Modifications to the Etherington Distance Duality Relation and Observational Limits

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- This is a purely kinematical result, and depends only on (i) conservation of light rays and (ii) metric spacetime geometry.
- Observational tests of the Etherington relation can thus be used to test the kinematical structure of spacetime.

Distance definitions: a short primer

- Given a standard candle of known luminosity L, and observed flux F, D_L is given by,

$$F = \frac{L}{4\pi D_L^2},$$

e.g. Supernovae of Type Ia (SNe Ia) act as standardizable candles.

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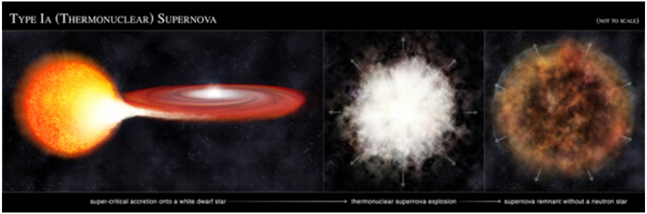
- Given a standard ruler (of known physical length d), and observed angular size θ, D_A is given by,

$$\theta = \frac{d}{D_A},$$

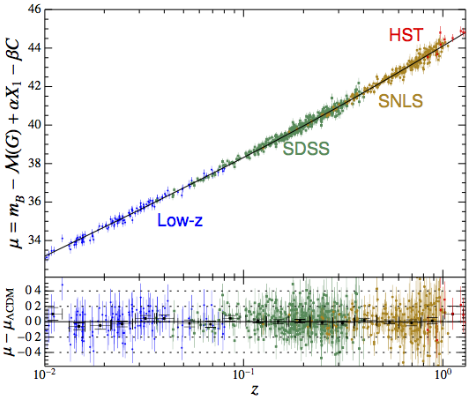
e.g. the baryon acoustic feature (BAF) in the clustering of galaxies acts as a standard ruler.

Observational probes: SN of Type Ia



- SNeIa are thermonuclear explosions of a white dwarf approaching the Chandrasekhar mass limit.
- SNeIa can be used as a standard candle, by correcting for observed variations in the luminosity and time scales of the SN light curves.

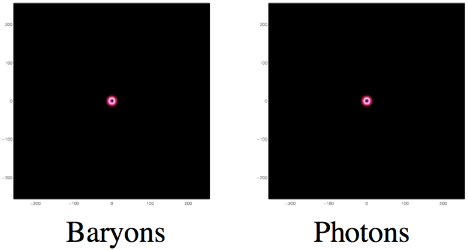
Observational measurements: SNeIa



- Joint Light-curve Analysis sample of SNeIa (Betoule et al. 2014)
- Distance modulus $\mu = 5 \log[D_{\rm L}/{\rm Mpc}]$

Observational probes: Baryon acoustic feature

Seo, Eisenstein and White (2005)



- Consider an overdensity in the initial density field in the early Universe.
- Baryons and photons are tightly coupled due to Thomson scattering.

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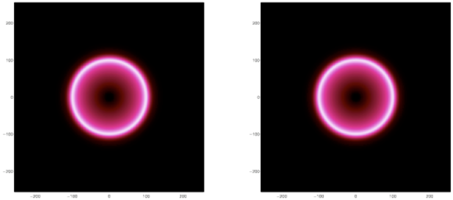

- The baryon photon plasma moves outward together due to high pressure.

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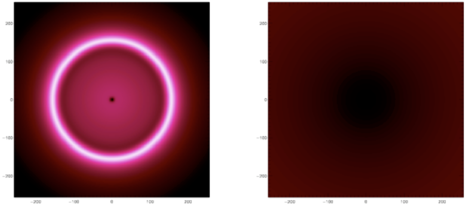
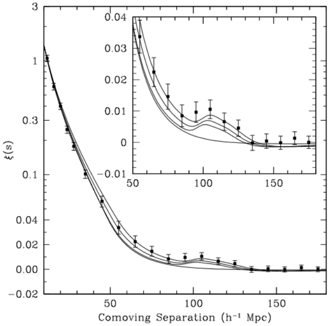


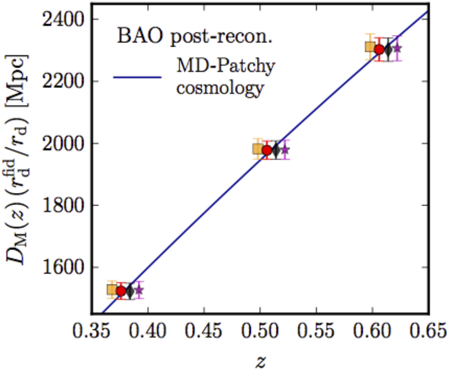
- At recombination, the photon free stream, and the baryon peak stalls.
- The scale can be measured in the cosmic microwave background radiation and is imprinted in the galaxy distribution.

Observational probes: Baryon acoustic feature



- The clustering of galaxies (two point correlation function) shows a distinct peak at the 100 h^{-1} Mpc scale.
- Large completed and ongoing surveys aim to detect and map out the BAF as a function of redshift.

Observational measurements: BAF



- Baryon acoustic feature measurements from SDSS-III BOSS (Alam et al. 2016), the middle redshift bin has overlapping volume with the bins on either end.
- The comoving angular diameter distance is defined as $D_{\rm M} = D_{\rm A}(1+z)$

Model independent test

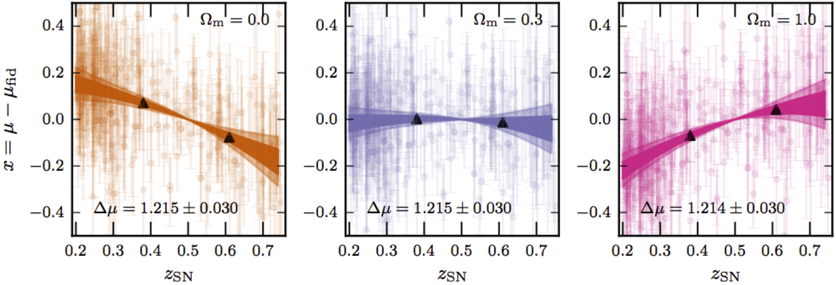
- Measurements of luminosity distances and angular diameter distances to two different redshifts can be used to test

$$\frac{D_{\mathrm{L},1}}{D_{\mathrm{A},1}}\frac{D_{\mathrm{A},2}}{D_{\mathrm{L},2}} = \left[\frac{(1+z_1)}{(1+z_2)}\right]^2$$

- Removes uncertainties in the standard scales and luminosities used to derive the distances (no assumptions about the cosmological model)
- The BAF measurements determine the redshifts to carry out the test
- Deviations parameterized as the optical depth τ between the two redshifts (e.g. due to absorption of light)

- How to get the luminosity distance measurement at the same redshift as the BAF measurement, $z_{\rm BAF}$?
- Common choices: Binning, or picking up the nearest supernova – neither is satisfactory.
- Our solution: Use SNeIa in the same volume as the galaxies used to measure the BAF.
- Subtract out the distance modulus from a fiducial cosmology – similar to the methodology adopted to analyze BAF.

Observational complication



- $\mu(z) - \mu^{\rm fid}(z)$ has reduced non-linearities, model these differences parametrically to obtain $\Delta\mu(z_{\rm BAF,1}, z_{\rm BAF,2})$, and compare it to the ratio of angular diameter distances.

- Using the SDSS BOSS for the BAF and JLA sample for the SNe Ia: [More, Niikura, Schneider, Schuller & Werner (2016)]

$$\tau(0.61) - \tau(0.38) = -0.006 \pm 0.046.$$

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- Etherington's relation is a kinematical relationship for any (convex normal neighbourhood of a) Lorentzian spacetime.

- Lorentzian spacetime kinematics stems from Maxwell theory in vacuum. Hence, consider generalized electromagnetism.

The Lagrangian of standard vacuum electromagnetism is

$$\mathcal{L}_0 \quad = \quad -\frac{1}{4}F^{ab}F_{ab} = -\frac{1}{8}(\eta^{ac}\eta^{bd} - \eta^{bc}\eta^{ad})F_{ab}F_{cd}$$

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introducing Petrov pair notation for the field strength tensor $F_{ab} = 2\partial_{[a}A_{b]} = F_{\bar{a}}$ with $\bar{a} \in \{[01], [02], [03], [23], [31], [12]\}$,

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and the corresponding constitutive tensor density in vacuum,

$$\chi_0^{\bar{a}\bar{b}} = \left[\begin{array}{c|c} -\boldsymbol{I} & \mathbf{0} \\ \hline \mathbf{0} & \boldsymbol{I} \end{array}\right]^{\bar{a}\bar{b}},$$

where $\boldsymbol{I}$ is the 3×3 identity.

The most general linear electromagnetism has the Lagrangian

[e.g. Post (1962)]

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whose (real) constitutive tensor density has symmetries,

$$\chi^{abcd} = -\chi^{bacd}, \quad \chi^{abcd} = -\chi^{abdc}, \quad \chi^{abcd} = \chi^{cdab} \text{ i.e. } \chi^{\bar{a}\bar{b}} = \chi^{\bar{b}\bar{a}}.$$

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$$\chi^{\bar{a}\bar{b}} = \left[\begin{array}{c|c} -\varepsilon & \boldsymbol{\varphi} \\ \hline \boldsymbol{\varphi}^T & \boldsymbol{\mu}^{-1} \end{array}\right]^{\bar{a}\bar{b}},$$

with 3×3 matrix blocks, where ε denotes electrical permittivity, $\boldsymbol{\mu}$ magnetic permeability and $\boldsymbol{\varphi}$ contains the Fresnel-Fizeau effect (tracefree part) and the axion (trace part).

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Null cones are governed by the principal polynomial,

$$P(x,p) = \mathcal{G}(x)^{abcd} p_a p_b p_c p_d = 0, \quad p \in T^*M,$$

with the Fresnel tensor

$$\mathcal{G}^{abcd} \propto \epsilon_{mnpq} \epsilon_{stuv} \chi^{mn(a} \chi^{b|pt|c} \chi^{d)quv}.$$

Light propagation

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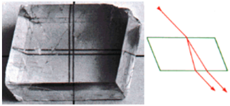
Since P is a quartic, light rays are not governed by Lorentzian metric geometry for general χ but are subject to birefringence.



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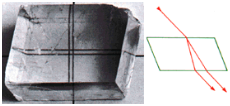

So far, there is no observational evidence for birefringence of the vacuum (10^{-38} from GRB polarization observations [cf. Ni (2015)]).

Exclusion of birefringence yields the most general constitutive

$$\chi^{abcd} = \sqrt{-\det g}\left(g^{ac}g^{bd} - g^{ad}g^{bc}\right)\psi + \phi\epsilon^{abcd},$$

where g_{ab} is a Lorentzian metric, ϕ the axion and ψ the dilaton.

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Absence of birefringence

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In this case, light ray geometry is metric since the Fresnel tensor

$$\mathcal{G}^{abcd} \propto g^{(ab}g^{cd)},$$

so that ray-geometrical quantities like D_A are as in standard theory.

Using the equations of motion i.e. modified Maxwell's equations

$$\partial_c \left(\chi^{abcd} F_{ab} \right) = 0, \quad \partial_{[a} F_{bc]} = 0,$$

we apply a WKB ansatz with eikonal phase function S,

$$F_{ab} = \mathrm{Re} \left[2 k_{[a} A_{b]} \exp \left(\mathrm{i} \frac{\mathrm{S}}{\epsilon} \right) \right], \quad k_a = -\partial_a \mathrm{S}.$$

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Then the transport of the scalar amplitude A along a light ray, where $A_a = A V_a$ with polarization $g^{ab} V_a V_b^* = -1$, satisfies

$$A(t) = A(0) \sqrt{\frac{\psi(t)}{\psi(0)}} \exp \left(- \int_0^t \theta \, \mathrm{d}t \right), \text{ with } \theta = \frac{1}{2} k^a{}_{;a}.$$

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In general, the two are related according to Gotay-Marsden,

$$T^i{}_j = C^i{}_j{}^{ab\dots} T_{ab\dots},$$

where $(\mathcal{L}_\xi G)^{ab\dots} = \xi^n G^{ab\dots}{}_{,n} + \xi^n{}_{,m} C^m_n{}^{ab\dots}$.

Hence, taking g in our χ as geometrical background,

$$T^i{}_j \quad = \quad C^i{}_j{}^{ab} \frac{\delta \mathcal{S}}{\delta g^{ab}} = -2 g^{ia} \delta^b{}_j T_{ab}$$

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so the Poynting vector $S^\mu = \langle T\rangle^\mu{}_0 \propto \psi^2$. Since for blackbody radiation $S \propto T^4$, the CMB with $T = 2.7255 \pm 0.0006$ K implies

$$\frac{\delta S}{S} = 4\frac{\delta T}{T} = 2\frac{\delta\psi}{\psi} \;\Rightarrow\; \frac{|\delta\psi|}{\psi} \leq 2\frac{0.0006}{2.7255} \simeq 4\cdot 10^{-4},$$

improving earlier estimates [Ni (2014)].

Consider a ray bundle in a spacetime domain $\mathcal{D}$ with 3-boundaries $\partial\mathcal{D}_S$ at source, $\partial\mathcal{D}_O$ at the observer. The excess photon number is

$$\begin{aligned}\Delta \cdot N &= \int_{\partial\mathcal{D}_O} \mathrm{d}^3x\sqrt{h}\ N^i n_i - \int_{\partial\mathcal{D}_S} \mathrm{d}^3x\sqrt{h}\ N^i(-n_i) \\ &= \int_{\partial\mathcal{D}} \mathrm{d}^3x\sqrt{h}\ N^i n_i = \int_{\mathcal{D}} \mathrm{d}^4x\sqrt{-g}\ N^i{}_{;i}\end{aligned}$$

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using Stokes' theorem, where h is the induced 3-metric and n_i the normal covector field. Then the Etherington relation becomes

$$D_L = \frac{(1+z)^2 D_A}{\sqrt{1+\Delta}}.$$

Applying Gotay-Marsden to a general $\mathcal{S}[A, G]$, diffeomorphism invariance implies covariant energy-momentum conservation,

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whence the photon current and excess obey

$$N^{i}{}_{;i} = 0 \;\Rightarrow\; \Delta = 0.$$

Hence, if one assumes generalized electromagnetism with χ excluding birefringence and g as background, then Etherington remains unchanged.

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Other kinematical setups are possible, too: separating dilatonic and electromagnetic matter, $T^i{}_j = \psi T^{\mathrm{EM}i}{}_j$, then $T^{\mathrm{EM}i}{}_j$ is not conserved and Etherington is modified.

[E.g. Brax et al. (2013), cf. Minazzoli & Hees (2014), Holanda & Pereira (2016)]

Hence, the detailed choice of kinematics determines modifications of Etherington. Our general formalism using Gotay-Marsden can even handle non-metric kinematics.

[Cf. Schuller & Werner (2017), forthcoming].

Conclusions

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Etherington relation as a test of spacetime kinematics: potentially interesting ($\Delta\tau < 0$) observational trends, but so far no evidence of deviations. We find

$$\tau(0.61) - \tau(0.38) = -0.006 \pm 0.046.$$

[More, Niikura, Schneider, Schuller & Werner (2016)]