Newtonian View of General Relativistic Stars

Hermano Velten

Federal University of Espírito Santo (Vitória-Brazil)

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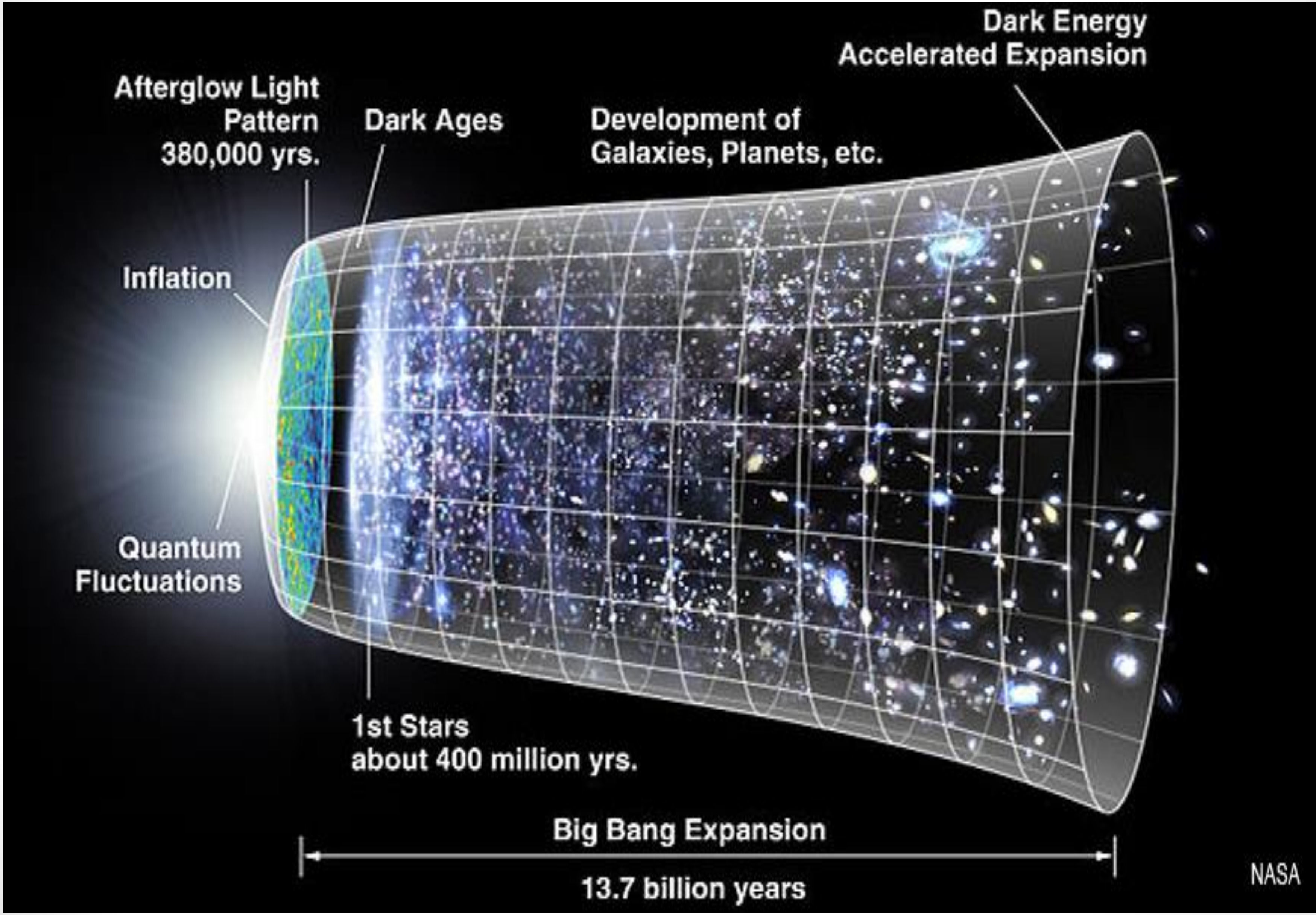
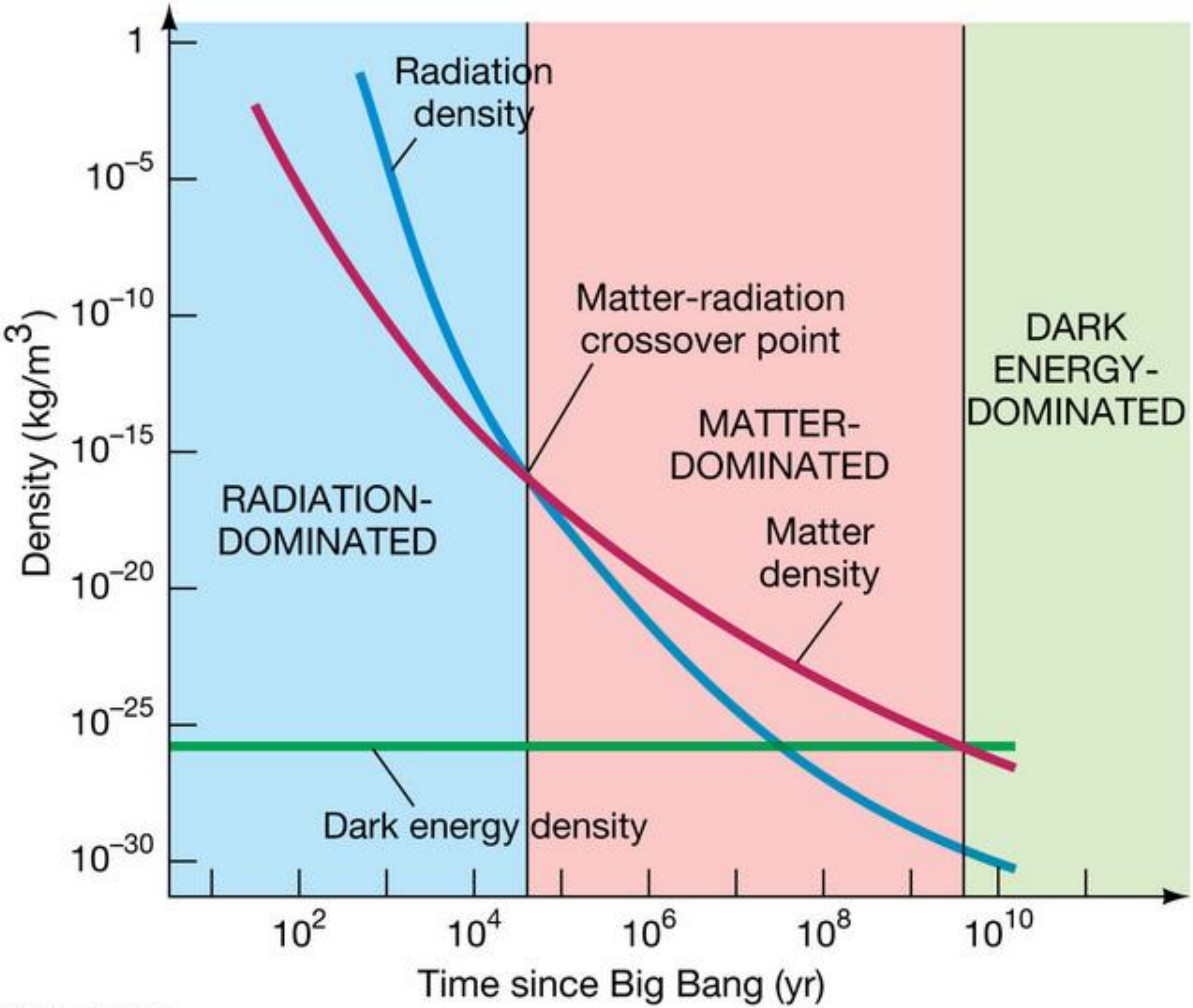
COSMOVIA LECTURE


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA



Radiation density
Matter-radiation crossover point
DARK ENERGY-DOMINATED
MATTER-DOMINATED
RADIATION-DOMINATED
Matter density
Dark energy density
Density (kg/m^3)
1
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
10^2
10^4
10^6
10^8
10^{10}
Time since Big Bang (yr)

Friedmann Equations (1922)

Friedmann-Lemaître-Robertson-Walker (FLRW):

$$ds^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \mathrm{sen}^2\theta d\phi^2)\right\}$$

$$\frac{\dot{a}^2}{a^2} + \frac{k\,c^2}{a^2} = \frac{8\pi G\rho}{3}, \qquad \frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k\,c^2}{a^2} = -\frac{8\pi G p}{c^2}.$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right).$$

$$\dot{a} = \partial_t a.$$

Fluids in a Gravitational Field
Newtonian Cosmology

E. A. Milne, *Quart. J. Math.* **5**, 64 (1934).

E. A. Milne, W. H. McCrea, *Quart. J. Math.* **5**, 73 (1934).

Edward Arthur Milne

William Hunter McCrea

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$$\dot{\rho} + \nabla \cdot (\rho \vec{v}) = 0,$$

$$\dot{\vec{v}} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{\nabla p}{\rho} - \nabla \Psi,$$

$$\nabla^2 \Psi = 4\pi G \rho.$$

Using a velocity field of a isotropic expansion which is proportional associated to the Hubble's law:

$$\vec{v} = H(t)\vec{r}.$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}.$$

$$\frac{\dot{a}^2}{a^2} + \frac{(-2E)}{a^2} = \frac{8\pi G}{3}\rho \quad \text{and} \quad \dot{H} + H^2 = -\frac{4\pi G}{3}\rho.$$

Fluids in a Gravitational Field
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E. A. Milne, *Quart. J. Math.* **5**, 64 (1934).

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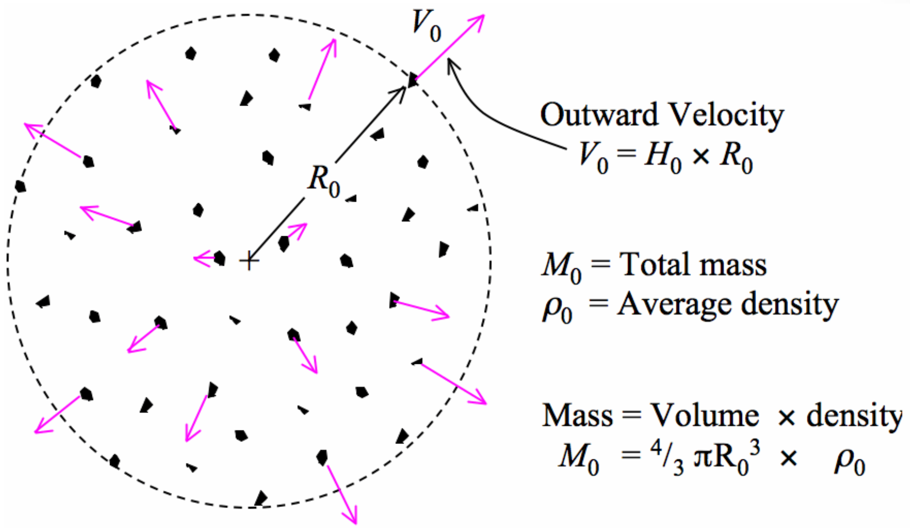
$$m\frac{d^2R}{dt^2} = -\frac{GMm}{R^2}$$



$$\rho = 3M/4\pi R^3$$

$$\ddot{R} = -\frac{4\pi G\rho R}{3}$$

$$\frac{\dot{R}^2}{R^2} + \frac{\left(\frac{-2E}{m}\right)}{R^2} = \frac{8\pi G\rho}{3}.$$

Newton

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right).$$

$$\frac{\dot{a}^2}{a^2} + \frac{k\,c^2}{a^2} = \frac{8\pi G\rho}{3}.$$

GR

Large scale structure formation

1. Cosmic structures formed during the matter dominated epoch, i.e., background dynamics effectively described by a Einstein-de-Sitter universe.

$$a(t) \propto t^{2/3}$$

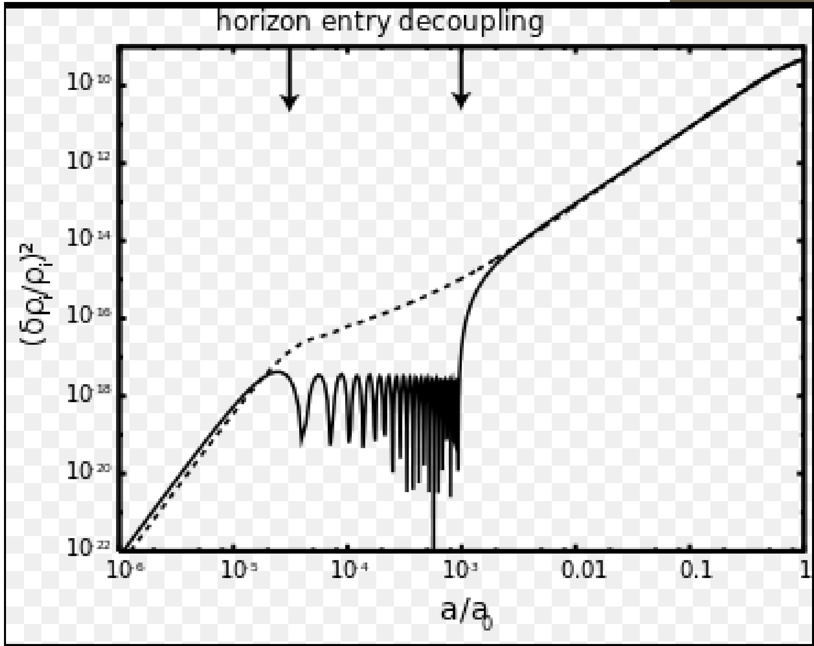
2. What type of cosmic structures?

-> Dark matter halos and baryons both assumed to be pressureless fluids, i.e., p=0.

-> Vanishing speed of sound!

$$\ddot{\Delta} + 2H\dot{\Delta} - 4\pi G\rho\,\Delta = 0.$$

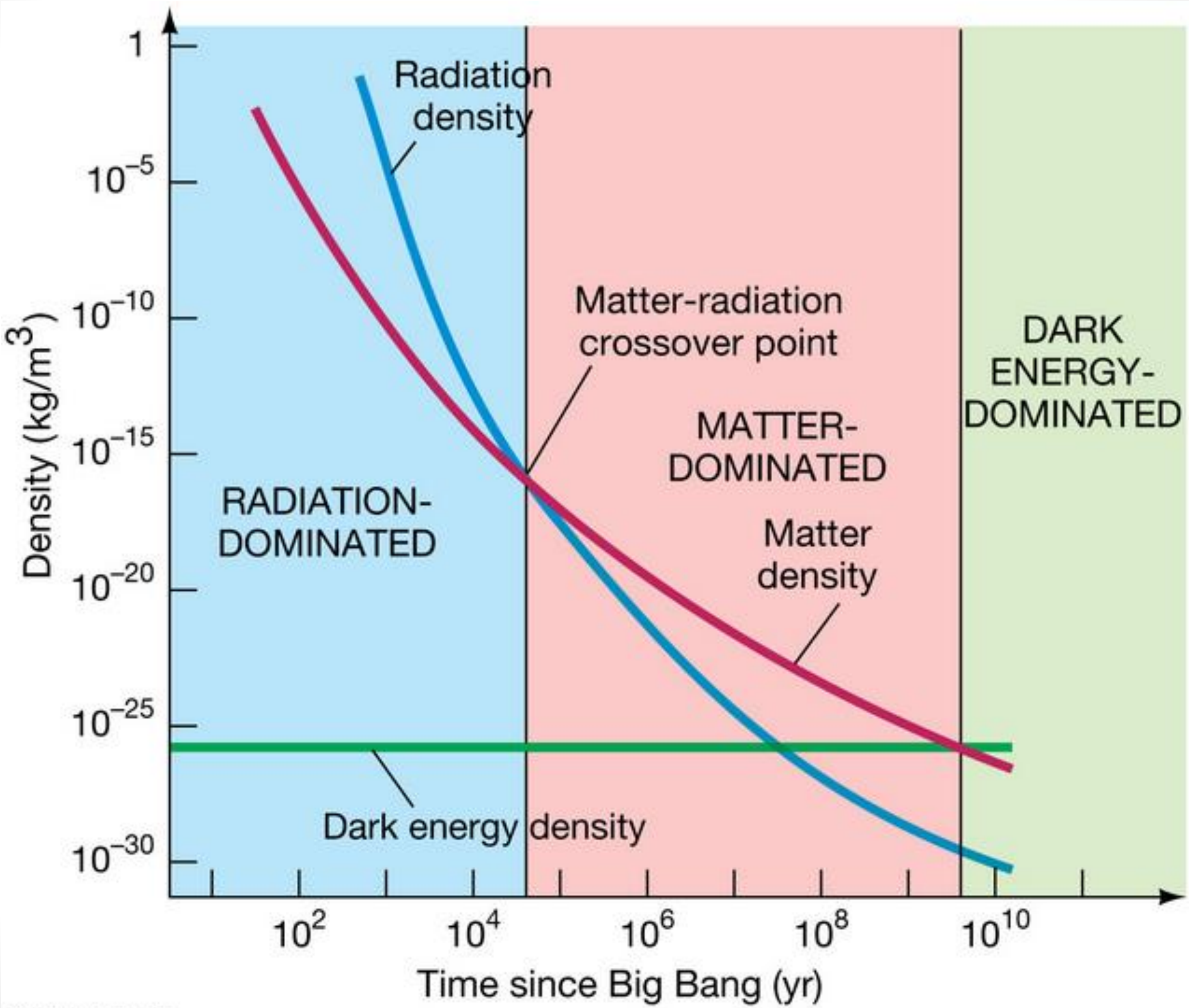
$$\Delta = \hat{\rho}_m/\rho_m.$$



Pressure effects?
neo-Newtonian Cosmology

W. H. McCrea, *Proc. R. Soc. London* **206**, 562 (1951).
E. R. Harrison, Ann. Phys (N.Y.) **35**, 437 (1965).

Edward Robert Harrison



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1. (Active)-Gravitational mass-energy density

$$\rho_g \rightarrow \rho + 3p. \qquad \nabla^2\phi = 4\pi G\rho_{ag}.$$

2. Inertial and (Passive)-Gravitational mass-energy density

$$\rho_i \rightarrow \rho + p.$$

Representing it as a set of fluid equations

$$\dot{\rho} + \nabla \cdot (\rho \vec{v}) + p \nabla \cdot \vec{v} = 0 \,,$$

On the Newtonian Cosmology Equations with Pressure

J.A.S. Lima, V. Zanchin, R. Brandenberger

Mon.Not.Roy.Astron.Soc. 291 (1997) L1-L4

$$\dot{\vec{v}} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla\Psi - \frac{\nabla p}{\rho + p} \,,$$

$$\nabla^2 \Psi = 4\pi G \,(\rho + 3p) \,.$$

Again, identifying the velocity field with the Hubble´s Law expansion:

$$\vec{v} = H(t)\vec{r}.$$

$$\left.\begin{aligned} &\frac{\dot{a}^2}{a^2} + \frac{(-2E)}{a^2} = \frac{8\pi G}{3}\rho, \\ &\dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3p). \end{aligned}\right\}$$

Neo-Newton

$$\left.\begin{aligned} &\frac{\dot{a}^2}{a^2} + \frac{k\,c^2}{a^2} = \frac{8\pi G\rho}{3}, \\ &\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right). \end{aligned}\right\}$$

GR

Is there the same equivalence for a non-expanding system?

arXiv:1409.7385

Newtonian View of General Relativistic Stars

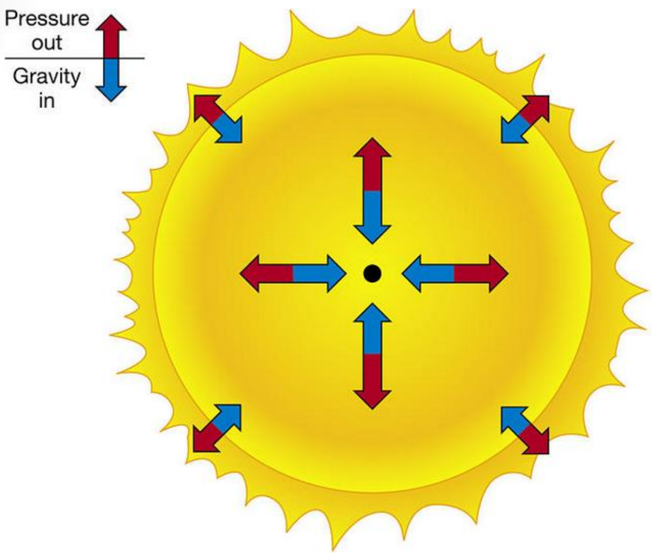
A. M. Oliveira[*]
Grupo de Ciências Ambientais e Recursos Naturais, Instituto Federal do Espírito Santo (IFES), Guarapari, Brazil

H. E. S. Velten[†] and J. C. Fabris[‡]
Departamento de Física, Universidade Federal do Espírito Santo (UFES), Vitória, Brazil

I. G. Salako[§]
Institut de Mathématiques et de Sciences Physiques (IMSP), Porto-Novo, Bénin

Although general relativistic cosmological solutions, even in the presence of pressure, can be mimicked by using neo-Newtonian hydrodynamics, it is not clear whether there exists the same Newtonian correspondence for spherical static configurations. General relativity solutions for stars are known as the Tolman-Oppenheimer-Volkoff (TOV) equations. On the other hand, the Newtonian description does not take into account the total pressure effects and therefore can not be used in strong field regimes. We discuss how to incorporate pressure in the stellar equilibrium equations within the neo-Newtonian framework. We compare the Newtonian, neo-Newtonian and the full relativistic theory by solving the equilibrium equations for both three approaches and calculating the mass-radius diagrams for some simple neutron stars equation of state.

- Newtonian hydrostatic equilibrium (Lane-Emden)



DENSITY

MASS WITHIN A SPHERE OF RADIUS r

$$\rho \frac{d^2 r}{dt^2} = -G \frac{M_r \rho}{r^2} - \frac{dP}{dr}$$

ACCELERATION

GRAVITATIONAL CONSTANT

PRESSURE GRADIENT

$$\frac{1}{\rho}\frac{dP}{dr} = -\nabla\Psi$$

$$\frac{1}{\rho}\frac{dP}{dr} = -\frac{G}{r^2}\int_0^r 4\pi\rho r'^2 dr'$$

$$\begin{cases} \dfrac{1}{\rho}\dfrac{dP}{dr} = -\dfrac{GM(r)}{r^2} \\ \dfrac{dM(r)}{dr} = 4\pi r^2 \rho. \end{cases}$$

- Hydrostatic equilibrium - GR

(Tolman-Oppeinheimer-Volkof - TOV)

$$ds^2 = -e^{2\psi}dt^2 + e^{2\lambda}dr^2 + r^2 d\Omega^2,$$

$$(\rho + p)\frac{d\psi}{dr} = -\frac{dP}{dr}.$$

Energy momentum conservation

(0-0) $$m(r) = \frac{1}{2}(1 - e^{-2\lambda}),$$

$$g_{rr} = e^{2\lambda} = \left(1 - \frac{2Gm(r)}{r}\right)^{-1}$$

$$\Rightarrow \quad \frac{dm(r)}{dr} = 4\pi r^2 \rho.$$

(r-r) $$\frac{d\psi}{dr} = \frac{m(r) + 4\pi r^3 P}{r(r - 2Gm(r)r)}$$

$$\frac{dP}{dr} = -\frac{G\rho m(r)}{r^2}\left(1 + \frac{P}{\rho}\right)\left(1 + \frac{4\pi r^3 P}{m(r)}\right)\left(1 - \frac{2Gm(r)}{r}\right)^{-1}$$

- Neo-Newtonian hydrostatic equilibrium
- arXiv: 1409.7385

$$\frac{1}{\rho_i}\frac{dp}{dr} = -\frac{G}{r^2}\int_0^r 4\pi r'^2(\rho + 3p)dr'.$$

$$\frac{dp}{dr} = -\frac{G(\rho + p)}{r^2}\tilde{M}(r).$$

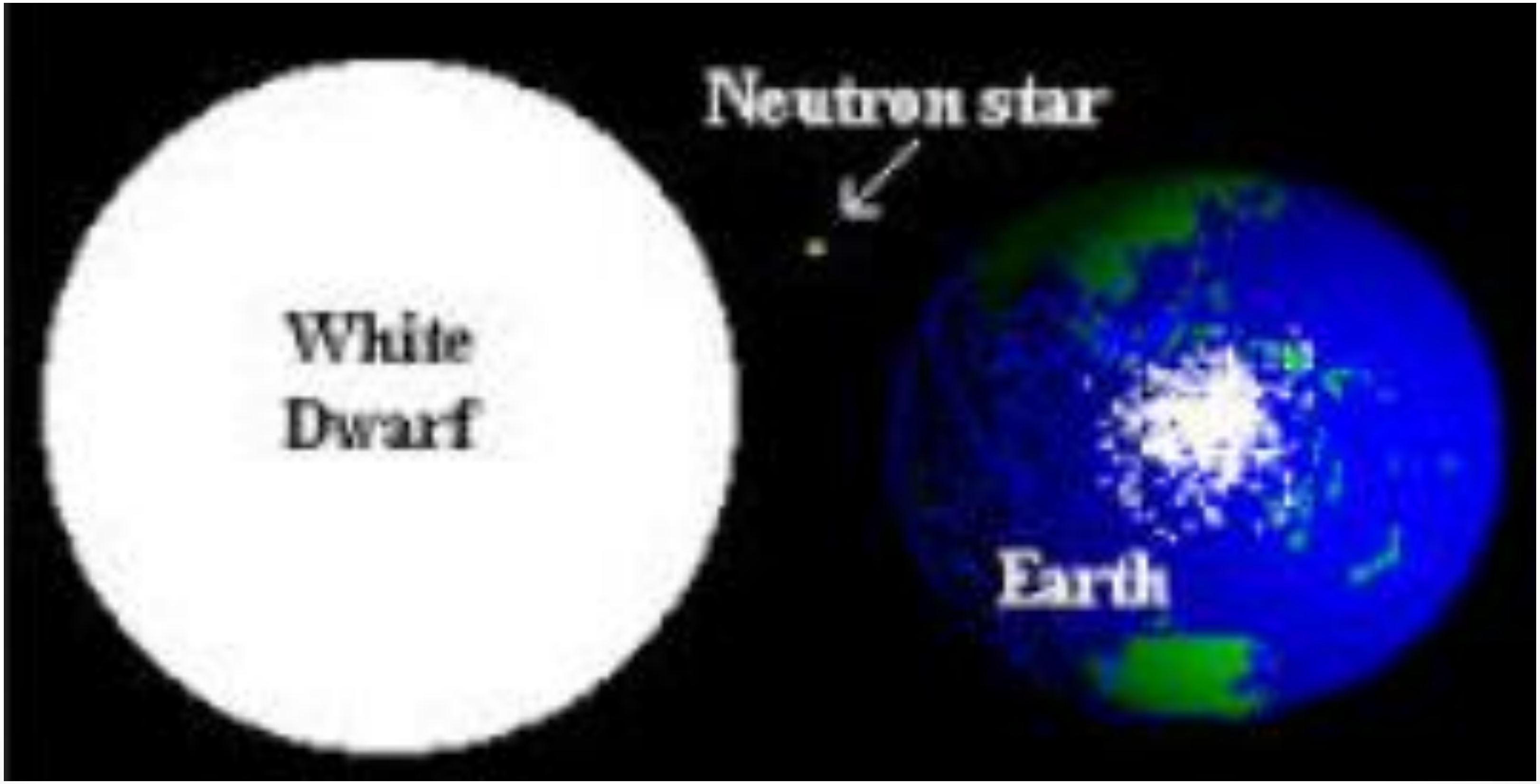
Where automatically there is an equivalent definition for mass (or a kind of)

$$\frac{d\tilde{M}(r)}{dr} = 4\pi r^2(\rho + 3P).$$

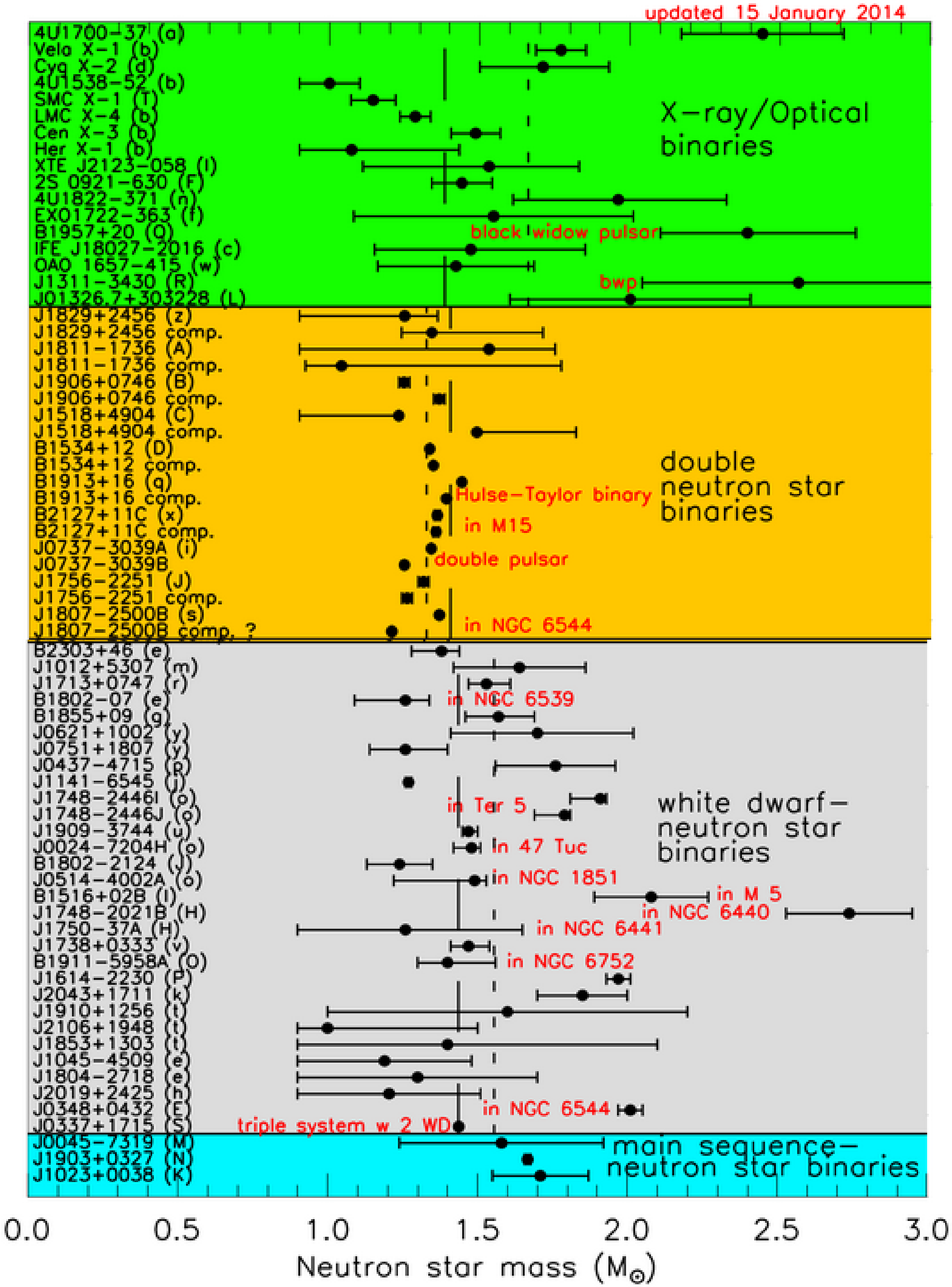
Having in mind the definition used in both the Newtonian and Relativistic cases

$$M = \int_0^R 4\pi r^2 \rho dr.$$

Neutron stars seem to be the ideal situation in which we can test whether the equivalence between GR and the neo-Newtonian formalism still holds for NON EXPANDING configurations.



stellarcollapse.org/nsmasses

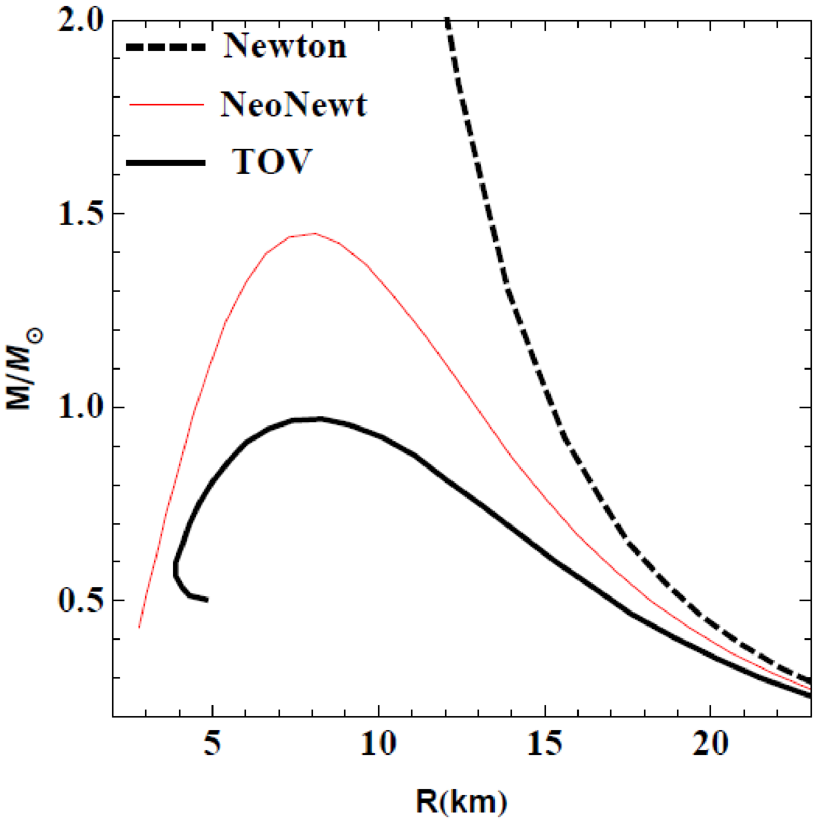


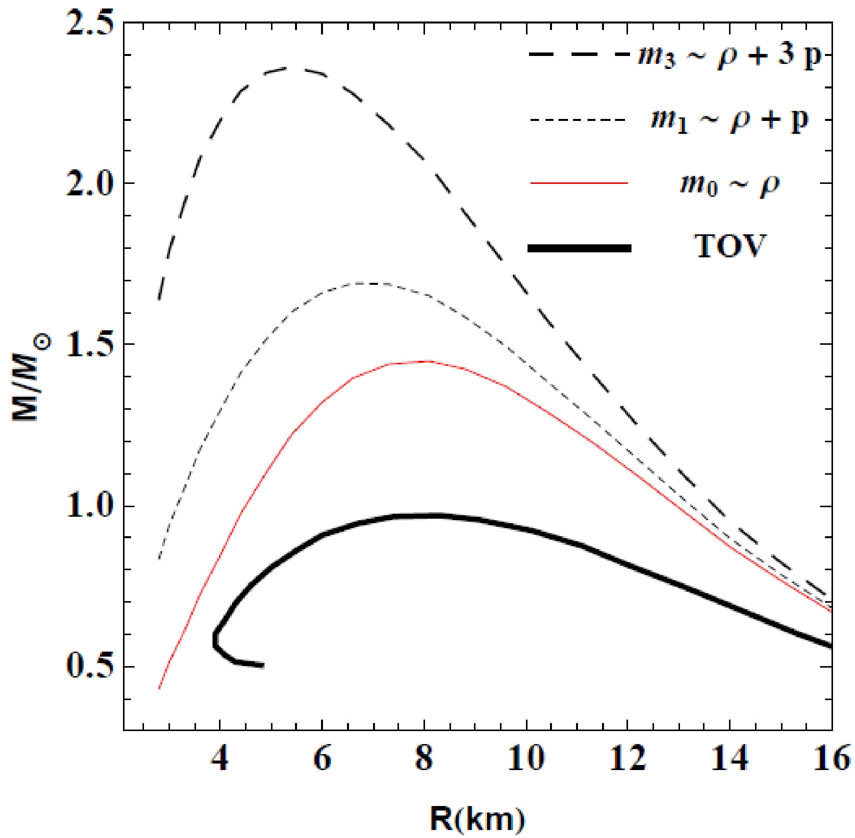
Testing simple Neutron star configurations: Pure neutrons - Fermi gas equation of state

$$\frac{\bar{\rho}(\bar{p})}{c^2} = \bar{K}^{-1}\,\bar{p}^{3/5} \qquad \bar{K} = 1.914$$

$$p = \epsilon_0 \bar{p} \qquad \rho = \epsilon_0 \bar{\rho}$$

$$\epsilon_0 = 1.603 \text{ x } 10^{38} ergs/cm^3.$$

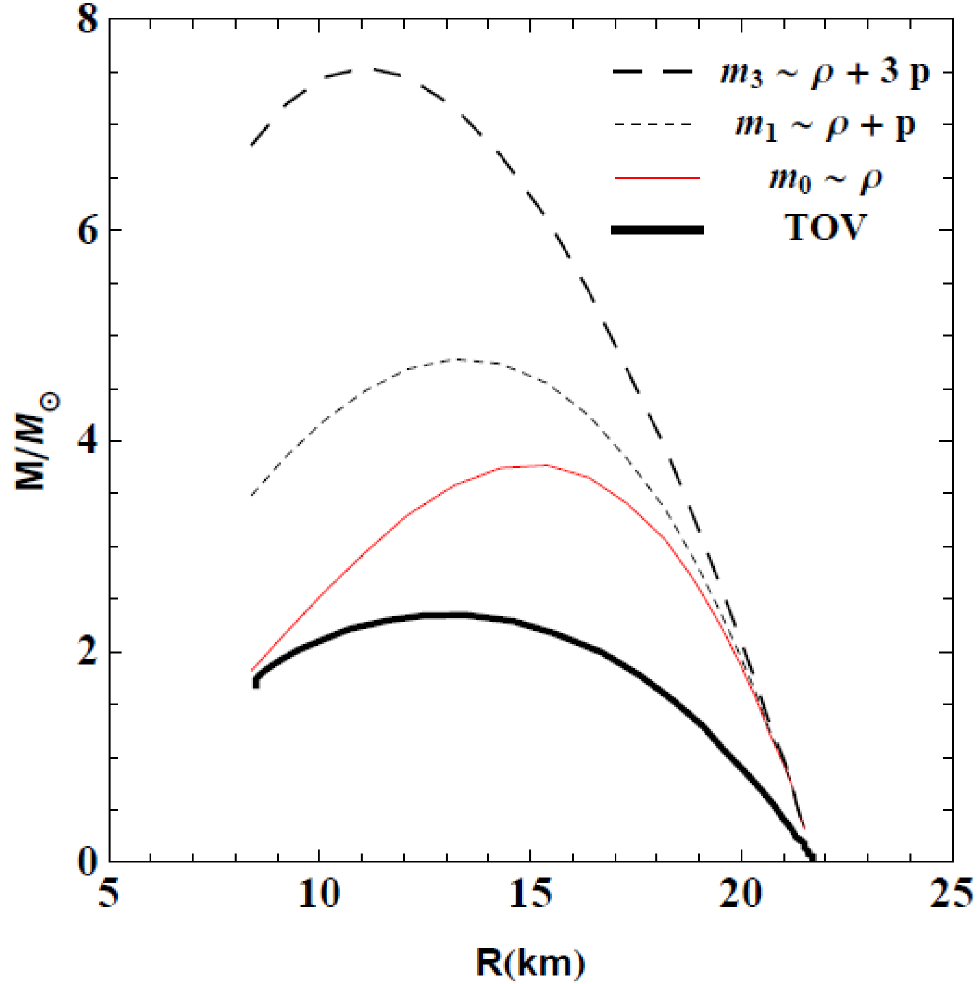




$$m_3 \sim \int (\rho + 3p) r^2 dr.$$

$$m_1 \sim \int (\rho + p) r^2 dr.$$

More realistic scenario for Neutron stars: Pure neutrons - Fermi gas equation of state with nucleon-nucleon interactions



$$\frac{\rho(\bar{p})}{c^2} = (\kappa_0 \epsilon_0)^{-1/2} \bar{p}^{1/2},$$

$$\epsilon_0 = m_n^4 c^5 / 3\pi^3 \hbar^3$$

$$\kappa_0 = 363 \ MeV$$

Final Remarks

- Cosmology can be studied in a much simpler formalism: Newtonian or neo-Newtonian
- This demands a deeper understanding of hydrodynamics., i.e., how to properly include pressure effects.
- The well known equivalence (classical-relativistic) in cosmology clearly breaks down for a non-expanding system.