Koide formula: beyond charged leptons

The waterfall in the quark sector

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By "Koide formula" we refer to a formula found by Y. Koide for charged leptons, exact enough to predict tau mass within experimental limits

$$(m_e + m_\mu + m_\tau) = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$$
$$\text{exp, in MeV:} \quad 1882.99 = 1882.97$$

It was found in the context of composite models of quarks and leptons, but can be produced more generally.
More informally, we also call "Koide formula" to its generalisations and look-alikes

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other unexplained fine-tuning...

Compare $1882.99/1882.97 = 1.00001$ with $m_{top}/174.1 = .995$

potential model

It is possible consider m_i as a composite of two entities with charge Q_0 and Q_i such that

$$m_i \propto \frac{1}{2}Q_0^2 + Q_0 Q_i + \frac{1}{2}Q_i^2$$

and asking the “matching conditions” $\sum Q_i = 0$ and $\sum Q_i^2 = \sum Q_0^2$.

- From here it is clear that $\sqrt{m}$ can have a negative sign sometimes.
- The generalisation to more than three particles simply substitutes $3/2$ by $n/2$. Of course, with more particles in the formula, the probability of finding a random coincidence increases.

circle "model"

[MDS, Rosen 2007] suggested to use the notation

$$m_k = M_0 \left(1 + \sqrt{2}\cos\left(\frac{2\pi}{3}k + \delta_0\right)\right)^2$$

And particularly for charged leptons:

$$M_{e\mu\tau} = 313.8 MeV,\ \delta_{e\mu\tau} = 0.222$$

F. Wilczek and A. Zee,
"Discrete Flavor Symmetries and a Formula for the Cabibbo Angle,"
Phys. Lett. B **70**, 418 (1977) [Erratum-ibid. **72B**, 504 (1978)].

$$\tan\theta_c = \sqrt{\frac{m_d}{m_s}}$$

H. Harari, H. Haut and J. Weyers,
"Quark Masses And Cabibbo Angles,"
Phys. Lett. B **78** (1978) 459.

$$m_u = 0;\ \frac{m_d}{m_s} = \frac{2-\sqrt{3}}{2+\sqrt{3}}$$

While trivial, this is the first example of a tuple that fulfils the formula

Y. Koide,
"A New Formula For The Cabibbo Angle And Composite Quarks And Leptons,"
Phys. Rev. Lett. **47** (1981) 1241.

"Quark And Lepton Masses Speculated From A Subquark Model,"
preprint (1981)

"A Fermion - Boson Composite Model Of Quarks And Leptons,"
Phys. Lett. B **120**, 161 (1983).

"A New View Of Quark And Lepton Mass Hierarchy,"
Phys. Rev. D **28** (1983) 252.

In 1981, measured τ lepton mass was still 1783 ± 4 MeV,
Koide simplest model predicted 1776.97

Y. Koide
Should The Renewed Tau Mass Value 1777-Mev Be Taken Seriously?
Mod.Phys.Lett. A8 (1993) 2071

R. Foot,
"A Note on Koide's lepton mass relation,"
MCGILL-94-09 [arXiv:hep-ph/9402242]

$$(\sqrt{m}_1, \sqrt{m}_2, \sqrt{m}_3)\angle(1,1,1) = 45^\circ$$

S. Esposito and P. Santorelli,
"A Geometric picture for fermion masses,"
Mod. Phys. Lett. A **10**, 3077 (1995) Considers quarks, family-wise, and also neutrinos.

From 1995 to 2005 there is a gap

N. Li and B. Q. Ma,
"Estimate of neutrino masses from Koide's relation,"
Phys. Lett. B **609**, 309 (2005) (received 15 October 2004)

April 23 2005

I comment on Li and Ma paper, and older ones on Koide formula, online at sci.physics.research and www.physicsforums.com. I am not the only one who is astonished, and other papers will follow.

A. Rivero and A. Gsponer, [hep-ph/0505220]
"The Strange formula of Dr. Koide,"

Y. Koide, [hep-ph/0506247].
"Challenge to the mystery of the charged lepton mass formula,"

J.-M. Gerard, F. Goffinet and M. Herquet,
"A New look at an old mass relation,"
Phys. Lett. B **633**, 563 (2006)

Sensibility to the renormalization group

Y Koide had already considered the sensibility to the running, but in early 2006 a pair of independent assessments are produced.

N. Li and B. Q. Ma,
"Energy scale independence of Koide's relation for quark and lepton masses,"
Phys. Rev. D **73**, 013009 (2006)

Z. z. Xing and H. Zhang,
"On the Koide-like relations for the running masses of charged leptons, neutrinos and quarks,"
Phys. Lett. B **635**, 107 (2006)

C. A. Brannen, [preprint (2006)]
"The Lepton Masses",
Alerts of the possibility of using negative signs in the formula and applies it to neutrinos

G. Rosen,
"Heuristic development of a Dirac-Goldhaber model for lepton and quark structure,"
Mod. Phys. Lett. A **22**, 283 (2007).
Promotes the use of the formula as $(313.85773\text{MeV})\left(1+\sqrt{2}\cos\theta_k\right)^2$.
In this way, the question of the signs is automatically considered.

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In this way, the question of the signs is automatically considered.

Curiously, nobody remarks that 313 MeV is the mass of a QCD quark.
As far as I know, this coincidence has not been useful for any model.

F. Goffinet,
"A Bottom-up approach to fermion masses",
These Univ. Cath. de Louvain, (2008)
Proposes some generalisations, as well as a formula containing the solutions to the equation jointly with spurious ones, so that the square roots are not needed.

Y. Sumino,
"Family Gauge Symmetry and Koide's Mass Formula,"
Phys. Lett. B **671**, 477 (2009)

Y. Sumino,
"Family Gauge Symmetry as an Origin of Koide's Mass Formula and Charged Lepton Spectrum,"
JHEP **0905**, 075 (2009)

Y. Koide
"Charged Lepton Mass Relations in a Supersymmetric Yukawaon Model"
Phys. Rev. D **79**, 033009 (2009)

M. D. Sheppeard, blog entries, (2010)
`pseudomonad.blogspot.com.es/2010/11/theory-update-19.html`
`pseudomonad.blogspot.com.es/2010/07/m-theory-lesson-342.html`
Notes that $\delta_{e\mu\tau} = 3\delta_{uct}$ and $\delta_{dsb} = 2\delta_{uct}$
(see also [Zenczykowski 2012], who uses $\delta_{e\mu\tau} = 3\delta_{uct}$)

W. Rodejohann and H. Zhang, [arXiv:1101.5525].
"Extended Empirical Fermion Mass Relation,"
Phys. Lett. B **698** (2011) 152

Only in the preprint version

Using PDG values, $m_t = 172.9$, $m_b = 4.19$, and $m_c = 1.29$ GeV,
$\sum^2 \sqrt{m} / \sum m$ is about 1.495

A. Kartavtsev, arXiv:1111.0480
"A remark on the Koide relation for quarks,"

F. G. Cao,
"Neutrino masses from lepton and quark mass relations..."
Phys. Rev. D **85**, 113003 (2012)
This is the first mention of the t, b, c tuple in the peer reviewed literature.

P. Zenczykowski,
"Remark on Koide's Z3-symmetric parametrization of quark masses,"
Phys. Rev. D **86**, 117303 (2012)

Empirical waterfall

Solve for Koide and **choose always the smaller solution**

Solve et Itera

$$m_3 = \left((\sqrt{m}_1 + \sqrt{m}_2) \left(2 - \sqrt{3 + 6 \frac{\sqrt{m_1 m_2}}{(\sqrt{m}_1 + \sqrt{m}_2)^2}} \right) \right)^2$$

	m_t		m_b		m_c	m_s	m_q	$m_{q'}$
(	173.21	,	4.18	)				

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	m_t		m_b		m_c	m_s	m_q	$m_{q'}$
(	173.21	,	4.18	) $\rightarrow$	1.3676			

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$$\begin{array}{cccccccccccc}
 & m_t & & m_b & & m_c & & m_s & & m_q & & m_{q'} \\
(& 173.21 & , & 4.18 &) \rightarrow & 1.3676 & & & & & & \\
 & & (& 4.18 & , & 1.3676 &) \rightarrow & .09312 & & & &
\end{array}$$

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 & & & & (& 1.3676 & , & .09312 &) \to & .00003187 & &
\end{array}$$

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 & & & & & & (& .09312 & , & .00003187 &) \rightarrow & .005441
\end{array}$$

$pdg2014$:	1.275	.095	.0023	.0048
	± 0.025	± 0.005	± 0.0006	± 0.0004

Empirical waterfall

Solve for Koide and **choose always the smaller solution**

Solve et Itera

$$m_3 = \left((\sqrt{m}_1 + \sqrt{m}_2) \left(2 - \sqrt{3 + 6 \frac{\sqrt{m_1 m_2}}{(\sqrt{m}_1 + \sqrt{m}_2)^2}} \right) \right)^2$$

		m_t		m_b		m_c		m_s		m_q		$m_{q'}$
	$\swarrow$ (	173.21	,	4.18	) $\rightarrow$	1.3676						
1346.84			(	4.18	,	1.3676	) $\rightarrow$	.09312				
60.31					$\swarrow$ (	1.3676	,	.09312	) $\rightarrow$	.00003187		
3558.3				34.72			$\swarrow$ (	.09312	,	.00003187	) $\rightarrow$	.005441
				1.996		1.227						.008156
				$pdg2014$:		1.275		.095		.0023		.0048
						± 0.025		± 0.005		± 0.0006		± 0.0004

As expected, we get good predictions for the charm and strange quarks.
We could interpret q as the up quark, q' as the down quark.
But the remarkable detail is that $m_q \approx 0$. We will use this fact later.

Are ther other tuples?

- with a 1% of tolerance: (cbt) and (scb).
- within a 10 % (usc), (dsb) and (dsc) can fit
- considering Renormalization Group
    - For (s, c, b), the quotient LHS/RHS of Koide formula using running masses from [XZ 2006] at M_Z is 0.949, at GUT scale it is 0.947.
    - For GUT-level masses within a 10% of tolerance, we have still the same triplets

$$S_4 = V_4 \ltimes S_3$$

$$\begin{array}{cccc} bds & usc & scb & cbt \\ uct & btd & tdb & dus \end{array}$$

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$$\begin{array}{cccc} bds & usc & scb & cbt \\ uct & btd & tdb & dus \end{array}$$

mixed families

$$\begin{array}{llll} \nu_2, & b_{rgb}, & e, & u_{rgb} \\ \tau, & c_{rgb}, & \nu_3, & d_{rgb} \\ \mu, & s_{rgb}, & \nu_1, & t_{rgb} \end{array}$$

Think $SU(4)$ Pati-Salam with a twist.
A similar rotation of the quarks was suggested in [HHW78]

remember $m_q = 0$?

If in some limit there is a massless particle, things are more predictive. We can just fix the scale of the most massive element.

For four levels, the solution is unique. We assume that the massless particle is the up quark.

$$\begin{array}{lll} & m_b & 4M_{scb} \\ m_\tau & m_c & \frac{2+\sqrt{3}}{2}M_{scb} \\ m_\mu & m_s & \frac{2-\sqrt{3}}{2}M_{scb} \\ m_e & m_u & 0 \end{array}$$

M_{scb} is $3M_{\tau\mu e}$
$\delta_{scb} = 45$, and $\delta_{\tau\mu e} = 15$

remember $m_q = 0$?

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	m_b	$4M_{scb}$
m_τ	m_c	$\frac{2+\sqrt{3}}{2}M_{scb}$
m_μ	m_s	$\frac{2-\sqrt{3}}{2}M_{scb}$
m_e	m_u	0

M_{scb} is $3M_{\tau\mu e}$
$\delta_{scb} = 45$, and $\delta_{\tau\mu e} = 15$

We can add an extra level above and below and fix the scale telling that the yukawa of the top is exactly $y_t = 1$

	m_t	174.1 GeV
	m_b	3.64 GeV
m_τ	m_c	1.698 GeV
m_μ	m_s	121.95 MeV
m_e	m_u	0 MeV
	m_d	8.75 MeV

remember $m_q = 0$?

If in some limit there is a massless particle, things are more predictive. We can just fix the scale of the most massive element.

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	m_b	$4M_{scb}$
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$$\delta_{scb} = 3\delta_{\tau\mu e}$$

In Foot's interpretation, $\tau\mu e$ and scb would be orthogonal vectors

Climbing the Cascade

We take m_e and m_τ as inputs, and the approximations detected in the previous slide:

$$M_{scb} = 3M_{\tau\mu e}$$

$$\delta_{scb} = 3\delta_{\tau\mu e}$$

Climbing the Cascade

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	$exp.pdg14$	$pred.$
t	173.21 ± 0.087	173.26
b	4.18 ± 0.03	4.197
c	1.275 ± 0.025	1.359
τ	$1.77682(16)$	1.776968
s	95 ± 5	92.275
d	~ 4.8	5.32
u	~ 2.3	$.0356$

tuples from heavy quark masses and the pion mass

Is QCD acting in Koide scene?

$$\frac{(0+\sqrt{M_{\pi^0}}+\sqrt{M_{D^0}})^2}{0+M_{\pi^0}+M_{D^0}}=1.5017 \quad \frac{(-\sqrt{M_{\pi^0}}+\sqrt{M_{D^0}}+\sqrt{M_{B^0}})^2}{M_{\pi^0}+M_{D^0}+M_{B^0}}=1.4923$$

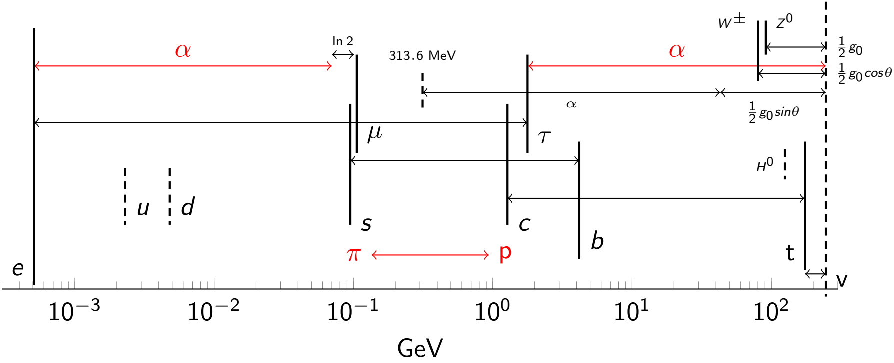
Not bad, even if due to the coincidence (fine-tuning?) $m_s \approx M_\pi$
We could add mesons and diquarks to our three-layer arrangement.

$\nu_2, b_{rgb}, e, u_{rgb}$	B^+, B_c^+	bu, bc	bs, bd	$B^0, B_s^0, \bar{B}^0, \bar{B}_s^0$	$5279.59 MeV$
$\tau, c_{rgb}, \nu_3, d_{rgb}$	D^+, D_s^+	sc, dc	bb, dd	$\eta_b, \eta_c, D^0, \bar{D}^0$	$1864.85 MeV$
$\mu, s_{rgb}, \nu_1, t_{rgb}$	π^+, K^+	su, du	ss, sd	$\eta_8, \pi^0, K^0, \bar{K}^0$	$134.9767 MeV$

Quark-Hadron Supersymmetry?
Same number of degrees of freedom.

The Small Seesaw

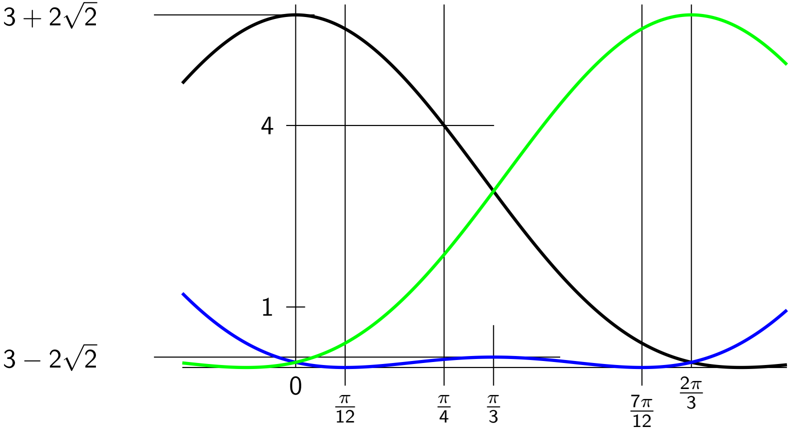
Is electroweak GWS acting in Koide scene?



From D. Lackey in a comment in [MDS]:
$M_Z * \sin\theta_W * \alpha = 313.66$ MeV.
(Using $\cos\theta_W = 0.8819$) The relationship $\mu - e$ is well known, used in the early 70s. From τ to the electroweak vacuum, I read it first in a comment of R. Yablon in USENET.

The Koide phase

Masses for δ_0 from 0 to $2\pi/3$, with $M_0 = 1$;



References

- arXiv:hep-ph/0505220 A. Rivero, A. Gsponer
  "The strange formula of Dr. Koide"
- arXiv:1111.7232 A. Rivero
  "A new Koide tuple: strange-charm-bottom"
- Another presentacion online, with a lot of zooms:
  `http://prezi.com/e2hba7tkygvj/koide-waterfall/`

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Thank You!