Radio Signatures from Conversion of axion-like particle to photons

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Axions, the strong CP problem and cosmology

In QCD an additional term of the form

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi}\,\theta\, G^\alpha_{\mu\nu}\tilde{G}^{\mu\nu}_\alpha = \frac{\alpha_s}{8\pi}\,\theta\,\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}G^\alpha_{\mu\nu}G^\alpha_{\lambda\sigma}\,,$$

with α_s the strong coupling constant and θ a CP-odd constant, is not forbidden by any symmetry, but would give rise to electric dipole moment for the neutron

$$d_n = 3.6\times 10^{-16}\theta\, e\,\mathrm{cm}$$

which upon comparing with experimental upper limit gives $\theta < 10^{-10}$.
A solution would be to promote θ to a pseudo-scalar field with a Lagrangian

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a\partial^\mu a + \frac{\alpha_s}{8\pi f_a}\,a\,G^\alpha_{\mu\nu}\tilde{G}^{\mu\nu}_\alpha + \frac{s\alpha_{\rm em}}{8\pi f_a}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu} - V_a(a)\,,$$

Non-perturbative QCD instantons lead to mixing with pions and gives zero-temperature potential of the form

$$V_a(a) \simeq m_u \Lambda^3_{\rm QCD}\left(1-\cos\frac{a}{f_a}\right)\,,$$

expanding in a gives the vacuum axion mass

$$m_a \simeq 6 \times 10^{-6} \left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right) \mathrm{eV}\,.$$

at finite temperature the axion mass is given by the topological susceptibility,

$$m_a^2(T) = \frac{\chi(T)}{f_a^2}\,, \quad \chi(T) = \frac{\partial^2 V_\theta}{\partial\theta^2}(\theta = 0, T) = \frac{\langle Q_5^2\rangle(\theta = 0, T)}{\mathcal{V}}\,,$$

This is essentially given by the fluctuations of the topological quantum number

$$Q_5(t) \equiv \int d^3\mathbf{r} j_5^0(\mathbf{r}) = N_L^q - N_R^q$$

which can be calculated approximately within the dilute instant approximation or numerically on the lattice.

Axion-like particles (ALPs) in general have independent mass m_a and coupling f_a and often only coupling to photons is considered.

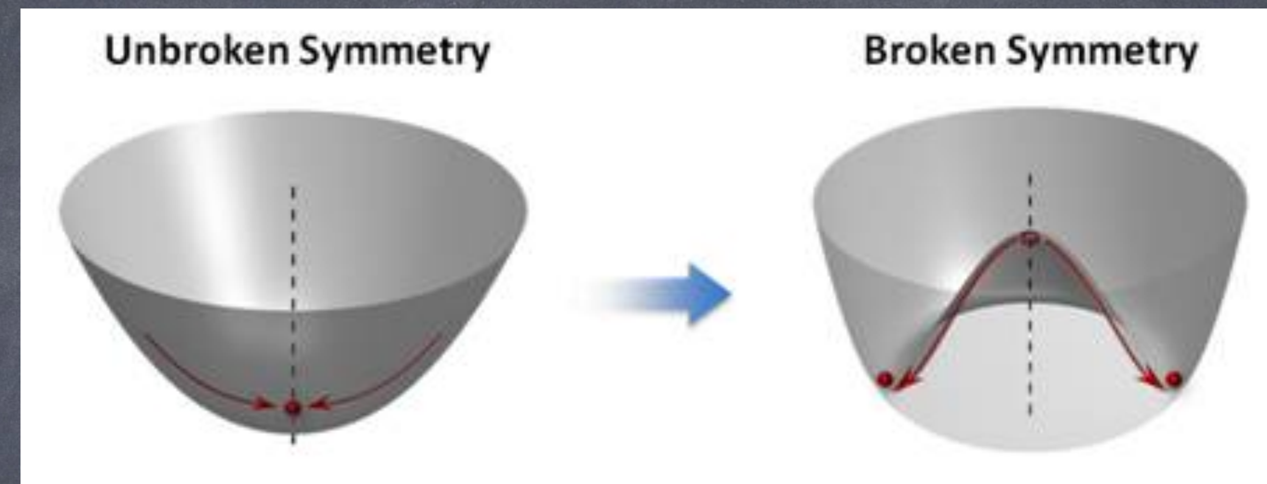
Axions, the strong CP problem and cosmology

a/f_a corresponds to an angular coordinate which at $T > f_a$ a chiral U(1) shift symmetry, known as Peccei-Quinn symmetry

spontaneous breaking of global Peccei-Quinn symmetry at temperature $T < f_a$: axion would be pseudo Nambu-Goldstone boson

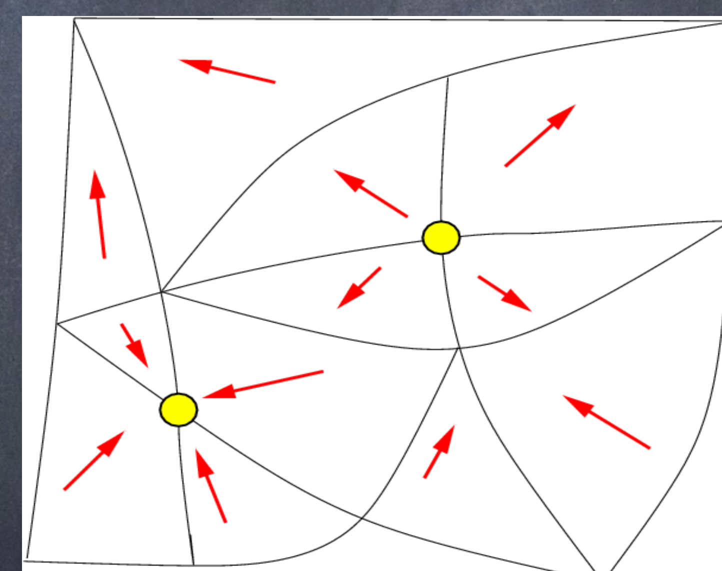
axion acquires mass at QCD scale due to mixing with pions -> tilted Mexican hat, solves strong CP-problem because axion field is naturally driven to zero

axion field is frozen for $H > m_a$ with random values uncorrelated over causal distances

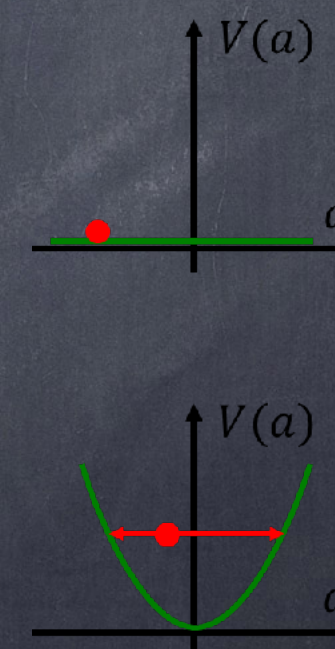


[Peking University]

$$\Phi(x) = \left[f_a + \rho(x)\right] e^{ia(x)/f_a}$$



[Uhlmann et al. `10]



[Raffelt]

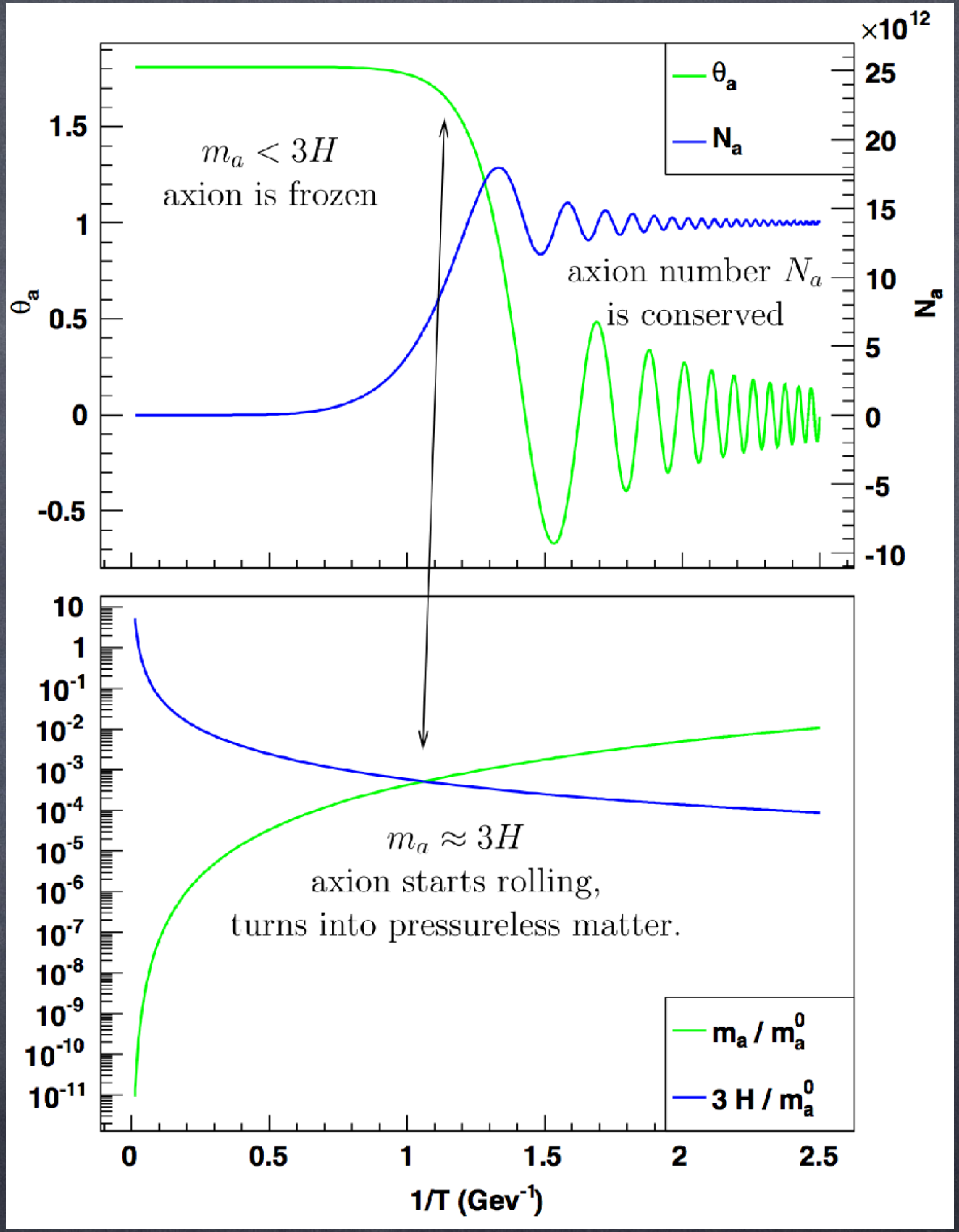
Once H < m_a axion field starts to oscillate in its potential and behaves as pressure-less non-relativistic cold dark matter when averaged over oscillations:

$$\rho = \frac{\dot{\phi}^2}{2} + V_a(a)\,, \quad p = \frac{\dot{\phi}^2}{2} - V_a(a)$$

resulting relic density has contributions from inflationary quantum fluctuations, possible cosmic string decays and the misalignment mechanism. The latter contributes

$$\Omega_a h^2 \simeq 7.4 \times 10^{-4} \left(\frac{m_a}{\mu\text{eV}}\right)^{1/2} \left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^2 \theta_{a,0}^2 \,.$$

Details depend on the temperature dependence of the axion mass



[Wantz,Shellard `09]

ALP-photon Coupling

fundamental coupling:

$$\frac{\alpha_{\rm em}}{8\pi}\frac{C_{a\gamma}}{f_a}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}=\frac{e^2}{32\pi^2}\frac{C_{a\gamma}}{f_a}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}=\frac{\alpha_{\rm em}}{8\pi}\frac{C_{a\gamma}}{f_a}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}=\frac{g_{a\gamma}}{4}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}\,, \tag{4}$$

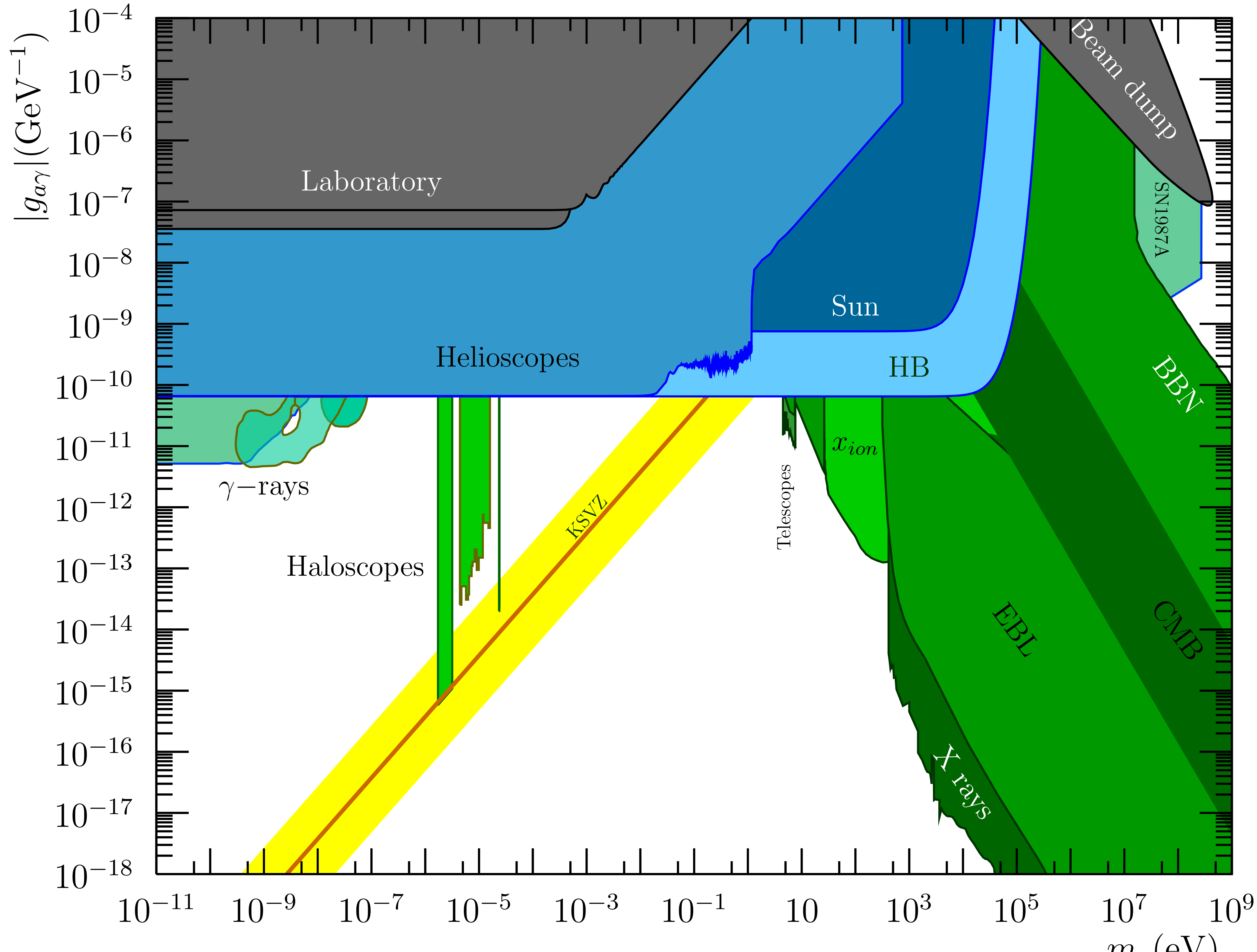
where $\alpha_{\rm em}=e^2/(4\pi\epsilon_0)$ and

$$g_{a\gamma}\equiv\frac{\alpha_{\rm em}C_{a\gamma}}{2\pi f_a}\,. \tag{5}$$

can give rise to following effects:

- Primakoff conversions between ALPs and photons in background electromagnetic fields -> shining light through a wall, helioscopes, haloscopes
- modified photon refraction in ALP background -> Mathieu-type equations
- parametric amplification of photon amplitudes in ALP background -> Mathieu-type equations

Overview Current Constraints



I. Irastorza, J. Redondo, Prog.Part.Nucl.Phys. 102 (2018) 89 [arXiv:1801.08127]

ALP-photon Conversion in Structured Magnetic Fields

The following is based on GS, PRD Phys.Rev. D96 (2017) 103014 [arXiv:1708.08908]

Energy-momentum conservation: quantities for ALP, photon and magnetic field carry subscript a, γ, or none, respectively:

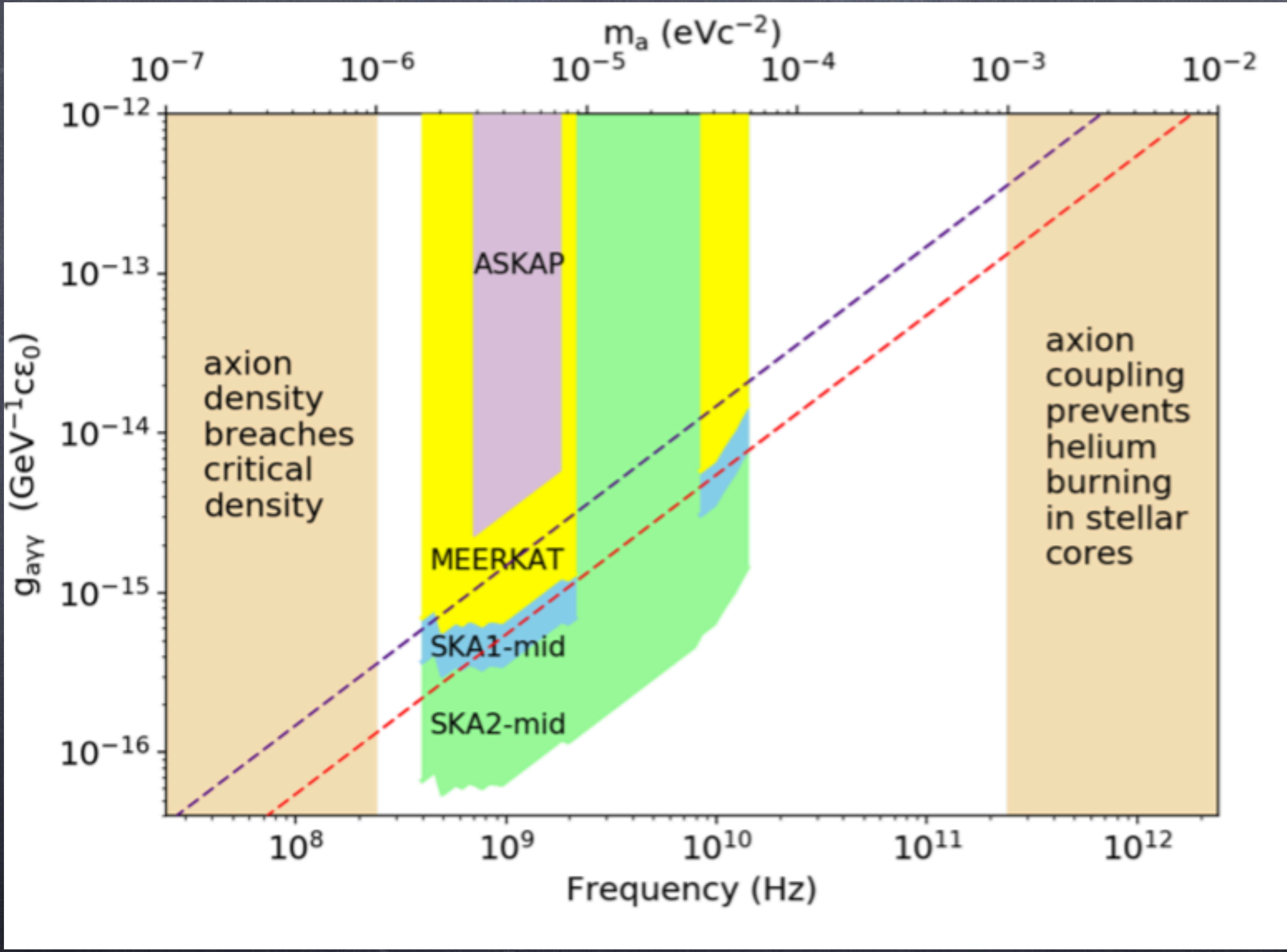
$$E_a = (m_a^2 + \mathbf{k}_a^2)^{1/2} = \omega_\gamma - \omega = (\omega_{\rm pl}^2 + \mathbf{k}_\gamma^2)^{1/2} - \omega\,, \quad \mathbf{k}_a = \mathbf{k}_\gamma - \mathbf{k}\,,$$

where the plasma frequency is given by:

$$\omega_{\rm pl} = \left(\frac{e^2 n_e}{\epsilon_0 m_e}\right)^{1/2} \simeq 5.6\times 10^4 \left(\frac{n_e}{\rm cm^{-3}}\right)^{1/2} {\rm rad\,s^{-1}}\,.$$

propagation of converted photons requires $m_a \gg \omega_{\rm pl}$. This will be the case for the objects considered here.

Also assume $n_e \sim$ constant here (non-resonant conversion) recently K. Kelley and P. J. Quinn, Astrophys. J. 845, 1 (2017) [arXiv:1708.01399] pointed out the possibility to search for ALP dark matter with radio telescopes; they used standard magnetic field estimates but assumed most of the power is on meter scales which is unlikely.



For non-relativistic (dark matter) ALPs, $k_a \ll m_a$, k_γ one can then show that the ALP-photon conversion rate can be written as

$$R_{a\to\gamma} = \frac{\pi\epsilon_0}{2} g_{a\gamma}^2 n_a \int \frac{d\omega}{T} d^3\mathbf{k}_\gamma \delta(\omega + E_a - \omega_\gamma) \sum_\lambda |\mathbf{B}(\omega, \mathbf{k}_\gamma - \mathbf{k}_a)\cdot \boldsymbol{\epsilon}_\lambda(\mathbf{k}_\gamma)|^2$$
$$= \frac{\pi\epsilon_0}{2} g_{a\gamma}^2 n_a \frac{1}{T} \int d^3\mathbf{k}_\gamma \sum_\lambda |\mathbf{B}(\omega_\gamma - E_a, \mathbf{k}_\gamma - \mathbf{k}_a)\cdot \boldsymbol{\epsilon}_\lambda(\mathbf{k}_\gamma)|^2 \,,$$

where ϵ_λ is the photon polarisation and T is the integration time. For MHD waves with dispersion relation $\omega = v_m k$, $v_m \ll 1$, this gives rise to a narrow line of relative width

$$\Delta \equiv \frac{\Delta k_\gamma}{k_\gamma} \simeq v_a^2/2 + v_m + \Delta v \lesssim 10^{-3} \,,$$

where $v_a \sim 10^{-3}$ = characteristic ALP velocity, Δv = velocity dispersion within the object and $v_m \lesssim$ Alfven velocity $\sim 10^{-5}\,(B/\mu G)(1\,\mathrm{cm}^{-3}/n)^{1/2}$
In the limit $\omega \ll k$ (almost static magnetic fields) this gives

$$R_{a\to\gamma} = \frac{\pi\epsilon_0}{2} g_{a\gamma}^2 n_a \int d^3\mathbf{k}_\gamma \delta(k_\gamma - E_a) \sum_\lambda |\mathbf{B}(\mathbf{k}_\gamma - \mathbf{k}_a)\cdot \boldsymbol{\epsilon}_\lambda(\mathbf{k}_\gamma)|^2 \,.$$

More compactly this can be written in terms of the magnetic field (static) power spectrum defined as

$$\rho_m = \frac{1}{2\mu_0 V}\int d^3\mathbf{r}|\mathbf{B}(\mathbf{r})|^2 = \frac{1}{2\mu_0 V}\int d^3\mathbf{k}|\mathbf{B}(\mathbf{k})|^2 = \int d\ln k\,\rho_m(k)\,,$$

Using $|\mathbf{k}_\gamma - \mathbf{k}_a| \sim k_\gamma \sim m_a$ and assuming a **homogeneous ALP distribution** with total mass $M_a = n_a m_a V$ this gives

$$R_{a\to\gamma} \simeq \pi g_{a\gamma}^2 \frac{M_a}{m_a^2}\rho_m(m_a)\,,$$

Integration over the line of sight dl this results in a **specific intensity per solid angle** [Jansky per steradian where 1 Jy = 10^{-26} W/(cm^2 Hz) = 10^{-23} erg/(cm^2 s Hz)]

$$I \simeq \pi \frac{g_{a\gamma}^2}{m_a^2}\frac{1}{\Delta}\int_{\rm l.o.s.} dl\,\rho_a(l)\rho_m(m_a,l)\,,$$

For a source at distance d containing total ALP mass M_a one similarly gets the **total flux density (Jansky)**

$$S \simeq \frac{\pi}{4d^2}\frac{g_{a\gamma}^2}{m_a^2}\frac{1}{\Delta}\int d^3\mathbf{r}\rho_a(\mathbf{r})\rho_m(m_a,\mathbf{r}) \simeq \frac{\pi}{4d^2}\frac{g_{a\gamma}^2}{m_a^2}\frac{1}{\Delta}M_a\rho_m(m_a)\,,$$

For a source radius r_s this corresponds to the **specific intensity** $I=S/\Omega_s$, which is independent of the distance:

$$I \simeq \frac{1}{4r_s^2}\frac{g_{a\gamma}^2}{m_a^2}\frac{1}{\Delta}M_a\rho_m(m_a)\,,$$

trade-off: significant small-scale magnetic field power favours small systems (stellar size) but small objects contain few ALPs -> most suitable objects ?

Application to Astrophysical Sources

radio photon frequency and wavenumber:

$$\nu = \omega_\gamma/(2\pi) = 242 \left(\frac{m_a}{\mu\text{eV}}\right) \text{MHz}\,, \quad \frac{1}{k} = 20 \left(\frac{m_a}{\mu\text{eV}}\right)^{-1} \text{cm}\,.$$

Ansatz for the magnetic field power spectrum:

$$\rho_m(k) = \frac{B^2}{2\mu_0} f(k)\,,$$

where f(k) is the fraction of the total power at k which is often written as a power law $f(k) \sim (kl_c)^n$ with l_c the coherence length, e.g. n=-2/3 for Kolmogorov turbulence, extending between l_c and the resistive scale

$$\lambda_r \simeq 0.9 \left(\frac{10^6\,\text{K}}{T}\right)^{3/2} \left(\frac{B_0}{\mu\text{G}}\right)^{-1} \left(\frac{n}{\text{cm}^{-3}}\right)^{1/2} \text{cm}\,.$$

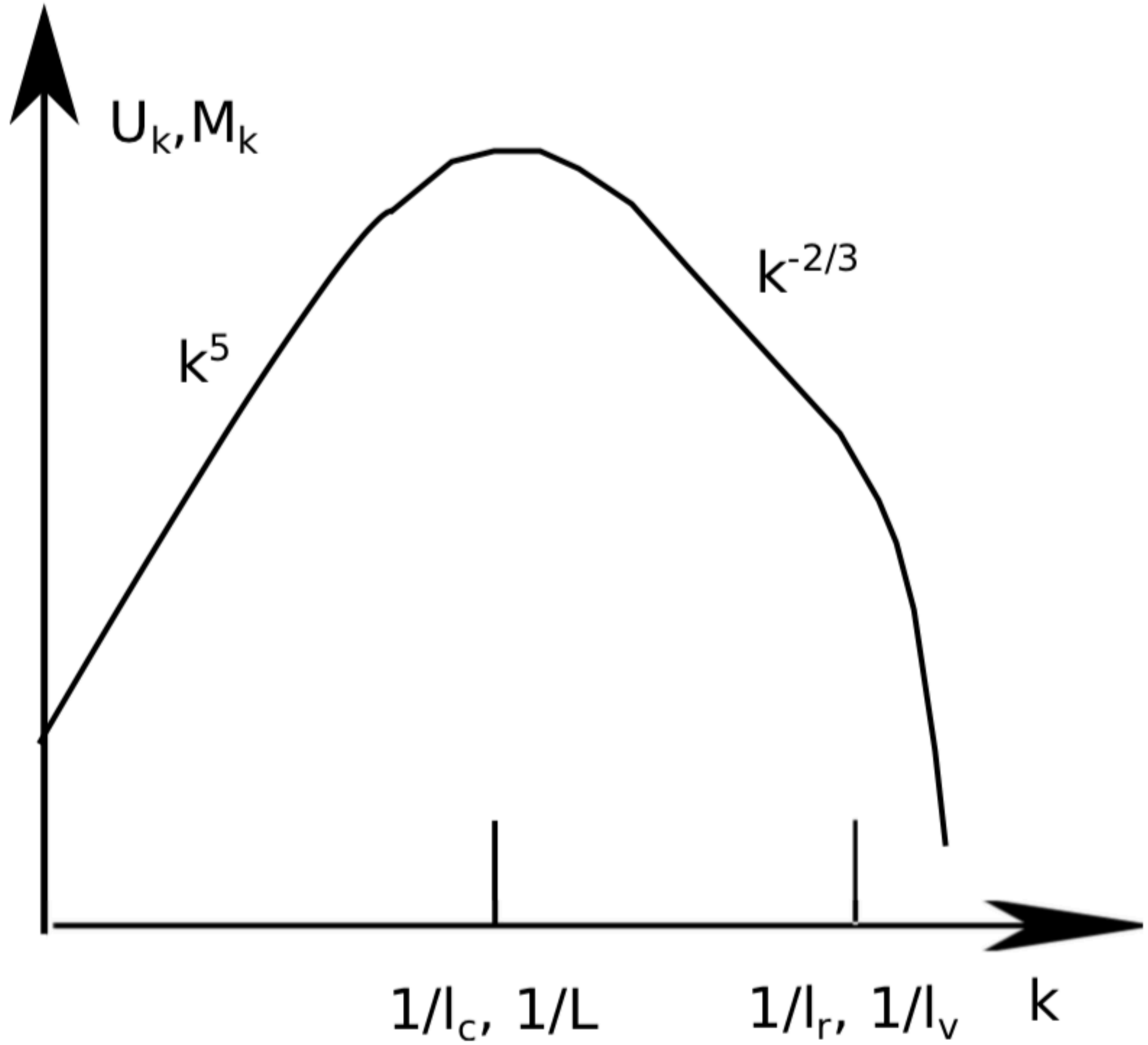


Fig. 3.4 A typical power spectrum of the magnetic field or kinetic fluid flow energy density per logarithm of wavenumber k, M_k and U_k, respectively, on logarithmic scales. In the inertial range between coherence length l_c of the magnetic field which is comparable to the energetically dominant eddy length L, and the resistive and viscous length scale l_r and l_v, respectively, a universal Kolmogorov turbulence spectrum $M_k, U_k \propto k^{-2/3}$ is indicated. At length scales $l \ll l_r, l_v$ the power spectrum is usually exponentially suppressed due to dissipation. At length scales $l \gg l_c, L$ the universal slope $M_k, U_k \propto k^5$ is indicated.

G. Sigl, book
"Astroparticle Physics:
Theory and Phenomenology",
Atlantis Press/Springer 2016

With these ansätze the conversion rate per ALP becomes

$$\frac{1}{\tau_a} \simeq \pi g_{a\gamma}^2 \frac{1}{m_a} \rho_m(m_a) \simeq 9.7 \times 10^{-29} \left(g_{a\gamma} 10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-1} \left(\frac{B}{\mathrm{G}}\right)^2 f(m_a)\,\mathrm{s}^{-1}\,,$$

Often one defines a brightness temperature by using the relation between temperature and specific intensity for black body radiation in the Rayleigh-Jeans limit:

$$T_b(\nu) \equiv \frac{c_0^2 I}{2\nu^2}\,,$$

which as I is independent of the distance. For our case $\nu = m_a/(2\pi)$ one gets

$$T_b(m_a) \equiv 2\pi^2 c_0^2 I / m_a^2 = 0.56 \left(\frac{I}{\mathrm{Jy/sr}}\right) \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-2} \mathrm{mK}\,.$$

For the galactic diffuse emission with galactic magnetic field amplitude B one gets

$$I \simeq 1.8\,\left(g_{a\gamma}10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-2} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{\rho_a}{0.3\,\mathrm{GeV cm^{-3}}}\right) \left(\frac{L}{8\,\mathrm{kpc}}\right) \times \left(\frac{B}{5\,\mu\mathrm{G}}\right)^2 f(m_a)\,\frac{\mathrm{mJy}}{\mathrm{sr}}\,,$$

where L is the characteristic length scale of the Milky Way. Advantage: **Insensitive to ALP inhomogeneities.**

For a discrete object at distance d containing total ALP mass M_a with characteristic magnetic field B the total flux density will be

$$S \simeq 2.8\times 10^{-11}\,\left(g_{a\gamma}10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-2} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{M_a}{10^{-10}\,M_\odot}\right) \left(\frac{d}{\mathrm{kpc}}\right)^{-2} \left(\frac{B}{\mathrm{G}}\right)^2 f(m_a)\,\mathrm{Jy}\,.$$

corresponding to a brightness temperature of

$$T_b \simeq 5\,\left(g_{a\gamma}10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-4} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{M_a}{10^{-10}\,M_\odot}\right) \left(\frac{r_s}{\mathrm{pc}}\right)^{-2} \left(\frac{B}{\mathrm{G}}\right)^2 f(m_a)\,\mathrm{nK}\,.$$

Supernova remnants and magnetized stellar winds have $r_s \sim$ pc, B ~ mG, d ~ kpc: Radius r_s of termination shock can be estimated by equating mass of interstellar medium swept up with ejecta mass:

$$r_s \sim \left(\frac{3M_e}{4\pi m_N n_0}\right)^{1/3} \simeq 2.1 \left(\frac{M_e}{M_\odot}\right)^{1/3} \left(\frac{1\,\mathrm{cm}^{-3}}{n_0}\right)^{1/3} \mathrm{pc}\,,$$

Magnetic field can be estimated from equipartition of magnetic and kinetic wind energy:

$$B(r_s) \sim \left(\frac{3\mu_0 M_e}{4\pi r_s^3}\right)^{1/2} v_w \simeq (\mu_0 m_N n_0)^{1/2} v_w \simeq 1.9\times 10^{-3} \left(\frac{n_0}{1\,\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{v_w}{10^{-2}}\right) \mathrm{G}\,,$$

This gives:

$$S \simeq 2.1\times 10^{-8} \left(g_{a\gamma} 10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-2} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{d}{2\,\mathrm{kpc}}\right)^{-2} \left(\frac{B}{10^{-3}\,\mathrm{G}}\right)^2 f(m_a)\,\mathrm{Jy}\,,$$

$$T_b \simeq 3.8 \left(g_{a\gamma} 10^{14}\,\mathrm{GeV}\right)^2 \left(\frac{m_a}{\mu\mathrm{eV}}\right)^{-4} \left(\frac{10^{-3}}{\Delta}\right) \left(\frac{B}{10^{-3}\,\mathrm{G}}\right)^2 f(m_a)\,\mu\mathrm{K}\,.$$

Supernova remnants are too radio-loud [10^3 Jy, brightness temperature ~ 10^5 K], but stellar winds, e.g. Wolf-Rayet stars could be sufficiently quiet.

Other possible ALP objects:

An "axion asteroid" would have ~ 10^{-13} M_{solar}, closest distance d ~ 100 AU ~ 5×10^{-7} kpc; for $g_{a\gamma}$ ~ 1/(10^{14} GeV), m_a ~ 10^{-6} eV and typical galactic field strength one gets 10^{-11} f(m_a) Jy at solid angles much smaller than the beam size. There would be many of them in the Milky Way, could this provide a measurable signature ? Probably comparable to diffuse flux (see D. Marsh, Cambridge)

Magnetic Field Structure

coherence length l_c is often of order size of the object unless instabilities act on small scales:

Weibel instability would develop on the Debye length scale

$$\lambda_D = \frac{\bar{v}}{\omega_{\rm pl}} \simeq \left(\frac{\epsilon_0 T_e}{e^2 n_e}\right)^{1/2} \simeq 6.9 \times 10^3 \left(\frac{T_e}{10^6{\rm K}}\right)^{1/2} \left(\frac{{\rm cm}^{-3}}{n_e}\right)^{1/2} {\rm cm}\,,$$

Bell instability can develop self-induced magnetic fields by accelerated cosmic rays and would develop on the gyro radius of accelerated cosmic ray ions

$$r_g \sim 3 \times 10^9 \left(\frac{10^{-3}{\rm G}}{B}\right) \left(\frac{p/Z}{\rm GeV}\right) {\rm cm}\,.$$

One needs MHD modes with $\omega = v_m k \ll k$ but their intensity in hot plasmas with magnetic fields coherent on the wavenumber scale m_a is hard to determine and needs to be worked out as a next step.

Example: MHD modes in cold, magnetised medium

Blandford+Thorne: Applications of classical physics (2003)

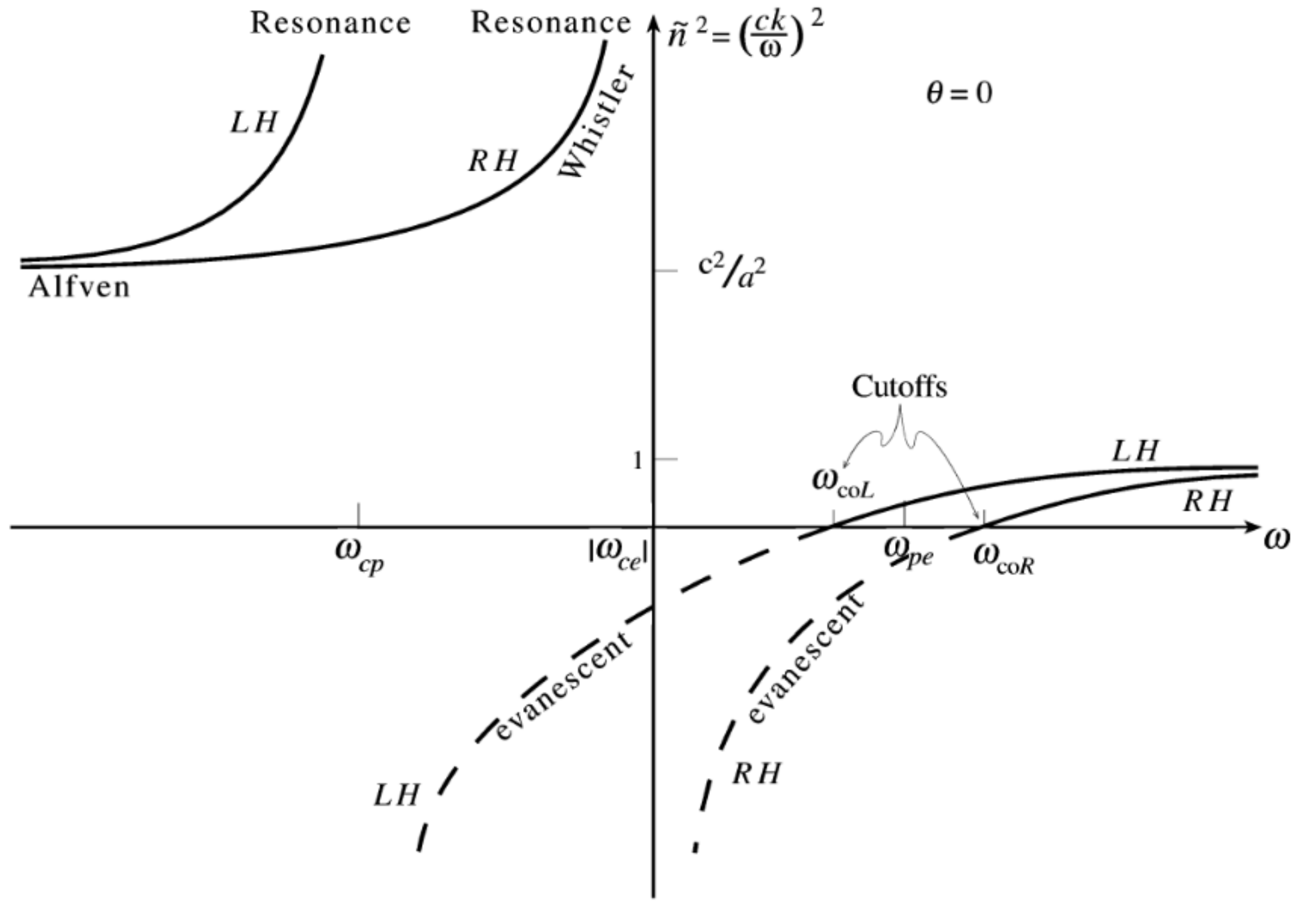


Fig. 20.3: Square of wave refractive index for circularly polarized waves propagating along the static magnetic field in a proton-electron plasma with $\omega_{pe} > \omega_{ce}$. (Remember, that we will regard both the electron and the proton cyclotron frequencies as positive numbers.) The angular frequency is plotted logarithmically in units of the modulus of the electron gyro frequency.

For $\omega < \omega_{ce}$ (electron cyclotron frequency) $< \omega_{pl}$ there are Whistler modes that could give significant power. However, this requires $v_s \sim (T/m_N)^{1/2} < \omega/k \sim \omega_{c,e}/m_a \sim 10^{-5}$ (B/mG)(μeV/m_a) which is hard to fulfil given that temperatures are T ~ eV-keV: need to better understand MHD modes in hot magnetised media

Detectability in Radio Telescopes

The effective solid angle of a single Gaussian beam is given by

$$\Omega_b \simeq \theta^2 \simeq \frac{1}{(l\nu)^2} = \frac{1}{A\nu^2}\,, \tag{36}$$

where θ is the angular radius of the beam, l is the effective length scale of the interferometer and $A = l^2$ its effective area. If a discrete source extends over several beams, the sensitivity in brightness temperature is increased by a factor $N_b^{1/2} = (\Omega_s/\Omega_b)^{1/2}$ relative to a single beam so that the minimal detectable brightness temperature is given by

$$T_{b,\mathrm{min}} \simeq \frac{T_{b,\mathrm{min0}}}{N_b^{1/2}} = T_{b,\mathrm{min0}} \left(\frac{\Omega_b}{\Omega_s}\right)^{1/2}\,, \tag{37}$$

where $T_{b,\mathrm{min0}}$ is the sensitivity for a single beam. In general one has

$$T_{b,\mathrm{min0}} \simeq \frac{T_{\mathrm{noise}}}{(Bt)^{1/2}}\,, \tag{38}$$

where T_{noise} is the effective noise temperature, resulting from system and sky temperature added in quadrature, B is the bandwidth and t is the observing time. One also often uses the antenna temperature induced by a total flux density S defined by

$$T_a \equiv \frac{AS}{2} = 0.36 \left(\frac{A}{10^3\,\mathrm{m}^2}\right)\left(\frac{S}{\mathrm{Jy}}\right)\,\mathrm{K}\,. \tag{39}$$

Combining this with Eqs. (25) and (36) and the relation $S = I\Omega_s$ this shows that

$$\frac{T_a}{T_b} = N_b = \frac{\Omega_s}{\Omega_b}\,. \tag{40}$$

If the noise in one beam is again characterized by the temperature $T_{b,\min 0}$, the noise in N_b beams corresponds to $N_b^{1/2} T_{b,\min 0}$. Comparing this with the total signal temperature T_a again gives a brightness temperature sensitivity improvement by a factor $N_b^{1/2}$. Equivalently, the smallest detectable total source flux density can be expressed as

$$S_{\min} = N_b^{1/2} S_b \,, \tag{41}$$

where S_b is the minimal detectable flux density per beam. Since $S_{\min}$ and S_b are proportional to $T_{b,\min 0}$ which according to Eq. (38) is proportional to $1/t^{1/2}$, one often denotes the minimal detectable source flux density in units of $\mathrm{Jy\,hr^{-1/2}}$.

LOFAR HBA:

at ν = 140 MHz, beam size 5 arcsec, $\Omega_b = 2\times10^{-9}$ sr, sensitivity per beam $S_b \sim 10^{-4}$ Jy
for canonical case r_s = 2 pc, d = 2 kpc this gives $T_b \sim 2$ [d/(2 kpc)] K,
$S_{min} \sim 4\times10^{-3}$ [d/(2 kpc)] Jy; comparing with prediction gives sensitivity to

$$g_{a\gamma} \gtrsim 2.5\times10^{-12}\,[m_a/(0.58\,\mu\mathrm{eV})][\Delta/10^{-3}]^{1/2}[d/(2\,\mathrm{kpc})]^{1/2}\,\mathrm{GeV}^{-1}/f(m_a)$$

SKA-low:

frequency range (50-250) MHz, beam size ~ square degree, $\Omega_b = 3\times10^{-4}$ sr, larger than typical source, flux density sensitivity S ~ 10^{-5} Jy/hr$^{1/2}$
this gives $T_b \sim 10^{-5}$ K/hr$^{1/2}$, corresponding to $T_{noise} \sim 10$ K and bandwidth B ~ 300 MHz; comparing with prediction for S gives sensitivity in one hour to

$$g_{a\gamma} \gtrsim 2\times10^{-13}\,[m_a/\mu\mathrm{eV}][\Delta/10^{-3}]^{1/2}[d/(2\,\mathrm{kpc})]^{1/2}\,\mathrm{GeV}^{-1}/f(m_a)$$

Resonant Primakoff Conversion

Full conversion (e.g. resonance between ALP mass and plasma frequency at distance r_s from neutron star center) gives

$$S_{\rm max} \simeq \frac{\rho_a}{m_a}\frac{v_a}{\Delta}\left(\frac{r_s}{d}\right)^2 \simeq 10^{-10}\left(\frac{m_a}{\mu{\rm eV}}\right)^{-1}\left(\frac{r_s}{10^6\,{\rm cm}}\right)^2\left(\frac{d}{\rm kpc}\right)^{-2}\,{\rm Jy}\,,$$

see also M.S.Pshirkov, J.Exp.Theor.Phys. 108 (2009) 384 [arXiv:0711.1264] who obtained higher fluxes, see also A. Hook et al., arXiv:1804.03145, F.P. Huang et al., arXiv:1803.08230

This would be detectable out to ~ pc distances, see also D. Marsh (Cambridge)

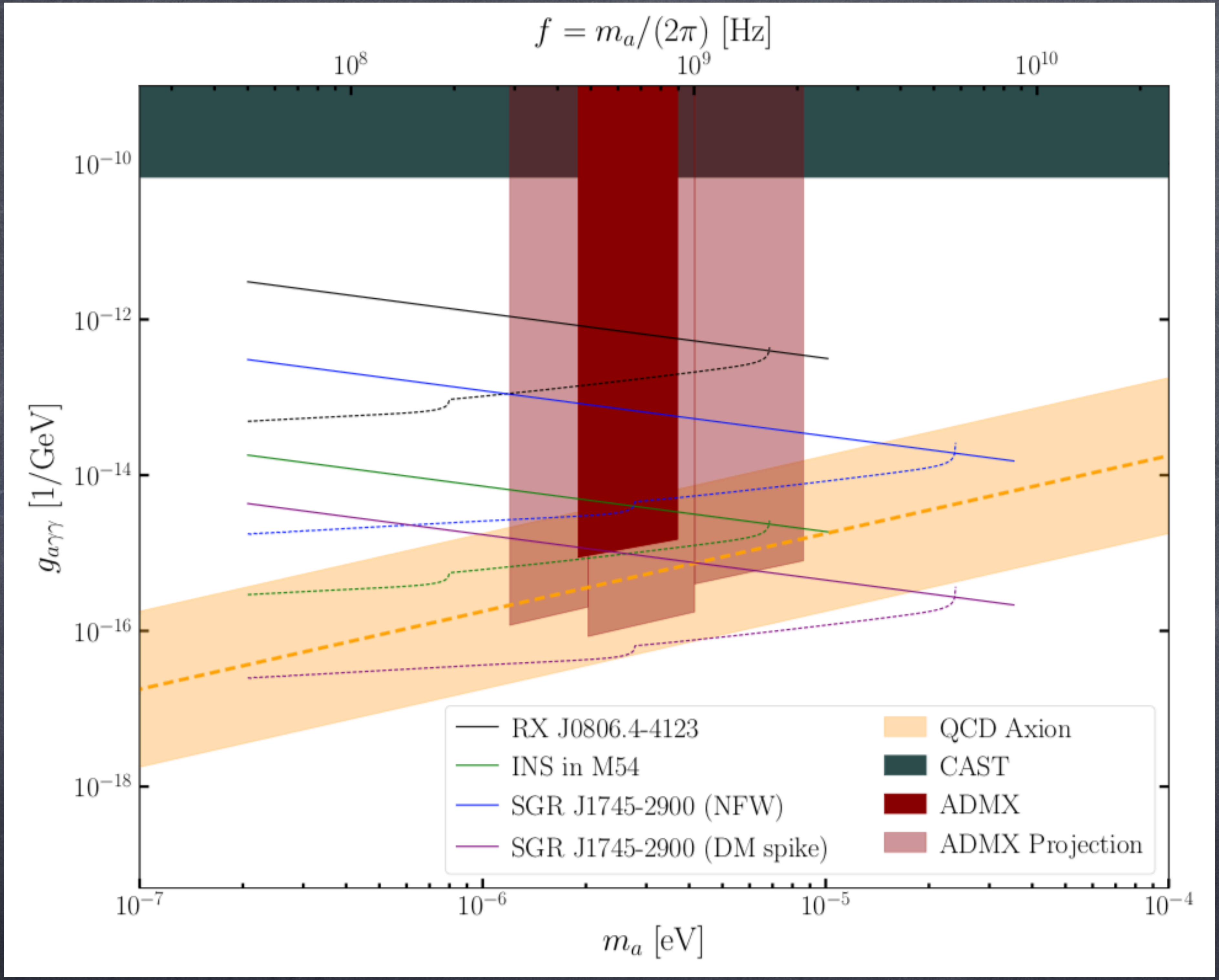
A. Hook et al., Phys.Rev.Lett 121 (2018) 241102 [arXiv:1804.03145] made a more detailed calculation of resonant conversion (when plasma frequency matches ALP mass) around neutron stars which results in

$$\frac{d\mathcal{P}(\theta=\frac{\pi}{2},\theta_m=0)}{d\Omega} \approx 4.5\times10^{8}\ \mathrm{W}\left(\frac{g_{a\gamma\gamma}}{10^{-12}\ \mathrm{GeV}^{-1}}\right)^2 \left(\frac{r_0}{10\ \mathrm{km}}\right)^2\left(\frac{m_a}{1\ \mathrm{GHz}}\right)^{5/3}\left(\frac{B_0}{10^{14}\ \mathrm{G}}\right)^{2/3}\left(\frac{P}{1\ \mathrm{sec}}\right)^{4/3} \left(\frac{\rho_\infty}{0.3\ \mathrm{GeV/cm^3}}\right)\left(\frac{M_{\mathrm{NS}}}{1\ M_\odot}\right)\left(\frac{200\ \mathrm{km/s}}{v_0}\right),$$

$$S = 6.7\times10^{-5}\ \mathrm{Jy}\left(\frac{100\ \mathrm{pc}}{d}\right)^2\left(\frac{1\ \mathrm{GHz}}{m_a}\right)\times \left(\frac{200\ \mathrm{km/s}}{v_0}\right)^2\left[\frac{d\mathcal{P}/d\Omega}{4.5\times10^{8}\ \mathrm{W}}\right].$$

Advantage: Depends on plasma and magnetic field structure only through adiabaticity of conversion (plasma scale height, mixing through magnetic field at resonance

A. Hook et al., Phys.Rev.Lett 121 (2018) 241102 [arXiv:1804.03145]



takes into account ALP density enhancement around galactic center (but spike may not be realistic)

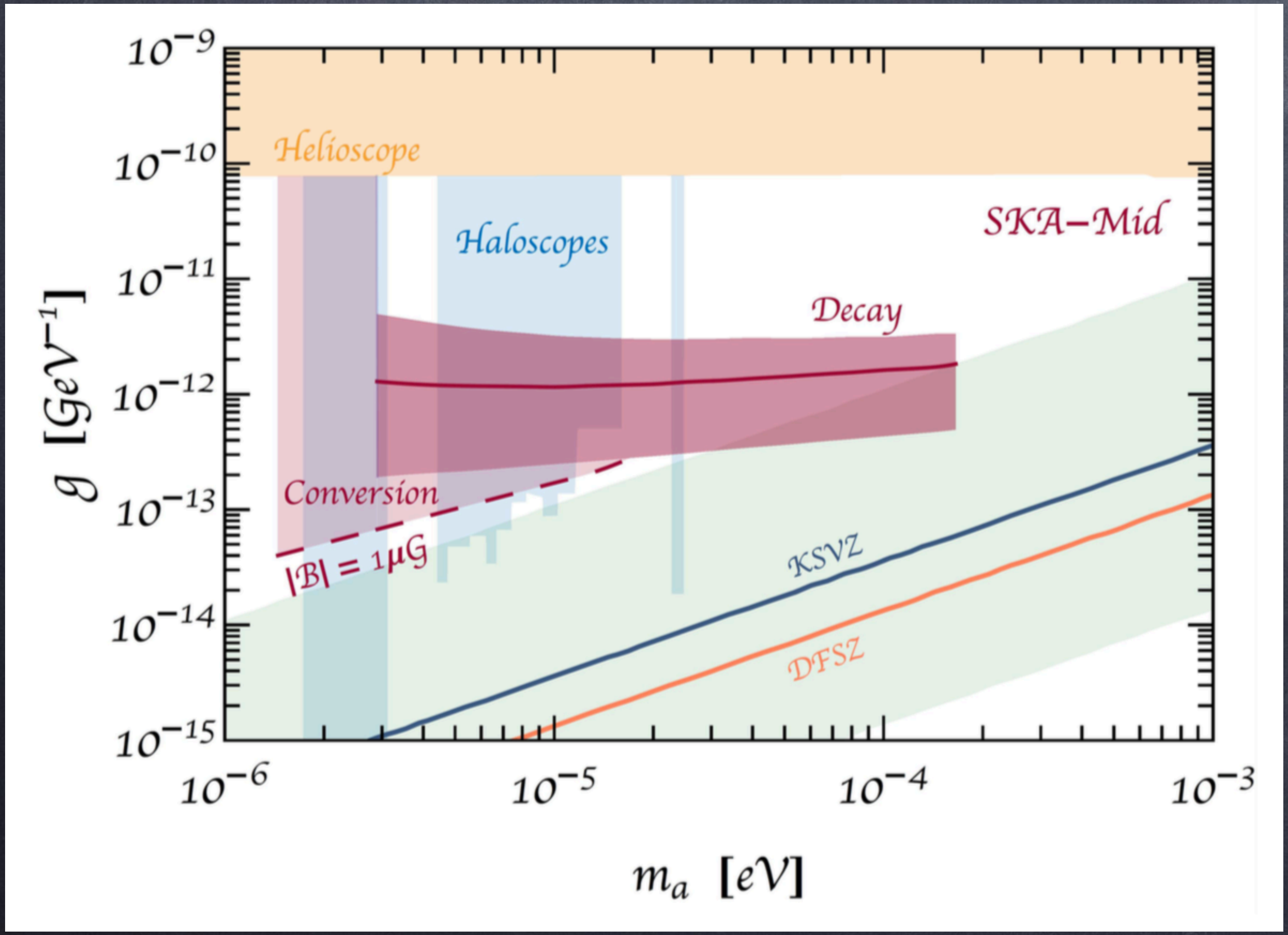
Spontaneous and Stimulated Decays

compare Primakoff conversion rate

$$\frac{1}{\tau_a} \simeq \frac{\pi g_{a\gamma}^2}{m_a}\rho_m(m_a) \simeq 9.7\times10^{-38}\,\left(g_{a\gamma}10^{14}\,\mathrm{GeV}\right)^2\left(\frac{m_a}{\mathrm{meV}}\right)^{-1}\left(\frac{B}{\mathrm{mG}}\right)^2 f(m_a)\,\mathrm{s}^{-1}\,,$$

with spontaneous decay rate

$$\frac{1}{\tau_a} = \frac{g_{a\gamma}^2 m_a^3}{64\pi} \simeq 1.5\times10^{-38}\,\left(g_{a\gamma}\,10^{14}\,\mathrm{GeV}\right)^2\left(\frac{m_a}{\mathrm{meV}}\right)^3\,\mathrm{s}^{-1}\,.$$



A. Caputo et al., arXiv:1805.08780, see also I.Tkachev, PLB 191 (1987) 41; T.W. Kephart and T.J. Weiler, PRD 52, 3226 (1995)

Modified Electrodynamics

Modified Maxwell equations in presence of photon-ALP coupling

$$\nabla \cdot \mathbf{E} = \frac{\rho_{\rm em}}{\epsilon_0} + \frac{\mathbf{B} \cdot \nabla a}{\epsilon_0 M_a}\,, \quad \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mu_0\, \mathbf{j}_{\rm em} + \frac{\mathbf{E} \times \nabla a - \mathbf{B}\partial_t a}{\epsilon_0 M_a}$$

Often one uses the coupling $g_{a\gamma} = 1/M_a$. In Lorentz gauge for a wave propagating in the z-direction with circular polarisation this yields

$$\left(\partial_t^2 - \partial_z^2\right) A_\pm = \pm\, i g_{a\gamma} \left[(\partial_z a)(\partial_t A_\pm) - (\partial_t a)(\partial_z A_\pm)\right]$$

Can be solved with the ansatz

$$A_\pm(t,z) = F_\pm(t,z) \exp\left[-i\omega t + ikz + iG_\pm(t,z)\right]$$

To first order in m_a/ω and $g_{a\gamma}$ this is solved by

$$\omega = k\,, \quad F = \text{const}\,, \quad G_\pm(t,z) = \mp \frac{g_{a\gamma}}{2} a(t,z) + f(z-t)$$

for an arbitrary function f(x).

M.A. Fedderke, P.W. Graham, S. Rajendran, arXiv:1903.02666

ALP-Photon Conversion through Parametric Resonance

Tkachev Sov.Astron.Lett. 12 (1986) 305, Pisma Astron.Zh. 12 (1986) 726,
see also M.P. Hertzberg and E.D. Schiappacasse, arXiv:1805.00430,

From the modified Maxwell equations for a homogeneous ALP field $a(t)=a_0 \sin m_a t$ for a photon momentum mode k one obtains a Mathieu-type equation of the form

$$\left[\frac{d^2}{dx^2} + A - 2q\cos(2x)\right] A_{\pm} = 0$$

for the two circularly polarised photon fields $A_{\pm}$ with $x=m_a t/2$ and

$$A = \frac{4k^2}{m_a^2}, \quad q = \pm\frac{2k}{\epsilon_0 M_a}\frac{a_0}{m_a}$$

For q<1 (narrow resonance) there are resonances at $A = 1 \pm q$ growing with a rate in x of ~ q/2. The resulting band width is $k = m_a(1 \pm q)/2$
This corresponds to the crossed spontaneous decay into $k=m_a/2$ photons.
For q>1 other resonances are at A~2q growing with a rate in x of ~1 (probably not relevant here)

Case 1: Diffuse galactic dark matter

$$\rho_a \simeq \frac{1}{2} m_a^2 a_0^2 \longrightarrow a_0 \simeq 2.2 \left(\frac{\mu\mathrm{eV}}{m_a} \right) \mathrm{keV}$$

which implies a narrow resonance with

$$q \sim 2.3 \times 10^{-19} \left(g_a \cdot 10^{14}\,\mathrm{GeV} \right) \left(\frac{\mu\mathrm{eV}}{m_a} \right)$$

This could give a few e-folds in Galaxy, but extremely narrow line. The condition that the observed photon background not be enhanced by more than factor unity reads

$$\int_0^1 \frac{d\tilde{q}}{2} \exp\left(m_a \int dl\, |\, q(l)\, |\, \Theta[\, |\, q(l)\, | - \tilde{q}]/\sqrt{2} \right) \lesssim f \frac{\Delta\nu}{\nu} ,$$

which up to corrections logarithmic in $g_{a\gamma}, m_a, \rho_a, f$ and $\Delta\nu/\nu$ gives

$$g_{a\gamma} \lesssim 1.9 \times 10^{-14} \left(\frac{10\,\mathrm{kpc}}{R} \right) \left(\frac{\rho_a}{0.3\,\mathrm{GeV cm}^{-3}} \right)^{-1/2} \mathrm{GeV}^{-1} ,$$

This constraint only applies to ALP conndensates that are in an energy ground state or vary on time scales larger than the growth rate.

Case 2: ALP stars

estimates based on Visinelli et al., Phys Lett. B 777 64 (2018) [arXiv:1710.08910]

$a_0 \sim f_a^2/M_{Pl}$ -> narrow resonance parameter

$$q \simeq C_{a\gamma} \frac{\alpha_{\rm em}}{2\pi} \frac{f_a}{M_{\rm Pl}} \sim 10^{-9} \left(\frac{f_a}{10^{12}\,{\rm GeV}} \right)$$

The radius of an axion star is $R \sim 1/(q m_a)$ and the kinetic energy of axions in an axion star is $\sim q m_a$. Therefore, impinging radio photon beams could be enhanced by $m_a\, q/(m_a q)$ thus potentially by several e-folds. But detailed numbers suggest no significant constraints.

see also A. Arza, arXiv:1810.03722

However, details depend on detailed structure of axion stars and their formation history: much interesting recent work on this

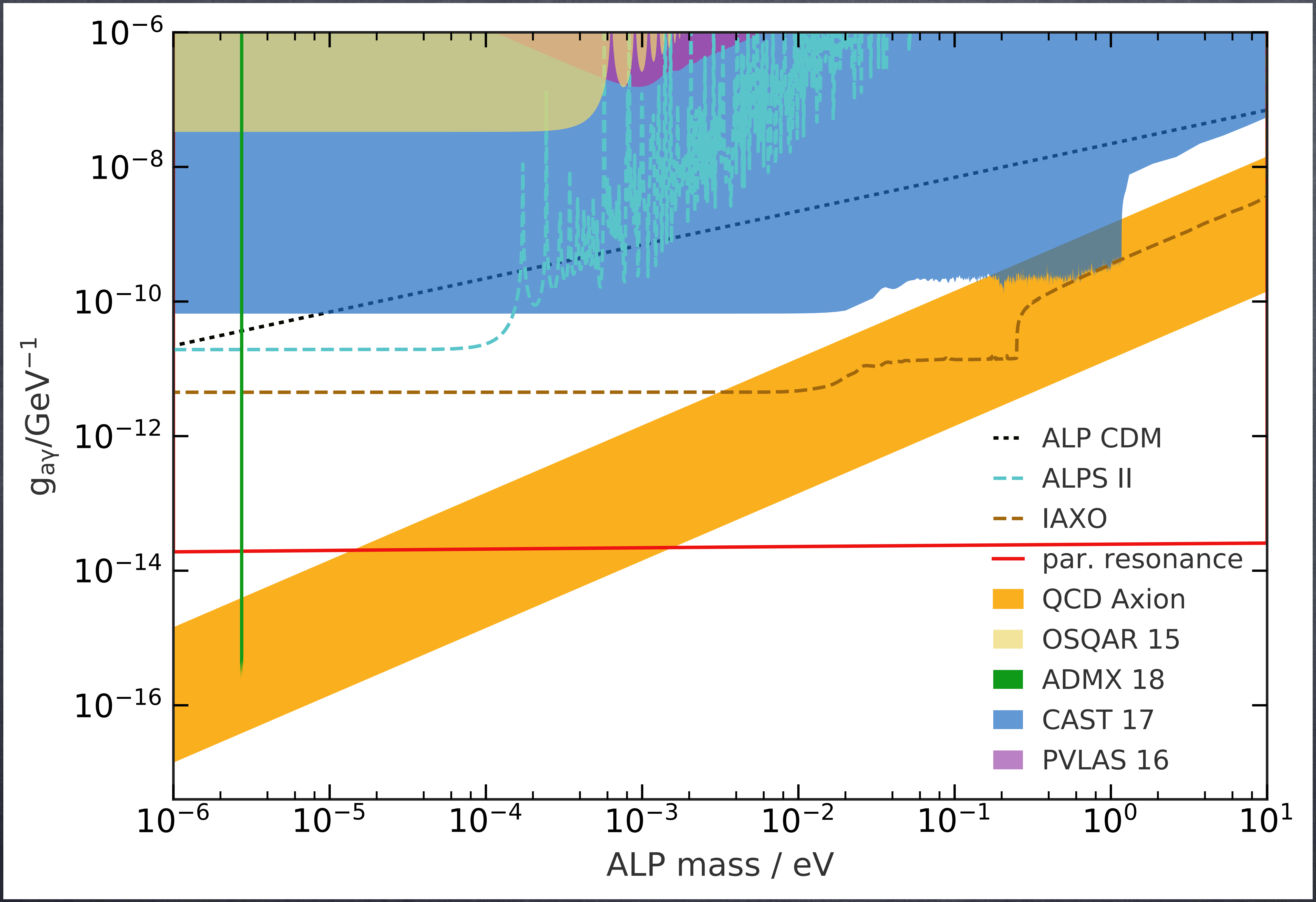
e.g. Levkov, Niemeyer, Pargner, Schwetz, Buschmann, …

Also axion stars probably cover too small a fraction of the sky to give observable effects. On the other hand, if a large fraction of the axion star could be converted to radio photons [Hertzberg and Schiappacasse, arXiv:1805.00430, Tkachev, PLB 191, 41]. Note that spontaneous ALP decay probably not crucial to seed this because radio photons are always around.

Interesting conceptual questions when back reaction becomes important

$$\Box a + \frac{\partial V_a}{\partial a}(a) = \frac{\mathbf{E} \cdot \mathbf{B}}{M_a}\,.$$

For example, does momentum conservation lead to recoil when photon beam is enhanced ? Energy conservation ?
See axion dark matter echo effect, A. Arza, P.Sikivie, arXiv:1902.00114



G. Sigl, P. Trivedi, in preparation

Birefringence in an ALP Background

on small length scales the Mathieu equation leads to the dispersion relation

$$\omega = k \mp \frac{m_a g_{a\gamma}}{2\epsilon_0} a_0 \cos(m_a t + \delta)\,,$$

which leads to birefringence with a phase shift

$$\Delta\phi_1 \simeq \frac{g_{a\gamma}}{\epsilon_0} a_0 \simeq 10^{-20} \left(g_{a\gamma} 10^{14}\,\mathrm{GeV} \right) \left(\frac{\mu\mathrm{eV}}{m_a} \right) .$$

Note that this does not depend on photon wavenumber and thus any waveband can be applied. Adding in quadrature phase shifts from domains $l_c > 1/m_a$ in which the axion field is coherent (i.e. phase δ ~ constant) yields

$$g_{a\gamma} \lesssim 3 \times 10^{-13}\, \Delta\phi\, v_a^{-1/2} \left(\frac{m_a}{10^{-22}\,\mathrm{eV}} \right)^{1/2} \left(\frac{10\,\mathrm{kpc}}{d} \right)^{1/2} \mathrm{GeV}^{-1}\,,$$

where $\Delta\phi$ is an upper limit on the observed phase shift.

Same effect also used in experimental approaches, e.g. birefringent cavities, arXiv:1809.01656

Effect of inhomogeneous ALP backgrounds: stochastic versus coherent polarisation rotation

The above solution to first order in m_a/ω and $g_{a\gamma}$

$$A_\pm(t,z) = F_\pm \exp\left[-i\omega(t-z) + i g_{a\gamma} a(t,z)/2 + f(t-z)\right]$$

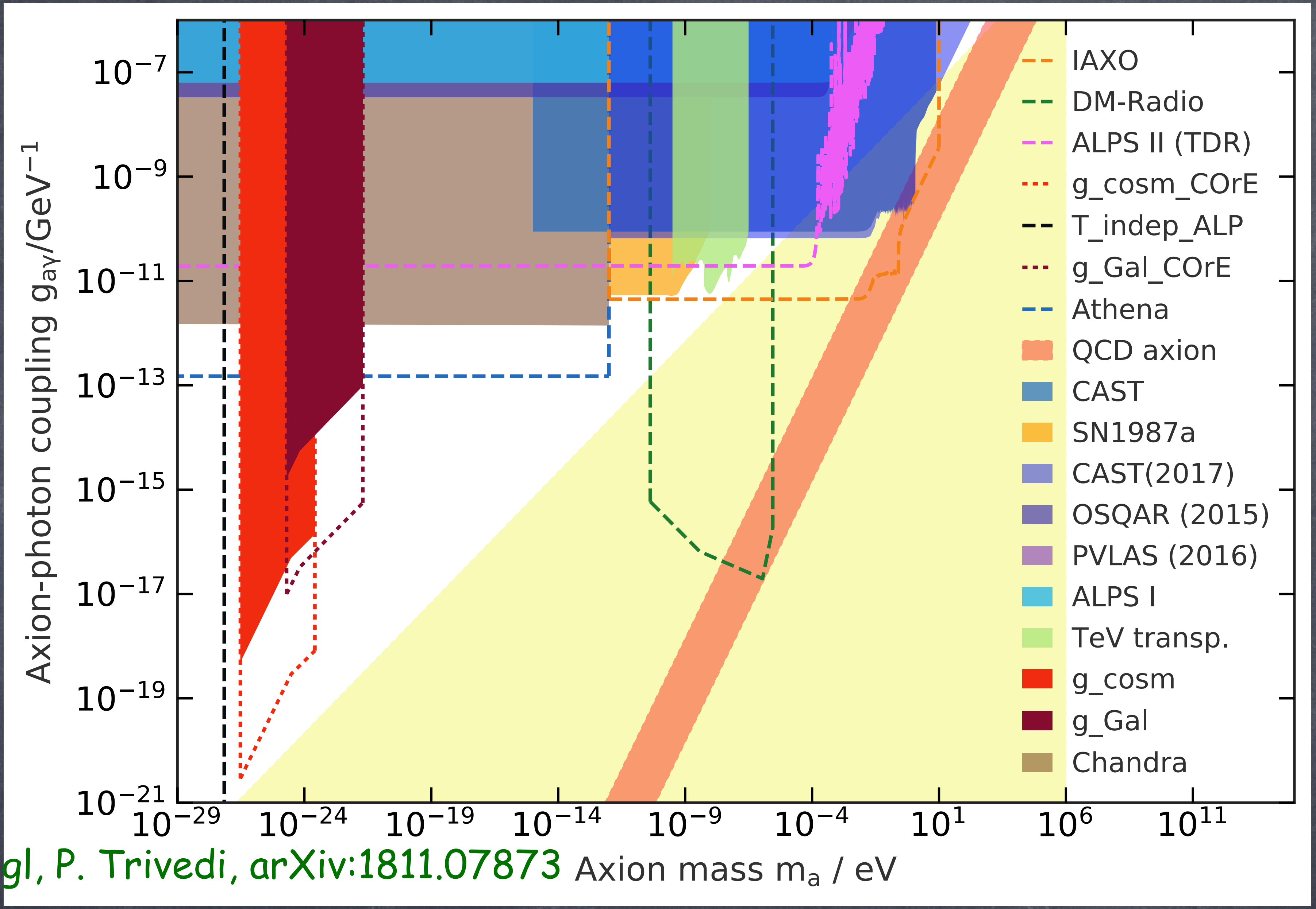
would imply for the rotation angle

$$\Delta\theta = \frac{g_{a\gamma}}{2}\int_{\mathscr{C}} ds n^\mu \partial_\mu a = \frac{g_{a\gamma}}{2}\left[a(t_f, z_f) - a(t_i, z_i)\right]$$

Thus, rotation angle would not depend on path, but only on values of ALP field at the endpoints.

M.A. Fedderke, P.W. Graham, S. Rajendran, arXiv:1903.02666

Application to CMB: Birefringence angle < 1°



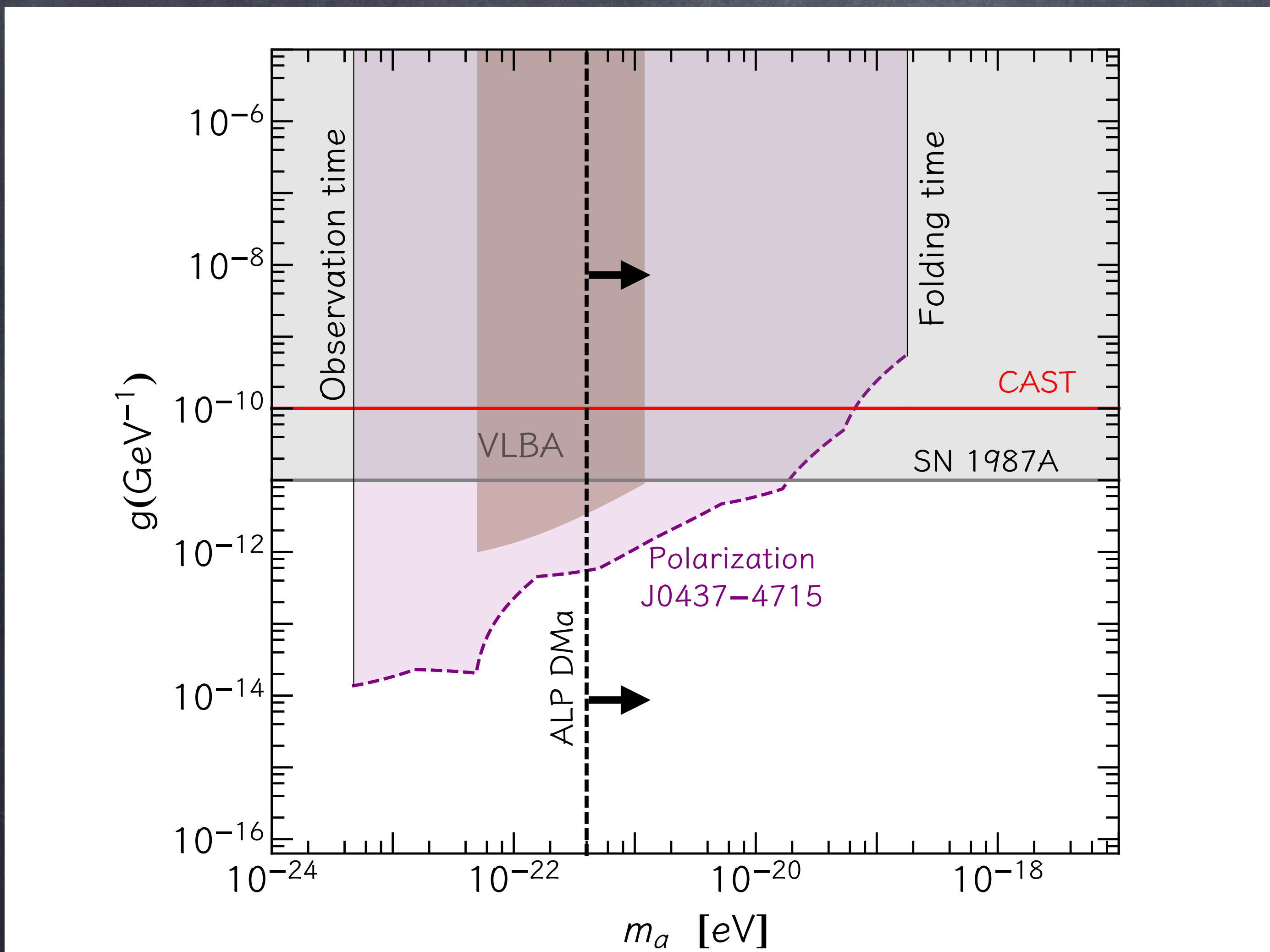
G. Sigl, P. Trivedi, arXiv:1811.07873

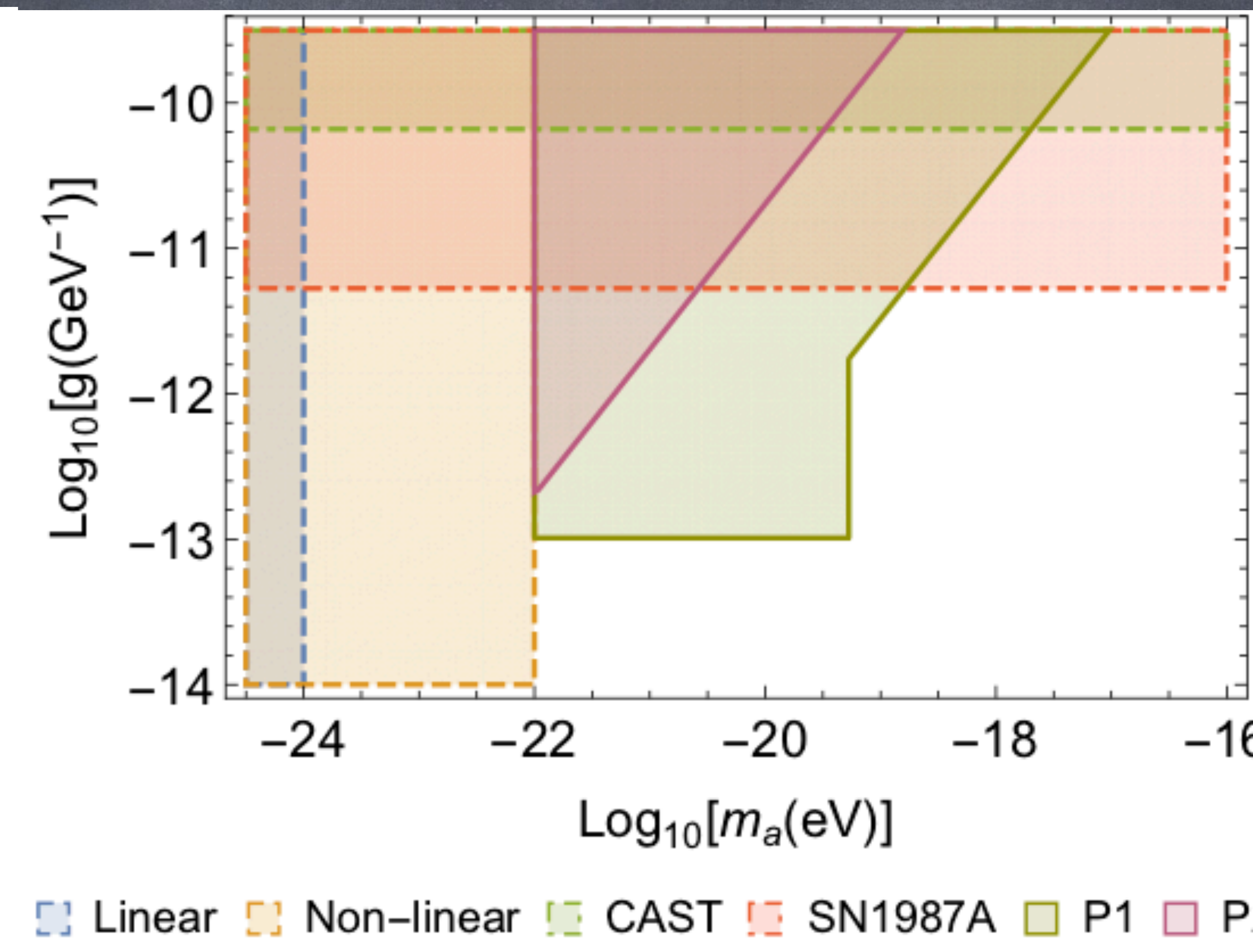
The range of constrained ALP masses comes from the requirement that the projection of phase shift variation onto the sky

$$\ell \sim \pi/\theta \sim \pi d_A/l_c \sim \pi d_A m_a v_a(z_{\rm rec}) \sim 2\pi m_a v_a(z_{\rm rec}/[H_0(1+z_{\rm rec})]$$

lies in the observed range of CMB multipoles 1 < l < 10³

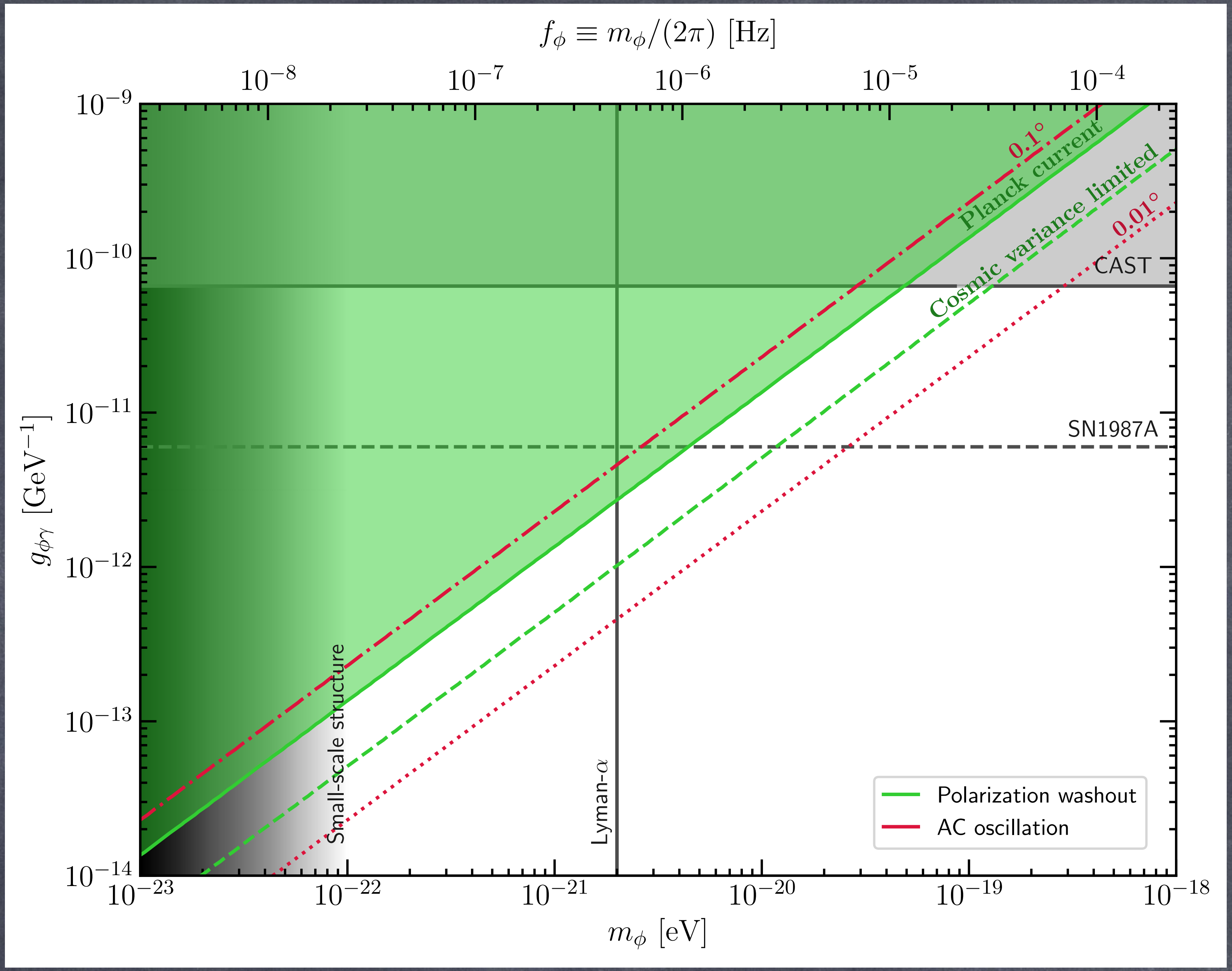
Other studies used polarisation data from other object





pulsar timing:
Caputo et al., arXiv:1902.02695

linearly polarised pulsar light:
T. Liu, G. Smoot, Y. Zhao, arXiv:1901.10981



CMB polarization: M.A. Fedderke, P.W. Graham, S. Rajendran, arXiv:1903.02666

polarisation of AGN jets: M.M. Ivanov et al., JCAP02(2019)059 [arXiv:1811.10997]

polarisation of protoplanetary disk emission: T. Fujita, R. Tazaki, K. Toma, arXiv: 1811.03525

Conclusions 1

1.) Linelike radio emissions from dark matter-ALP conversion into photons in magnetic fields may be detectable with current and future radio telescopes such as LOFAR and SKA

2.) However, the most crucial (and least known) parameter is the magnetic field power on the ALP mass scale which is in the meter regime for μeV ALP masses. MHD modes in the presence of coherent magnetic fields would play an important role but their intensity is currently unclear

3.) Resonant conversion around compact stellar objects may give interesting signals less dependent on magnetic field structure

Conclusions 2

4.) Spontaneous decay (interesting above $\sim 10^{-5}$ eV) and parametric amplification in ALP stars are independent of magnetic fields, but the latter depends a lot on ALP star structure and their formation (not well understood yet but many opportunities for collaboration !)

5.) Parametric resonance in a Galactic scale condensate may give interesting constraints from observed diffuse photon backgrounds on the level of $g_{a\gamma} \sim 1/(10^{14}$ GeV) over wide mass range

6.) Birefringence induced in photons propagating in an oscillating axion background should be wavelength independent (thus also relevant e.g. for X-rays) and can lead to further constraints