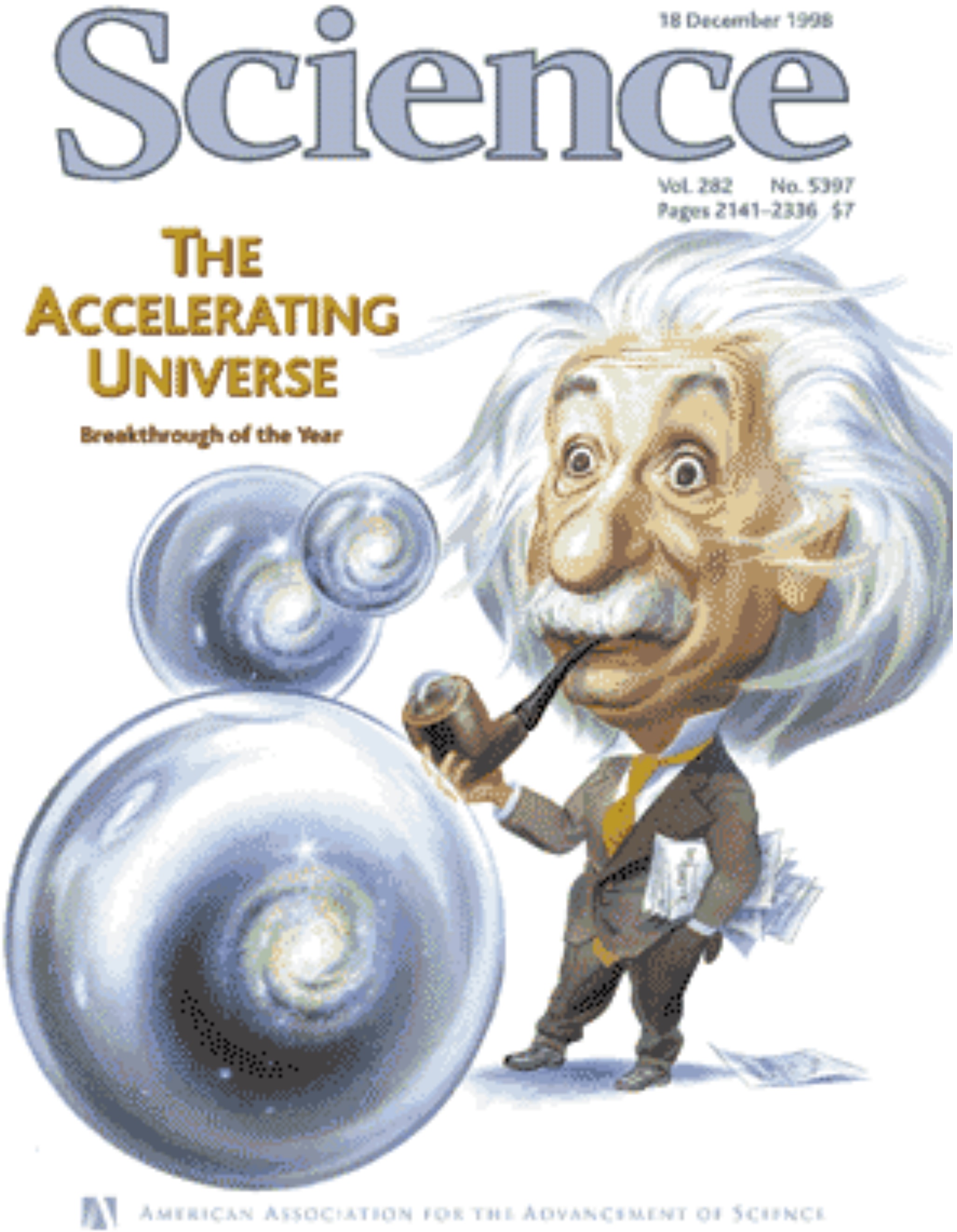
Ruling out inhomogeneous void models

VIA lecture
APC Seminar
13th April 2012

arXiv:1201.2790

Juan García-Bellido
IFT-UAM/CSIC
Madrid


Science
18 December 1998
Vol. 282 No. 5397
Pages 2141–2336 $7
The Accelerating Universe
Breakthrough of the Year
American Association for the Advancement of Science

The Nobel Prize in Physics 2011

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"



Saul Perlmutter   Brian P. Schmidt   Adam G. Riess

What is the Nature of Dark Energy?

There are many alternatives:

- **Cosmological constant Λ (vacuum energy)**
- **Scalar field (quintessence, tachyon,…)**
- **Relativistic Ether fluids (Chaplygin gas, VDE,…)**
- **Modifications of GR on UV scales (f(R), GB)**
- **Weyl gravity, Horava gravity, massive graviton**
- **Extra dimensions (DGP, KK, …)**
- **Effective interactions (Chameleon, Galileon,…)**
- **Inhomogeneous universes (backreaction, LTB large Voids,,…)**

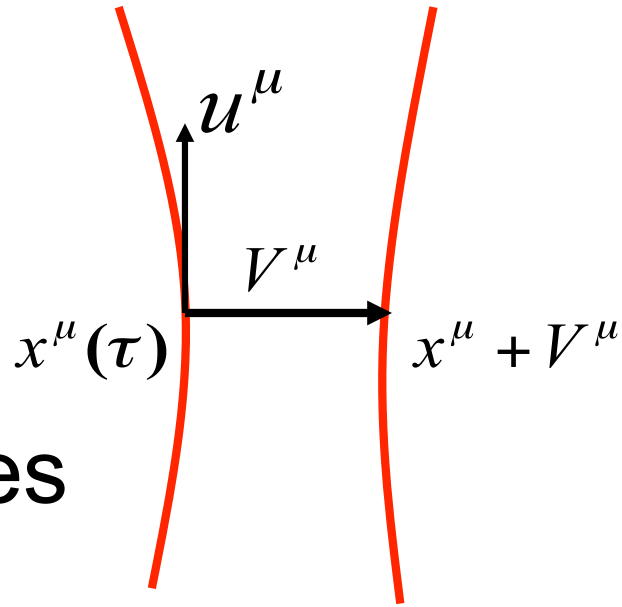
Basic notions of Geometry

Metric signature: $g_{\mu\nu} = diag(-,+,+,+)$

$$u^\mu \equiv \frac{dx^\mu}{d\tau}, \quad \text{normalization} \quad u_\mu u^\mu = -1$$

$$\boxed{\frac{Du^\mu}{d\tau} \equiv \frac{du^\mu}{d\tau} + \Gamma^\mu_{\nu\lambda}\, u^\nu u^\lambda = 0}$$

Geodesic Eq.



Geodesic Deviation

$$\frac{D^2 V^\mu}{d\tau^2} \equiv R^\mu{}_{\nu\lambda\rho}\, u^\nu u^\rho\, V^\lambda \quad \text{tidal forces}$$

Congruence of timelike geodesics

$$\frac{DV^{\mu}}{d\tau} = u^{\nu} D_{\nu} V^{\mu} \equiv \Theta^{\mu}{}_{\nu} V^{\nu}$$

$$\Theta_{\mu\nu} = \frac{1}{3}\Theta P_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu}$$

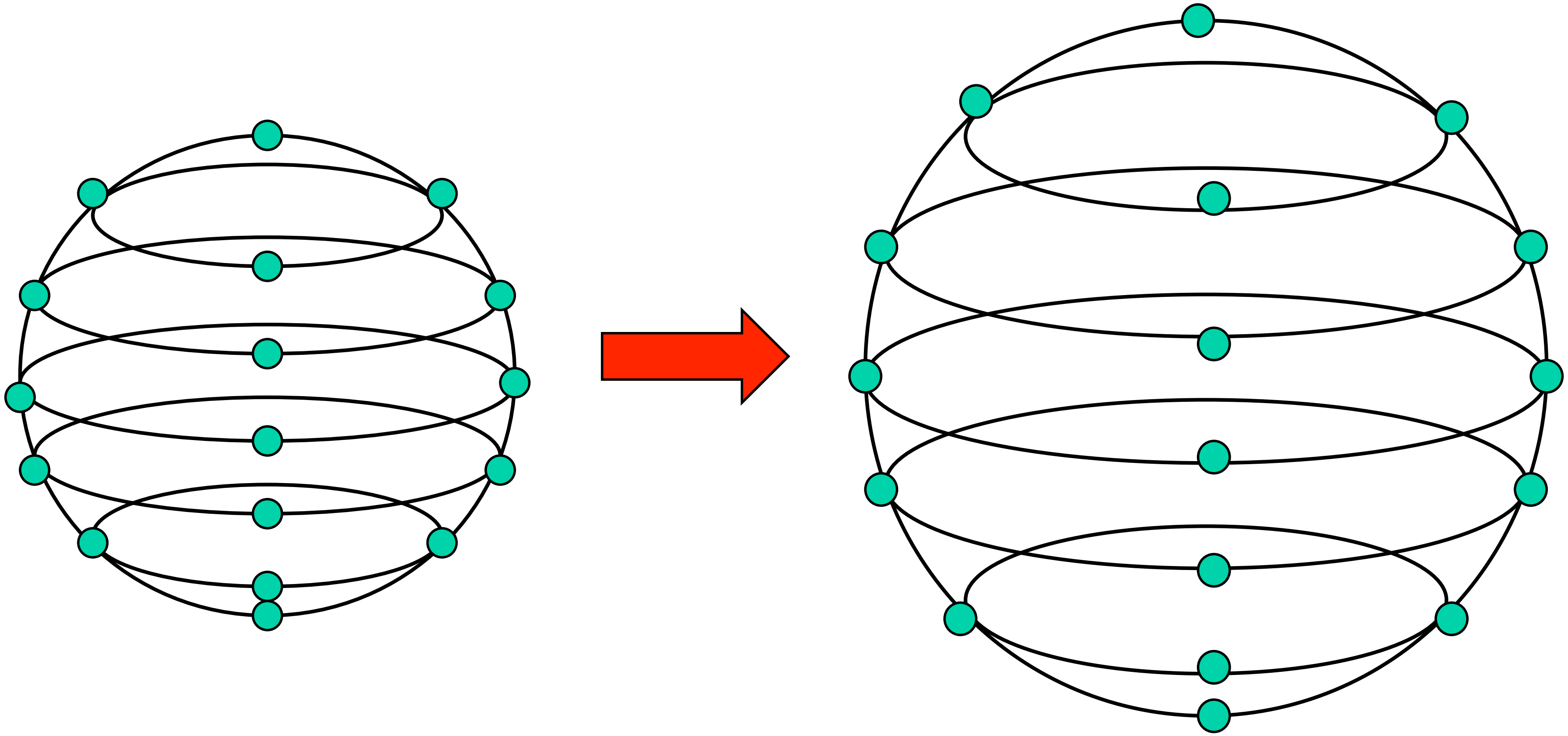
Describes the extent to which neighboring geodesics deviate from remaining parallel

$\Theta = D_{\mu} u^{\mu}$ trace

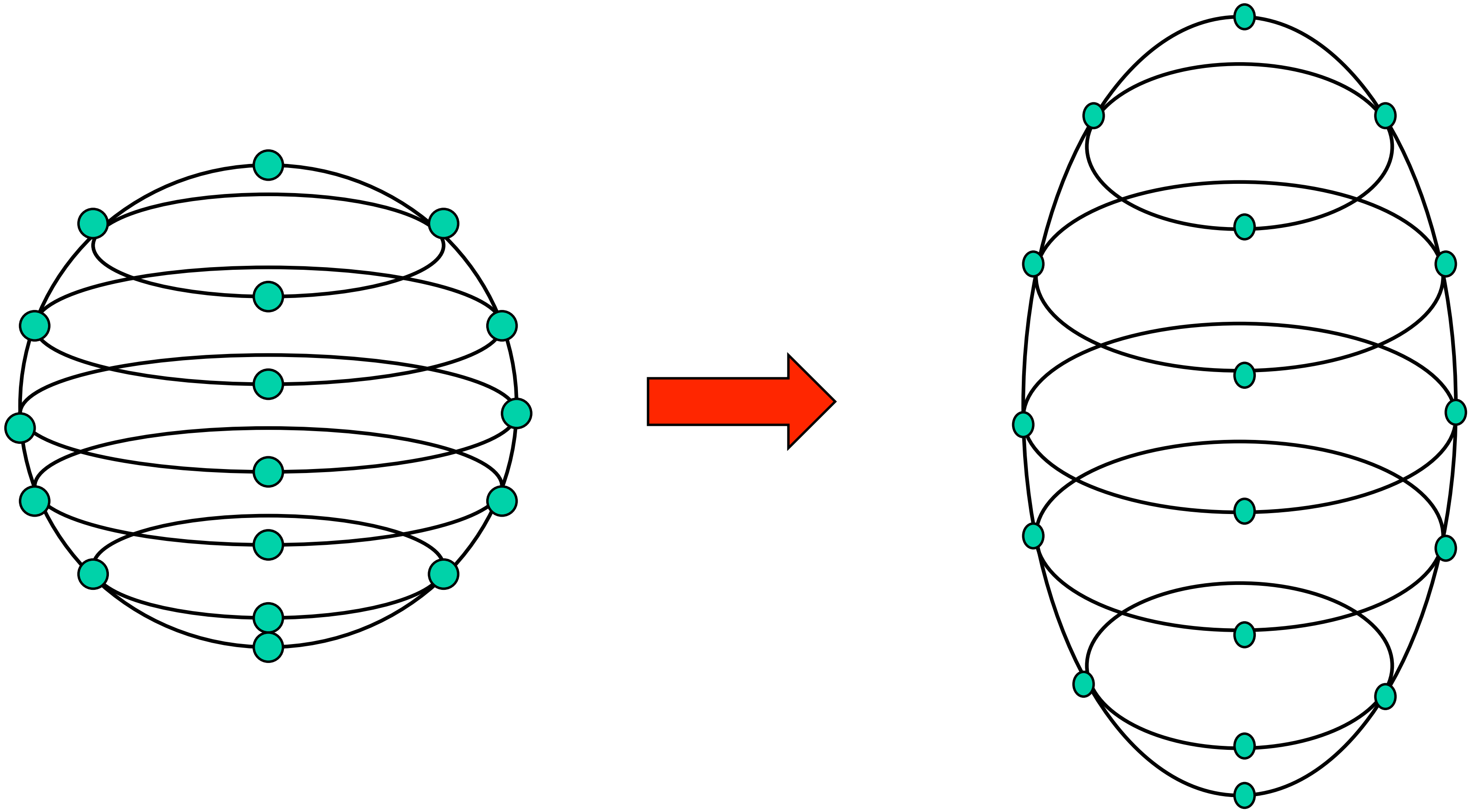
$\sigma_{\mu\nu} = \Theta_{(\mu\nu)} - \frac{1}{3}\Theta P_{\mu\nu}$ traceless symmetric

$\omega_{\mu\nu} = \Theta_{[\mu\nu]}$ antisymmetric

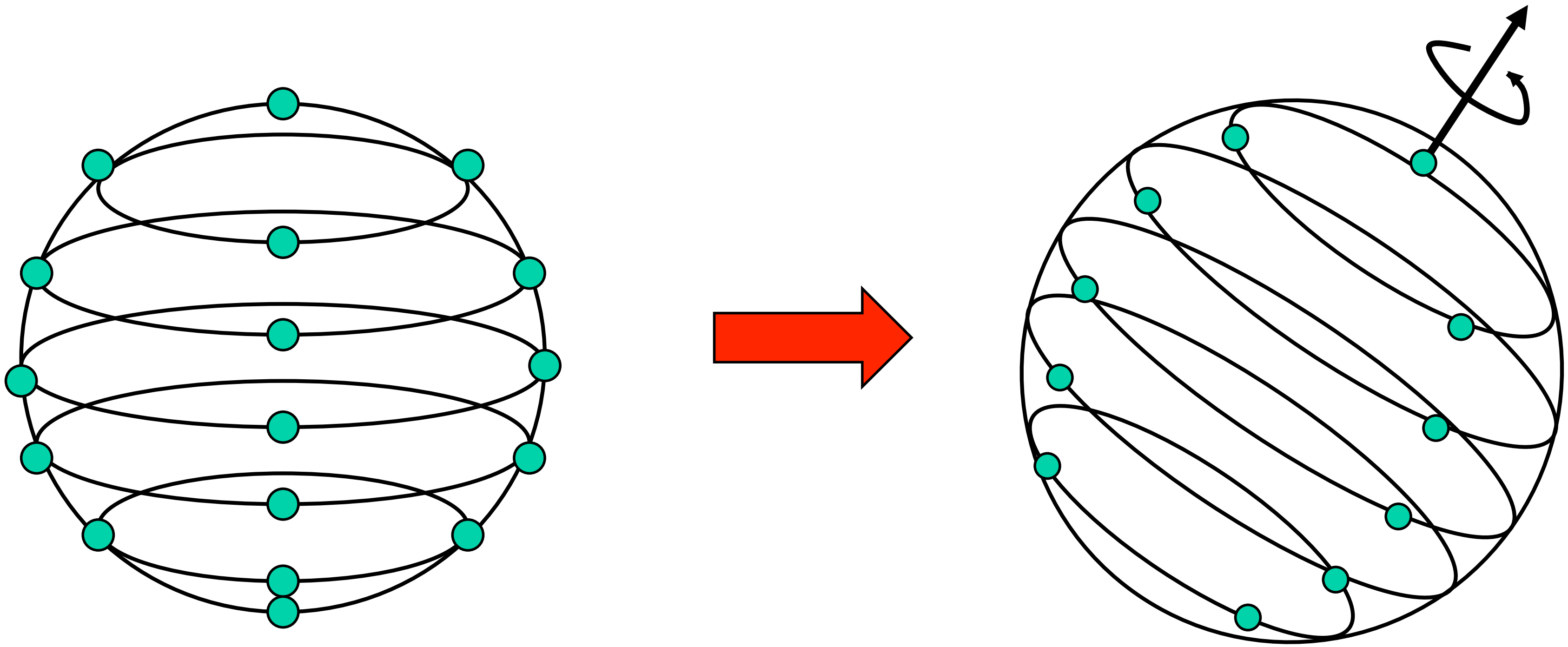
Θ expansion of congruence



$\sigma_{\mu\nu}$ shear of congruence



$\omega_{\mu\nu}$ vorticity of congruence



Evolution of Congruence

$$\frac{D}{d\tau}\Theta_{\mu\nu} = u^{\sigma} D_{\sigma} D_{\nu}\, u_{\mu} = u^{\sigma} D_{\nu} D_{\sigma}\, u_{\mu} + u^{\sigma} R^{\lambda}_{\ \mu\nu\sigma}\, u_{\lambda}$$

$$= -\Theta^{\sigma}_{\nu}\, \Theta_{\mu\sigma} - R_{\lambda\mu\sigma\nu}\, u^{\sigma} u^{\lambda}$$

Raychaudhuri Equation (trace)

Characterizes expansion's evolution

$$\boxed{\frac{d\Theta}{d\tau} = -\frac{1}{3}\Theta^{2} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}\, u^{\mu} u^{\nu}}$$

$\sigma_{\mu\nu}\sigma^{\mu\nu} \geq 0, \quad \omega_{\mu\nu}\omega^{\mu\nu} \geq 0,$ spatial tensors

For an Expanding Universe

$$H(t,\bar{x}) = \frac{1}{3}\Theta = \frac{1}{3} D_\mu u^\mu$$ Hubble parameter

$$q = -1 - H^{-2} u^\mu D_\mu H$$ deceleration parameter

R.E. $\Rightarrow$ $$qH^2 = \frac{1}{3}(\sigma_{\mu\nu}\sigma^{\mu\nu} - \omega_{\mu\nu}\omega^{\mu\nu}) + \frac{1}{3} R_{\mu\nu} u^\mu u^\nu$$

Einstein eqs. $\downarrow$ Perfect Fluid $\downarrow$

$$R_{\mu\nu} u^\mu u^\nu = 8\pi G (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) u^\mu u^\nu = 4\pi G (\rho + 3p)$$

$$\boxed{-\frac{\ddot{a}}{a} = \frac{4\pi G}{3}(\rho + 3p) \quad \text{Homog.} + \text{Isotrop. Universe}}$$

Conditions for acceleration (q<0)

One of the following must be violated:

1. The Strong Energy Condition:

$$(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) u^{\mu} u^{\nu} \geq 0, \quad u^{\mu} \text{ timelike}$$

2. Gravity is described by General Relativity:

$$R_{\mu\nu} = 8\pi G \, (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T)$$

3. The universe is homogeneous and isotropic:

$$T^{\mu\nu} = p(t) g^{\mu\nu} + [\rho(t) + p(t)] \, u^{\mu} u^{\nu}$$

Conditions for acceleration

Usually one drops assumptions 1. or 2.

1. Strong EC for a homogeneous universe:

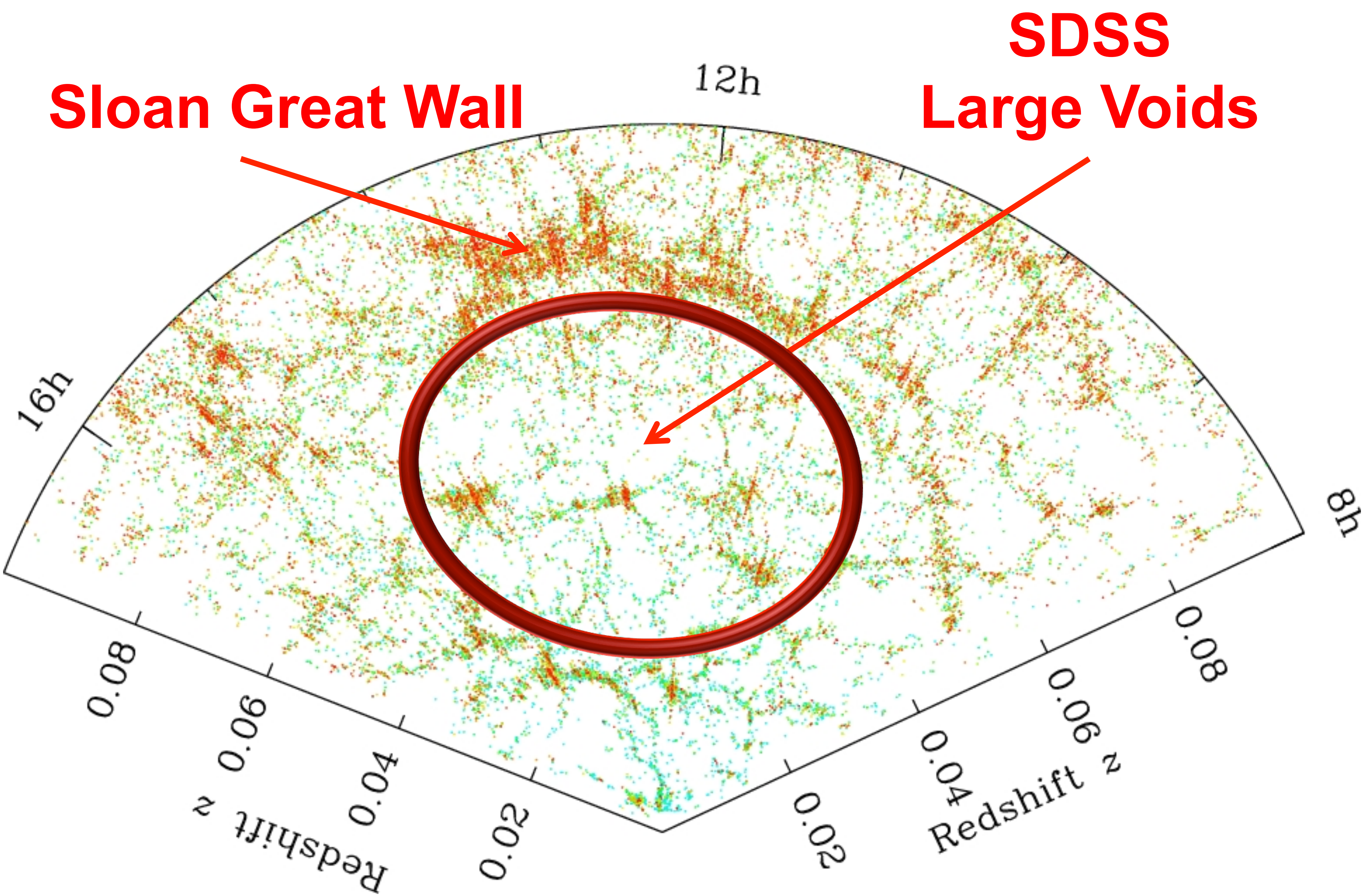
$$(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) u^\mu u^\nu = \rho + 3p \geq 0$$

Dark Energy violates SEC: $p = -\rho \Rightarrow \rho + 3p < 0$

2. Modified Gravity on large scales

$$R_{\mu\nu}\, u^\mu u^\nu = f(T_{\mu\nu}, G_{\mu\nu}, D_\mu D_\nu \Phi) u^\mu u^\nu < 0$$

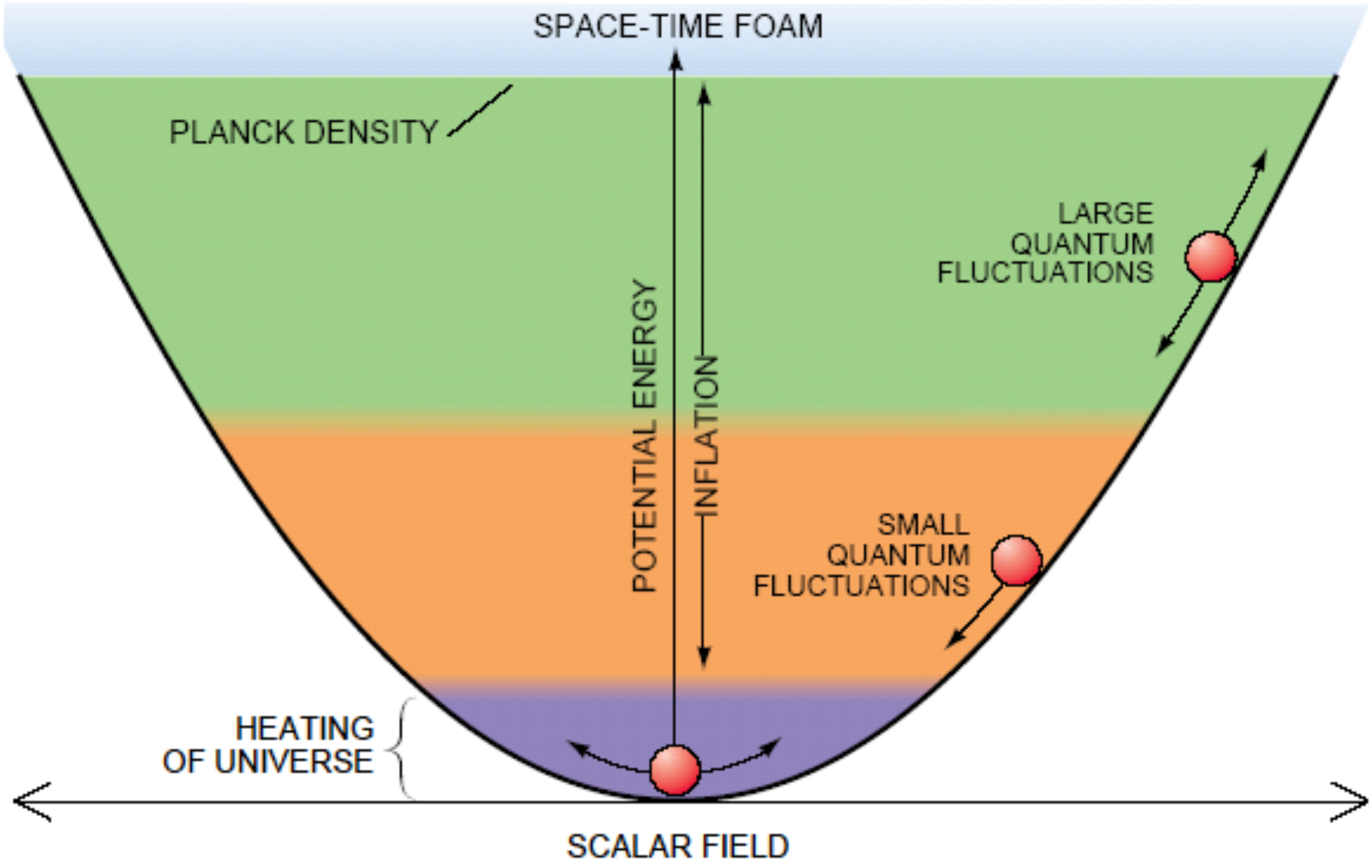
Assumption 3. is only approx. valid in the real universe, deviations are small on large scales

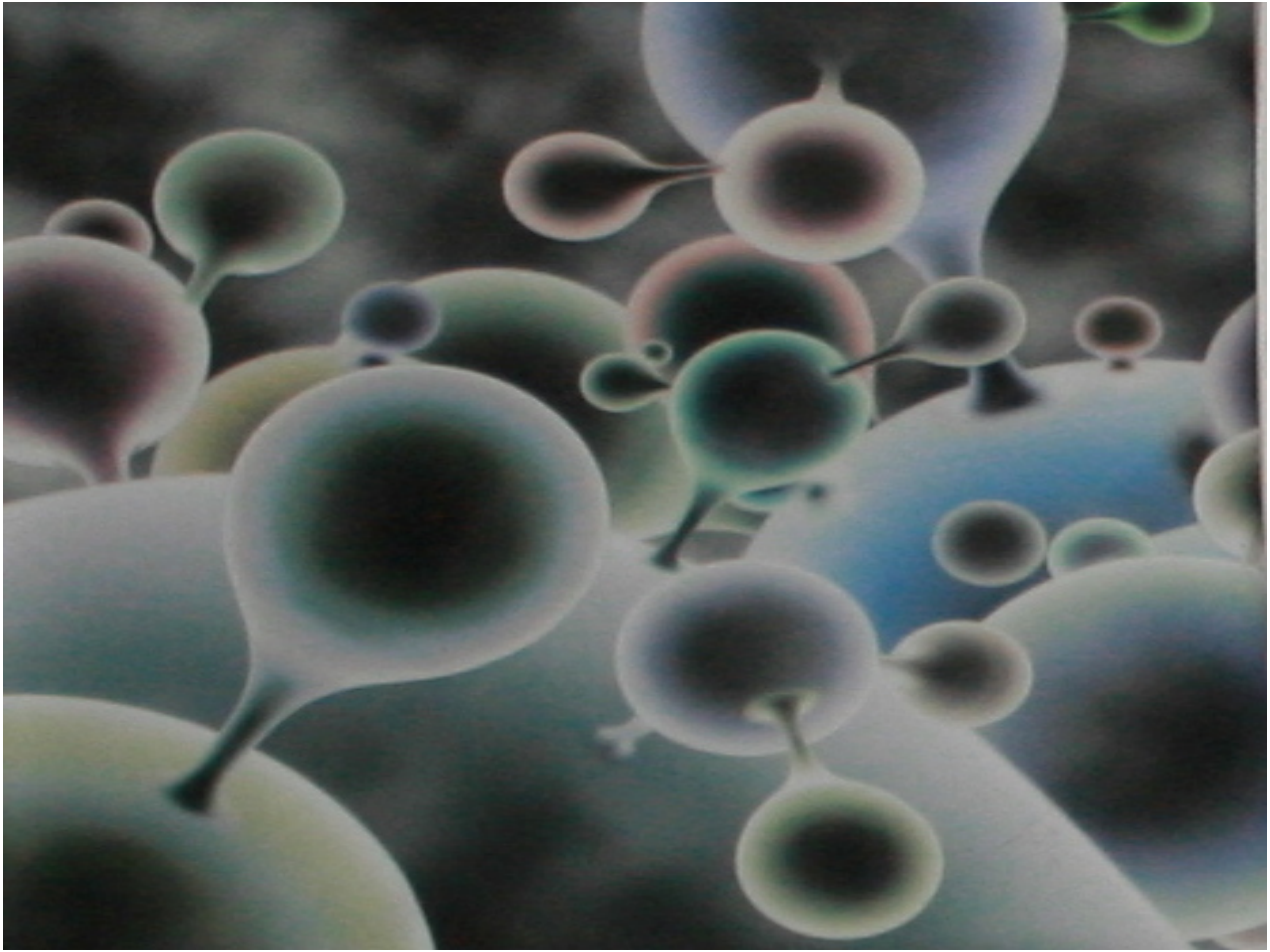

Sloan Great Wall
SDSS
Large Voids
12h
16h
8h
0.08
0.06
0.04
0.02
Redshift z
0.02
0.04
0.06
0.08
Redshift z

If we live in a highly inhomogeneous Universe…

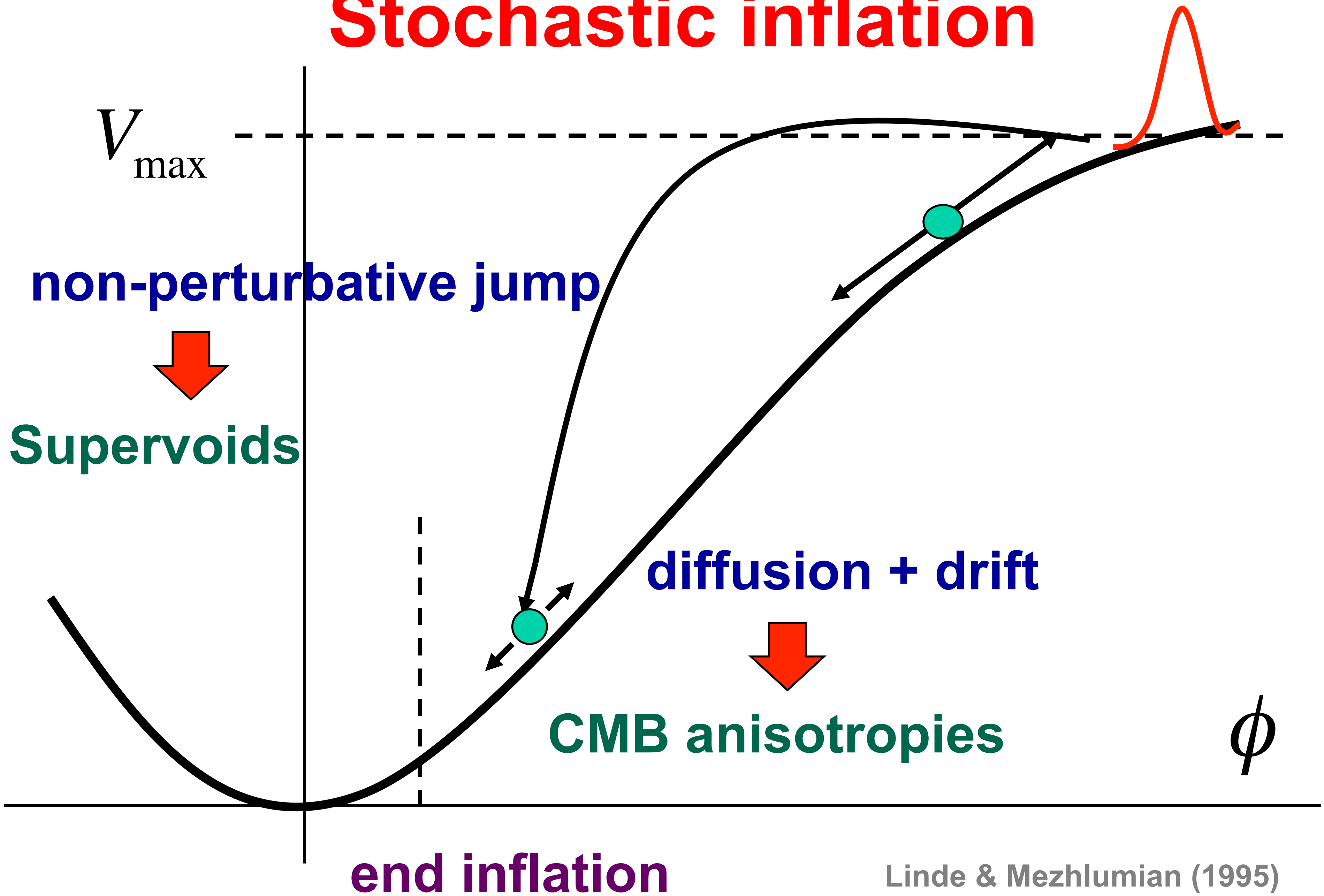
what is
the origin of
dens. perturbations?

Chaotic Inflation



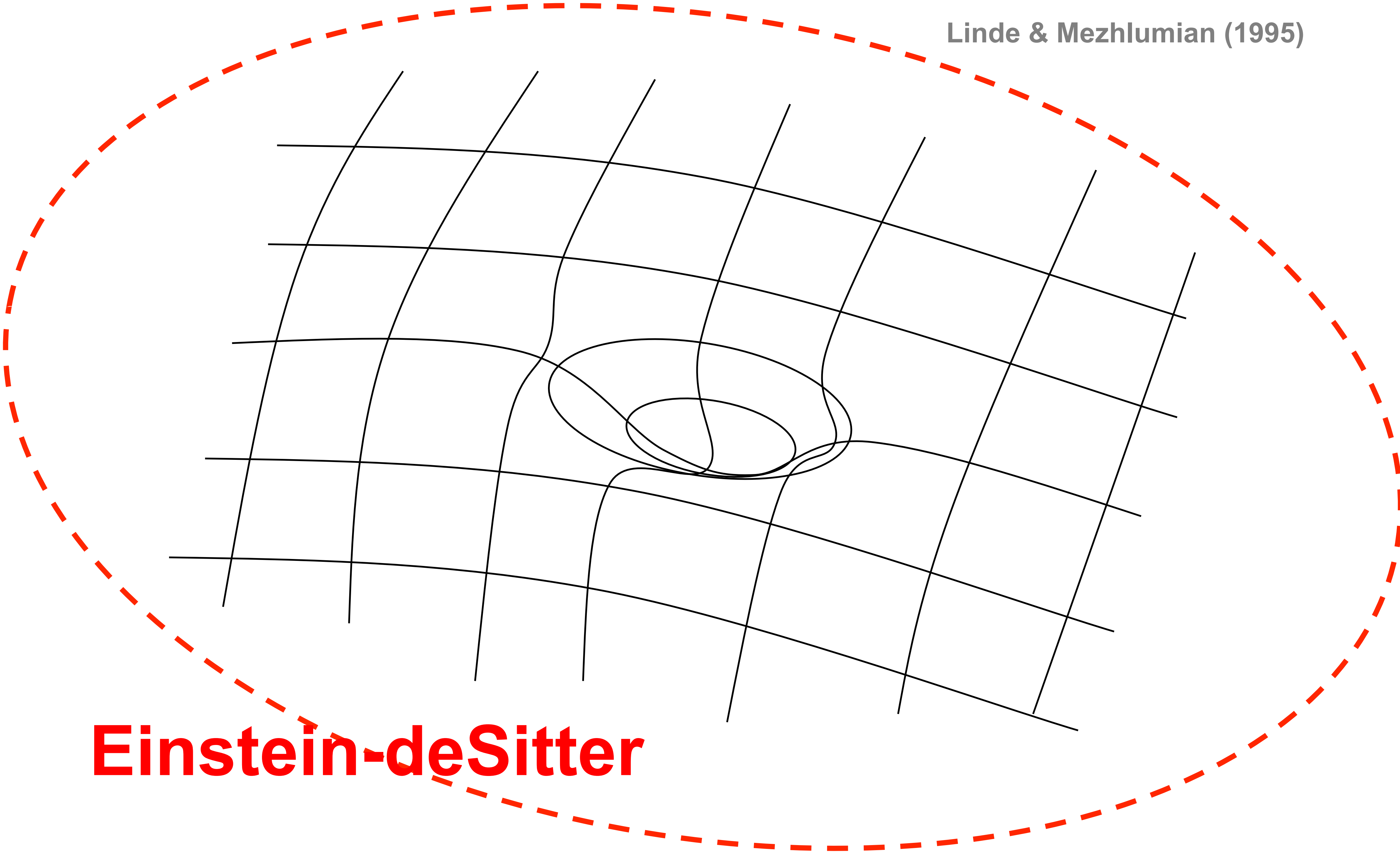


Stochastic inflation

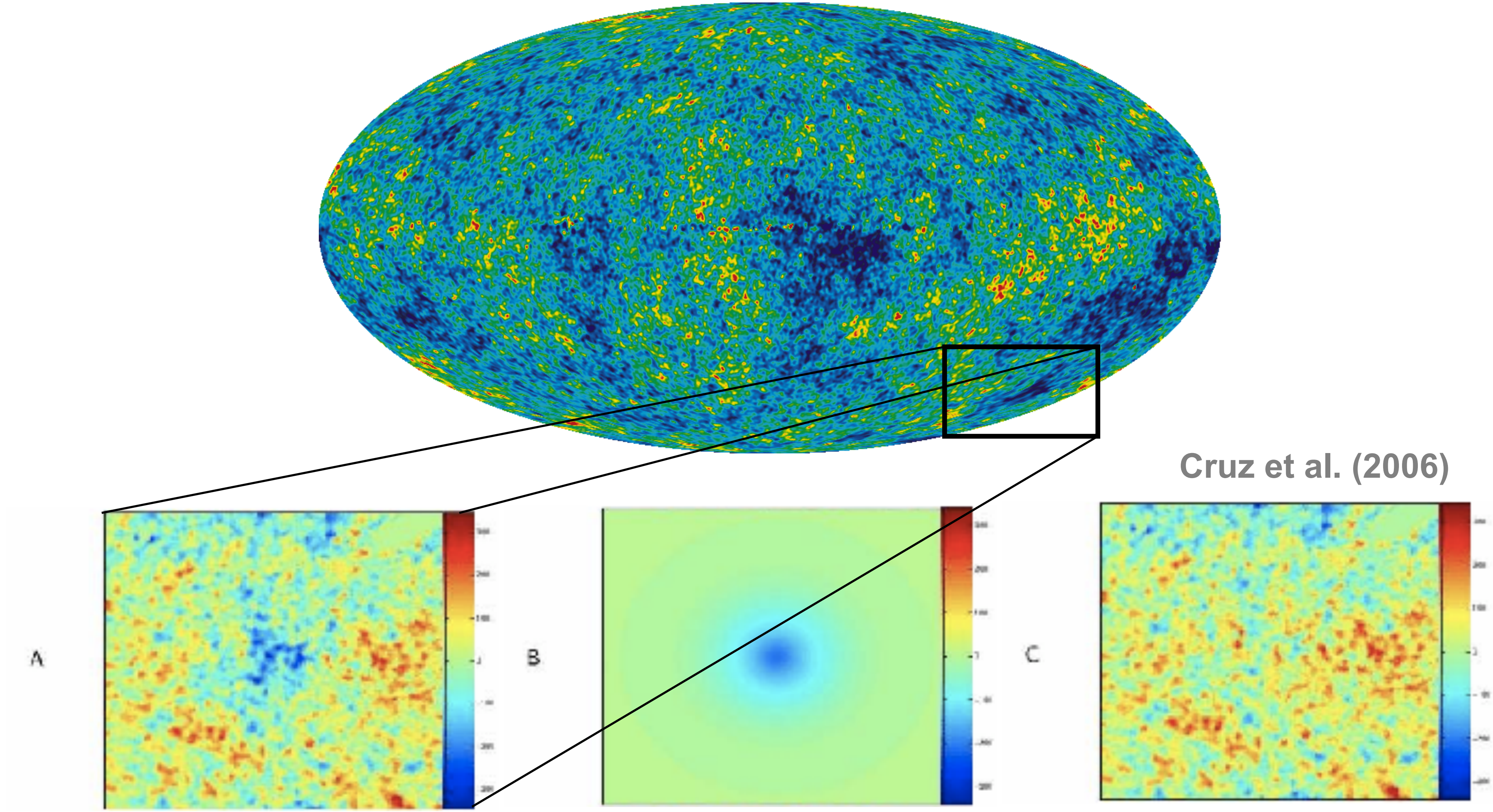


Infloid = Lemaitre-Tolman-Bondi Model

Linde & Mezhlumian (1995)



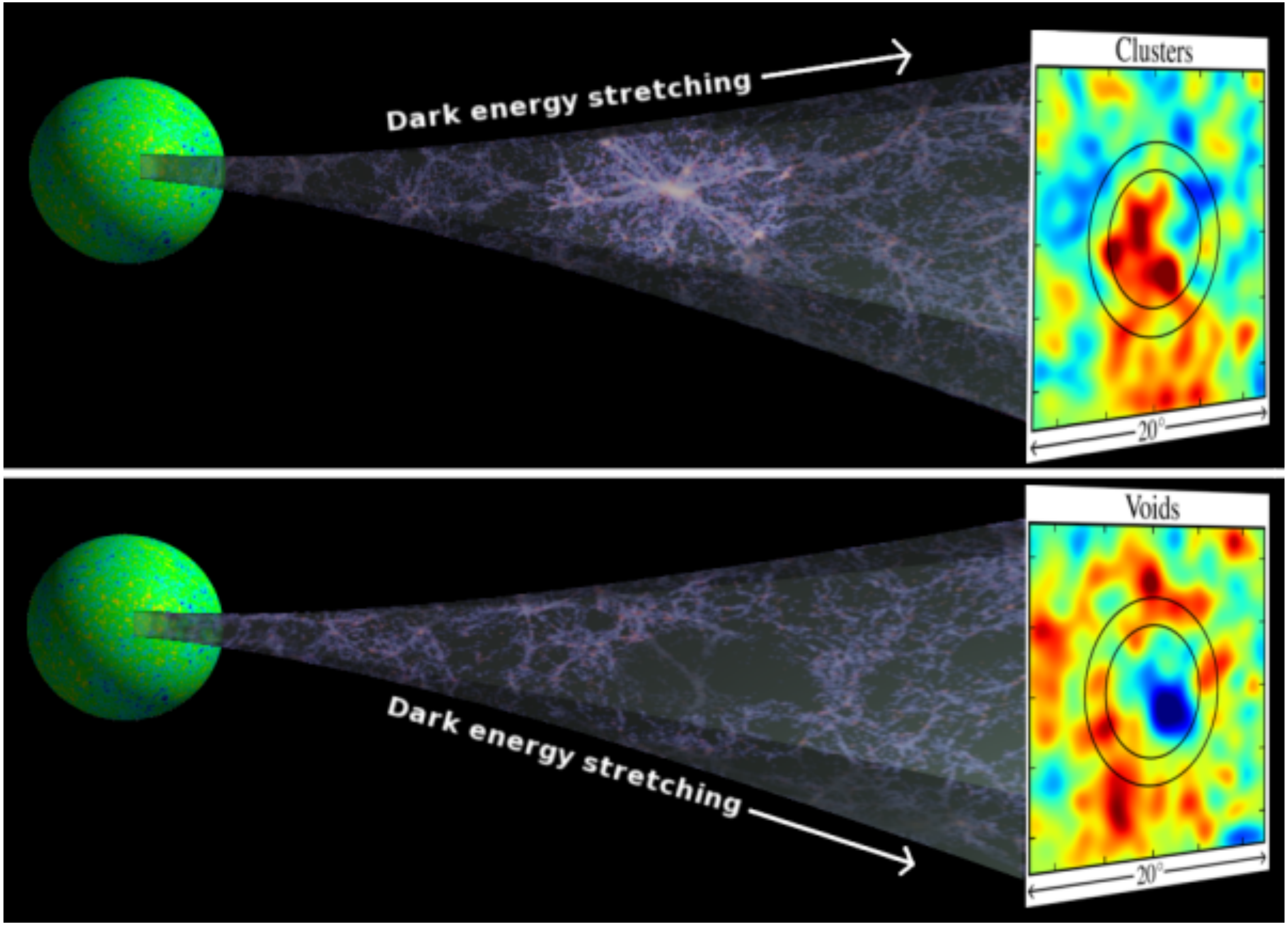
Could the Cold Spot in CMB be an "infloid" ?



A large void, approximately 2 Gpc in size

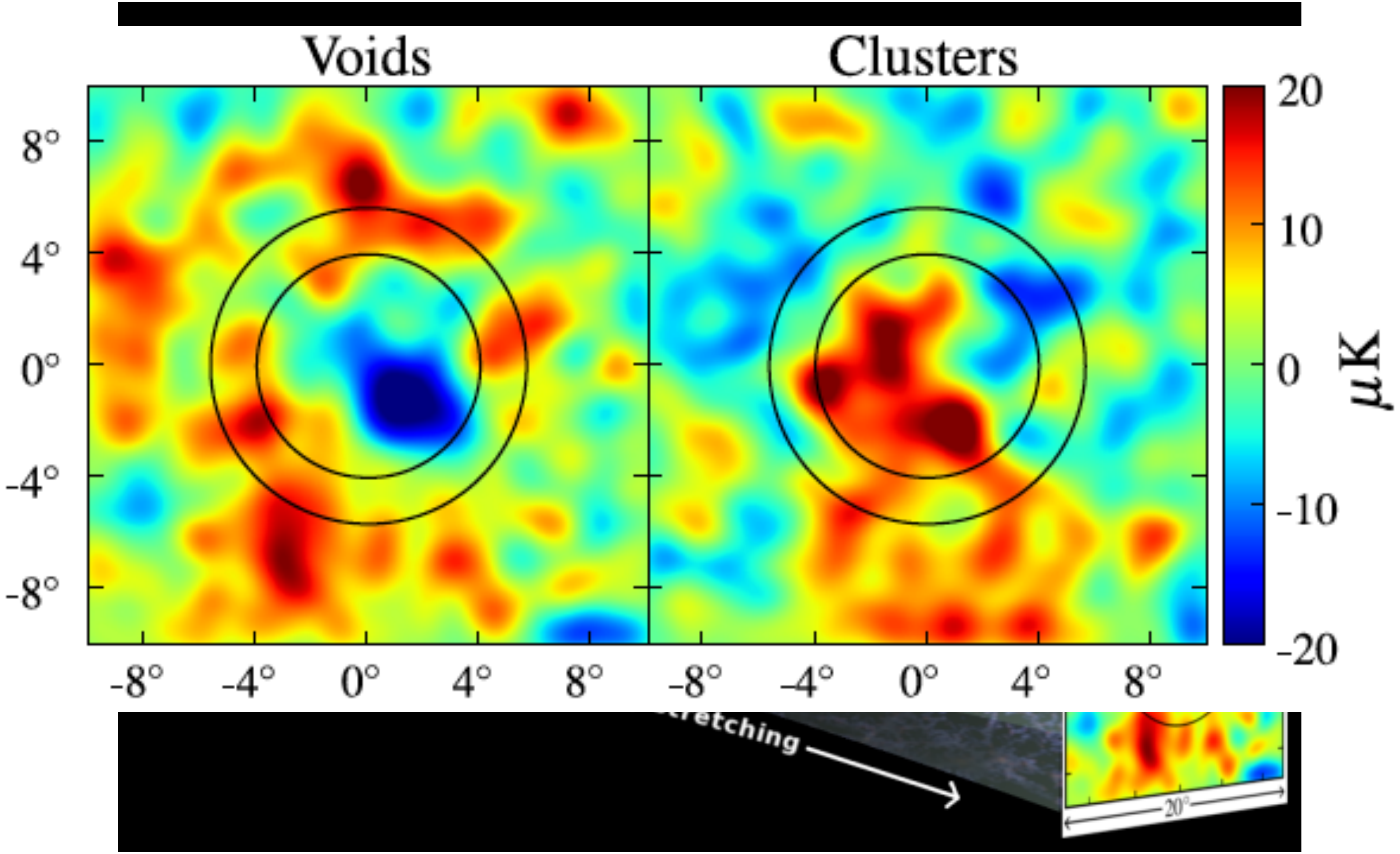
Voids and Superclusters in SDSS

Granett et al. (2008)



Voids and Superclusters in SDSS

Granett et al. (2008)



If Homogeneity and Isotropy is NOT assumed, How do we test it?

Independently of what model is behind inhomogeneity, one has several key tests:

- **Cosmic background shear**

$$\varepsilon \equiv \sqrt{\frac{3}{2}}\frac{\sigma}{\Theta} = \frac{H_T - H_L}{H_L + 2H_T}$$

- **Growth function f(a)**
- **Bulk velocities and kSZ effect on clusters**
- **Fractal dimension dim(z)**
- **ISW effect, LSS simulations**

The Lemaître-Tolman-Bondi Model

Celerier (1999), Tomita(2000), Moffat (2005), Alnes et al. (2005)

- Describes a space-time which has spherical symmetry in the spatial dimensions, but with time and radial dependence:

$$ds^2 = -dt^2 + X^2(r,t)\,dr^2 + A^2(r,t)\,d\Omega^2$$

- From the 0-r part of the Einstein-Equations we get:

$$X(r,t) \quad = \quad A'(r,t)/\sqrt{1-k(r)}$$

- One can recover the FRW model setting:

$$A(r,t) = a(t)\,r \qquad k(r) = k\,r^2$$

The Lemaitre-Tolman-Bondi Model

- Matter content:

$$T^{\mu}_{\nu} = -\rho_M(r,t)\,\delta^{\mu}_{0}\,\delta^{0}_{\nu}$$

- The other Einstein equations give:

$$\frac{\dot{A}^2 + k}{A^2} + 2\frac{\dot{A}\dot{A}'}{AA'} + \frac{k'(r)}{AA'} = 8\pi G\,\rho_M$$

$$\dot{A}^2 + 2A\ddot{A} + k(r) = 0$$

- Integrating the last equation:

Enqvist & Mattsson(2006)

$$\frac{\dot{A}^2}{A^2} = \frac{F(r)}{A^3} - \frac{k(r)}{A^2}$$

The Lemaitre-Tolman-Bondi Model

García-Bellido & Haugbølle (2008)

- All we need to specify:

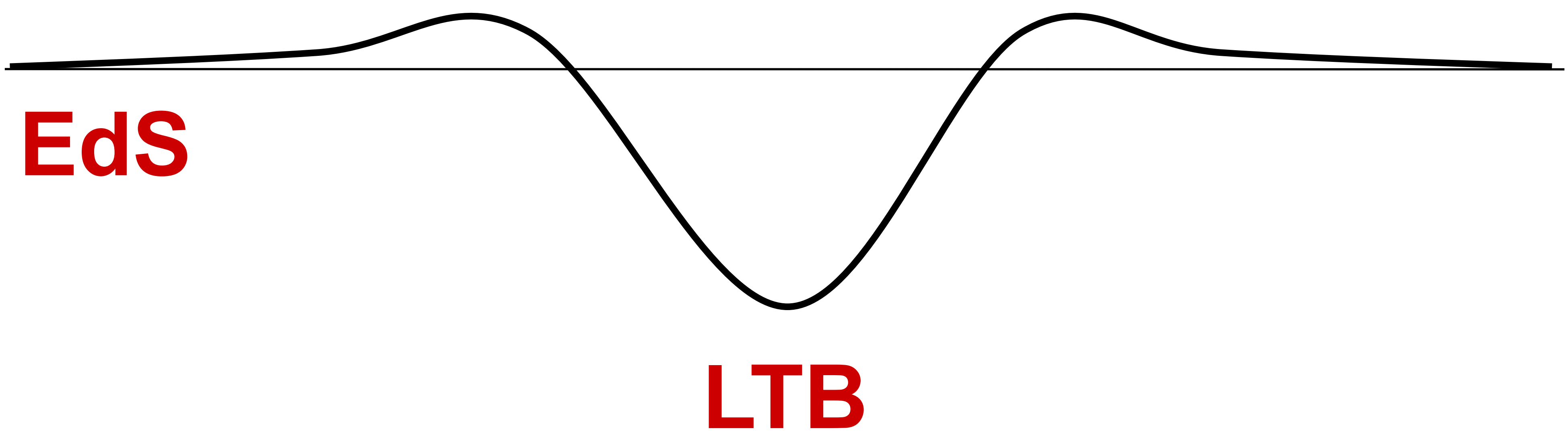
$$F(r) = H_0^2(r)\,\Omega_M(r)\,A_0^3(r)$$

$$k(r) = H_0^2(r)\Big(\Omega_M(r) - 1\Big)\,A_0^2(r)$$

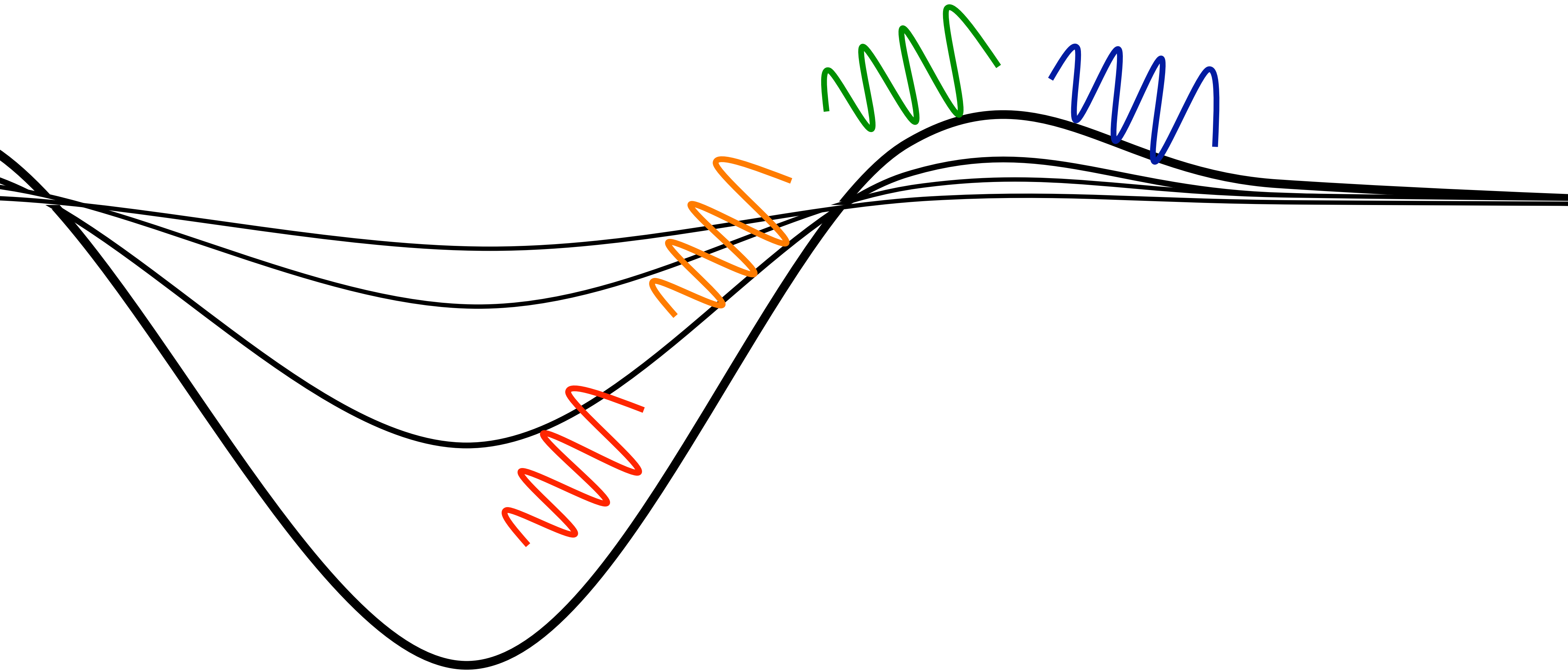
- Then the Hubble rate can be integrated to give A(r,t):

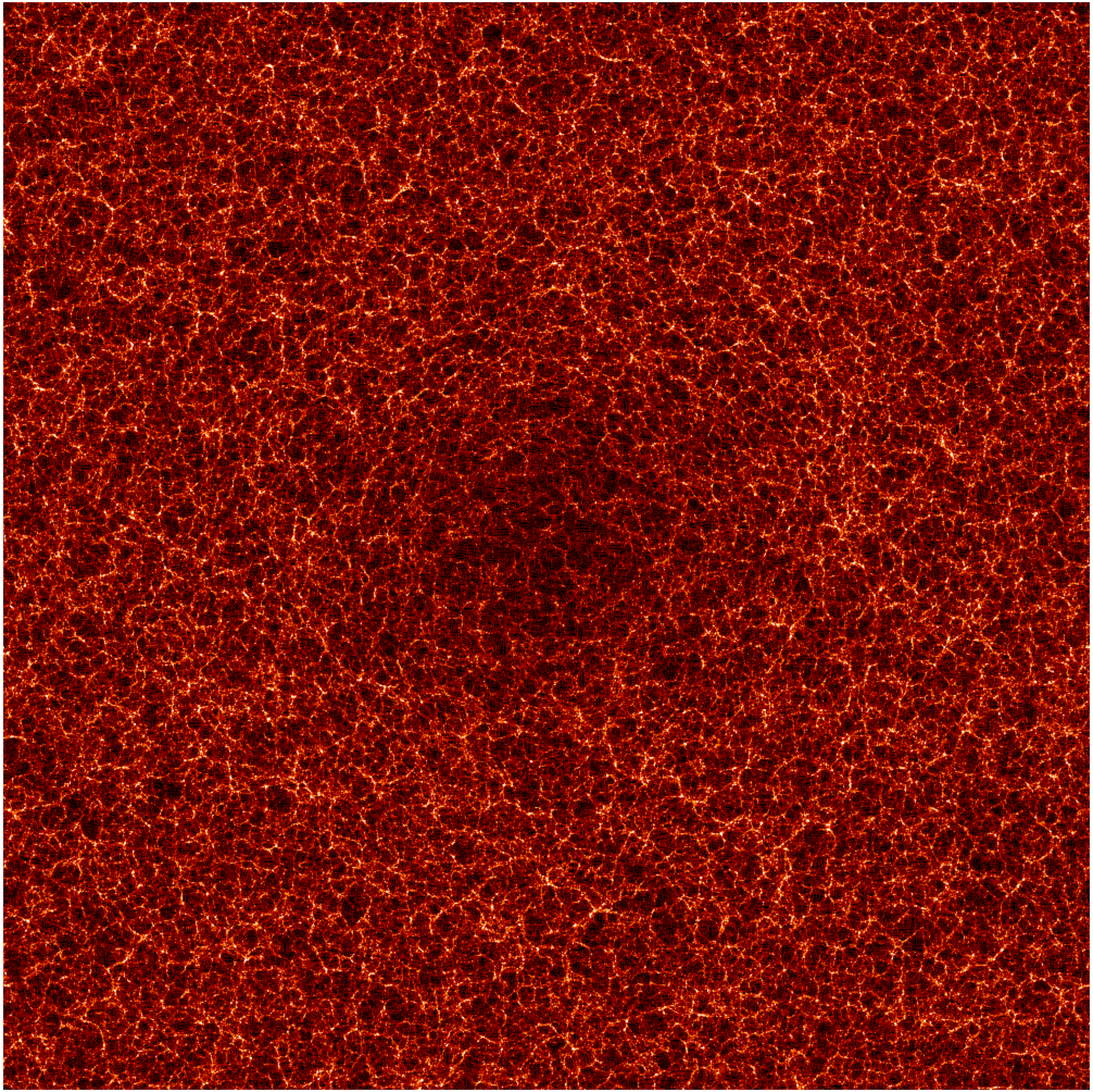
$$H^2(r,t) = H_0^2(r)\left[\Omega_M(r)\left(\frac{A_0(r)}{A(r,t)}\right)^3 + (1-\Omega_M(r))\left(\frac{A_0(r)}{A(r,t)}\right)^2\right]$$

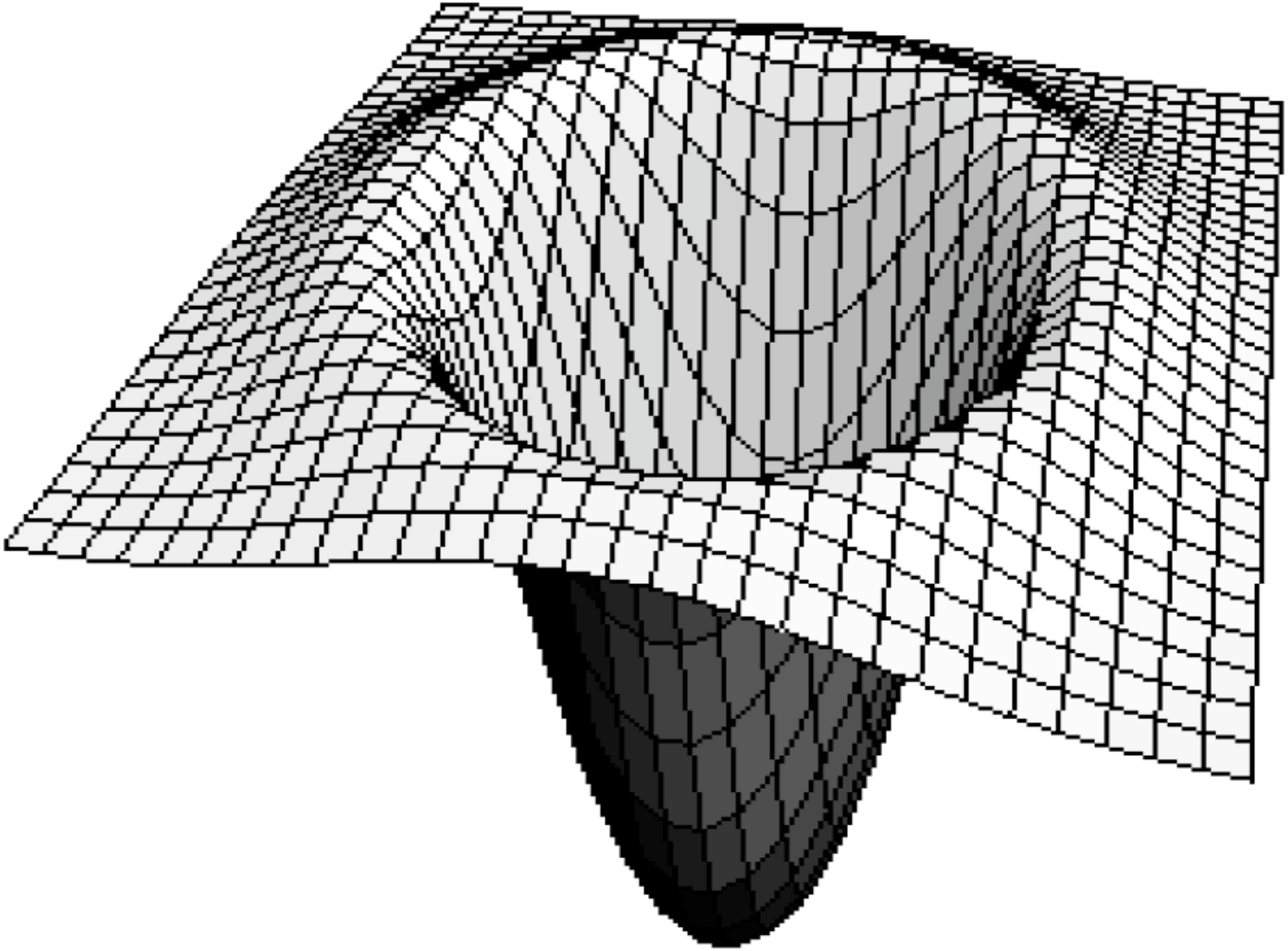
Density profile



Density profile







Alonso, JGB, Haugbølle, Vicente, PRD 82 (2010) 123530

Light Ray Propagation

- By looking at the geodesic equation, we can find the equation of motion for light rays:

$$\frac{dt}{dN} = -\frac{A'(r,t)}{\dot{A}'(r,t)} \qquad \frac{dr}{dN} = \frac{\sqrt{1-k(r)}}{\dot{A}'(r,t)}$$

where N = ln(1+z) are the # e-folds before present time.

- The various distances as a function of redshift are:

$$d_L(z) = (1+z)^2 A[r(z), t(z)]$$

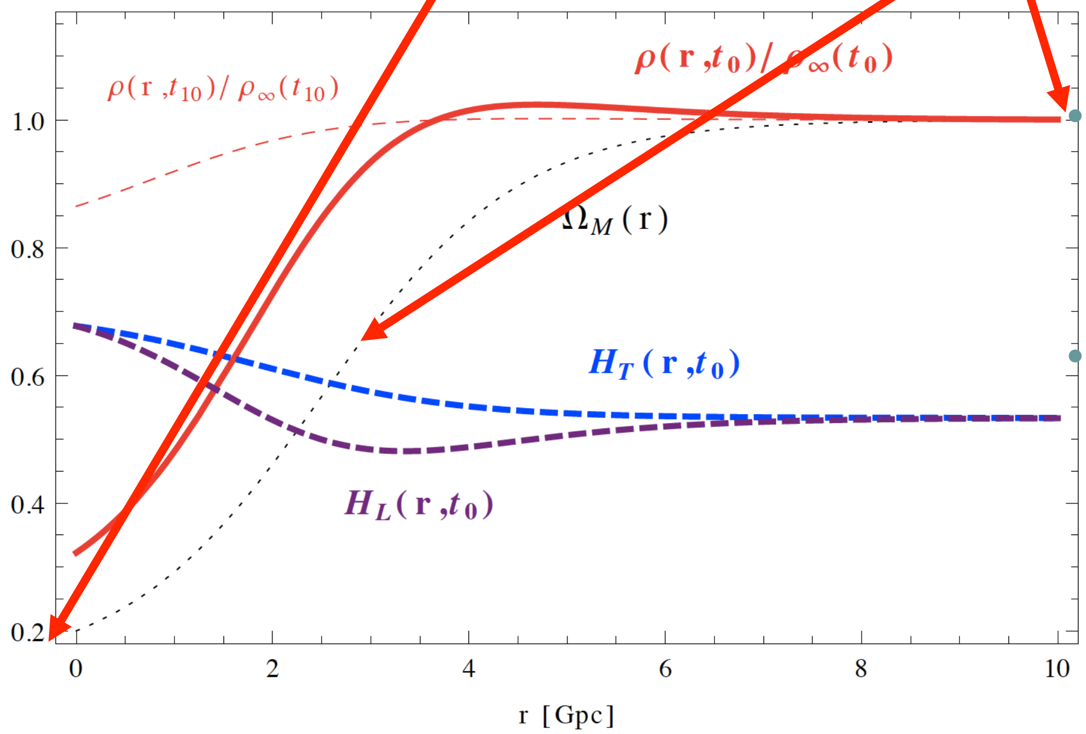
$$d_C(z) = (1+z)\, A[r(z), t(z)]$$

$$d_A(z) = A[r(z), t(z)]$$

The LTB-GBH model

$$\Omega_M(r) = \Omega_{\rm out} + \left(\boxed{\Omega_{\rm in}} - \boxed{\Omega_{\rm out}}\right)\left(\frac{1-\tanh[(r-\boxed{r_0})/2\Delta r]}{1+\tanh[r_0/2\Delta r]}\right)$$

$$H_0(r) = H_{\rm out} + \left(H_{\rm in} - H_{\rm out}\right)\left(\frac{1-\tanh[(r-r_0)/2\Delta r]}{1+\tanh[r_0/2\Delta r]}\right)$$



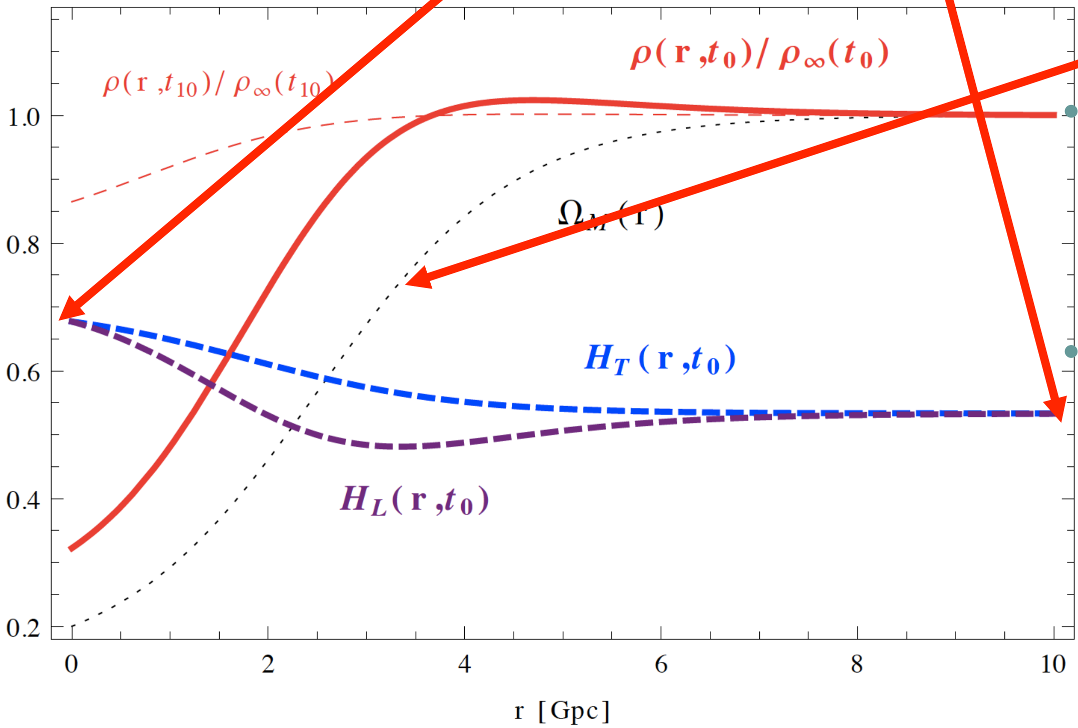
- If we assume asymptotic flatness, then the model has **5** parameters
- If we require a homogeneous Big Bang then the model has **4** parameters

García-Bellido & Haugbølle (2008)

The LTB-GBH model

$$\Omega_M(r) = \Omega_{\rm out} + \left(\Omega_{\rm in} - \Omega_{\rm out}\right)\left(\frac{1-\tanh[(r-r_0)/2\Delta r]}{1+\tanh[r_0/2\Delta r]}\right)$$

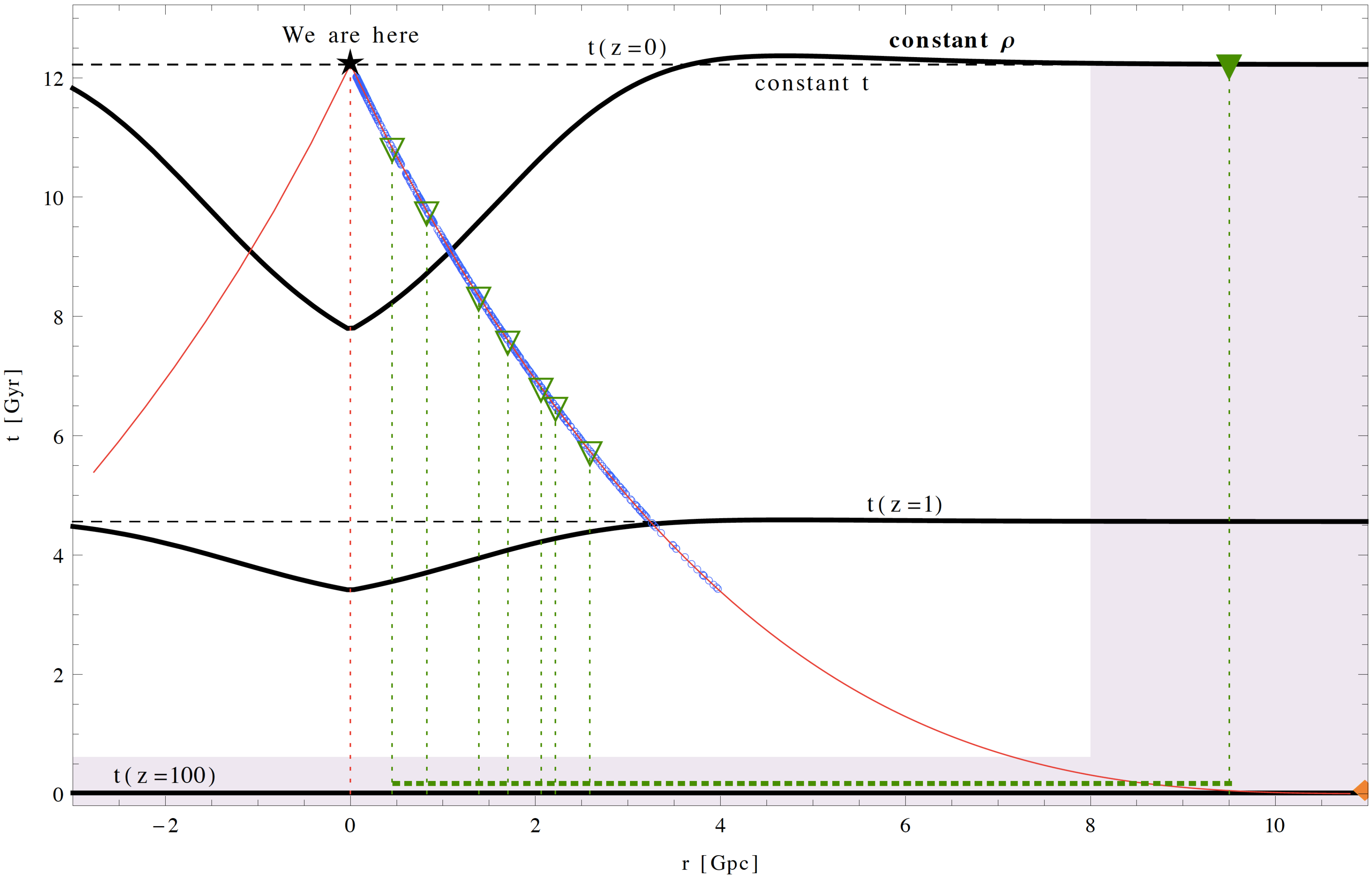
$$H_0(r) = H_{\rm out} + \left(\boxed{H_{\rm in}} - \boxed{H_{\rm out}}\right)\left(\frac{1-\tanh[(r-r_0)/2\Delta r]}{1+\tanh[r_0/2\boxed{\Delta r}]}\right)$$



- If we assume asymptotic flatness, then the model has **5** parameters
- If we require a homogeneous Big Bang then the model has **4** parameters

García-Bellido & Haugbølle (2008)

The LTB-GBH model



A useful observable: cosmic shear

García-Bellido & Haugbølle (2009)

$$\frac{d\Theta}{d\tau} = -\frac{1}{3}\Theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}n^\mu n^\nu$$

$$\varepsilon \equiv \sqrt{\frac{3}{2}}\frac{\sigma}{\Theta} = \frac{H_T - H_L}{H_L + 2H_T}$$ normalized shear

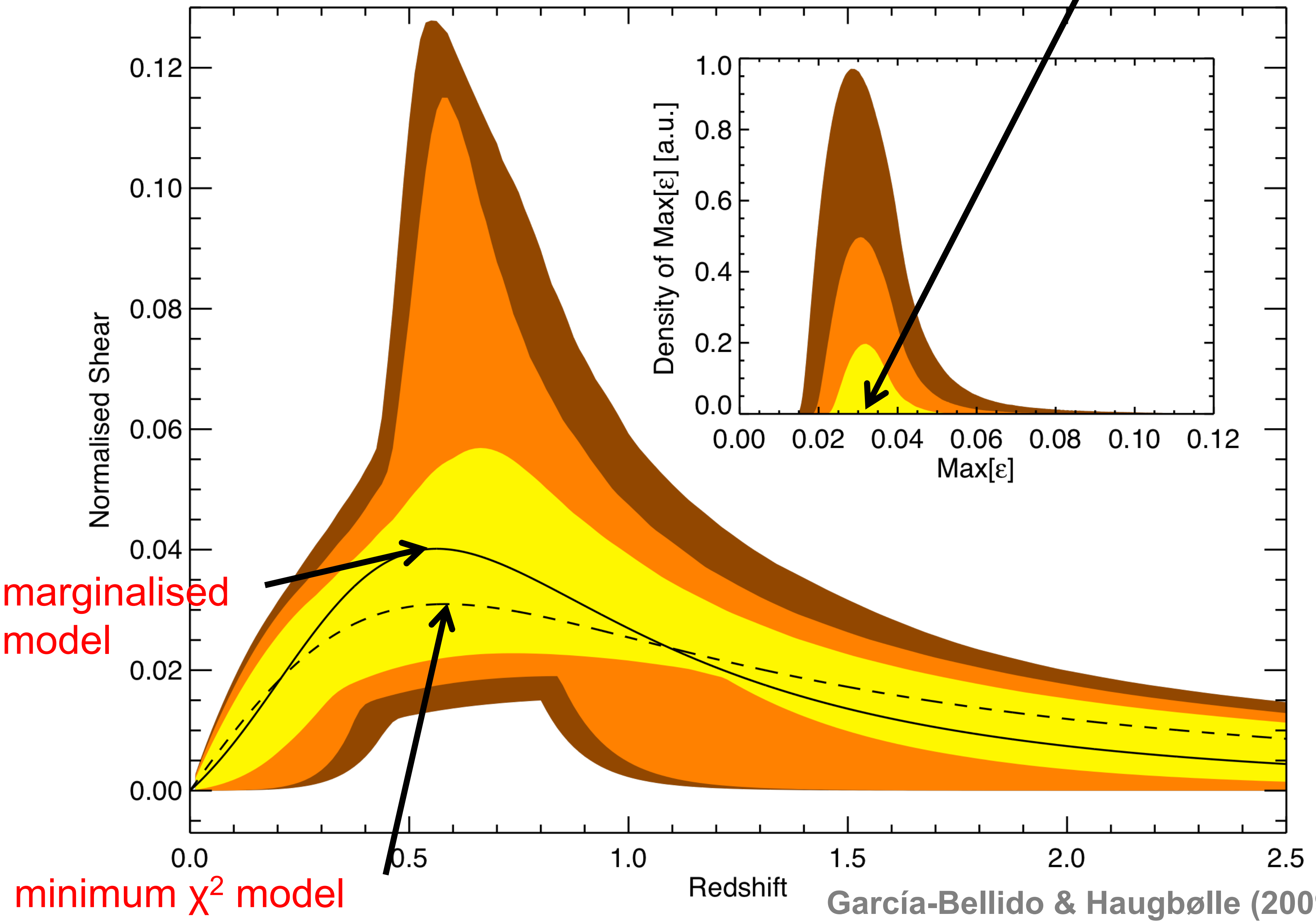
$$\boxed{\varepsilon(z) = \frac{1 - H_L(z)[(1+z)\,d_A(z)]'}{3H_L(z)d_A(z) + 2 - 2H_L(z)[(1+z)\,d_A(z)]'}}$$

FRW: $H_L = H_T = H$ shearless

$$(1+z)d_A = \int dz/H(z) \quad \Rightarrow \quad \varepsilon(z) = 0$$

Normalized Shear

$\varepsilon \approx 2$ - 5% at maximum



García-Bellido & Haugbølle (2009)

Constraining Cosmological Data

- Type Ia Supernovae: 557 SNIa Union 2 Supernovae

  Simple to do since we just fit against $d_L(z)$

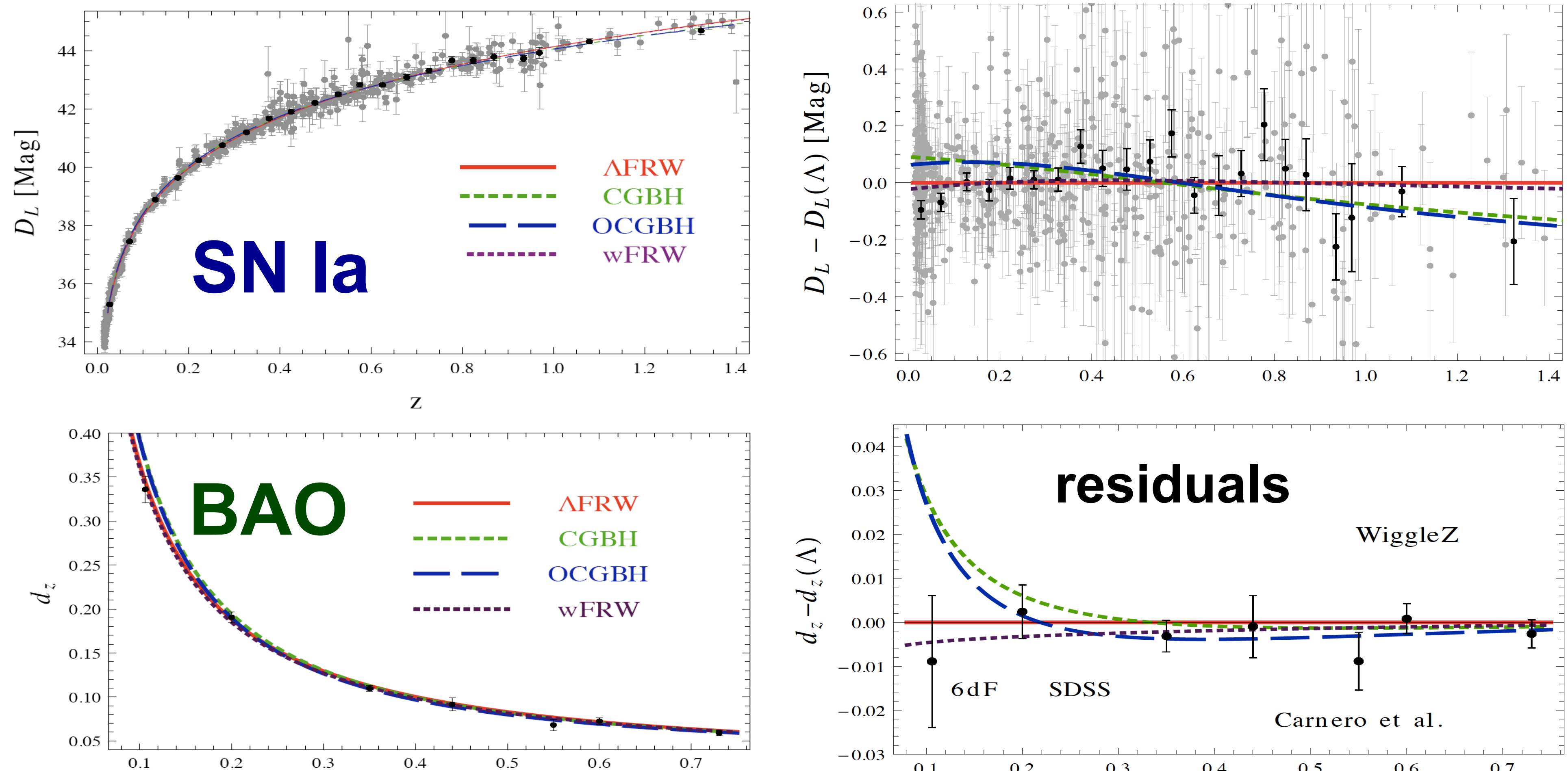
- Acoustic peaks in the CMB: $d(z_{rec})$, sound horizon $r_s(z)$

3 distances

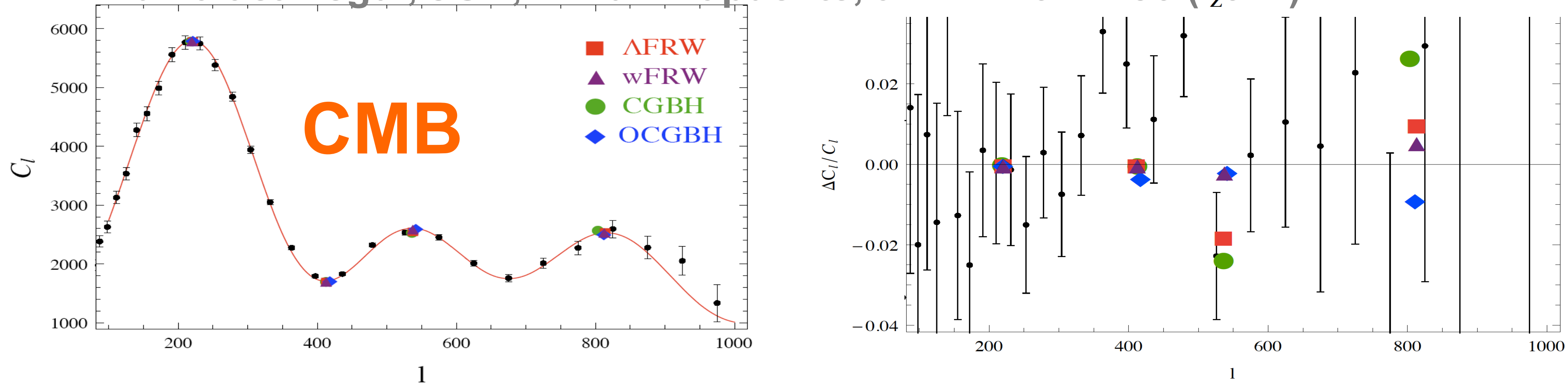
- Baryon Acoustic Oscillations:

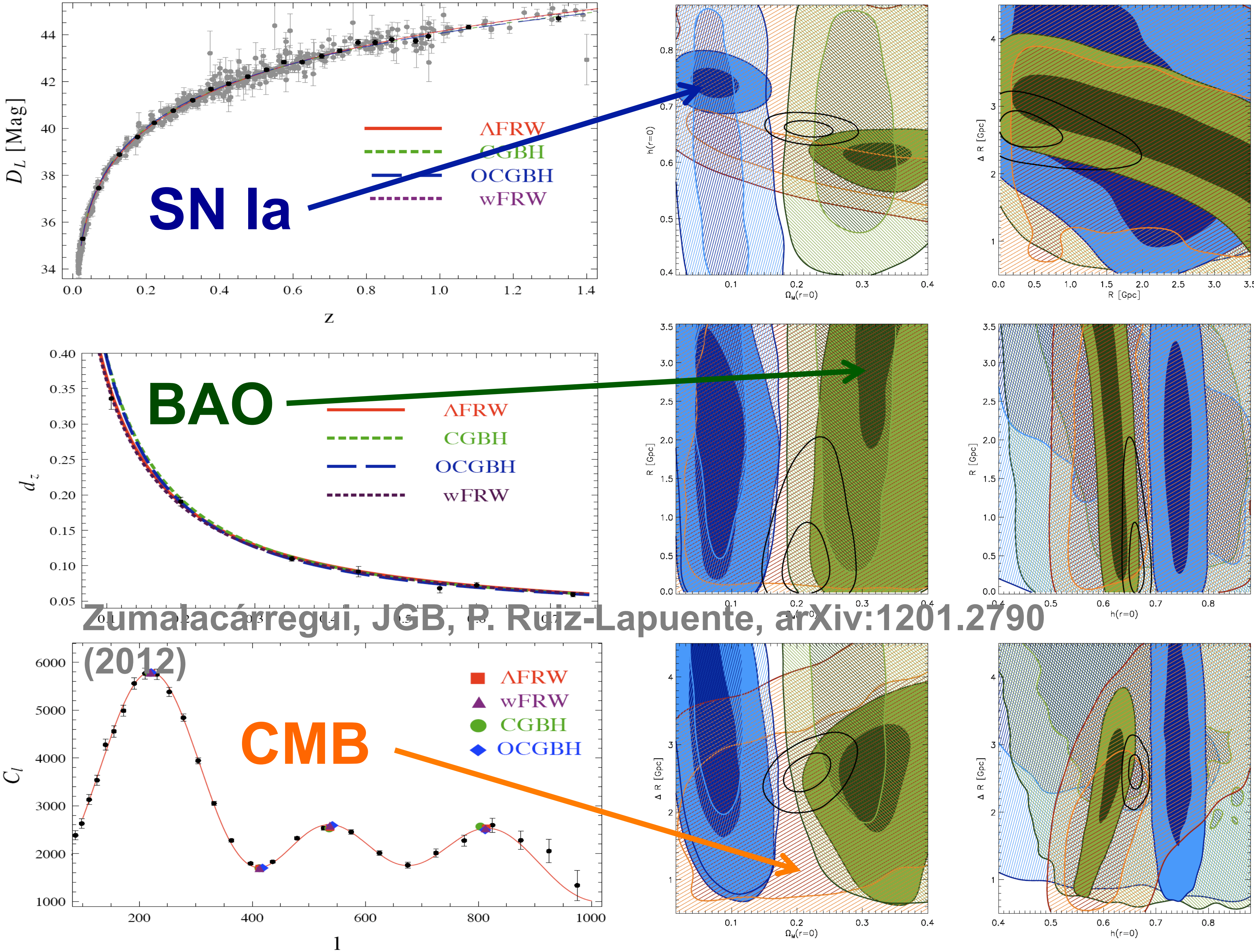
  Sound horizon $$D_V(z) = \left[d_A^2(z)(1+z)^2 \frac{cz}{H_L(z)} \right]^{1/3}$$

- Other constraints:
  - $f_{gas} = \rho_b / \rho_m = \omega_b / (\Omega_m h^2)$ adiabatic
  - HST+Cepheids (Riess): $H_0 = 73.8 \pm 2.4$ km/s/Mpc (1σ)
  - Globular cluster lifetimes ($t_{BB} > 11.2$ Gyr)



Zumalacárregui, JGB, P. Ruiz-Lapuente, arXiv:1201.2790 (2012)

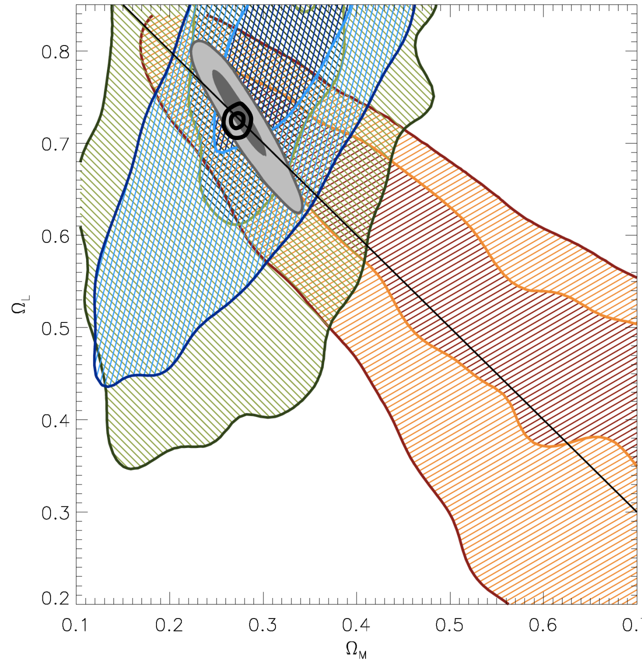




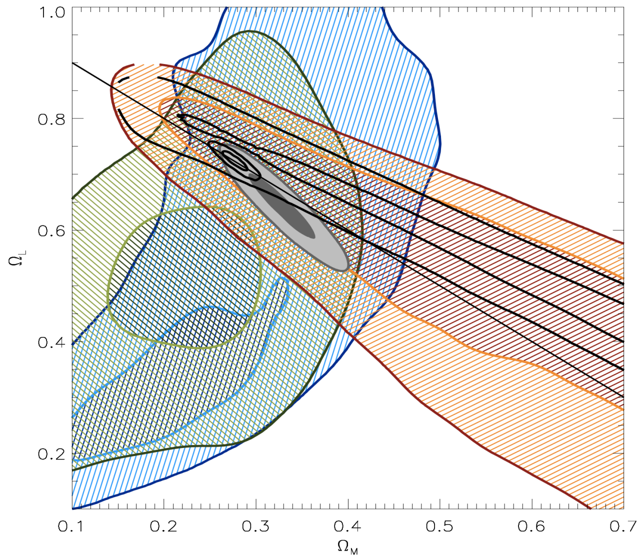
Zumalacárregui, JGB, P. Ruiz-Lapuente, arXiv:1201.2790 (2012)

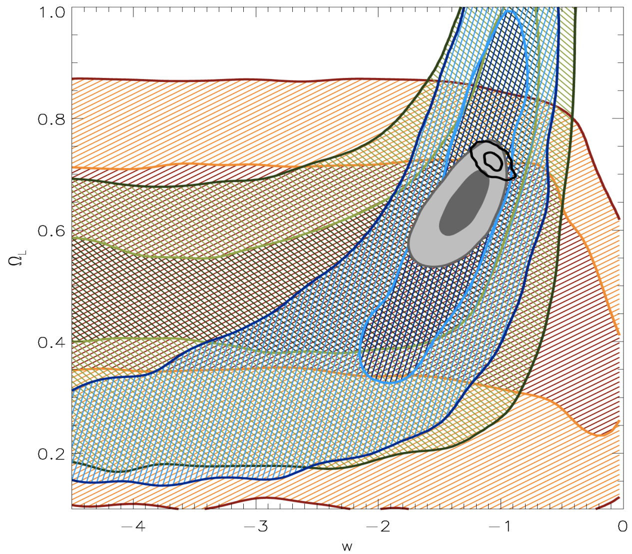
ΛCDM

For comparison (same data)



wCDM





ΛCDM

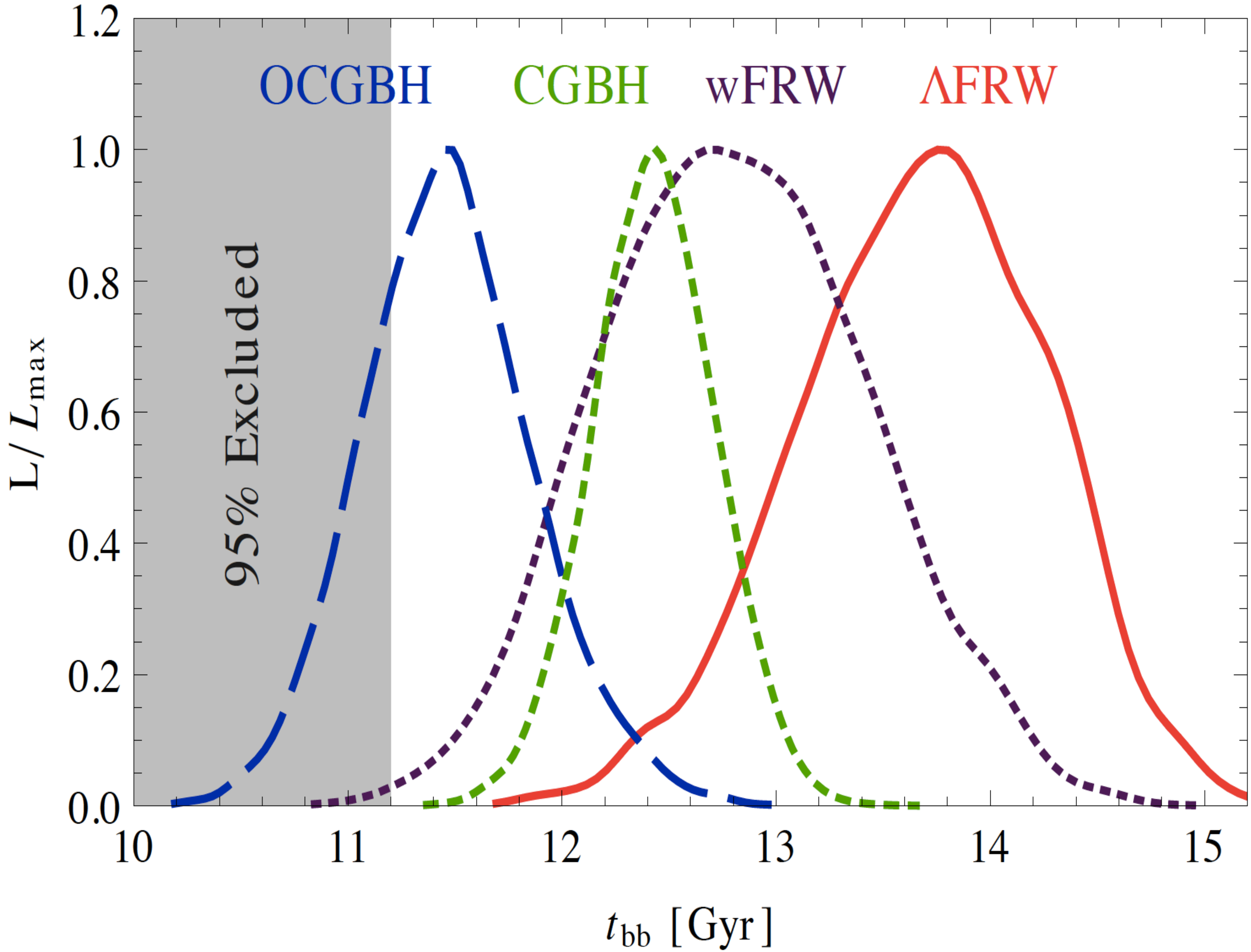
	H_0 [Mpc/km/s]	Ω_M	Ω_Λ	$100 f_b$	n_s
All - Min χ^2	70.7	0.28	0.72	17	0.97
Marginalized	$70.3^{+1.7}_{-1.5}$	0.27 ± 0.03	0.73 ± 0.05	17 ± 0.04	$0.99^{+0.06}_{-0.09}$

wCDM

H_0 [Mpc/km/s]	Ω_M	Ω_Λ	w	$100 f_b$	n_s
72.8	0.32	0.66	-1.26	12	0.87
73.5 ± 2.3	0.33 ± 0.04	0.64 ± 0.06	$-1.26^{+0.17}_{-0.22}$	$0.10^{+0.03}_{-0.02}$	$0.82^{+0.08}_{-0.06}$

CGBH

	H_{in}	Ω_{in}	R [Gpc]	dR [Gpc]	$100 f_b$	n_s
All - Min χ^2	66.4	0.21	0.02	2.78	7.7	0.74
Marginalized	66.0 ± 1.4	0.22 ± 0.04	$0.18^{+0.64}_{-0.18}$	$2.56^{+0.28}_{-0.24}$	7.7 ± 0.4	0.74 ± 0.03
BAO+CMB	61.6 ± 2.4	$0.32^{+0.06}_{-0.04}$	$3.92^{+0.48}_{-3.71}$	$2.76^{+0.50}_{-0.88}$	7.8 ± 0.8	0.73 ± 0.04
SNe+H0	74.0 ± 2.6	0.07 ± 0.04	$1.95^{+1.22}_{-1.82}$	$3.19^{+1.63}_{-1.66}$	-	-



OCGBH
CGBH
wFRW
ΛFRW
95% Excluded
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10
11
12
13
14
15
L / $L_{\max}$
t_{bb} [Gyr]

	CGBH	OCGBH	ΛCDM	wCDM
Union SNe	**539.94**	**539.06**	**530.70**	**530.40**
Hubble μ_0	**6.97**	**0.38**	**2.17**	**0.14**
6dF	5.35	4.73	0.35	0.09
SDSS	0.73	0.04	1.29	1.24
WiggleZ	0.65	1.2	0.93	0.63
Carnero et al.	0.78	0.12	0.61	0.34
Total BAO	**7.51**	**6.09**	**3.18**	**2.30**
Peak positions	0.87	0.3	0.96	0.07
Peak heights	1.13	0.11	0.24	0.04
Total CMB	**2.00**	**0.41**	**0.50**	**0.06**
Total χ^2	**556.45**	**545.96**	**536.56**	**532.94**
χ^2/d.o.f.	0.985	0.968	0.948	0.943
Akaike IC	568.6	560.1	546.6	545.0
Bayesian IC	594.5	584.0	568.3	571.0
$-\log E$	292.2	288.2	282.2	284.8

Bayesian analysis

- Posterior distribution for param θ, given model M & data D

$$\mathcal{P}(\theta, \mathcal{M}|\mathbf{D}) = \frac{\mathcal{L}(\mathbf{D}|\theta, \mathcal{M})\, \pi(\theta, \mathcal{M})}{E(\mathbf{D}|\mathcal{M})}$$

- Bayesian evidence: average likelihood L over priors π

$$E(\mathbf{D}|\mathcal{M}) = \int d\theta\ \mathcal{L}(\mathbf{D}|\theta, \mathcal{M})\, \pi(\theta, \mathcal{M})$$

- Bayes factor between competing models { i, j } :

$$B_{ij} \equiv \frac{E(\mathbf{D}|\mathcal{M}_i)}{E(\mathbf{D}|\mathcal{M}_j)}$$

Jeffreys' scale

- Occam's razor: Arbitrary scale on log Bij with unit steps

$\ln B_{ij} = 0$ undecisive

$\ln B_{ij} = 1$ weakly disfavoured

...

$\ln B_{ij} = 5$ strongly ruled out

- LTB-GBH model versus FRW-ΛCDM

$$\ln E(\Lambda CDM) = -\ 282.2$$

$$\ln E(GBH) = -\ 292.2$$

$$\ln B_{12} = 10$$

Discussion

- Void models, observationally, seem a natural alternative to the standard model. While they break away from the Copernican Principle, they do not need dark energy.
- There is no “coincidence problem”: Void was there always. The question “Why Now?” becomes “Why Here?”
- A void model with a size of ~ 2 Gpc yields a reasonable fit to observations constraining the geometry of the universe.
- However, comparing the model to observations there is Bayesian evidence against Large Void Models:
  - $\Delta E(\text{CGBH}-\Lambda\text{CDM}) = 10$ “Strongly ruled out”
  - Cosmic shear + bulk flows near z~0.5 (DES, PAU)
  - Remote measurements of the CMB: The kinematic Sunyaev-Zeldovich effect (ACT, SPT, Planck)