AN ALTERNATIVE TO THE ΛCDM MODEL: THE CASE OF SCALE INVARIANCE

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The laws of Physics are generally not unchanged under a change of scale (Feynman, 1963). Material content.

Even the vacuum at quantum level has some units ℓ, t

But, the empty space at large scales has no preferred scales

HYPOTHESIS: EMPTY SPACE IS SCALE INVARIANT AT LARGE

In the same way as we may use Eisntein's theory at large scales, even if we do not have a quantum theory of gravitation, we may consider that the large scale empty space is scale invariant even if this is not true at the quantum level.

This hypothesis implies some properties for Λ

Scale invariance has attracted great attention: Weyl, 1923; Eddington, 1923; Dirac, 1973; Canuto et al. 1977.

Maxwell equations of electrodynamics are scale invariant.

New interest: it generates an acceleration of the expansion consistent with observations.

A scale transformation GR $ds' = \lambda(x^{\mu})\, ds$ Sc. inv.

$$g'_{\mu\nu} = \lambda^2\, g_{\mu\nu}$$

Scale invariance enlarges the group of invariances of GR

Maxwell equation are scale invariant.

COTENSOR ANALYSIS (Weyl 1923, Eddington 1923, Dirac 1973, Canuto et al. 1977). Covariant derivatives, modified Christoffel symbols, Riemann-Christoffel tensor, Ricci tensor, …

A theory of insuperable beauty (Dirac 1973)

Scale invariant field equation

$$R'_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R' - \kappa_{\mu;\nu} - \kappa_{\nu;\mu} - 2\kappa_{\mu}\kappa_{\nu} + 2 g_{\mu\nu}\kappa^{\alpha}_{;\alpha} - g_{\mu\nu}\kappa^{\alpha}\kappa_{\alpha} = -8\pi G T_{\mu\nu} - \lambda^2 \Lambda_{\mathrm{E}}\, g_{\mu\nu}$$

GR Additional terms

$$\kappa_{\nu} = -\frac{\partial}{\partial x^{\nu}} \ln \lambda$$

PROPERTIES OF THE EMPTY SPACE

NEW !

HYPOTHESIS:
EMPTY SPACE IS SCALE INVARIANT AT LARGE

$$R'_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R' - \kappa_{\mu;\nu} - \kappa_{\nu;\mu} - 2\kappa_{\mu}\kappa_{\nu} + 2 g_{\mu\nu}\kappa^{\alpha}_{;\alpha} - g_{\mu\nu}\kappa^{\alpha}\kappa_{\alpha} = -8\pi G T_{\mu\nu} - \lambda^2 \Lambda_{\mathrm{E}}\, g_{\mu\nu}$$

GR

Additional terms

$$\kappa_{\nu} = -\frac{\partial}{\partial x^{\nu}} \ln \lambda$$

This is what remains in empty space

$$\kappa_0 = -\dot{\lambda}/\lambda$$

$$\kappa_{\mu;\nu} = \kappa_{0;0} = \partial_0 \kappa_0 = \frac{d\kappa_0}{dt} \equiv \dot{\kappa}_0$$

$$3\,\frac{\dot{\lambda}^2}{\lambda^2} = \lambda^2\,\Lambda_{\mathrm{E}} \quad \text{and} \quad 2\,\frac{\ddot{\lambda}}{\lambda} - \frac{\dot{\lambda}^2}{\lambda^2} = \lambda^2\,\Lambda_{\mathrm{E}}$$

→ Scale factor $\lambda \sim 1/t$

$$\dot{\lambda}/\lambda = -\frac{1}{t}$$

- Bondi remark (Bertotti et al. 1990): *«Einstein's disenchantment with the cosmological constant was partially motivated by a desire to preserve scale invariance of the empty space.»*

In General Relativity, empty space with Λ_E is not scale invariant.

The scale invariant framework conciliates Λ_E and the scale invariance of the empty space.

SCALE INVARIANT COSMOLOGICAL MODELS

The scale invariant field equation with the R-W metric
→ cosmological equations (Canuto et al. 1977)

$$\frac{8\,\pi G \varrho}{3} = \frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + 2\,\frac{\dot{\lambda}\,\dot{R}}{\lambda\,R} + \frac{\dot{\lambda}^2}{\lambda^2} - \frac{\Lambda_{\rm E}\lambda^2}{3}$$

$$-8\,\pi G p = \frac{k}{R^2} + 2\frac{\ddot{R}}{R} + 2\frac{\ddot{\lambda}}{\lambda} + \frac{\dot{R}}{R}^2 + 4\frac{\dot{R}\,\dot{\lambda}}{R\,\lambda} - \frac{\dot{\lambda}^2}{\lambda^2} - \Lambda_{\rm E}\,\lambda^2$$

<u>In General Relativity: gravitation couples universally to all energy and momentum, thus we apply</u>

$$3\,\frac{\dot{\lambda}^2}{\lambda^2} = \lambda^2\,\Lambda_{\rm E}$$

$$2\,\frac{\ddot{\lambda}}{\lambda} - \frac{\dot{\lambda}^2}{\lambda^2} = \lambda^2\,\Lambda_{\rm E}$$

→

$$\frac{8\,\pi G \varrho}{3} = \frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + 2\,\frac{\dot{R}\dot{\lambda}}{R\lambda}\,, \qquad \text{I}$$

$$-8\,\pi G p = \frac{k}{R^2} + 2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + 4\frac{\dot{R}\dot{\lambda}}{R\lambda} \qquad \text{II}$$

If $\lambda = \text{const.}$ → usual equations. The effects that do not depend on time evolution are the same as in G R.

The two equations →

III $$-\frac{4\pi G}{3}(3p+\varrho)=\frac{\ddot{R}}{R}+\frac{\dot{R}\dot{\lambda}}{R\lambda}$$ $$-\frac{H}{t}$$

- The additional term is an acceleration that opposes to gravity.
- Acceleration variable in time. **No need of unknown particles.**

Empty space: R ~ t^2

DENSITIES:

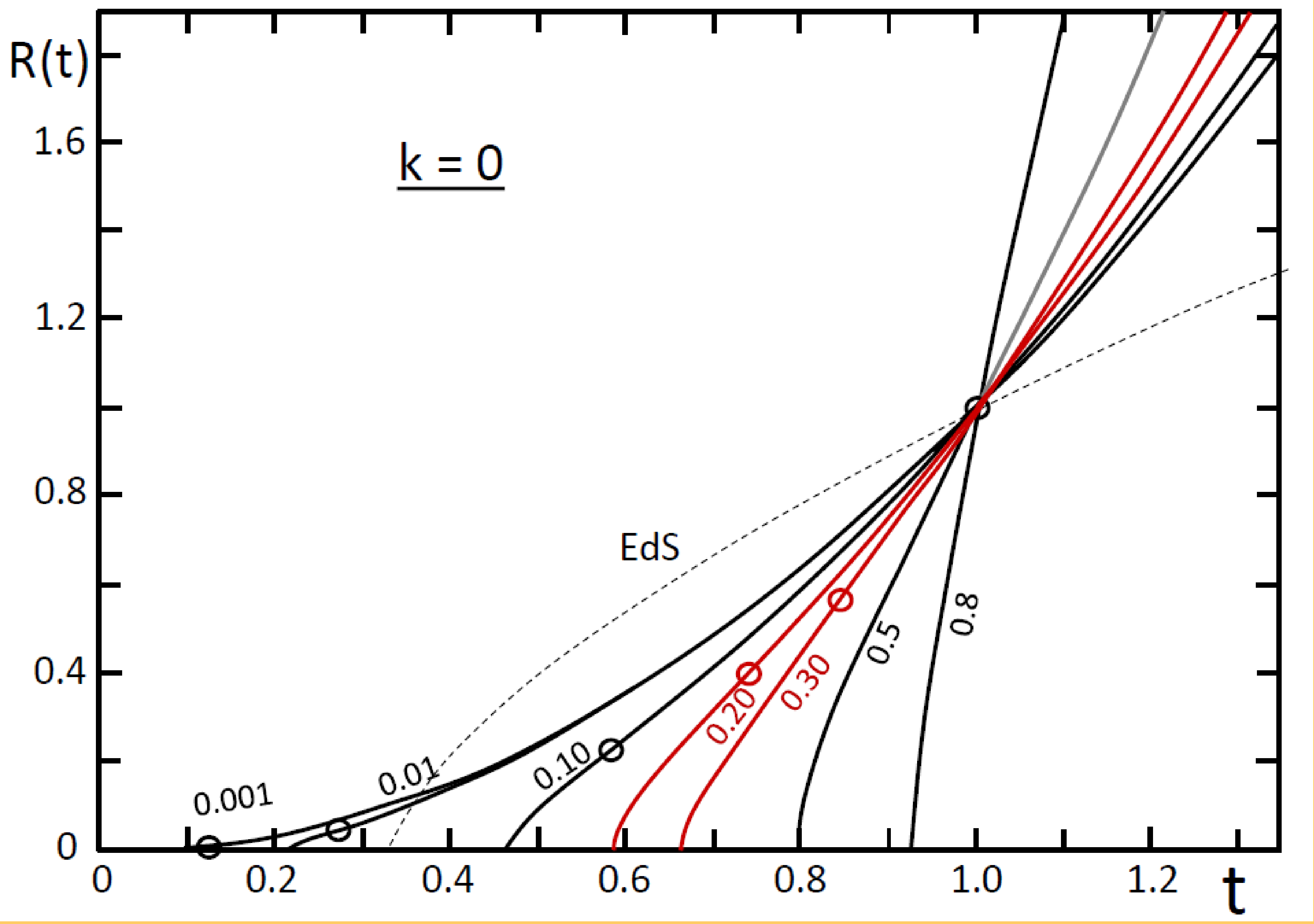
$$\Omega_{\mathrm{m}}+\Omega_{\mathrm{k}}+\Omega_{\lambda}=1$$

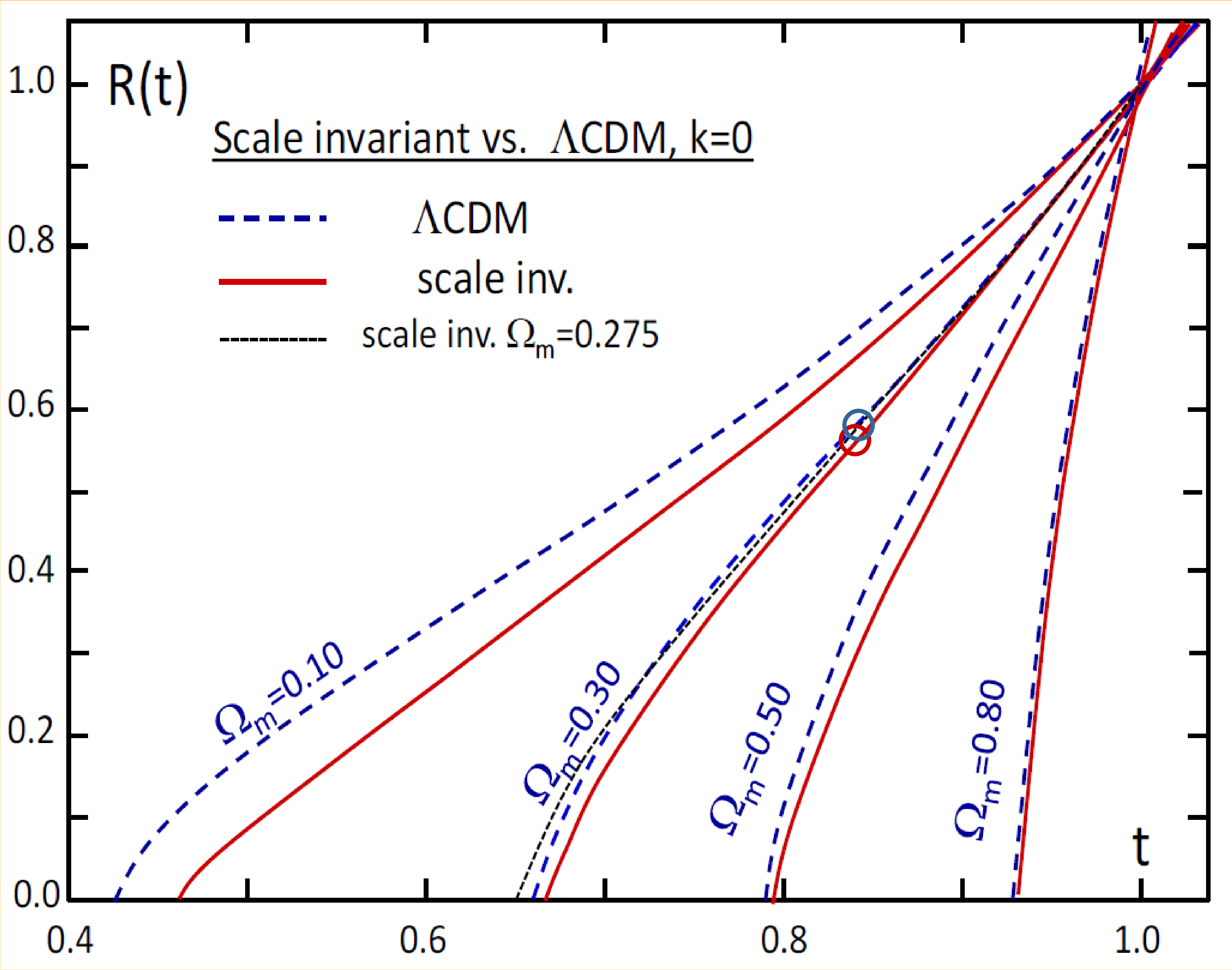
$$\Omega_{\lambda}\equiv\frac{2}{H\,t}$$

Major physical difference with ΛCDM: Ω_{λ} expresses the energy-density from the variations of the scale factor (**acceleration**)

Solving the equation

$$\frac{8\pi G\varrho}{3} = \frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + 2\,\frac{\dot{R}\dot{\lambda}}{R\lambda}\,,$$





R(t)
Scale invariant vs. ΛCDM, k=0
ΛCDM
scale inv.
scale inv. Ω_m=0.275
Ω_m=0.10
Ω_m=0.30
Ω_m=0.50
Ω_m=0.80
t
1.0
0.8
0.6
0.4
0.2
0.0
0.4
0.6
0.8
1.0

Geometry:

$$q_0 = \frac{\Omega_\mathrm{m}}{2} - \frac{\Omega_\lambda}{2}$$

q = 0 for $\Omega_m = \Omega_\lambda$

$$q_0 = \frac{1}{2}\Omega_\mathrm{m} - \Omega_\Lambda$$

For ΛCDM

NEW INVARIANCE → conservation laws

$$\frac{d(\varrho R^3)}{dR} + 3\,pR^2 + (\varrho + 3\,p)\frac{R^3}{\lambda}\frac{d\lambda}{dR} = 0\,.$$

Integral

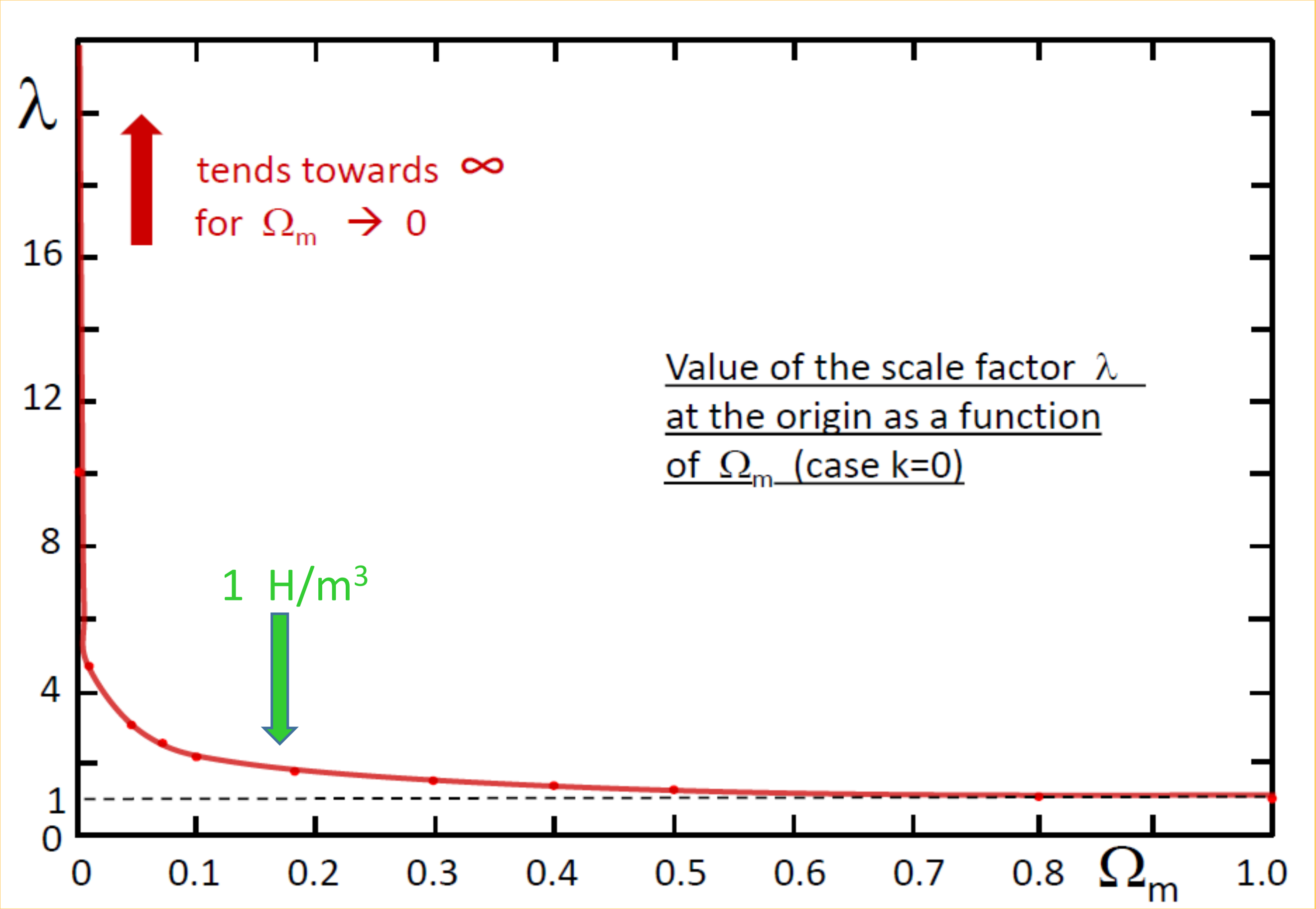
$$\varrho\, R^{3(w+1)}\, \lambda^{(3w+1)} = const$$

The curvature of space-time is slightly changing with time.

For $\Omega_m = 0.3$, the variations of λ are very small.

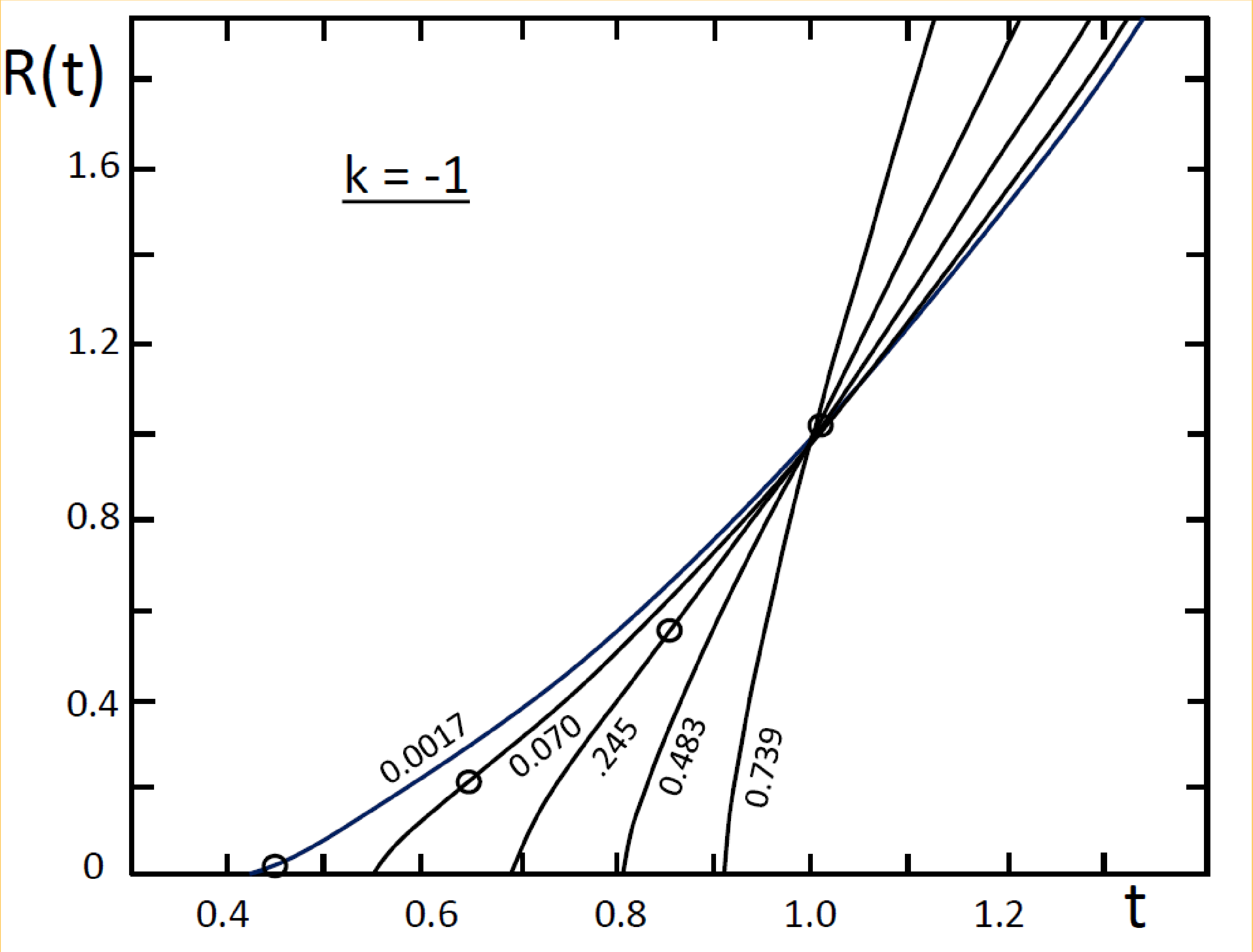
For $\Omega_m > 1$, $\lambda = $ const.

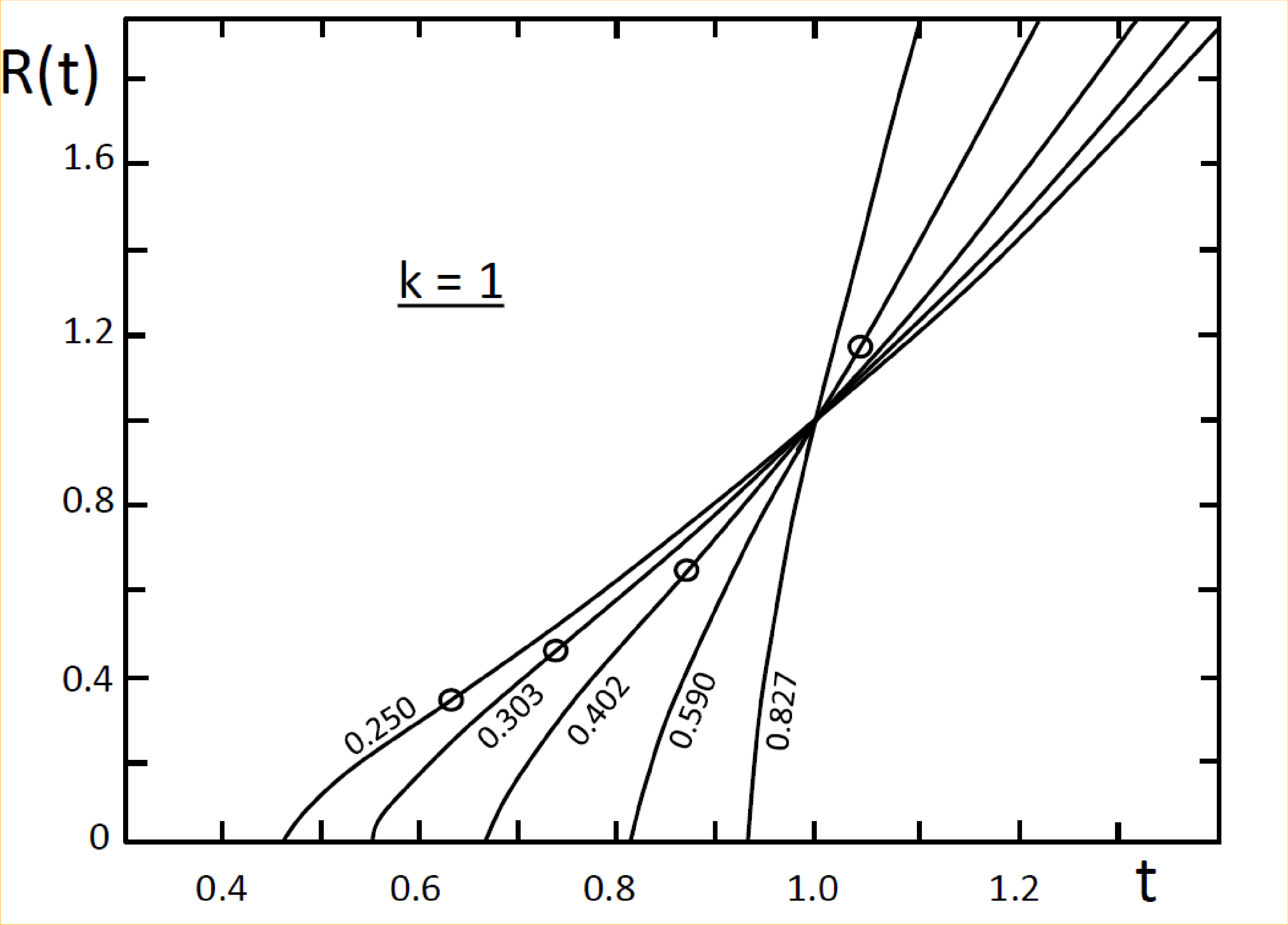
Amplitude of the variations of λ



A non-zero matter density tends to kills scale invariance.

At Ω_m = 0.3, the effect is not yet completely killed.


R(t)
k = -1
1.6
1.2
0.8
0.4
0
0.0017
0.070
.245
0.483
0.739
0.4
0.6
0.8
1.0
1.2
t


R(t)
1.6
1.2
0.8
0.4
0
k = 1
0.250
0.303
0.402
0.590
0.827
0.4
0.6
0.8
1.0
1.2
t

OBSERVATIONAL PROPERTIES

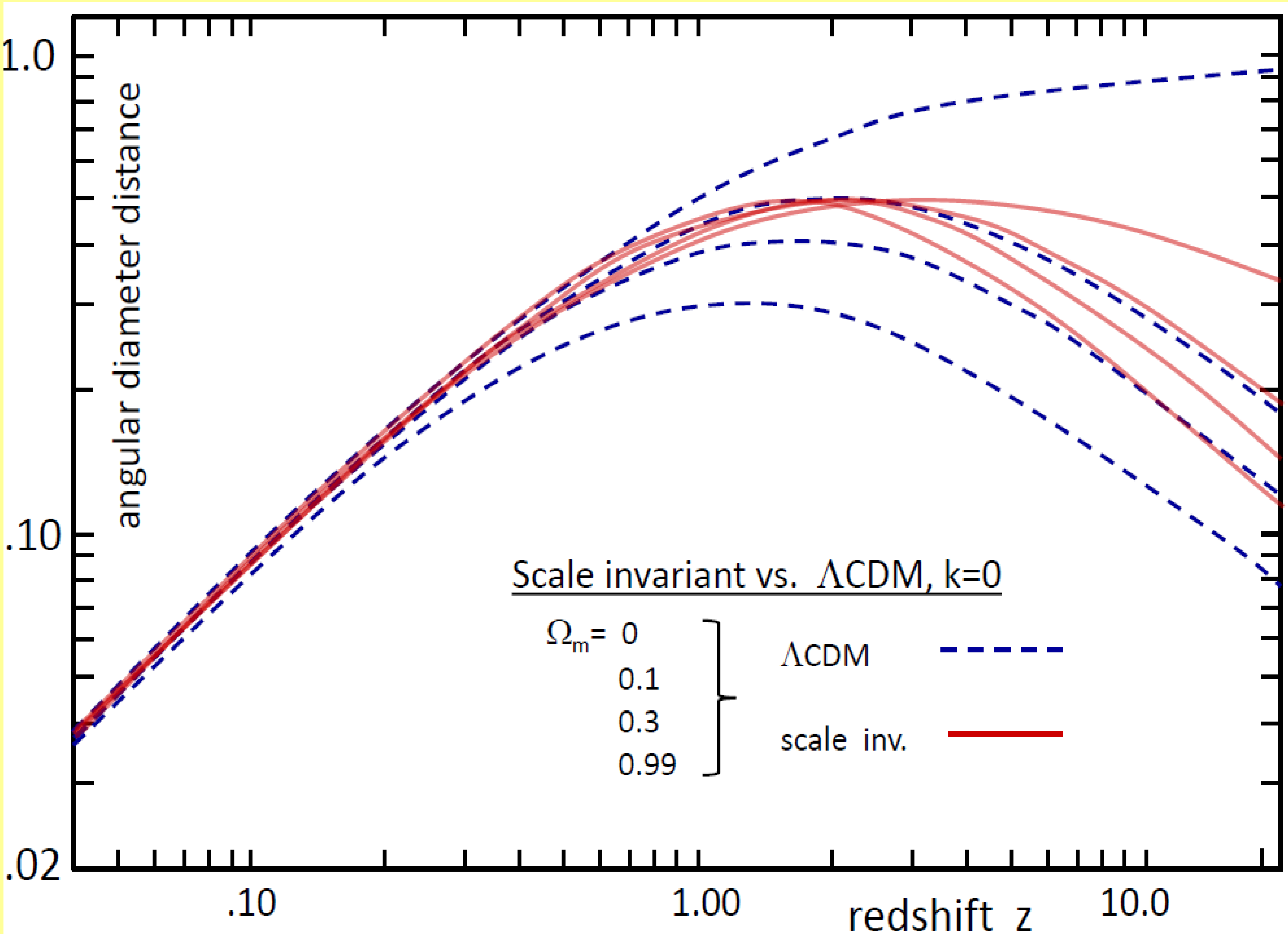
Distances
m – z diagram

Distances

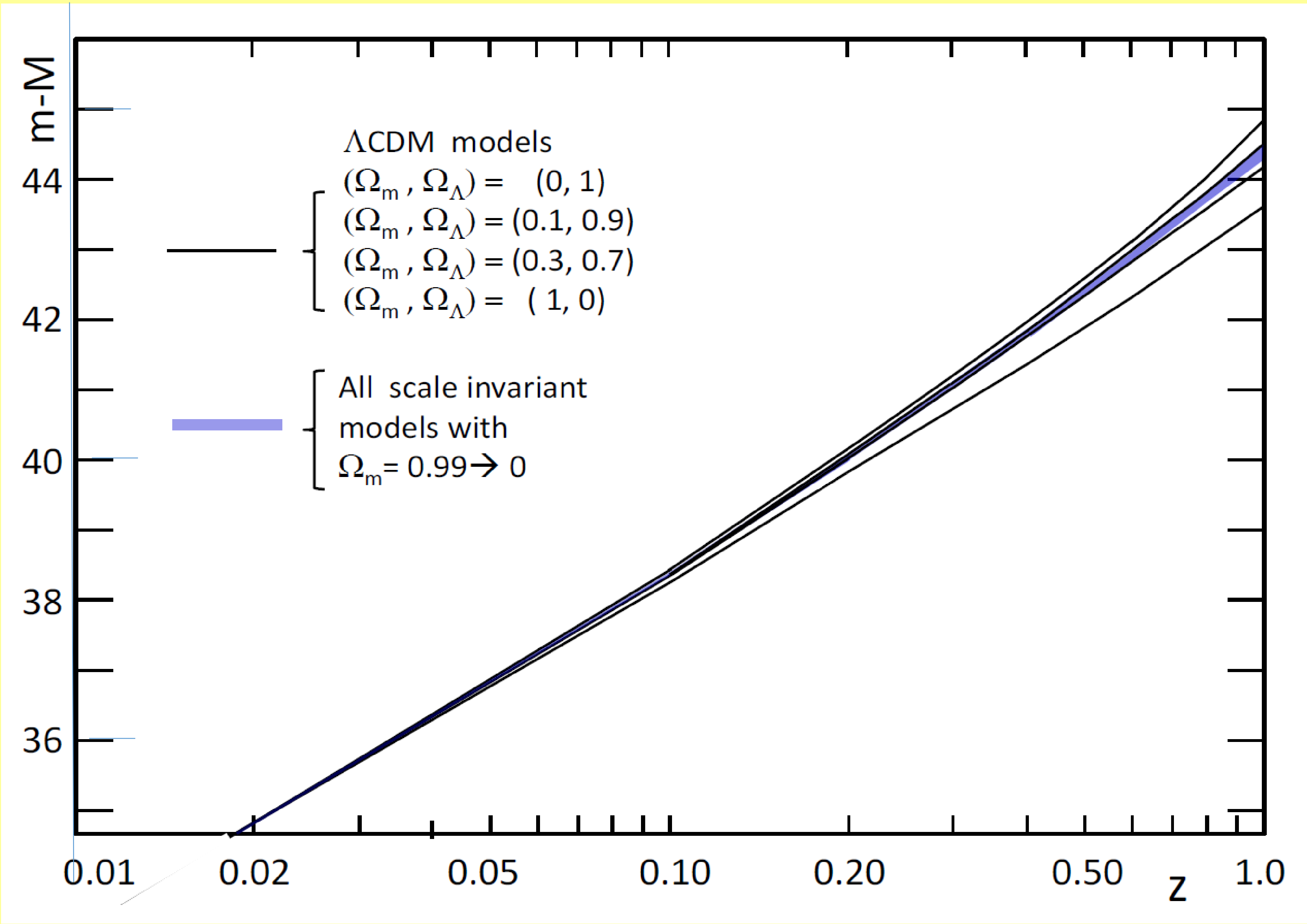
$$d_L = (1+z)d_M = (1+z)^2 d_A\,, \quad \text{with} \quad d_M = R_0\, r_1$$

$$R_0\, r_1 = \frac{c}{H_0}\int_0^z \frac{dz}{H(z)}$$

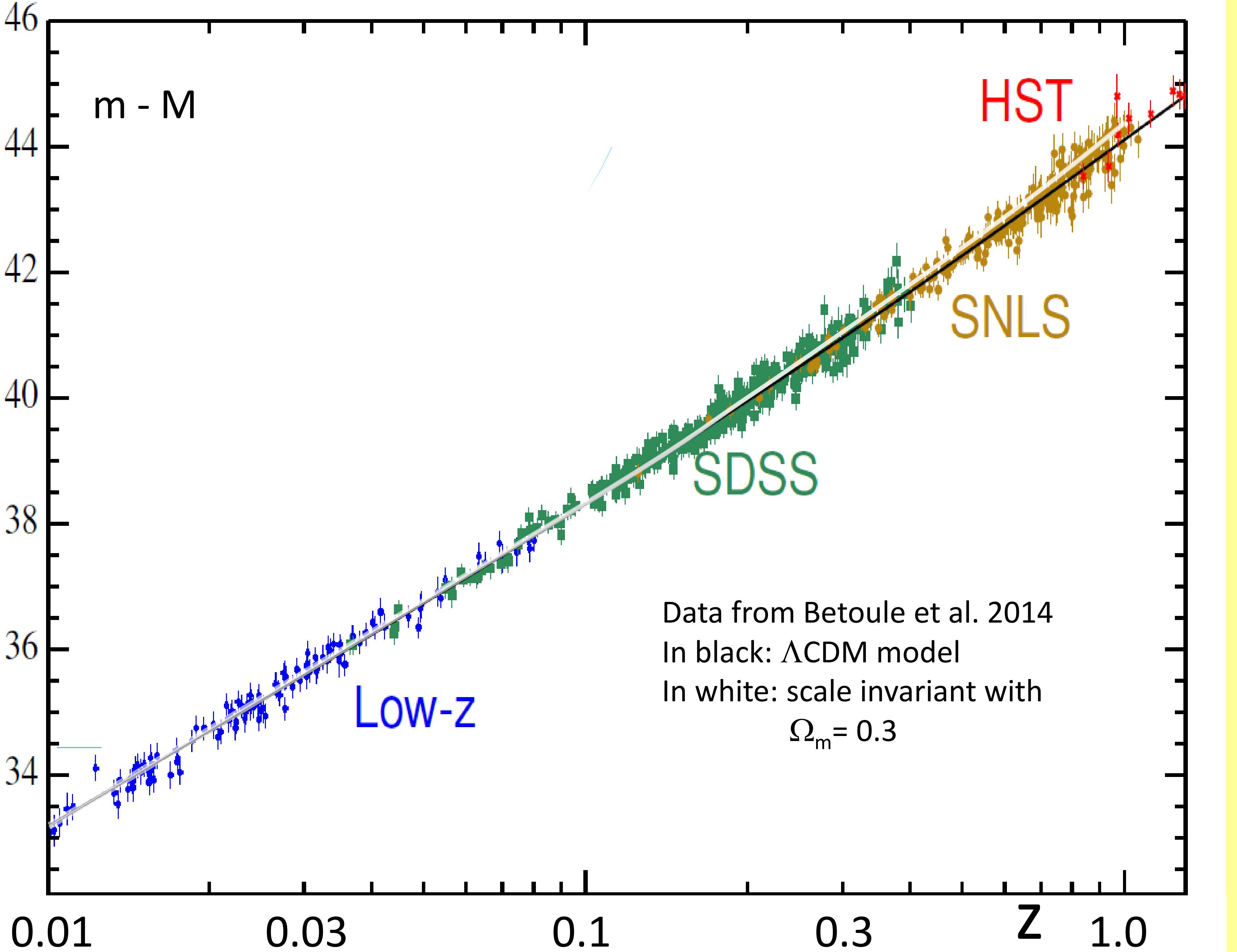
$$\frac{H^2(z)}{H_0^2} = \Omega_m (1+z)^3\, t + \Omega_k (1+z)^2 + 2\frac{H(z)}{H_0}\left(\frac{1}{H_0\, t}\right)$$



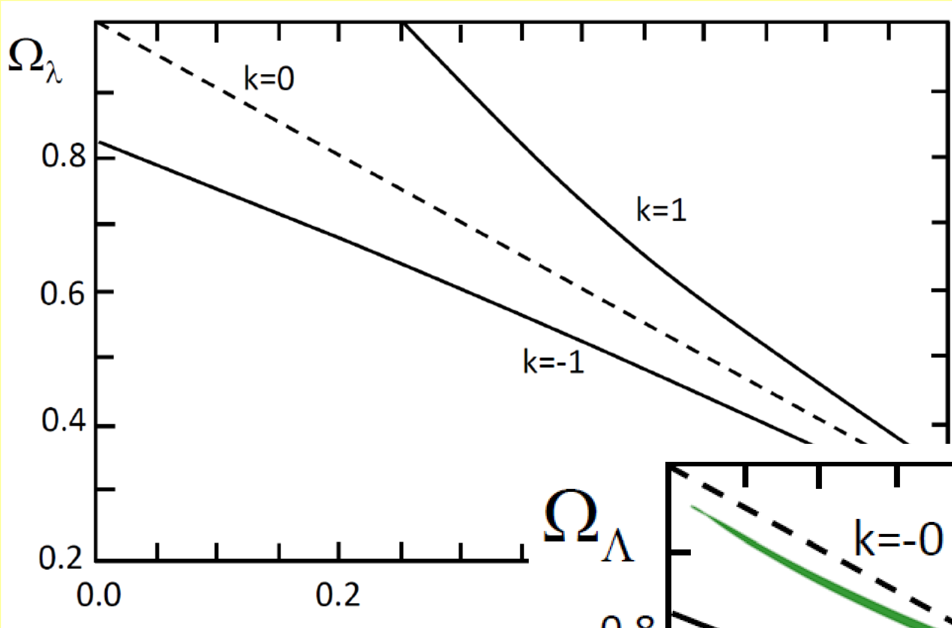
Distances intervene in most major tests



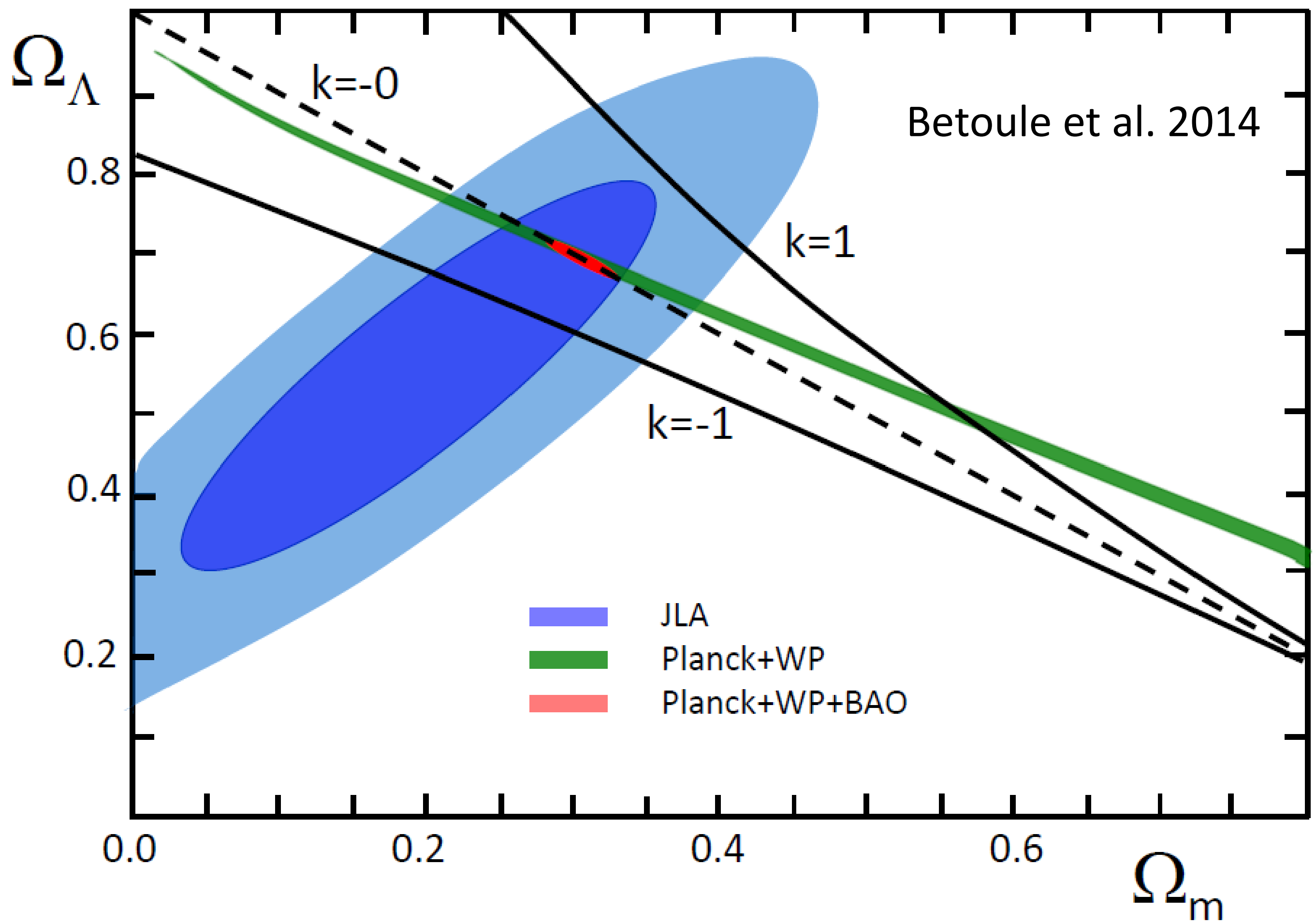
m-M
44
42
40
38
36
ΛCDM models
(Ωm , ΩΛ) = (0, 1)
(Ωm , ΩΛ) = (0.1, 0.9)
(Ωm , ΩΛ) = (0.3, 0.7)
(Ωm , ΩΛ) = (1, 0)
All scale invariant
models with
Ωm= 0.99→ 0
0.01
0.02
0.05
0.10
0.20
0.50
z
1.0



m - M
46
44
42
40
38
36
34
HST
SNLS
SDSS
Low-z
Data from Betoule et al. 2014
In black: ΛCDM model
In white: scale invariant with
Ωm= 0.3
0.01
0.03
0.1
0.3
z
1.0



Need to have model independent data.

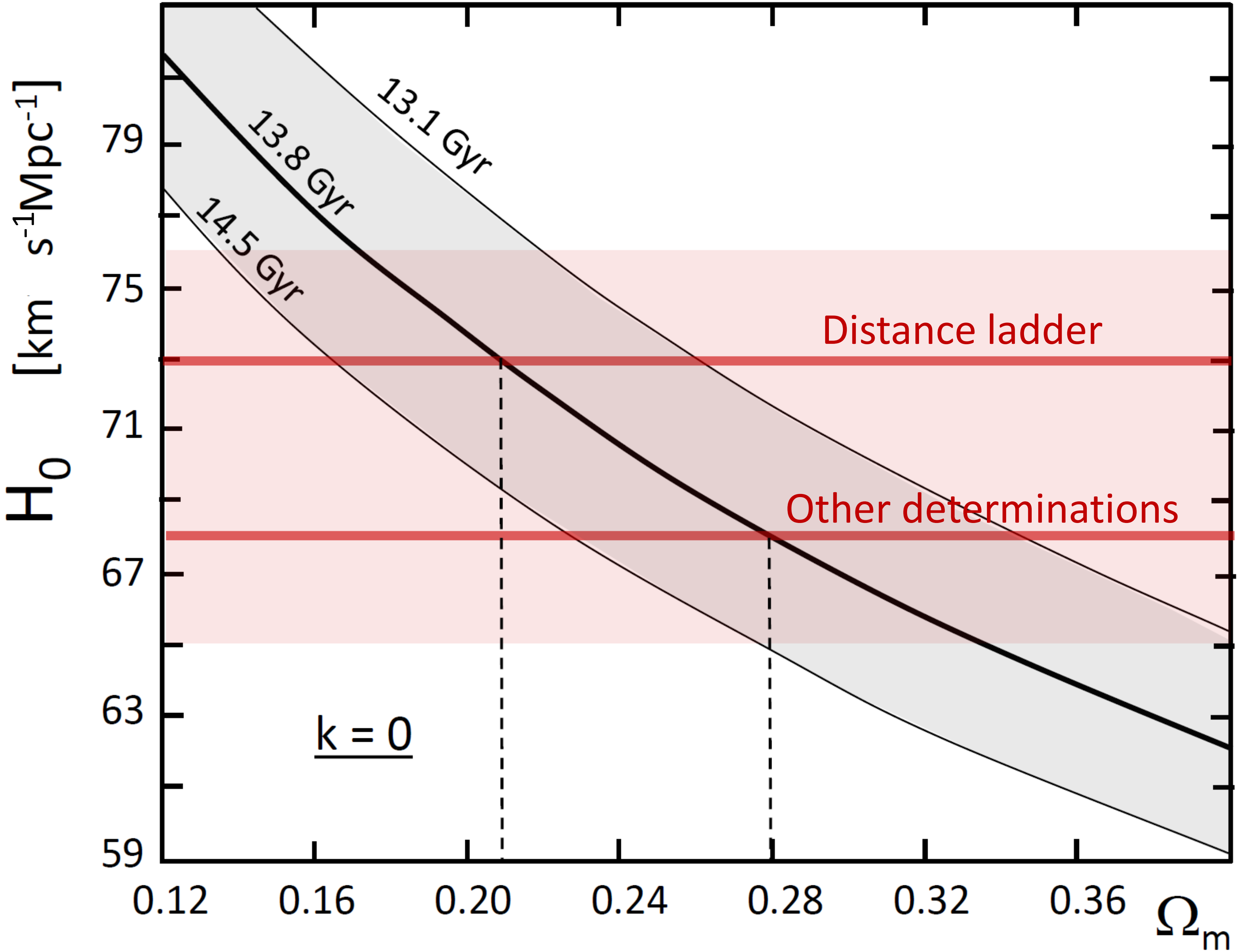


Many observations are in fact derived in ΛCDM models

Consistency of the age of the Universe and H_0

For a given age of the Universe, H_0 depends on Ω_m.

$$\left.\begin{array}{l}\text{Age} \\ \\ H_0\end{array}\right\} \quad \Omega_m$$



13.1 Gyr
13.8 Gyr
14.5 Gyr
Distance ladder
Other determinations
k = 0
H0 [km s-1Mpc-1]
79
75
71
67
63
59
0.12
0.16
0.20
0.24
0.28
0.32
0.36
Ωm

Dynamical tests at past epochs

The expansion rates H(z) are a direct test on   R(t).

Goes to much higher redshifts   z than other tests.

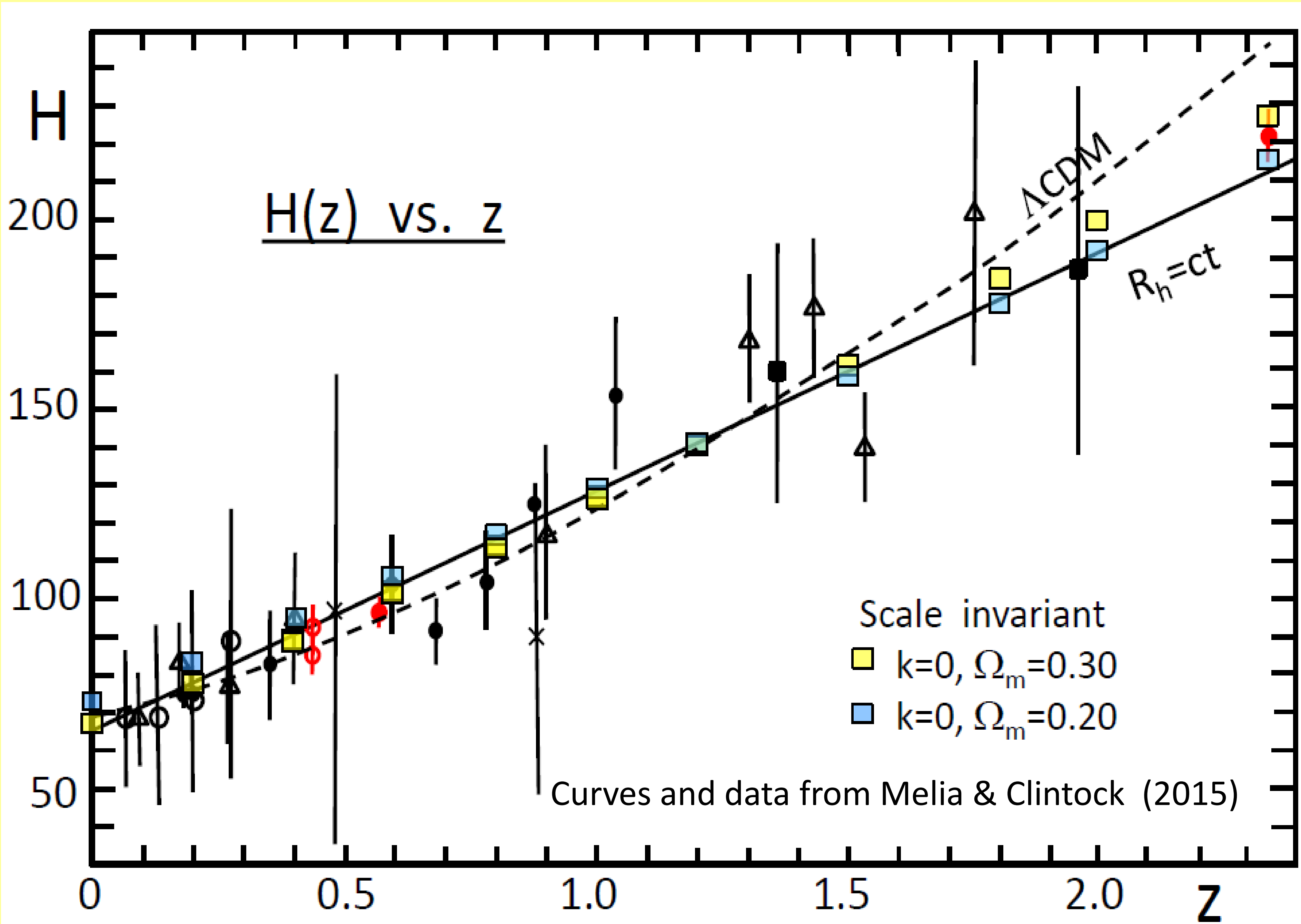
Method of cosmic chronometer (Jimenez & Loeb 2002; Simon et al. 2005; Stern et al. 2010; Melia & Clintock 2015; Moresco et al 2016).

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}$$

model independent

From $R_0/R = 1+z$ and $H = dR/(dt\ R)$

dz/dt estimated from a sample of passive galaxies of different z and age estimates.



H
H(z) vs. z
200
150
100
50
0
0.5
1.0
1.5
2.0
z
ΛCDM
R_h=ct
Scale invariant
k=0, Ω_m=0.30
k=0, Ω_m=0.20
Curves and data from Melia & Clintock (2015)

Observations show some tension with ΛCDM models

- Delubac et al. (2015) point a 2.5 sigma difference at z= 2.34

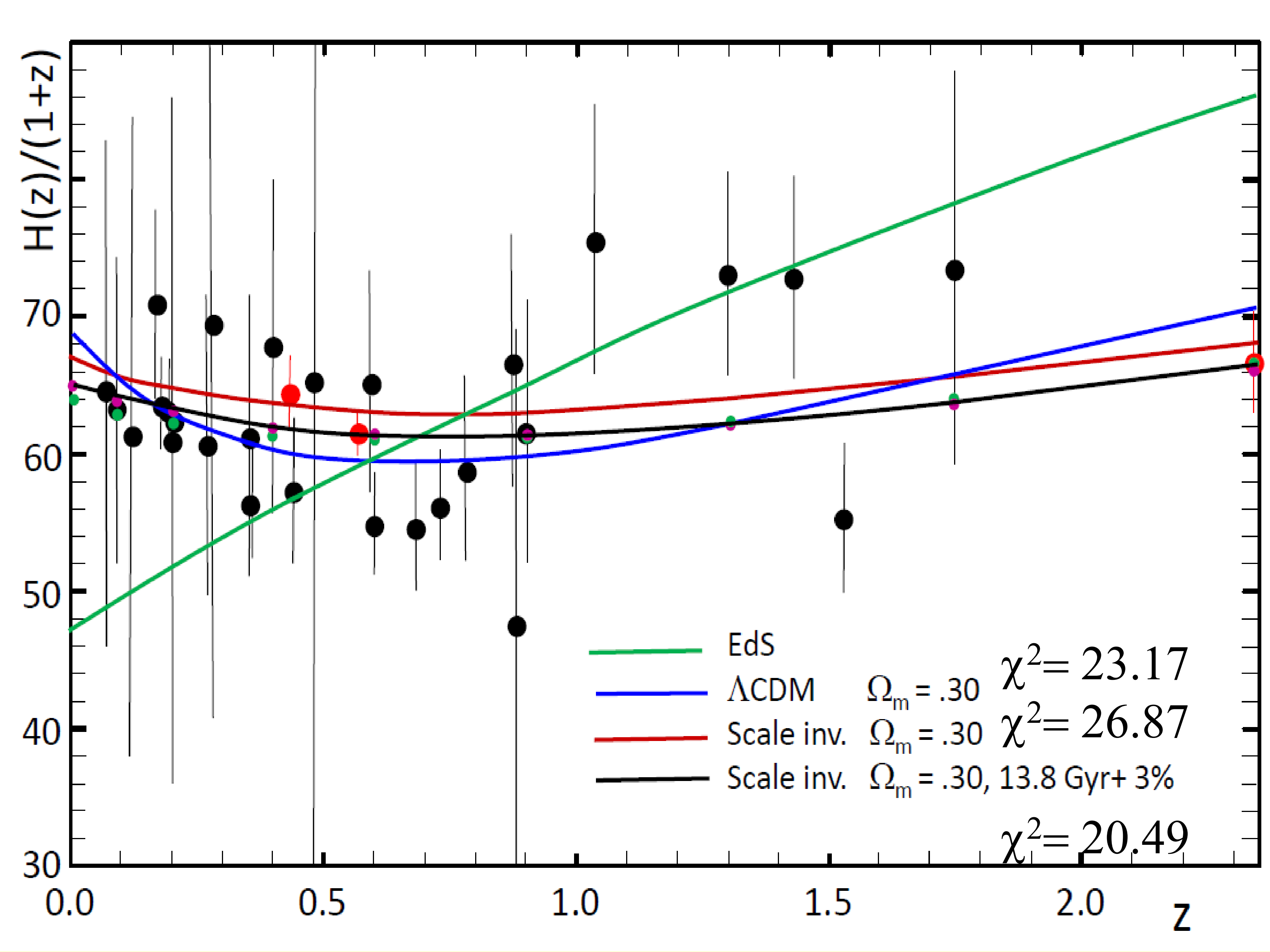
- Sahni et al. (2015); Ding et al. (2015): «*allowing dark energy to evolve seems to be the most plausible approach to this problem*»

- Sola et al. (2015, 2016) find a better agreement with a time-evolving Λ Depending on H^2 and dH/dt. Constancy of Λ ?

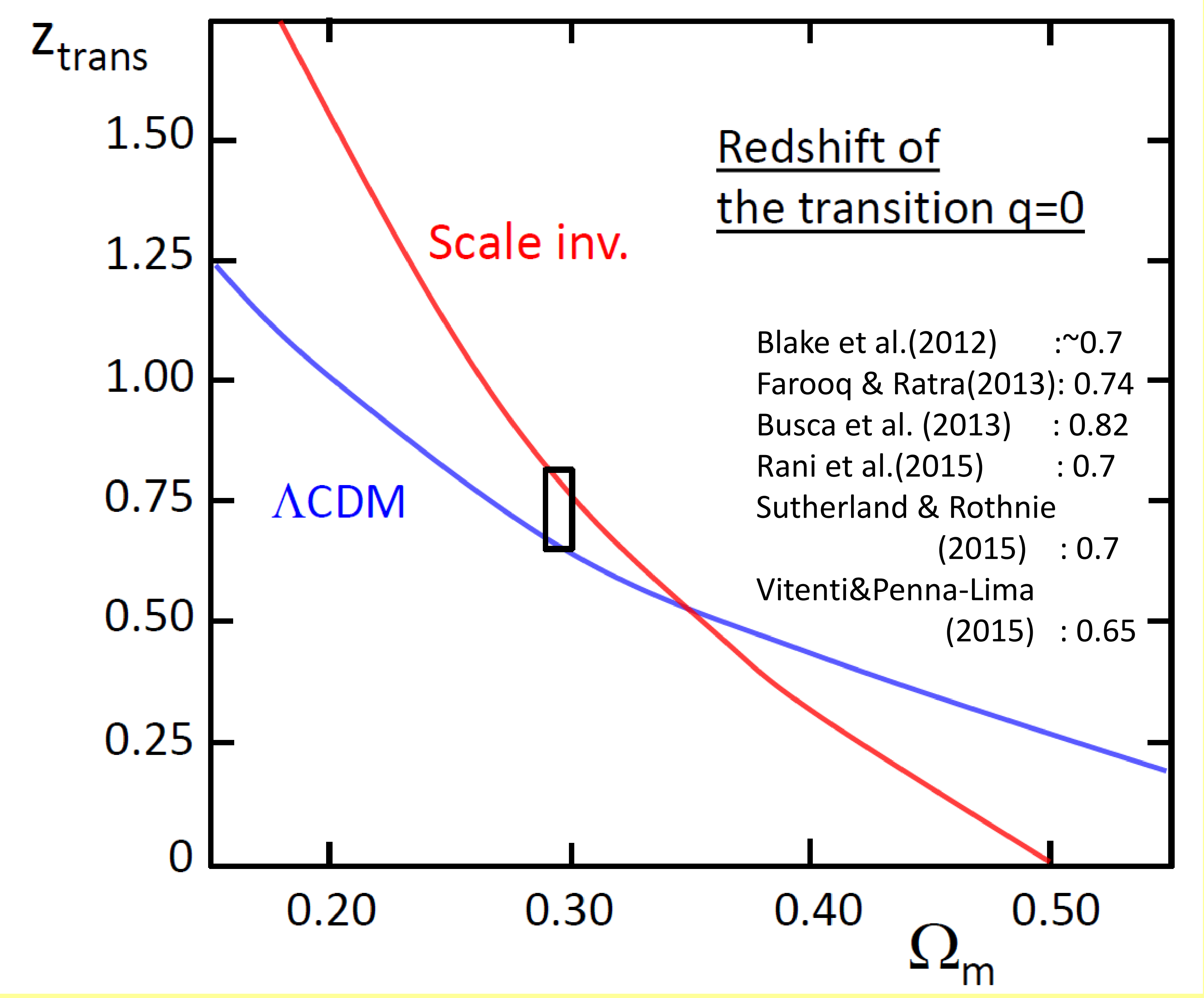
Comparisons are better performed with H(z)/(z+1)

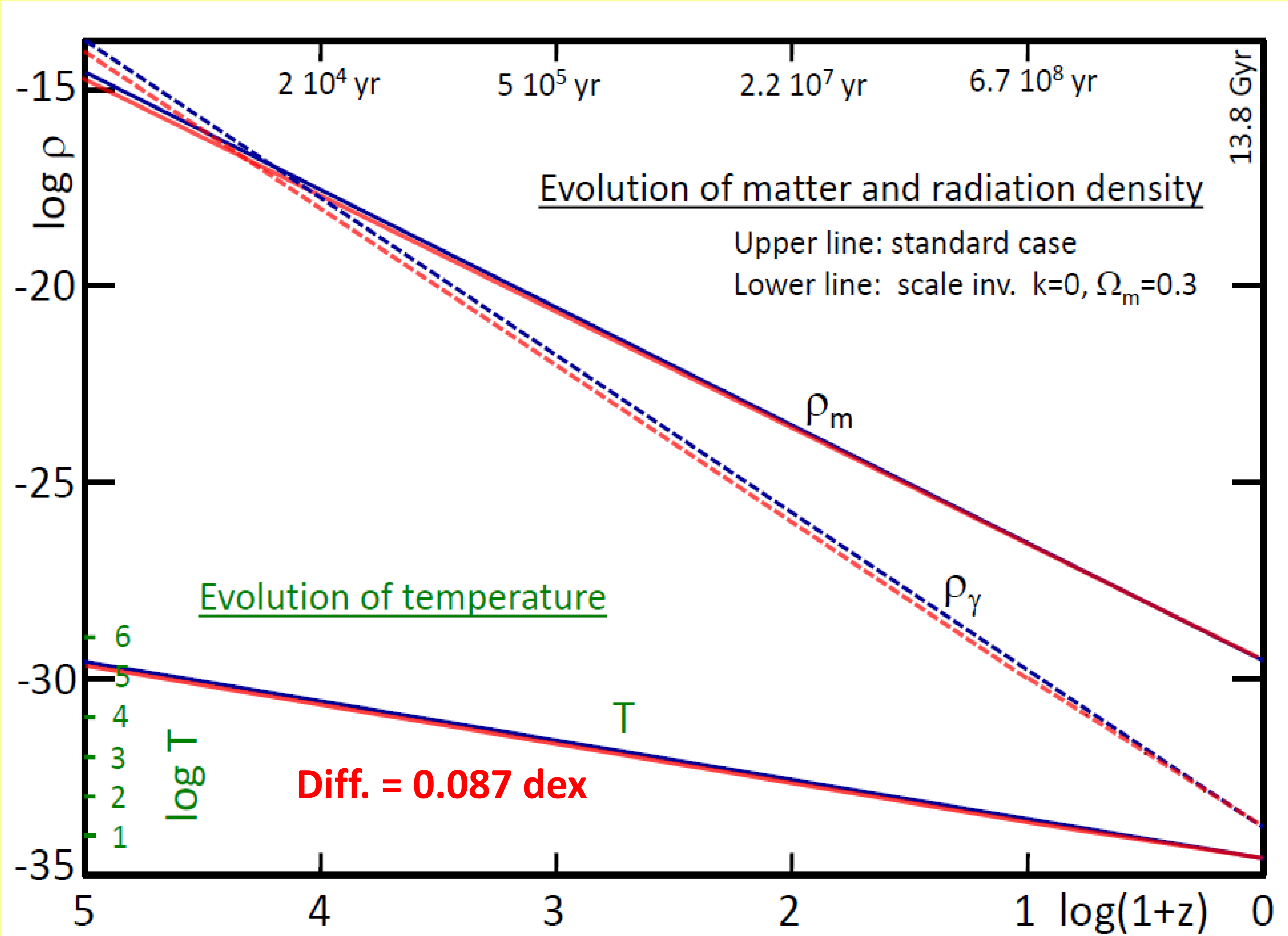
$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -\frac{dH}{dz}\frac{dz}{dt}\frac{1}{H^2} - 1 = \frac{dH}{dz}\frac{1+z}{H} - 1$$

$$\frac{d}{dz}\left(\frac{H(z)}{1+z}\right) = \frac{1}{1+z}\left(\frac{dH}{dz} - \frac{H(z)}{1+z}\right)$$



Data from Farooq & Ratra (2013)


z_{trans}
1.50
1.25
1.00
0.75
0.50
0.25
0
0.20
0.30
0.40
0.50
Ω_m
Scale inv.
ΛCDM
Redshift of
the transition q=0
Blake et al.(2012) :~0.7
Farooq & Ratra(2013): 0.74
Busca et al. (2013) : 0.82
Rani et al.(2015) : 0.7
Sutherland & Rothnie (2015) : 0.7
Vitenti&Penna-Lima (2015) : 0.65



Evolution of matter and radiation density
Upper line: standard case
Lower line: scale inv. k=0, Ωm=0.3
2 10^4 yr
5 10^5 yr
2.2 10^7 yr
6.7 10^8 yr
13.8 Gyr
log ρ
-15
-20
-25
-30
-35
ρm
ργ
Evolution of temperature
T
log T
6
5
4
3
2
1
Diff. = 0.087 dex
5
4
3
2
1
log(1+z)
0

$$\frac{8\,\pi G \varrho}{3} = \frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + 2\,\frac{\dot{R}\dot{\lambda}}{R\lambda}\,,$$

$$-8\,\pi G p = \frac{k}{R^2} + 2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + 4\frac{\dot{R}\dot{\lambda}}{R\lambda}$$

$$-\frac{4\,\pi G}{3}\,(3p+\varrho) = \frac{\ddot{R}}{R} + \frac{\dot{R}\dot{\lambda}}{R\lambda}$$

SCALE INVARIANCE ACCOUNTS FOR:

ACCELERATION OF COSMIC EXPANSION

Distances

m – z diagram

H_0 vs. age and Ω_m

History of expansion H(z)/(z+1)

Transition from braking to acceleration, etc…

The tests on scale invariant cosmology are positive

Further tests need to be explored

A. Maeder, 2017, ApJ 834, 194

APPLICATION TO EMPTY SPACE

In GR, the $g'_{\mu\nu}$ represent de Sitter metric. A particular form is

$$ds'^2 = dt^2 - e^{2kt}[dx^2 + dy^2 + dz^2]$$

$$k^2 = \Lambda_E/3 \text{ and } c = 1$$

A transformation of coordinates:

τ is a new time coordinate

$$\tau = \int e^{\left(-\sqrt{\frac{\Lambda_E}{3}}\,t\right)} dt$$

De Sitter metric is conformal to Minkowski metric.

With above transformations, one has

$$ds'^2 = e^{\psi(\tau)}[d\tau^2 - (dx^2 + dy^2 + dz^2)]\,, \quad \text{with} \quad e^{\psi(\tau)} = e^{\left(2\sqrt{\frac{\Lambda_E}{3}}\,t\right)}$$

$$e^{\psi(\tau)} = \frac{3}{\Lambda_E\,\tau^2}$$

$$ds^2 = \lambda^{-2} ds'^2 = \frac{3\,\lambda^{-2}}{\Lambda_E\,\tau^2}[c^2\,d\tau^2 - (dx^2 + dy^2 + dz^2)]$$

Minkowsik metric is compatible with scale inv. framework

$$3\,\frac{\dot{\lambda}^2}{\lambda^2} = \lambda^2\,\Lambda_E \quad \text{and} \quad 2\,\frac{\ddot{\lambda}}{\lambda} - \frac{\dot{\lambda}^2}{\lambda^2} = \lambda^2\,\Lambda_E$$