Prospects of detecting spacetime torsion

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Outline

Introduction

- Crash-course in differential geometry:
- Spacetime = 4-dimensional smooth manifold
- Coframe $\vartheta^\alpha = e_i^\alpha dx^i$; dual frame field $e_\alpha = e^i_\alpha \partial_i$ (observer)
- Metric $\mathbf{g}$: $g_{\alpha\beta} = \mathbf{g}(e_\alpha, e_\beta)$ (lengths and angles)

$$ds^2 = g_{\alpha\beta}\vartheta^\alpha \otimes \vartheta^\beta$$

- Linear connection: 1-form $\Gamma_{i\alpha}{}^\beta$ (parallel transport)

$$\delta_{||}V^\alpha = -\delta x^i \Gamma_{i\beta}{}^\alpha V^\beta$$

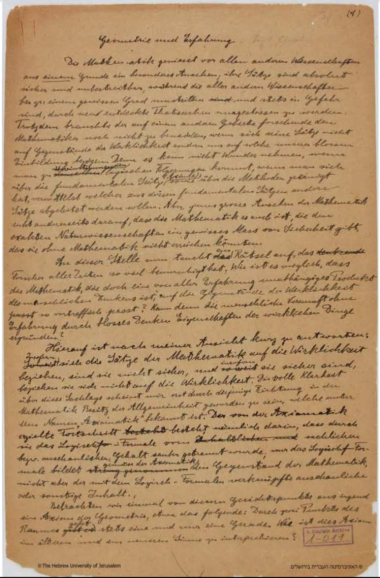
- Notation: $\alpha, \beta, \dots = 0, 1, 2, 3$ anholonomic components; $i, j, \dots = 0, 1, 2, 3$ coordinate components
- Lorentzian signature of the metric is $(+1, -1, -1, -1)$
- Local coordinates on manifold: $x^i = (t, x^a)$ t time and spatial coordinates x^a, $a = 1, 2, 3$

- Spacetime metric and linear connection are a priori *independent structures*
- *Riemannian constraints:*
- $$D_i g_{\alpha\beta} = 0 \qquad \text{(metricity)}$$
- $$D_i e_j^\alpha - D_j e_i^\alpha = 0 \qquad \text{(no torsion)}$$
- Then connection is expressed in terms of metric (coframe):

$$\Gamma_{i\alpha}{}^{\beta} = \widetilde{\Gamma}_{i\alpha}{}^{\beta} = e_k^\beta \left\{ {}_i{}^k{}_j \right\} e_\alpha^j + e_k^\beta \partial_i e_\alpha^k$$

$\left\{ {}_i{}^k{}_j \right\} = \frac{1}{2} g^{kl} \left(\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij} \right)$ Christoffel symbols.

- Einstein's GR is based on the Riemannian geometry.
- Why torsion? Physics is an experimental science
- Test foundations: Lorentz symmetry violation?
- Probe spacetime: which geometry?


Geometrie und Erfahrung

"...The question whether this continuum has a Euclidean, Riemannian, or any other structure is a question of physics proper which must be answered by experience, and not a question of a convention to be chosen on grounds of mere expediency."

A. Einstein, Geometrie und Erfahrung, Sitzungsber. preuss. Akad. Wiss. **1** (1921) 123-130.

Poincaré gauge gravity: history and reviews

- *Timeline*: Weyl (1929,1950), Utiyama (1956), Kibble (1961), Sciama (1962), supergravity (mid-70s), . . .
- F.W. Hehl et al, *Rev. Mod. Phys.* **48** (1976) 393
- A. Trautman, in: *One hundred years after the birth of A. Einstein*, (NY, Plenum, 1980) vol. 1, p. 287
- D. Ivanenko, G. Sardanashvily, *Phys. Rep.* **94** (1983) 1
- F.W. Hehl et al, *Phys. Rep.* **258** (1995) 1
- I.L. Shapiro, *Phys. Rep.* **357** (2002) 113
- M.Blagojević, *Gravitation and Gauge Symmetries* (2002)
- Y.N. Obukhov, *Int. J. Geom. Meth. Mod. Phys.* **3** (2006) 95
- A. Trautman, in: *Encyclopedia of Mathematical Physics*, (Elsevier, Oxford, 2006) vol. 2, p. 189
- M.Blagojević, F.W.Hehl, *Gauge Theories of Gravitation. A Reader with Commentaries* (IC Press, London 2013)

The analogy between charge and spin (Sciama)

- Yang-Mills theory for *internal symmetry* group G with N parameters ε^I. Invariance of action $S = \int \mathcal{L}(\Psi, \partial_i \Psi)$ under *global* transformations $\delta\Psi = \varepsilon^I \rho_I \Psi$ ($\rho_I \in \mathcal{G}$) $\Longrightarrow$ current J^i_I

$$\text{Noether theorem} \quad \Longrightarrow \partial_i J^i_I = 0 \qquad I = 1, \dots, N$$

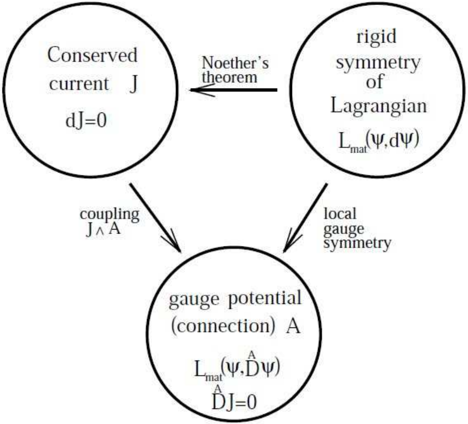
- Local symmetry $\varepsilon^I(x) \Rightarrow$ gauge field $A^I_i \Rightarrow$ Lagrangian

$$\mathcal{L}(\Psi, \partial_i \Psi) \qquad \Longrightarrow \qquad \mathcal{L}(\Psi, D_i \Psi)$$

Covariant derivative

$$D_i \Psi = \partial_i \Psi - A^I_i \rho_I \Psi$$

- Potential $\delta A^I_i = D_i \varepsilon^I = \partial_i \varepsilon^I + f^I{}_{JK} \varepsilon^J A^K_i$ with structure constants $f^I{}_{JK}$ of Lie algebra $\mathcal{G}$
- $\Longrightarrow$ covariant conservation law $D_i J^i_I = 0$



Gauge principle:

conserved current J from symmetry group (Noether)

Local symmetry $\Longrightarrow$ gauge potential A: $\partial_i\Psi \longrightarrow D_i\Psi$

Interaction $J \wedge A = J^i_I\, A^I_i$

- Geometrically, A_i^I is a connection in a fibre bundle with structure group G over spacetime manifold
- Field strength is a curvature in this bundle

$$F_{ij}{}^I = \partial_i A_j^I - \partial_j A_i^I - f^I{}_{JK} A_i^J A_j^K$$

Yang-Mills field equations generalize Maxwell's theory

$$D_j H^{ij}{}_I = J_I^i, \qquad D_i F_{jk}{}^I + D_j F_{ki}{}^I + D_k F_{ij}{}^I = 0$$

- Constitutive relation $H = H(F)$: $\quad H^{ij}{}_I = -\partial V(F)/\partial F_{ij}{}^I$
- *Gravity*: Poincaré symmetry group $G = T_4 \rtimes SO(1,3)$
- Global symmetry $\varepsilon^I = \{\varepsilon^\alpha, \varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}\} \Longrightarrow$ currents

$$J_I^i = \{\Sigma_\alpha{}^i, \quad S_{\alpha\beta}{}^i = -S_{\beta\alpha}{}^i\}$$

- Conservation of energy-momentum & angular momentum

$$\partial_i \Sigma_\alpha{}^i = 0, \qquad \partial_i S_{\alpha\beta}{}^i = \Sigma_{\alpha\beta} - \Sigma_{\beta\alpha}$$

- Local symmetry ⇒ gauge fields (translational & rotational)

$$A_i^I = \left\{ e_i^\alpha, \quad \Gamma_i{}^{\alpha\beta} = -\Gamma_i{}^{\beta\alpha} \right\}$$

- Covariant derivative for matter fields

$$D_\alpha \Psi = e_\alpha^i \left(\partial_i \Psi - \frac{1}{2} \Gamma_i{}^{\beta\gamma} \rho_{\beta\gamma} \Psi \right)$$

Poincaré gauge field strengths $F_{ij}{}^I$: torsion and curvature

$$\begin{aligned} T_{ij}{}^\alpha &= \partial_i e_j^\alpha - \partial_j e_i^\alpha + \Gamma_{i\beta}{}^\alpha e_j^\beta - \Gamma_{j\beta}{}^\alpha e_i^\beta \\ R_{ij}{}^{\alpha\beta} &= \partial_i \Gamma_j{}^{\alpha\beta} - \partial_j \Gamma_i{}^{\alpha\beta} + \Gamma_{i\gamma}{}^\beta \Gamma_j{}^{\alpha\gamma} - \Gamma_{j\gamma}{}^\beta \Gamma_i{}^{\alpha\gamma} \end{aligned}$$

- Geometrical structure on the spacetime introduced:
- Riemann-Cartan manifold – curved & contorted

Einstein-Cartan-(Sciama-Kibble) theory

- Gravitational (Hilbert-Einstein) Lagrangian
$$V_{EC} = \frac{1}{2\kappa} e_i^\alpha e_j^\beta R_{\alpha\beta}{}^{ij}(\Gamma) = \frac{1}{2\kappa} R$$
- Field equations
$$\begin{aligned} \mathrm{Ric}_\alpha{}^i - \frac{1}{2} e^i_\alpha R &= \kappa \Sigma_\alpha{}^i \\ T_{\alpha\beta}{}^i - e^i_\alpha T_{\beta k}{}^k + e^i_\beta T_{\alpha k}{}^k &= \kappa S_{\alpha\beta}{}^i \end{aligned}$$
- Ricci $\mathrm{Ric}_\alpha{}^i = R_{\alpha\beta}{}^{ij} e_j^\beta$, curvature scalar $R = \mathrm{Ric}_\alpha{}^i e_i^\alpha$
  Einstein's gravitational coupling constant $\kappa = 8\pi G/c^4$
- Torsion is algebraically related to spin. Expressing $T = T(S)$, we derive Einstein's equation
$$\widetilde{\mathrm{Ric}}_{ij} - \frac{1}{2} g_{ij} \widetilde{R} = \kappa \Sigma^{eff}_{ij}$$
  with effective energy-momentum $\Sigma^{eff}_{ij} = \Sigma_{ij} + (S \cdot S)_{ij}$

Einstein-Cartan Theory = GR + contact spin-spin interaction

- Probing the geometrical structure of spacetime: How?
- $\Longrightarrow$ dynamics of matter (particles, fluids, bodies)
- Confusing claims about 'torsion effects' on body's motion
- $\#1$: GR: particle moves along geodesics $\dot{u}^i + \widetilde{\Gamma}_{jk}{}^i u^j u^k = 0$
  Non-Riemannian geometry: particle follows autoparallel
  $\dot{u}^i + \Gamma_{jk}{}^i u^j u^k = 0$ (H.Kleinert 2000, R. March et al 2011)
- Compare: *neutral particle do not feel Lorentz force!*
- $\#2$: Macroscopic massive body is affected by torsion
  (Gravity Probe B: Y. Mao, M.Tegmark, A. Guth, S. Cabi, Phys. Rev. **D76** (2007) 104029)
- $\#3$: Equivalence principle violates gauge symmetry
  (C. Mukku, W.A. Sayed 1979)

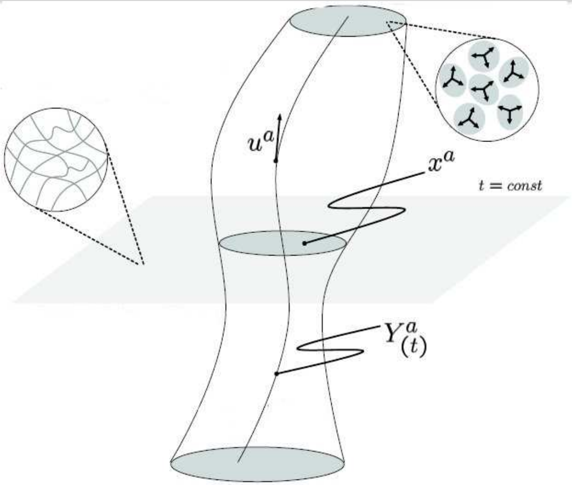
Minimal coupling: conservation laws

- In gravity, equations of motion need not be postulated separately, they are derived from conservation laws
- Noether: Symmetry $\Longrightarrow$ conservation law (YM: $D_i J^i_I = 0$)
- Poincaré gravity accounts for *microstructure* of matter

Conservation of energy-momentum and angular momentum

$$\overset{*}{D}_i \Sigma_k{}^i = \Sigma_j{}^i T_{ki}{}^j - S^m{}_n{}^l R_{klm}{}^n, \qquad \overset{*}{D}_i S_{[jk]}{}^i = -\Sigma_{[jk]}$$

- We assume *Minimal Coupling!* $\qquad \overset{*}{D}_i = D_i - T_{ki}{}^k$
- Many ways to equations of motion. *Multipole expansion*: replace body of cont. matter by a ‘particle’ with moments
- Example: electrodynamical multipoles $\int \delta x^{i_1} \cdots \delta x^{i_n} J$ with $\delta x^i = x^i - Y^i(\tau)$, where $Y^i(\tau)$ is body’s world-line
- $\Longrightarrow$ charge $\mathcal{Q} = \int J$, dipole $\mathcal{D}^i = \int \delta x^i J$, quadrupole etc



Multipole method:
In body's world tube choose reference world-line Y^a. Coords x^a label cross-section $t =$const. Velocity $u^a = dY^a/d\tau$.

Equations of motion for extended test body

- Use analogy between charge and spin. Gravity multipoles
$$p^{i_1\dots i_n}{}_\alpha = \int \delta x^{i_1}\cdots\delta x^{i_n}\Sigma_\alpha, \qquad s^{i_1\dots i_n}{}_{\alpha\beta} = \int \delta x^{i_1}\cdots\delta x^{i_n} S_{\alpha\beta}$$
- [Here symbolic notation; for precise definitions see D. Puetzfeld & YNO, Phys. Rev. **D88** (2013) 064025]
- Integrate conservation laws $\Longrightarrow$ multipole equations
- *Monopole*: p_α nontrivial only, neglect higher multipoles
$$u^i\widetilde{D}_i p_\alpha = 0, \qquad u^{[i}p^{j]} = 0$$
- $\Longrightarrow p_i = mu_i \Longrightarrow$ geodesic $u^i\widetilde{D}_i u^j = 0$ motion!

Dipole equations of motion

$$u^i\widetilde{D}_i\mathcal{J}^{ab} = -2u^{[a}\mathcal{P}^{b]} + 2Q^{cd[a}T_{cd}{}^{b]} + 4Q^{[a}{}_{cd}T^{b]cd}$$
$$u^i\widetilde{D}_i\mathcal{P}^a = \frac{1}{2}\widetilde{R}^a{}_{bcd}\mathcal{J}^{cd}v^b + Q^{bc}{}_d\widetilde{\nabla}^a T_{bc}{}^d$$

- Total momenta: $\mathcal{P}^a = p^a - \frac{1}{2}K^a{}_{cd}S^{cd}$ and $\mathcal{J}^{ab} = p^{[ab]} + S^{ab}$

Nonminimal coupling: a loophole for torsion?

- *Minimal coupling: torsion can be probed only by spin*
- Nonminimal coupling model (Goenner 1984) attracts attention. Nonminimal Lagrangian $F\,L_{\rm mat}(\Psi, D_i\Psi, g_{ij})$ depends on *coupling function* $F = F(g_{ij}, R_{kli}{}^j, T_{kl}{}^i)$

Noether theorem yields conservation laws:

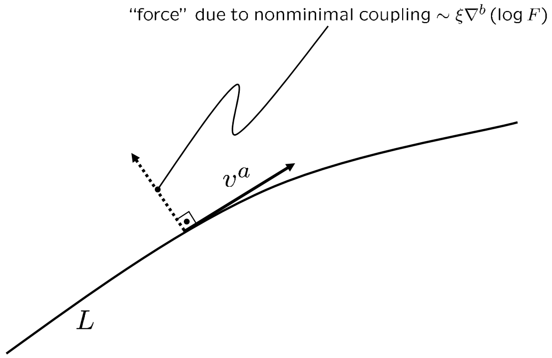
$$\begin{aligned} \overset{*}{D}_i\left(FS_{jk}{}^i\right) &= -F\Sigma_{[jk]}, \\ \overset{*}{D}_i\left(F\Sigma_k{}^i\right) &= F\Sigma_l{}^i T_{ki}{}^l - FS^m{}_n{}^i R_{kim}{}^n - L_{\rm mat} D_k F \end{aligned}$$

- Multipole equations derived. Lowest monopole order:

Propagation of structureless point particle

$$mu^i\widetilde{D}_i u^j = \xi\left(\delta_i^j - u^j u_i\right)\nabla^i \log F$$

- "Pressure"-like force, with $\xi = \int L_{\rm mat} \neq 0$, $\Longrightarrow$ *nongeodetic* motion due to coupling function $F(R_{ijk}{}^l, T_{ij}{}^k)$



Motion of single-pole (structureless) test body, in lowest order of multipole approximation, is non-geodetic, but surprisingly simple: non-geodetic "force" is proportional to gradient of nonminimal coupling function F.

Dirac particle in gravitational (& electromagnetic) field

- For Dirac field Lorentz generators are $\rho_{\alpha\beta} = -\frac{i}{2}\sigma_{\alpha\beta}$ with $\sigma_{\alpha\beta} = i\gamma_{[\alpha}\gamma_{\beta]}$. Hence, covariant gauge derivative reads

$$D_\alpha \Psi = e^i_\alpha \left(\partial_i \Psi - \frac{iq}{\hbar} A_i \Psi + \frac{i}{4} \Gamma_i{}^{\beta\gamma} \sigma_{\beta\gamma} \Psi \right)$$

- (+ electromagnetism included). Lagrangian for Dirac field

$$L = \frac{i\hbar}{2} \left(\overline{\Psi} \gamma^\alpha D_\alpha \Psi - D_\alpha \overline{\Psi} \gamma^\alpha \Psi \right) - mc \overline{\Psi} \Psi$$

- Poincaré gauge translational & rotational field ($a = 1, 2, 3$):

$$e_i^{\widehat{0}} = V\,\delta^0_i, \qquad e_i^{\widehat{a}} = W^{\widehat{a}}{}_b \left(\delta^b_i - cK^b\,\delta^0_i \right), \qquad \Gamma_i{}^{\alpha\beta}$$

V and K^a, and 3×3 matrix $W^{\widehat{a}}{}_b$ depend arbitrarily on t, x^a

$\Longrightarrow$ Dirac wave equation in Schrödinger form $i\hbar \frac{\partial \psi}{\partial t} = \mathcal{H}\psi$

- Note: rescaling of Ψ needed to get Hermitian Hamiltonian

- Denote ${\mathcal F}^b{}_a = VW^b{}_{\widehat{a}}$, $\Upsilon = V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{a}\widehat{b}\widehat{c}}$ and $\Xi_{\widehat{a}} = \frac{V}{c}\,\epsilon_{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{0}}{}^{\widehat{b}\widehat{c}}$

Hermitian Dirac Hamiltonian (with $\boldsymbol{\pi} = \boldsymbol{p} - q\boldsymbol{A}$)

$$\begin{aligned}\mathcal{H} &= \beta mc^2 V + \frac{c}{2}\left(\pi_b\,{\mathcal F}^b{}_a\alpha^a + \alpha^a{\mathcal F}^b{}_a\pi_b\right)\\ &\quad + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma} - \Upsilon\gamma_5\right)\end{aligned}$$

- Foldy-Wouthuysen transformation to reveal physics

$$\psi_{FW} = U\psi, \qquad \mathcal{H}_{FW} = U\mathcal{H}U^{-1} - i\hbar U\partial_t U^{-1}$$

- Recast Hamiltonian into

$$\mathcal{H} = \beta\mathcal{M} + \mathcal{E} + \mathcal{O}, \qquad \beta\mathcal{M} = \mathcal{M}\beta, \quad \beta\mathcal{E} = \mathcal{E}\beta, \quad \beta\mathcal{O} = -\mathcal{O}\beta$$

- FW transformation is then constructed as

$$U = \frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}\,\beta, \qquad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}$$

- Computation yields FW Hamiltonian $\mathcal{H}_{FW} = \mathcal{H}_{FW}^{(1)} + \mathcal{H}_{FW}^{(2)}$
- The two terms read explicitly

$$\begin{aligned}
\mathcal{H}_{FW}^{(1)} &= \beta\epsilon + \frac{\hbar c^2}{16}\left\{\frac{1}{\epsilon},\left(2\epsilon^{cae}\Pi_e\{p_b,\mathcal{F}^d{}_c\partial_d\mathcal{F}^b{}_a\} + \Pi^a\{p_b,\mathcal{F}^b{}_a\Upsilon\}\right)\right\} \\
&\quad + \frac{\hbar m c^4}{4}\epsilon^{cae}\Pi_e\left\{\frac{1}{\mathcal{T}},\left\{p_d,\mathcal{F}^d{}_c\mathcal{F}^b{}_a\partial_b V\right\}\right\}, \\
\mathcal{H}_{FW}^{(2)} &= \frac{\hbar c^2}{16}\left\{\frac{1}{\mathcal{T}},\left\{\Sigma_a\{p_n,\mathcal{F}^n{}_b\},\{p_k,\left[\epsilon^{abc}\left(\frac{1}{c}\dot{\mathcal{F}}^k{}_c - \mathcal{F}^d{}_c\partial_d K^k + K^d\partial_d\mathcal{F}^k{}_c\right)\right.\right.\right. \\
&\quad \left.\left.\left. - \frac{1}{2}\mathcal{F}^k{}_d\left(\delta^{db}\Xi^a - \delta^{da}\Xi^b\right)\right]\}\right\}\right\} + \frac{c}{2}\left(K^a p_a + p_a K^a\right) + \frac{\hbar c}{4}\Sigma_a\Xi^a
\end{aligned}$$

- Here $\{\ ,\ \}$ denote anticommutators, $\mathcal{T} = 2\epsilon^2 + \{\epsilon, mc^2 V\}$, $\Pi^a = \beta\Sigma^a$, and $\epsilon = \sqrt{m^2c^4V^2 + \frac{c^2}{4}\delta^{ac}\{p_b,\mathcal{F}^b{}_a\}\{p_d,\mathcal{F}^d{}_c\}}$
- This result is exact – no (weak field etc) approximations for $V, W^{\widehat{a}}{}_b, K^a, \Gamma_i{}^{\alpha\beta}$. Planck $\hbar$ is the only small parameter

Experimental bounds on torsion

- Evolution of spin (polarization operator $\boldsymbol{\Pi} = \beta\boldsymbol{\Sigma}$)

$$\frac{d\boldsymbol{\Pi}}{dt} = \frac{i}{\hbar}[\mathcal{H}_{FW}, \boldsymbol{\Pi}] = \boldsymbol{\Omega} \times \boldsymbol{\Pi}$$

- *Theory*: spin precession to probe torsion: Adamowicz (1975), Rumpf (1980), Audretsch (1981), Lämmerzahl (1997); review W.T.Ni, Rep.Prog.Phys. 73 (2010) 056901
- *Experiment*: effect of Earth's gravity on nuclear spins Hg
- Spin Hamiltonian (torsion $\check{T}^\alpha = \frac{1}{2}\eta^{\mu\nu\lambda\alpha}T_{\mu\nu\lambda}$, $\check{\boldsymbol{T}} = \{\check{T}^a\}$)

$$\mathcal{H}_{FW} = -g_N\mu_N\boldsymbol{B}\cdot\boldsymbol{\Pi} - \frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma} - \frac{\hbar c}{4}\check{\boldsymbol{T}}\cdot\boldsymbol{\Sigma}.$$

- B.J. Venema et al, Phys. Rev. Lett. **68** (1992) 135

Limits on torsion from Zeeman energy levels measurements

- $$|\check{\boldsymbol{T}}| < 4.3 \times 10^{-14}\mathrm{m}^{-1}$$

- Recent: C. Gemmel et al, Phys. Rev. **D82** (2010) 111901

Conclusions and Outlook

- Poincaré gauge gravity is natural extension of GR
- Einstein: geometry of spacetime is a *physical question*
- Spinless matter *cannot detect or place limits* on torsion
- *Nonminimal* coupling: loophole to detect torsion
- Experimental limit on torsion from spin dynamics. Agrees with estimates C. Lämmerzahl, Phys. Lett. **A228** (1997) 223; V.A. Kostelecký et al, PRL **100** (2008) 111102
- Torsion effects elsewhere? Early cosmology modified: singularity can be avoided due to spin-spin interaction (Trautman 1973, Poplawski 2012)
- Work partly done together with Dirk Puetzfeld (Bremen), Alexander Silenko (Minsk) and Oleg Teryaev (Dubna)

Thanks !

YNO, *Phys. Rev. Lett.* **86** (2001) 192
YNO, *Int. J. Geom. Meth. Mod. Phys.* **3** (2006) 95
D. Puetzfeld, YNO, *Phys. Rev.* **D87** (2013) 044045
F.W. Hehl, YNO, D. Puetzfeld, *Phys. Lett.* **A377** (2013) 1775
D. Puetzfeld, YNO, *Phys. Rev.* **D88** (2013) 064025
YNO, A.J. Silenko, O.V. Teryaev, *Phys. Rev.* **D80** (2009) 064044, *Phys. Rev.* **D84** (2011) 024025, *Phys. Rev.* **D 88** (2013) 084014