Is this the end of dark energy?

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Contents

Theoretical basis

Space-Time geometry

Cosmological Principle

The Universe homogeneous and isotropic at large scales.

$$\Downarrow$$

Robertson-Walker metric

$$ds^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

$a(t)$: scale factor;
K: spatial curvature ($k = -1, 0, 1$).

Matter contents

Weyl's postulate

Galaxies are the fundamental particles of the substract.

$$\Downarrow$$

Perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu},$$

ρ: energy density of the fluid;
p: pressure of the fluid;
$u^\mu = dx^\mu/d\tau$: 4-velocity

Dynamics of the Universe

General relativity

$$G_{\mu\nu} \equiv \underbrace{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R}_{\text{geometry}} = \underbrace{8\pi G T_{\mu\nu}}_{\text{matter}}$$

PC+PW+RG

$$\Downarrow$$

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho; \quad \text{(Friedmann's eq.)}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad \text{(aceleration eq.)}.$$

Theoretical basis

Conservation laws

$$\underbrace{\nabla_\mu G^{\mu\nu} = 0}_{\text{Bianchi identity}} \Rightarrow \underbrace{\nabla_\mu T^{\mu\nu} = 0}_{\text{E-M conservation}}$$

$$\Downarrow$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0.$$

EoS

$$p = w\rho$$

$$\Downarrow$$

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^3 \exp\left[-3\int_{a_0}^{a} \frac{w(a')}{a'} da'\right]$$

Universe' contents

Ordinary matter: w=0

$$\rho_m = \rho_{m,0}\left(\frac{a_0}{a}\right)^3$$

Radiation: w=1/3

$$\rho_\gamma = \rho_{\gamma,0}\left(\frac{a_0}{a}\right)^4$$

Quantum vacuum: w=-1

$$\rho_\Lambda = \rho_{\Lambda,0} = cte$$

$w = cte$ dark energy?

$$\rho = \rho_0\left(\frac{a_0}{a}\right)^{3(1+w)}$$

$$H^2 = H_0^2\left[\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{de,0} f(z)\right],$$

$$f(z) = (1+z)^3 \exp\left[3\int_0^z \frac{w(z')dz'}{1+z'}\right], \qquad \frac{a_0}{a} = 1 + z$$

Type Ia supernovae

Physical mechanisms

$$(M_{AB} > M_{Ch} \approx 1,4 M_\odot)$$
$$\Downarrow$$
SN Ia explodes
$$\Downarrow$$
$$\underbrace{\text{standard candles}}$$
$M = cte.$ for any SN Ia
$$\Downarrow$$
measurements of m provides d_L

SNs Ia distances

Apparent magnitude

$$m \equiv -2,5\log(f/f_x),\ \ f_x = 2,58\times 10^{-8}\,\mathrm{W\cdot m^{-2}}$$

Absolute magnitude

$$M \equiv -2,5\log(L/L_x),\ \ L_x = 78,7 L_\odot$$

Luminosity distance

$$d_L \equiv \sqrt{\frac{L}{4\pi f}} = (1+z)\int_0^z \frac{dz'}{H(z')}$$

Evidence of cosmic acceleration

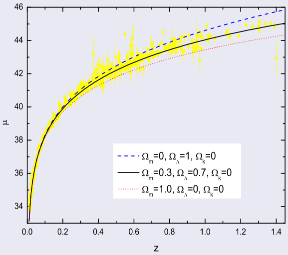
Distance modulus

$$\mu \equiv m - M = 5\log\left(\frac{d_L}{1\mathrm{Mpc}}\right) + 25$$
$$= 25 - 5\log\left(\frac{H_0}{km\cdot s^{-1}\cdot Mpc^{-1}}\right) +$$
$$+5\log\left(\frac{cz}{Km\cdot s^{-1}}\right) + 1.086(1-q_0)z + \cdots$$

$q = -\frac{\ddot{a}}{H^2 a}$: deceleration parameter

SNe Ia $\Rightarrow q_0 < 0$. The Universe's expansion rate is speed up.

SCP: Amanullah *et al.* APJ **716** (2010)



Main candidates to explain the cosmic aceleration

ΛCDM model

$$\nabla_\nu G^\nu{}_\mu = \nabla_\nu T^\nu{}_\mu = 0;\ \ \nabla_\nu \delta^\nu{}_\mu = 0$$

$$\Downarrow$$

$$G^\nu{}_\mu + \Lambda\delta^\nu{}_\mu = 8\pi G T^\nu{}_\mu + \Lambda\delta^\nu{}_\mu$$

$$\underbrace{\begin{array}{l}\Lambda:\ \text{fund. constant}\\ \Lambda = 8\pi G\rho_{vac}:\ \text{vacuum energy}\end{array}}_{w = p_{vac}/\rho_{vac} = -1}$$

$$\rho_{vac}^{teo} = (10^{43} - 10^{121})\rho_{de}^{obs}$$

$$\Downarrow$$

$$\Lambda_{ef} = \Lambda + 8\,\pi\, G\rho_{vac},$$

$$\Downarrow$$

$$\underbrace{\rho_{de}^{obs} = \rho_{\Lambda_{ef}}}_{\text{CC problem}}$$

ϕCDM model

$$\mathcal{L} = \sqrt{-g}\big[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\big]$$

$$\Downarrow$$

$$T_{\mu\nu}^{(\phi)} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\big[\frac{1}{2}g^{\kappa\lambda}\partial_\kappa\phi\partial_\lambda\phi - V(\phi)\big]$$

$$\Downarrow$$

$$\rho_\phi = \frac{\dot\phi^2}{2} + V(\phi) \text{ e } p_\phi = \frac{\dot\phi^2}{2} - V(\phi)$$

$$\Downarrow$$

$$w_\phi = \frac{\dot\phi^2 - 2V(\phi)}{\dot\phi^2 + 2V(\phi)}$$

Others theories of gravitation

$$\mathcal{L}_{EH} = \sqrt{-g}R \rightarrow \mathcal{L} = \sqrt{-g}f(R)$$

Probing the dark energy hypothesis

Basic assumptions

1. General relativity is the right theory of gravitation;
2. The contents of the universe (matter, radiation, etc.) can be described by perfect fluids

Consequence

The universe is homogeneously pervaded by a fluid with pressure negative enough to ensure that $\rho + 3p < 0$ ($q < 0$). This fluid is called dark energy (DE) and characterized by its EoS parameter $w = p/\rho$;

Question

Homogenous fluids tends to reach thermodynamical stabilities conditions, so can thermodynamical stabilities requirements $C_V, C_P, \kappa_T, \kappa_S > 0$ constrain or even rule out DE models?

Thermodynamics of the cosmic fluids

Internal energy and temperature law

Internal energy of the i-th fluid component

$$U_i = \rho_i c^2 V, \; V = a^3(t) V_0$$

First law of thermodynamics

$$T_i dS_i = dU_i + p_i dV$$

$$\Downarrow$$

$$d \ln \rho_i + 3(1 + w_i) d \ln a = 0$$

$$\Downarrow$$

$$\frac{T_i}{T_{i,0}} = \frac{\rho_i}{\rho_{i,0}} a^3 \left(\frac{1}{w_i} \frac{p_i V}{T_i} = \frac{1}{w_{i,0}} \frac{p_{i,0} V_0}{T_{i,0}} \right)$$

$$\Downarrow$$

$$U_i = U_{i,0} \frac{T_i}{T_{i,0}}$$

Enthalpy

$$h_i = U_i + p_i V = h_i = (1 + w_i) U_i$$

The Universe's heat capacity

Heat capacity at constant volume

$$C_{iV} = \left(\frac{\partial U_i}{\partial T_i} \right)_V = \frac{U_{i,0}}{T_{i,0}} = \text{constant}$$

Heat capacity at constant pressure

$$C_{p_i} = \left(\frac{\partial h_i}{\partial T_i} \right)_{p_i} = \left(1 + w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \right) C_{iV}$$

Compressibility

Isothermal compressibility

$$\kappa_{T_i} = -\frac{1}{V} \left(\frac{\partial V}{\partial p_i} \right)_{T_i} = \frac{1}{w_i p_i} \left(w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \right)$$

Adiabatic compressibility

$$\kappa_{S_i} = -\frac{1}{V} \left(\frac{\partial V}{\partial p_i} \right)_{S_i} = \frac{C_{iV}}{w_i p_i C_{p_i}} \left(w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \right)$$

Constraints on dark fluid

Stability conditions

Thermal stability

$$C_{iV}, C_{p_i} \geq 0$$

Thermal stability

$$\kappa_{S_i}, \kappa_{T_i} \geq 0$$

$$+$$

$$C_{p_i} \geq C_{iV} \text{ and } \kappa_{T_i} \geq \kappa_{S_i}$$

$$\Downarrow$$

$$w_i - \frac{1}{3}\frac{d\ln|w_i|}{d\ln a} \geq 0$$

$$w_i = cte. \Rightarrow w_i \geq 0$$

$$\Downarrow$$

Vacuum energy can not be described by a fluid with a negative constant pressure!

Stability conditions

w	$w - \frac{1}{3}\frac{d\ln\|w\|}{d\ln a} \geq 0$
$\frac{w_0}{(1-b\ln a)^2}$	No
$w_f + \frac{\Delta w a_t^{1/\tau}}{a_t^{1/\tau}+a^{1/\tau}}$	No
$w_f w_i \frac{a^l + a_t^l}{w_i a^l + w_f a_t^l}$	No
$w_0 + w_0'(a - a^2)$	No
$w_0 + w_0' \frac{a-1}{1-2a+2a^2}$	No
$w_0 + w_0' \frac{a^\beta - 1}{\beta}$	No

Constraints on dark fluid

Probing the EoS time-dependence

Present time constraints

$$w_i - \frac{1}{3}\frac{d\ln|w_i|}{d\ln a} \geq 0 \Rightarrow w_0' \geq 3w_0^2$$

Hypothesis : $\rho_{de}(z)$ admits a Taylor expansion in the range $(\tilde{a}-\epsilon_-, \tilde{a}+\epsilon_+)$

$$\begin{aligned}\rho_{\rm DE}(a) &= \rho_{\rm DE}(\tilde{a}) + \frac{d\rho_{\rm DE}}{da}\Big|_{a=\tilde{a}}(a-\tilde{a}) + \\ &+ \frac{1}{2}\frac{d^2\rho_{\rm DE}}{da^2}\Big|_{a=\tilde{a}}(a-\tilde{a})^2 + \cdots .\end{aligned}$$

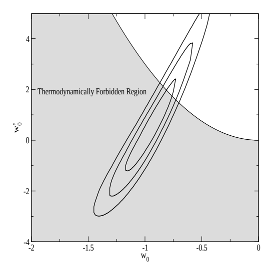
$$\frac{d\rho_{\rm DE}}{da} = -\frac{3}{a}(1+w)\rho_{DE} \text{ (recurrence)}$$

$$\Downarrow$$

$$\frac{d^2\rho_{\rm DE}}{da^2} = \left[\frac{3}{a^2}(1+w)+\frac{9}{a^2}(1+w)^2-\frac{3}{a}\frac{dw}{da}\right]\rho_{DE}$$

$$\vdots$$

Probing the EoS time-dependence



$$a = a_0 = 1$$

$$\Downarrow$$

$$\begin{aligned}\rho_{\rm DE}(a) &= \rho_{\rm DE,0}\Big\{1+3(1+w_0)(1-a) \\ &+ \frac{1}{2}\left[3(1+w_0)-3w_0'+9(1+w_0)^2\right] \\ &\times (1-a)^2\Big\}.\end{aligned}$$

Conclusions

- The Universe's specific heat is greater than $10^{13}\,\mathrm{cal}\cdot\mathrm{g}^{-1}\cdot\mathrm{K}^{-1}$ (relativistic matter estimate);
- A negative pressure fluid with a constant EoS parameter which including the vacuum energy cannot meet the thermal and mechanical stability conditions;
- Time-dependent EoS parameter also are in conflict with the thermodynamic stability conditions;
- If stability conditions are a nature requirement the dark energy hypothesis is fail.
- Relaxing the mechanical stability condition can save dark energy quintessential models $-1 \leq w \leq 0$ and relaxing both, thermal and mechanical stability conditions, will save the phantom dark energy hypothesis $w < -1$.

Is the stability conditions a nature requirement for all homogeneous systems?

If so, dark energy is not the answer to the cosmic acceleration problem.

If no, dark energy remains in the game.