $\mu_q = \left(1 + \frac{8\alpha_c}{3\pi}\right) c p_{Fq}$

$$\epsilon^{\mathrm{ud-DUrca}} = \frac{914}{315} \frac{G_F^2 \cos^2\theta_C}{\hbar^{10} c^9} \alpha_c \, \mu_u \mu_d \mu_e (k_B T)^6$$

**Deconfined Quarks will result in DUrca processes if they can be present**

# The Two Basic Neutrino Processes

**SLOW**



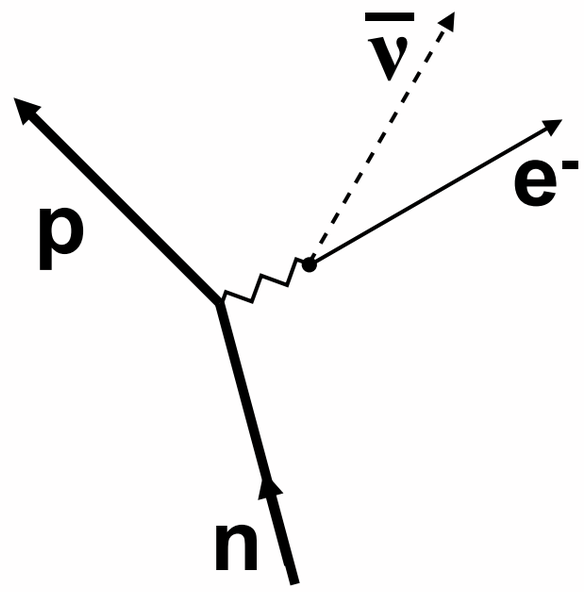

Modified Urca

**MEDIUM**




Cooper pair formation ("PBF")

**FAST**




Direct Urca

# A sample of neutrino emission processes

| Name | Process | Emissivity (erg cm$^{-3}$ s$^{-1}$) | |
|---|---|---|---|
| Modified Urca cycle (neutron branch) | $n+n \to n+p+e^- + \bar{\nu}_e$<br>$n+p+e^- \to n+n+\nu_e$ | $\sim 2{\times}10^{21}\, R\, T_9^8$ | Slow |
| Modified Urca cycle (proton branch) | $p+n \to p+p+e^- + \bar{\nu}_e$<br>$p+p+e^- \to p+n+\nu_e$ | $\sim 10^{21}\, R\, T_9^8$ | Slow |
| Bremsstrahlung | $n+n \to n+n+\nu+\bar{\nu}$<br>$n+p \to n+p+\nu+\bar{\nu}$<br>$p+p \to p+p+\nu+\bar{\nu}$ | $\sim 10^{19}\, R\, T_9^8$ | Slow |
| Cooper pair formations | $n+n \to [nn]+\nu+\bar{\nu}$<br>$p+p \to [pp]+\nu+\bar{\nu}$ | $\sim 5{\times}10^{21}\, R\, T_9^7$<br>$\sim 5{\times}10^{19}\, R\, T_9^7$ | Medium |
| Direct Urca cycle (nucleons) | $n \to p+e^- + \bar{\nu}_e$<br>$p+e^- \to n+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| Direct Urca cycle ($\Lambda$ hyperons) | $\Lambda \to p+e^- + \bar{\nu}_e$<br>$p+e^- \to \Lambda+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| Direct Urca cycle ($\Sigma^-$ hyperons) | $\Sigma^- \to n+e^- + \bar{\nu}_e$<br>$n+e^- \to \Sigma^-+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| $\pi^-$ condensate | $n+<\pi^->\to n+e^-+\bar{\nu}_e$ | $\sim 10^{26}\, R\, T_9^6$ | Fast |
| $K^-$ condensate | $n+<K^->\to n+e^-+\bar{\nu}_e$ | $\sim 10^{25}\, R\, T_9^6$ | Fast |
| Direct Urca cycle (u-d quarks) | $d \to u+e^- + \bar{\nu}_e$<br>$u+e^- \to d+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| Direct Urca cycle (u-s quarks) | $s \to u+e^- + \bar{\nu}_e$<br>$u+e^- \to s+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |

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| Modified Urca cycle (proton branch) | $p+n \to p+p+e^- + \bar{\nu}_e$<br>$p+p+e^- \to p+n+\nu_e$ | | Slow |
| Bremsstrahlung | $n+n \to n+n+\nu+\bar{\nu}$<br>$n+p \to n+p$<br>$p+p \to$ | | Slow |
| Cooper pair formations | $n$ | $R\, T_9^7$ | Medium |
| Direct Urca cycle (nucleons) | | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| Dir | $+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| D (Σ | $n+e^- + \bar{\nu}_e$<br>$n+e^- \to \Sigma^- + \nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| $\pi^-$ c | $n+<\pi^->\to n+e^- + \bar{\nu}_e$ | $\sim 10^{26}\, R\, T_9^6$ | Fast |
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Anything beyond just neutrons and protons (and only a small amount of them) results in enhanced neutrino emission

# Neutron Star Cooling:

# Neutron Star Cooling on a Napkin

*Assume the star's interior is isothermal and neglect GR effects.*

Thermal Energy, $E_{th}$, balance:

$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu + H$$

⇒ 3 essential ingredients are needed:

- $C_v$ = total stellar specific heat
- $L_\gamma$ = total surface photon luminosity
- $L_\nu$ = total stellar neutrino luminosity

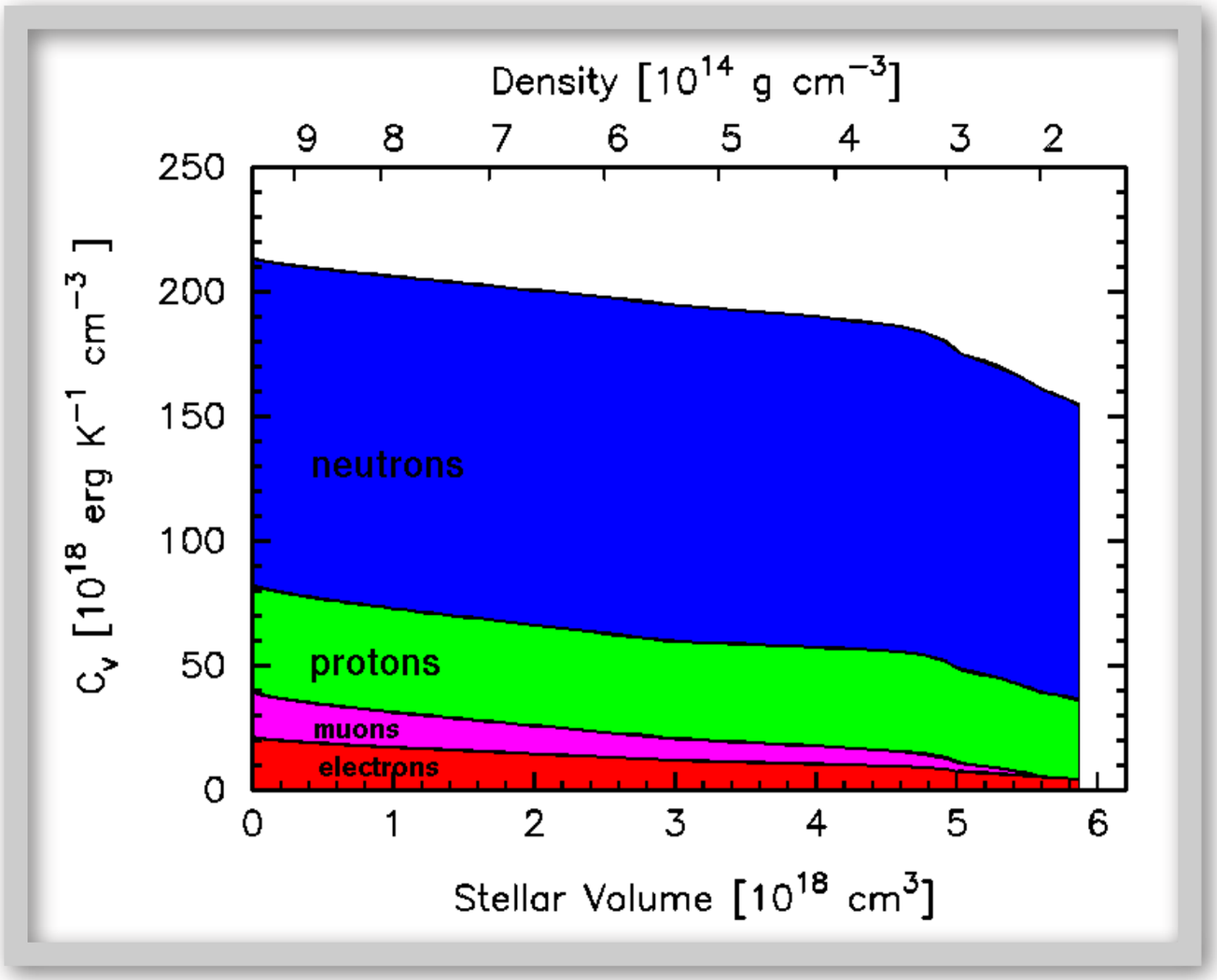
$H$ = "heating", from B field decay, friction, etc ...

# Specific Heat on a Napkin

Sum over all degenerate fermions: $C_V = \sum_i C_{V\,i}$ $\qquad c_{v\,i} = N(0)\frac{\pi^2}{3}k_B^2 T$ with $N(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$

$$C_v = \iiint c_v \, dV \simeq 10^{38} - 10^{39} \times T_9 \ \text{erg K}^{-1} \equiv C_9 T_9$$

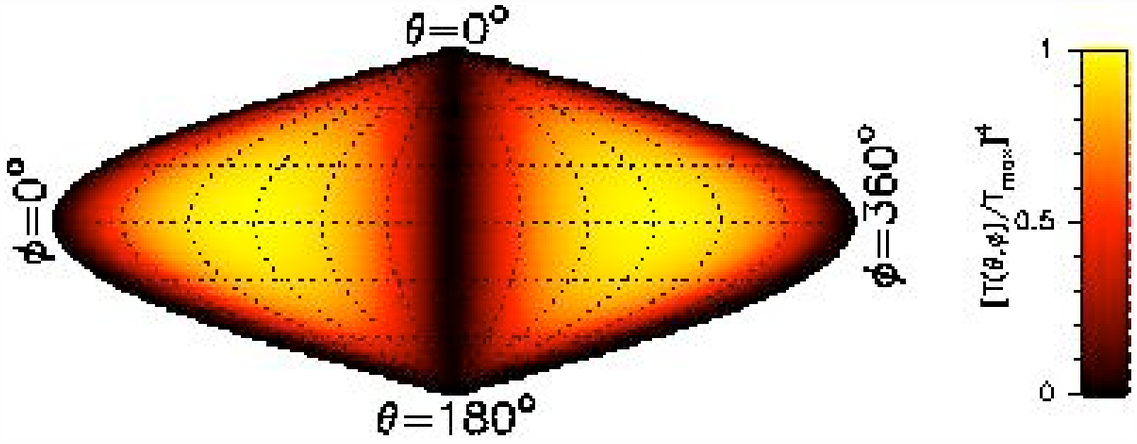
(lowest value corresponds to the case where extensive pairing of baryons in the core suppresses their $c_v$ and only the leptons, e & μ, contribute)




Distribution of $c_v$ in the core of a 1.4 $M_{Sun}$ neutron star build with the APR EOS (Akmal, Pandharipande, & Ravenhall, 1998), at

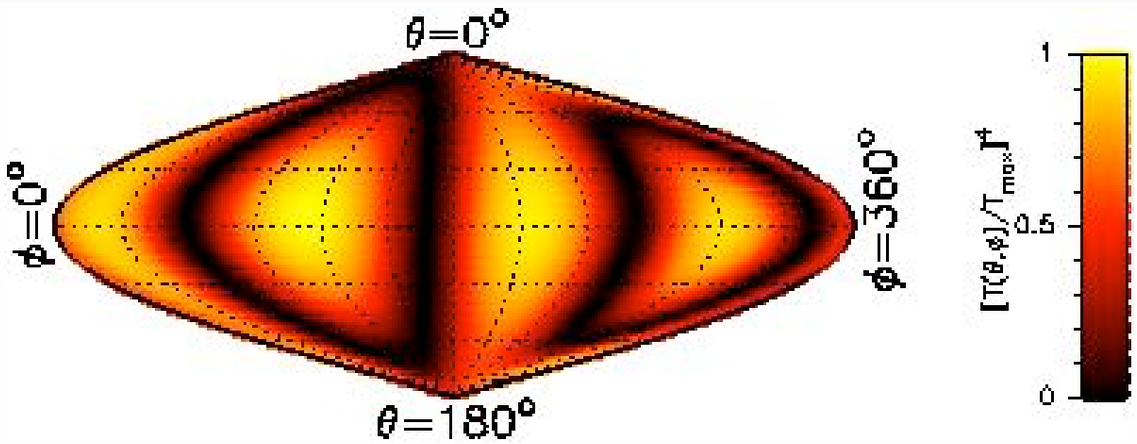
$T = 10^9$ K

# Neutron Star Surface with Magnetic Fields




Purely dipolar field

(oriented on the equatorial plane
to make a prettier picture !)




Dipolar + quadrupolar field

Surface temperature of a magnetized neutron star and interpretation of the ROSAT data. I. Dipolar fields

Page, D.

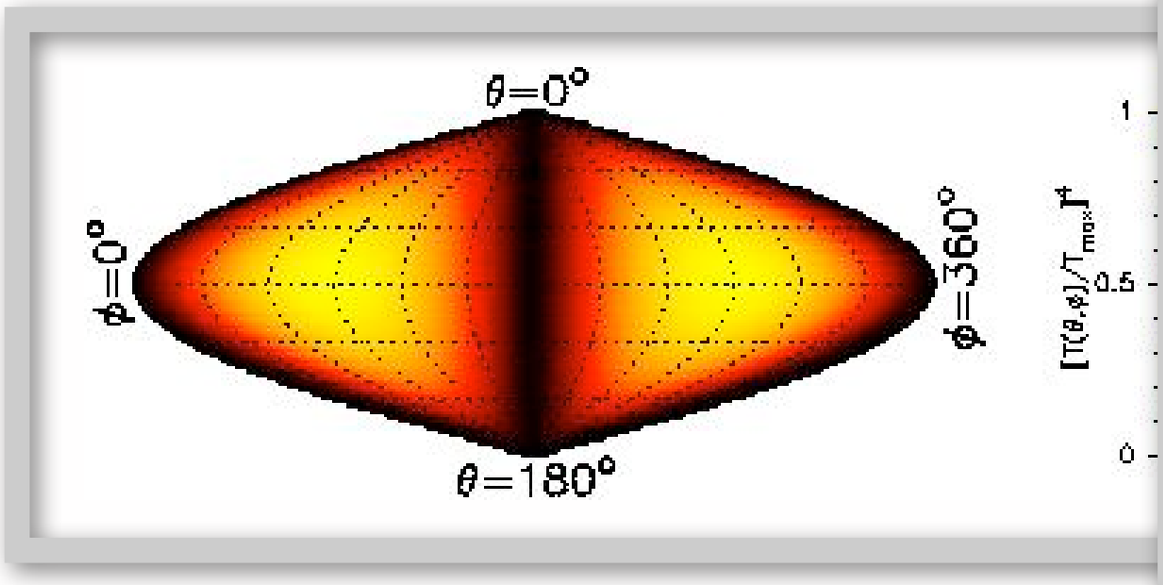
1995ApJ...442..273P

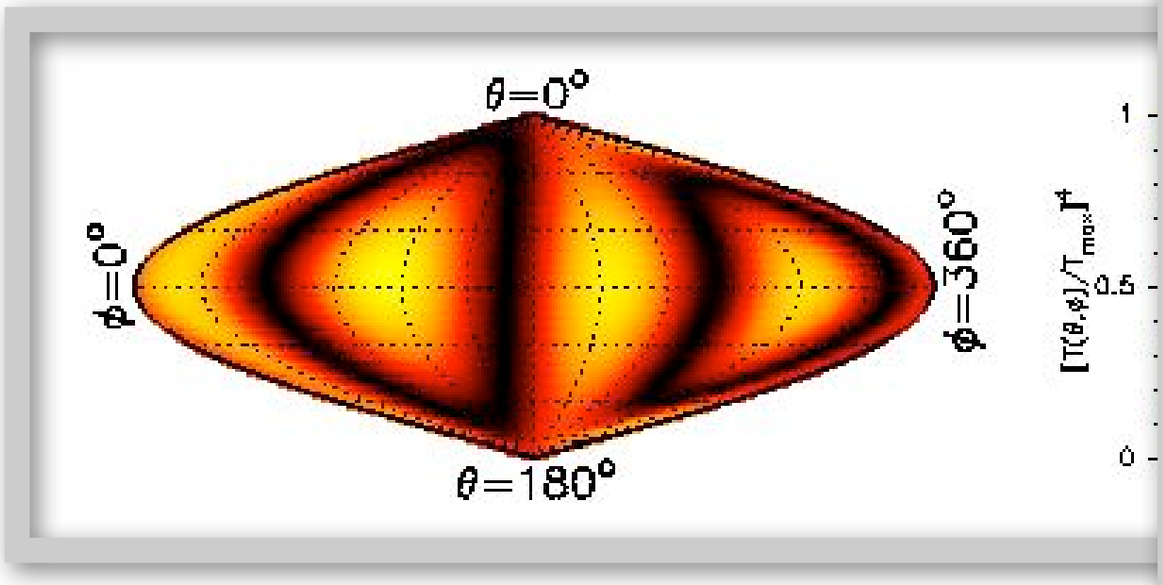
Surface temperature of a magnetized neutron star and interpretation of the ROSAT data. II.

Page, D.; Sarmiento, A.

1996ApJ...473.1067P

# Neutron Star Surface with Magnetic Fields




[Surface temperature of a magnetized neutron star and interpretation of the ROSAT data. I. Dipolar fields](1995ApJ...442..273P)
Page, D.
[1995ApJ...442..273P](1995ApJ...442..273P)

## Luminosity and effective temperature

$$L \equiv 4\pi R^2 \, \sigma T_e^4$$

$$L = \iint \sigma \, T_s(\theta, \phi)^4 \, dS$$

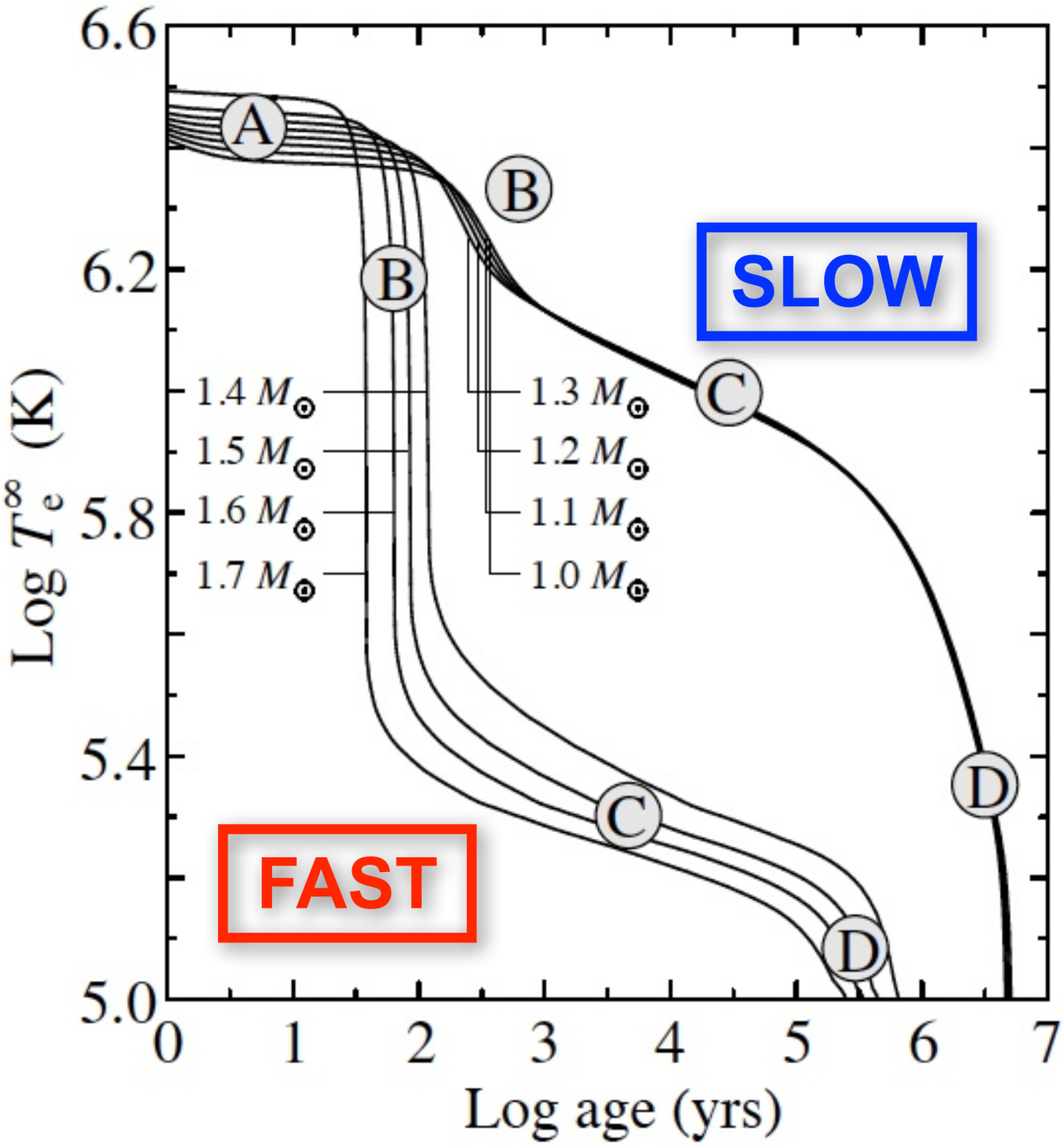
$$(dS = R^2 \cdot d\Omega)$$

$$T_e^4 \equiv \frac{1}{4\pi} \iint \sigma \, T_s(\theta, \phi)^4 \, d\Omega$$

[1996ApJ...473.1067P](1996ApJ...473.1067P)

# Slow vs Fast Cooling

# Direct vs Modified Urca Cooling




Models based on the PAL EOS:

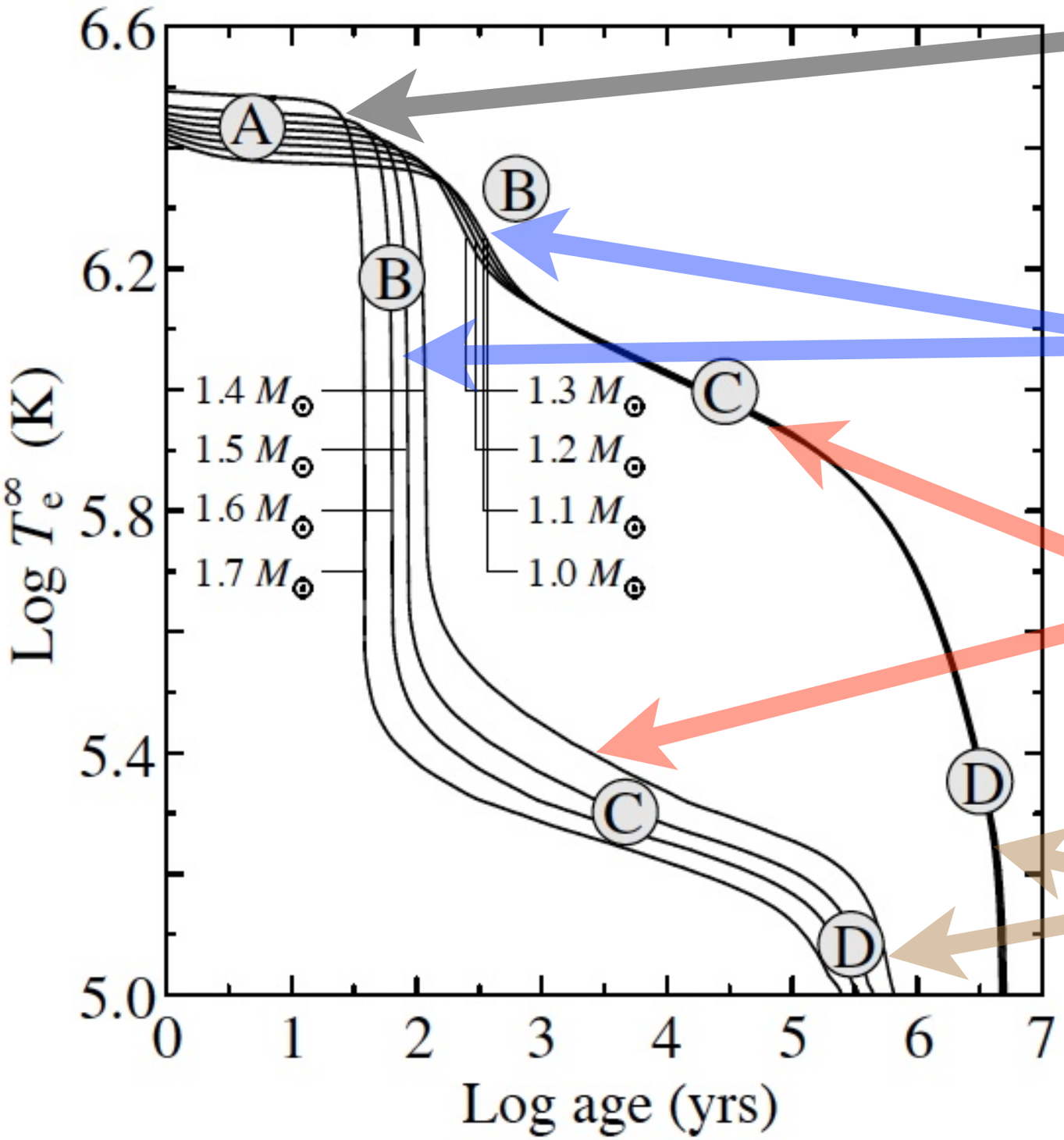
adjusted (by hand) so that DURCA becomes allowed (triangle rule !) at M > 1.35 $M_{Sun}$.

This value is arbitrary:
we DO NOT know the value of this critical mass, and hopefully observations will, some day, tell us what it is !

GR models, solving numerically the energy balance and heat transport equations, with lots of microphysics involved.

"The Cooling of Neutron Stars by the Direct Urca Process", Page & Applegate, ApJ 394, L17 (1992)

# Direct vs Modified Urca Cooling



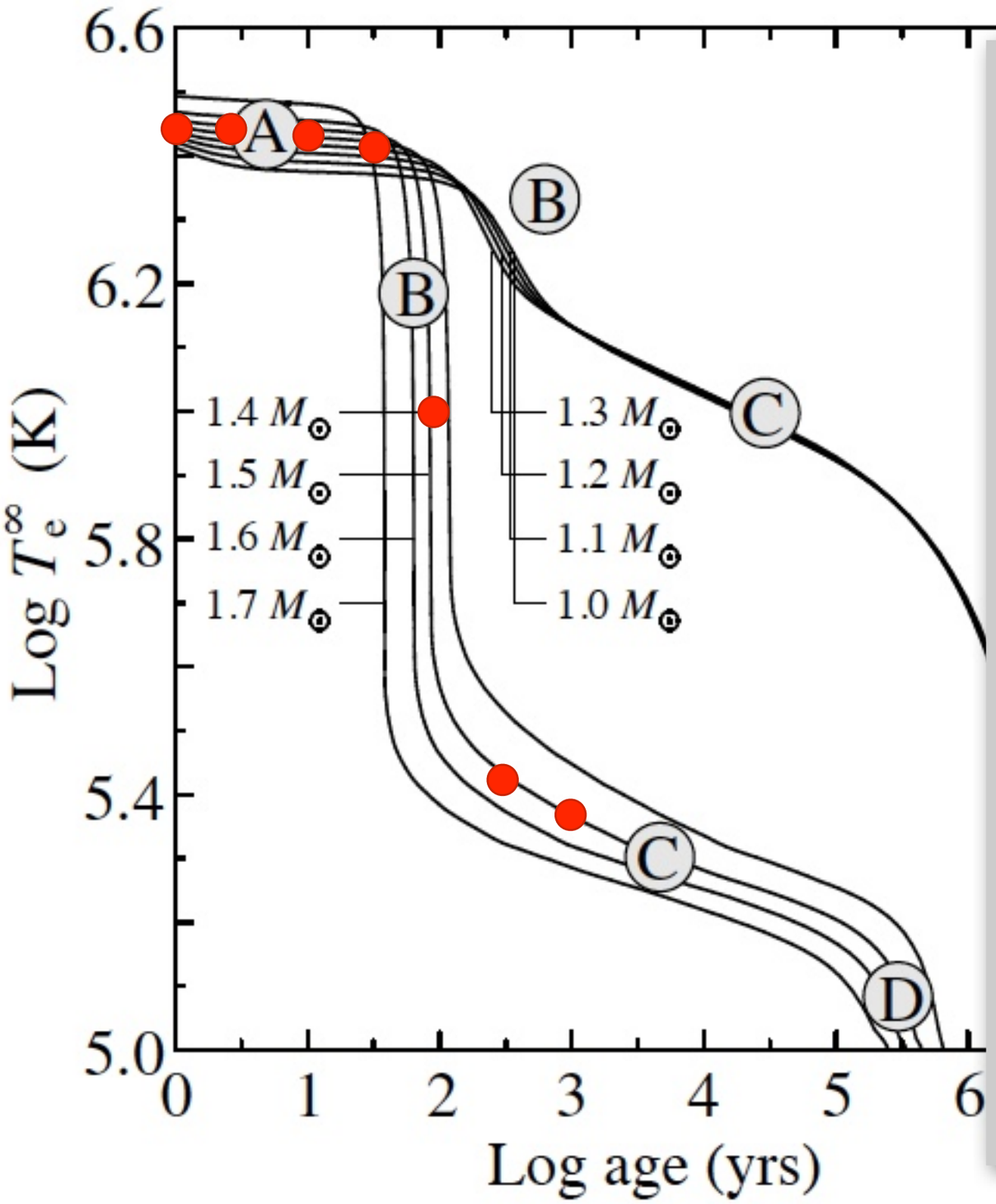

A: Early Time:
$T_e$ controlled by
$\nu$ emission
in the crust

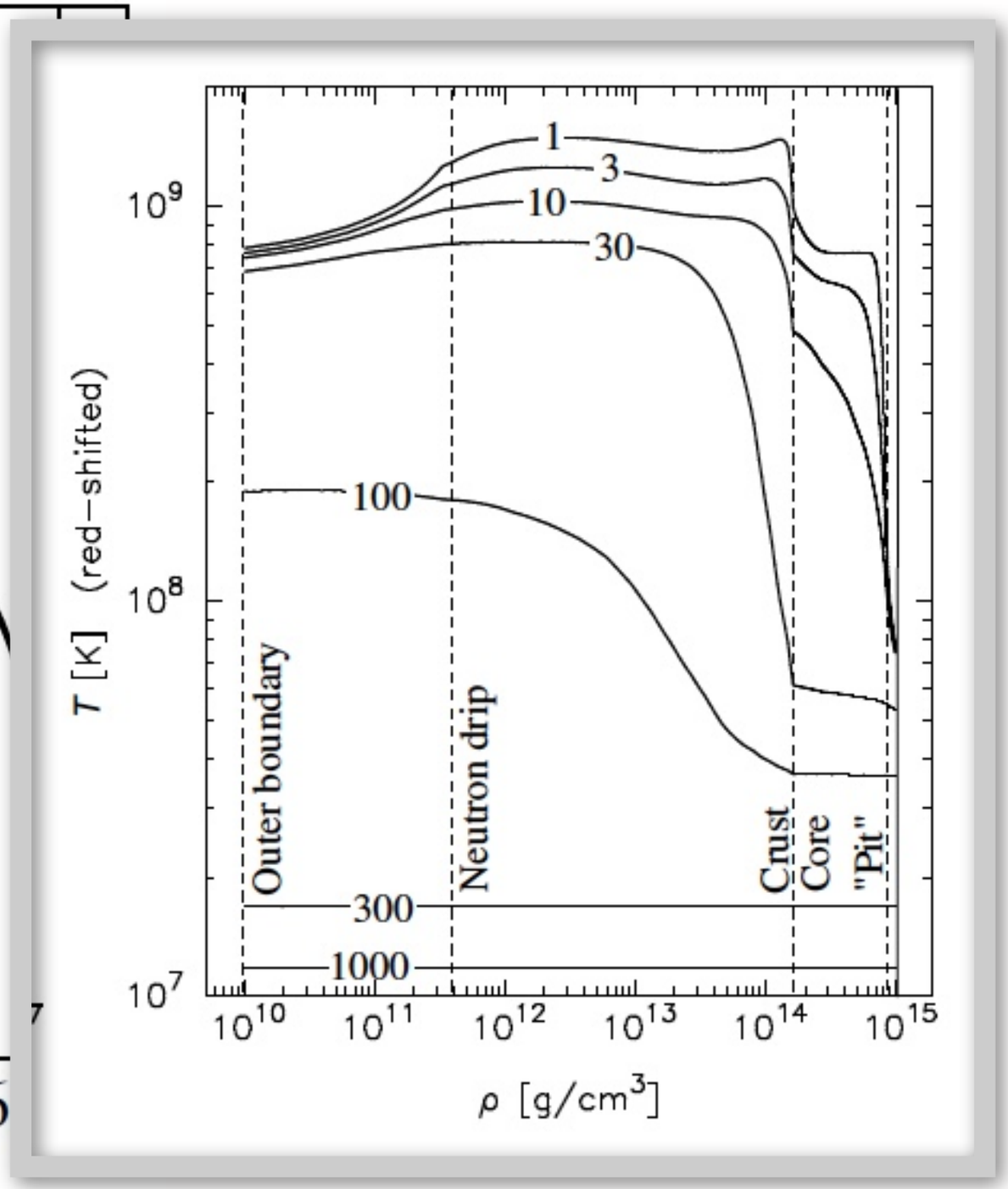
B: Crust relaxation:
crustal heat flows
into the core.

C: Isothermal Star:
$T_e$ controlled by
$\nu$ emission
from the core

D:Isothermal Star:
$T_e$ driven by
photon emission
from the surface

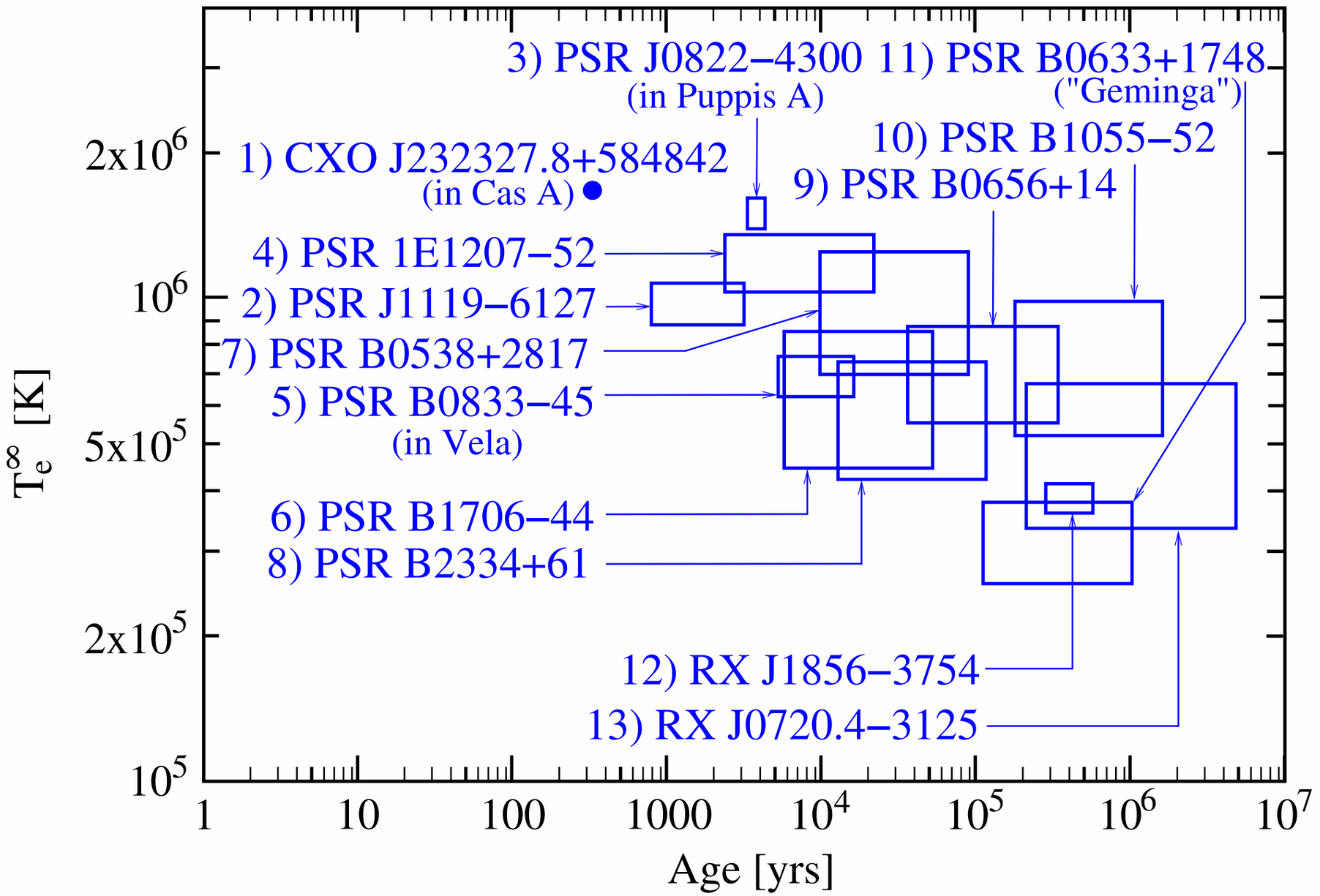
# Crust Thermal Relaxation



# Cooling Isolated Neutron Stars: The Data

# Isolated Cooling Neutron Stars I




Neutron stars with clearly detected thermal emission

# One Example: PSR 0656+14

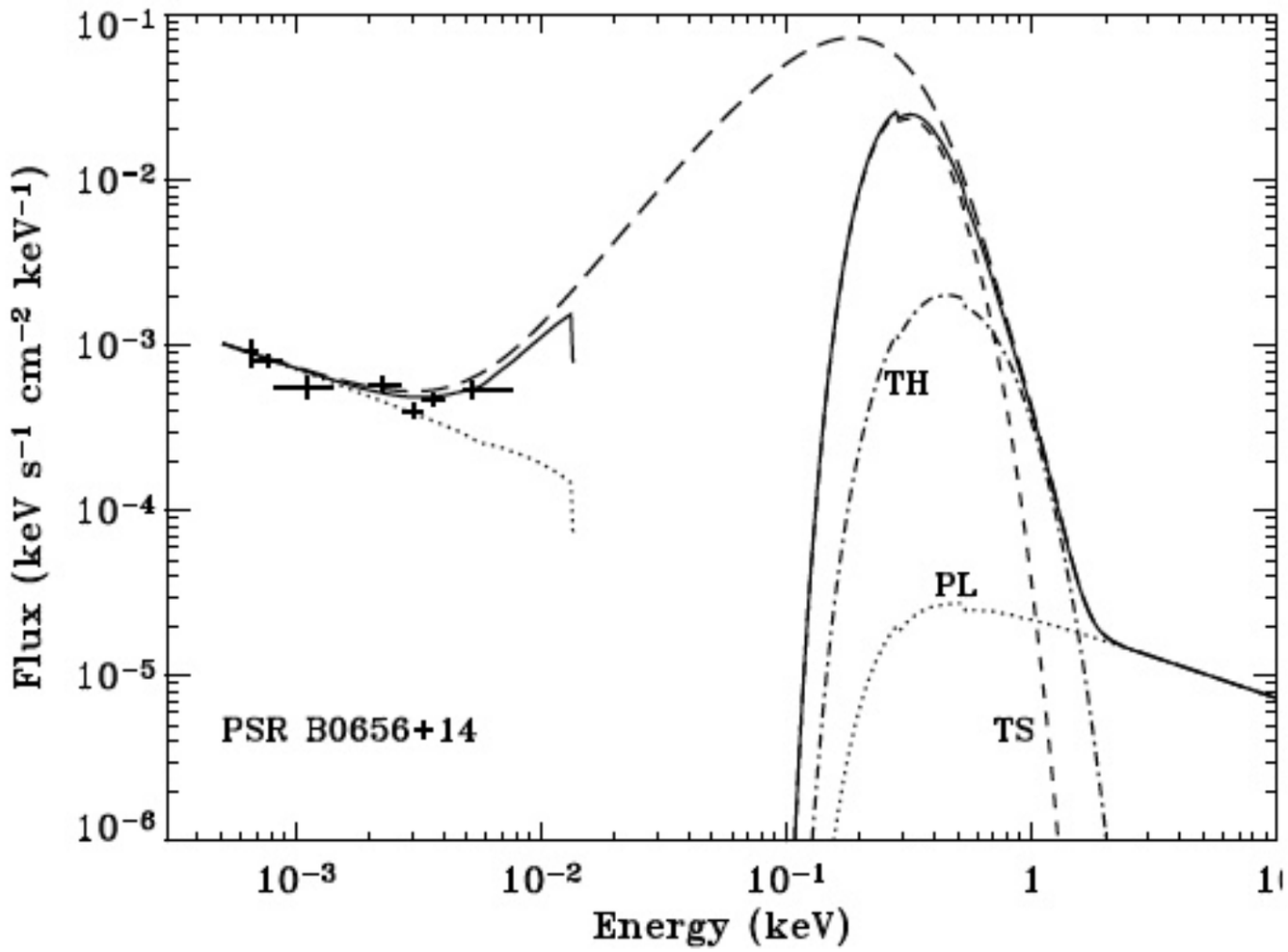
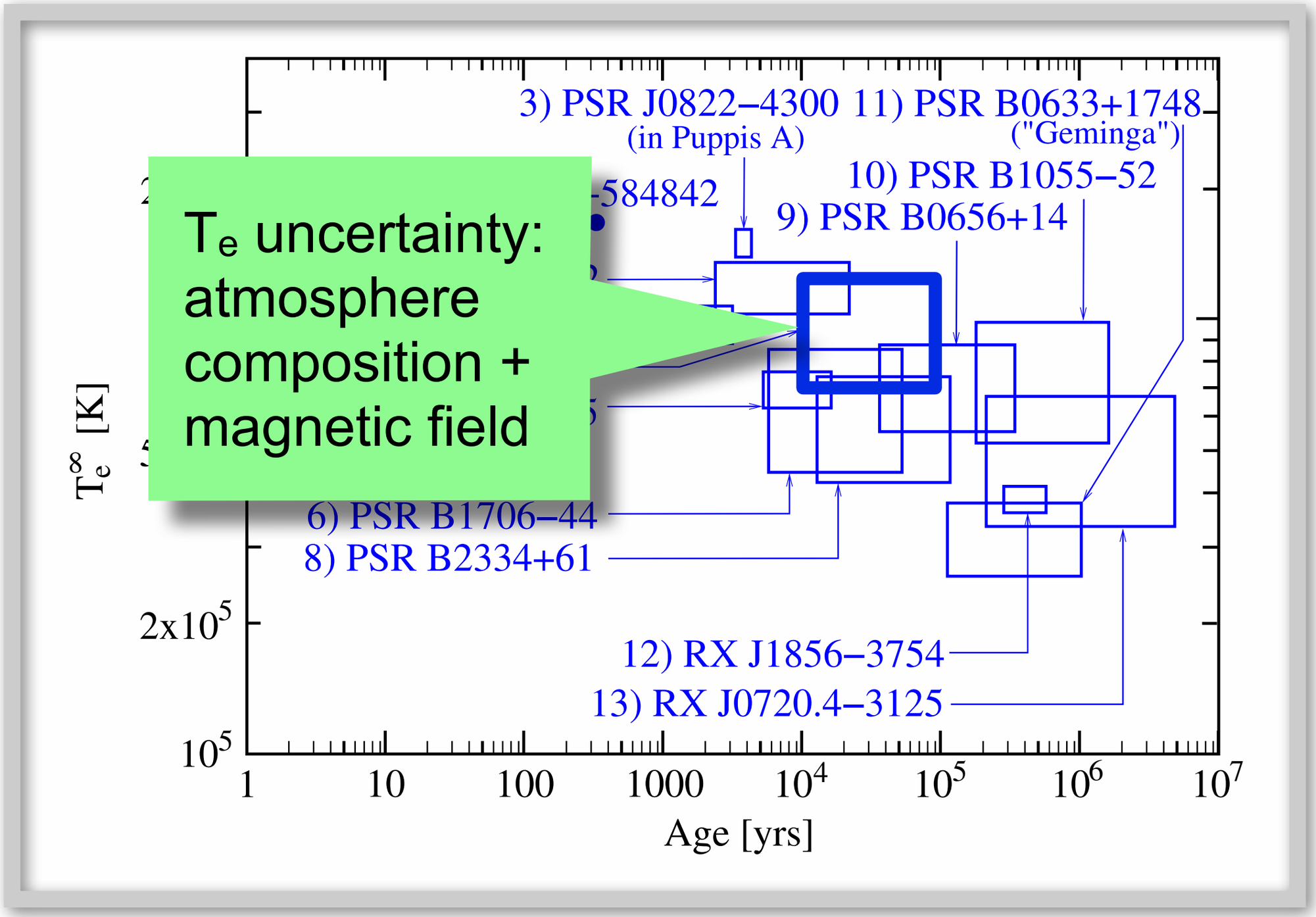



**Fig. 9.** Broadband spectrum of PSR B0656+14 for a three-component model (TS+TH+PL; see § 4.4) extrapolated in optical. The solid and long-dashed curves show the absorbed and unabsorbed spectra, respectively. Crosses indicate the IR-optical fluxes.

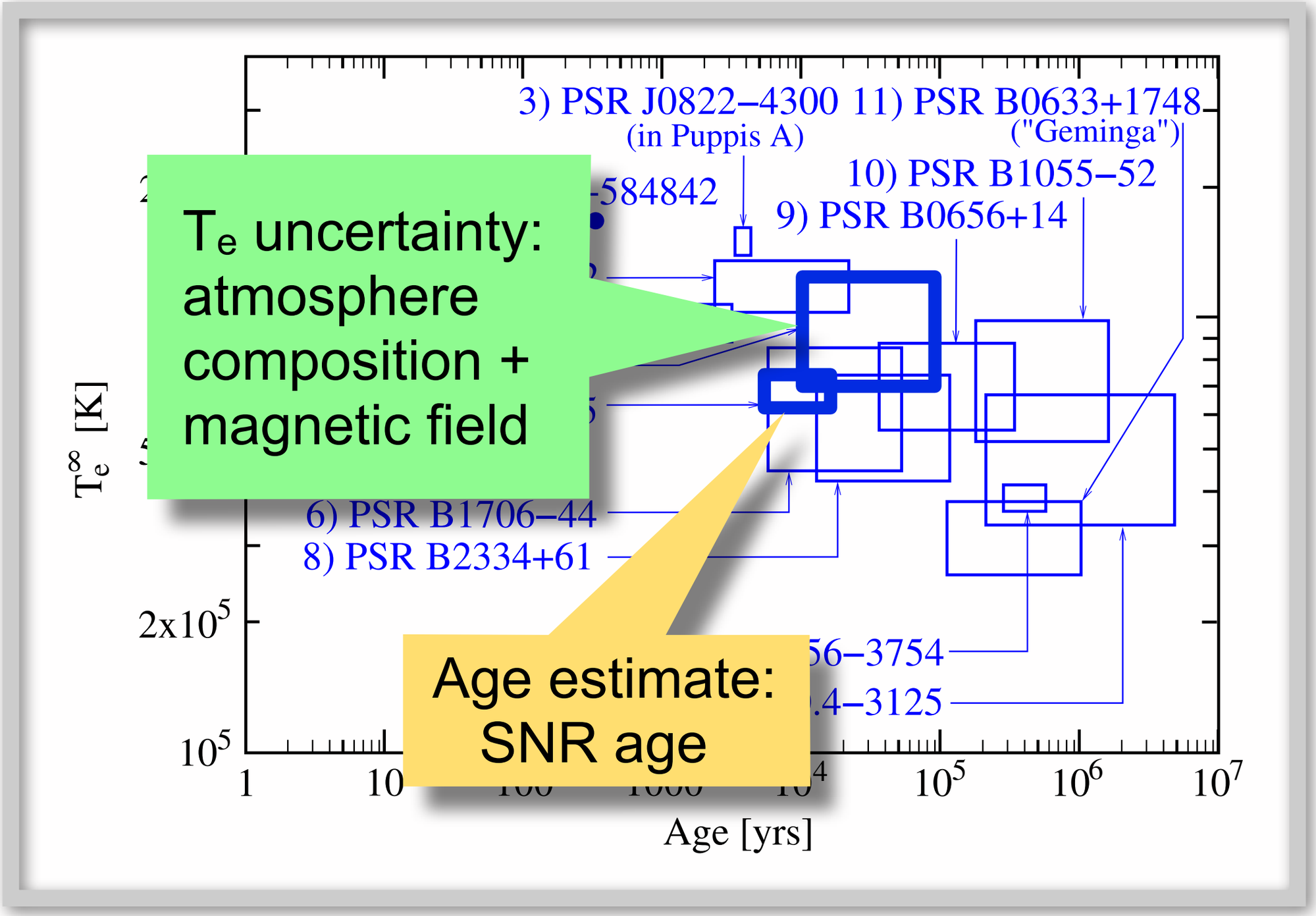
Thermal emission from isolated neutron stars: theoretical and observational aspects
V. E. Zavlin
in “Neutron Stars and Pulsars”, Ed. W. Becker, Astrophysics and Space Library, vol. 357, p. 181 (2008).

# Isolated Cooling Neutron Stars I



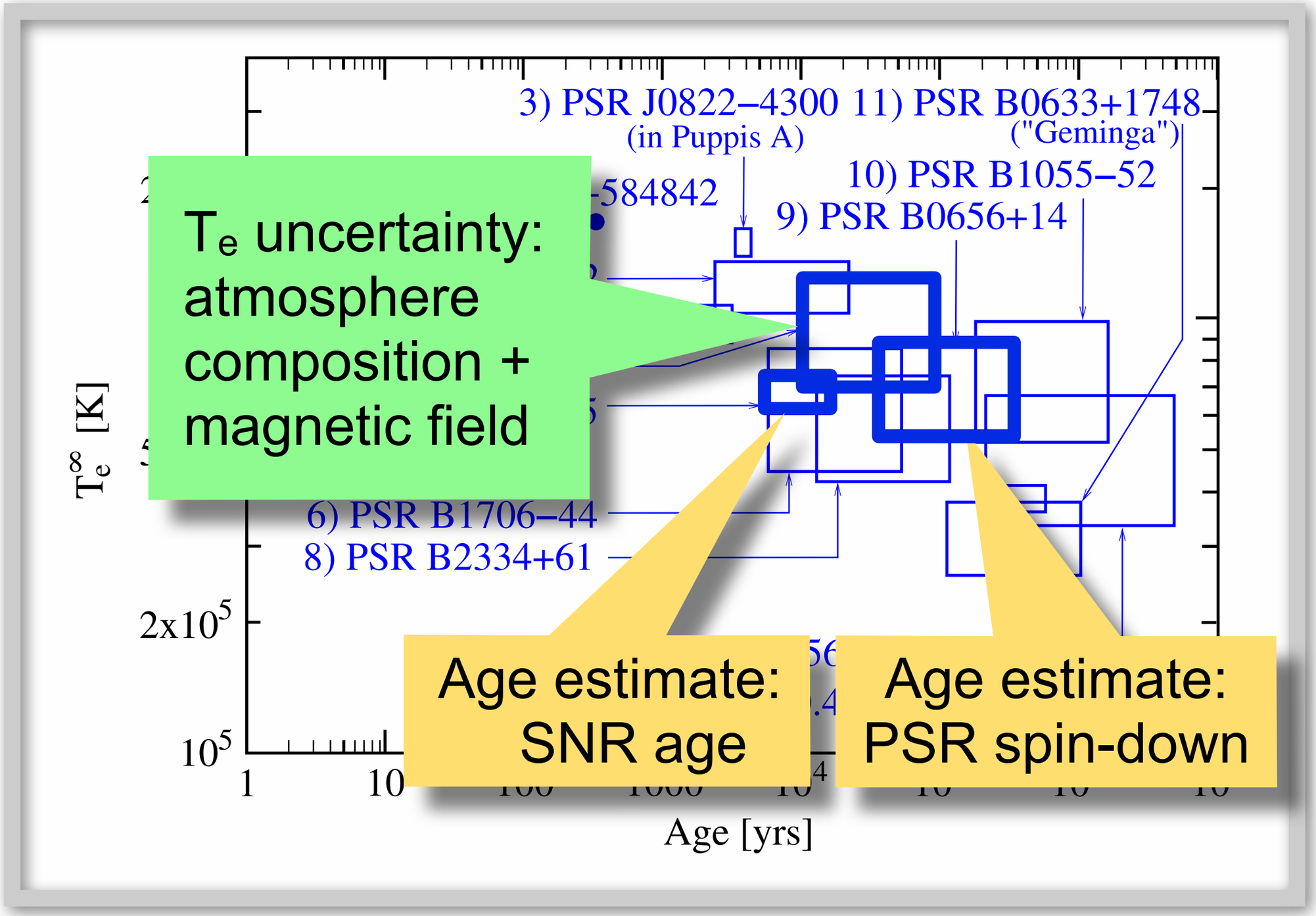

Neutron stars with clearly detected thermal emission

# Isolated Cooling Neutron Stars I



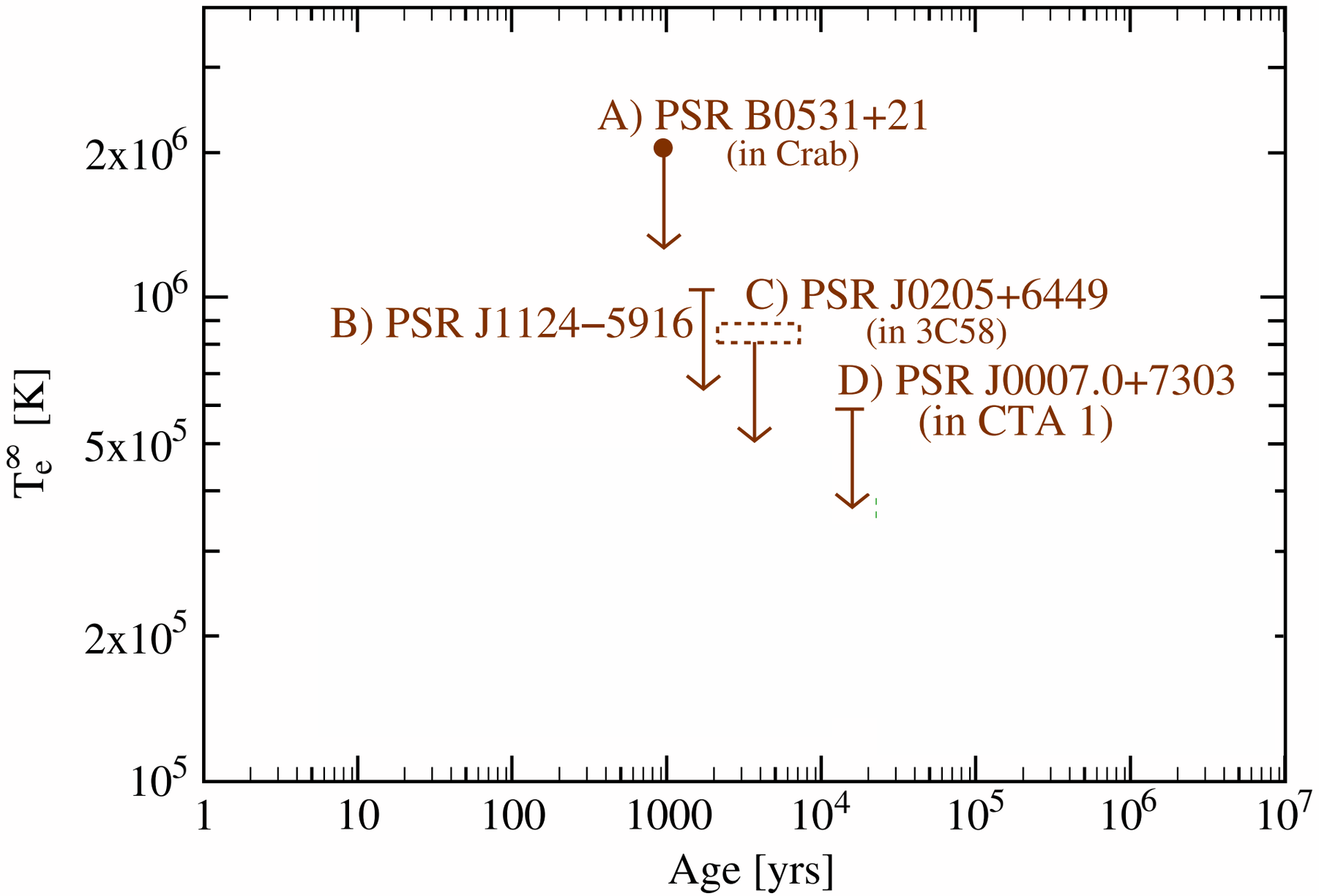

Neutron stars with clearly detected thermal emission

# Isolated Isolated Cooling Neutron Stars I



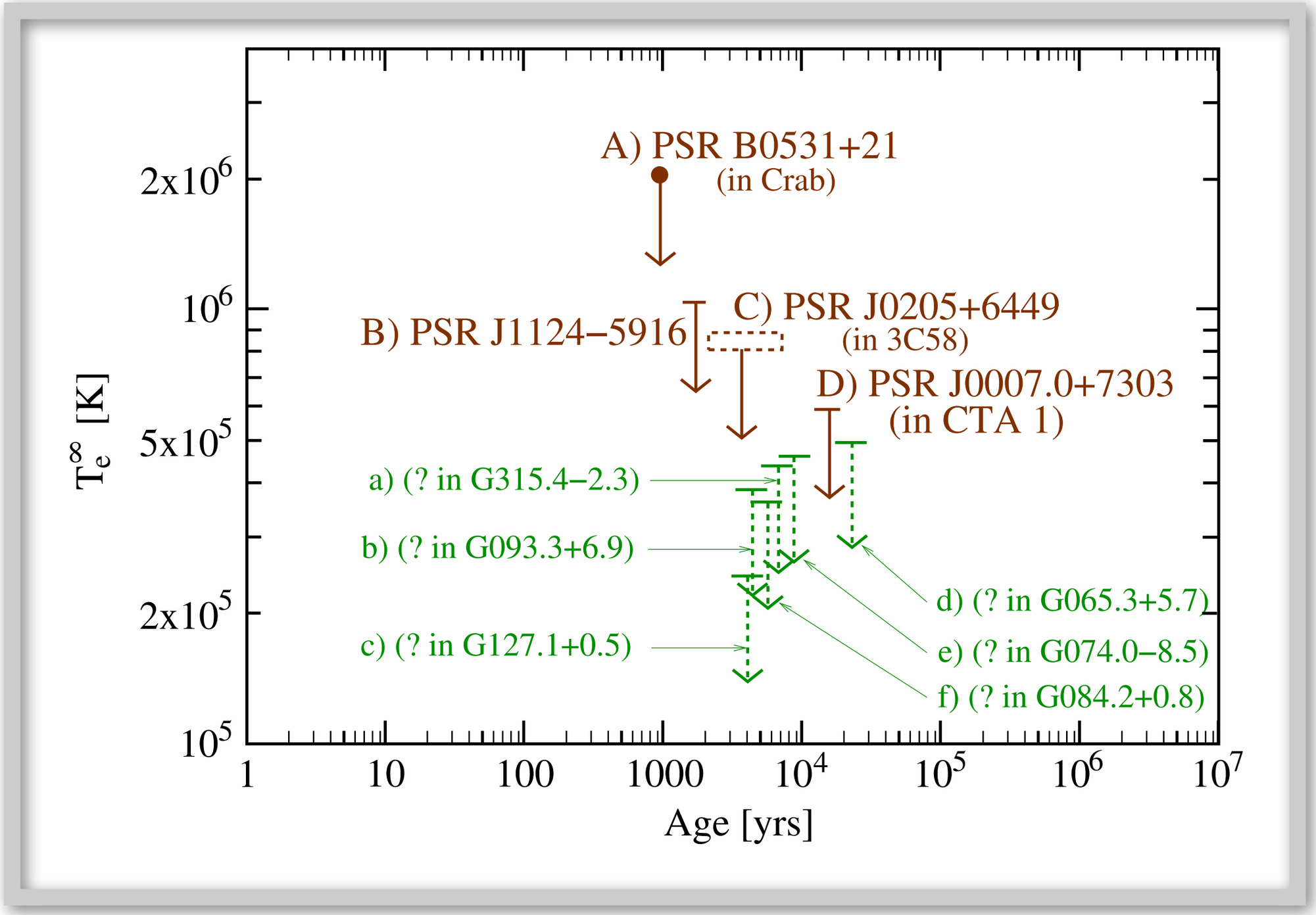

Neutron stars with clearly detected thermal emission

# Isolated Cooling Neutron Stars II




Detected neutron stars with undetected thermal emission

# Isolated Cooling Neutron Stars ?



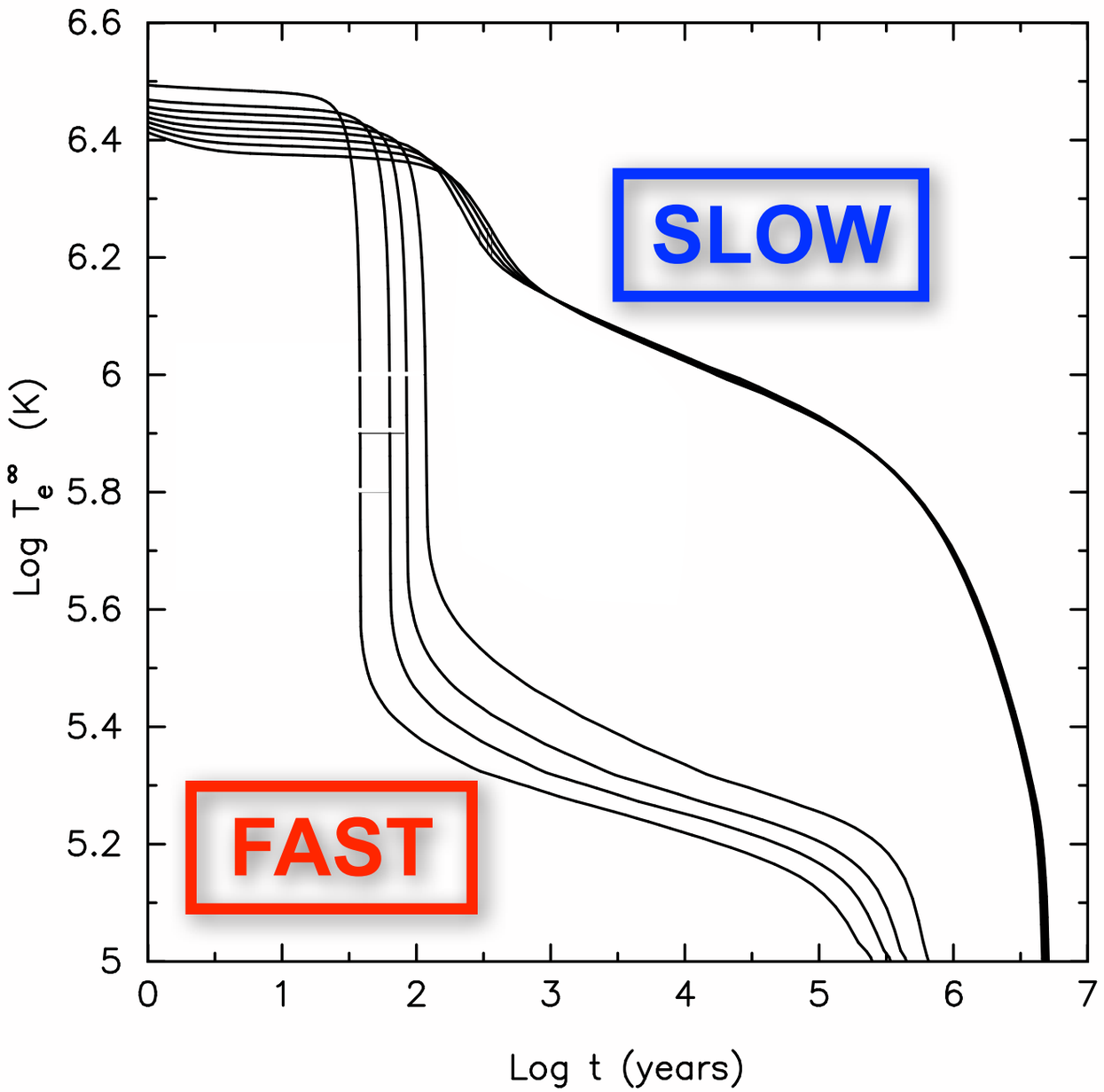

Upper limits on undetected neutron stars

# Simple Cooling Models

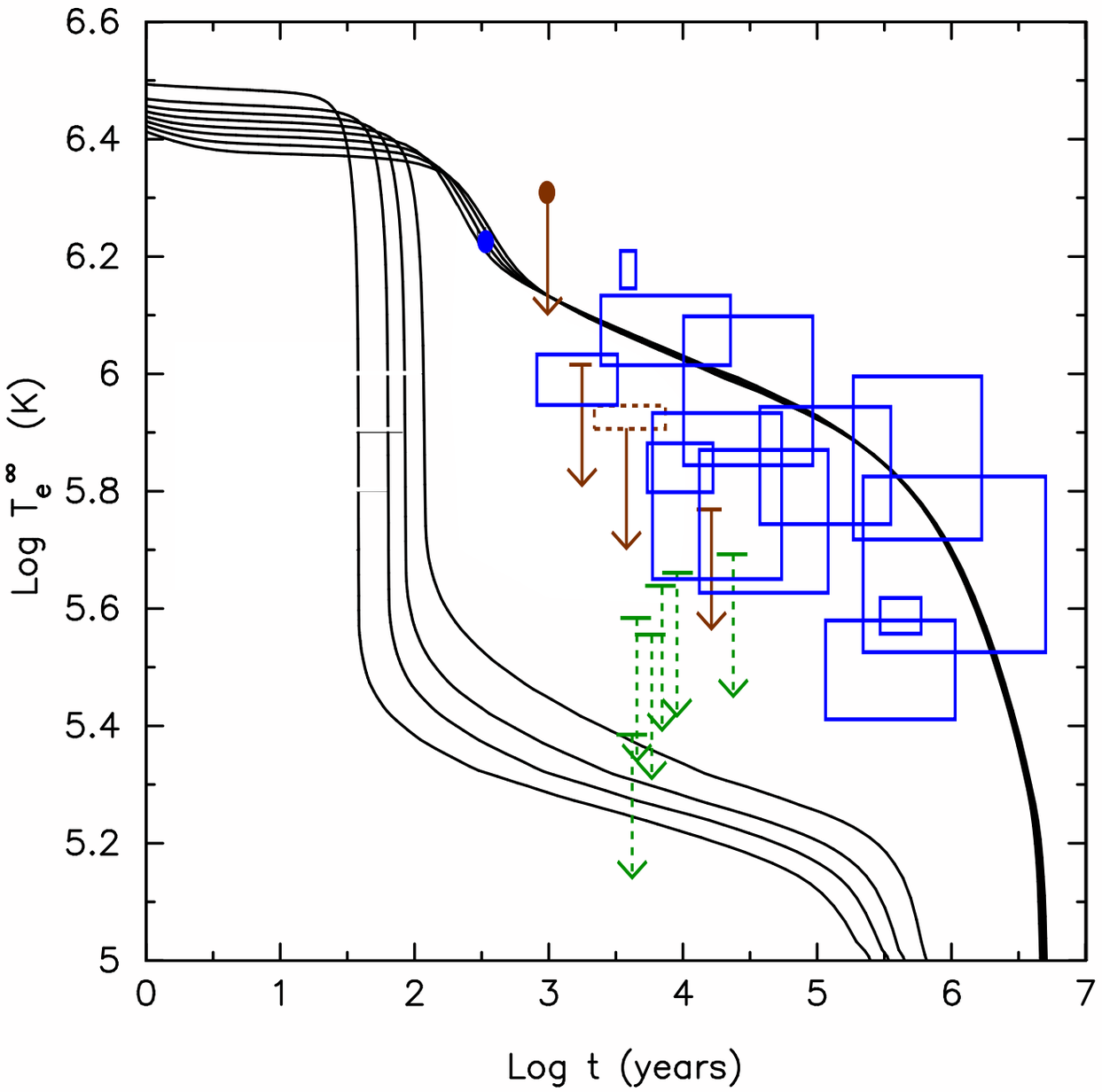
# versus

# Data

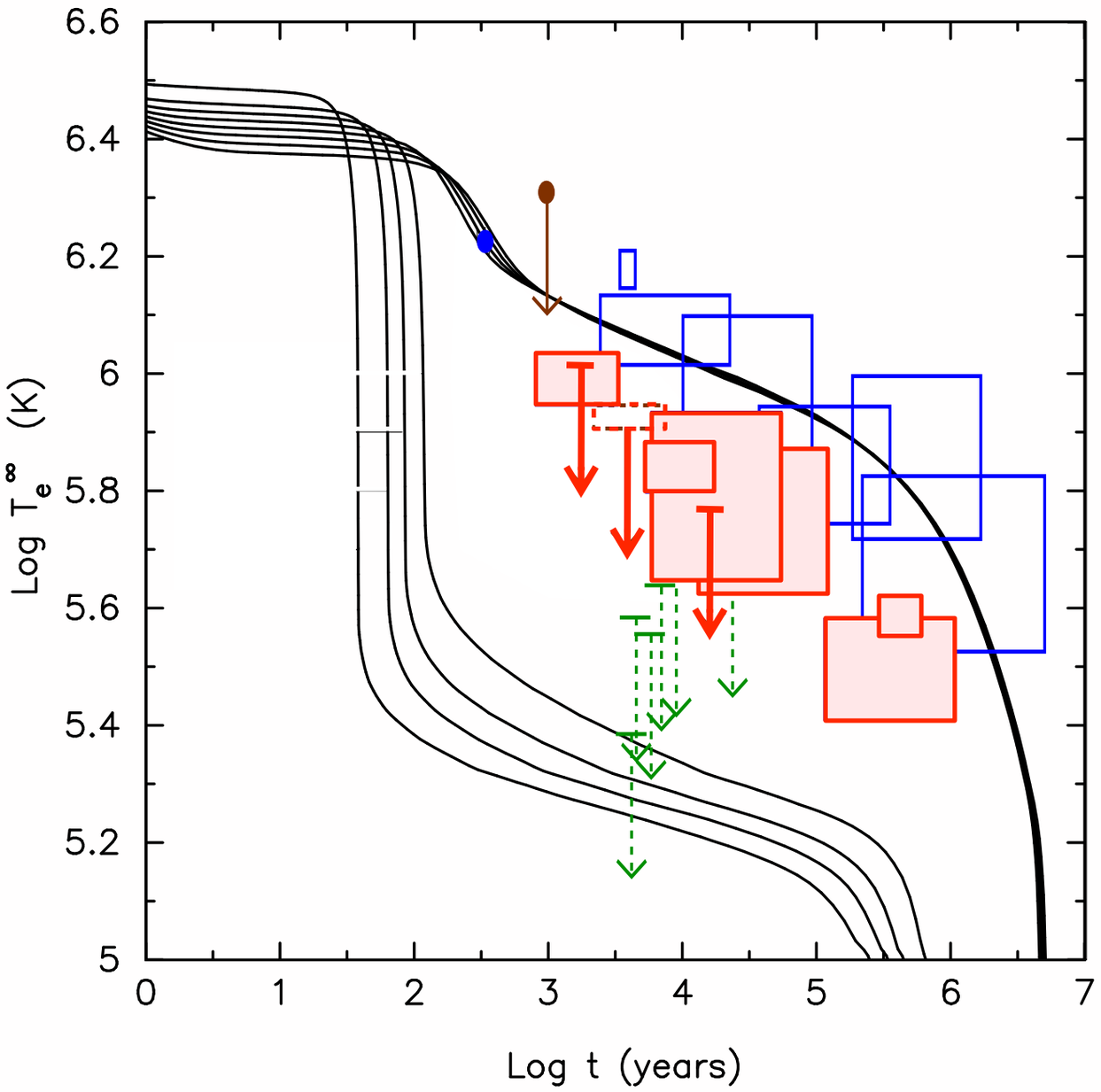
# DUrca & MUrca vs Data



# DUrca & MUrca vs Data



# DUrca & MUrca vs Data




**9 out of 17** young NSs (age<$10^6$ yrs) appear to require fast neutrino emission

(Plus 6 stars to be confirmed)

The majority of observed young cooling neutron stars appear to require some form of enhanced neutrino emission (i.e., some form of “exotic” matter):

# THAT IS INTERESTING !

... is it really true ?

# Pairing: Superfluidity & Superconductivity

# Brief History of Superconductivity & Superconductivity

- 1911: Heike Kamerlingh Onnes discovers superconductivity
- 1937: Pyotr Kapitsa discovers superfluidity of $^4$He
- 1940: Lev Landau proposes his theory of superfluidity
- 1954: F. London proposes a simple explanation of superconductivity
- 1956: Leon Cooper ‘discovers’ the Cooper pairs
- 1957: J. Bardeen, L.N. Cooper, & J.R. Schrieffer present the `BCS´ theory of superconductivity.
- 1958: A. Bohr, B.R. Mottelson, & D. Pines argue that neutrons and protons in nuclei form Cooper pairs
- 1959: A. Migdal suggests that neutrons in neutron stars may be superfluid
- 1960´s: theoretical work on possibility of $^3$He superfluidity *à la* BCS
- 1972: D.D. Osheroff, R.C. Richardson, & D. Lee discover $^3$He superfluidity
- 1967: J. Bell & A. Hewish discover pulsars (= neutron stars)
- → numerous theoretical works on neutron/proton superfluidity/superconductivity in neutron stars ...

# Fermion Superfluidity/Superconductivity

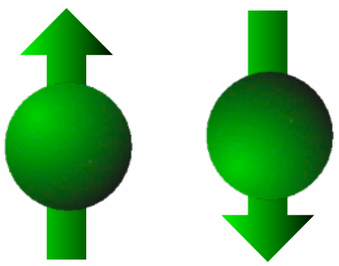
Superfluidity/superconductivity o fermions:
**fermionic condensation**

**BCS Theory** (Bardeen-Cooper-Schrieffer):
1 - fermions form Cooper pairs (=bosons)
2 - and then the pairs can condense

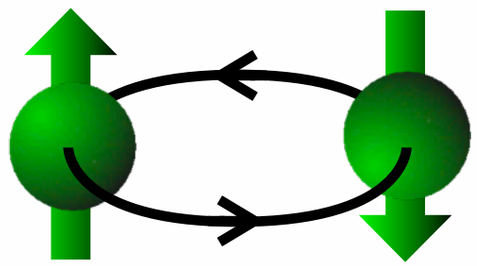
## Cooper pairs spin-angular momentum

spin singlet pairs

L = 0

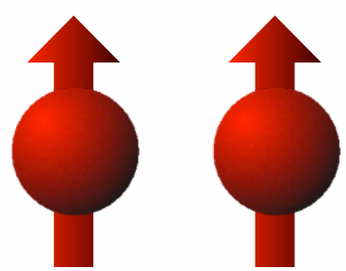


L > 0

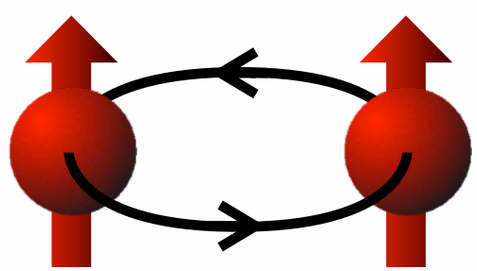


spin triplet pairs

L = 0

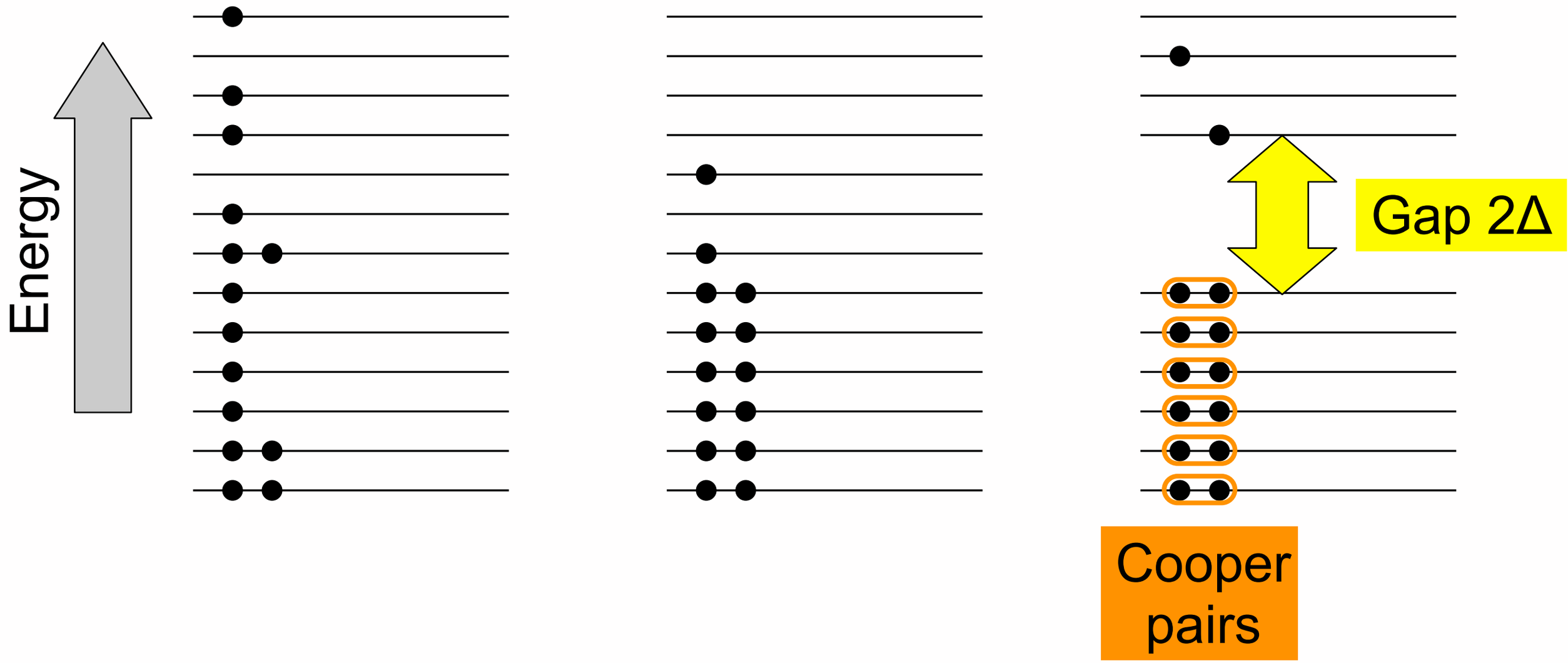


L > 0



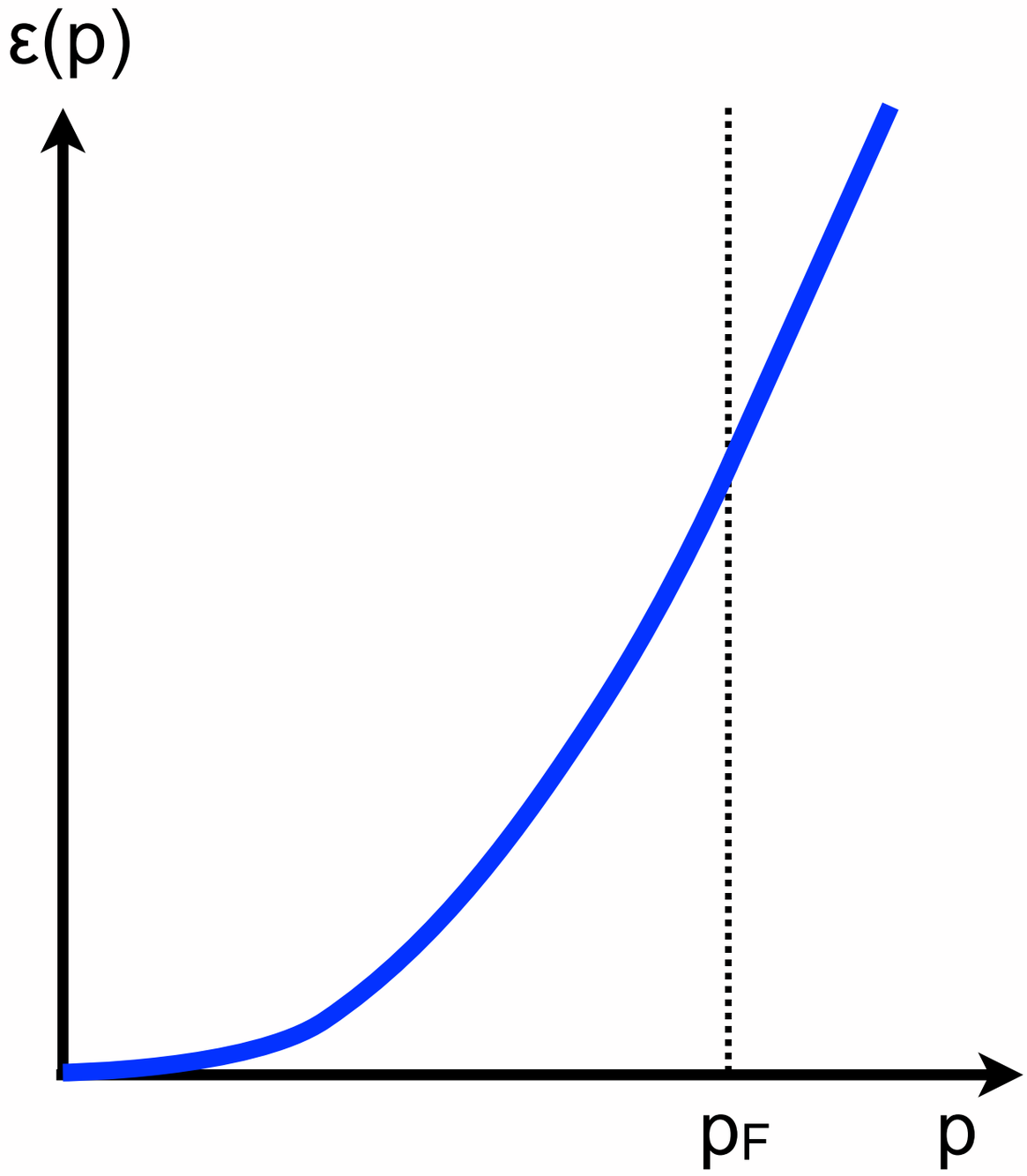
# Fermions Energy Levels and BCS

| Non degenerate Fluid | Normal Degenerate Fluid | BCS Superfluid |
|---|---|---|
| $T >> T_F$ | $T << T_F$ | $T < T_c << T_F$ |

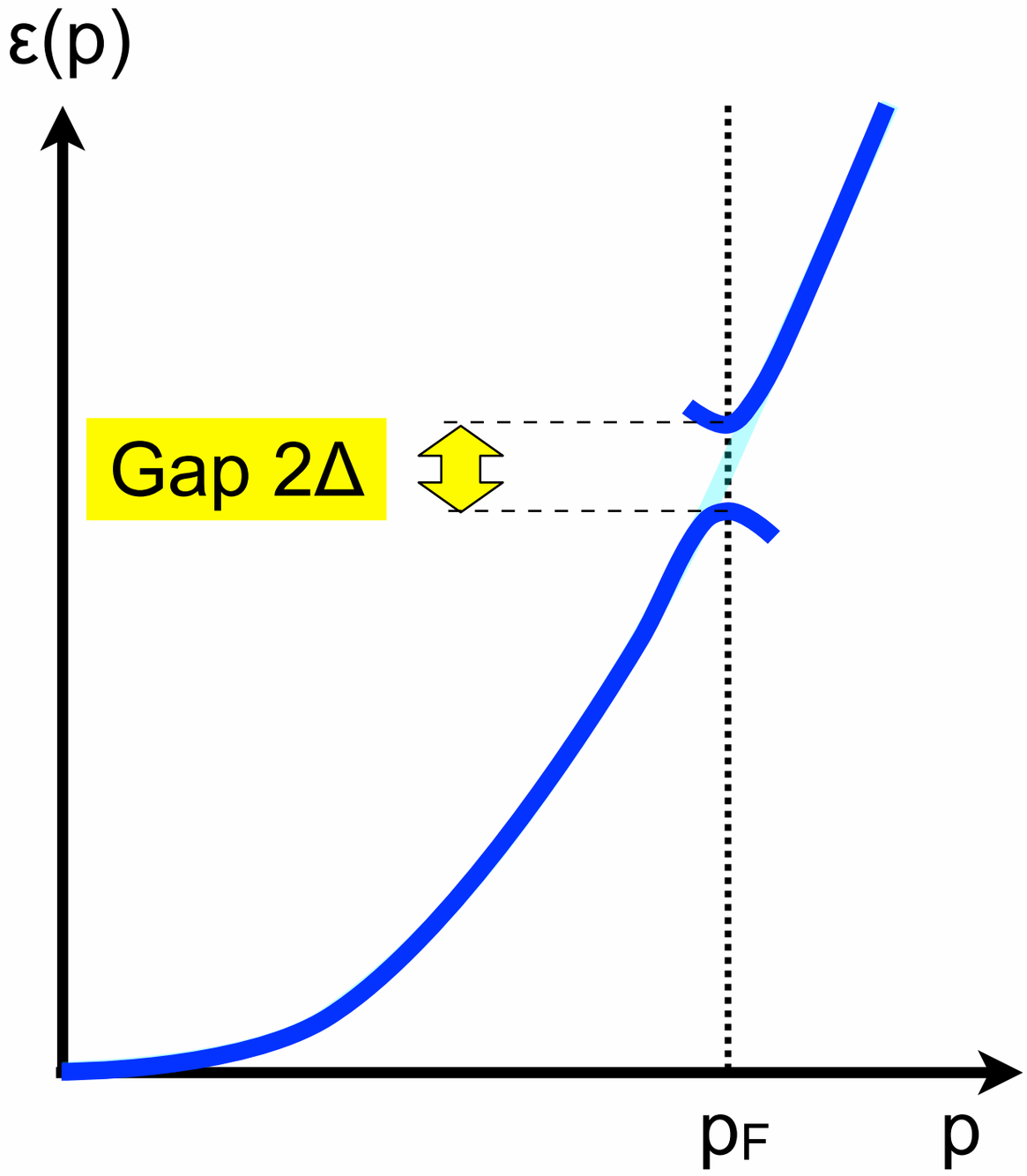


# Single-Particle Excitations Spectra

## Normal Fermion Liquid



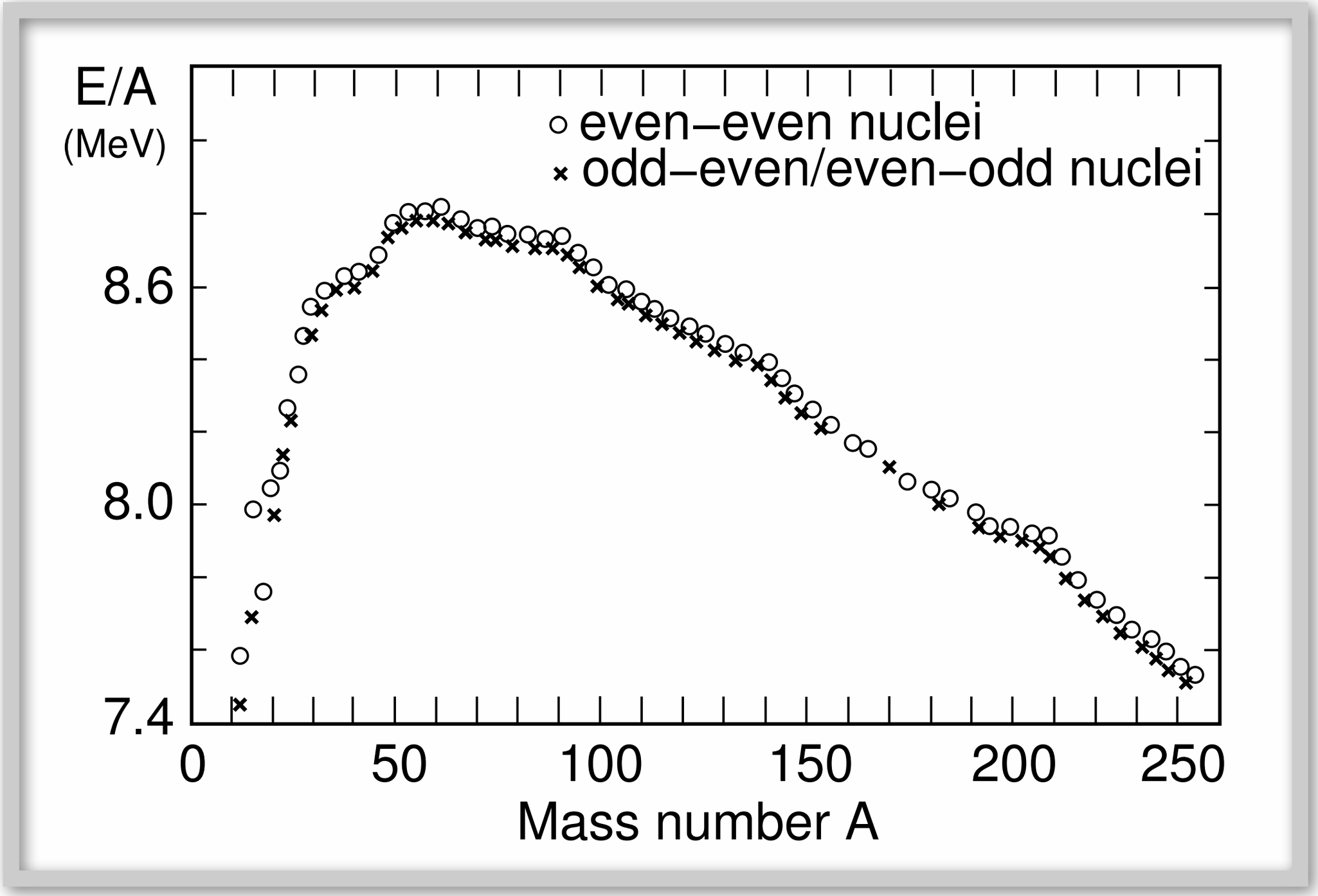

## Superfluid Fermions



# Pairing in Nuclei

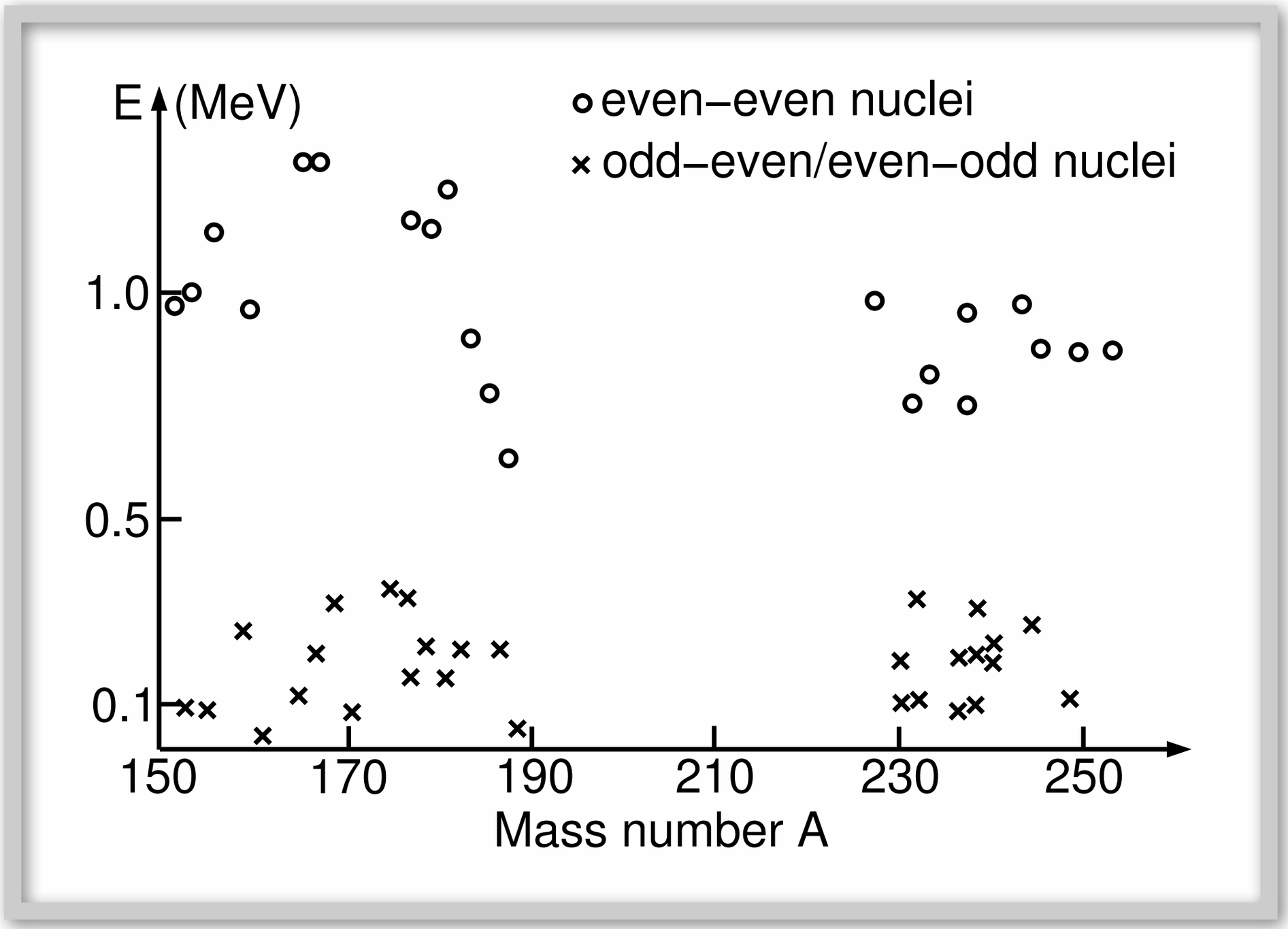
# Nuclear Binding Energy

Binding energy per nucleon of the most beta-stable isobars



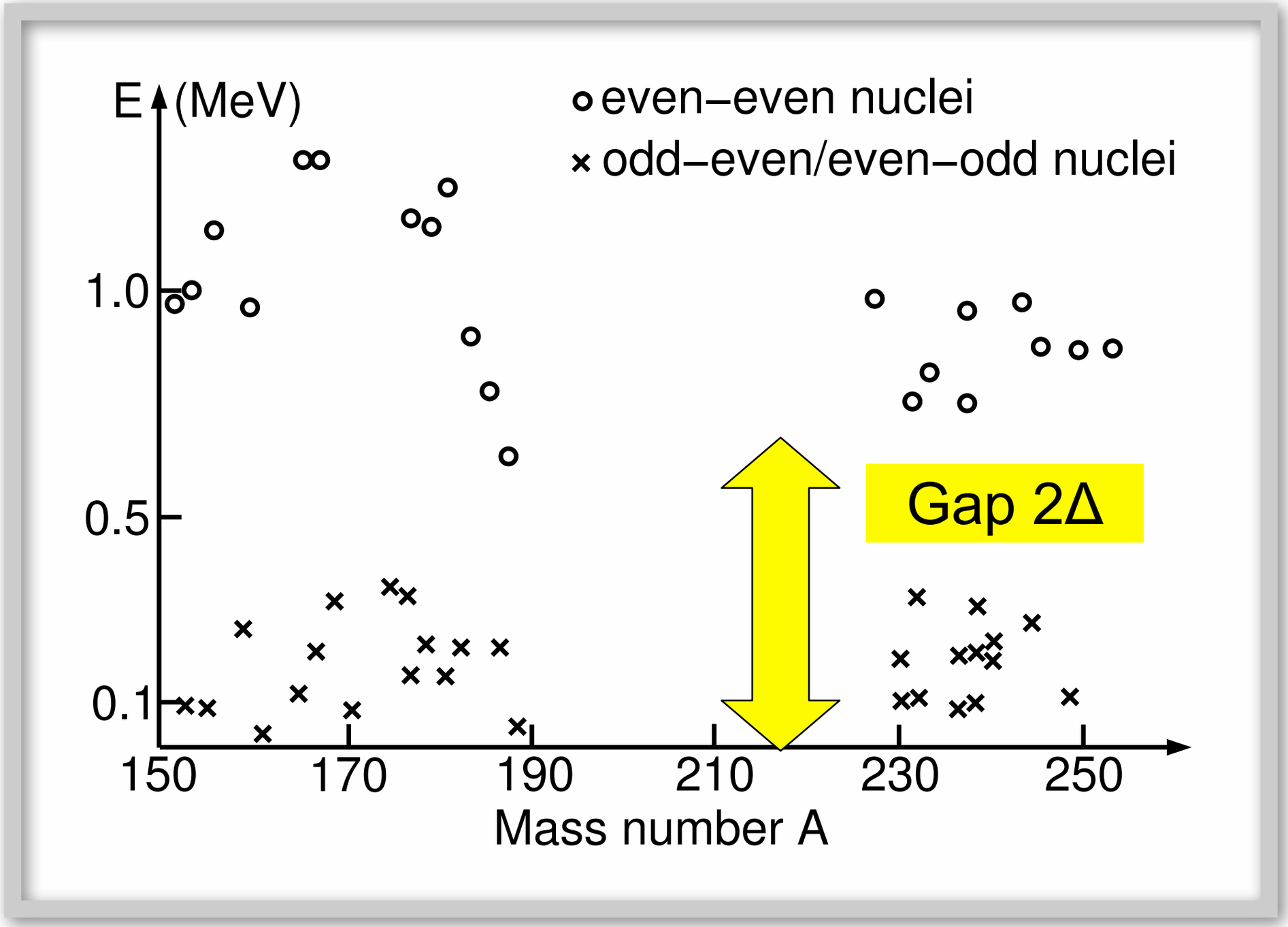

odd-odd nuclei: $^{2}$H (1,1), $^{6}$Li (3,3), $^{10}$B (5,5) and $^{14}$N (7,7)

# Lowest Excitation Levels in Nuclei




Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State
Bohr, A.; Mottelson, B. R.; Pines, D. (1958), Phys. Rev. 110, p.936

# Lowest Excitation Levels in Nuclei




Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State
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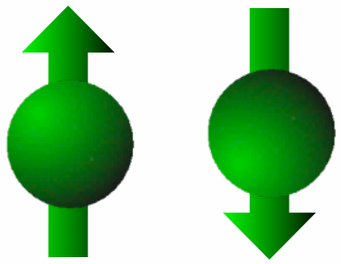
# Pairing in Neutron Stars

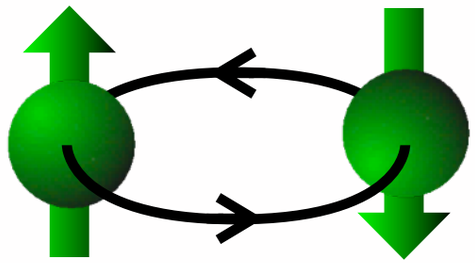
# Pairing of Nucleons

spin singlet pairs

L = 0

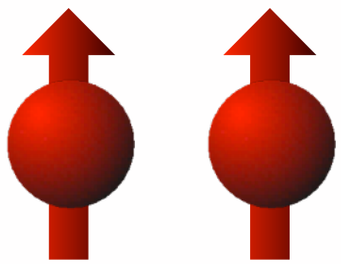


L > 0

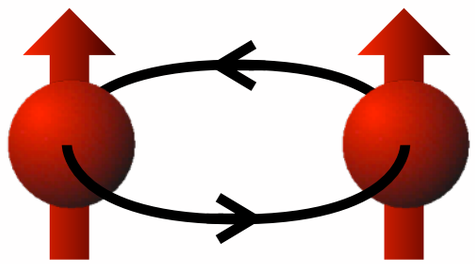


spin triplet pairs

L = 0



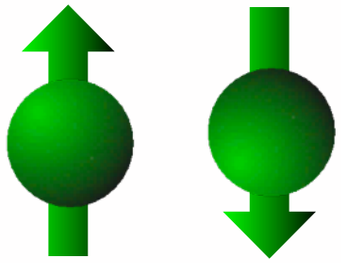
L > 0



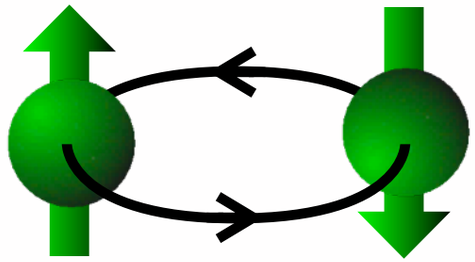
# Pairing of Nucleons

## spin singlet pairs

L = 0

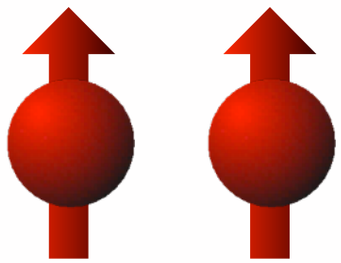


L > 0

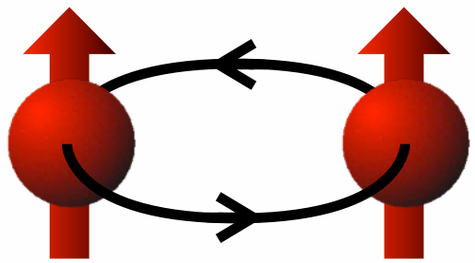


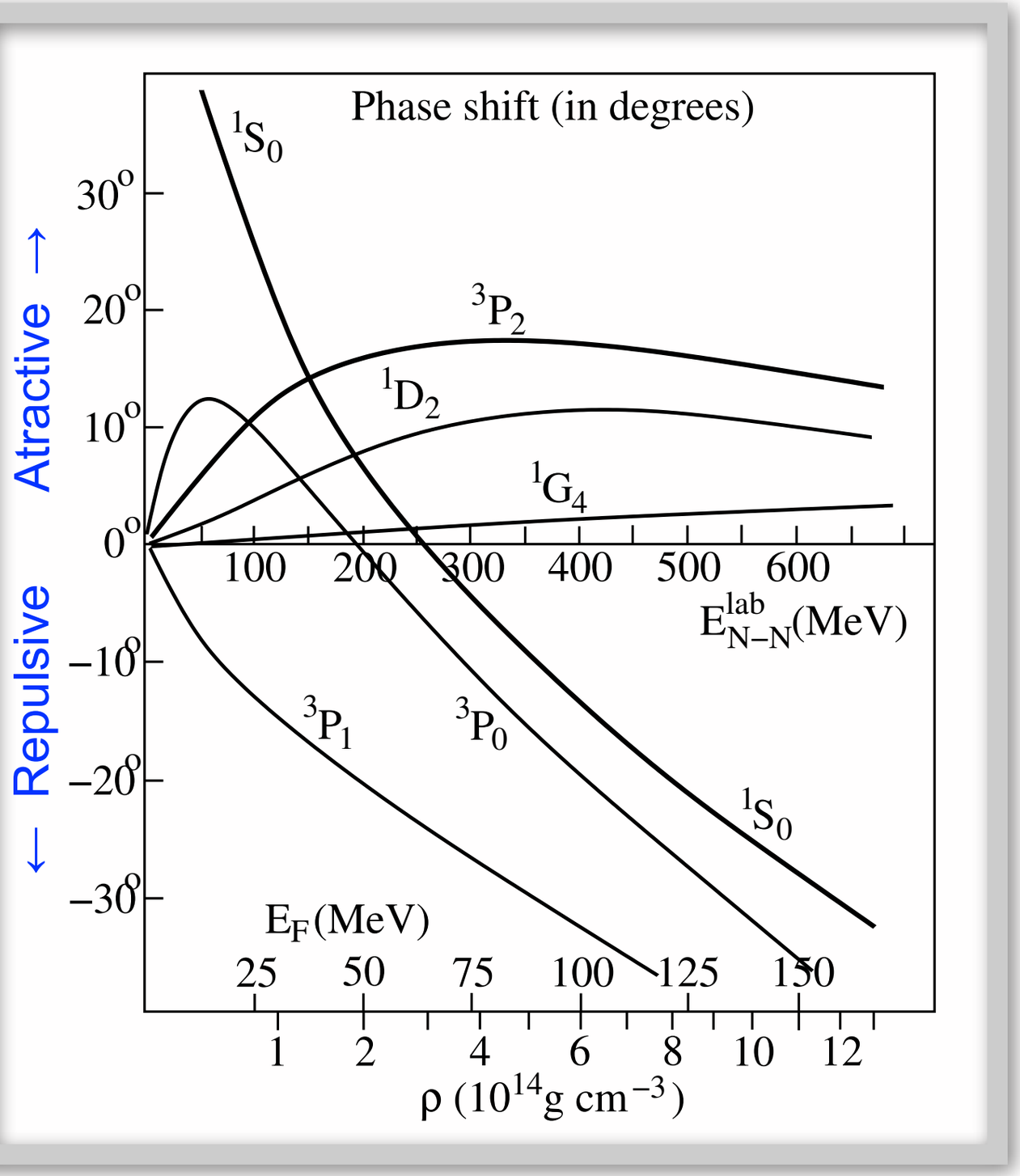
## spin triplet pairs

L = 0



L > 0



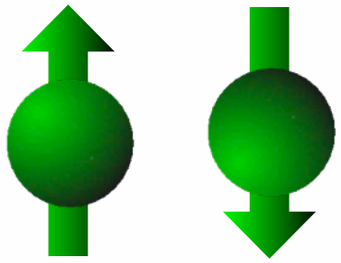



Superfluid State in Neutron Star Matter. I Generalized Bogoliubov Transformation and Existence of $^3P_2$ Gap at High Density
Tamagaki, R., 1970PThPh..44..905T

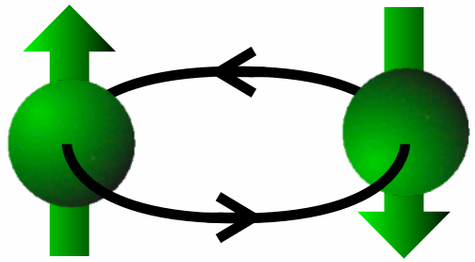
# Pairing of Nucleons

## spin singlet pairs

L = 0

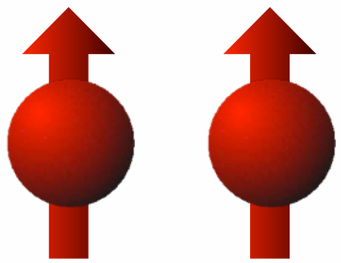


L > 0

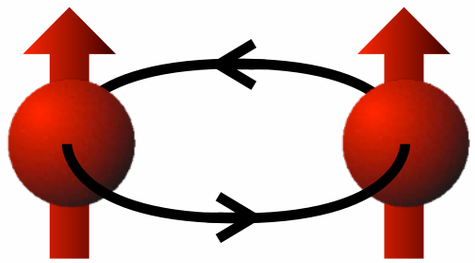


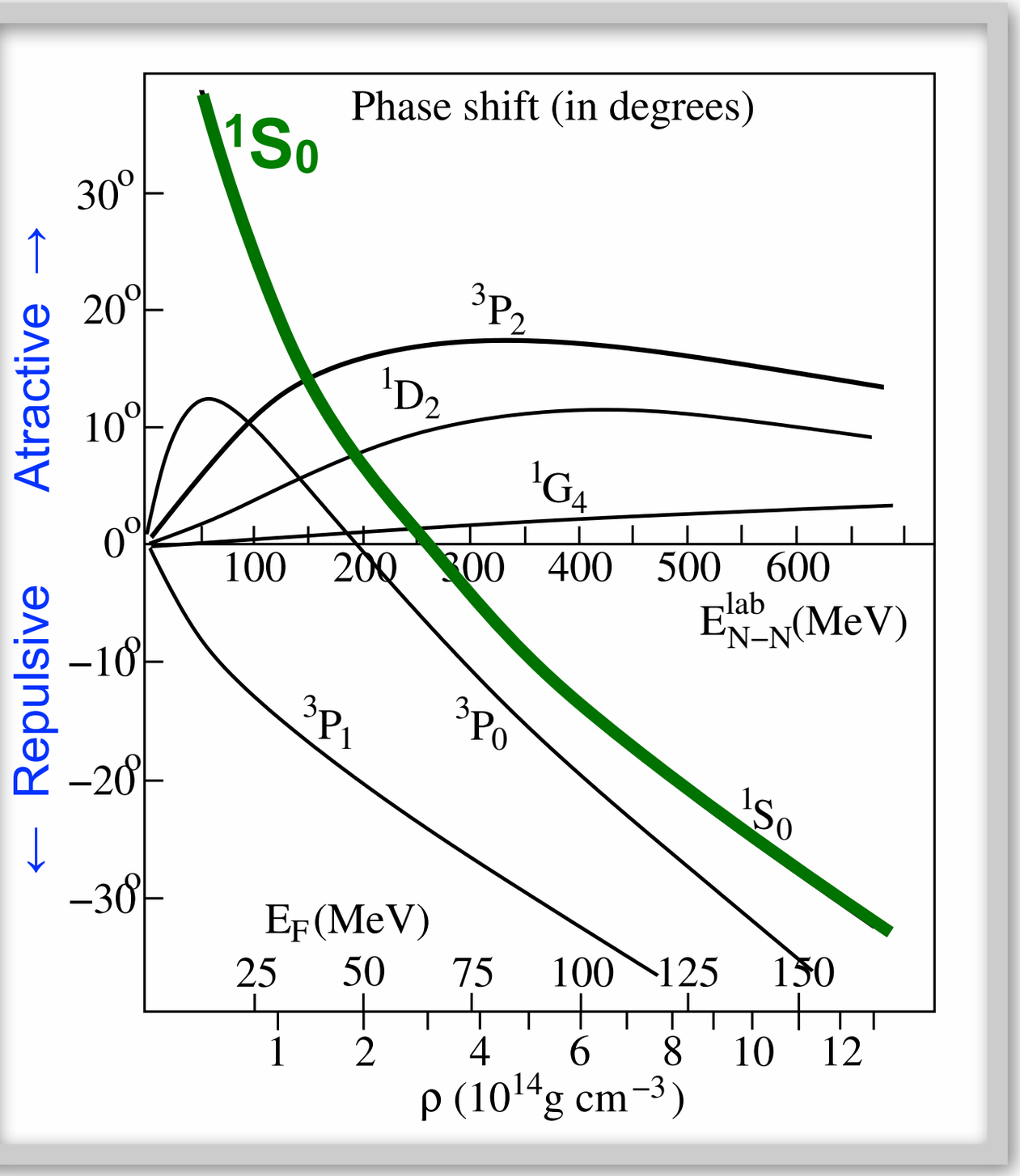
## spin triplet pairs

L = 0



L > 0



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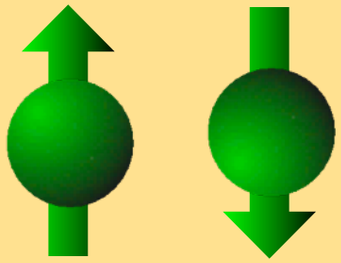
# Pairing of Nucleons

spin singlet pairs

S=0 L=0 J=0

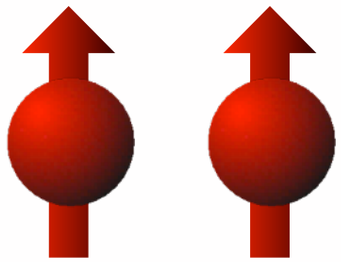


At low $k_F$:

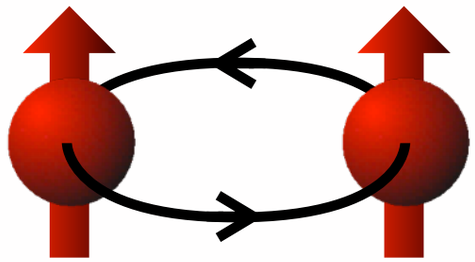
$^1S_0$ pairing

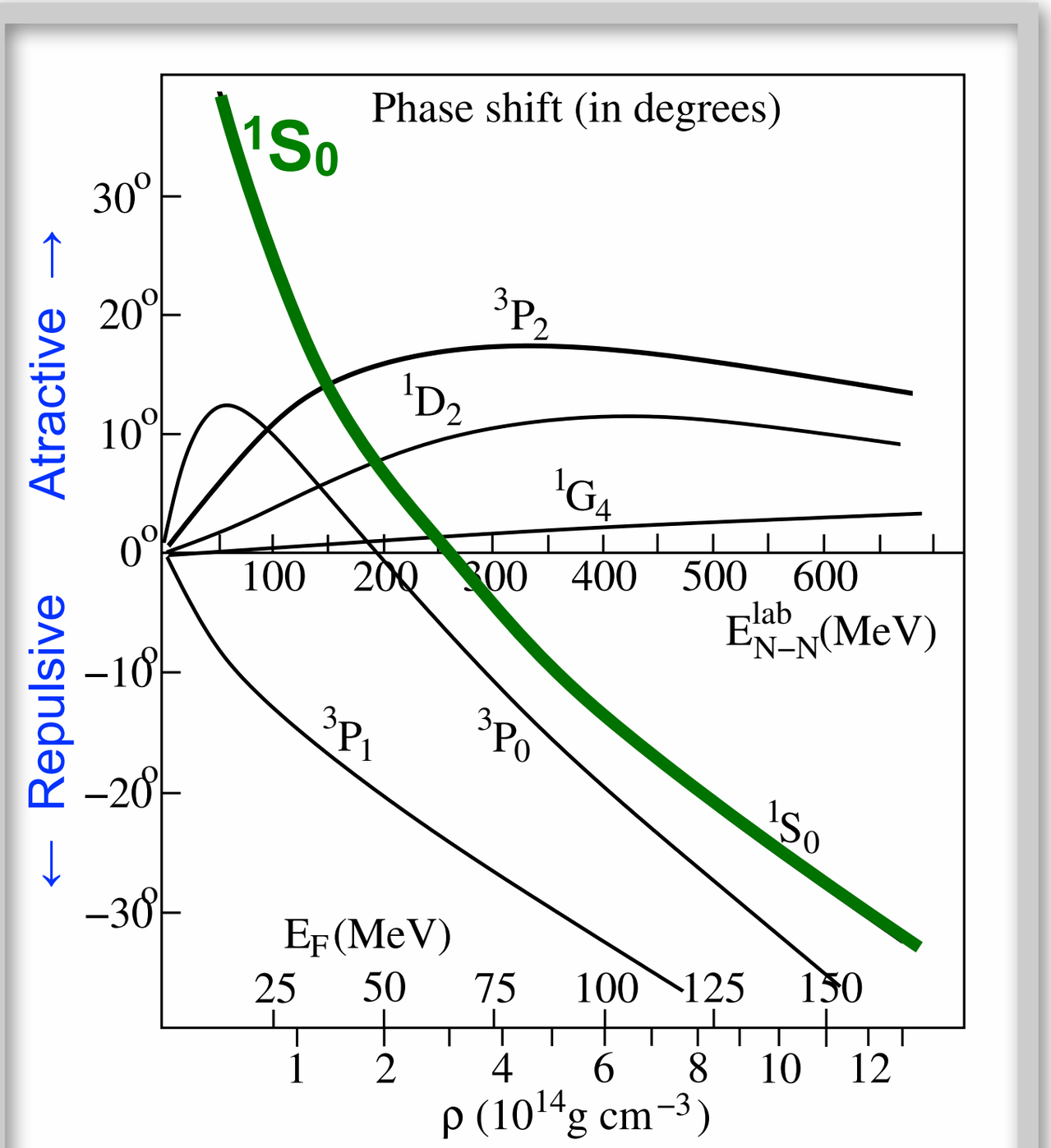
spin triplet pairs

L = 0



L > 0



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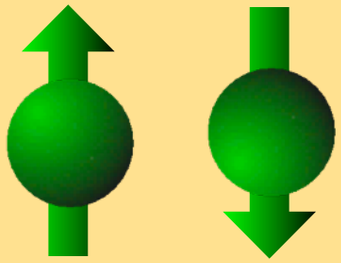
# Pairing of Nucleons

spin singlet pairs

S=0 L=0 J=0

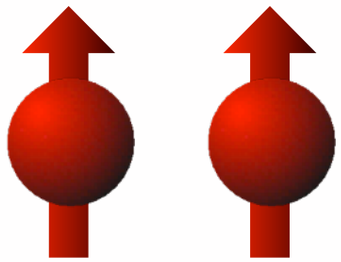


At low $k_F$:

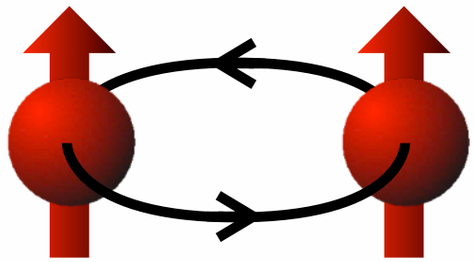
$^1S_0$ pairing

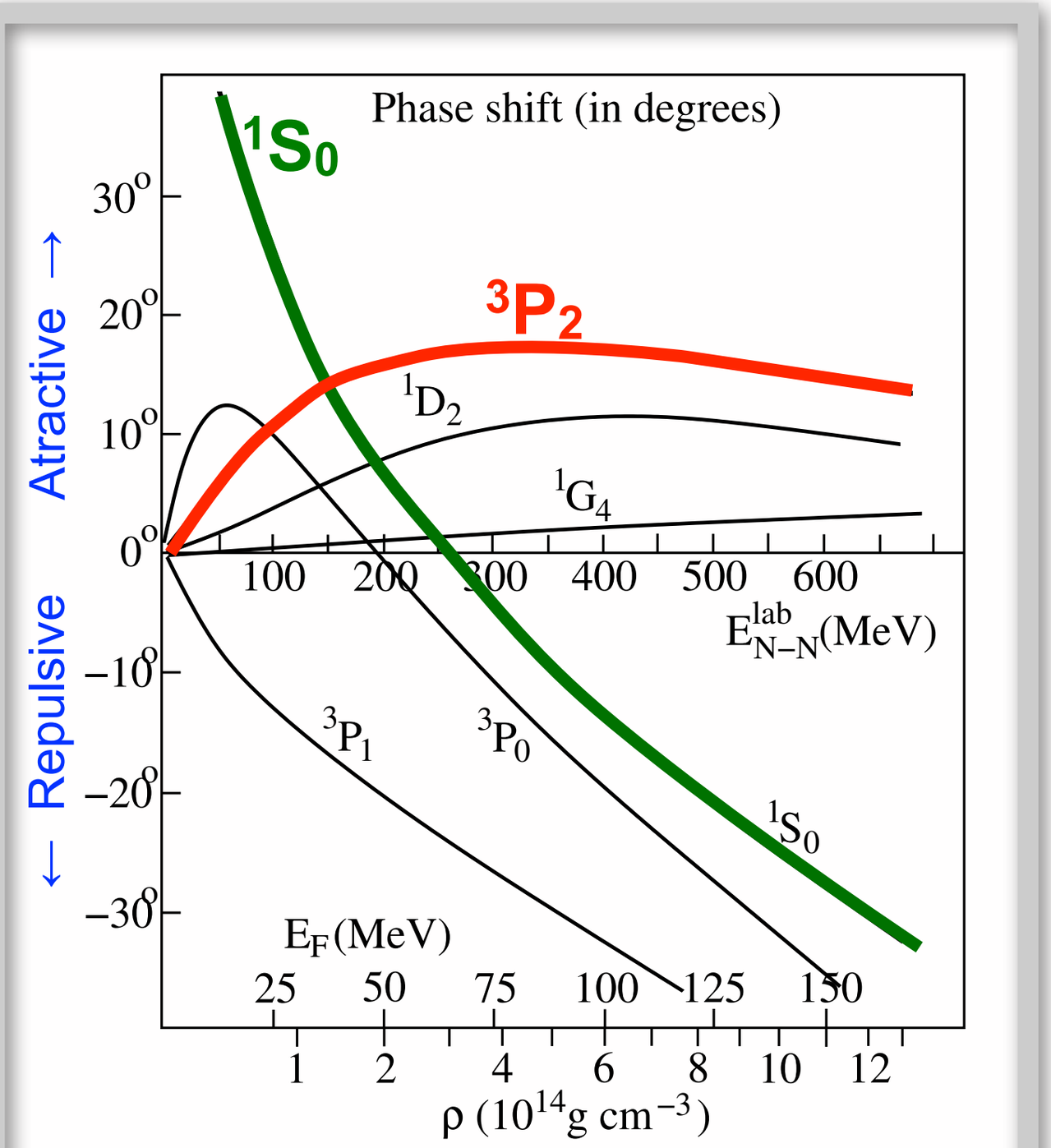
spin triplet pairs

L = 0



L > 0



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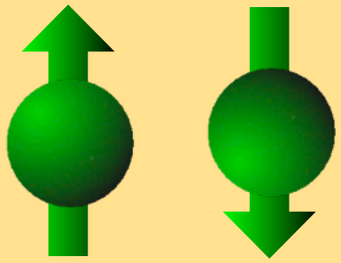
# Pairing of Nucleons

spin singlet pairs

S=0 L=0 J=0



At low $k_F$:

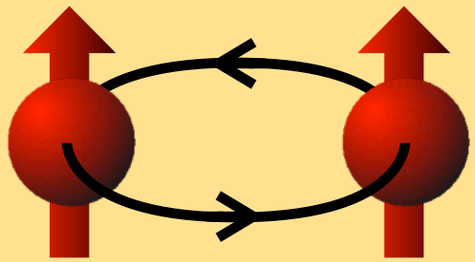
$^1S_0$ pairing

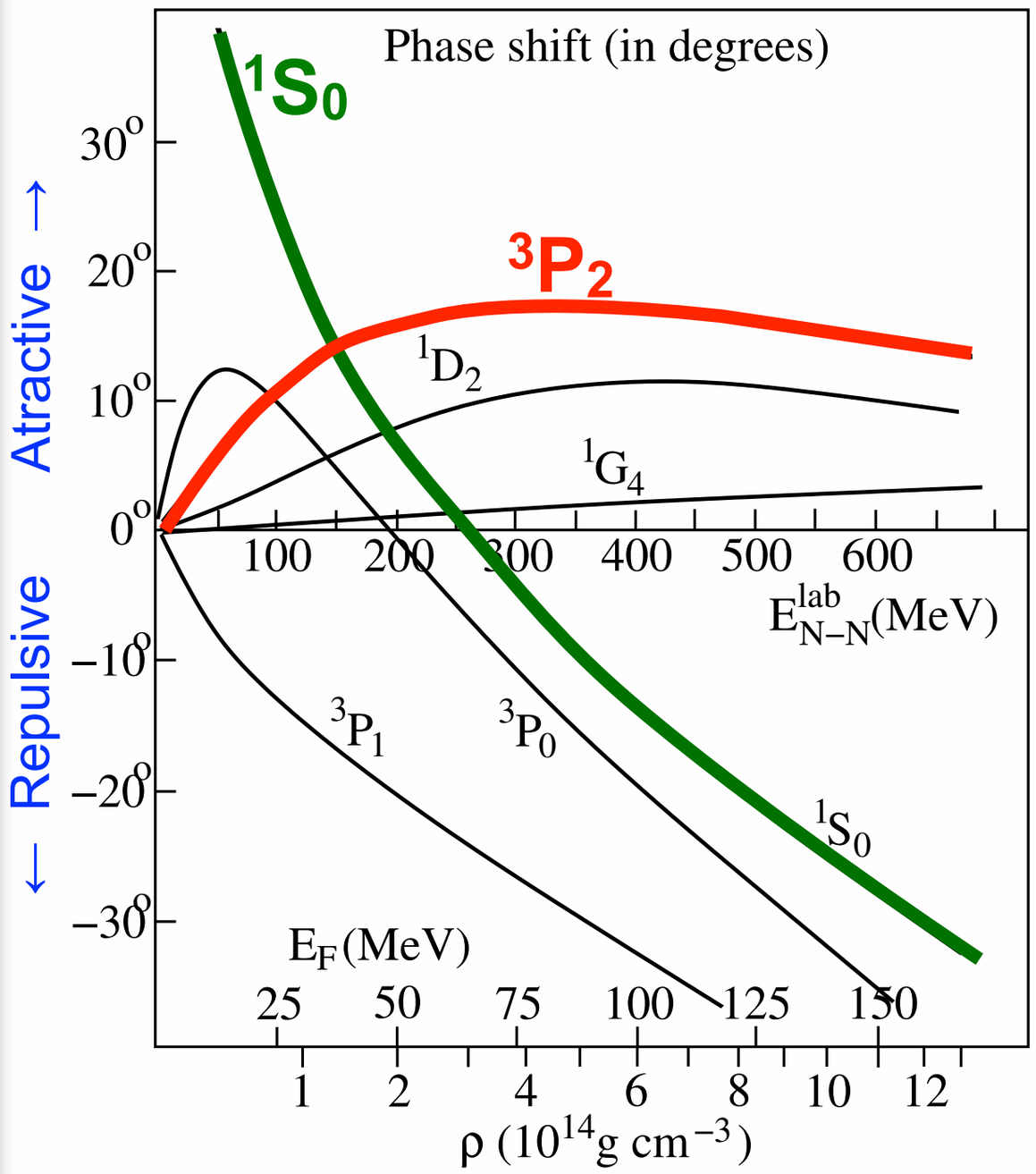
spin triplet pairs

S=1 L=1 J=2

At high $k_F$:

$^3P_2$ pairing






Superfluid State in Neutron Star Matter. I Generalized Bogoliubov Transformation and Existence of $^3P_2$ Gap at High Density
Tamagaki, R., 1970PThPh..44..905T

# Δ(T=0) and $T_c$ and Neutron Star Matter

Simple BCS estimate:

$$\Delta(T=0) \simeq E_F \, \mathrm{e}^{-1/N_0 V_P}$$

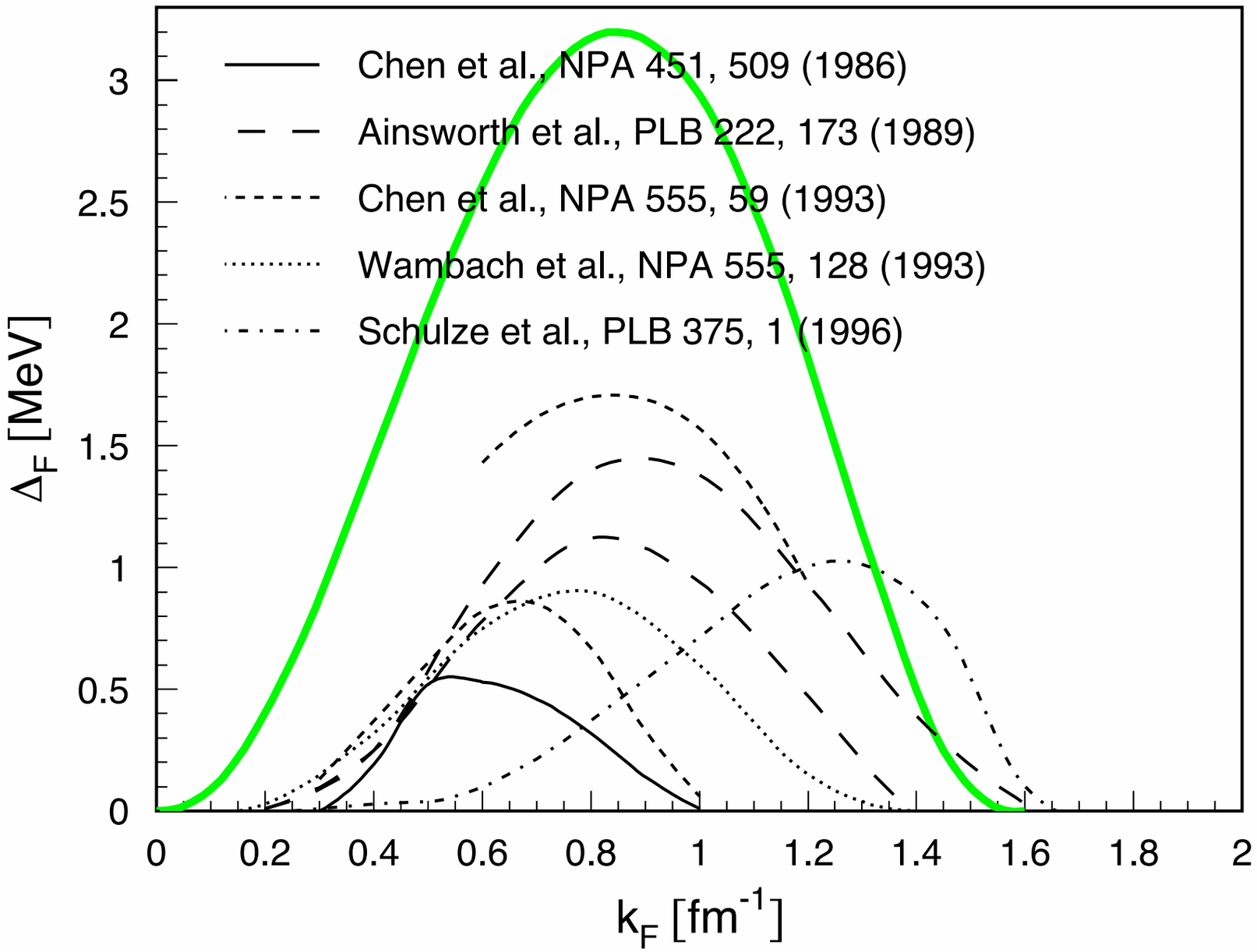
$N_0$ = density of states
$V_P$ = pairing potential

$V_P$ is *enormously* messy and goes into an exponential.

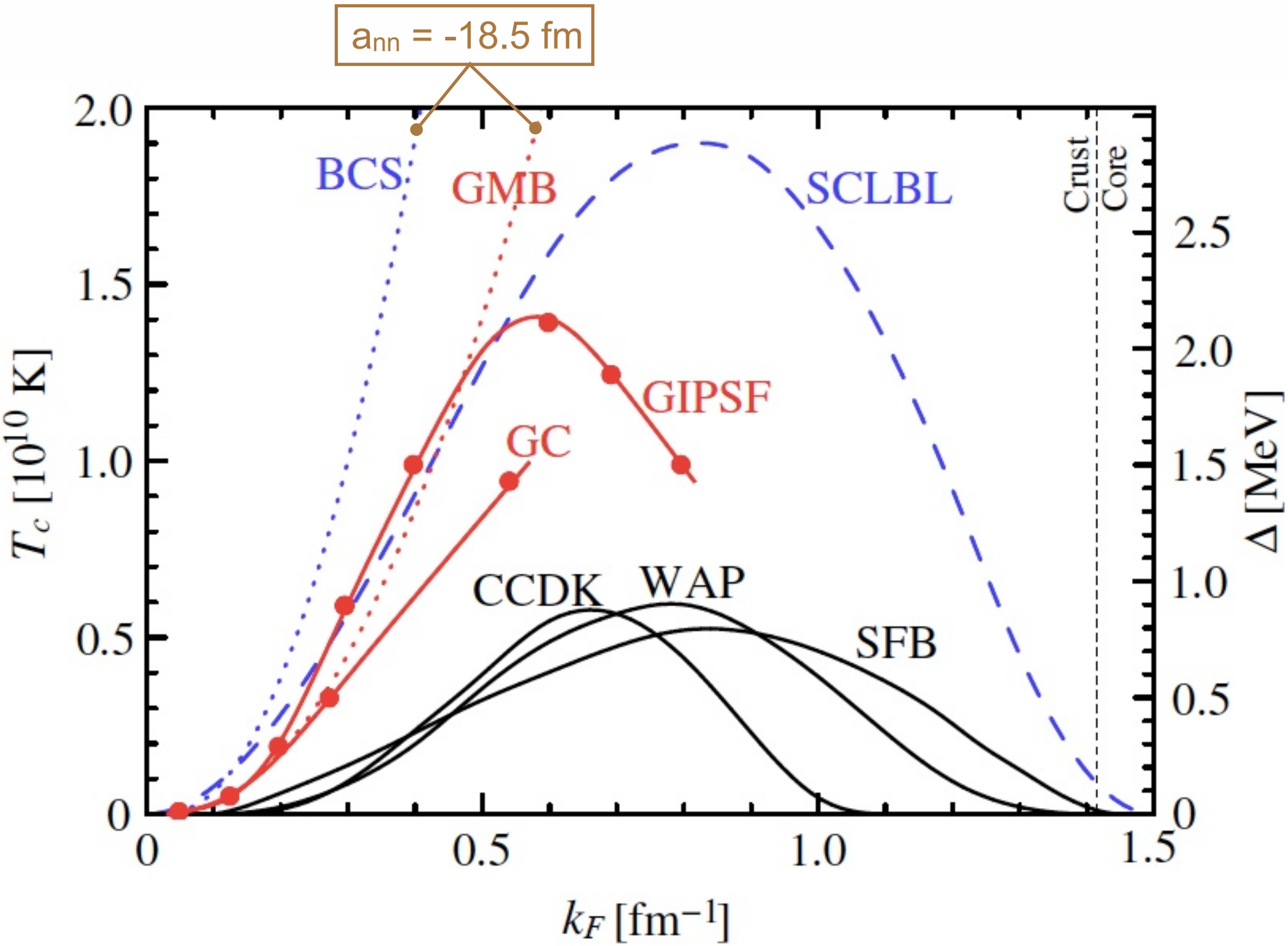
Critical temperature of the phase transition:

$$\Delta(T=0) \simeq 1.75 k_B T_c$$

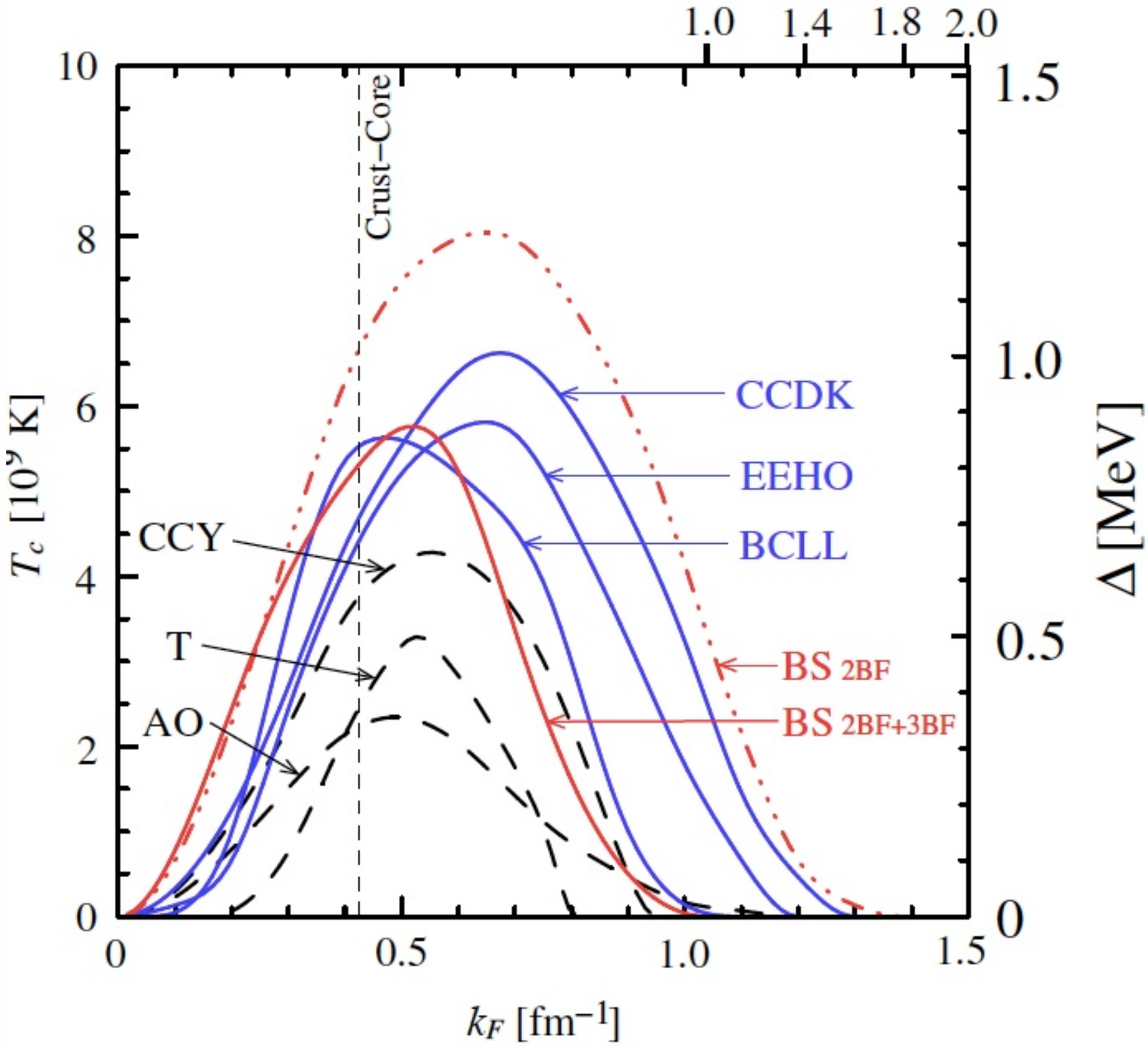
# BCS+Polarization: Neutron $^1S_0$ Gaps



# Prediction for Neutron $^1S_0$ Pairing



# Prediction for Proton $^1S_0$ Pairing



# Uncertainty on the $^3P_2$ Interaction Model

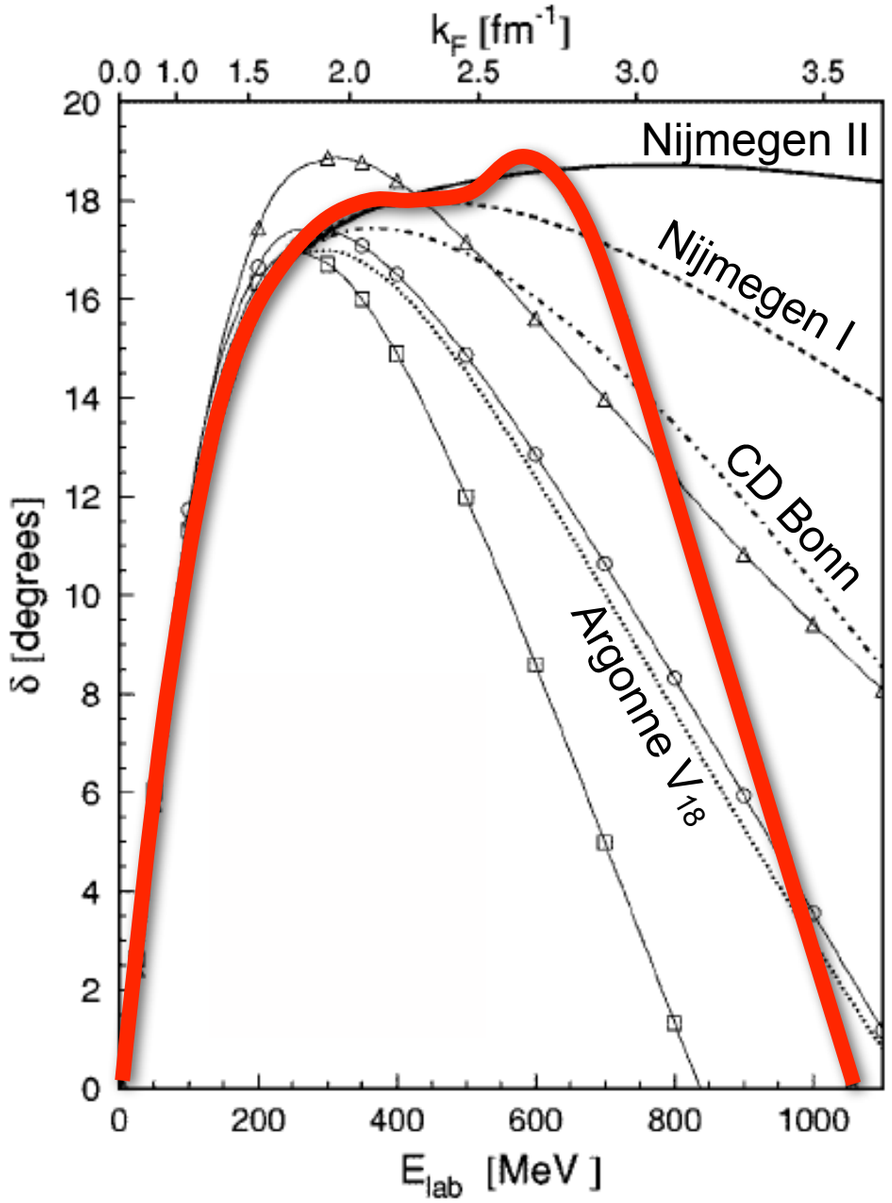
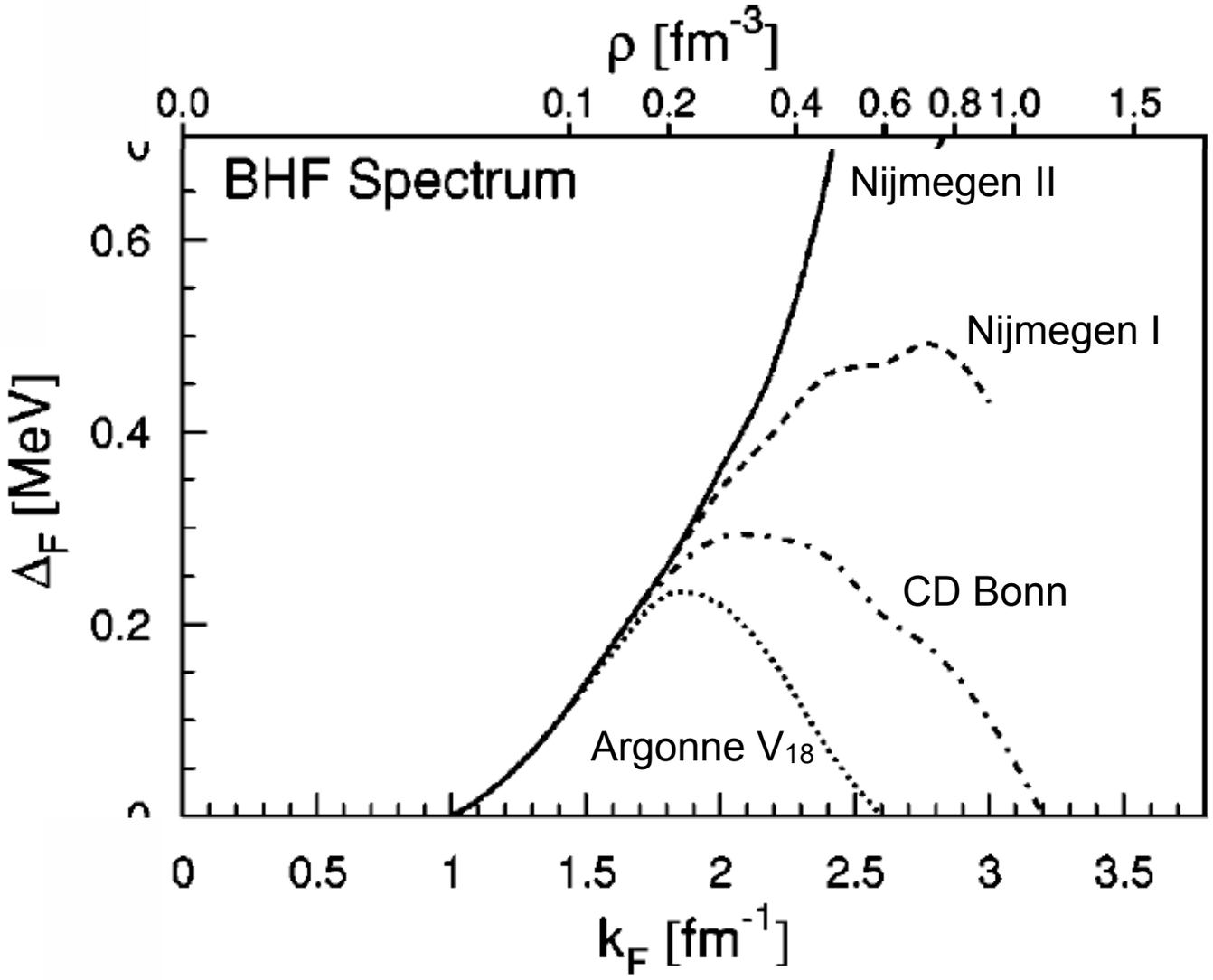



FIG. 1. $^3P_2$ phase-shift predictions of different potentials up to $E_{lab}=1.1$ GeV, compared with the phase-shift analysis of Arndt *et al.* [27]. The ''old'' potentials are denoted by different symbols, the ''modern'' potentials by different line styles.




[$^3P_2$-$^3F_2$ pairing in neutron mater with modern nucleon-nucleon potentials](1998PhRvC..58.1921B)
M. Baldo, Ø. Elgarøy, L. Engvik, M. Hjorth-Jensen, and H.-J. Schulze
[1998PhRvC..58.1921B]

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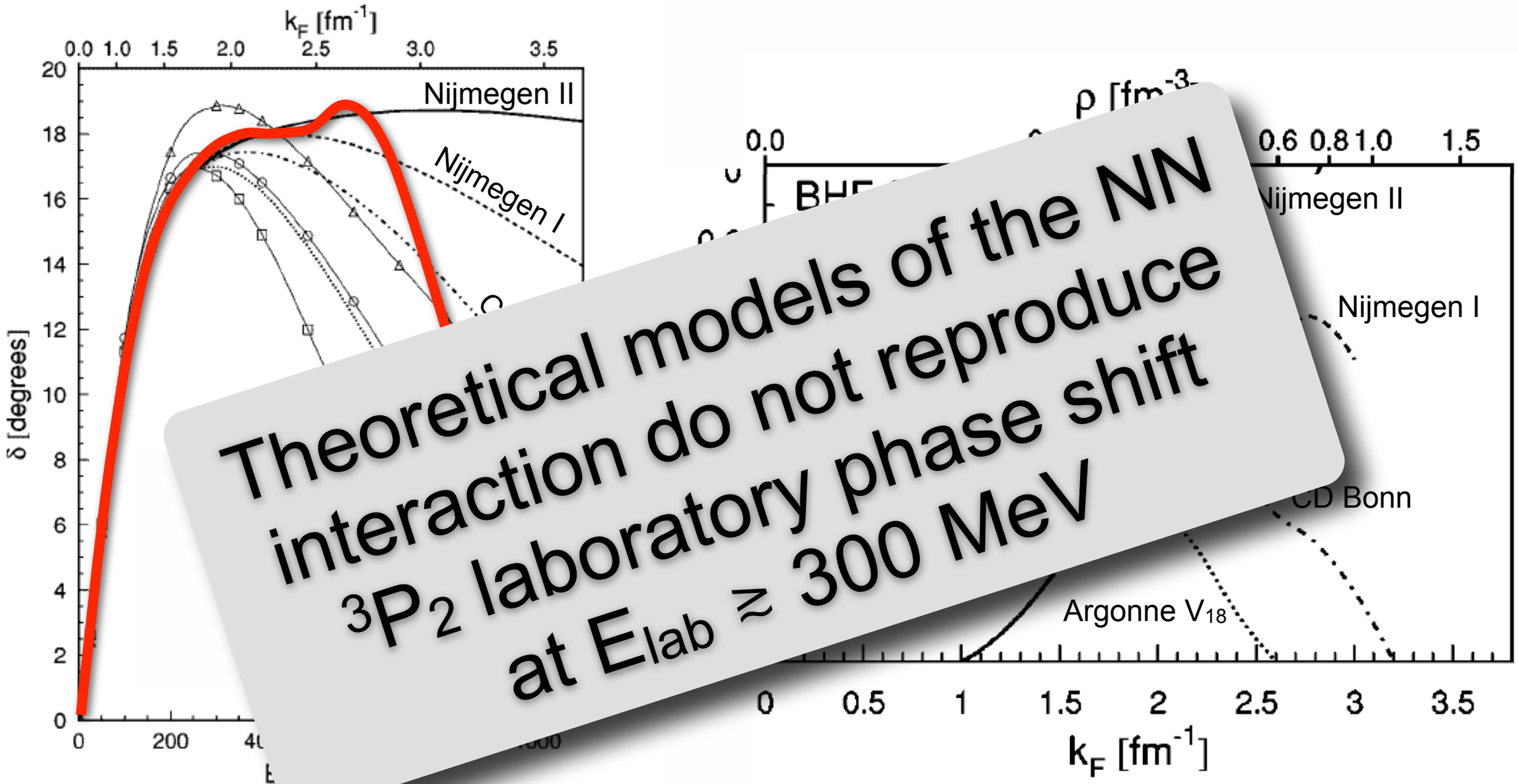


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$^3P_2$-$^3F_2$ pairing in neutron mater with modern nucleon-nucleon potentials
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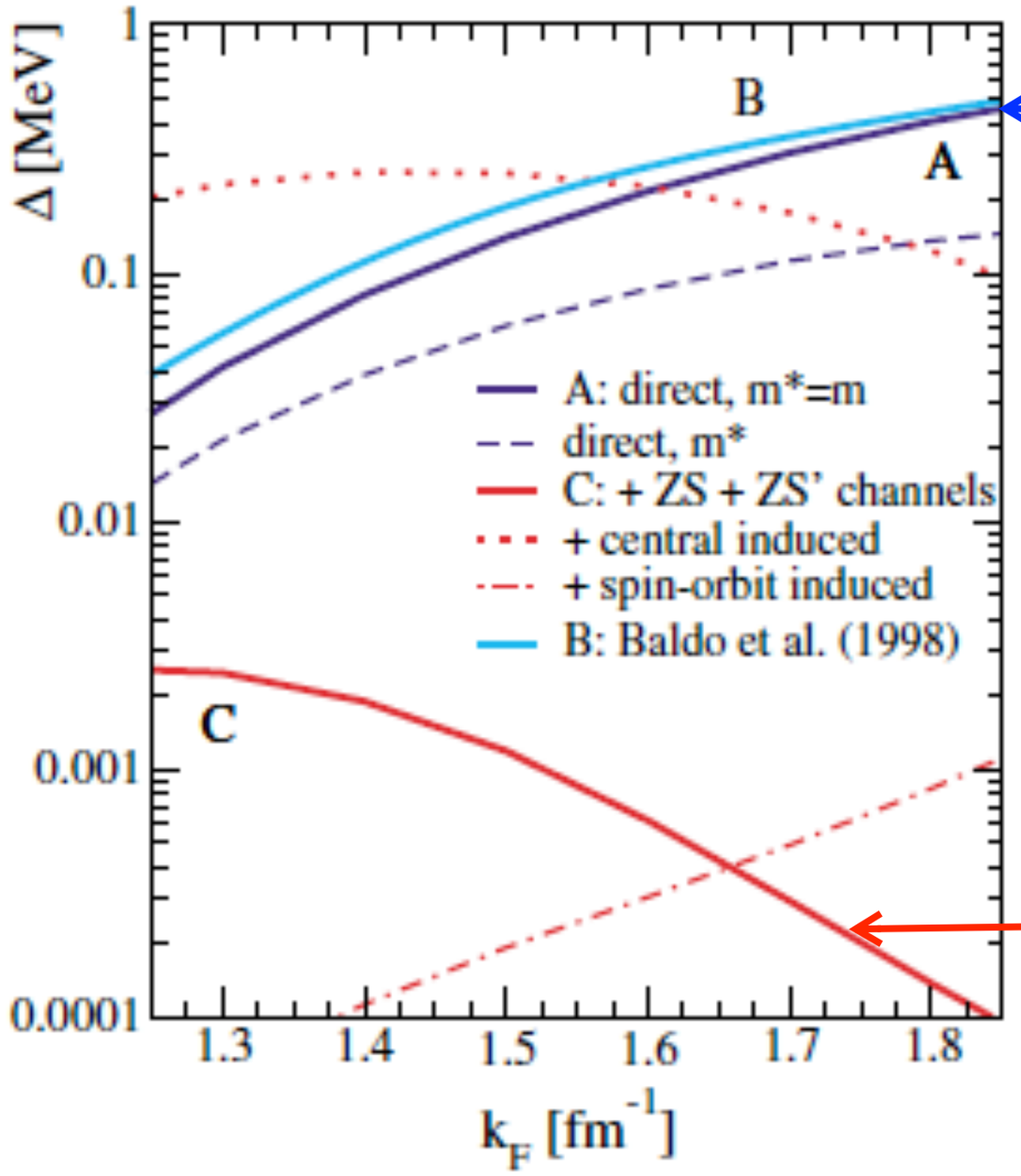
# Medium Polarization and the $^3P_2$ Gap

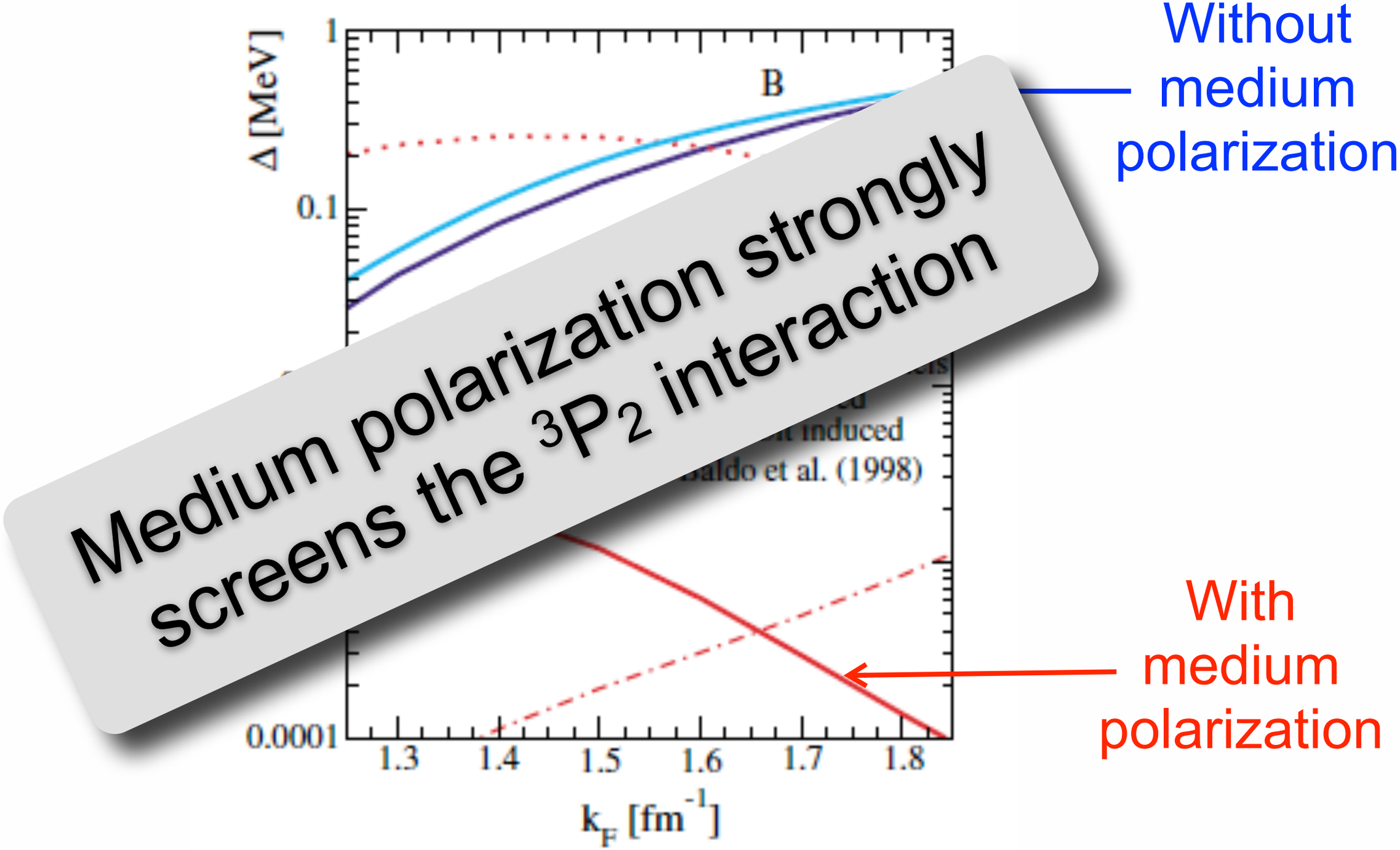



Without medium polarization

With medium polarization

Polarization contribution to the spin dependence of the effective interaction in neutron matter
Schwenk, Achim; Friman, Bengt 2004PhRvL..92h2501S

# Medium Polarization and the $^3P_2$ Gap




Polarization contribution to the spin dependence of the effective interaction in neutron matter
Schwenk, Achim; Friman, Bengt 2004PhRvL..92h2501S

# 3-Body Forces and the $^3P_2$ Gap

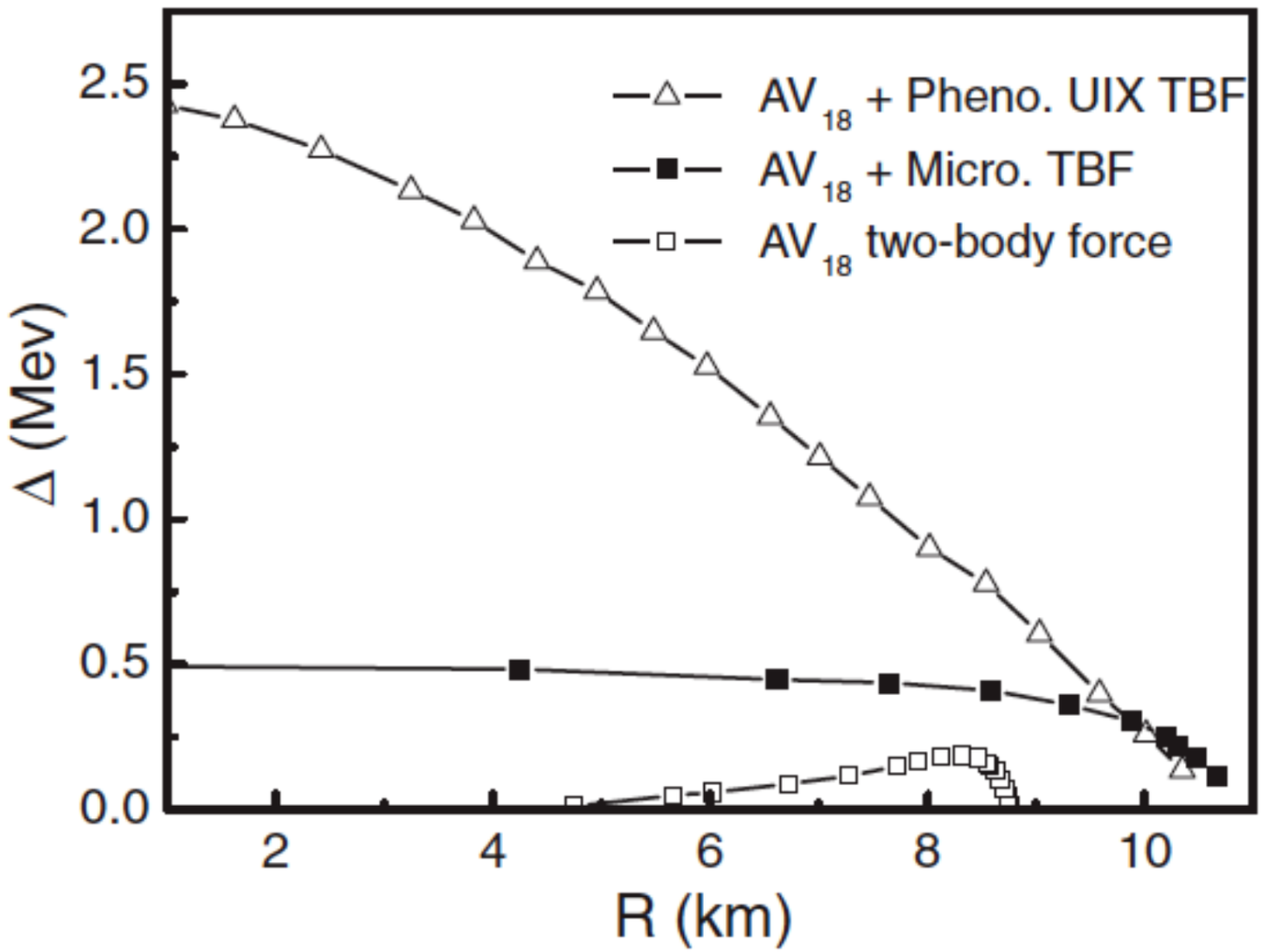
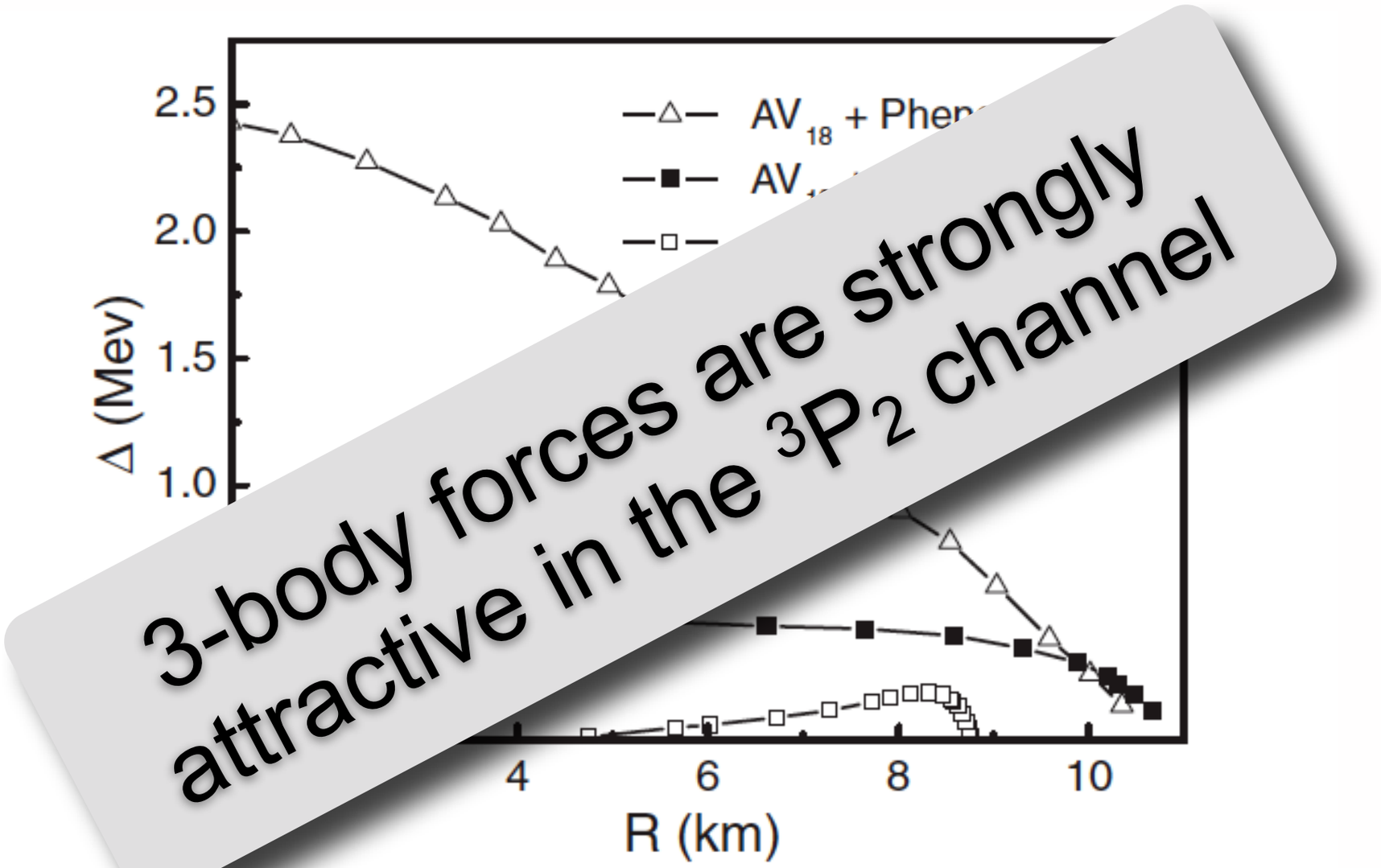



FIG. 8. Distribution of neutron $^3PF_2$ pairing gap in a typical neutron star.

Three-body force effect on P3 $F_2$ neutron superfluidity in neutron matter, neutron star matter, and neutron stars
Zuo, W.; Cui, C. X.; Lombardo, U.; Schulze, H.-J. 2008PhRvC..78a5805Z

# 3-Body Forces and the $^3P_2$ Gap

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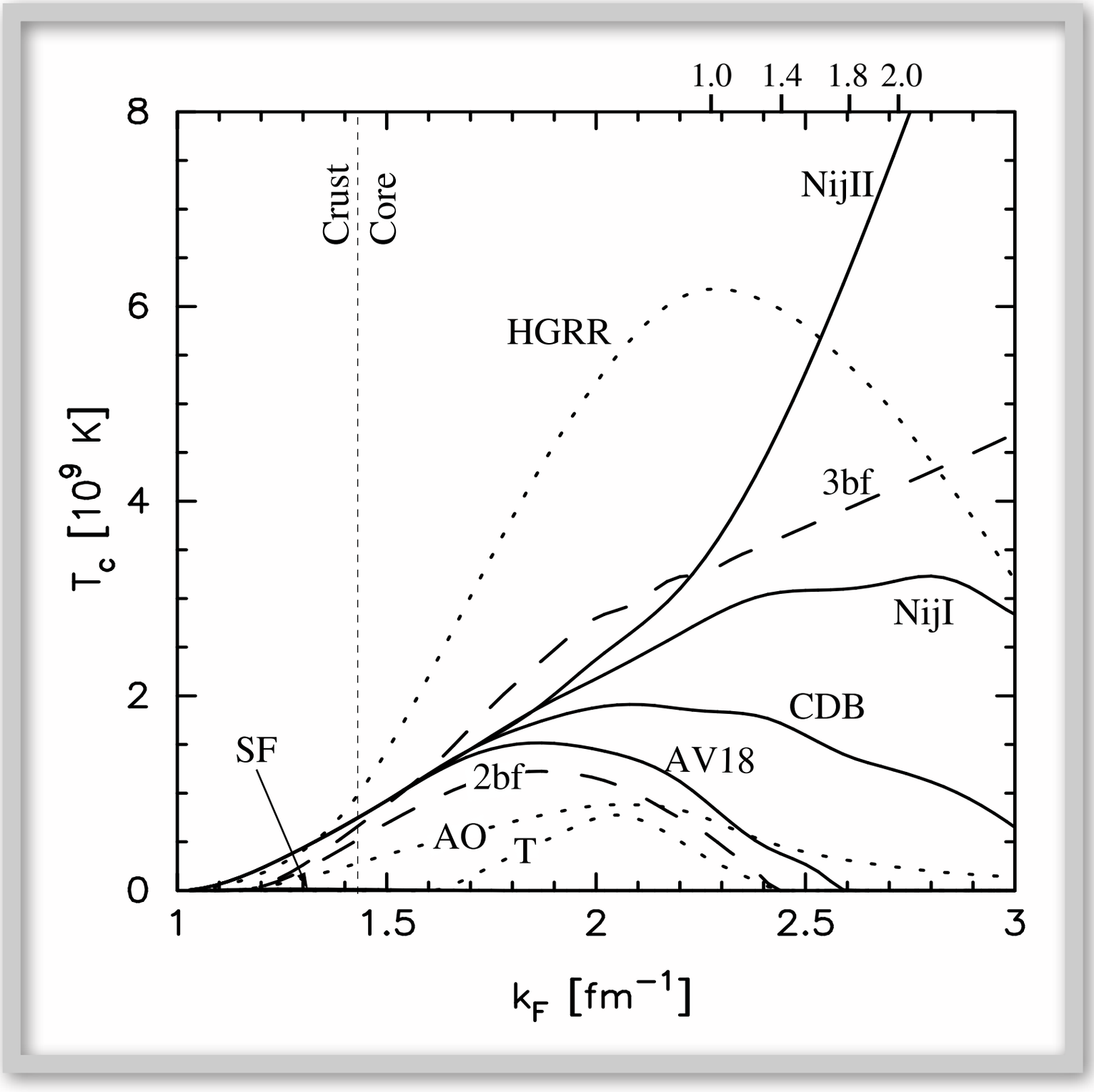
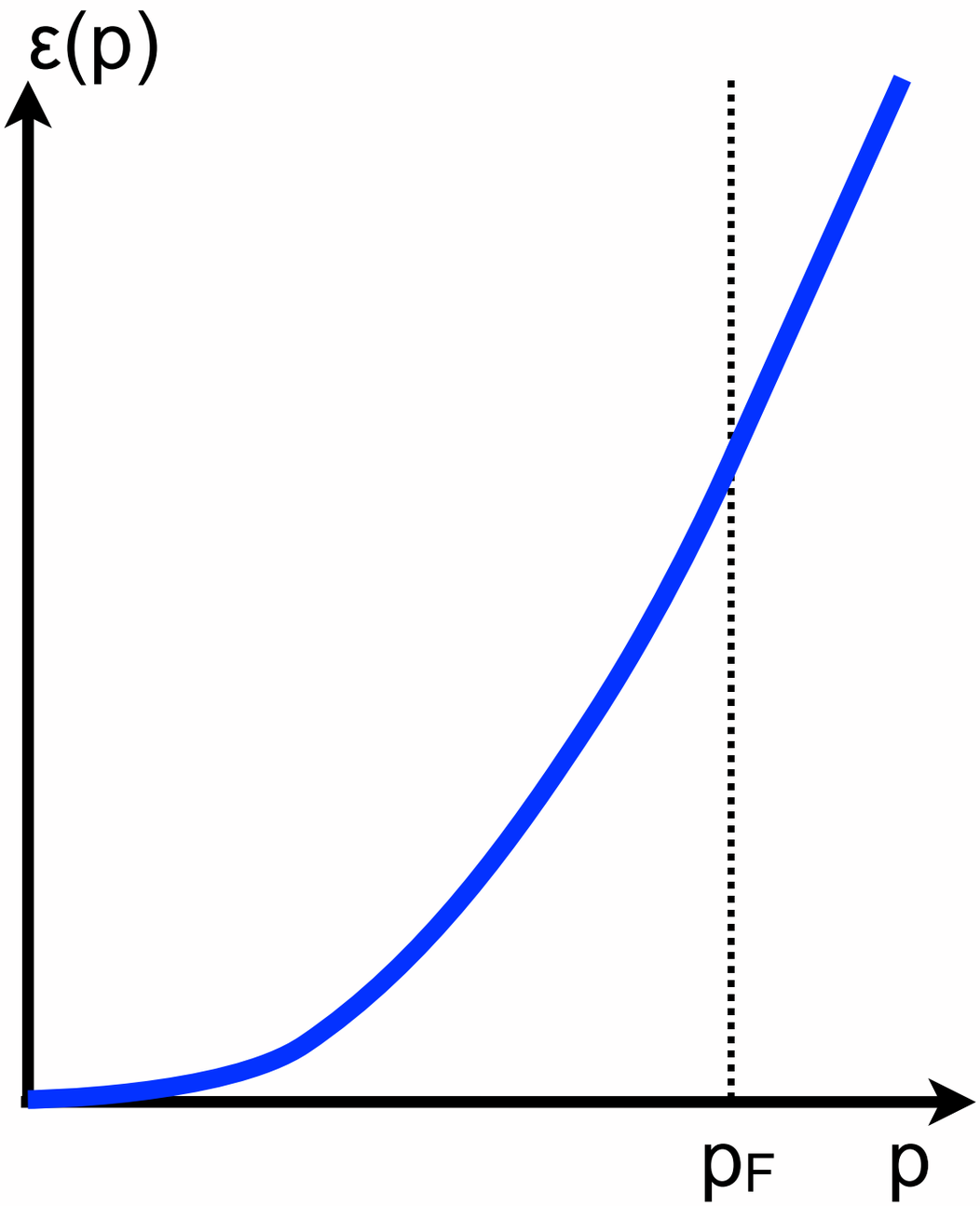
# Neutron $^3P_2$: $T_C$ Theoretical Predictions



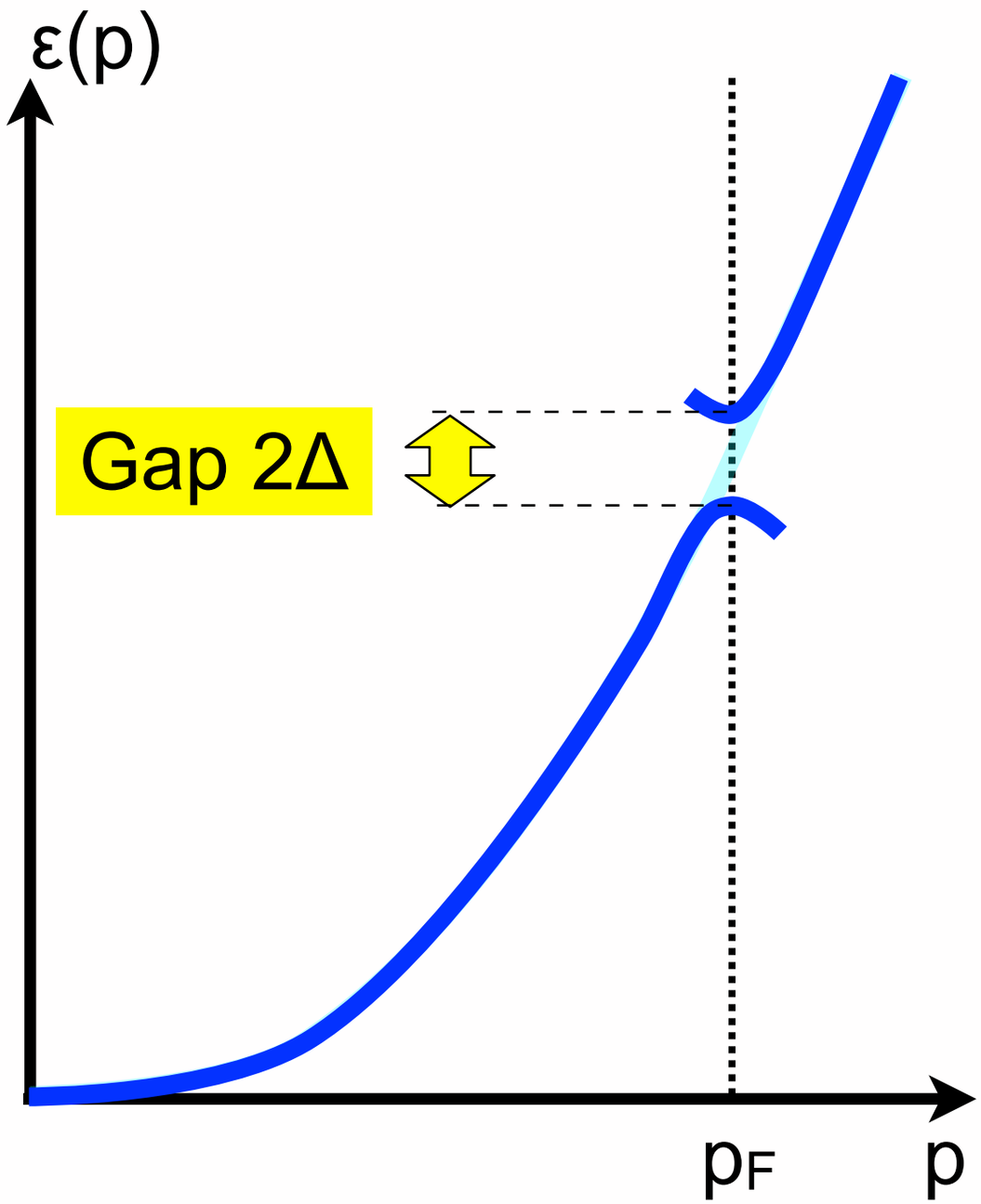
# Pairing in Neutron Stars: Effects

# Single-Particle Excitations Spectra

## Normal Fermion Liquid



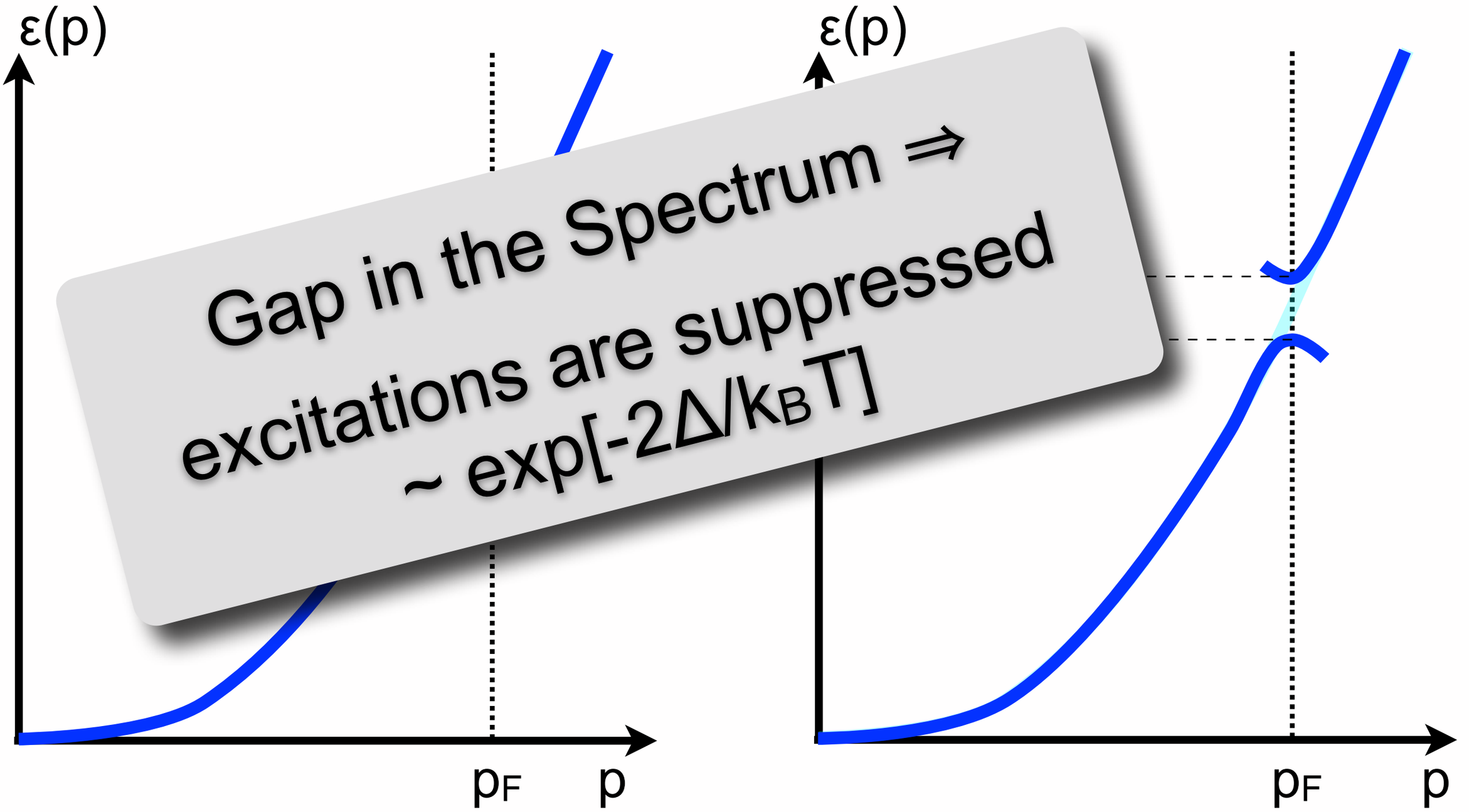

## Superfluid Fermions



# Single-Particle Excitations Spectra

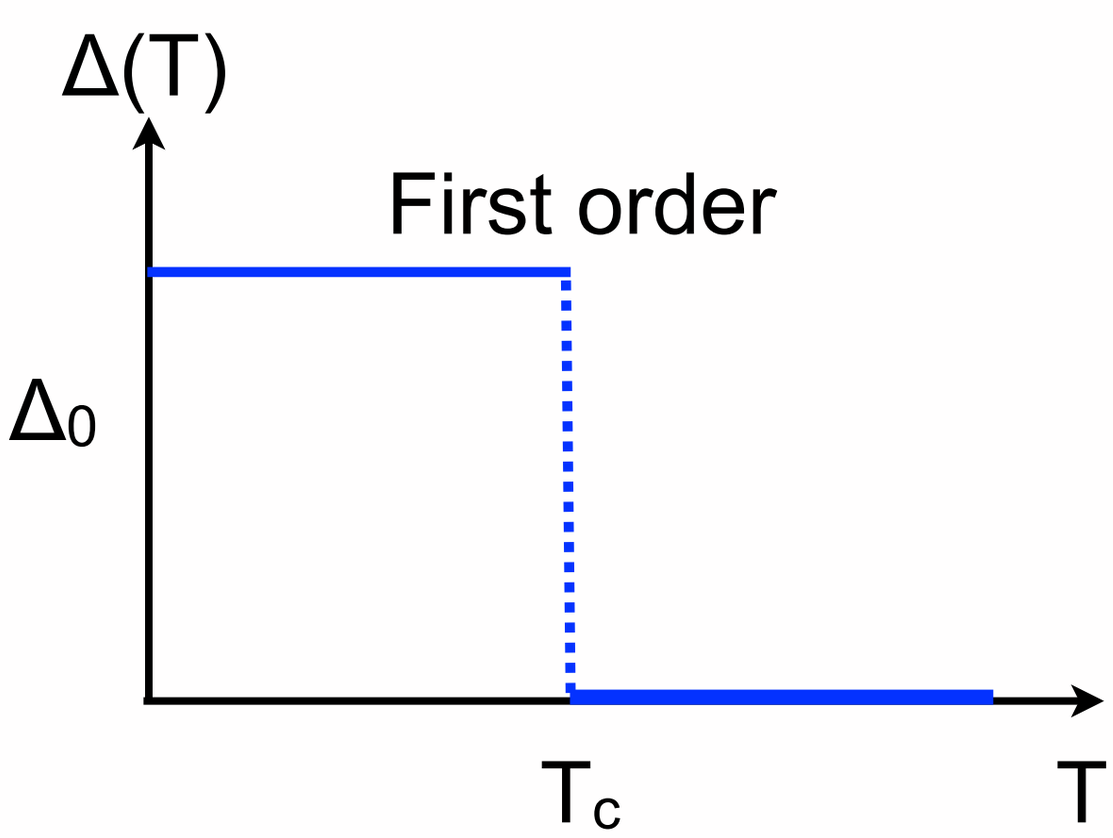
Normal Fermion Liquid

Superfluid Fermions

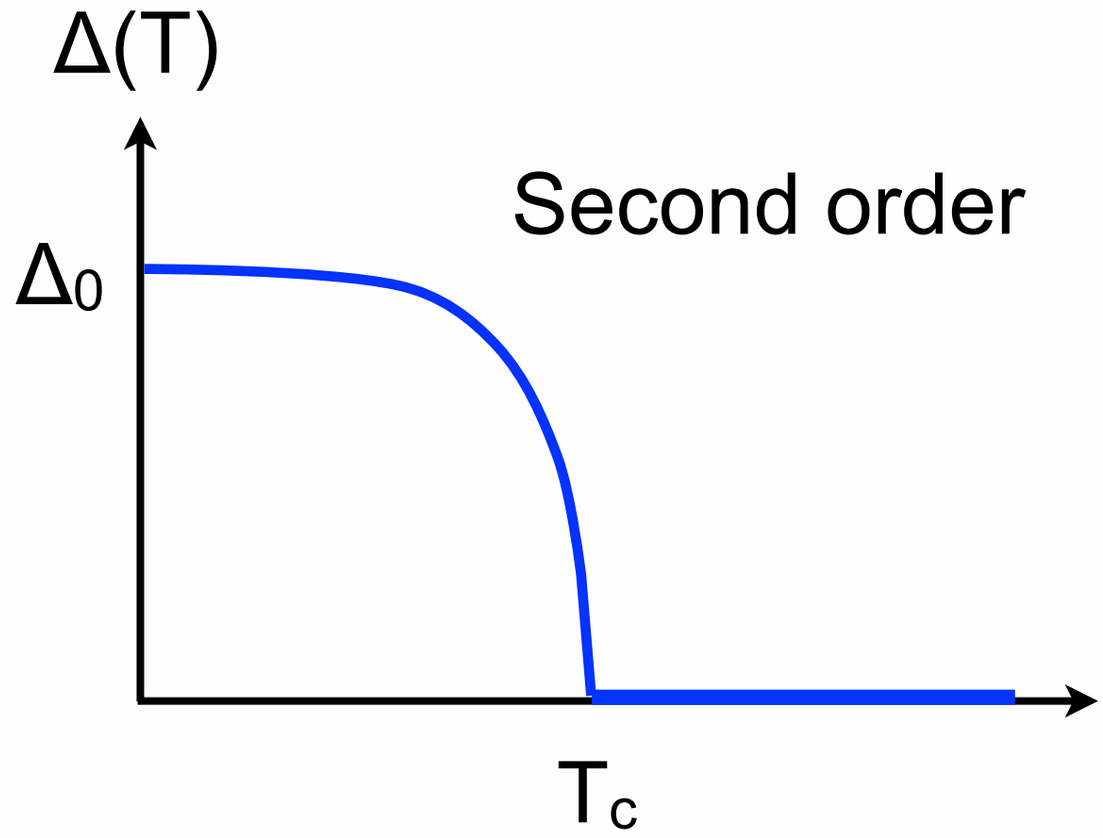


# First vs Second Order Phase Transitions

Landau description: “order” parameter Δ



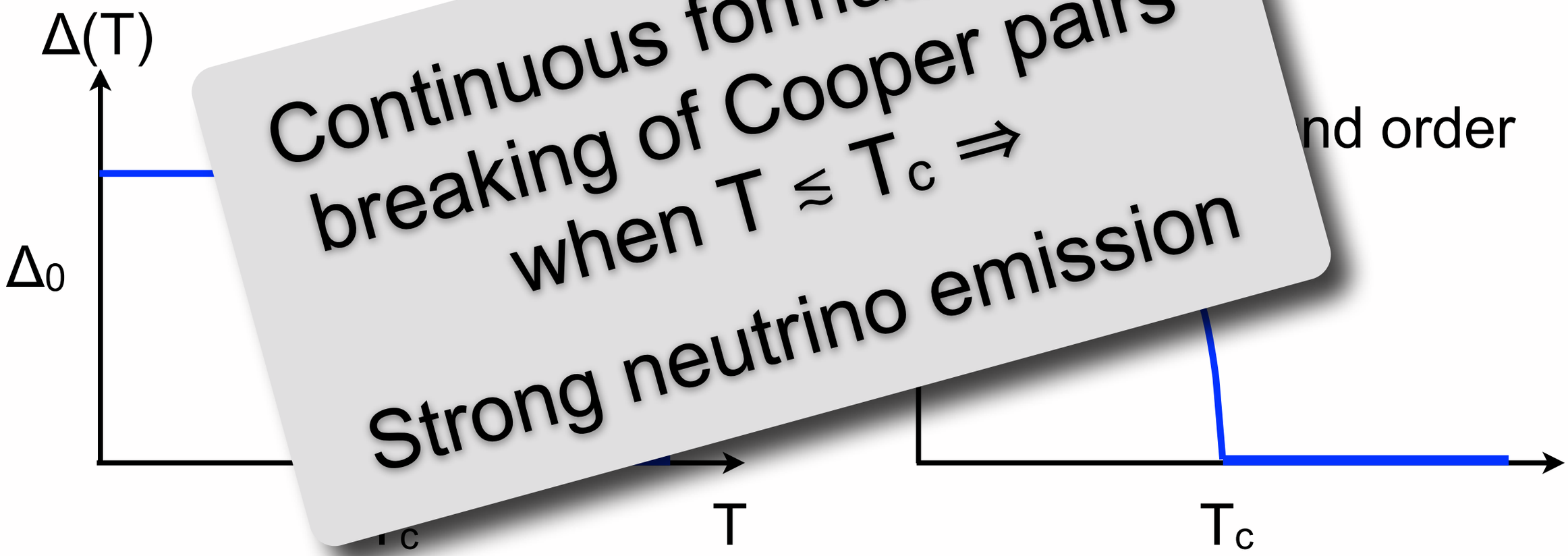

Δ changes discontinuously:
new phase appears abruptly




Δ changes continuously:
new phase develops smoothly

# First vs Second Order Phase Transitions

Landau description: “order” parameter Δ




Continuous formation and breaking of Cooper pairs when $T \lesssim T_c \Rightarrow$

Strong neutrino emission

Δ changes discontinuously:
new phase appears abruptly

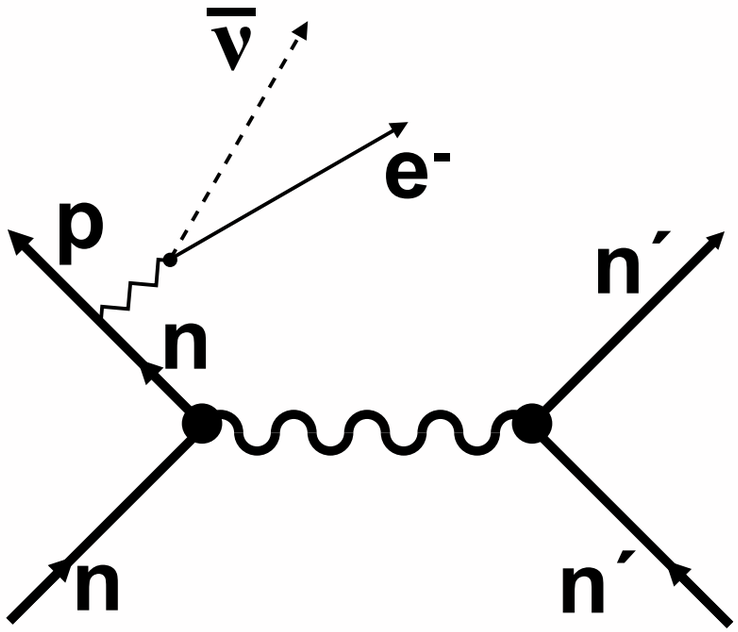
Δ changes continuously:
new phase develops smoothly

# A sample of neutrino emission processes

| Name | Process | Emissivity (erg cm$^{-3}$ s$^{-1}$) | |
|---|---|---|---|
| Modified Urca cycle (neutron branch) | $n+n \to n+p+e^- + \bar{\nu}_e$<br>$n+p+e^- \to n+n+\nu_e$ | $\sim 2{\times}10^{21}\, R\, T_9^8$ | Slow |
| Modified Urca cycle (proton branch) | $p+n \to p+p+e^- + \bar{\nu}_e$<br>$p+p+e^- \to p+n+\nu_e$ | $\sim 10^{21}\, R\, T_9^8$ | Slow |
| Bremsstrahlung | $n+n \to n+n+\nu+\bar{\nu}$<br>$n+p \to n+p+\nu+\bar{\nu}$<br>$p+p \to p+p+\nu+\bar{\nu}$ | $\sim 10^{19}\, R\, T_9^8$ | Slow |
| Cooper pair formations | $n+n \to [nn]+\nu+\bar{\nu}$<br>$p+p \to [pp]+\nu+\bar{\nu}$ | $\sim 5{\times}10^{21}\, R\, T_9^7$<br>$\sim 5{\times}10^{19}\, R\, T_9^7$ | Medium |
| Direct Urca cycle (nucleons) | $n \to p+e^- + \bar{\nu}_e$<br>$p+e^- \to n+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| Direct Urca cycle ($\Lambda$ hyperons) | $\Lambda \to p+e^- + \bar{\nu}_e$<br>$p+e^- \to \Lambda+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| Direct Urca cycle ($\Sigma^-$ hyperons) | $\Sigma^- \to n+e^- + \bar{\nu}_e$<br>$n+e^- \to \Sigma^-+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| $\pi^-$ condensate | $n+<\pi^->\to n+e^-+\bar{\nu}_e$ | $\sim 10^{26}\, R\, T_9^6$ | Fast |
| $K^-$ condensate | $n+<K^->\to n+e^-+\bar{\nu}_e$ | $\sim 10^{25}\, R\, T_9^6$ | Fast |
| Direct Urca cycle (u-d quarks) | $d \to u+e^- + \bar{\nu}_e$<br>$u+e^- \to d+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |
| Direct Urca cycle (u-s quarks) | $s \to u+e^- + \bar{\nu}_e$<br>$u+e^- \to s+\nu_e$ | $\sim 10^{27}\, R\, T_9^6$ | Fast |

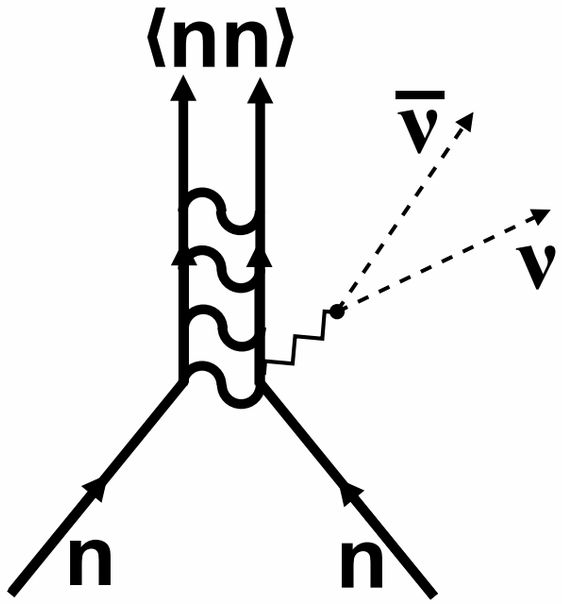
# The Three Basic Neutrino Processes

**SLOW**



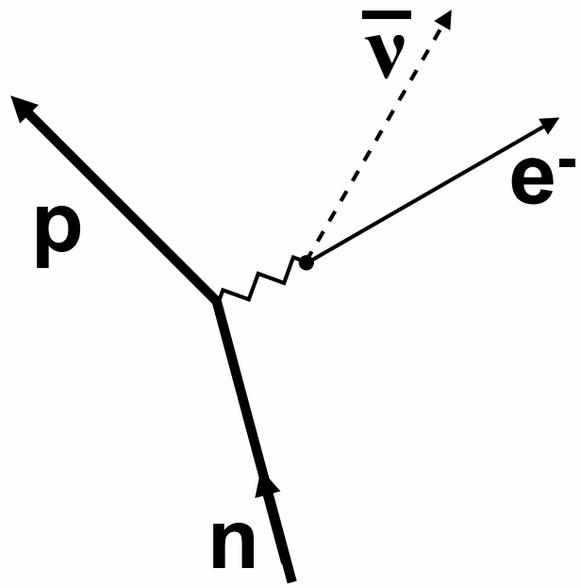

Modified Urca

**MEDIUM**



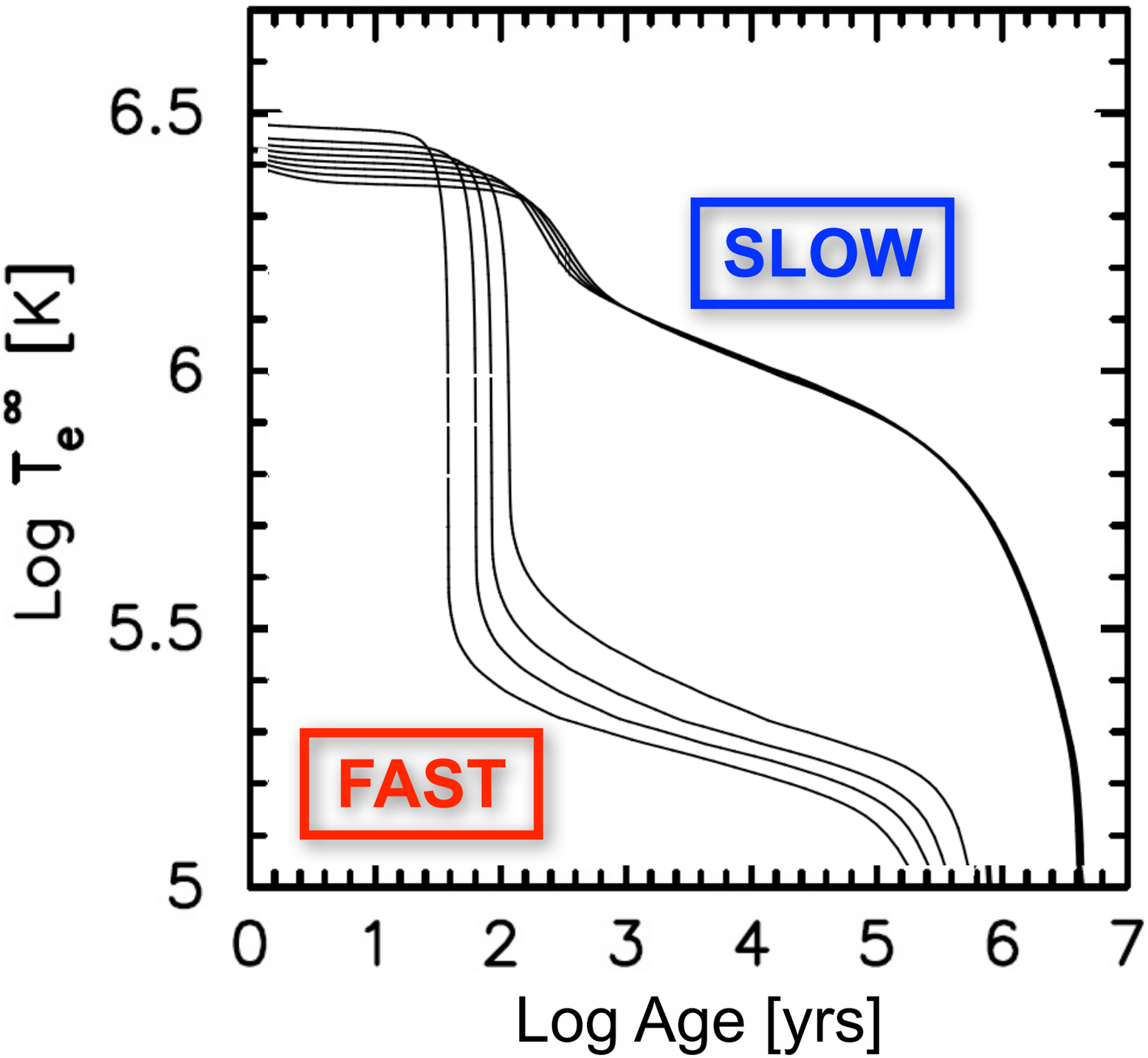

Cooper pair formation ("PBF")

**FAST**

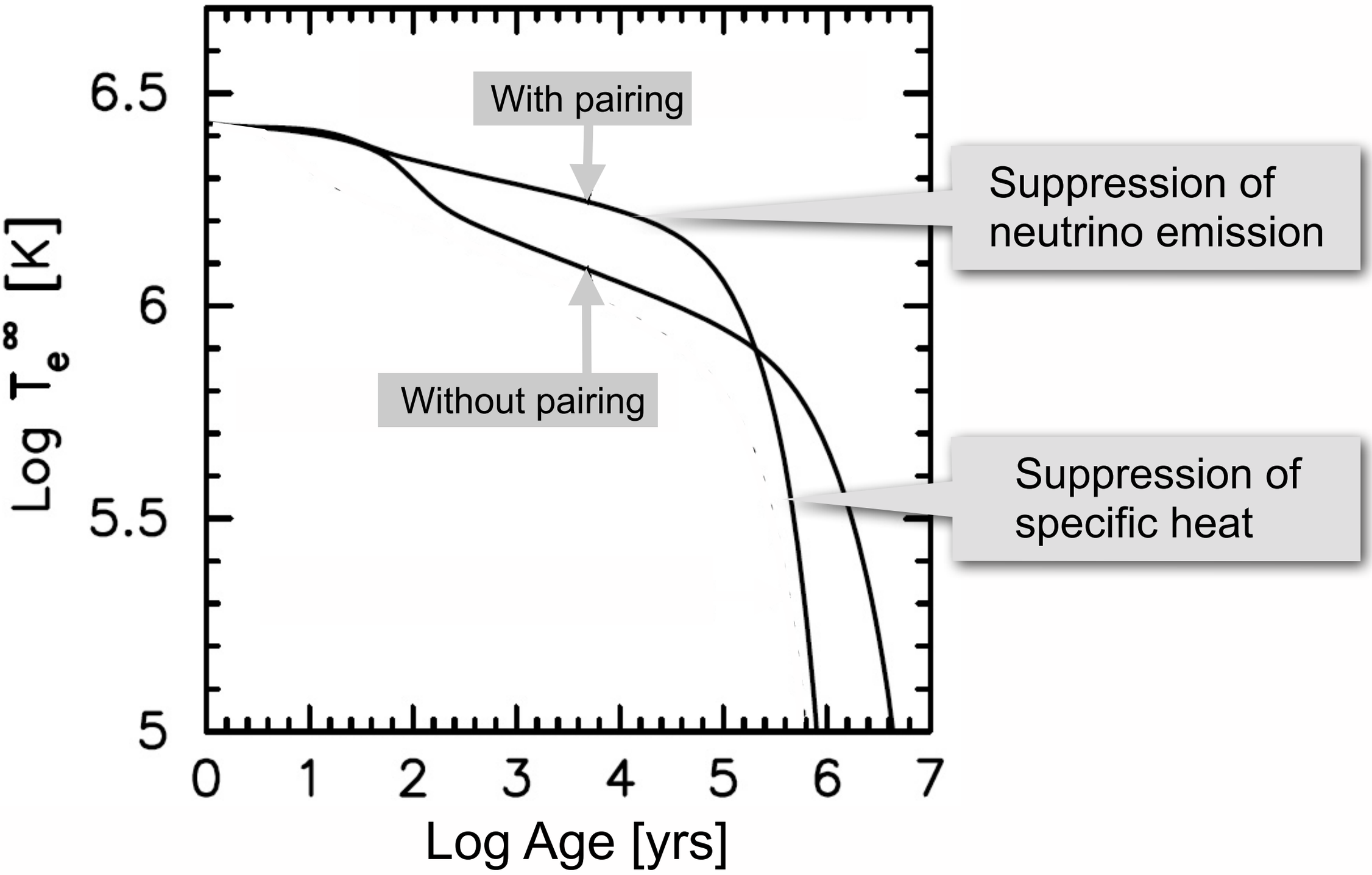



Direct Urca

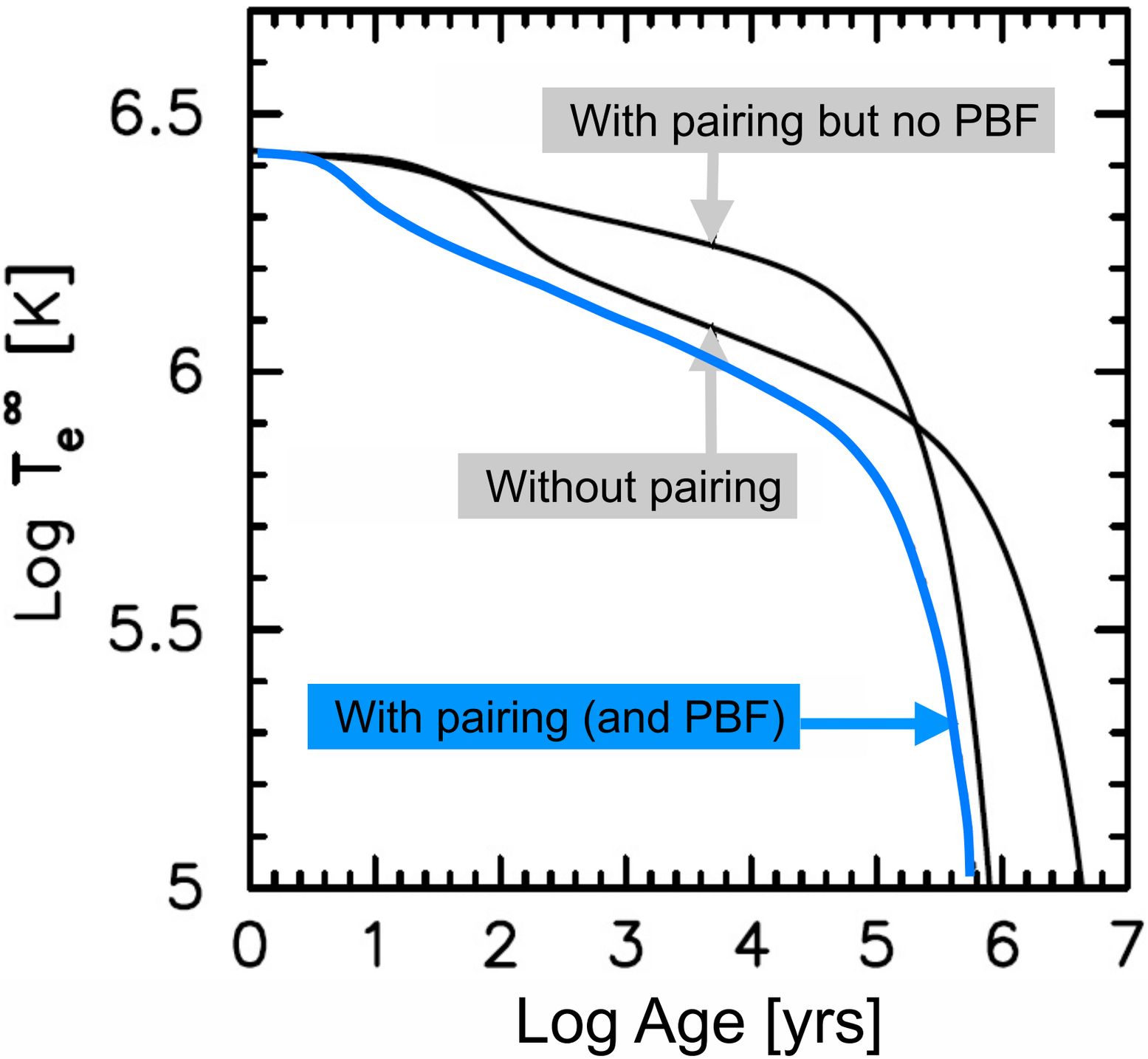
# Slow Neutrino Cooling without Pairing



# Slow Neutrino Cooling with Pairing



# Slow Neutrino Cooling and Pairing



# Minimal Cooling of Isolated Neutron Stars

# Minimal Cooling

Minimal Neutron Star:

- No “exotic” matter in the deep core:
  only neutrons and protons
- Take into account pairing and uncertainties on $T_c$’s
- Take into account Cooper pair neutrino emission
- Take into account uncertainty on surface chemical composition

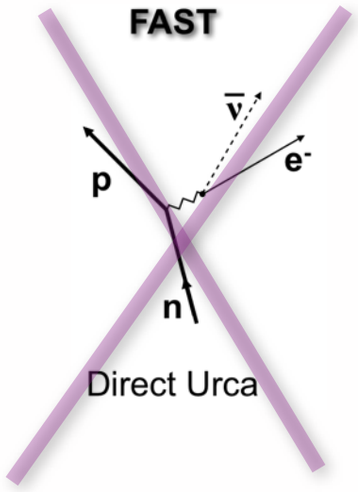
**A neutron star with a $T_e$ incompatible with minimal cooling is a strong candidate for some “exotic” form of matter**

Minimal Cooling of Neutron Stars: A New Paradigm
D. Page, J.M. Lattimer, M. Prakash & A.W. Steiner 2004ApJS..155..623P

Neutrino Emission from Cooper Pairs and Minimal Cooling of Neutron Stars
D. Page, M. Prakash, J.M. Lattimer & A.W. Steiner 2009ApJ...707.1131P

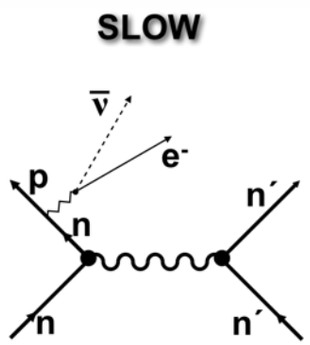
# Minimal Cooling: bottom line argument

Fast processes are excluded *a priori*.



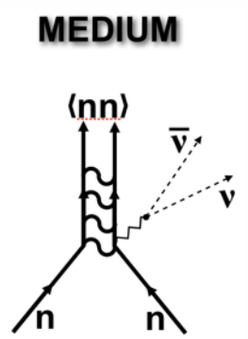

## Three regimes of T vs neutron pairing $T_c$:

### $T>T_c$




Modified Urca

### $T\sim T_c$




Cooper pair formation ("PBF")

### $T\ll T_c$

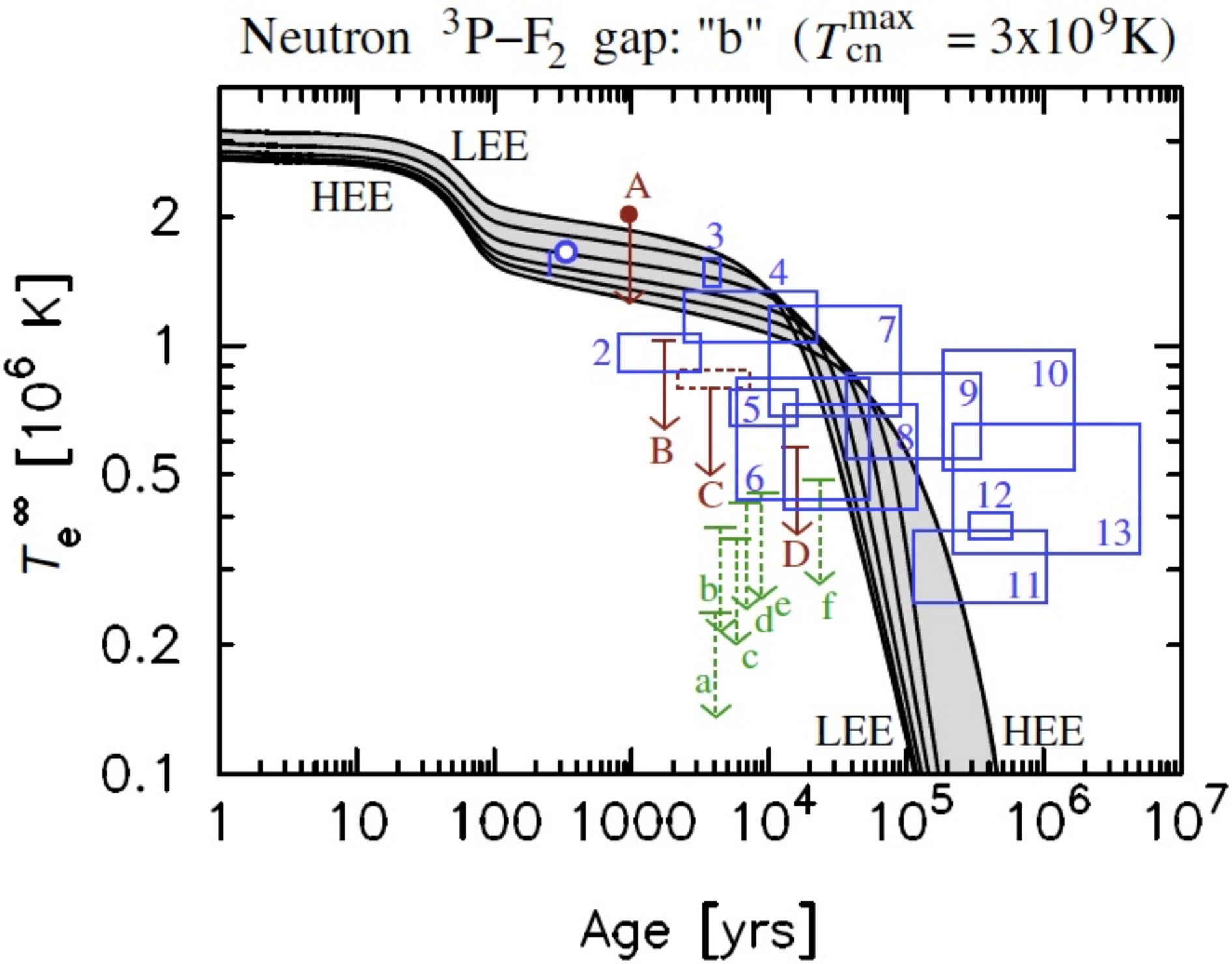
neutrino emission is practically shut off.

$T_c$ very large: neutrino emission shut off very early

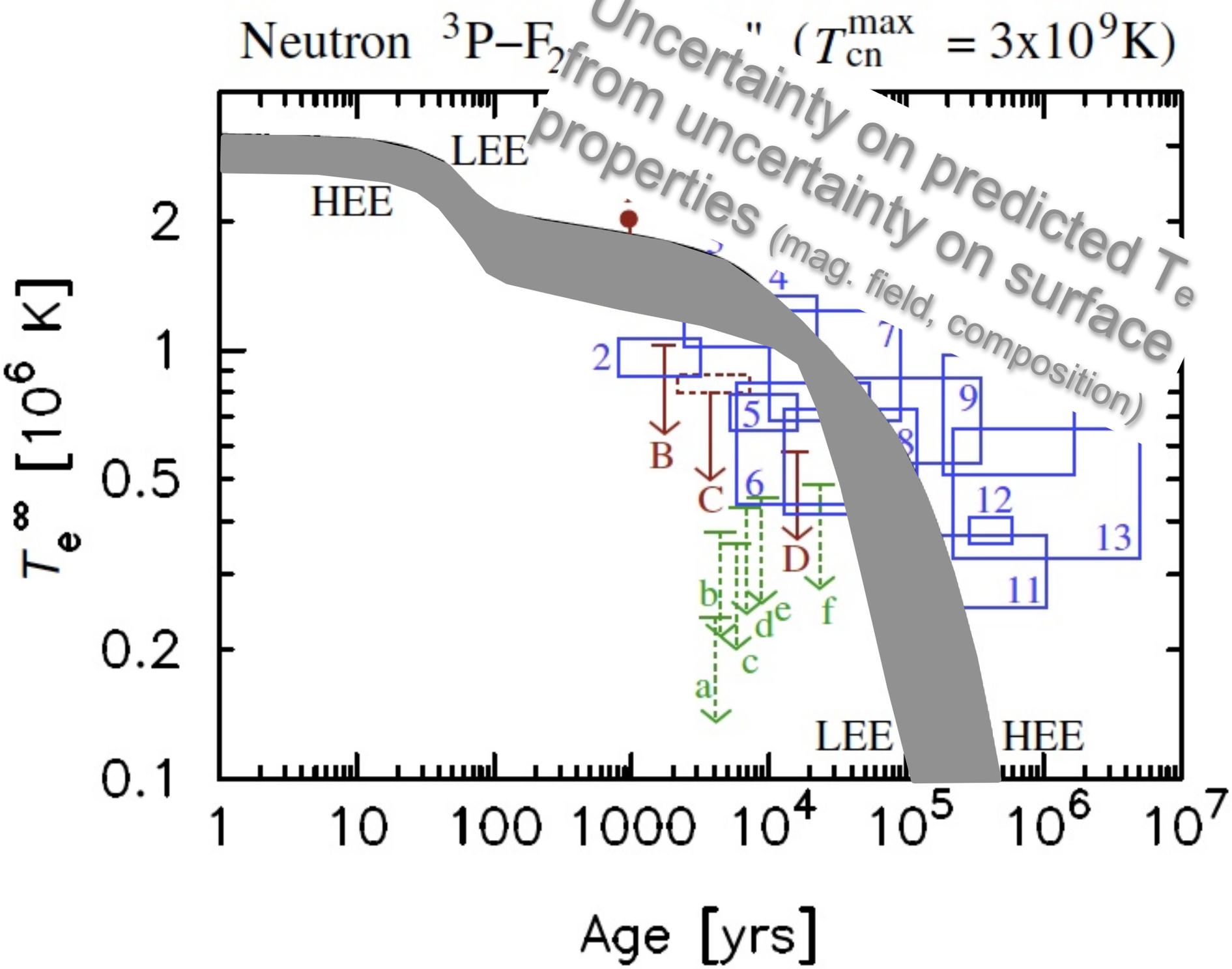
$T_c$ small: pairing sets in too late and only slow neutrino cooling.

$T_c$ medium size: enhanced cooling from the PBF process !

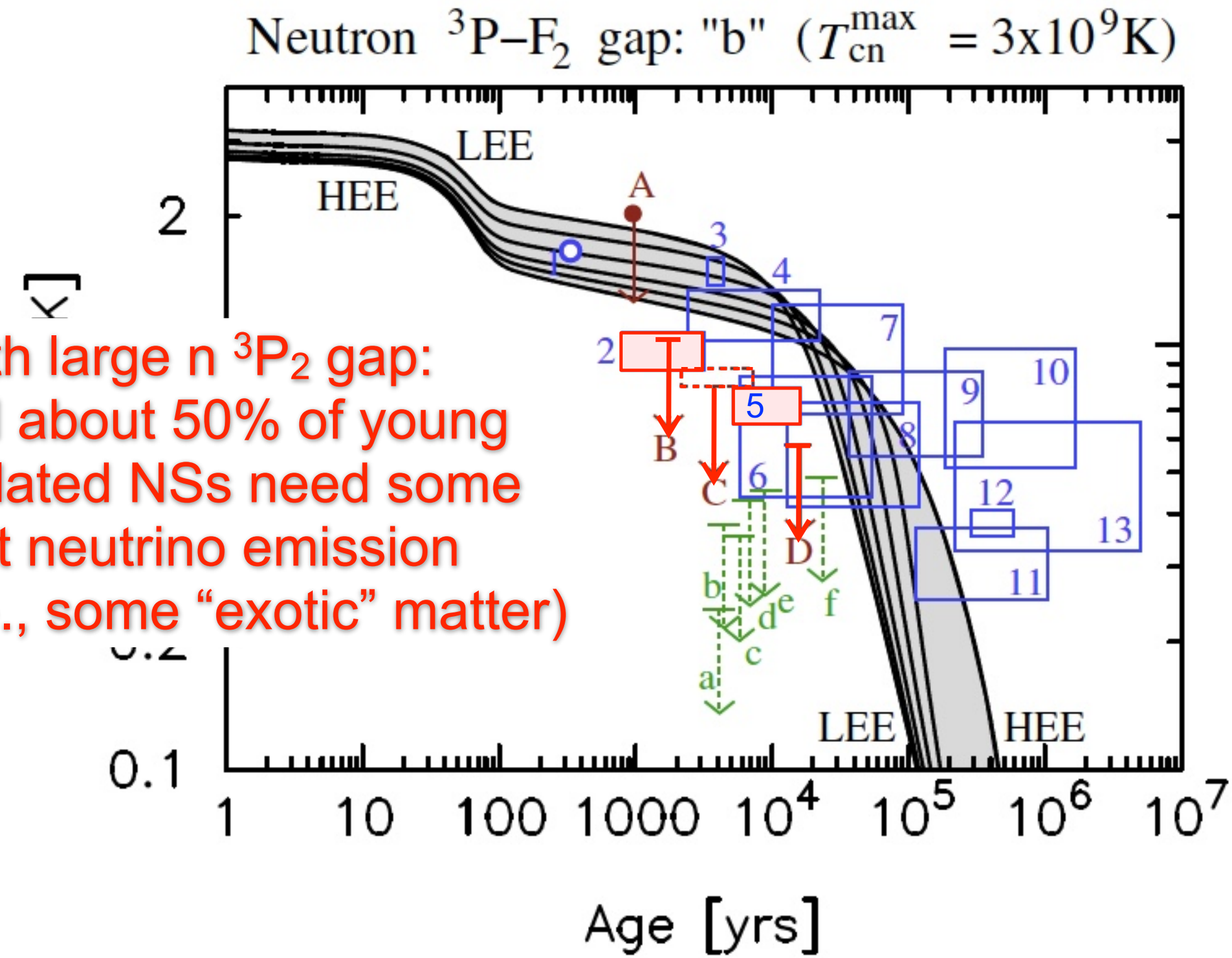
# Minimal Cooling with Large n $^3P_2$ Gap



# Minimal Cooling with Large n $^3P_2$ Gap

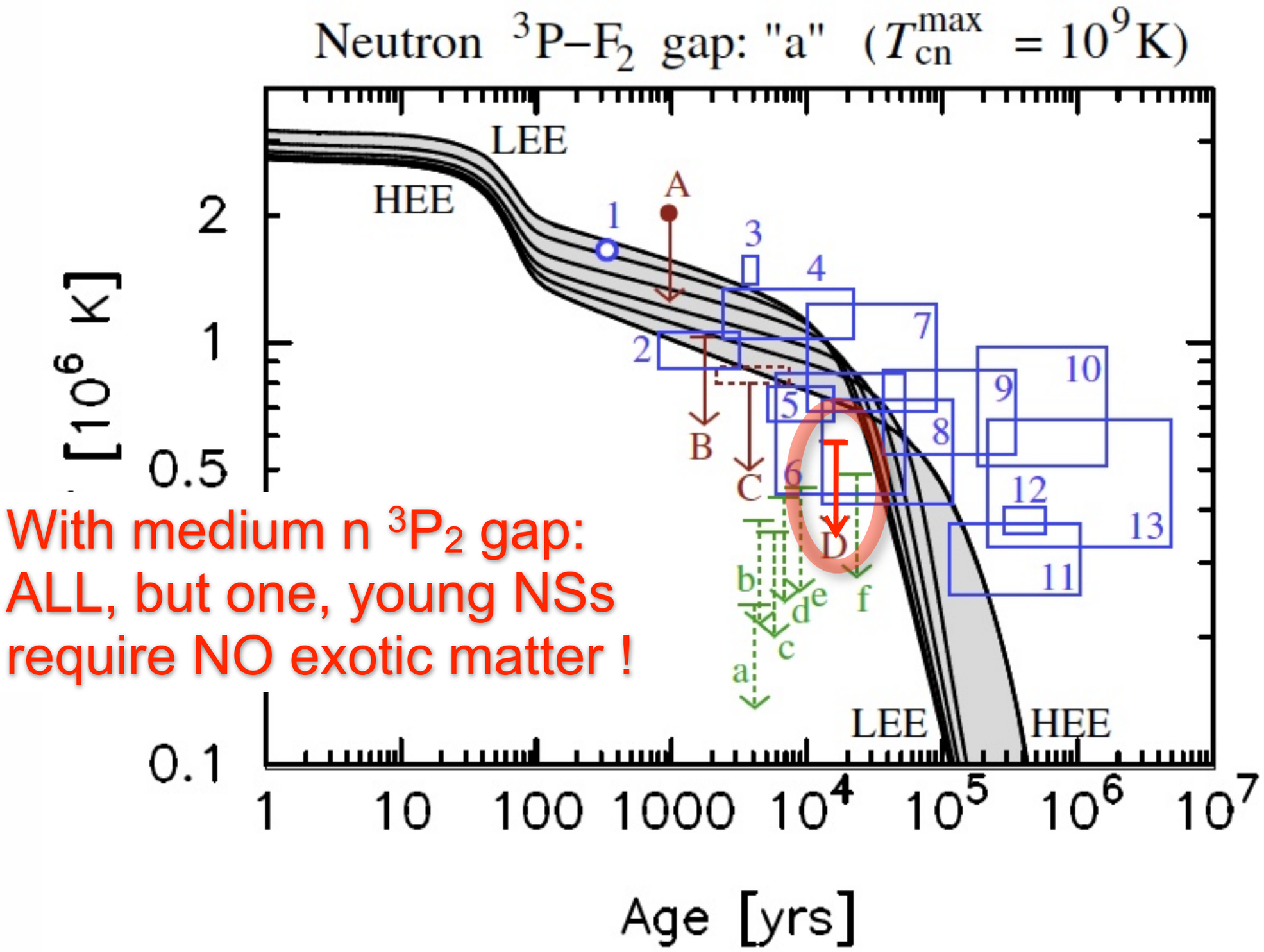


# Minimal Cooling with Large n $^3P_2$ Gap




With large n $^3P_2$ gap:
still about 50% of young isolated NSs need some fast neutrino emission (i.e., some "exotic" matter)

# Minimal Cooling with Medium n $^3P_2$ Gap




With medium n $^3P_2$ gap:
ALL, but one, young NSs
require NO exotic matter !

If the neutron $^3P_2$ gap has the appropriate size:

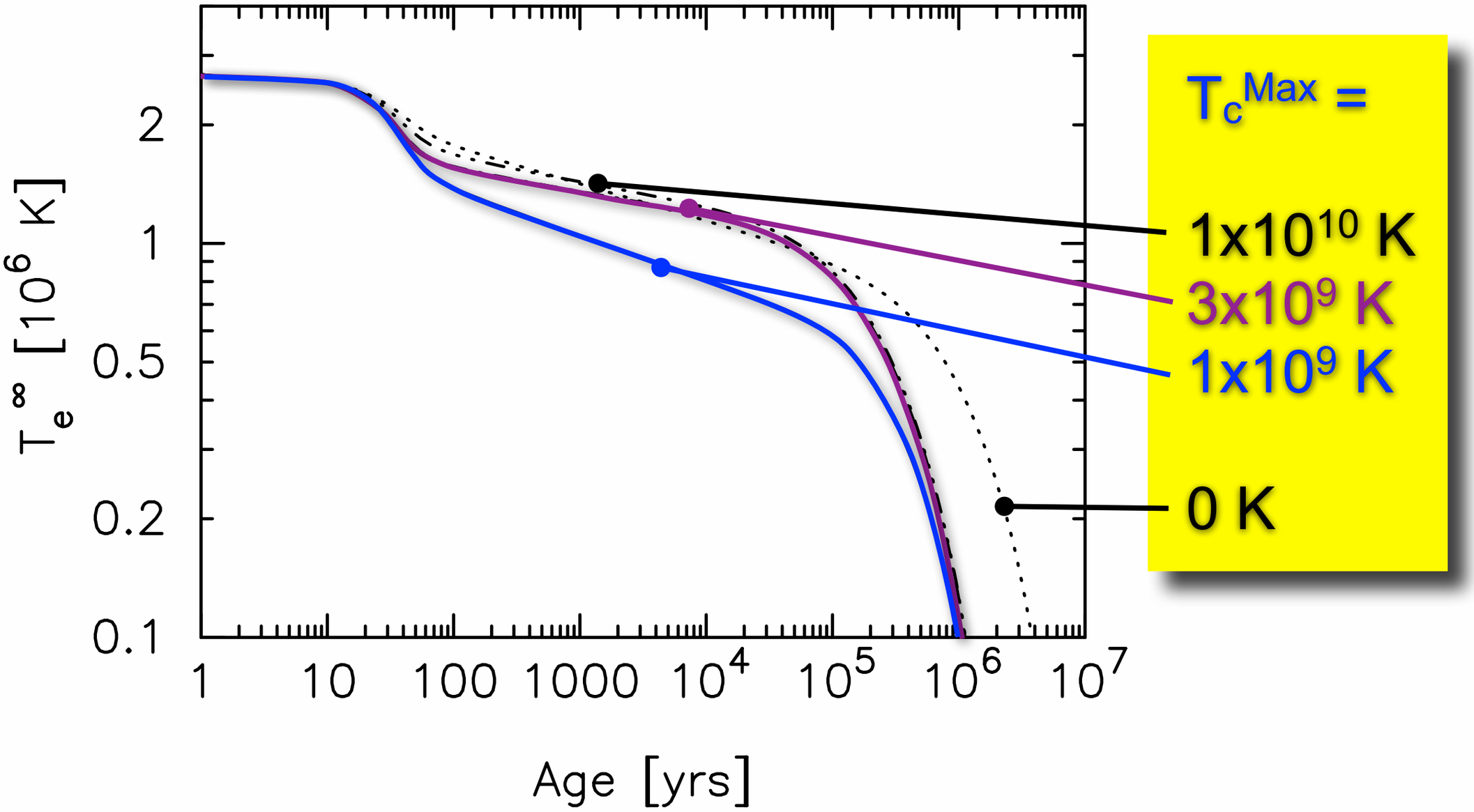
$$\Delta \sim 0.2 \text{ MeV} \quad \text{and} \quad T_c^{max} \sim 10^9 \text{ K}$$

only ONE young cooling neutron star shows strong evidence for the occurrence of fast neutrino emission

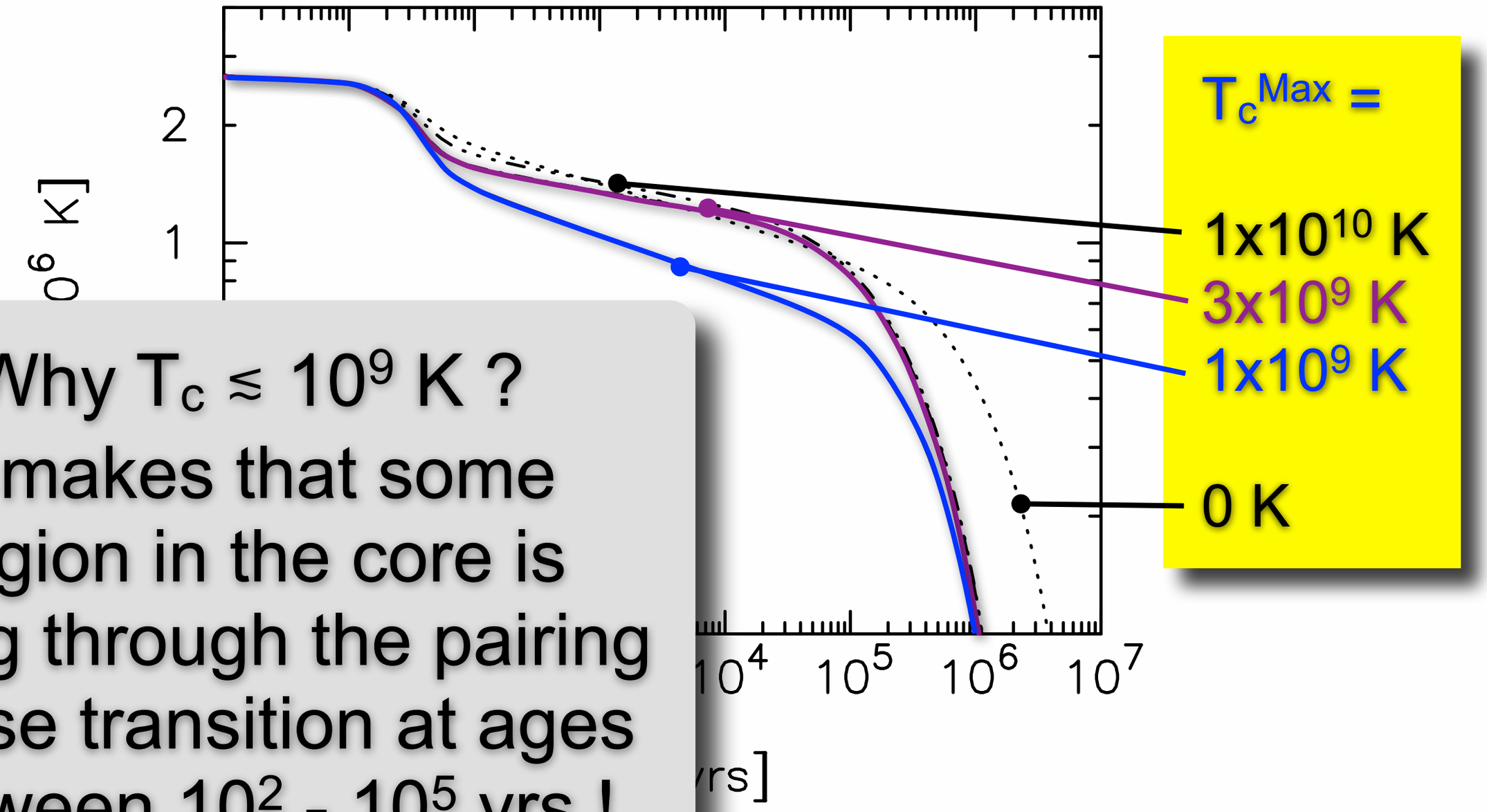
# THAT IS INTERESTING !

... is it really true ?

# Cooper Pair Neutrinos: dependency on $T_c$



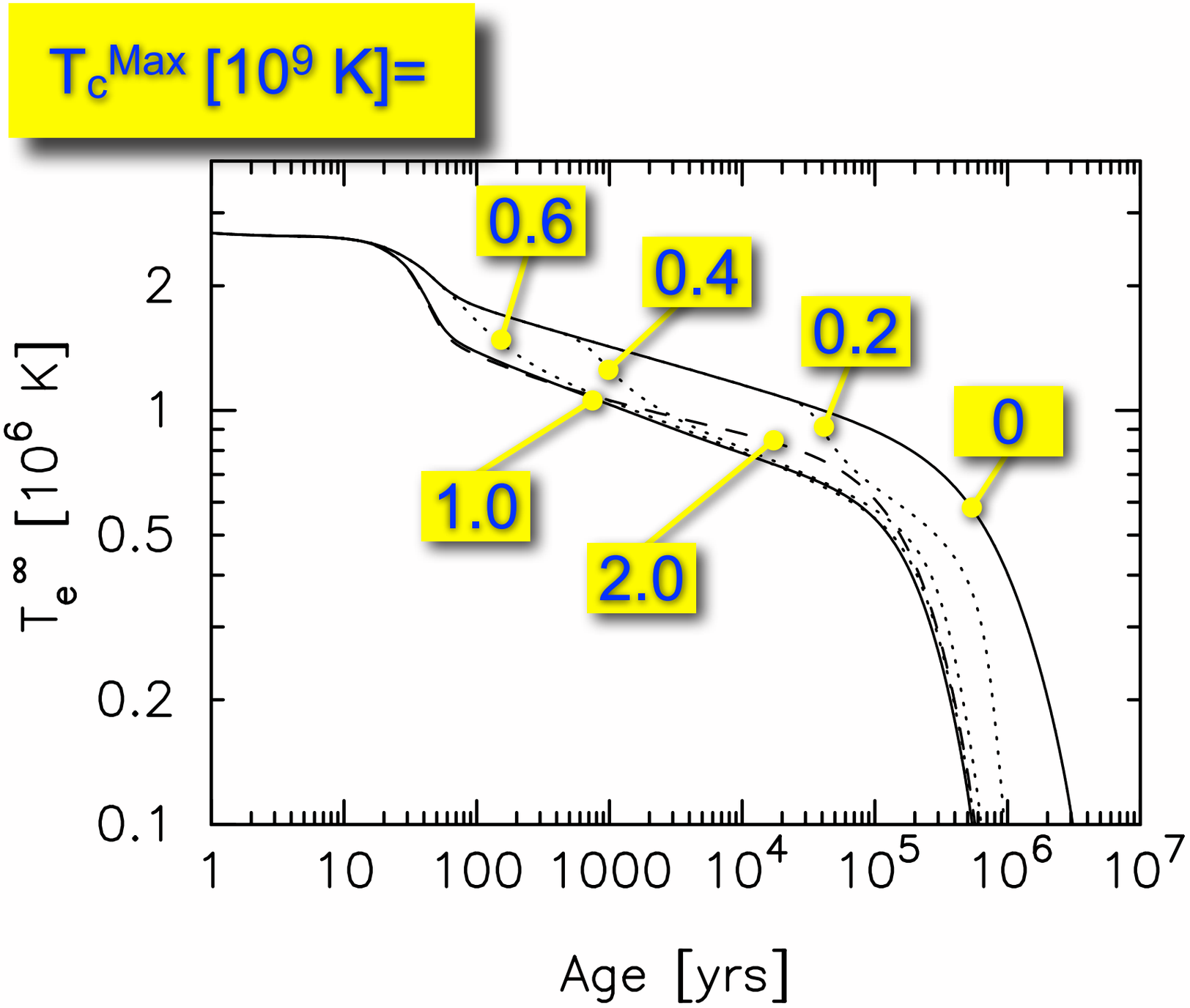
# Cooper Pair Neutrinos: dependency on $T_c$




Why $T_c \lesssim 10^9$ K ?

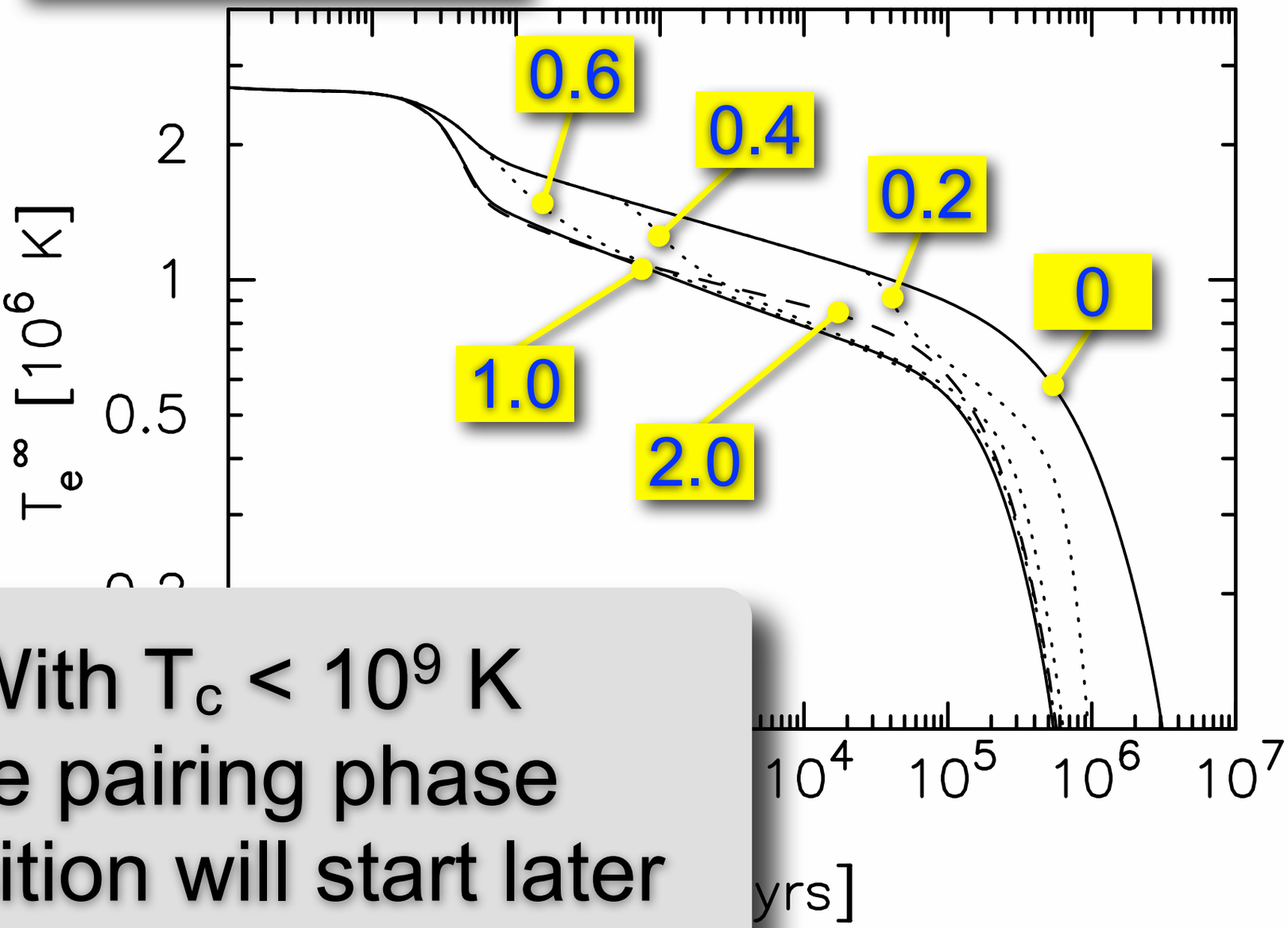
It makes that some region in the core is going through the pairing phase transition at ages between $10^2$ - $10^5$ yrs !

# Cooper Pair Neutrinos with Low $^3P_2$ $T_C$



# Cooper Pair Neutrinos with Low $^3P_2$ $T_C$

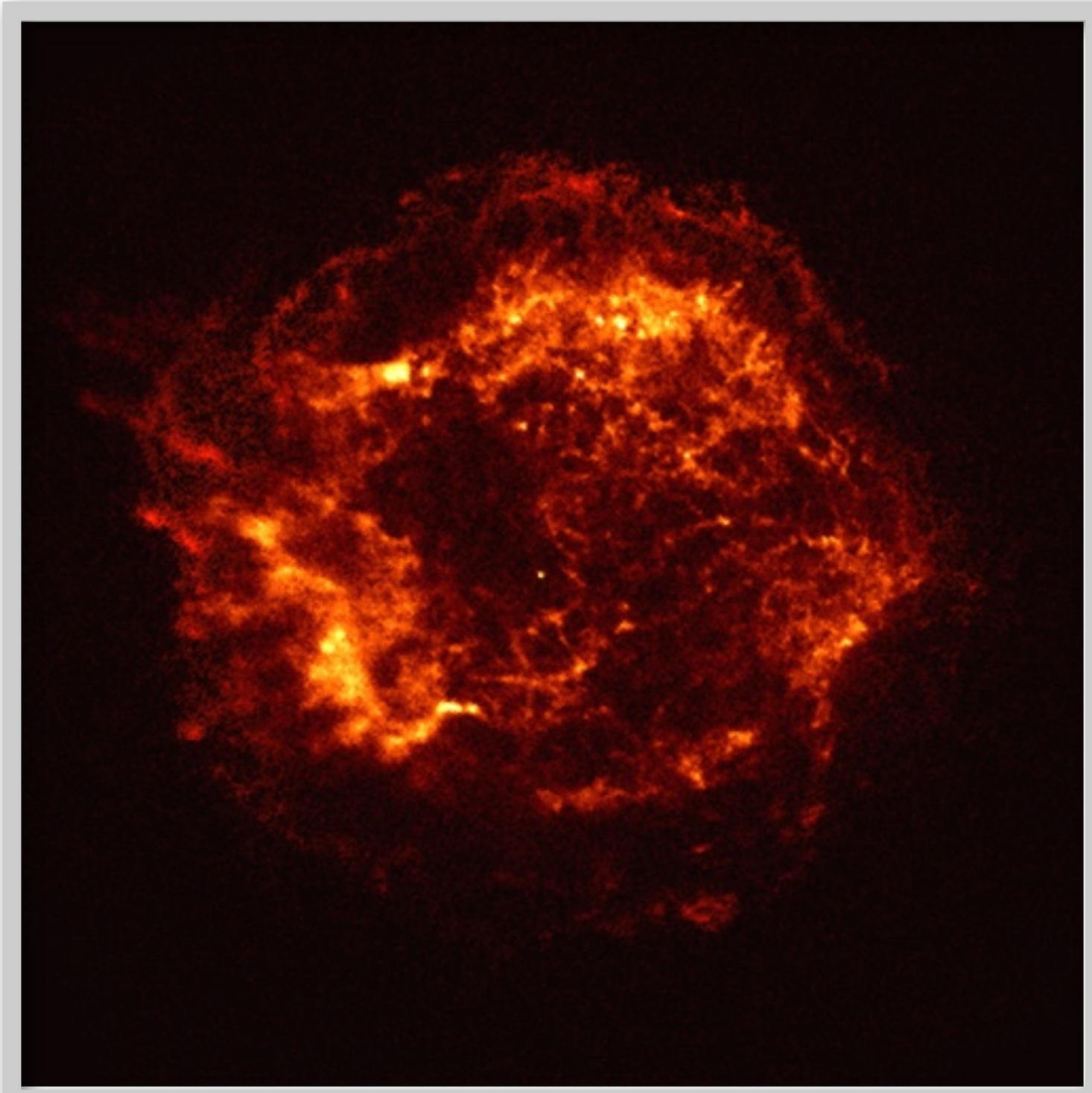
$T_c^{Max}$ [$10^9$ K]=




With $T_c < 10^9$ K
the pairing phase
transition will start later

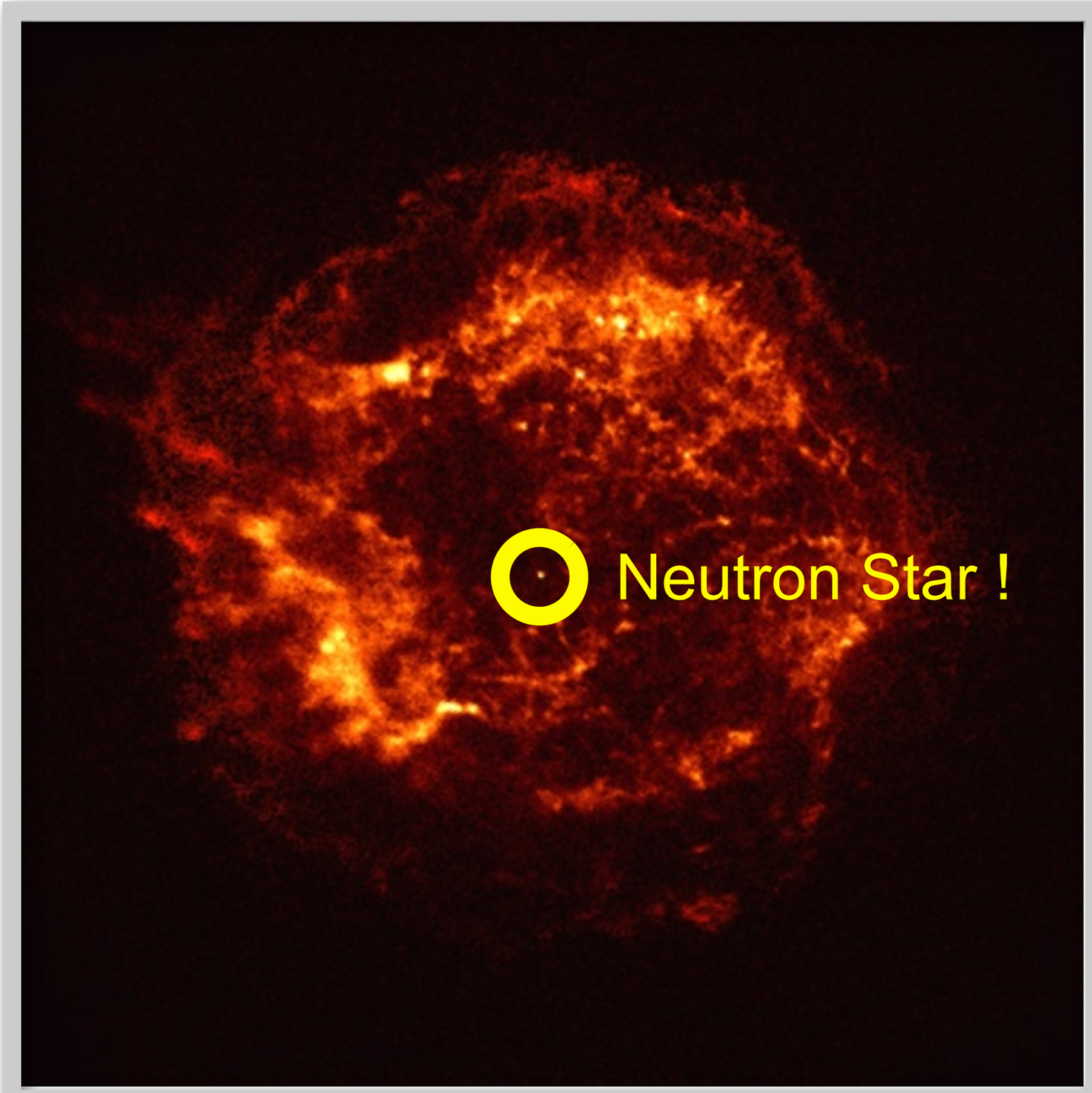
# Rapid Cooling of the Cassiopeia A Neutron Star

# Chandra's First Light: Cassiopeia A



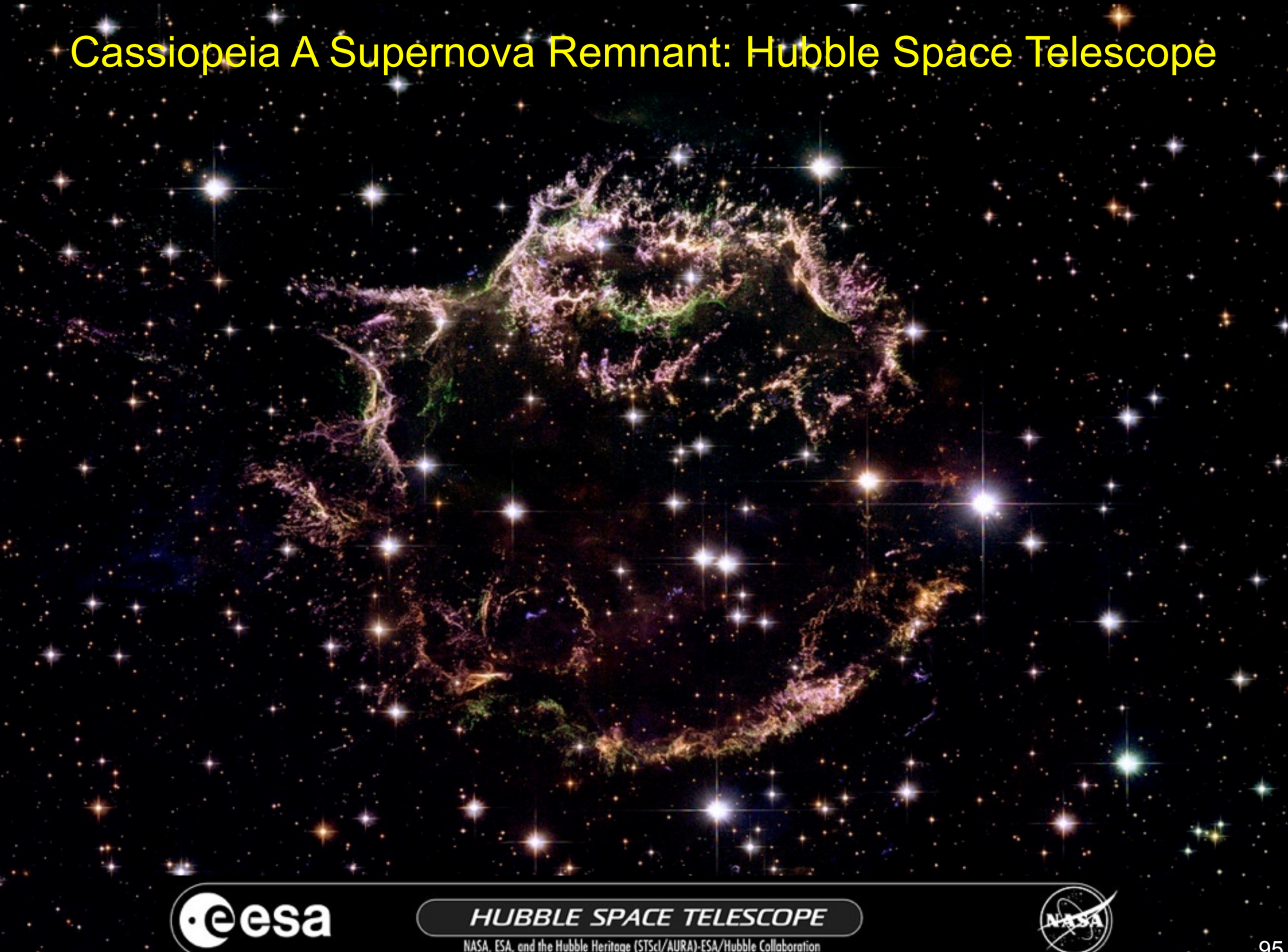
August 1999

# Chandra's First Light: Cassiopeia A




August 1999

# Cassiopeia A Supernova Remnant: Hubble Space Telescope



Cassiopeia A

Caph

γ Cas

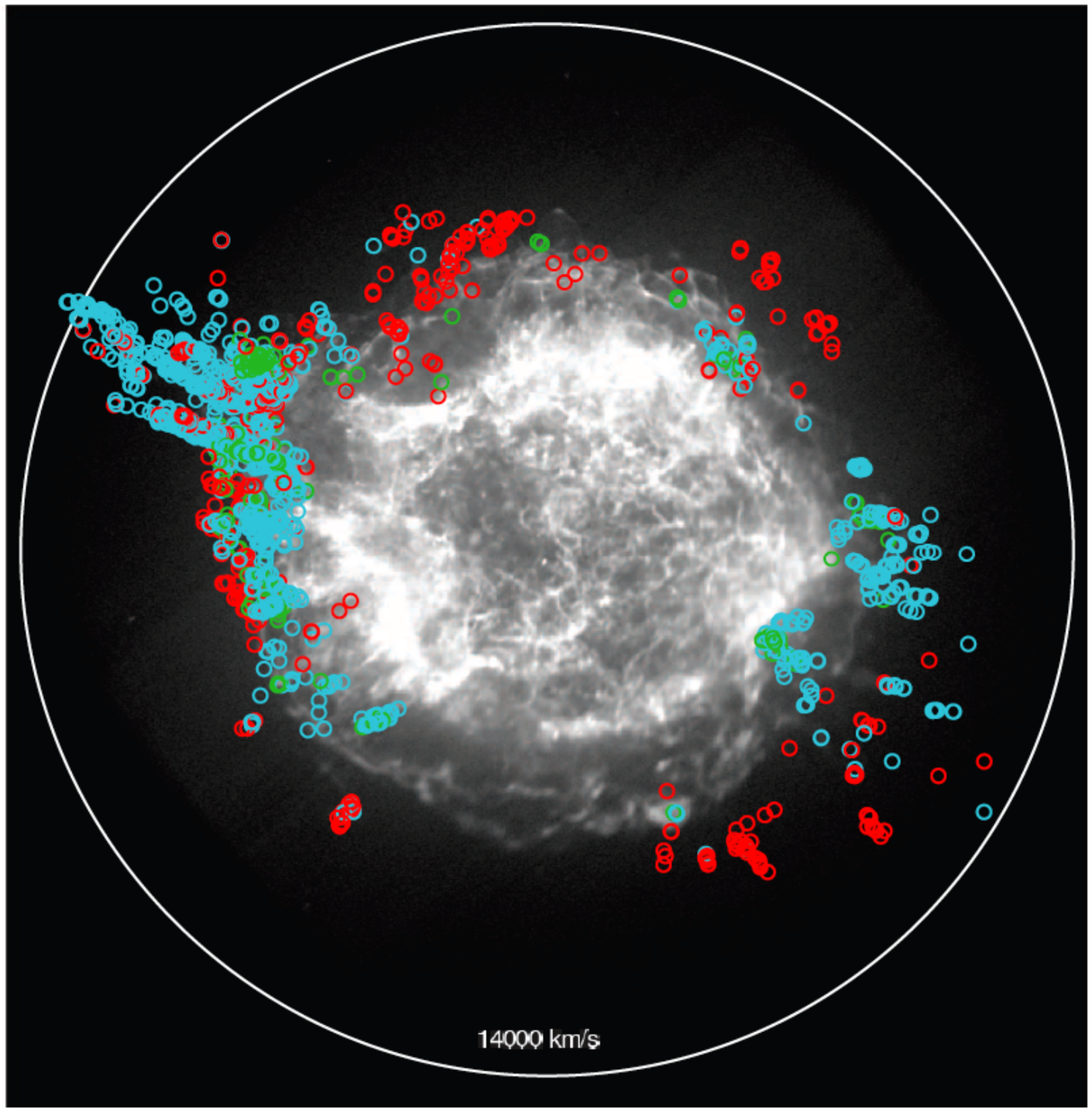
Shedir

Ruchbah

# SNR Expansion and Age:

Chandra image + Hubble knots:
NII OII SII emission lines

March to December 2004:
knots expansion clearly seen.




The Expansion Asymmetry and Age of the Cassiopeia A Supernova Remnant Fesen, R. A. et al. 2006ApJ...645..283F

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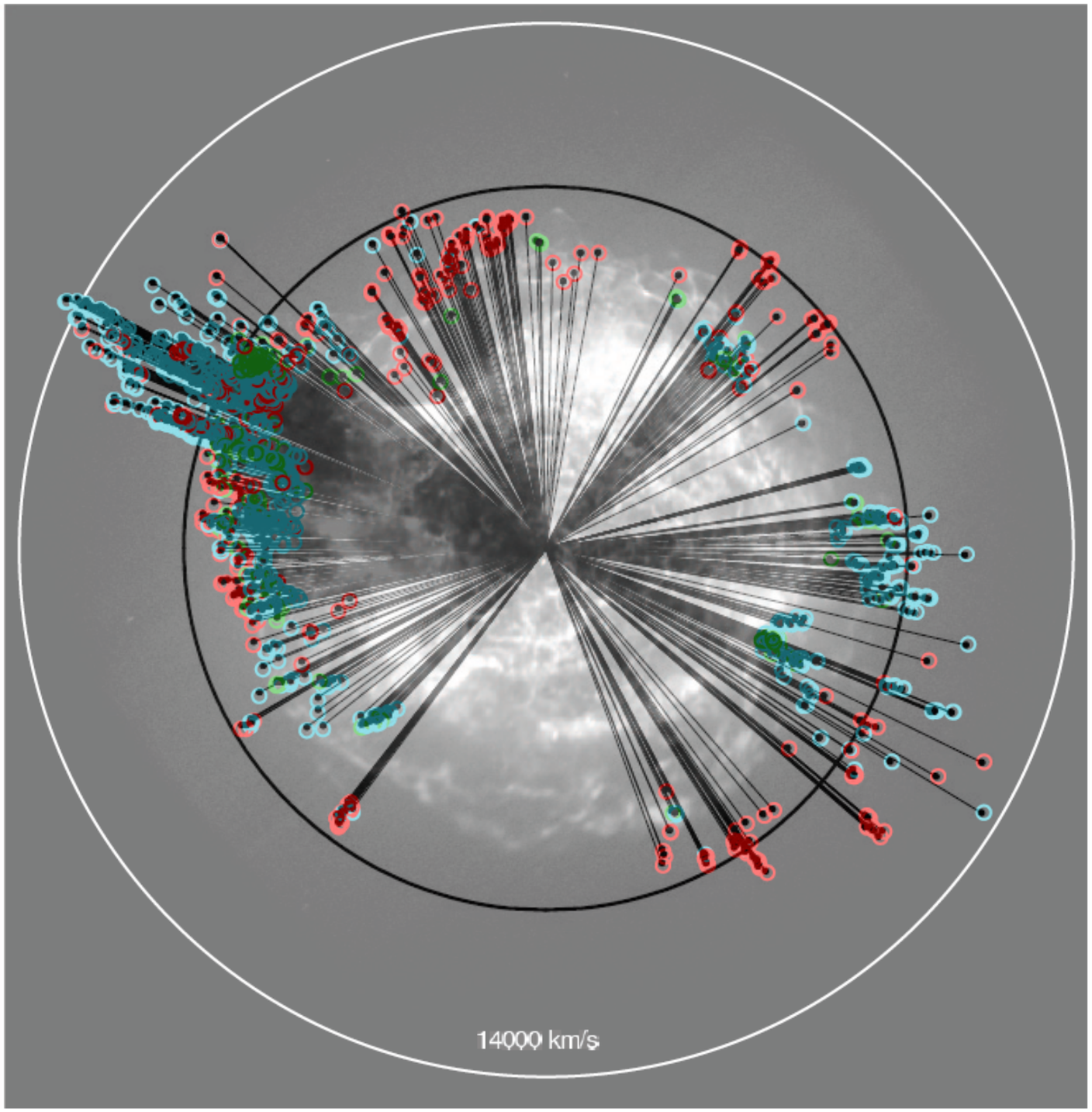
COE (Center Of Expansion)

Supernova explosion time:

- 1825 outer ejecta knots give
  AD **1662 ± 27**
- 72 bright compact knots give
  AD **1672 ± 18**

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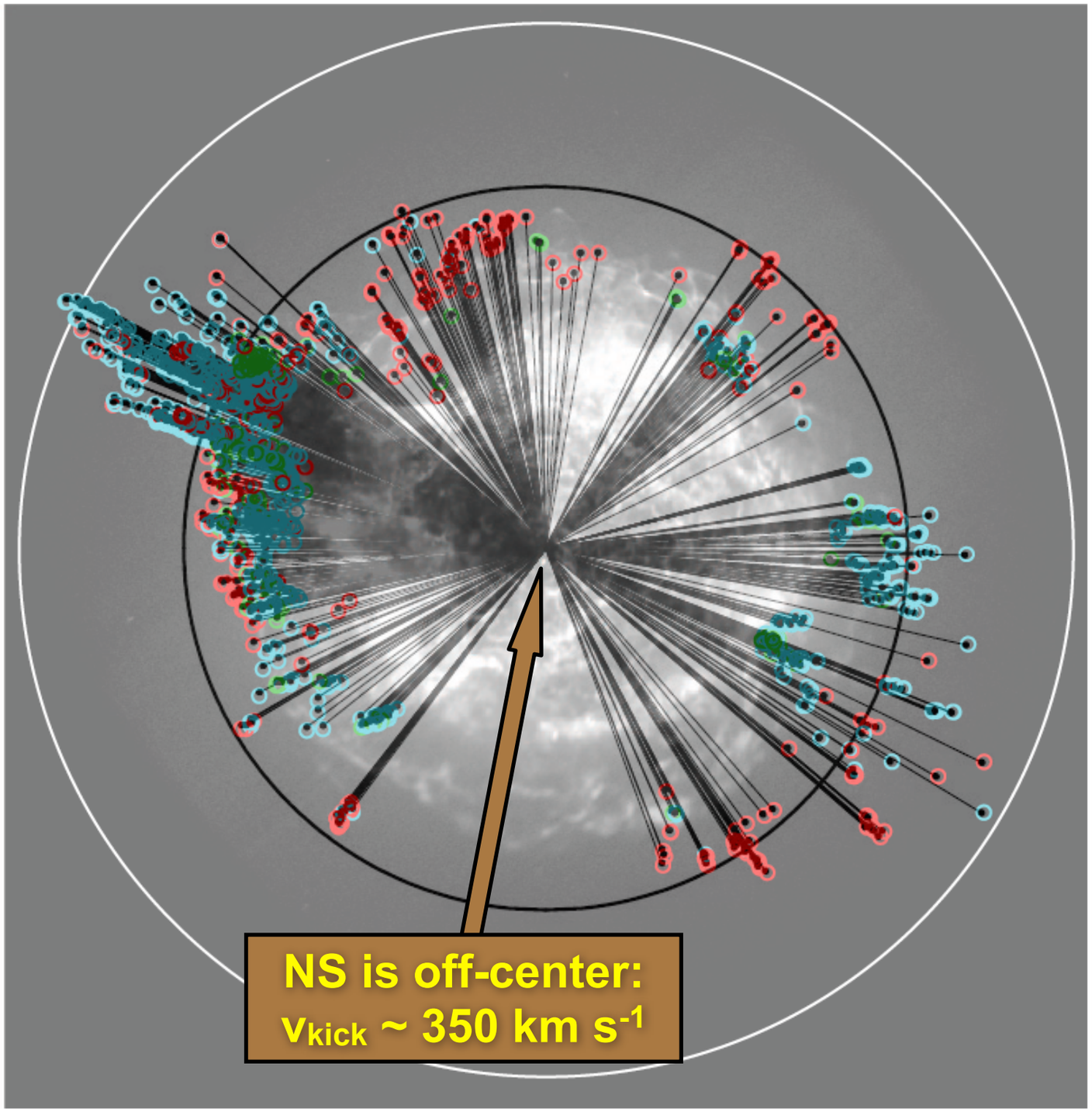
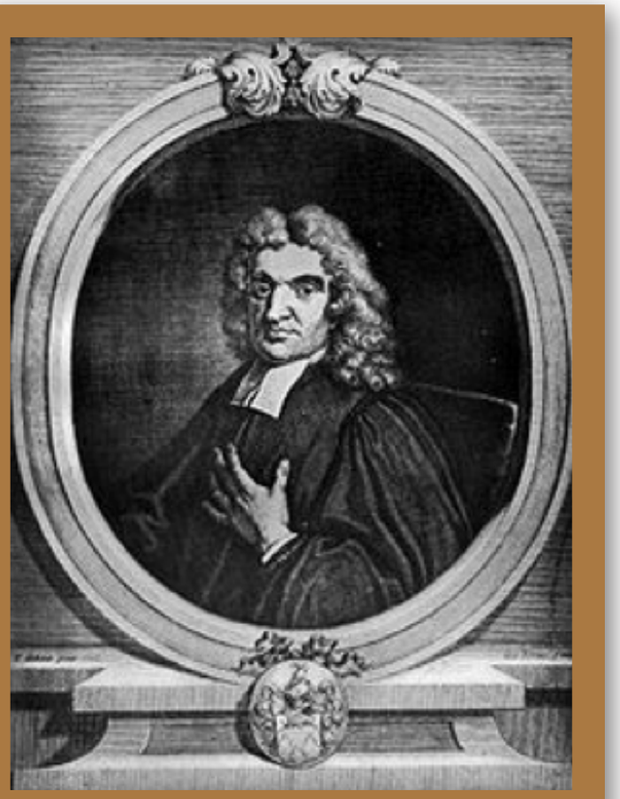



The Expansion Asymmetry and Age of the Cassiopeia A Supernova Remnant Fesen, R. A. et al. 2006ApJ...645..283F

# Historical SN: John Flamsteed (1680)

## John Flamsteed

(19 Aug. 1646 - 31 Dec. 1719)
First **Astronomer Royal**
(1675 - 1719)



In Conſtellatione CASSIOPEÆ.

| ORDO Ptol. | ORDO Tych. | STELLARUM Denominatio. | Bayer. Cha. | Aſcenſio Recta. | Diſtantia à Polo B. | Longitudo. | Latitudo. | Varia. Aſc.R. | Varia. D. à P. | Magnitud. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | o ′ ″ | o ′ ″ | s o ′ ″ | o ′ ″ | ′ ″ | ′ ″ | |
| | | Quæ eſt | e | 343 25 0 | 32 14 20 | ♈ 21 6 45 | 56 46 0 B. | 44 4 | 22 48 | 6 |
| | | | | 344 7 30 | 32 20 0 | 21 29 16 | 56 26 10 | 44 32 | 22 55 | 7 |
| | | | | 347 37 0 | 33 1 30 | 23 5 5 | 54 38 32 | 46 57 | 22 20 | 6 |
| | | | d | 347 49 30 | 29 24 30 | 27 42 49 | 57 10 12 | 45 53 | 23 17 | 5 |
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FIG. 1. The positions of the first five Cassiopeia stars, as given in Flamsteed's 1725 catalogue and as depicted in his atlas of 1729.[5] The Flamsteed numbers on the star map have been added.

A Probable Flamsteed Observations of the Cassiopeia-A Supernova Ashworth, W. B., Jr. 1980JHA....11....1A

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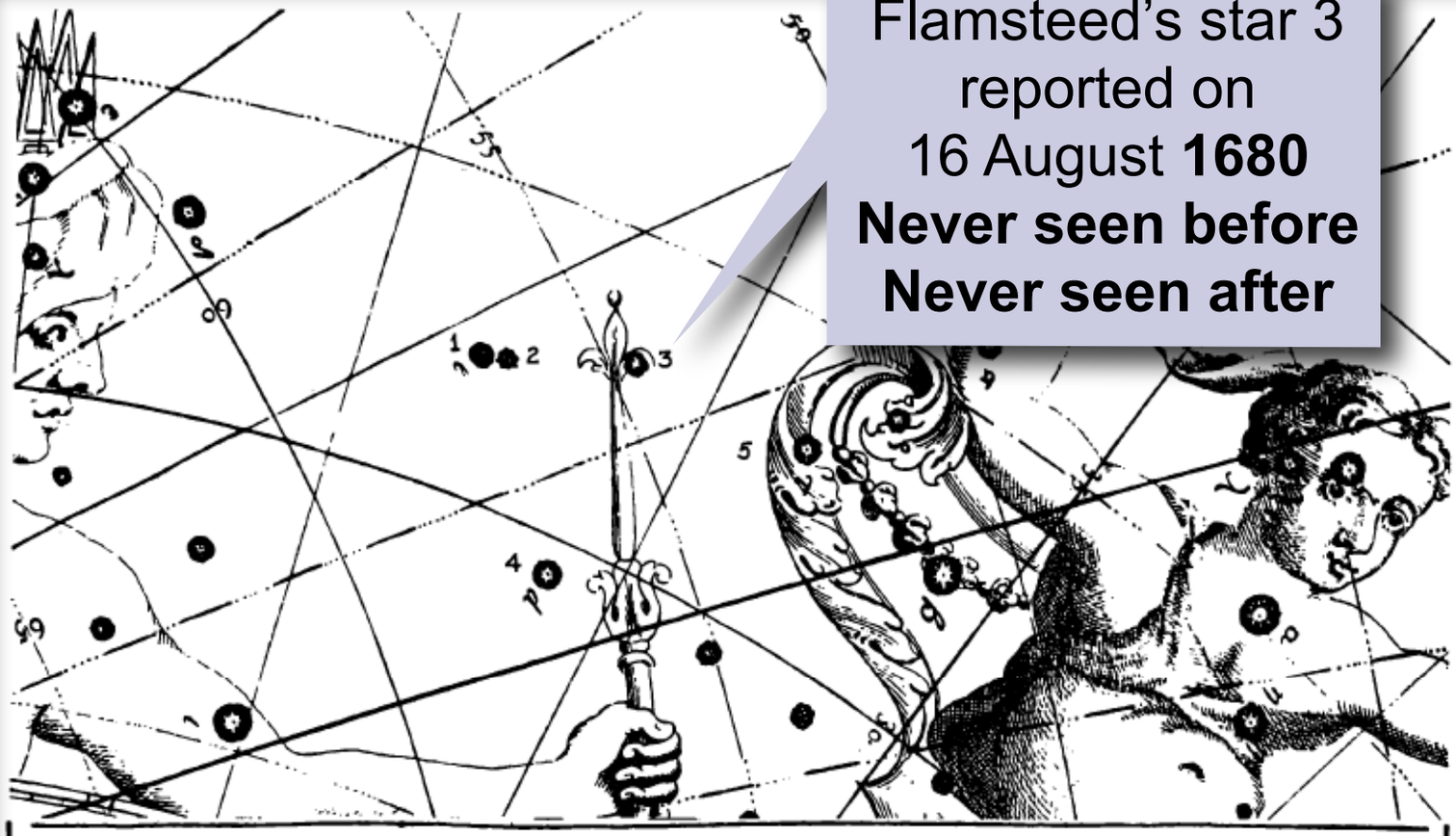
Flamsteed's star 3
reported on
16 August **1680**
**Never seen before**
**Never seen after**



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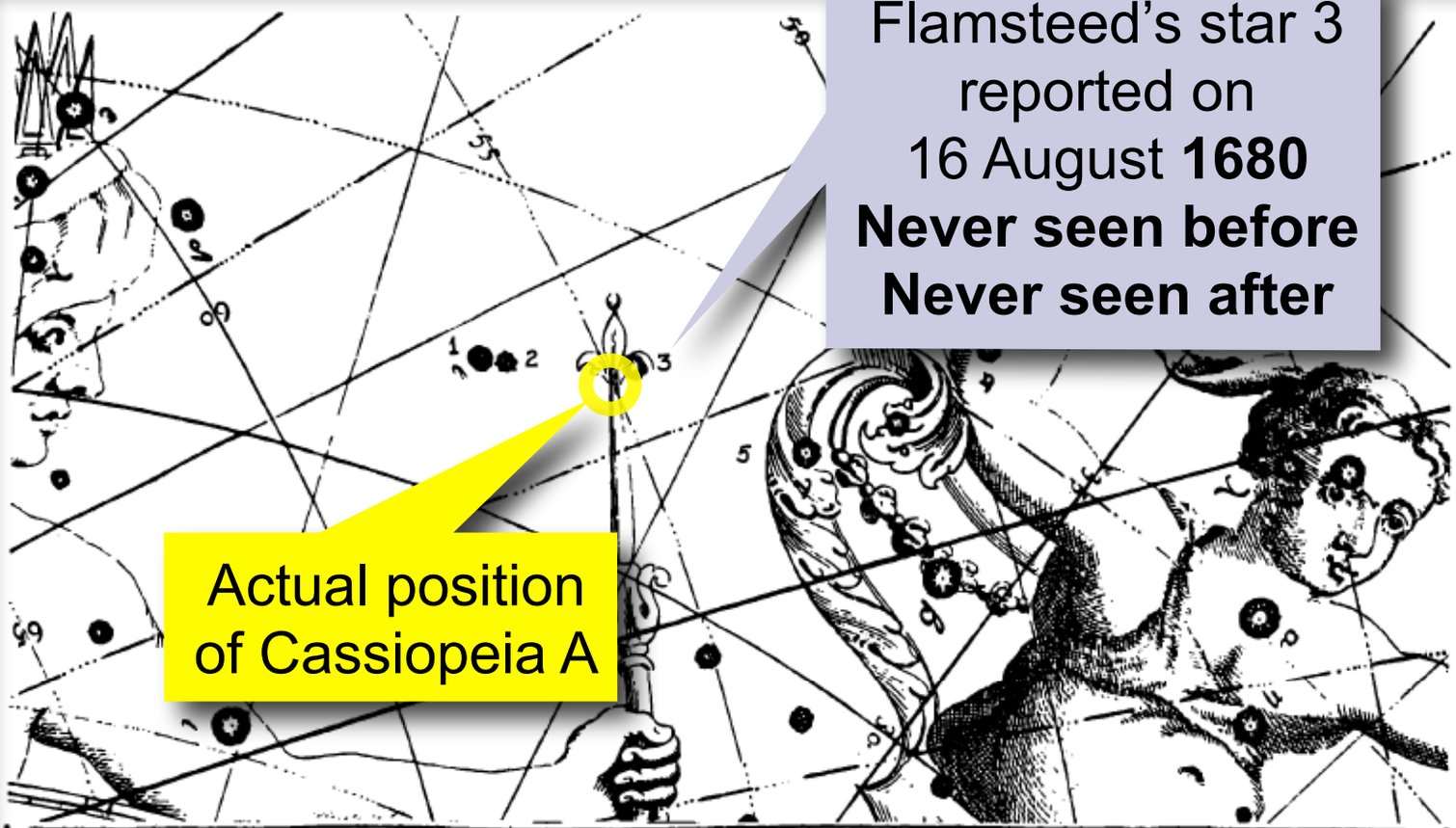


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|---|---|---|---|---|---|---|---|---|---|---|
| | | | | o ′ ″ | o ′ ″ | s o ′ ″ | o ′ ″ | ′ ″ | ′ ″ | |
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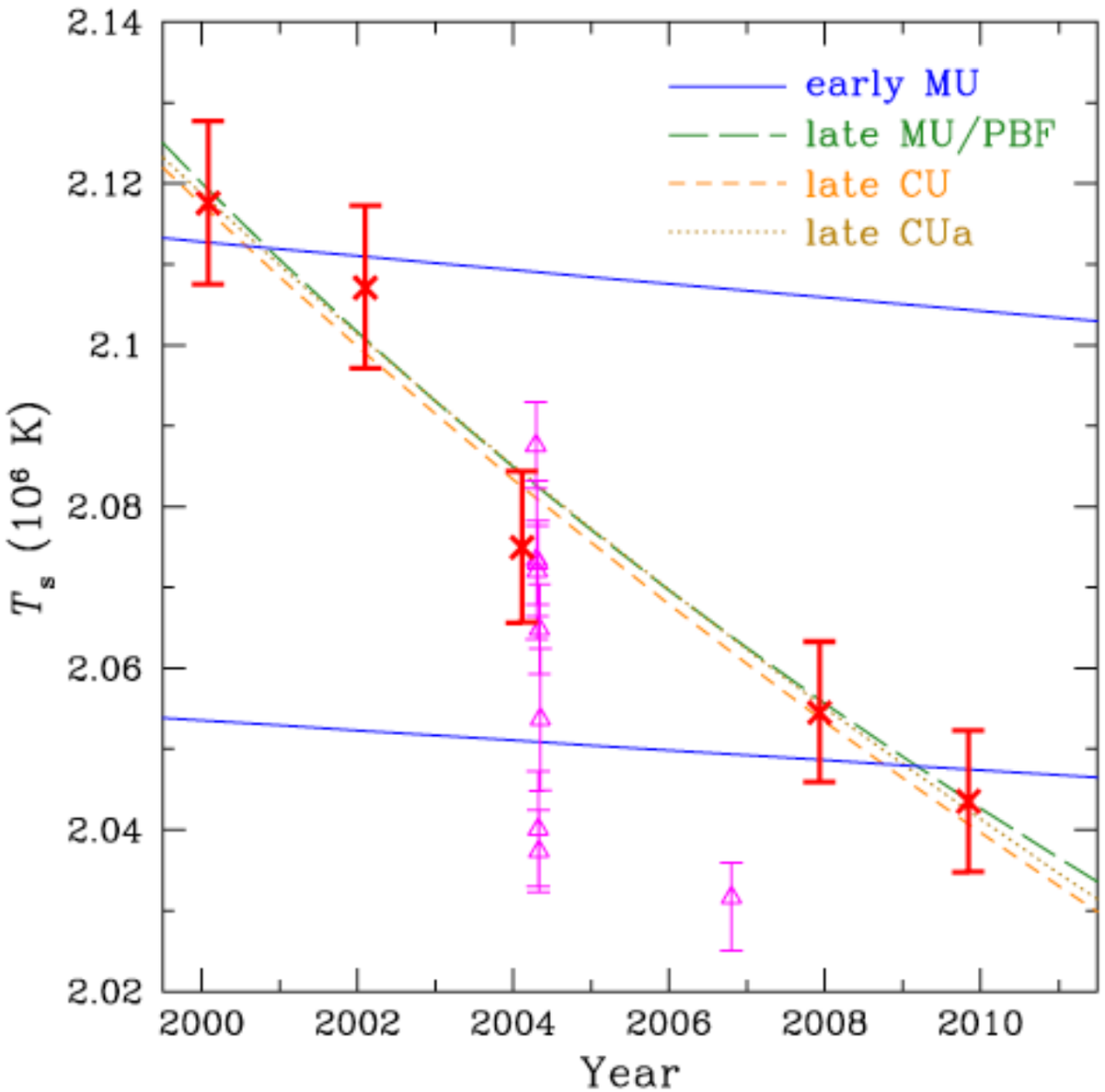
# Cassiopeia A Supernova Remnant

- Brightest radio source in the sky (after the Sun).
  Discovered in 1947.
- Radio source name: **3C461**
  SNR name: **G111.7-2.1**
- Discovered in optical in 1950
- Discoverd in X-ray in 1965 (Aerobie rocket): **Cas X-1**
- Distance: 3.4 kpc (11,000 light years)
- Expansion velocities: 5,500 - 14,500 km $s^{-1}$
- Age: 350 ± 30 yrs (Flamsteed: 334 yrs)
- From a Type IIb SN: 15-20 $M_\odot$ progenitor.

# Rapid Cooling of Cas A

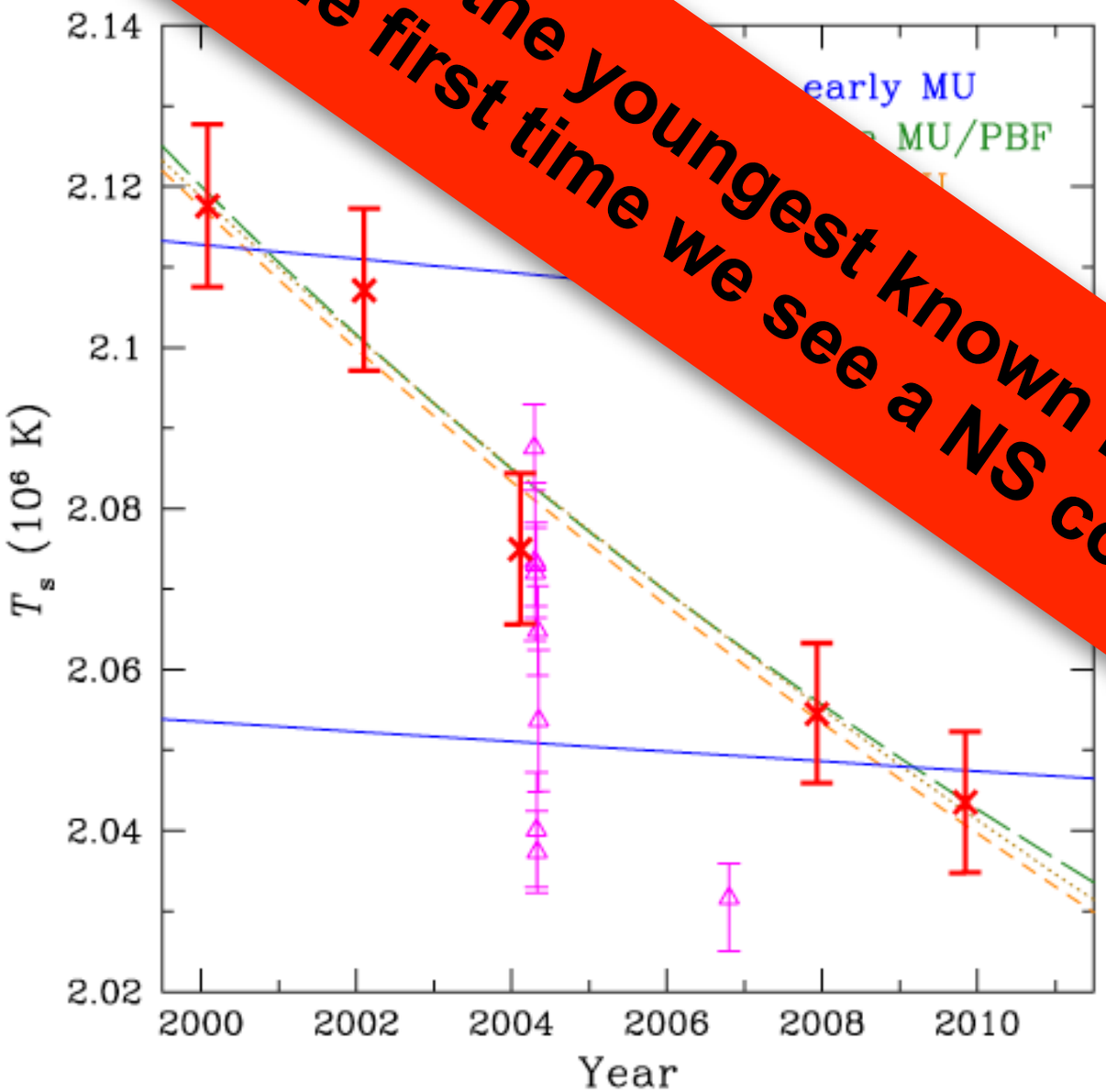
4% decrease in $T_e$
20% decrease in $L_\gamma$




Direct Observation of the Cooling of the Cassiopeia A Neutron Star
Heinke, Craig O.; Ho, Wynn C. G. 2010ApJ...719L.167H

# d Cooling of Cas A

**It is the youngest known neutron star**
**It is the first time we see a NS cool in real time !**

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Observed cooling rate + $C_v$ estimate:
need thermal energy loss

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**$L_X \sim 10^{34}$ erg s$^{-1}$**

Observed cooling rate + $C_v$ estimate:
need thermal energy loss

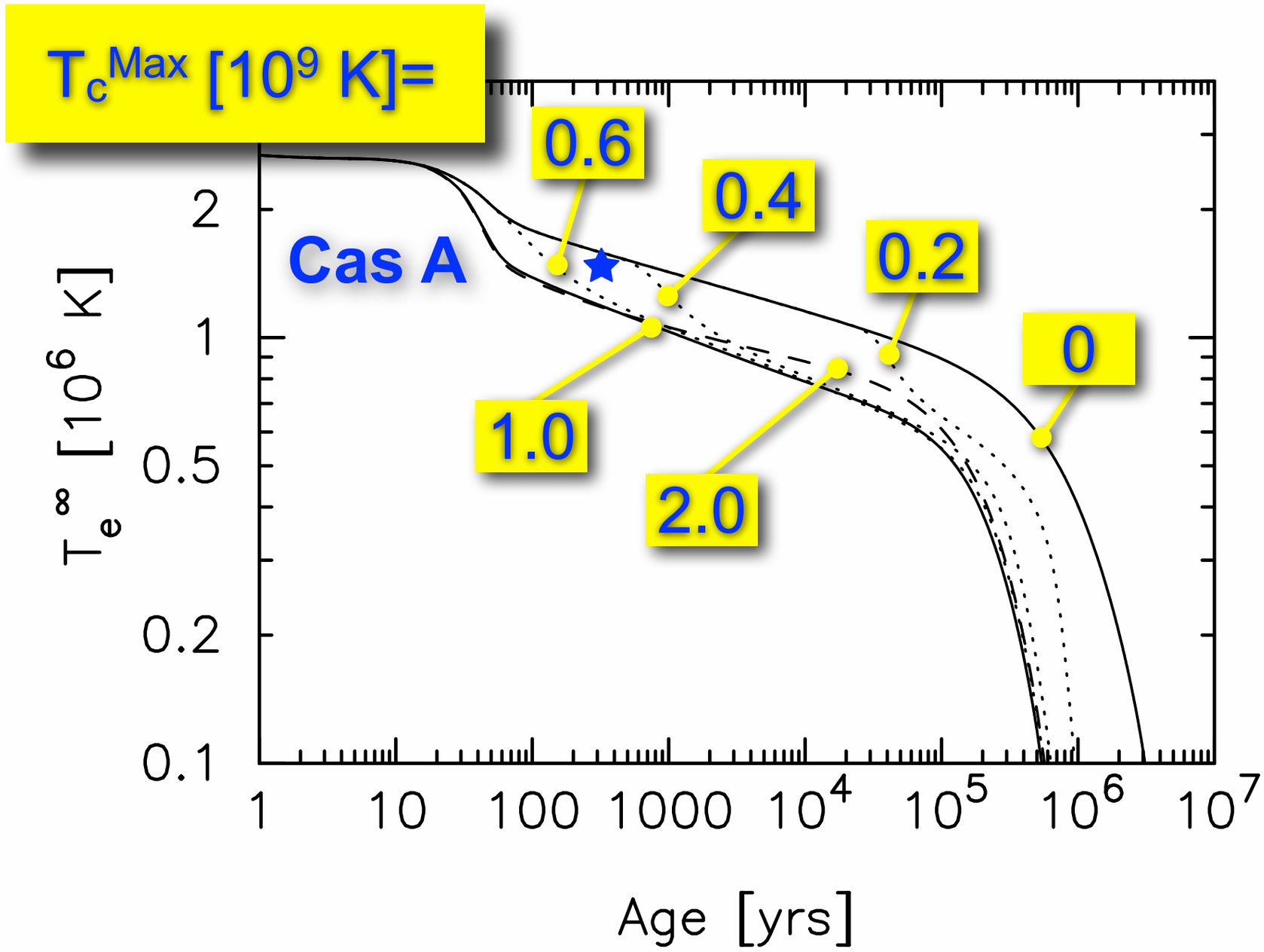
**$\dot{E}_{th} \sim 10^{38}$ erg s$^{-1}$**

4 orders of magnitude: NEUTRINOS

So rapid it must be transitory:
started recently ⇒ critical X

X = Temperature most natural
⇒ Phase transition

# Cooper Pair Neutrinos with Low $^3P_2$ $T_C$

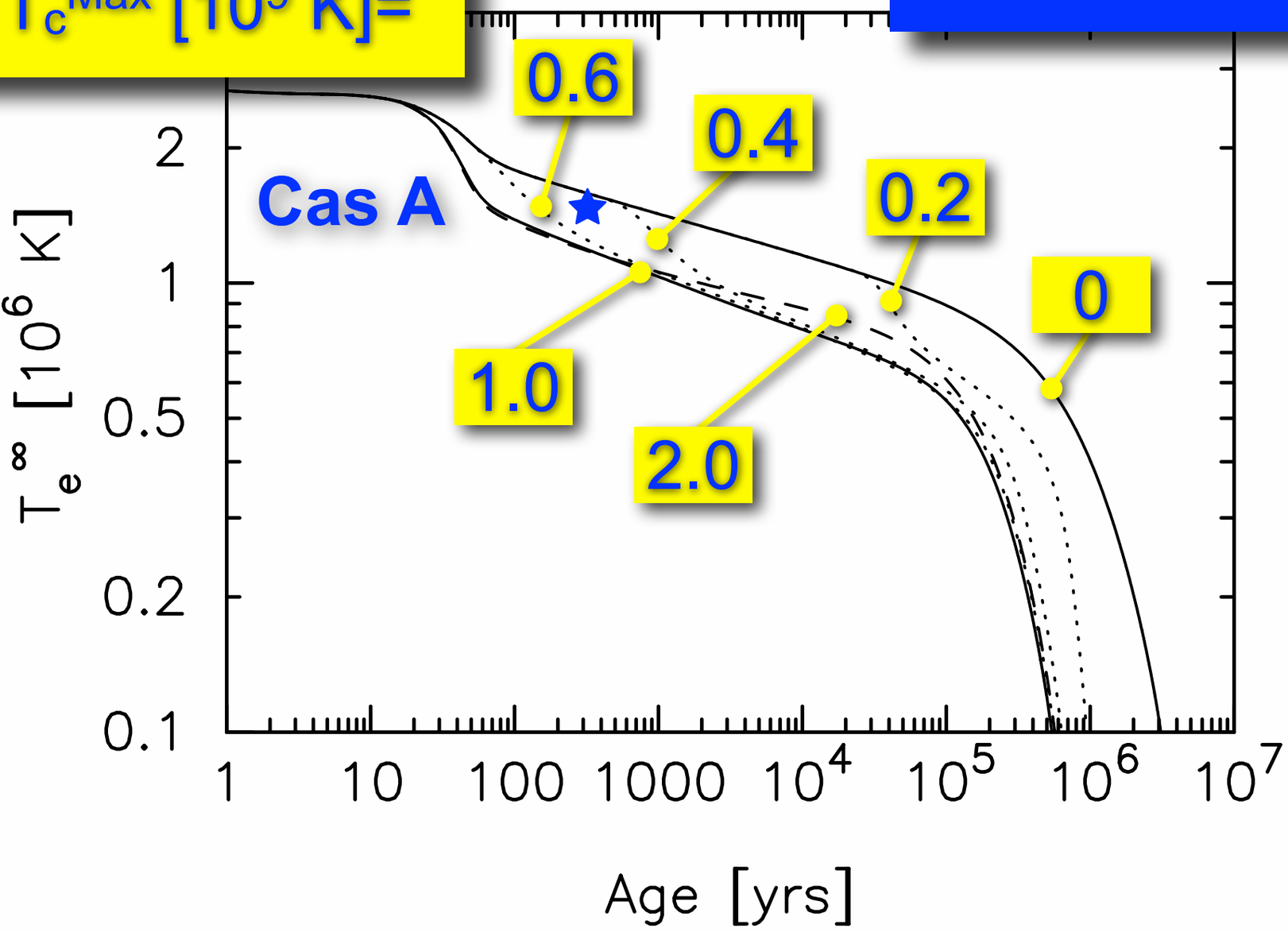



Rapid Cooling of the Neutron Star in Cassiopeia A Triggered by Neutron Superfluidity in Dense Matter
Page, Dany; Prakash, Madappa; Lattimer, James M.; Steiner, Andrew W.

# Cooper Pair Neutrinos with Low $^3P_2$ $T_C$

$T_c^{Max}$ [$10^9$ K]=

$T_c^{Max}$ = 5·$10^8$ K !

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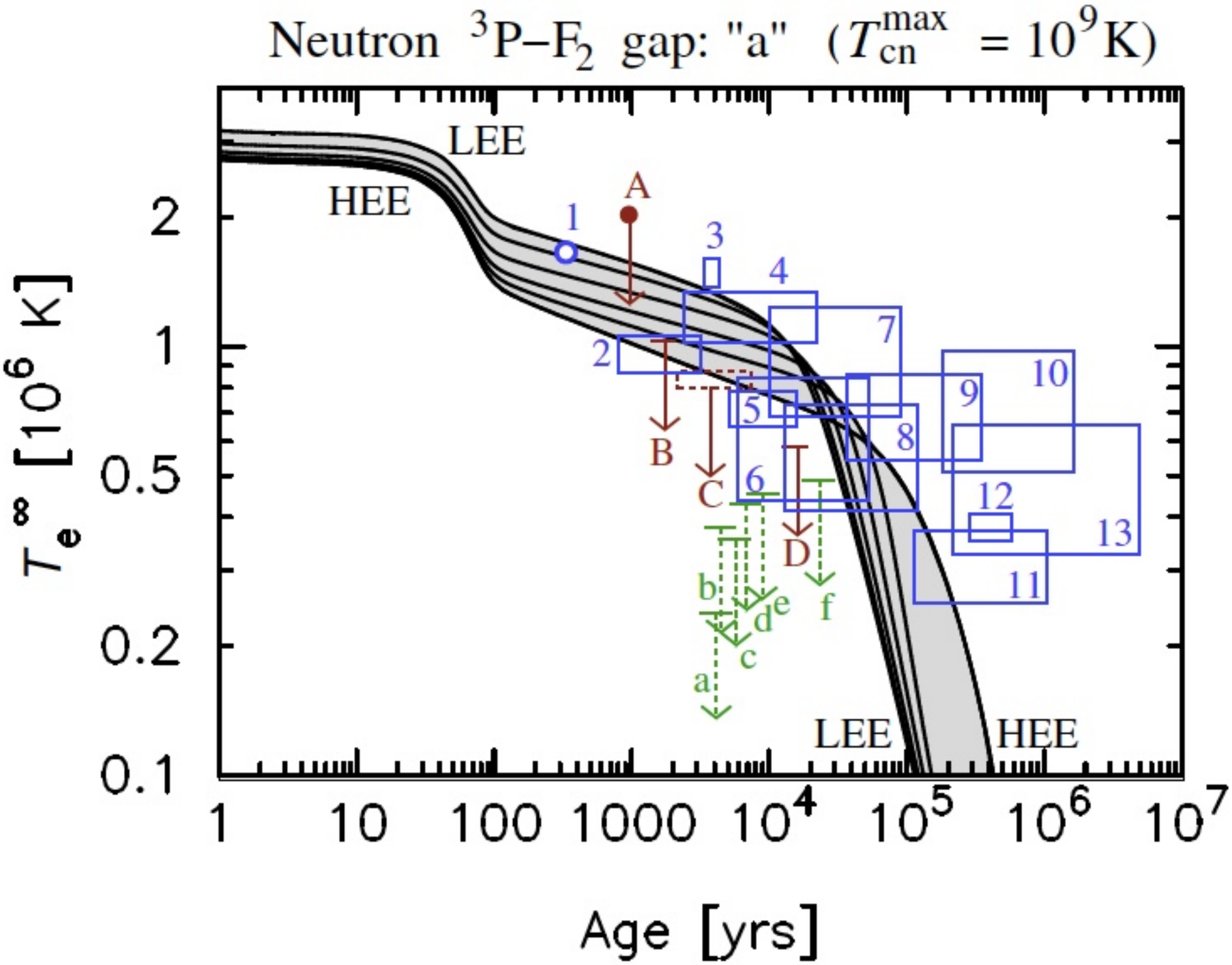
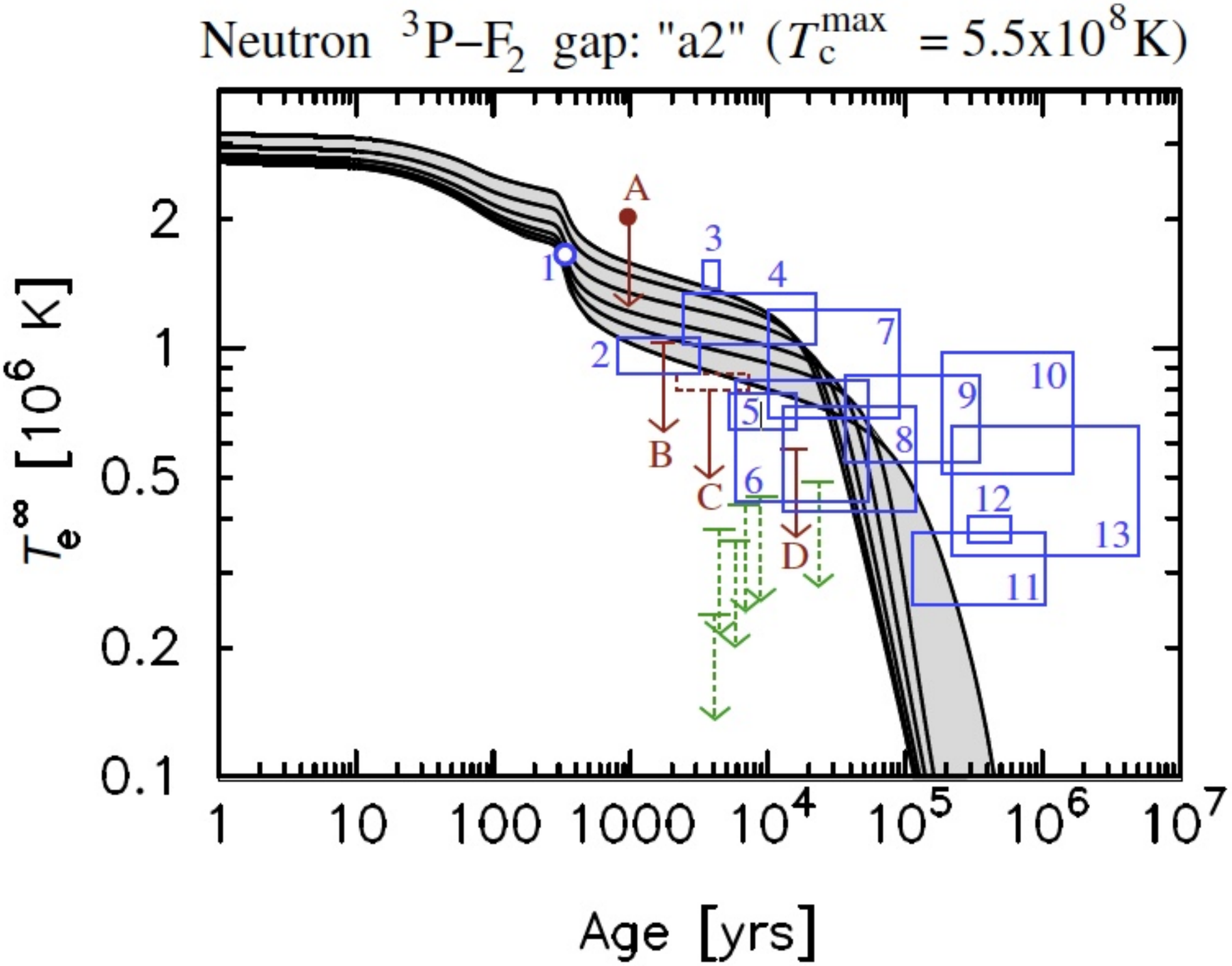
# Our previous “best” Minimal Cooling Model



# Minimal Cooling and “Cas A”



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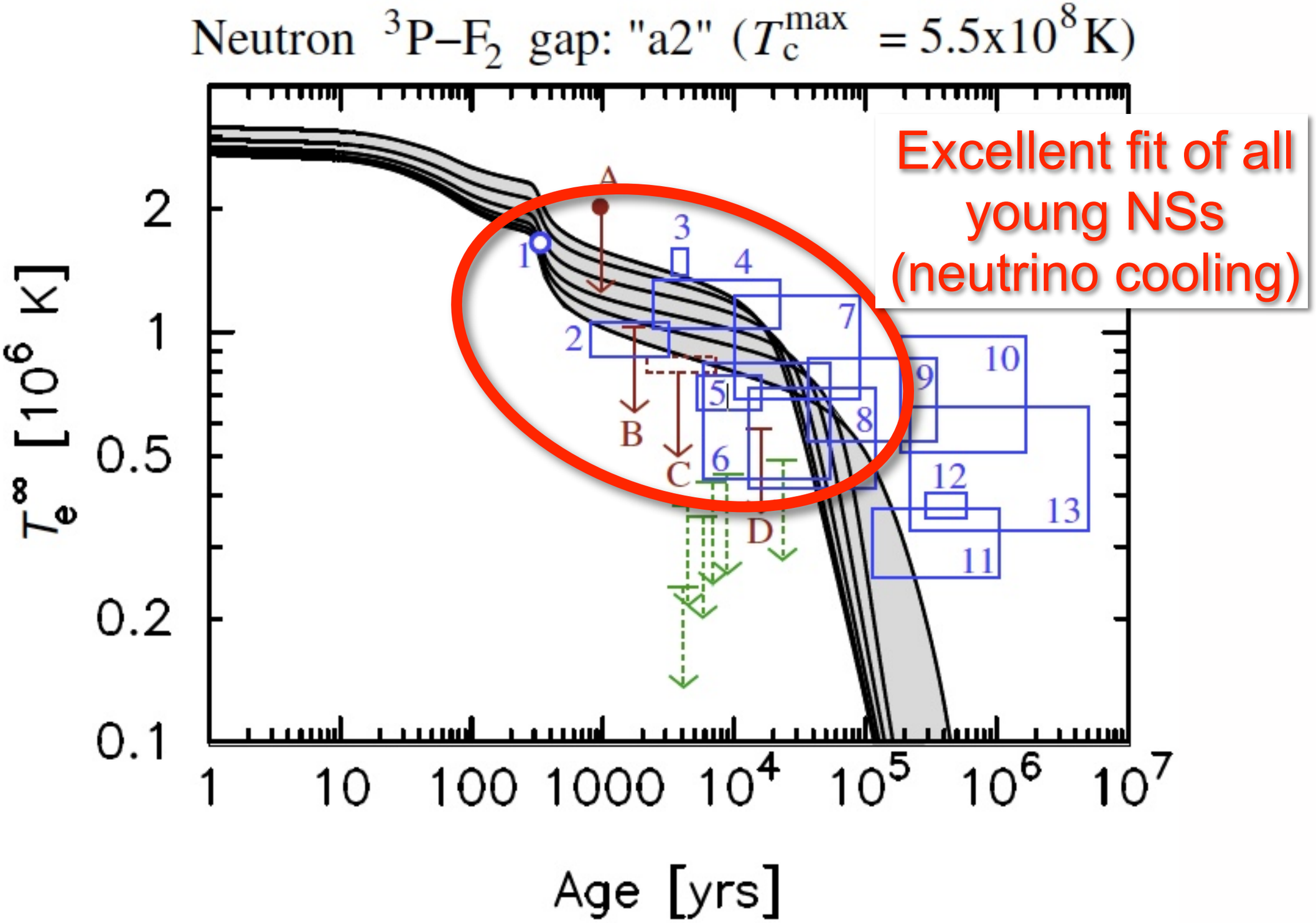

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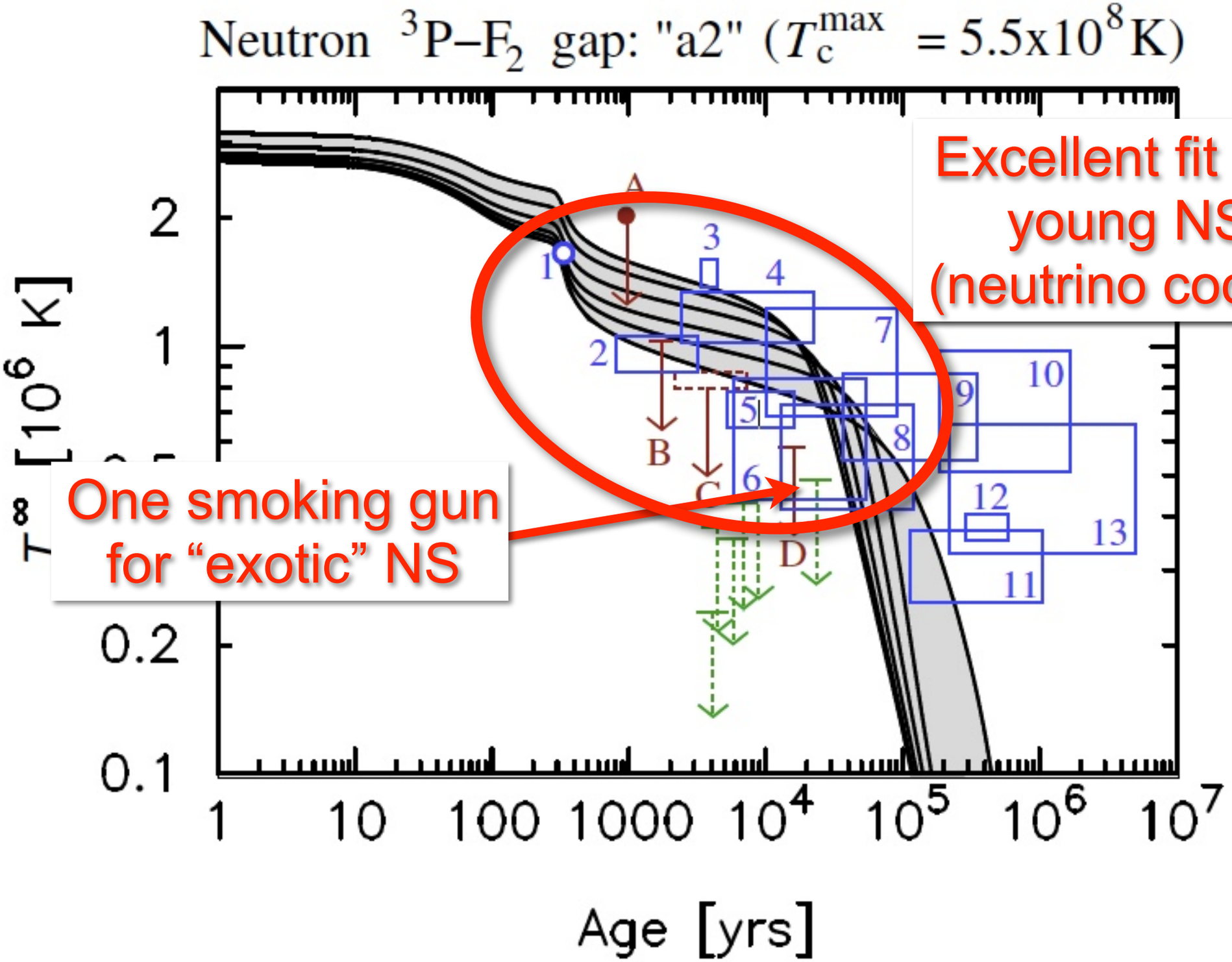

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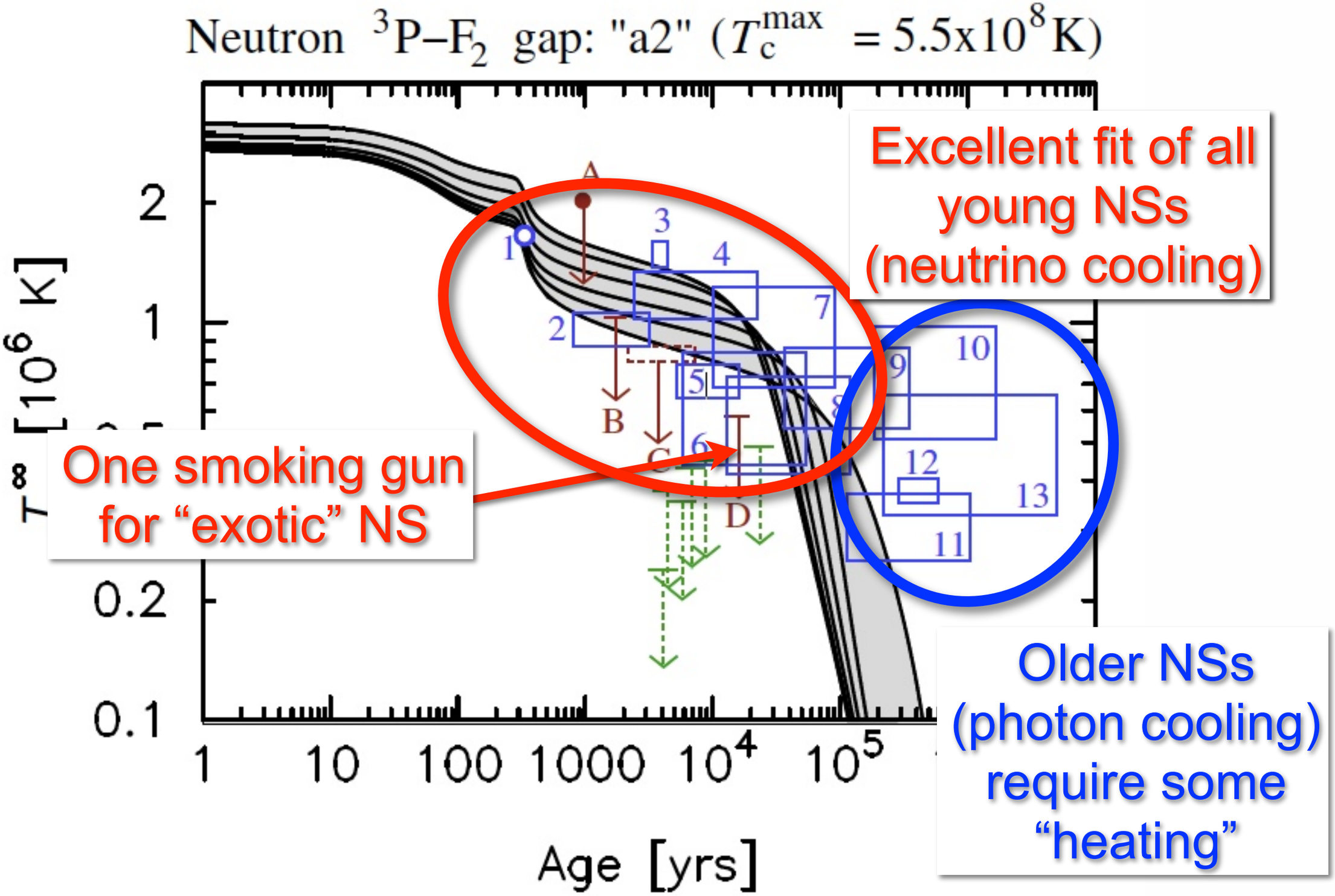

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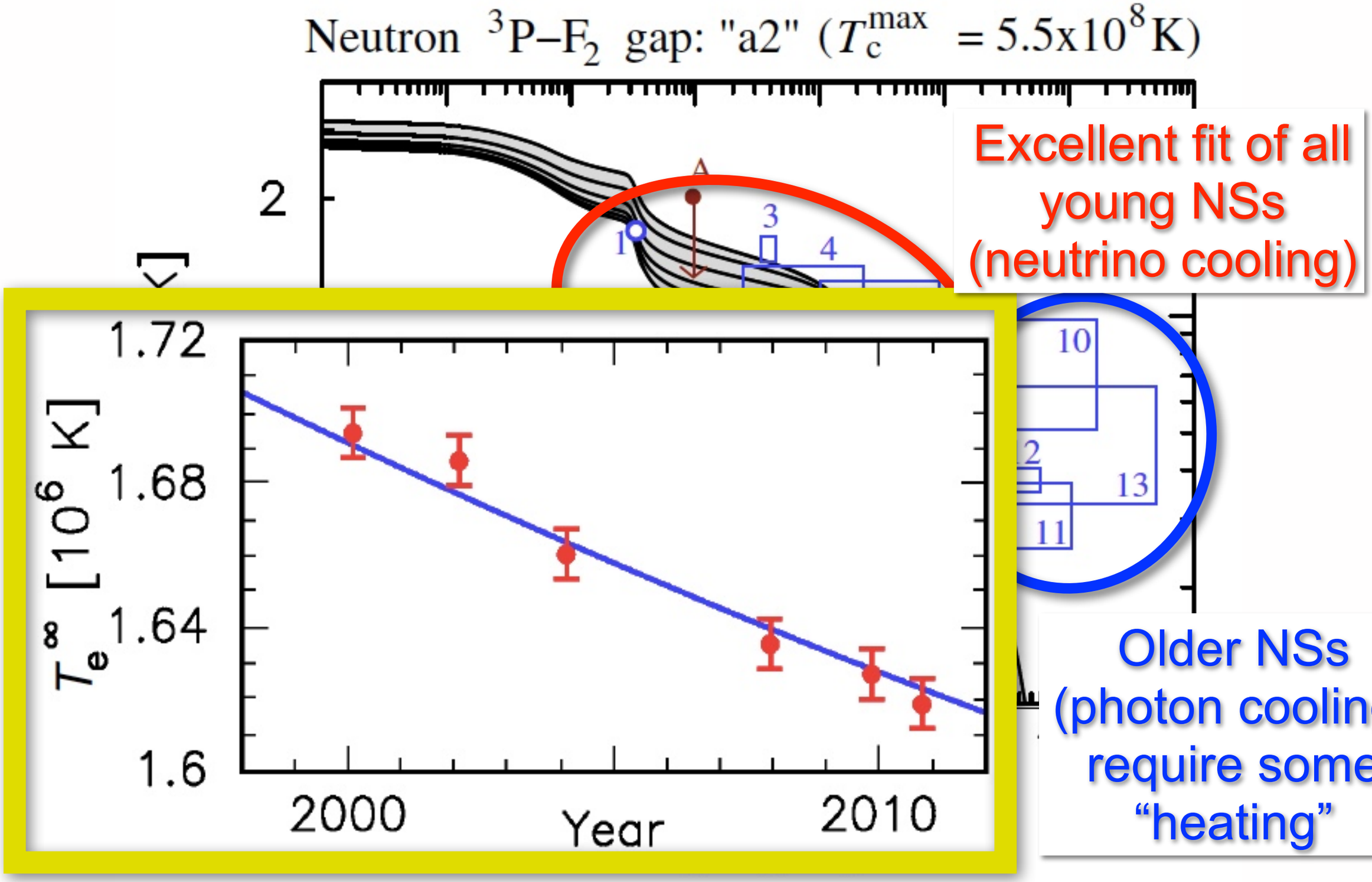
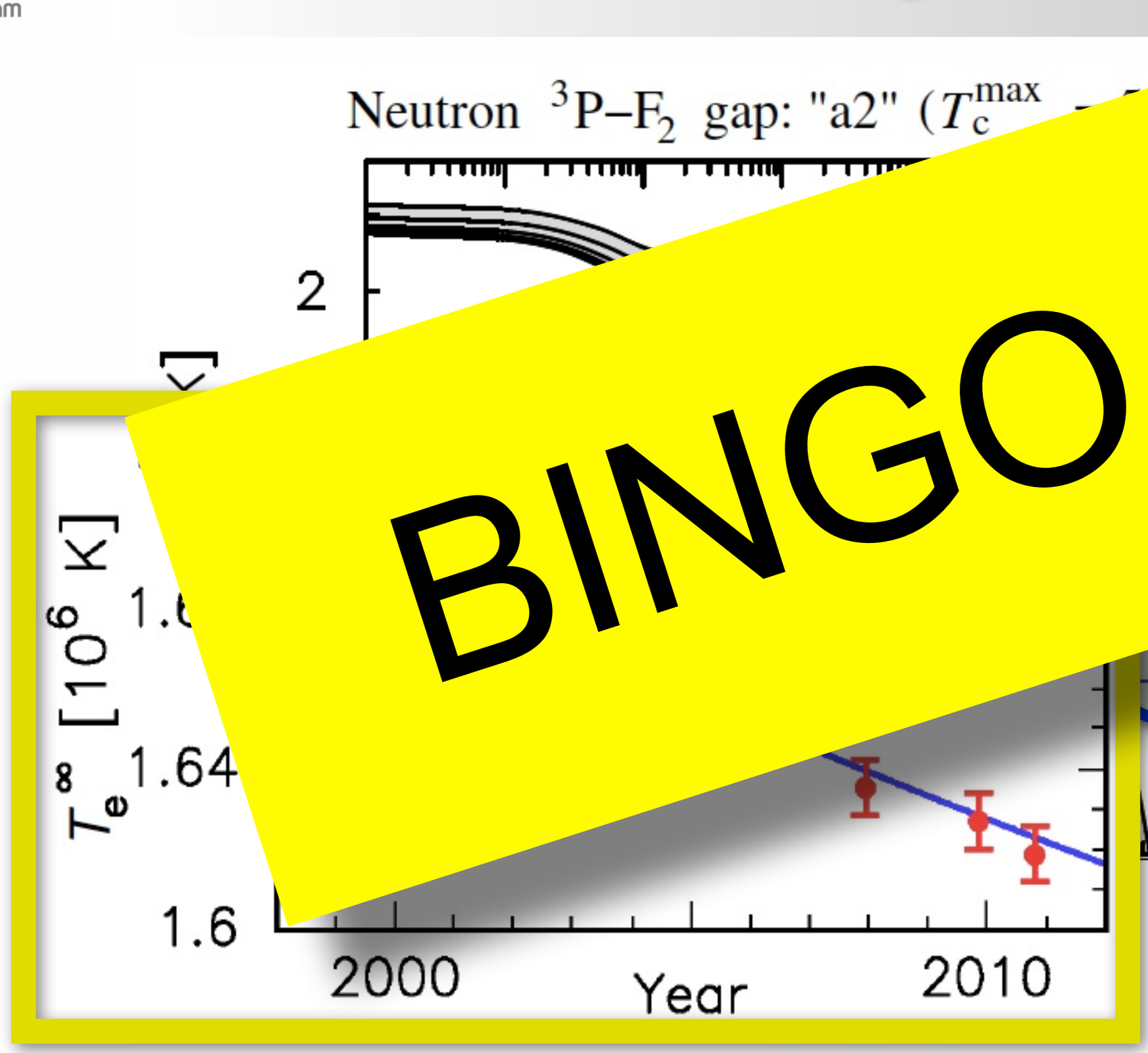



Excellent fit of all young NSs (neutrino cooling)

Older NSs (photon cooling) require some “heating”

# Minimal Cooling and “Cas A”




BINGO !

Older NSs (photon cooling) require some “heating”

# The first direct evidence that superfluidity and superconductivity occur at supranuclear densities within neutron stars.

## Cooling neutron star in the Cassiopeia A supernova remnant: Evidence for superfluidity in the core

Peter S. Shternin[1⋆], Dmitry G. Yakovlev[1], Craig O. Heinke[2], Wynn C. G. Ho[3]†, Daniel J. Patnaude[4]
[1]*Ioffe Physical Technical Institute, Politekhnicheskaya 26, 194021 St. Petersburg, Russia*
[2]*Department of Physics, University of Alberta, Room 238 CEB, 11322-89 Avenue, Edmonton, AB, T6G 2G7, Canada*
[3]*School of Mathematics, University of Southampton, Southampton, SO17 1BJ, United Kingdom*
[4]*Smithsonian Astrophysical Observatory, Cambridge, MA 02138, USA*

Month. Not. R. Astron. Soc. Lett. **412** (2011), p. 108

## Rapid Cooling of the Neutron Star in Cassiopeia A Triggered by Neutron Superfluidity in Dense Matter

Dany Page,[1] Madappa Prakash,[2] James M. Lattimer,[3] and Andrew W. Steiner[4]
[1]*Instituto de Astronomía, Universidad Nacional Autónoma de México, Mexico D.F. 04510, Mexico*
[2]*Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701-2979, USA*
[3]*Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, New York 11794-3800, USA*
[4]*Joint Institute for Nuclear Astrophysics, National Superconducting Cyclotron Laboratory and, Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*

Phys. Rev. Lett. 106 (2011), p. 08101

# C O N C L U S I O N S

With “exotic” matter in the inner core, and pairing, “everything is compatible with everything”. Good, ... or bad ?

(Requires hypothetical “exotic” matter and hypothetical superfluidity/superconductivity)

Within Minimal Cooling: IF the neutron $^3P_2$ pairing $T_c$ has the right value (~$10^9$ K), all thermally detected young NSs are compatible with minimal cooling, except the NSs in CTA1.

Rapid cooling of Cas A: fits perfectly within minimal cooling and provides first direct observational evidence for superfluidity in the NS core and gives us $T_c$ ~0.5-0.7x$10^9$ K

# WHAT'S WRONG WITH THIS ?

The data: spectacular, but observationally it is a small effect. Could be just aging of the Chandra detector.

There are alternative explanations of Cas A, but they are ad hoc and explain noghing else than Cas A

# WHAT'S WRONG WITH THIS ?

The data: spectacular, but observationally it is a [illegible] effect.
Could be just aging of the Chandra [illegible]

There are alt[illegible]re ad hoc a[illegible]

Maybe

NATURE is finally cooperating

What happens when

# Paris fits in a teaspoon

**it becomes a Superfluid !**

"That's all Folks!"

*Cooling neutron star in the Cassiopeia-A supernova remnant: Evidence for superfluidity in the core*
**Shternin, P. S., Yakovlev, D. G., Heinke, C. O., Ho, W. C. G., & Patnaude, D. J.**
**2011, MNRAS Lett. 412, p. 108**

*Rapid Cooling of the Neutron Star in Cassiopeia-A Triggered by Neutron Superfluidity in Dense Matter*
**Page, D., Prakash, M., Lattimer, J. M., & Steiner, A. W.**
**2011, Phys. Rev. Lett. 106, 081101**

*Stellar Superfluids*
**Page, D., Lattimer, J. M., Prakash, M., & Steiner, A. W.**
**in "Novel Superfluids", Ed. K. H. Bennemann and J. B. Ketterson (Oxford University Press, 2014).**
**arXiv: 1302.6626**

Some reviews on neutron star cooling:

*Neutron Star Cooling*
Yakovlev, D. G.; Pethick, C. J.
2004, Annu. Rev. Astron. & Astrophys. **42**, p. 169

*Dense Matter in Compact Stars: Theoretical Developments and Observational Constraints*
Page, D., Reddy, S.
2006, Annu. Rev. Nucl. & Part. Sci. **56**, p. 327

*The Cooling of Compact Stars*
Page, D., Geppert, U., Weber, F.
2006, Nucl. Phys. **A777**, p. 497