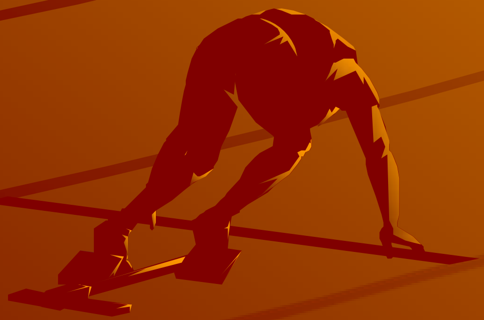
Multidimensional gravity

S. G. Rubin



Motivation:

Attempt to construct the Universe from multidimensional world provided any matter is absent other than gravity.

Supposition:

Quantum fluctuations produce any manifolds of different dimensionality, geometry and topology. S. G. Rubin, JETP, to be published in May, 2008

Quantum fluctuations are responsible for nonlinear terms in lagrangians

Main question:

What observational effects could we expect considering different **pure gravitational** Lagrangians – K. A. Bronnikov, R.V. Konoplich, S. G. Rubin, QCG, 24, 1261, 2007

We investigate abilities of this approach but not a final version.

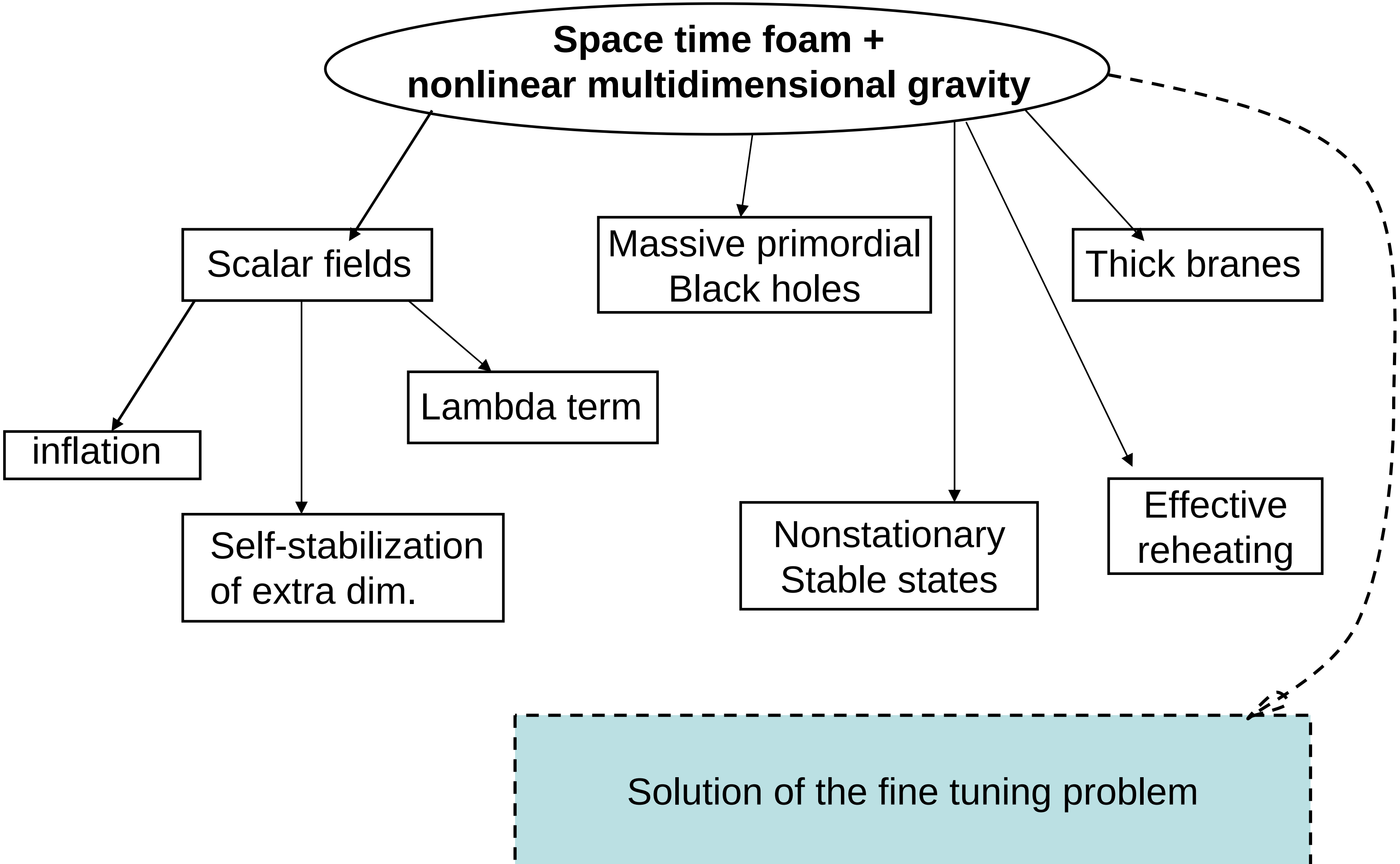
Tools:

The method of slow motion elaborated in

see K. A. Bronnikov, S. G. Rubin, PR D73, 124019, 2006;

and apply it to reduce lagrangians to 4-dim low energy theory.

(Multidimensional) gravity should contain higher derivatives.
The gravitational action, in general, includes terms containing **powers of *R*** (Ricci scalar), **Ricci tensor squared, Kretschmann scalar etc** due to **quantum effects in curved space-time.**



Space $M_4 \times M_{\mathbf{d}}$

Specific assumptions:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \mathrm{e}^{2\beta(x)} h_{ab} dx^a dx^b$$

g: metric of external space;
h: metric of internal space of constant curvature ***k* = ±1.**

$\beta(x)$ is seen as a scalar field from 4-dim external space, inevitable consequence of multidimensionality.

Consider action in the form

$$S = \frac{1}{2\kappa^2} \int \sqrt{{}^D g} d^D x F(R)$$

Extensions are supposed

We elaborate the method to treat general form of gravitational action in low energy limit. The last is not very restrictive, e.g. inflation starts with $R \sim 10^{-11}$ in Planck units.

L^2

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \mathrm{e}^{2\beta(x)} h_{ab} dx^a dx^b$$

$$R = \overline{R}[g] + \mathrm{e}^{-2\beta} \overline{R}[h] + 2d_1 \Box \beta + d_1(d_1+1)(\partial\beta)^2$$

$$L^2 \sim \frac{1}{\phi}$$

$$R = R_4 + \phi + f_1$$

Scalar field made from metric tensor of extra space: $\boxed{\phi = k d_1 (d_1 - 1) e^{-2\beta}}$

Steps to simplify the equations:

1. Reduce to 4D
2. Invoke the slow change approximation: $\phi >> R_4, f_1$
3. Pass to Einstein frame in 4D (if necessary)

Let us start with: $$S = \frac{1}{2\kappa^2}\int \sqrt{{}^D g}\, d^D x F(R)$$

$$F(R) = F(\phi + R_4 + f_1) \approx F(\phi) + F'(\phi)\left(R_4 + f_1\right) + \mathrm{O}\left(\left(R_4 + f_1\right)^2\right)$$

$$f_1 = 2d_1 \Box \beta + d_1(d_1+1)(\partial\beta)^2$$

$$S = \frac{\mathcal{V}[d_1]}{2\kappa^2}\int \sqrt{{}^4 g}\, d^4 x e^{d_1\beta}[F'(\phi)R_4 + F(\phi) + F'(\phi)f_1]$$

$$V_J(\phi) = -\frac{e^{d_1(\beta-\beta_m)}}{2\kappa_N^2 F'(\phi_m)}F(\phi)$$

This reduced action is characteristic of STT (Jordan frame)

Einstein frame

$$g_{\mu\nu} \mapsto \widetilde{g}_{\mu\nu} = |f(\phi)| g_{\mu\nu}, \quad f(\phi) = e^{d_1\beta} F'(\phi)$$

$$S = \frac{V[d_1]}{2} m_D^2 \int \sqrt{\widetilde{g}}\,(\mathrm{sign}\,F') L d^4x \longleftarrow \boxed{S = \frac{1}{2\kappa^2}\int \sqrt{^D g}\, d^D x F(R)}$$

$$L = \widetilde{R}_4 + \frac{1}{2} K_{\mathrm{Ein}}(\phi)(\partial\phi)^2 - V_{\mathrm{Ein}}(\phi)$$

$$K_{\mathrm{Ein}}(\phi) = \frac{1}{2\phi^2}\left[6\phi^2\left(\frac{F''}{F'}\right)^2 - 2d_1\phi\frac{F''}{F'} + \frac{1}{2}d_1(d_1+2)\right]$$

$$V_{\mathrm{Ein}}(\phi) = -(\mathrm{sign}\,F')\left[\frac{|\phi|}{d_1(d_1-1)}\right]^{d_1/2} \frac{F(\phi)}{F'(\phi)^2}$$

Promising Lagrangian of scalar field (radion) has utter geometrical origin.
What kind of phenomena could be described in this way?
Application for different F(R).

Effects of pure multidimensional world

for different parameters, number of extra dimensions
and their topological structure

1. Selfstabilization, Ads space

$$S = \frac{1}{2\kappa^2}\int \sqrt{{}^{D}g}\, d^D x (R + cR^2 - 2\Lambda)$$

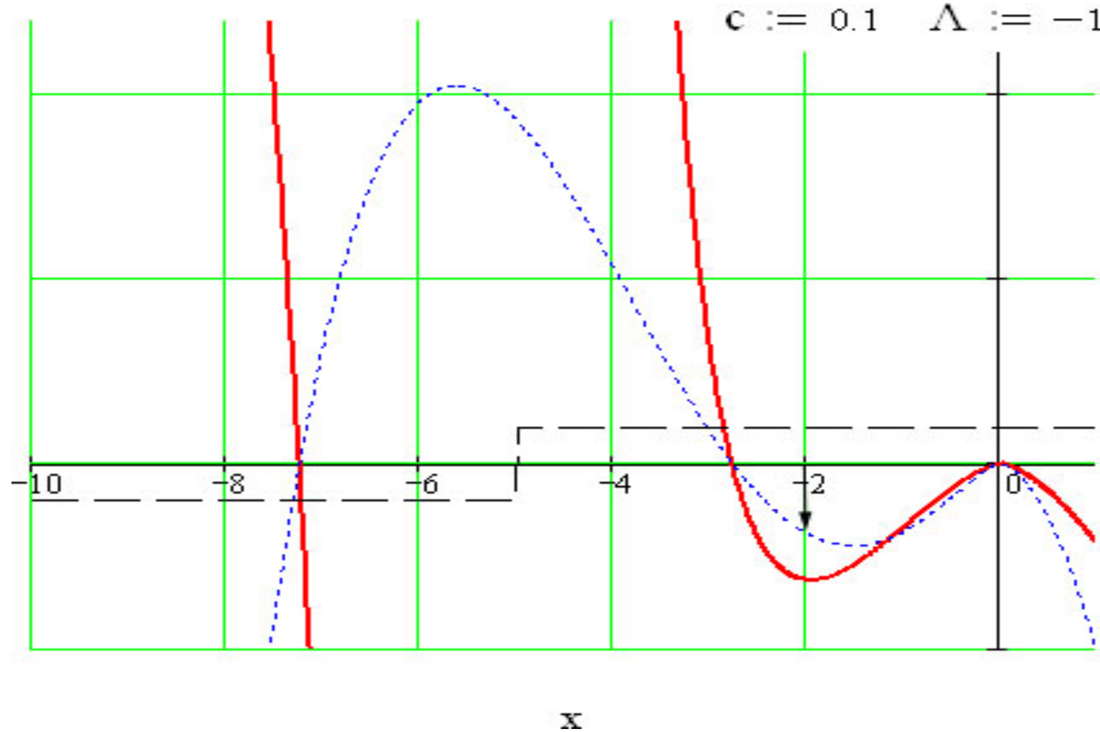
The scalar field is settled at the minimum of its potential $L^2 \sim \frac{1}{\phi}$

Red: ***V*** **[Einst. frame]**

Blue: ***V*** **[Jordan frame]**

Dashed: **sign** ***K***

The only min ***V:*** **AdS**

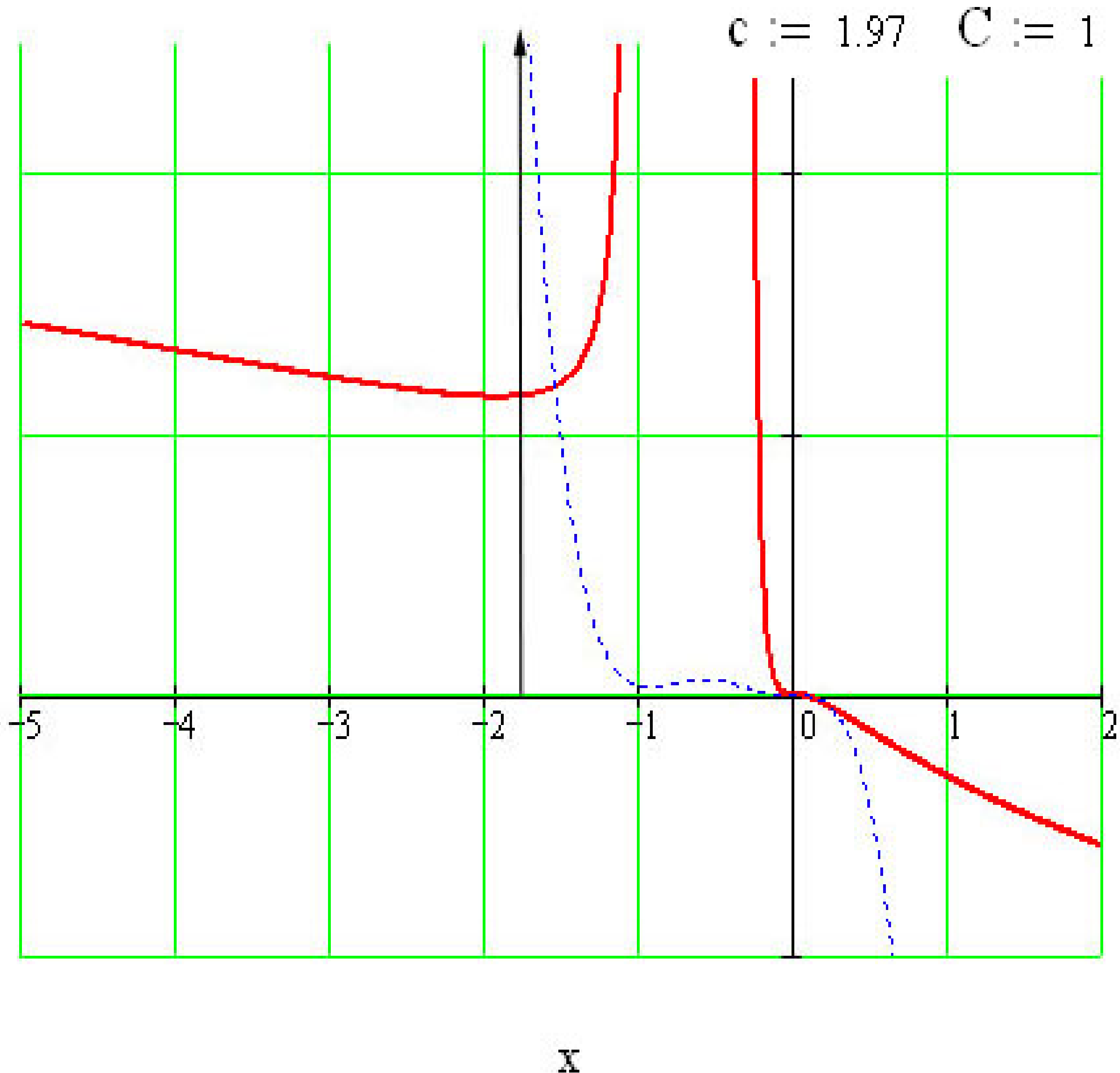


2. Selfstabilization, De Sitter space

$$F(R) = R + cR^2 + CR^3$$

Red: ***V***
[Einst. frame]

Blue: ***V***
[Jordan frame]



3. Quick particle production after the end of inflation

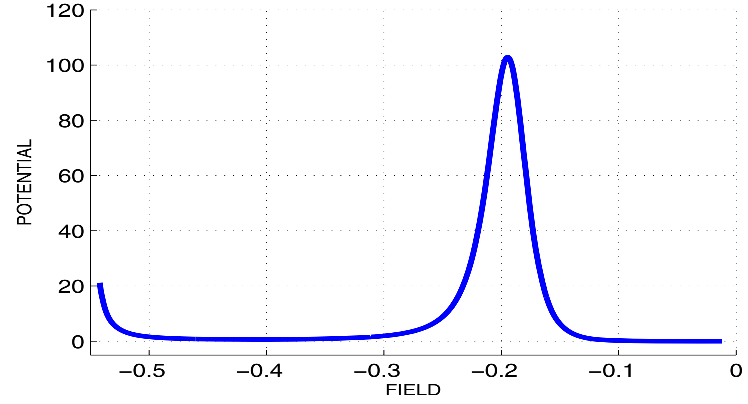
The role of kinetic term

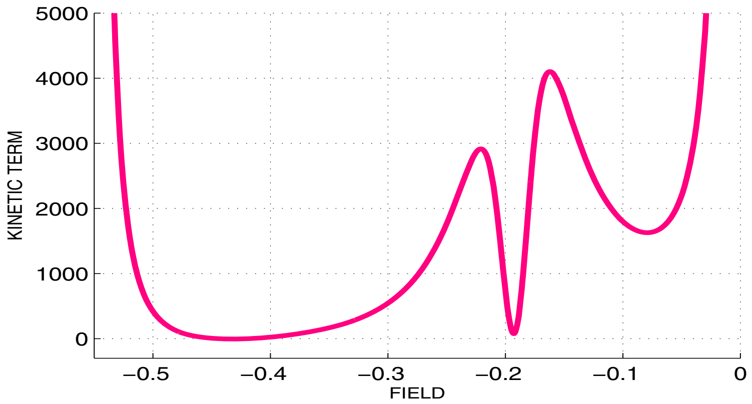
$$F(R) = R + cR^2 + w_1 R^3 + w_2 R^4 - 2\Lambda$$

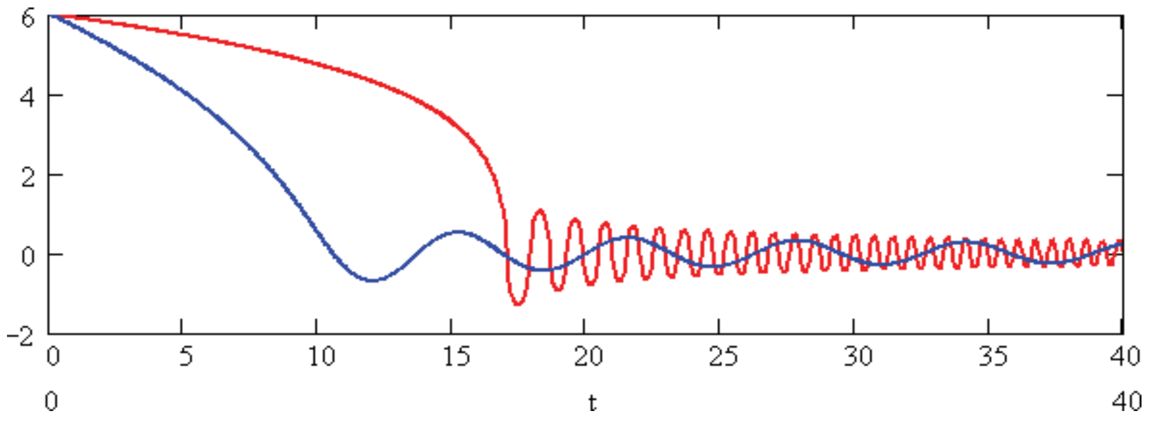
$$L_{\text{eff}} \simeq \frac{1}{2} K_{\min} \dot{\phi}^2 + g\phi\chi\chi$$

$$\downarrow$$

$$L_{\text{eff}} \simeq \frac{1}{2} \dot{\phi}^2 + \frac{g}{\sqrt{K_{\min}}} \phi\chi\chi$$



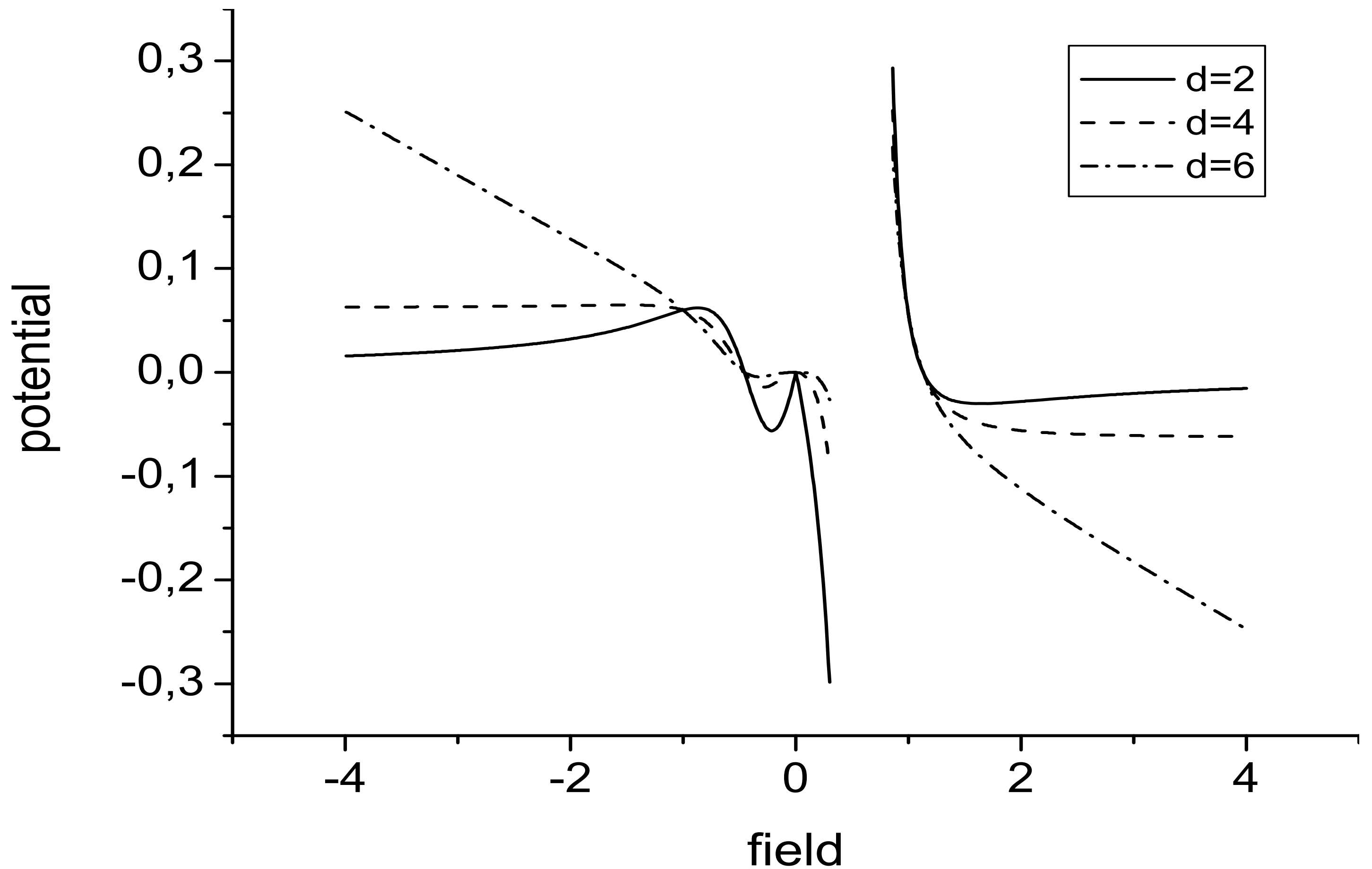




4. Effect of dimensionality of extra space

$$F(R) = R + cR^2 + w_1 R^3 + w_2 R^4 - 2\Lambda$$

Variation of dimensionality of extra space for the same parameters leads to significantly different low energy physics.



Generalization

$$S = \frac{\mathcal{V}[d_1]}{2\kappa^2} \int d^4x \sqrt{{}^4g}\, \mathrm{e}^{d_1\beta}(R + cR^2 - 2\Lambda + c_1 R_{AB}R^{AB} + c_2\, R_{ABCD}R^{ABCD})$$

$$V(\phi) = -\frac{1}{[d_1(d_1-1)]^{d_1/2}} \phi^{d_1} \frac{c_{tot}\phi^4 + \phi^2 - 2\Lambda}{(1+2c\phi^2)^2}$$

$$K_{\mathrm{Ein}}^{(2)}(\phi) = \frac{1}{(1+2c\phi)^2\phi^2}\Big[c^2\phi^2(d_1^2 - 2d_1 + 12) + d_1^2 c\phi + \frac{1}{4} d_1(d_1+2)\Big] \boxed{+ \frac{c_1 + c_2}{2\phi(1+2c\phi)}}$$

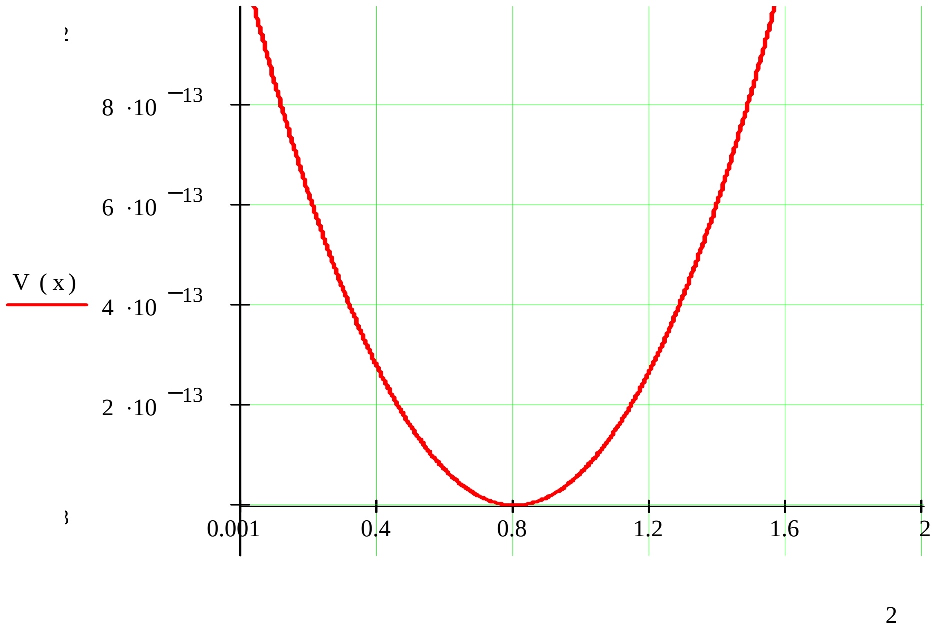
More freedom in choosing parameters

5. Fitting the dark energy density and the inflaton mass

$$S = \int d^D x \sqrt{{}^D g}\,\left[F(R) + c_1 R_{AB} R^{AB} + c_2 \mathcal{K}\right]$$

$F(R) = R + cR^2 - 2\Lambda$ c = 25000, $\Lambda = 0.2015$, $c_{tot} = -0.62$

d=4



The potential reproduces Inflation with the potential $\sim\phi^2$

Nodes of kinetic term

6. Branes (point A)

$$S = \int d^D x \sqrt{{}^D g}\,\left[F(R) + c_1 R_{AB} R^{AB} + c_2 \mathcal{K}\right]$$

$$F(R) = R + cR^2 - 2\Lambda$$

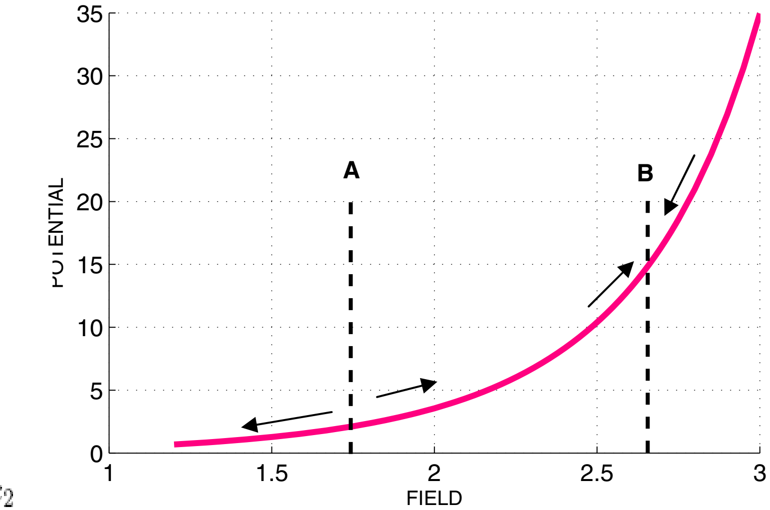
Low energy physics

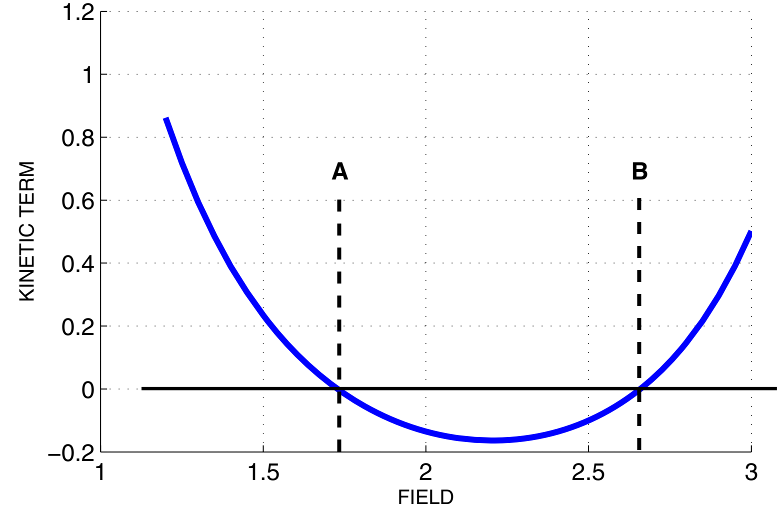
$$V_{\rm Ein}(\phi) = -\,{\rm sign}(1+2c\phi)[d(d-1)]^{-d/2}\cdot|\phi|^{d/2}\frac{c'\phi^2+\phi-2\Lambda}{(1+2c\phi)^2},$$

$$c' = c + \frac{c_1}{d} + \frac{2c_2}{d(d-1)},$$

$$K_{\rm Ein}(\phi) = \frac{1}{\phi^2(1+2c\phi)^2}\left[c^2\phi^2(d^2-2d+12)+d^2c\phi+\frac{1}{4}d(d+2)\right]+\frac{c_1+c_2}{2\phi(1+2c\phi)}$$

$$S = \int d^4x \left[\frac{1}{2}K(\phi)(\partial\phi)^2 - V(\phi)\right]$$





7. Nonstationary stable state (point B).

$$S = \int d^4x \left[\frac{1}{2}K(\varphi)(\partial\varphi)^2 - V(\varphi)\right]$$

$$K(\varphi)\partial^2\varphi + \frac{1}{2}K'(\varphi)(\partial\varphi)^2 = -V'(\varphi)$$

Critical point – $K(\varphi_{crit})=0$

Near the critical point (let φ_{crit} =0):

$$K(\varphi) = k\varphi + o(\varphi), V(\varphi) = V(0) + h\varphi + o(\varphi), \quad k > 0, h > 0$$

Main equation is reduced to the equation

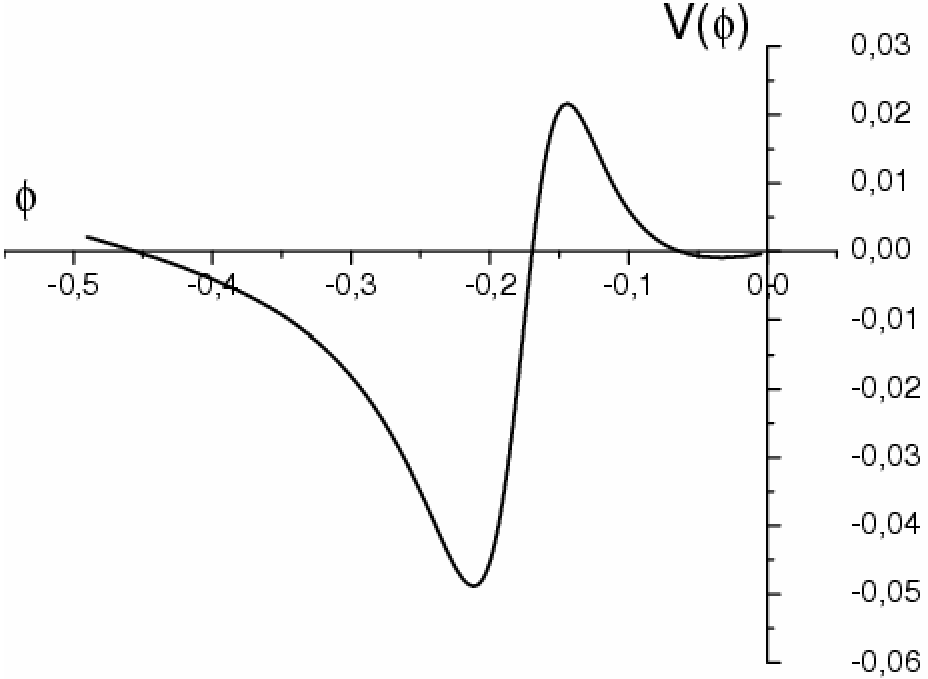
$$k\varphi\partial^2\varphi = -\left(h + \frac{1}{2}k(\partial\varphi)^2\right) + o(\varphi)$$

which has no stationary uniform solutions (!?).

8. Thick brane

$$S = \int d^D x \sqrt{^D g}\left[F(R) + c_1 R_{AB}R^{AB} + c_2 \mathcal{K}\right]$$

2-dim internal (compact) space,
4+1 external (large) space



d0 := 5 　 c := 6 　 Λ := −0.02

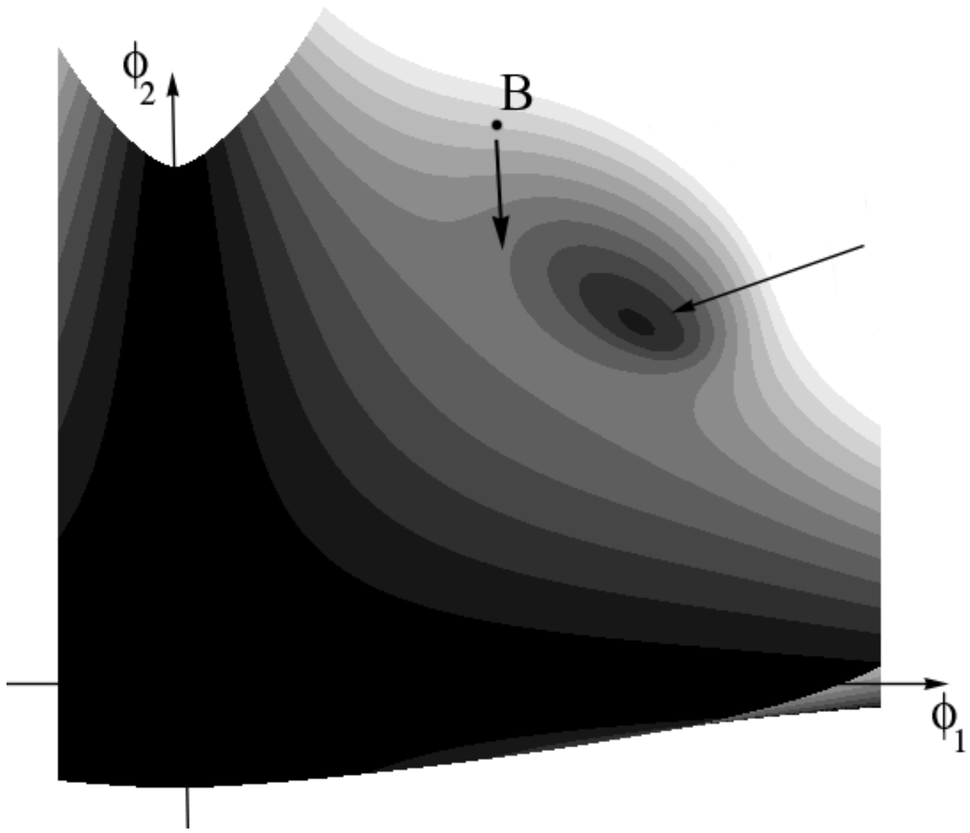
d1 := 2

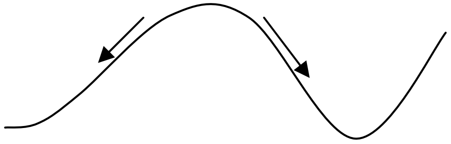
w1 := 15 　 cv := 1.5 　 ck := 5

W2=10

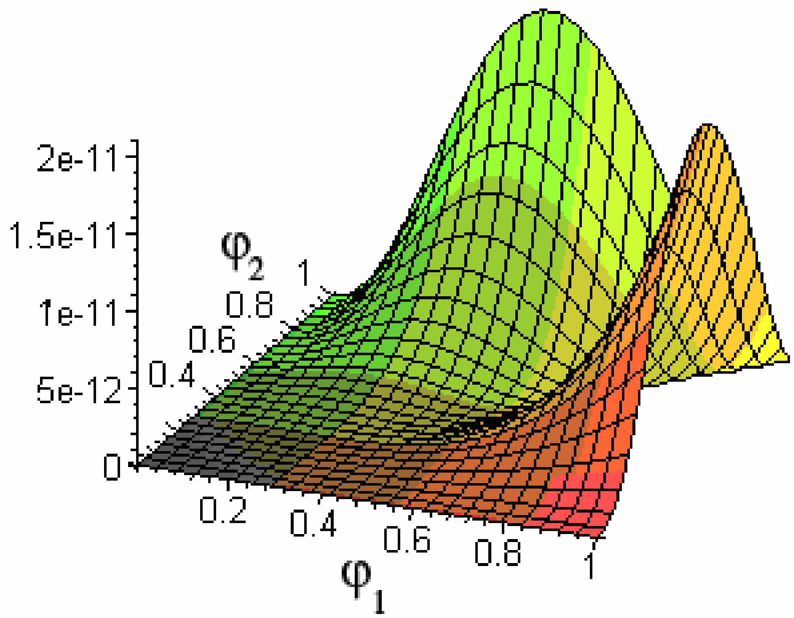
Space $R_4 \times R_3 \times R_3$

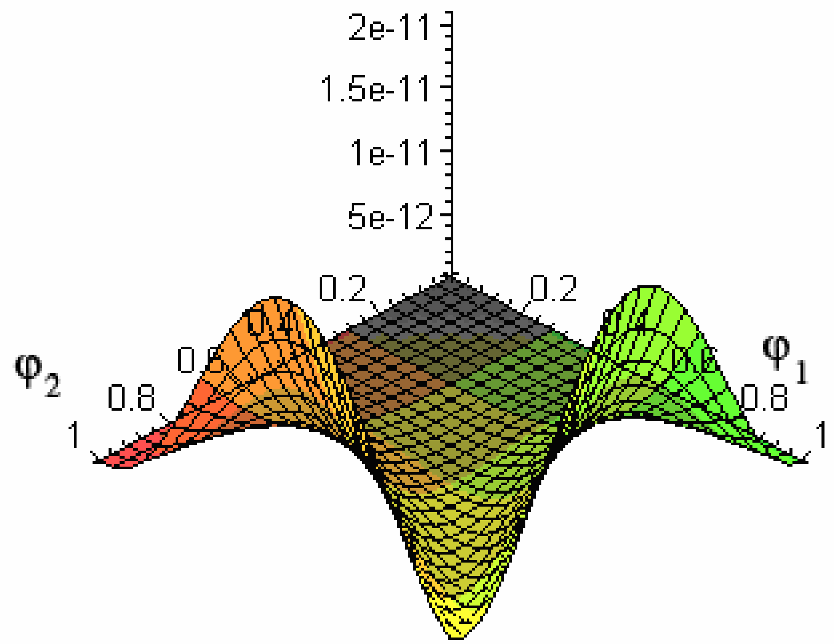
Metastable Universe
Closed walls made from pure gravity
(with infinite size of one of extra space)





Inflationary potential + Λ term from pure gravity





Conclusion

We suggested the method of treating the nonlinear multidimensional gravity [slow change approximation, compared to Planck scale].

The method can be applied to arbitrary nonlinear terms like e.g. the Kretschmann scalar. The number of extra dimensions may be arbitrary.

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There is no necessity in artificial scalar fields to stabilize a size of extra dimensions and/or produce branes

The kinetic term also has a nontrivial form and gives additional opportunities in the effective field dynamics.

Cosmology: the potentials possess minima able to describe both inflationary period and modern de Sitter stage of our Universe .

Brane world concept: there are segments of the range of ϕ able to describe a thick brane

The concept of multidimensional world could be the basis of modern physics without additional postulate.

The problem of fine tuning is not solved yet...

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