Interpretation of Mössbauer experiment in a rotating system: a new proof for general relativity

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Abstract

A historical experiment by Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect (Mössbauer rotor experiment) has been recently first re-analyzed and then replied by an experimental research group. The results of re-analyzing the experiment have shown that a correct re-processing of Kündig's experimental data gives an interesting deviation of a relative redshift between emission and absorption resonant lines from the standard prediction based on the relativistic dilatation of time. That prediction gives a redshift

$$\frac{\nabla E}{E} \simeq -\frac{1}{2}\frac{v^2}{c^2}$$

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where **v** is the tangential velocity of the absorber of resonant radiation, **c** is the velocity of light in vacuum and the result is given to the accuracy of first-order in $(v/c)^2$. Data re-processing gave

$$\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2},$$

with **k = 0.596 ± 0.006**. Subsequent new experimental results by the reply of Kündig experiment have shown a redshift with **k = 0.68 ± 0.03** instead.

By using **Einstein Equivalence Principle**, which states the equivalence between the gravitational "force" and the pseudo-force experienced by an observer in a non-inertial frame of reference (**included a rotating frame of reference**) here we re-analyze the theoretical framework

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of Mössbauer rotor experiments directly in the rotating frame of reference by using a general relativistic treatment. It will be shown that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision in the rotating frame gives $k \simeq 2/3$ in perfect agreement with the new experimental results. Such an effect of clock synchronization has been missed in various papers in the literature with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects. Our general relativistic interpretation shows, instead, that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent, proof of Einstein general relativity.

Introduction

The **Mössbauer effect** (discovered by R. Mössbauer in 1958, see **R. L. Mössbauer, Zeitschrift für Physik A (in German) 151, 124 (1958)**) consists in resonant and recoil-free emission and absorption of **gamma rays**, without loss of energy, by atomic nuclei bound in a solid. It resulted and currently results very important for basic research in physics and chemistry. In this Lecture we will focus on the so called **Mössbauer rotor experiment**. In this particular experiment, the Mössbauer effect works through an absorber orbited around a source of resonant radiation (or vice versa). The aim is to verify the relativistic time dilation time for a moving resonant absorber (the source) inducing a relative energy shift between emission and absorption lines.

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In the recent papers A. L. Kholmetskii, T. Yarman and O. V. Missevitch, Phys. Scr. 77, 035302 (2008) and A. L. Kholmetskii, T. Yarman, O.V. Missevitch and B. I. Rogozev, Phys. Scr. 79, 065007 (2009), the authors first re-anayzed the data of a known experiment of Kündig on the transverse Doppler shift in a rotating system measured with the Mössbauer effect, see W. Kündig, Phys. Rev. 129, 2371 (1963), and second, they carried out their own experiment on the time dilation effect in a rotating system. In the first paper, they found that the original experiment by Kündig contained errors in the data processing. A puzzling fact is that, after correction of the errors of Kündig, the experimental data gave the value

$$\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2},$$

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where **k = 0.596 ± 0.006**, instead of the standard relativistic prediction **k = 0.5** due to time dilatation. **Kholmetskii et al.** stressed that the deviation of the coefficient **k** from 0.5 exceeds by almost 20 times the measuring error and that the revealed deviation cannot be attributed to the influence of rotor vibrations and other disturbing factors. All these potential disturbing factors have been indeed excluded by a perfect methodological trick applied by Kündig, i.e. a first-order Doppler modulation of the energy of **gamma–quanta** on a rotor at each fixed rotation frequency. In that way, Kündig's experiment can be considered as the most precise among other experiments of the same kind, i.e. **H. J. Hay et al, Phys. Rev. Lett. 4, 165 (1960), H. J. Hay, in Proc. 2nd Conf. Mössbauer Effect, ed A Schoen and D M T Compton (New York: Wiley) p 225 (1962), T. E. Granshaw and H. J. Hay, in Proc. Int. School of Physics, 'Enrico Fermi' (New York:**

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Academic) p 220 (1963), D. C. Champeney and P. B. Moon, Proc. Phys. Soc. 77, 350 (1961), D. C. Champeney, G. R. Isaak and A. M. Khan, Proc. Phys. Soc. 85, 583 (1965), where the experimenters measured only the count rate of detected gamma–quanta as a function of rotation frequency. Kholmetskii et al. have also shown that the last experiment of Champeney et al., which contains much more data than previous papers, also confirms the supposition k > 0.5. Motived by their results in reanalyzing Kündig experiment, Kholmetskii et al. carried out their own experiment. They decided to repeat neither the scheme of the Kündig experiment nor the schemes of other known experiments on the subject previously mentioned above. In that way, they got independent information on the value of k.
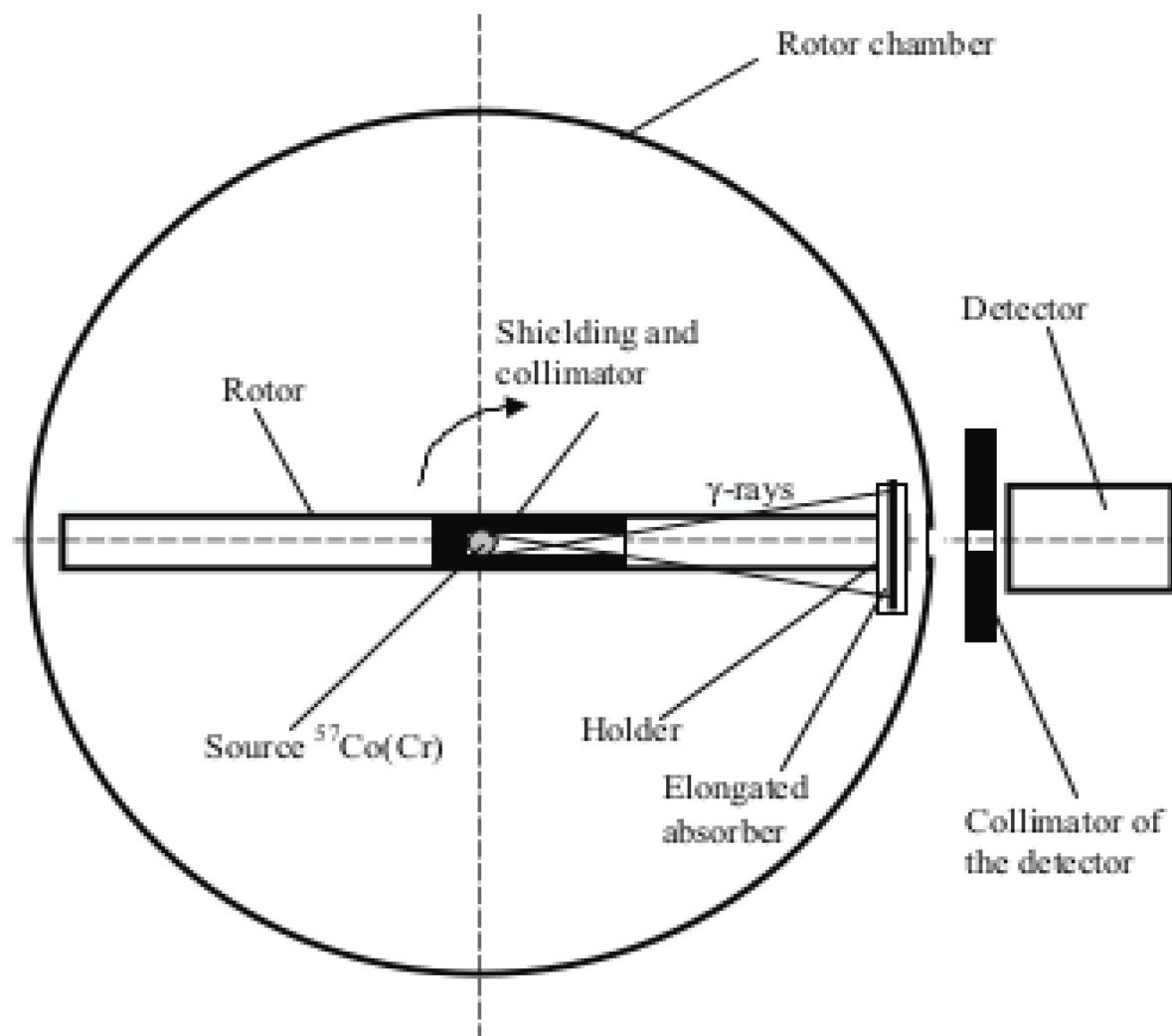
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They followed the standard scheme of previously cited experiments, where the count rate of detected gamma–quanta N as a function of the rotation frequency is measured. On the other hand, differently from previously cited experiments, they evaluated the influence of chaotic vibrations on the measured value of k. Their developed method involved a joint processing of the data collected for two selected resonant absorbers with the specified difference of resonant line positions in the Mössbauer spectra. The result obtained in the independent experiment by Kholmetskii et al. is k = 0.68±0.03, confirming that the coefficient k substantially exceeds 0.5. The scheme of the new Mössbauer rotor experiment is in Figure 1, while technical details on it can be found the papers of Kholmetskii et al.

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Scheme of the new Mössbauer rotor experiment, adapted from A. L. Kholmetskii, T. Yarman, O.V. Missevitch and B. I. Rogozev, Phys. Scr. 79, 065007 (2009).

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In this Lecture, Einstein Equivalence Principle, which states the equivalence between the gravitational "force" and the pseudo-force experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) will be used to re-analyze the theoretical framework of Mössbauer rotor experiments directly in the rotating frame of reference by using a general relativistic treatment. The results will show that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives k ≃ 2/3 in perfect agreement with the new experimental results of Kholmetskii et al. In that way, the general relativistic interpretation of this paper shows that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent proof of Einstein general relativity.

General relativistic interpretation of time dilatation

Following N. Ashby, Liv. Rev. Rel. 6, 1 (2003), let us consider a transformation from an inertial frame, in which the space-time is Minkowskian, to a rotating frame of reference. Using cylindrical coordinates, the line element in the starting inertial frame is

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2.$$

The transformation to a frame of reference rotating at the uniform angular rate omega with respect to the starting inertial frame is given by

$$t = t' \quad r = r' \quad \phi = \phi' + \omega t' \quad z = z' \ .$$

General relativistic interpretation of time dilatation

Thus, one gets the following well-known line element (Langevin metric) in the rotating frame

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - 2\omega r'^2 d\phi' \, dt' - dr'^2 - r'^2 d\phi'^2 - dz'^2.$$

That, considering light propagating in the radial direction, reduces to

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - dr'^2.$$

General relativistic interpretation of time dilatation

Einstein Equivalence Principle permits to interpret the last line element in terms of a curved spacetime in presence of a static gravitational field. Setting the origin of the rotating frame in the source of the emitting radiation, we have a first contribution which arises from the "gravitational redshift" that can be directly computed using eq. (25.26) in C. W. Misner , K. S. Thorne, J. A. Wheeler, "Gravitation" , Feeman and Company (1973), (MTW) which in its twentieth printing of 1997 is written as

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{received} - \lambda_{emitted}}{\lambda_{emitted}} = |g_{00}(r_1')|^{-\frac{1}{2}} - 1$$

General relativistic interpretation of time dilatation

and represents the redshift of a photon emitted by an atom a rest an a gravitational field and received by an observer at rest at infinity. Here we use a slightly different equation with respect the equation in the book MTW because here we are considering a gravitational field which increases with increasing radial coordinate while eq. (25.26) in the book MTW concerns a gravitational field which decreases with increasing radial coordinate. Also, we set the zero potential in the origin instead of at infinity and we use the proper time instead of the wavelength. Thus, combining previous equation with the last line element one gets

General relativistic interpretation of time dilatation

$$z_1 \equiv \frac{\nabla \tau_{10} - \nabla \tau_{11}}{\tau} = 1 - |g_{00}(r_1')|^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{1 - \frac{(r_1')^2 \omega^2}{c^2}}}$$

$$= 1 - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \simeq -\frac{1}{2} \frac{v^2}{c^2},$$

Hence, we find a first contribution, say $k_1 = 1/2$, to k.

Clock synchronization

We stress that we calculated the variations of proper time ∇_{10} and ∇_{11} in the origin of the rotating frame which is located in the source of the radiation. But **the detector is moving with respect to the origin in the rotating frame.** Thus, **the clock in the detector must be synchronized with the clock in the origin**, and this gives a **second contribution** to the redshift. To compute this second contribution we use eq. (10) of **N. Ashby, Liv. Rev. Rel. 6, 1 (2003),** which represents the proper time increment d on the moving clock having radial coordinate **r'** for values **v ≪ c:**

$$d\tau = dt' \left(1 - \frac{r'^2 \omega^2}{c^2} \right).$$

Clock synchronization

Inserting the **condition of null geodesics ds = 0** in previous equation one gets

$$cdt' = \frac{dr'}{\sqrt{1 - \frac{r'^2\omega^2}{c^2}}},$$

where we take the positive sign in the square root because the radiation is propagating in **the positive r direction**. Combining the last two equations one gets

$$cd\tau = \sqrt{1 - \frac{r'^2\omega^2}{c^2}}dr'.$$

Clock synchronization

Previous equation is well approximated by

$$cd\tau \simeq \left(1 - \frac{1}{2}\frac{r'^2\omega^2}{c^2} + ...\right)dr',$$

which permits to find the second contribution of order $(v/c)^2$ to the variation of proper time as

$$c\nabla\tau_2 = \int_0^{r_1'}\left(1 - \frac{1}{2}\frac{(r_1')^2\omega^2}{c^2}\right)dr' - r_1' = -\frac{1}{6}\frac{(r_1')^3\omega^2}{c^2} = -\frac{1}{6}r_1'\frac{v^2}{c^2}.$$

Thus, we get the second contribution of order $(v/c)^2$ to the redshift as

Clock synchronization

$$z_2 \equiv \frac{\nabla \tau_2}{\tau} = -k_2 \frac{v^2}{c^2} = -\frac{1}{6}\frac{v^2}{c^2}.$$

Then, we obtain $k_2 = 1/6$ and the total redshift is

$$z \equiv z_1 + z_2 = \frac{\nabla \tau_{10} - \nabla \tau_{11} + \nabla \tau_2}{\tau} = -(k_1 + k_2)\frac{v^2}{c^2}$$

$$= -\left(\frac{1}{2} + \frac{1}{6}\right)\frac{v^2}{c^2} = -k\frac{v^2}{c^2} = -\frac{2}{3}\frac{v^2}{c^2} = 0.\bar{6}\frac{v^2}{c^2},$$

which is completely consistent with the result k = 0.68 ± 0.03 in the paper A. L. Kholmetskii, T. Yarman, O.V. Missevitch and B. I. Rogozev, Phys. Scr. 79, 065007 (2009).

Discussion

We stress that the additional factor 1/6 that we found comes from clock synchronization. In other words, its theoretical absence in previous works reflected the incorrect comparison of clock rates between a clock at the origin and one at the detector. This generated wrong claims of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects, for example V. O. de Haan, Jour. Comp. Meth. Scien. Eng., 13, 51 (2013), or Kholmetskii et al., AIP Conf. Proc. 1648, 510011 (2015), which, instead, must be rejected. We evoked the appropriate work of Ashby for a discussion of the Langevin metric. This is dedicated to the use of general relativity in Global Positioning Systems (GPS), which leads to the following interesting realization the correction of 1/6 to the total redshift is analog to the correction that one must consider in GPS when

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accounting for the difference between the time measured in a frame co-rotating with the Earth geoid and the time measured in a non-rotating (locally inertial) Earth centered frame (and also the difference between the proper time of an observer at the surface of the Earth and at infinity). Indeed, if one simply considers the gravitational redshift due to the Earth's gravitational field, but neglects the effect of the Earth's rotation, GPS would not work! The key point is that the proper time elapsing on the orbiting GPS clocks cannot be simply used to transfer time from one transmission event to another because path-dependent effects must be taken into due account, exactly like in the above discussion of clock synchronization. In other words, the obtained correction 1/6 to the redshift is not an obscure mathematical or physical detail, but a fundamental ingredient that must be taken into due account.

Conclusion remarks

In this Lecture, **Einstein Equivalence Principle**, stating the equivalence between the gravitational "force" and the pseudo-force experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference) has been used to re-analyze the theoretical framework of the new Mössbauer rotor experiment in found by **Kholmetskii et al.** directly in the rotating frame of reference by using a general relativistic treatment. The results have shown that previous analyses **missed an important effect of clock synchronization and that the correct general relativistic prevision gives k ≃ 2/3** in perfect agreement with the new experimental results by **Kholmetskii et al**. Thus, the general relativistic interpretation in this Lecture shows that **the new experimental results of the Mössbauer rotor experiment are a new, strong and independent proof of Einstein general relativity**.

Conclusion remarks

The importance of our results is stressed by the issue that various papers in the literature, comprised a paper published in Phys. Rev. Lett. 4, 165 (1960), missed the effect of clock synchronization with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" or crackpot effects. An analogy with the use of General Relativity in Global Positioning Systems has been highlight in the final discussion of this Lecture.

Acknowledgements