

# Neutrino Masses in Noncommutative Geometry: An Overview

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## Overview

- ① Basic Ideas
- ② Geometrical and Physical Obstructions
- ③ Five Scenarios for Neutrino Masses

## Overview

① Basic Ideas

② Geometrical and Physical Obstructions

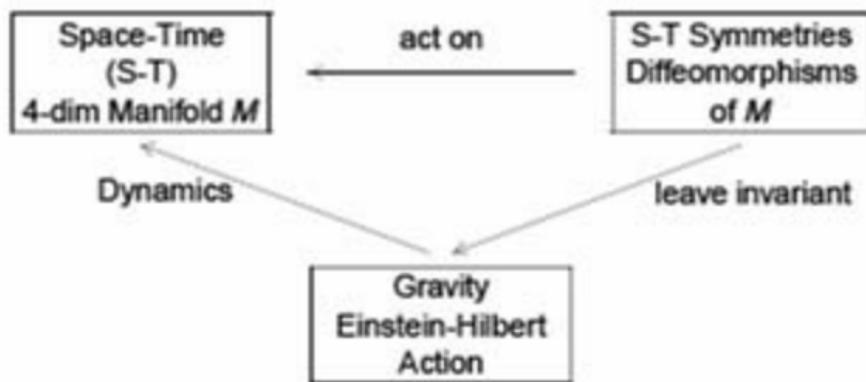
③ Five Scenarios for Neutrino Masses

### The aim of Noncommutative Geometry à la Connes:

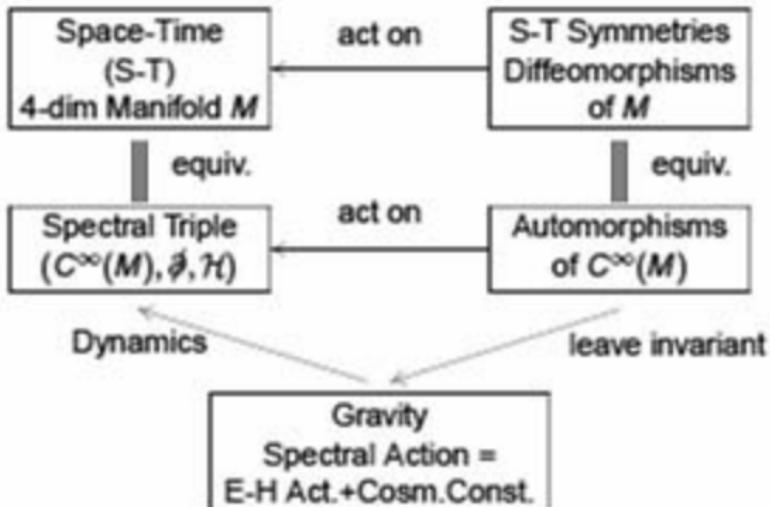
To unify general relativity (GR) and the standard model of particle physics (SM) on the same geometrical level.

This means to describe gravity and the electro-weak and strong forces as gravitational forces of a unified space-time.

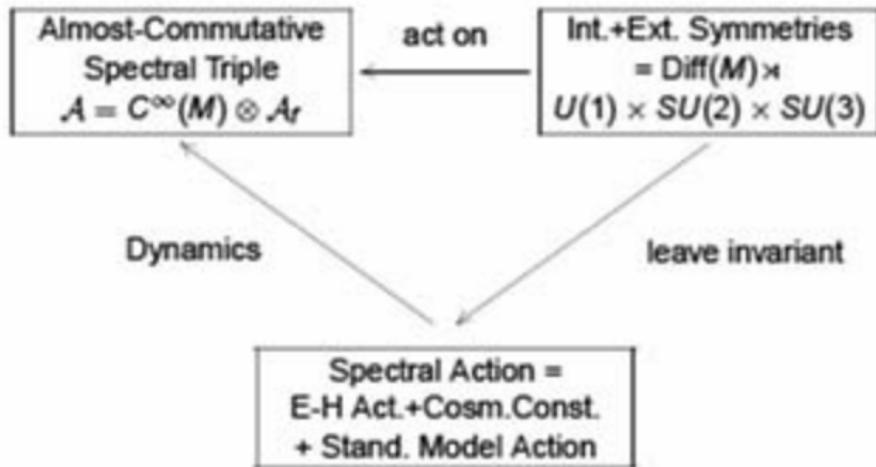
Gravity emerges as a pseudo-force associated to the space-time symmetries, i.e. the diffeomorphisms of the manifold  $M$ .



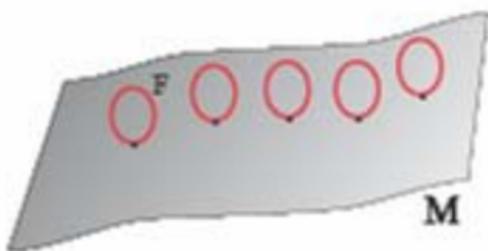
## Euclidean space-time!



**Almost-Commutative Spectral Action (A.Chamseddine, A.Connes 1996):**

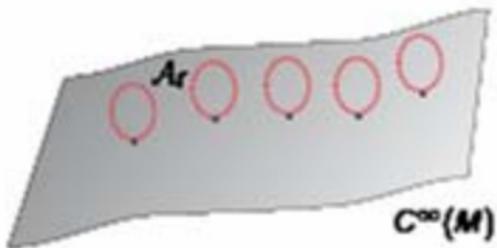


Analogy: Almost-comm. geometry  $\leftrightarrow$  Kaluza-Klein space



Idea:  $M \rightarrow C^\infty(M)$ ,  $F \rightarrow$  some "finite space",  
differential geometry  $\rightarrow$  spectral triple

## Almost-commutative Geometry



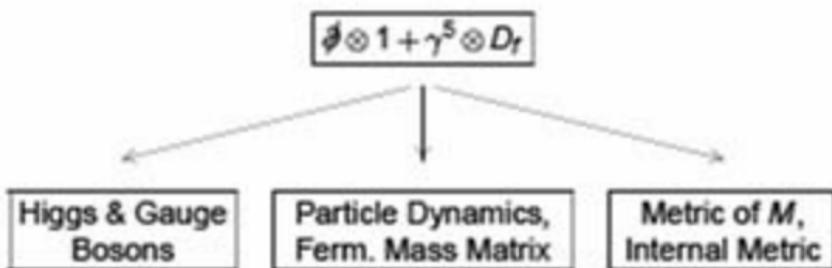
"finite space"  $\rightarrow \mathcal{A}_r = M_1(\mathbb{K}) \oplus M_2(\mathbb{K}) \oplus \dots$

Kaluza-Klein space  $\rightarrow$  almost-com. geometry,  $\mathcal{A} = C^\infty(M) \otimes \mathcal{A}_r$

The almost-commutative standard model automatically produces:

- The combined Einstein-Hilbert and standard model action
  - A cosmological constant
  - The Higgs boson with the correct quartic Higgs potential

The Dirac operator plays a multiple role:



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An even, real spectral triple  $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ ; the ingredients  
(A. Connes):

- A real, associative, unital pre- $C^*$ -algebra  $\mathcal{A}$
- A Hilbert space  $\mathcal{H}$  on which the algebra  $\mathcal{A}$  is faithfully represented via a representation  $\rho$
- A self-adjoint operator  $\mathcal{D}$  with compact resolvent, the Dirac operator
- An anti-unitary operator  $J$  on  $\mathcal{H}$ , the real structure (charge conjugation operator)
- A unitary operator  $\gamma$  on  $\mathcal{H}$ , the chirality

**The conditions or axioms of noncommutative geometry**  
(A. Connes 1996):

Condition 1: Classical Dimension  $n$  ( $n = 0$  for the finite part)

Condition 2: Regularity

Condition 3: Finiteness

Condition 4: First Order of the Dirac Operator

Condition 5: Poincaré Duality

Condition 6: Orientability

Condition 7: Reality ( $\rightarrow \text{KO-dim} = 0$  or 6 for finite part)

## Condition 7: (Reality)

The anti-unitary operator  $J$  obeys the following commutation relations in  $KO\text{-dim. } (\text{mod } 8, \text{ even})$ :

$$[\rho(a), J\rho(a')J^{-1}] = 0 \text{ for all } a, a' \in \mathcal{A}$$

$p \bmod 8$	0	2	4	6
$J^2 = \pm 1$	+	-	-	+
$J\mathcal{D}J^{-1} = \pm \mathcal{D}$	+	+	+	+
$J\gamma J^{-1} = \pm \gamma$	+	-	+	-

The spectral action (A. Connes & A. Chamseddine 1996):

The spectral action is defined to be the number of eigenvalues of the Dirac operator up to a cut-off  $\Lambda$ .

$$S_{sp.} = \text{Tr}(f(\frac{D}{\Lambda})) + (\Psi, D\Psi)$$

$f$  is a positive test function, only its momenta play a role. The effective action is obtained by a heat-kernel expansion of  $S_{sp.}$  of the trace (bosonic part of the action).

Constraints on the SM parameters at the cut-off  $\Lambda$ :

$$N_{SM} g_2^2 = N_{SM} g_3^2 = 3 \frac{Y_2^2}{H} \frac{\lambda}{24} = \frac{3}{4} Y_2$$

- $g_1, g_2, g_3$ :  $U_Y(1)$ ,  $SU_W(2)$ ,  $SU_C(3)$  gauge couplings
- $\lambda$ : quartic Higgs coupling
- $Y_2$ : sum of all Yukawa couplings squared
- $H$ : sum of all Yukawa couplings to the fourth power
- $N_{SM}$ : number of standard model generations

Robust predictions:  $\Lambda \sim 10^{17} \text{ GeV}$  and  $m_{\text{Higgs}} \sim 170 \text{ GeV}$

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Pure SM with KO-dim. = 0 (Connes, Chamseddine 1996)

finite Algebra:  $C \oplus H \oplus M_3(C)$

- $N_{\mathcal{H}} \neq N_{SM}$  (Poincaré duality)
- Neutrino masses are Dirac masses
- No SeeSaw mechanism
- Constraint  $Y_2 = 4 g_2^2$  at  $\Lambda \sim 10^{17} \text{ GeV}$   
 $\Rightarrow m_{top} \sim 190 \text{ GeV}$
- Solution I: Need another Yukawa coupling  $g_Y \sim 1$  in  $Y_2$
- Solution II: New particles

Pure SM with KO-dim. = 6 (Connes, Barrett 2006)

finite Algebra:  $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

- $N_{\nu_R}$  arbitrary
- Dirac and Majorana masses are allowed
- SeeSaw mechanism is *natural* with  $M_{M_W} \sim 10^{13} \text{ GeV}$  and  $g_\nu \sim 1.6 \Rightarrow m_{top} \sim 170 \text{ GeV}$
- Problem: Leptoquark masses (are put to zero by hand)
- Poincaré duality needs to be modified  
consider Leptons and Quarks separately
- Finite spectral triple violates Orientability axiom

Solution (Connes 2006): enlarge finite Algebra to

$\mathbb{C} \oplus \mathbb{H} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

Pure SM with KO-dim. = 0 (Jureit, Schücker, C.S. 2005)

finite Algebra:  $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \oplus \mathbb{C}$

- $N_{\nu_R}$  arbitrary
- Neutrino masses are Dirac masses
- No SeeSaw mechanism
- Constraint  $Y_2 = 4 g_2^2$  at  $\Lambda \sim 10^{17} \text{ GeV}$   
 $\Rightarrow m_{top} \sim 190 \text{ GeV}$

### Pure SM with KO-dim. = 6 (Jureit, C.S. 2006)

finite Algebra:  $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \oplus \mathbb{C}$

- $N_{\nu_R}$  arbitrary
- Dirac and Majorana masses are allowed
- SeeSaw mechanism is *natural*  $\Rightarrow m_{top} \sim 170\text{GeV}$
- No Leptoquark masses!
- Poincaré duality needs **not** to be modified
- Finite spectral triple violates Orientability axiom  
(generic feature of right-handed neutrinos with Majorana mass)

SM + 2 neutral Fermions, KO-dim. = 6 (C.S., to appear)

finite Algebra:  $C \oplus H \oplus M_3(C) \oplus C \oplus C \oplus C \oplus C \oplus C$

- $N_{\nu_R}$  arbitrary
- two new neutral particles  $X$  and  $Y$  (possibly in every generation)
- Dirac masses for all particles
- $X$  and  $Y$  masses are vectorlike  $\Rightarrow m_X \sim m_Y \sim \Lambda$
- vectorlike mass terms between  $X, Y$  and  $\nu_R$   
SeeSaw-like mechanism
- no problems with Axioms or Constraints

New part in the SM-Langrangian:

$$\begin{aligned}\mathcal{L}_{\text{new}} = & g_\nu \phi^0 \bar{\nu}_L \nu_R + m_X \bar{X}_L X_R + M_1 \bar{\nu}_R X_L + M_2 \bar{\nu}_R \bar{X}_R \\ & + m_Y \bar{Y}_R Y_L + h.c.\end{aligned}$$

Mass eigenvalues for  $M = M_1 = M_2 \sim \Lambda$

$$m_{1/2} \sim \pm m_\nu^2 \frac{m_X}{2M} \quad m_{3..6} \sim \pm m_X \quad m_{7/8} \sim \pm 2 \frac{M^2}{m_X}$$

Successful SeeSaw mechanism with a detour!

### Conclusions:

- Majorana masses and the SeeSaw mechanism problematic in Noncommutative Geometry à la Connes
- Physical constraint  $Y_2 = 4g_2$  at  $\Lambda$  seems to suggest particles beyond the Standard Model
- SeeSaw mechanism requires either modification of Axioms or new particles

### Open Questions & Outlook:

- How to distinguish the different models experimentally?
- Underlying theory? → Quantisation?
- Lorentzian spectral triples (M. Paschke, A. Rennie, R. Verch to appear)