Standard Model and Gravity from Spinors

Left-Right Graviweak Unifications

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with R. Percacci

Outline

1. Problem
   - Unify gravity with other forces?
   - GR is in broken phase
   - Fermion quantum numbers: hint for unification
   - Unifications
2. Botton-Up: Graviweak unification
   - Graviweak: a simple chiral world
   - Extended vierbein for extended group
   - Actions
   - Vierbein Higgs mechanism
   - Particle content
3. Top-down: SM + GR from algebraic spinors
   - Left-Right symmetric algebraic spinors
   - Particle content
   - The Higgs field

Sensible to unify Gravity with other forces?

- Much different energy scales
  (GR: 10^{-20}-10^{-3} eV and Planck; Weak: 10^{-1}-10^{11} eV)
- Different actions
  (GR linear in curvature, GAUGE quadratic)
- GR works well!
  (At low energy)

+ Fermions demand it!
+ GR is a broken gauge theory: can we extend the group?
+ High energy and quantization modified.
+ Emergent metric: new insight on spacetime and scales.
+ Possible direct and indirect observable phenomena
  (new particles, Lorentz violation, exotic decays).

Not the first time this question is posed...

Sensible to unify Gravity with other forces?

Previous investigations, after Einstein '45:

- Gravity as strong interactions for confinement (before QCD...!)
  [Salam, +Isham-Strathdee '65–'72, +Chamseddine '78]
- Complex gravity, matrix gravity (complex vierbeins)
  [Chamseddine '01 – '04]
- Palatini formulation (bispinor vierbeins)
  [Cahill '82, Percacci FN '07]
- Palatini as quantum theory (vector vierbein)
  [Peldan '92, +Chakraborthy '94, Gambini Olson Pullin '04]
- McDowell-Mansouri (wilson line)
  [Wilczek '98, Lisi '07]
- Plebanski formulation (two-form)
  [Smolin '07]
- Algebraic spinors (bispinor)
  [Chisolm Farewell '87, Woit '88, Baylis Trayling '01, FN '07]

Hints from quantum numbers. . .

GR is a SO(1,3) gauge theory, in a broken phase

- Einstein gravity, highly nonpolynomial:

$$L_{EH}(g) = M_P^2 \int \sqrt{g}\, R[\Gamma(g)]\,, \qquad \Gamma_{christoffel} \sim g^{-1}\partial g$$

- Palatini-Cartan: polynomial in vierbein and connection θ_μ^m, ω_μ^{mn}
  With local-Lorentz gauge invariance:

$$L_R(\theta,\omega) = \int \epsilon_{mnrs}\theta^m\theta^n R^{rs}[\omega] = \int \epsilon_{mnrs}\theta^m\theta^n(d\omega^{rs} + \omega^{rt}\omega_t^s)$$

EOMs for a background $\theta_\mu^m = M\, e_\mu^m$:

$\delta\omega$(Torsion=0) : $de + \omega e = 0 \quad \to \omega = \Gamma_{christoffel} \sim e^{-1}de$

δe(Einstein eqs) : $R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R = 0$

- The metric is effective: $g_{\mu\nu} = e_\mu^n e_\nu^n \eta_{mn}$; and at Planck scale the VEV e_μ^m breaks the gauge:

$$SO(1,3)_{local} \times \text{Diff} \to SO(1,3)_{global} \qquad \text{(in minkowsky!)}$$

The vierbein acts as a higgs field for local Lorentz...

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GR is a SO(1,3) gauge theory, in a broken phase

- Vierbein and connection 1-forms of SO(1,3)$\sim$ SL(2,$\mathbb{C}$):

$$\theta^m = M\, e_\mu^m dx^\mu \qquad \omega^m_n = \omega_\mu{}^m{}_n dx^\mu \qquad m,n = 1\ldots 4$$

- Fluctuations and Higgs mechanism:

$$\theta^m_\mu = M(\bar{e}^m_\mu + h^m_\mu) \qquad \text{16 real fluctuations}$$

The *anti*symmetric part $h_{[\mu\nu]} = \bar{e}_{[\mu n} h^n_{\nu]}$ are the 6 goldstones of Lorentz, eaten by ω in unitary gauge:

$$\begin{aligned} \omega &= \Gamma(\bar{e}) + \text{massive fluctuations} \\ h_{[\mu\nu]} &= 0 \quad \text{goldstone (6)} \\ h_{(\mu\nu)} &= \text{massless graviton (10)} \end{aligned}$$

In this form gravity is unification-ready...
...simply extend the group and the vierbein θ.

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Actions

- Cartan actions: (torsion $T^m = D\theta^m$)

$$\mathcal{S} = \int \theta^m \wedge \theta^n \wedge R^{rs}(\omega)\epsilon_{mnrs} + T^m \wedge {}^*T_m + R^{mn} \wedge {}^*R_{mn}$$

…in broken phase, neglecting R^2, T^2 at low energy:

$$\mathcal{S} \to \mathcal{S}_{EH} = M^2 \int e \wedge e \wedge R(e)\, \epsilon = M^2 \int \sqrt{g}\, R(g)$$

- Fermion kinetic term: (volume form in spinor basis)

$$\mathcal{S}_\psi = \int \psi^* \hat\sigma^m d\psi\, \theta^n \theta^r \theta^s \epsilon_{mnrs} = \int \psi^A d\psi^{A'}\, \theta^{BB'}\, \theta^{CC'}\, \theta^{DD'} \epsilon_{(AA')(BB')(CC')(DD')}$$

…in broken phase: $\to M^3 \int |e|\, \psi^* e^\mu_m \hat\sigma^m \partial_\mu \psi = \int \sqrt{g}\, \psi_c^* \partial\!\!\!/ \psi_c$

While spinors do not partecipate in Higgs…
…they reveal the higher group structure.

Hint from Fermion quantum numbers

	SL(2,$\mathbb{C}$) $=$ SO(1,3)	Q $(Y+T_{3L})$	Y $(T_{3R}+\frac{(B-L)}{2})$	T_{3L}	T_{3R}	$B-L$	$SU(3)$	$SU(4)$
u_L	$\mathbf{2}$	$2/3$	$1/6$	$1/2$	0	$1/3$	$\mathbf{3}$	$\mathbf{4}$
d_L	$\mathbf{2}$	$-1/3$	$1/6$	$-1/2$	0	$1/3$	$\mathbf{3}$	
ν_L	$\mathbf{2}$	0	$-1/2$	$1/2$	0	-1	$\mathbf{1}$	
e_L	$\mathbf{2}$	-1	$-1/2$	$-1/2$	0	-1	$\mathbf{1}$	
u_R	$\overline{\mathbf{2}}$	$2/3$	$2/3$	0	$1/2$	$1/3$	$\mathbf{3}$	$\mathbf{4}$
d_R	$\overline{\mathbf{2}}$	$-1/3$	$-1/3$	0	$-1/2$	$1/3$	$\mathbf{3}$	
ν_R	$\overline{\mathbf{2}}$	0	0	0	$1/2$	-1	$\mathbf{1}$	
e_R	$\overline{\mathbf{2}}$	-1	$-1/2$	0	$-1/2$	-1	$\mathbf{1}$	

So let's start from the minimal unification, Pati-Salam. [Pati Salam '74]

$$\mathrm{SL}(2,\mathbb{C})_{lorentz} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$$

$$\psi_L \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) \qquad \psi_R \in (\overline{\mathbf{2}},\mathbf{1},\mathbf{2},\mathbf{4})\,.$$

Then one naturally tries to unify different factors...

Unification schemes

$$\mathrm{SO}(1,3)_{lorentz} \times \mathrm{SU}(2)_L{\times}\mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$$
$$\psi_L \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) \qquad \psi_R \in (\bar{\mathbf{2}},\mathbf{1},\mathbf{2},\mathbf{4})\,.$$

Unify only gauge (GUT):

- SO(1,3)×SO(10): $\psi_L + \psi_R^c \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) + (\mathbf{2},\mathbf{1},\mathbf{2},\bar{\mathbf{4}}) = (\mathbf{2},\mathbf{16})$

Partially with Lorentz:

- SO(1,7)×SU(4): $\psi_L + \psi_R \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) + (\bar{\mathbf{2}},\mathbf{1},\mathbf{2},\mathbf{4}) = (\mathbf{8},\mathbf{4})$
- SO(7,$\mathbb{C}$)×SU(4): $\psi_L + \psi_R \in (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{4}) + (\bar{\mathbf{2}},\mathbf{1},\bar{\mathbf{2}},\mathbf{4}) = (\mathbf{8},\mathbf{4})$

Unifying all?

- SO(1,13) [Cahill '82]
  or SO(13,$\mathbb{C}$): $\mathbf{64}_+ \rightarrow (\mathbf{2},\mathbf{16}) + (\bar{\mathbf{2}},\overline{\mathbf{16}})$ (extra 'mirror' family)

Leave color aside, consider first left spinors; "Graviweak unification":

- SO(4,$\mathbb{C}$)×SU(4): $\psi_L \in (\mathbf{2},\mathbf{2}) \times \mathbf{4} = (\mathbf{4},\mathbf{4})$
- GL(4,$\mathbb{C}$)×SU(4): $\psi_L \in (\mathbf{2},\mathbf{2}) \times \mathbf{4} = (\mathbf{4},\mathbf{4})$

A simple chiral world

- Left fermions are doublets of Lorentz and Isospin

$$\psi_L^{A\alpha} \in (\mathbf{2},\mathbf{2}) \qquad A = 1,2_{(spin)}\ , \quad \alpha = 1,2_{(isospin)}$$

- Extend the isospin to “isolorentz”:

$$\mathrm{SL}(2,\mathbb{C})_{lorentz} \times \mathrm{SL}(2,\mathbb{C})_{weak} = \mathrm{SO}(4,\mathbb{C})$$

$$\underset{e^{i(\theta^i+i\nu^i)\sigma_i}}{[\text{Spin + Boosts}]} \qquad \underset{e^{i(\alpha^i+i\beta^i)\sigma_i}}{[\text{Isospin + “Isoboosts”}]}$$

- Now ψ_L is a (complex) vector of SO(4,$\mathbb{C}$):

$$\psi_L^{A\alpha} \in (\mathbf{2},\mathbf{2}) \sim \psi_L^a \in \mathbf{4}_C\ , \qquad \text{via } \hat\sigma^{A\alpha}_{a=1\ldots4}\ .$$

Gauge theory of SO(4,$\mathbb{C}$)...

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Gauge theory of SO(4,$\mathbb{C}$)...

Graviweak unification

$$SO(4,\mathbb{C}) \equiv SL(2,\mathbb{C}) \times SL(2,\mathbb{C})$$

Self-dual factors: one used for lorentz, the other for isospin.

- Connection and curvature of $SO(4,\mathbb{C})$:

$$A^a_b = \omega^i_L(\sigma_i \otimes \mathbf{1}_2) + (W^i_L + iK^i_L)(\mathbf{1}_2 \otimes \sigma_i)$$

$$R^a_b = (dA + A \wedge A) = R^i_{\omega_L}(\sigma_i \otimes \mathbf{1}_2) + R^i_{W_L+iK_L}(\mathbf{1}_2 \otimes \sigma_i)$$

- The fermion bilinears dictate that the vierbein is a bivector:

$$\psi^{\bar{a}}\partial_\mu\psi^a \quad \rightarrow \quad \theta^{\bar{a}a}_\mu \in \mathbf{16}_R \qquad \text{16 real components}$$

As a matrix in bi-spinor basis: (via $\hat{\sigma}^{A'A}_{m=1\ldots4}$ and $\hat{\sigma}^{\alpha'\alpha}_{u=1\ldots4}$)

$$\theta^{\bar{a}a} \sim \theta^{A'\alpha'A\alpha}_\mu \sim \theta^{mu}_\mu(\hat{\sigma}_m \otimes \hat{\sigma}_u)$$

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Breaking

- Now we see the right VEV - in the 'timelike' isospin-direction:

$$\langle\theta_\mu\rangle = M\,\bar{e}_\mu^m(\hat{\sigma}_m \otimes \mathbf{1}_2) \qquad \langle\theta_\mu^{mu}\rangle = M\,\bar{e}_\mu^m\delta^{u4}$$

- Breaks Diff, local Lorentz and 'isoboosts'; but $\mathbf{1}_2$ preserves the compact part, i.e. standard weak interactions:

$$Diff \times \mathrm{SO}(4,\mathbb{C}) \;\rightarrow\; \mathrm{SU}(2)_L\,.$$

Local Lorentz is broken as in Palatini-Cartan gravity.

- Standard global $\mathrm{SO}(1{,}3)_{lorentz}$ appears when $\bar{e}_\mu^m$ is minkowski.

A single VEV $\hat{\sigma}_m \otimes \mathbf{1}_2$ gives the correct breaking.

Actions. . . ?

Actions I: The dual

First we need the epsilon, extending the Lorentz one:

$$\epsilon_{mnrs} \sim \epsilon_{(A'A)(B'B)(C'C)(D'D)} \;\longrightarrow\; \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} = ?$$

- An SO(4,$\mathbb{C}$) invariant dual is:

$$\epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} \sim \epsilon_{(mu)(nv)(rw)(sz)} =$$
$$= \epsilon_{mnrs}(\eta_{uv}\eta_{wz} + \eta_{uw}\eta_{vz} + \eta_{uz}\eta_{vw}) + (\eta_{mn}\eta_{rs} + \eta_{mr}\eta_{ns} + \eta_{ms}\eta_{nr})\epsilon_{uvwz}$$

  this 4-index antisymmetric tensor in 16 dimensions is inherited from the duals of SL(2,$\mathbb{C}$)×SL(2,$\mathbb{C}$), in a symmetric fashion.

- For larger nontrivial groups it has to be provided as a new field ϕ_{MNRS} (like Plebanski BF models).

Actions II: Fermions

Then one can start from the fermions:

$$\mathcal{S}_\psi = \int \psi_L^{*\,\bar a} D\psi_L^a \wedge \theta^{\bar b b} \wedge \theta^{\bar c c} \wedge \theta^{\bar d d} \epsilon_{(\bar a a)(\bar b b)(\bar c c)(\bar d d)}$$

- When $\theta = \bar{e}^m(\hat\sigma_m \otimes \mathbf{1}_2)$ this correctly gives the $\mathrm{SU}(2)_L$ action:

$$\begin{aligned} \mathcal{S}_\psi \quad &\to \quad \int \psi_L^{*\,A'\alpha'}\, \mathcal{D}\psi_L^{A\alpha} \hat\sigma^m_{A'A} \delta_{\alpha'\alpha} \wedge e^n \wedge e^r \wedge e^s \epsilon_{mnrs} \\ &= \quad \int d^4x\, |e|\, e^\mu_m \psi_L^{*\,\alpha} \hat\sigma^m \mathcal{D}_\mu \psi_L^\alpha\,, \end{aligned}$$

- In the covariant derivative only the low-energy gauge fields should appear: spin connection and W's

$$\mathcal{D} = d + \omega_L(\bar e) + W_L\,.$$

Actions III: Gauge+Gravity

First-order actions for the gauge part: $(T^{\bar{a}a} = D\theta^{\bar{a}a})$

$$\mathcal{S}_R = \frac{g_1}{16\pi} \int R^{\bar{a}a\,\bar{b}b} \wedge \theta^{\bar{c}c} \wedge \theta^{\bar{d}d}\, \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} \qquad \sim \int R$$

$$\mathcal{S}_{T^2} = a_1 \int \left[t_{\bar{e}e}^{\bar{a}a\,\bar{b}b}\, T^{\bar{e}e} + (t^2)\, \theta^{\bar{a}a} \wedge \theta^{\bar{b}b} \right] \wedge \theta^{\bar{c}c} \wedge \theta^{\bar{d}d} \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} \sim \int T^2$$

- Equations of motion around the VEV $\theta = \bar{e}(\hat{\sigma} \otimes \mathbf{1}_2)$:

  $\delta\omega$ - Zero classical torsion: $\omega =$ Christoffel$(\bar{e})$;

  $\delta(W + iK)$ - Zero isoboost connection: $K = 0$;

  $\delta\theta$ - Einstein equations (in vacuum here)

- Insertion of the VEV is also instructive:

$$\mathcal{S}_R + \mathcal{S}_{T^2} \to \int d^4x \sqrt{g} \left[\frac{g_1}{16\pi} M^2 R + 4a_1 M^2 \left(T^m_{\mu\nu}\, T^{\mu\nu}_m + 10\, K^j_\mu\, K^\mu_j \right) \right].$$

...i.e. torsion is zero and 'isoboosts' K^j have Planck mass.

Actions III: Gauge+Gravity

- Cosmological constant extends simply:

$$\mathcal{S}_\Lambda = \lambda \int \theta^{\bar{a}a} \wedge \theta^{\bar{b}b} \wedge \theta^{\bar{c}c} \wedge \theta^{\bar{d}d} \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)} \,.$$

- Then, we need terms quadratic in curvature, $\sim \int R^2_{\mu\nu}$:

$$\mathcal{S}_{R^2} = \frac{1}{g_2^2} \int \left[r^{\bar{a}a\,\bar{b}b}_{\bar{e}e\,\bar{f}f}\, R^{\bar{e}e\,\bar{f}f} + (r^2)\, \theta^{\bar{a}a} \wedge \theta^{\bar{b}b} \right] \wedge \theta^{\bar{c}c} \wedge \theta^{\bar{d}d} \epsilon_{(\bar{a}a)(\bar{b}b)(\bar{c}c)(\bar{d}d)}$$

$$\mathcal{S}_{R^2} \to \int d^4x \sqrt{g}\, \frac{1}{g_2^2} \left(R^2_{\mu\nu} + W^2_{\mu\nu} + K^2_{\mu\nu} \right)$$

…weak & gravitational quadratic-curvature terms unified at M.

Goldstone counting…

Higgs mechanism?

In addition to the 6 complex gauge fields of SO(4,$\mathbb{C}$):

$$A_L = \omega^i_L(\sigma_i \otimes \mathbf{1}_2) + (W^i_L + iK^i_L)(\mathbf{1}_2 \otimes \sigma_i)\,, \qquad (i = 1,2,3)$$

we can decompose the 4×16=64 fluctuations of θ:

$$\theta_\mu = M(\bar{e}^m_\mu + \underset{[16]}{h^m_\mu})(\hat{\sigma}_m \otimes \mathbf{1}_2) + \underset{[48]}{\Delta^{mi}_\mu}(\hat{\sigma}_m \otimes \sigma_i)\,.$$

- $h_{[\mu\nu]}$ are the goldstones of lorentz - eaten by ω^i
- $h_{(\mu\nu)}$ is the graviton [10]
- $\Delta^{i\mu}_{\mu}$ goldstones of isoboosts - eaten by K^i (if no other field.)
- $\Delta^i_{[\mu\nu]}$ nondynamical [similar to Chamseddine '03]
- $\Delta^i_{(\mu\nu)}$ a new traceless spin-two, isospin-triplet [3×9]

At low energy we have an additional graviton, isospin-triplet!

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Isospin-triplet graviton - phenomenology

The triplet graviton is $(\Delta^+_{L\mu\nu}, \Delta^0_{L\mu\nu}, \Delta^-_{L\mu\nu})$. Observable?

- It couples to (left) matter, but its coupling is Planck-suppressed. However:
- It is charged under $\mathrm{SU}(2)_L$ - it interacts with the W's...
  If it is light ($\sim$TeV) it will be seen at LHC!?
  - Pairwise production: $qq \to W^+ \to \Delta^+ \Delta^0$

    Δ^+ is charged and visible. Decays through the mass difference:
    $\to$ long-lived... displaced decay... $\sim$ cm!

    (Later about its mass with a nonchiral model.)
- It would be a first manifestation of gravity at accelerator energies!
- Not only: it's a weakly interacting massive particle: dark matter? (abundance, signals, ...)

To conclude, graviweak extensions of gravity are possible...
...and may lead also to interesting phenomenology!

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Algebraic spinors [Cartan'37, Kæhler '62, Graf '78]

- In addition to Dirac's $\partial\!\!\!/^2 = \Box$, there is an other well known 'square root' of the laplacian:

$$(d+\delta)^2 = \Box$$

- Like $\partial\!\!\!/$ acts on (and defines) Dirac spinors, $(d+\delta)$ acts on inhomogeneous differential forms $\Lambda = \oplus\Lambda^k$:

$$\Psi = \psi_{.} + \psi_\mu dx^\mu + \psi_{\mu\nu} dx^{\mu\nu} + \psi_{\mu\nu\rho} dx^{\mu\nu\rho} + \psi_{\mu\nu\rho\sigma} dx^{\mu\nu\rho\sigma}\,.$$

- Λ splits into 4 subspaces (ideals), each representing a Dirac spinor. In practice four columns of a 4×4 matrix:

$$\Psi = \psi_{.} + \psi_\mu \gamma^\mu + \psi_{\mu\nu} \gamma^{\mu\nu} + \psi_{\mu\nu\rho} \gamma^{\mu\nu\rho} + \psi_{\mu\nu\rho\sigma} \gamma^{\mu\nu\rho\sigma}$$

$$= \begin{pmatrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \\ \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} \end{pmatrix}.$$

Left-Right Algebraic spinors

Ψ contains 4 Dirac spinors, e.g.

$$\Psi = \begin{pmatrix} \psi_{L1} & \psi_{L2} & \psi_{L3} & \psi_{L4} \\ \psi_{L1} & \psi_{L2} & \psi_{L3} & \psi_{L4} \\ \psi_{R1} & \psi_{R2} & \psi_{R3} & \psi_{R4} \\ \psi_{R1} & \psi_{R2} & \psi_{R3} & \psi_{R4} \end{pmatrix}$$

- General transformations of Ψ are of $\mathrm{GL}(4,\mathbb{C}) \times \widetilde{\mathrm{GL}(4,\mathbb{C})}$

$$\Psi \to \epsilon^{\Lambda} \Psi \epsilon^{\tilde{\Lambda}} .$$

Λ (left-side) actually contain Lorentz: $\mathrm{GL}(4,\mathbb{C}) \supset \mathrm{SO}(1,3)$.
$\tilde{\Lambda}$ (right-side) are an internal symmetry, e.g. $\widetilde{\mathrm{GL}(4,\mathbb{C})} \supset \mathrm{SU}(4)$.

- However this spinor is too small for the SM family (that needs 8 Dirac spinors).

Need to change approach to chirality: LR-symmetry. . .

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Left-Right Algebraic spinors

Let's use Left-Right symmetric algebraic spinors. Then each column can accomodate up/down fermions, i.e. a graviweak "vector".
A Standard-Model family is accomodated suggestively:

$$\Psi_L = \left(\begin{array}{c|c|c|c} \nu_{\rm L} & u_{{\rm L},r} & u_{{\rm L},g} & u_{{\rm L},b} \\ \nu_{\rm L} & u_{{\rm L},r} & u_{{\rm L},g} & u_{{\rm L},b} \\ \hline e_{\rm L} & d_{{\rm L},r} & d_{{\rm L},g} & d_{{\rm L},b} \\ e_{\rm L} & d_{{\rm L},r} & d_{{\rm L},g} & d_{{\rm L},b} \end{array}\right), \quad \Psi_R = \left(\begin{array}{c|c|c|c} \nu_{\rm R1} & u_{{\rm R},r} & u_{{\rm R1},g} & u_{{\rm R},b} \\ \nu_{\rm R} & u_{{\rm R},r} & u_{{\rm R},g} & u_{{\rm R},b} \\ \hline e_{\rm R} & d_{{\rm R},r} & d_{{\rm R},g} & d_{{\rm R},b} \\ e_{\rm R} & d_{{\rm R},r} & d_{{\rm R},g} & d_{{\rm R},b} \end{array}\right).$$

- Transformations of $\Psi_{L,R}$ are $\Psi_{L,R} \to \epsilon^{\Lambda_{L,R}} \Psi_{L,R} \epsilon^{\tilde\Lambda_{L,R}}$:

$$\mathrm{GL}(4,\mathbb{C})_L \times \mathrm{GL}(4,\mathbb{C})_R \times \widetilde{\mathrm{GL}(4,\mathbb{C})}_L \times \widetilde{\mathrm{GL}(4,\mathbb{C})}_R$$

- $\mathrm{GL}(4,\mathbb{C}) \supset \mathrm{SO}(4,\mathbb{C})$: Graviweak$_{L,R}$, $\widetilde{\mathrm{GL}(4,\mathbb{C})} \supset \mathrm{SU}(4)$:Color$_{L,R}$.

...a Pati-Salam group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)$ is emerging!

Let's try to gauge all...

Extended vierbeins again

We are gauging *separately* the Left and Right GL(4,$\mathbb{C}$) groups:

- One vierbein for each group:

$$\theta_{L,R} = \theta^{mu}_{L,R}(\hat{\sigma}_m \otimes \hat{\sigma}_u) \qquad \text{(16 real components each)}$$

- Separate VEVs: (Aligned! $\bar{e}^m_L = \eta^{mm}\bar{e}^m_R$)

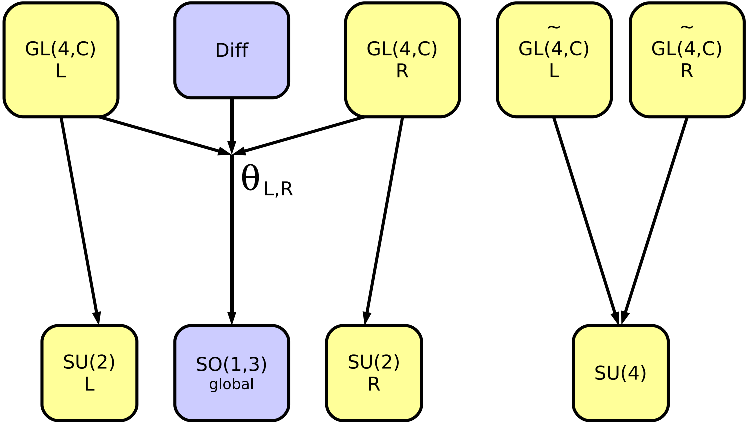
$$\theta_L = \bar{e}^m_L(\hat{\sigma}_m \otimes \mathbf{1}_2) \qquad \theta_R = \bar{e}^m_R(\hat{\sigma}_m \otimes \mathbf{1}_2)$$

This breaks $\text{GL}(4,\mathbb{C})_L \times \text{GL}(4,\mathbb{C})_R \to \text{SU}(2)_L \times \text{SU}(2)_R$.

- Since Diff is unique $\to$ unique global Lorentz symmetry.
- In curved space $\to$ conjugate spin connects.: $\omega_L = \bar{\omega}_R$ (!).

Again the L, R weak-isospin groups remain.

Breaking pattern



What about low energy states?

States

Connections and vierbeins of $\mathrm{GL}(4,\mathbb{C})_L \times \mathrm{GL}(4,\mathbb{C})_R$:

$$A_L = \omega^i_L(\sigma_i \otimes \mathbf{1}_2) + (W_L + iK_L)^i(\mathbf{1}_2 \otimes \sigma_i) + X^{ij}_L(\sigma_i \times \sigma_j)$$

$$\theta_{L\,\mu} = (\bar{e} + h)^m_\mu(\hat{\sigma}_m \otimes \mathbf{1}_2) + \Delta^{mi}_\mu(\hat{\sigma}_m \otimes \sigma_i)$$

(and same for R).

In the broken phase, in unitary gauge:

- $h_{L,R\,[\mu\nu]}$ eaten by $\omega^i_{L,R}$ [6+6];
- $\Delta^i_{L,R}{}^\mu_\mu$ eaten by $K^i_{L,R}$ [3+3];
- $\Delta^i_{L,R\,[\mu\nu]}$ eaten by $X^{ij}_{L,R}$ [18+18];

- $h_{L\,(\mu\nu)}$, $h_{R\,(\mu\nu)}$ gravitons, interacting with L/R matter [10+10];
- $\Delta^i_{L\,(\mu\nu)}$, $\Delta^i_{R\,(\mu\nu)}$: L/R isospin-triplet traceless gravitons: [27+27].

Gravitons

We have two singlet-gravitons h_L, h_R, two triplet-gravitons Δ_L, Δ_R.

- Triplet gravitons as before:
  Δ_L, Δ_R with different masses, linked to the scales of L,R breakings.
- Singlet gravitons:

$$h_+ = h_L + h_R \qquad h_- = h_L - h_R \qquad \text{parity even/odd}$$

- h_+ is a standard graviton, massless by linearized Diffs.
  It will couple equally to L,R matter.
- h_- is not protected by Diffs and may be massive ($> 10^{-3}$eV).
  Its mass, linked to the scale of L-R coupling, may be low.
  If low enough, it will bring parity breaking in gravitational waves or polarization effects (e.g. [Contaldi et al '08]).
  On the other hand matter is mostly unpolarized: h_- hidden.

Finally, one would like massive fermions - guess the higgs field

The Higgs field

Any isospin-doublet would also be a lorentz-doublet, i.e. a spinor.

. . . how to find a *scalar* doublet?

Under $GL(4,\mathbb{C})_L \times GL(4,\mathbb{C})_R \to SO(1,3)_{global} \times SU(2)_L \times SU(2)_R$

- The Higgs bidoublet ϕ is in a link field

$$H_{LR} \in (\mathbf{4}_L, \mathbf{4}_R) \to (\mathbf{1}_\ell + \mathbf{3}_\ell, \mathbf{2}_L, \mathbf{2}_R)$$

- Couplings L-R are restricted; defining $\hat{\theta}_L = H_{LR}\theta_R H^\dagger_{LR}$:

$$\begin{aligned}
L_0 &= \lambda_0 \theta_L\theta_L\theta_L\theta_L\epsilon_L &\sim\quad& (\lambda_0 M^2 + \lambda_1\phi^2)[M^2 + \Delta^2_{\mu\nu}] \quad \to \text{C.C.}\\
L_1 &= \lambda_1 \theta_L\theta_L\theta_L\hat{\theta}_L\epsilon_L &\sim\quad& \lambda_1[\phi\Delta_{\mu\nu}]^2 \quad \to m^2_\Delta \sim v^2?\\
L_2 &= \lambda_2 \theta_L\theta_L\hat{\theta}_L\hat{\theta}_L\epsilon_L &\sim\quad& \lambda_2\phi^4[1 + \Delta^2_{\mu\nu}/M^2] \to \text{Quartic...}\\
L_3 &= \cdots
\end{aligned}$$

Work in progress. . . Some predictivity expected!

$\mathrm{GL}(4,\mathbb{C})^4$ Breakings

- Fields . . . (under $\mathrm{GL4}_L\times\mathrm{GL4}_R\times\mathrm{GL4}_{\tilde L}\times\mathrm{GL4}_{\tilde R}\to G_{\ell 224}$)
  $\Psi_L \in (\mathbf{4}_L,\mathbf{4}_{\tilde L})$
  $H_{LR} \in (\mathbf{4}_L,\mathbf{4}_R) \to (\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})+(\mathbf{3},\mathbf{2},\mathbf{2},\mathbf{1})$
  $\Sigma_{\tilde L\tilde R} \in (\mathbf{4}_{\tilde R},\mathbf{4}_{\tilde L}) \to (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})+\cdots$
  $\Delta^R_{R\tilde R\tilde R} \in (\mathbf{16}^R_R,\mathbf{10}_{\tilde R\tilde R}) \to (\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{10})+\cdots$
  $\Delta^{R\tilde R}_{R\tilde R} \in (\mathbf{16}^R_R,\mathbf{16}^{\tilde R}_{\tilde R}) \to (\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{15})+\cdots$
- Breaking chain to the SM:
  $\langle\Sigma_{RL}\rangle = \alpha+\beta P(1)$ breaks $\mathrm{U}(4)_L\times\mathrm{U}(4)_R \to \mathrm{SU}(3)_{color}\times\mathrm{U}(1)_{B-L}$;
  $\langle\Delta\text{'s}\rangle = (\cdots)$ break $\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}\to\mathrm{U}(1)_Y$;
  $\langle H_{LR}\rangle = \alpha+\beta\gamma_5$ breaks finally $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\to\mathrm{U}(1)_{em}$.
- Kinetic terms:

$$\mathcal{L} = \mathrm{tr}\left\{\Psi_L^\dagger(\theta_L\theta_L\theta_L\epsilon)D\Psi_L\right\} \;+\; (L\leftrightarrow R)$$

- A mass term for fermions may be written as:

$$\mathcal{L}_{mass} \sim \mathrm{tr}\left\{\Psi_L^\dagger H_{LR}\Delta_R\Psi_R\Sigma_{RL}\right\}$$

Outlook

- Natural to unify lorentz with gauge interactions.
- Graviweak unification possible.
- Extra isospin-triplet gravitons likely to appear (adding to the dream-list for LHC!)
- In Left-Right setup a parity-odd graviton (with possible parity-breaking effects)
- Algebraic spinors in LR way give a Standard Model family;
- Suggests gauging copies of GL(4,$\mathbb{C}$) for graviweak and color: (a geometrical way to Pati-Salam)

Then:

- L, R scales and Higgs fields. . . (Model building)
- e_L, e_R - Two metrics. . . (Classical and quantum)
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- Thanks!

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