Is this the end of dark energy?

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### Theoretical basis

#### Space-Time geometry

**Cosmological Principle**

The Universe homogeneous and isotropic at large scales.

\[ ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

- \( a(t) \): scale factor;
- \( K \): spatial curvature (\( k = -1, 0, 1 \)).

#### Matter contents

**Weyl’s postulate**

Galaxies are the fundamental particles of the substract.

**Perfect fluid**

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \]

- \( \rho \): energy density of the fluid;
- \( p \): pressure of the fluid;
- \( u^\mu = dx^\mu/d\tau \): 4-velocity

#### Dynamics of the Universe

**General relativity**

\[ G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R = 8\pi GT_{\mu\nu} \]

- geometry
- matter

**PC+PW+RG**

\[ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \rho; \quad \text{(Friedmann’s eq.)} \]

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad \text{(acceleration eq.)} \]
Theoretical basis

Conservation laws

\[ \nabla_\mu G^{\mu \nu} = 0 \Rightarrow \nabla_\mu T^{\mu \nu} = 0 \]

\[ \Delta \]

Bianchi identity \quad \text{E-M conservation}

\[ \dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0. \]

\[ \Delta \]

EoS

\[ p = w \rho \]

\[ \Delta \]

\[ \rho = \rho_0 \left( \frac{a_0}{a} \right)^3 \exp \left[ -3 \int_{a_0}^a \frac{w(a')}{a'} da' \right] \]

Universe’ contents

\[ \text{Ordinary matter: } w=0 \]

\[ \rho_m = \rho_{m,0} \left( \frac{a_0}{a} \right)^3 \]

\[ \text{Radiation: } w=1/3 \]

\[ \rho_\gamma = \rho_{\gamma,0} \left( \frac{a_0}{a} \right)^4 \]

\[ \text{Quantum vacuum: } w=-1 \]

\[ \rho_\Lambda = \rho_{\Lambda,0} = \text{cte} \]

\[ w = \text{cte dark energy?} \]

\[ \rho = \rho_0 \left( \frac{a_0}{a} \right)^{3(1+w)} \]

\[ H^2 = H_0^2 \left[ \Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{de,0} f(z) \right], \]

\[ f(z) = (1 + z)^3 \exp \left[ 3 \int_0^z \frac{w(z')dz'}{1 + z'} \right], \quad \frac{a_0}{a} = 1 + z \]
**Type Ia supernovae**

**Physical mechanisms**

\((M_{AB} > M_{Ch} \approx 1, 4M_\odot)\)
\[\implies\]
SN Ia explodes
\[\implies\]
standard candles
\(M = \text{cte.}\) for any SN Ia
\[\implies\]
measurements of \(m\) provides \(d_L\)

**SNs Ia distances**

**Apparent magnitude**

\(m \equiv -2, 5 \log(f/f_x), \ f_x = 2, 58 \times 10^{-8} \ W \cdot m^{-2}\)

**Absolute magnitude**

\(M \equiv -2, 5 \log(L/L_x), \ L_x = 78, 7L_\odot\)

**Luminosity distance**

\(d_L \equiv \sqrt{\frac{L}{4\pi f}} = (1 + z) \int_0^z \frac{dz'}{H(z')}\)

**Evidence of cosmic acceleration**

**Distance modulus**

\(\mu \equiv m - M = 5 \log \left(\frac{d_L}{1 \text{Mpc}}\right) + 25\)
\[= 25 - 5 \log \left(\frac{H_0}{\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}}\right) +
+ 5 \log \left(\frac{cz}{\text{km} \cdot \text{s}^{-1}}\right) + 1.086(1 - q_0)z + \cdots\)

\(q = -\frac{\ddot{a}}{H^2 a}\): deceleration parameter

SNe Ia \(\Rightarrow q_0 < 0\). The Universe’s expansion rate is speed up.

**SCP: Amanullah et al. APJ 716 (2010)**

![Graph showing distance modulus vs. redshift with different cosmological models]
Main candidates to explain the cosmic acceleration

**ΛCDM model**

\[ \nabla_\nu G^\nu_\mu = \nabla_\nu T^\nu_\mu = 0; \quad \nabla_\nu \delta^\nu_\mu = 0 \]

\[ \nabla_\nu G^\nu_\mu + \Lambda \delta^\nu_\mu = 8\pi G T^\nu_\mu + \Lambda \delta^\nu_\mu \]

\( \Lambda \): fundamental constant

\( \Lambda = 8\pi G \rho_{\text{vac}} \): vacuum energy

\[ w = \rho_{\text{vac}} / \rho_{\text{vac}} = -1 \]

\[ \rho^{\text{teo}}_{\text{vac}} = (10^{43} - 10^{121}) \rho^{\text{obs}}_{\text{de}} \]

\[ \Lambda_{\text{ef}} = \Lambda + 8\pi G \rho_{\text{vac}}, \]

\[ \rho^{\text{obs}}_{\text{de}} = \rho_{\Lambda_{\text{ef}}} \]

**φCDM model**

\[ \mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \]

\[ T^{\phi}_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[ \frac{1}{2} g^{\kappa\lambda} \partial_\kappa \phi \partial_\lambda \phi - V(\phi) \right] \]

\[ \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \quad \text{e} \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \]

\[ w_\phi = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \]

**Others theories of gravitation**

\[ \mathcal{L}_{\text{EH}} = \sqrt{-g} R \rightarrow \mathcal{L} = \sqrt{-g} f(R) \]
Basic assumptions

1. General relativity is the right theory of gravitation;
2. The contents of the universe (matter, radiation, etc.) can be described by perfect fluids

Consequence

The universe is homogeneously pervaded by a fluid with pressure negative enough to ensure that \( \rho + 3p < 0 \) (\( q < 0 \)). This fluid is called dark energy (DE) and characterized by its EoS parameter \( w = p/\rho \);

Question

Homogenous fluids tends to reach thermodynamical stabilities conditions, so can thermodynamical stabilities requirements \( C_V, C_P, \kappa_T, \kappa_S > 0 \) constrain or even rule out DE models?
Thermodynamics of the cosmic fluids

**Internal energy and temperature law**

**Internal energy of the i-th fluid component**

\[ U_i = \rho_i c^2 V, \quad V = a^3(t)V_0 \]

**First law of thermodynamics**

\[ T_i dS_i = dU_i + p_i dV \]

\[ \downarrow \]

\[ d\ln \rho_i + 3(1 + w_i)d\ln a = 0 \]

\[ \downarrow \]

\[ \frac{T_i}{T_{i,0}} = \frac{\rho_i}{\rho_{i,0}}a^3 \left( \frac{1}{w_i} \frac{p_i V}{T_i} = \frac{1}{w_{i,0}} \frac{p_{i,0} V_0}{T_{i,0}} \right) \]

\[ \downarrow \]

\[ U_i = U_{i,0} \frac{T_i}{T_{i,0}} \]

**Enthalpy**

\[ h_i = U_i + p_i V = h_i = (1 + w_i)U_i \]

**The Universe’s heat capacity**

**Heat capacity at constant volume**

\[ C_{iV} = \left( \frac{\partial U_i}{\partial T_i} \right)_V = \frac{U_{i,0}}{T_{i,0}} = \text{constant} \]

**Heat capacity at constant pressure**

\[ C_{p_i} = \left( \frac{\partial h_i}{\partial T_i} \right)_{p_i} = \left( 1 + w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \right) C_{iV} \]

**Compressibility**

**Isothermal compressibility**

\[ \kappa_{T_i} = -\frac{1}{V} \left( \frac{\partial V}{\partial p_i} \right)_{T_i} = \frac{1}{w_i p_i} \left( w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \right) \]

**Adiabatic compressibility**

\[ \kappa_{S_i} = -\frac{1}{V} \left( \frac{\partial V}{\partial p_i} \right)_{S_i} = \frac{C_{iV}}{w_i p_i C_{p_i}} \left( w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \right) \]
Constraints on dark fluid

### Stability conditions

#### Thermal stability

\[ C_{iV}, C_{p_i} \geq 0 \]

\[ \kappa_{S_i}, \kappa_{T_i} \geq 0 \]

\[ + \]

\[ C_{p_i} \geq C_{iV} \quad \text{and} \quad \kappa_{T_i} \geq \kappa_{S_i} \]

\[ \Downarrow \]

\[ w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \geq 0 \]

\[ w_i = \text{cte.} \Rightarrow w_i \geq 0 \]

\[ \Downarrow \]

Vacuum energy can not be described by a fluid with a negative constant pressure!

#### Stability conditions

| \( w \)                                                                 | \( w - \frac{1}{3} \frac{d \ln |w|}{d \ln a} \geq 0 \) |
|------------------------------------------------------------------------|--------------------------------------------------------|
| \( w_0 \)                                                             | No                                                     |
| \( w_f + \frac{\Delta w a_t^{1/\tau}}{a_t^{1/\tau} + a_t^{1/\tau}} \) | No                                                     |
| \( w_f w_i \frac{a_i^{1/\tau} + a_t^{1/\tau}}{w_i a_t^{1/\tau} + w_f a_t^{1/\tau}} \) | No                                                     |
| \( w_0 + w_0'(a - a^2) \)                                            | No                                                     |
| \( w_0 + w_0'\frac{a-1}{1-2a+2a^2} \)                                | No                                                     |
| \( w_0 + w_0'\frac{a^\beta - 1}{\beta} \)                            | No                                                     |
**Constraints on dark fluid**

### Probing the EoS time-dependence

**Present time constraints**

\[ w_i - \frac{1}{3} \frac{d \ln |w_i|}{d \ln a} \geq 0 \Rightarrow w'_0 \geq 3w_0^2 \]

**Hypothesis** : \( \rho_{de}(z) \) admits a Taylor expansion in the range \((\tilde{a} - \epsilon_-, \tilde{a} + \epsilon_+)\)

\[
\rho_{DE}(a) = \rho_{DE}(\tilde{a}) + \frac{d \rho_{DE}}{da} \bigg|_{a=\tilde{a}} (a - \tilde{a}) + \\
+ \frac{1}{2} \frac{d^2 \rho_{DE}}{da^2} \bigg|_{a=\tilde{a}} (a - \tilde{a})^2 + \cdots.
\]

\[
\frac{d \rho_{DE}}{da} = -\frac{3}{a} (1 + w) \rho_{DE} \text{ (recurrence)}
\]

\[
\frac{d^2 \rho_{DE}}{da^2} = \left[ \frac{3}{a^2} (1+w) + \frac{9}{a^2} (1+w)^2 - \frac{3}{a} \frac{dw}{da} \right] \rho_{DE}
\]

\[
\vdots
\]

**Thermodynamically Forbidden Region**

\[
a = a_0 = 1
\]

\[
\Downarrow
\]

\[
\rho_{DE}(a) = \rho_{DE,0} \left\{ 1 + 3(1 + w_0)(1 - a) \\
+ \frac{1}{2} \left[ 3(1 + w_0) - 3w'_0 + 9(1 + w_0)^2 \right] \times (1 - a)^2 \right\}.
\]
Conclusions

- The Universe's specific heat is greater than $10^{13} \text{cal} \cdot \text{g}^{-1} \cdot \text{K}^{-1}$ (relativistic matter estimate);
- A negative pressure fluid with a constant EoS parameter which including the vacuum energy cannot meet the thermal and mechanical stability conditions;
- Time-dependent EoS parameter also are in conflict with the thermodynamic stability conditions;
- If stability conditions are a nature requirement the dark energy hypothesis is fail.
- Relaxing the mechanical stability condition can save dark energy quintessential models $-1 \leq w \leq 0$ and relaxing both, thermal and mechanical stability conditions, will save the phantom dark energy hypothesis $w < -1$.

Is the stability conditions a nature requirement for all homogeneous systems?

If so, dark energy is not the answer to the cosmic acceleration problem.

If no, dark energy remains in the game.