Information retrieval from black holes : Quantum correlations in non-vacuum distortions

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Credits

- **Main references :**
  arXiv:1604.04987 [gr-qc]
  arXiv:1507.06402 [gr-qc] *[Phys. Rev. Lett. 116, 051301 (2016)]*

Introduction

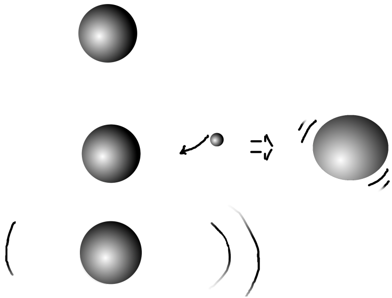
- The information loss in Black Holes
- The Semi-classical Scheme
- Quantum correlations : Non vacuum structure
- Retrieving the information.

Black Holes

- Compliment of the past domain of dependence of future null infinity.
- Spacetime region, denied to asymptotic future observers.
- Boundary of such a region is the Event horizon.
- Event horizon : One way traffic.
- Whatever enters becomes eternally trapped $\Rightarrow$ Information loss ?

Black Holes : Information Sink ?

- Black Holes : Thermodynamic objects : Temperature decides by the mass.
- Throwing something into the black hole increases its mass, decreases the temperature, increases the entropy.
- Perturbing black holes : Shivering in spacetime : Gravitational radiation



Black Holes : Information Sink ?

- Black Holes : Thermodynamic objects : Temperature decides by the mass.
- Throwing something into the black hole increases its mass, decreases the temperature.
- Perturbing black holes : Shivering in spacetime : Gravitational radiation
- Black hole settles in a new configuration.
- The (exact) nature of perturbation is essentially hidden from late time observers.

Black Holes : Information Sink ?

- Classical Level : No information loss.
- Semiclassical Level : Black Holes radiate [S. W. Hawking'74] $\rightarrow$ Evaporate
- The black hole shrinks $\rightarrow$ Thermal radiation
- The thermal (Hawking radiation) is characterized by the black hole mass.
- At very late times, the spacetime is filled by this radiation.
  The Black hole gets converted into thermal radiation.
  A two fold problem !

Black Holes : Information Sink ?

- Thermal radiation : Correlation free $\Rightarrow$ Contains *no information* at all !
- After black hole settles down it shrinks via thermal radiation.
- Eats up matter, churns out gravitational waves(classically), thermal radiation (semi-classically). QG - a distant player [Mathur'07]!
- No way for future asymptotic observers to reconstruct the initial data.

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- No way for future asymptotic observers to reconstruct the initial data **?**

The Hawking Radiation : The genesis

- General Covariance + QFT : Vacuum of different observers are different.
- Field description in terms of mode functions

$$\phi(x) = \sum_k (\hat{a}_k u_k(x) + \hat{a}_k^\dagger u_k^*(x))$$

- Mode function of any observer is linear combination of mode functions suited for another observer.

$$u_k(x) = \sum_{k'} \alpha_{kk'} \tilde{u}_{k'}(x) + \beta_{kk'} \tilde{u}_{k'}^*(x)$$

- The coefficients of transformation $\alpha_{kk'}, \beta_{kk'}$ are the Bogoliubov coefficients.

Black hole : Bogoliubov Coefficients

- As a result the creation/annihilation operators also transform

$$\hat{a}_k = \sum_{k'} \alpha^*_{kk'} \hat{\tilde{a}}_{k'} - \beta^*_{kk'} \hat{\tilde{a}}^\dagger_{k'}$$

- Vacuum of $\hat{\tilde{a}}_{k'}$ is not the vacuum of $\hat{a}_k$.

$$\langle \tilde{0} | \hat{a}^\dagger_k \hat{a}_k | \tilde{0} \rangle = \sum_{k'} |\beta_{kk'}|^2.$$

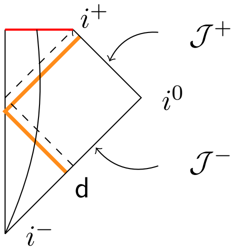

Figure: Penrose Diagram For Schwarzschild collapse

Black hole Information : Bogoliubov Coefficients

- The out-going modes v_ω on $\mathcal{J}^+$ derive their causal support completely from the portion $u < d$ on $\mathcal{J}^-$.

$$v_\omega \to \frac{1}{\sqrt{2\omega}} e^{-i\omega u}; \qquad u \in (-\infty, \infty).$$

- The in-moving modes v_ω on $\mathcal{J}^-$

$$u_\omega \to \frac{1}{\sqrt{2\omega}} e^{-i\omega v}; \qquad v \in (-\infty, \infty).$$

- Ray tracing close to the horizon relates u to v,

$$\alpha_{\omega\omega'} = -2i \int_{-\infty}^{\infty} dv u_\omega \partial_- u^*_{\omega'},$$
$$\beta_{\omega\omega'} = 2i \int_{-\infty}^{\infty} dv u_\omega \partial_- u_{\omega'}.$$

Black hole : Bogoliubov coefficients

$$\alpha_{\Omega\omega} = \frac{1}{2\pi\kappa}\sqrt{\frac{\Omega}{\omega}}\exp\left[\frac{\pi\Omega}{2\kappa}\right]\exp\left[\frac{i\Omega}{\kappa}\log\frac{\omega}{C}\right]\Gamma\left[-\frac{i\Omega}{\kappa}\right],$$

$$\beta_{\Omega\omega} = -\frac{1}{2\pi\kappa}\sqrt{\frac{\Omega}{\omega}}\exp\left[-\frac{\pi\Omega}{2\kappa}\right]\exp\left[\frac{i\Omega}{\kappa}\log\frac{\omega}{C}\right]\Gamma\left[-\frac{i\Omega}{\kappa}\right].$$

- For late time observers : A vacuum $\Rightarrow$ For late time observers : Thermal spectrum.
- The radiation knows only about the mass (change).

Black hole : Bogoliubov coefficients

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- For early time observers : A vacuum $\Rightarrow$ For late time observers : Thermal spectrum.
- The radiation knows only about the mass (change).
- How could have the black hole come about ?
- Some matter fields in non vacuum states (mass, energy) collapsed.

Black Hole :

- QFT with non vacuum states and Self gravity.
- Exact calculation : Technically formidable.
  1. Perturbation Analysis
  2. Exact calculation in lower dimension.

  Perturbation Analysis

  Perturb a *classical* black hole with a small mass ($|\Psi\rangle$).

$$\hat{\phi}(x) = \phi_0 \mathbb{I} + \delta\hat{\phi},$$

  with

$$\langle\Psi|T_{\mu\nu}[\phi]|\Psi\rangle \sim \langle\Psi|T_{\mu\nu}[\phi_0]|\Psi\rangle$$

- Reconstruct the perturbation $\delta\hat{\phi}$ data, i.e. $|\Psi\rangle$.

$$|\Psi\rangle_{in} = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} f(\omega)\hat{a}^\dagger(\omega)|0\rangle_{in}$$

- Late time : Geometry change, Hawking radiation will not help.

Black Hole : Non vacuum correlations

- Quantum information resides in

$$C(x,y) \equiv \langle\Psi|\delta\hat{\phi}(x)\delta\hat{\phi}(y)|\Psi\rangle.$$

- For an observer with mode functions $u_k(x)$,

$$N_k = -(u_k(x),(C(x,y),u_k^*(y))$$

- Picks up correction over thermal spectrum if $|\Psi\rangle$ is not vacuum state.
- Initial data reconstruction from this non-vacuum distortion.

Black Hole : Non vacuum correlations

- Correlations in frequency space

$$\hat{N}_{\mathbf{k_1}\mathbf{k_2}} \equiv \hat{\bar{\phi}}_{\mathbf{k_1}}\hat{\bar{\phi}}^{\dagger}_{\mathbf{k_2}} + \hat{\bar{\phi}}_{\mathbf{k_2}}\hat{\bar{\phi}}^{\dagger}_{\mathbf{k_1}}$$
$$= \hat{a}_{\mathbf{k_1}}\hat{a}^{\dagger}_{\mathbf{k_2}}e^{-i(\omega_{\mathbf{k_1}}-\omega_{\mathbf{k_2}})t} + \hat{a}_{\mathbf{k_2}}\hat{a}^{\dagger}_{\mathbf{k_1}}e^{i(\omega_{\mathbf{k_1}}-\omega_{\mathbf{k_2}})t}.$$

  with $\hat{\bar{\phi}}_{\mathbf{k}} = \hat{a}_{\mathbf{k}}e^{-i\omega_{\mathbf{k}}t}$.
- The Diagonal elements gives the spectrum operator and the off diagonal elements the correlation.
- Focus on this observable corresponding to the late time observers.

Black Hole : Non vacuum correlations

- Off diagonal elements vanish for in-vacuum state. Non vanishing for any non vacuum state.
- Diagonal elements : Spectrum operators
  Off diagonal elements : the correlation.
- Focus on this observable corresponding to the late time observers.
- For the fields "constituting" the black holes,
  Off diagonal elements :Non-zero.
  Diagonal elements :Non thermal.

Black Hole : Information Retrieval

- Transformation between the late time and the early time observers

$$\hat{b}_\Omega = \int d\omega \left(\alpha^*_{\Omega\omega} \hat{a}_\omega - \beta^*_{\Omega\omega} \hat{a}^\dagger_\omega \right)$$

$$_{\text{in}}\langle \Psi | \hat{N}_{\Omega_1 \Omega_2} | \Psi \rangle_{\text{in}} = \Bigg[\left(\int \frac{d\omega}{\sqrt{4\pi\omega}} f(\omega) \alpha^*_{\Omega_2 \omega} \right) \times \left(\int \frac{d\bar{\omega}}{\sqrt{4\pi\bar{\omega}}} f^*(\bar{\omega}) \alpha_{\Omega_1 \bar{\omega}} \right)$$

$$+ \left(\int \frac{d\omega}{\sqrt{4\pi\omega}} f^*(\omega) \beta^*_{\Omega_2 \omega} \right) \times \left(\int \frac{d\bar{\omega}}{\sqrt{4\pi\bar{\omega}}} f(\bar{\omega}) \beta_{\Omega_1 \bar{\omega}} \right) \Bigg] e^{-i(\Omega_1 - \Omega_2)t} + \text{c.c}$$

- Series of transformations : Frequency re-parameterization and Fourier transform on $f(\bar{\omega})$ we obtain $F(\Omega)$.

$$\begin{aligned} \log \frac{\tilde{\omega}}{C} &= z \Rightarrow f(Ce^z) = g(z), \\ F(y) &= \int_{-\infty}^{\infty} dz g(z) e^{iyz}. \end{aligned}$$

- Using the Bogoliubov coefficients

$$
{}_{\mathsf{in}}\langle\Psi|\hat{N}_{\Omega_1\Omega_2}|\Psi\rangle_{\mathsf{in}} = \frac{1}{4\pi}\left[A(\Omega_1)A(\Omega_2)^* + c.c.\right] + \frac{1}{4\pi}\left[B(\Omega_1)B(\Omega_2)^* + c.c.\right].
$$

with

$$
A(\Omega) = e^{-\frac{\pi\Omega}{2\kappa}}\frac{\sqrt{\Omega}}{2\pi\kappa}\Gamma\left[-i\frac{\Omega}{\kappa}\right]F\left(\frac{\Omega}{\kappa}\right)e^{-i\Omega t},
$$

and

$$
B(\Omega) = e^{\frac{\pi\Omega}{2\kappa}}\frac{\sqrt{\Omega}}{2\pi\kappa}\Gamma\left[-i\frac{\Omega}{\kappa}\right]F^*\left(-\frac{\Omega}{\kappa}\right)e^{-i\Omega t}
$$

- Using the Bogoliubov coefficients

$$\mathcal{D}_{\Omega_1\Omega_2} \equiv N_{\Omega_1\Omega_2} + \frac{i}{\Delta\Omega}\frac{\partial}{\partial t}N_{\Omega_1\Omega_2},$$

where $\Delta\Omega = \Omega_1 - \Omega_2$.

$$\mathcal{D}_{\Omega_1\Omega_2} = \frac{1}{2\pi}\frac{\sqrt{\Omega_1\Omega_2}}{4\pi^2\kappa^2}\Gamma\left[-i\frac{\Omega_1}{\kappa}\right]\Gamma\left[i\frac{\Omega_2}{\kappa}\right]e^{-i(\Omega_1-\Omega_2)t}\times$$
$$\left\{e^{\frac{\pi(\Omega_1+\Omega_2)}{2\kappa}}F\left(-\frac{\Omega_2}{\kappa}\right)F^*\left(-\frac{\Omega_1}{\kappa}\right) + (\Omega_1, \Omega_2 \leftrightarrow -\Omega_2, -\Omega_1)\right\}.$$

Black Hole : Information Retrieval

- Symmetries become important for partial/complete retrieval.
- e.g. for real initial data $F(\Omega/\kappa) = F^*(-\Omega/\kappa)$

$$S_{\Omega_1\Omega_2} \equiv \frac{4\pi^3\kappa^2}{\sqrt{\Omega_1\Omega_2}} \frac{\mathcal{D}_{\Omega_1\Omega_2} e^{i(\Omega_1-\Omega_2)t}}{\Gamma\left[-i\frac{\Omega_1}{\kappa}\right]\Gamma\left[i\frac{\Omega_2}{\kappa}\right]\cosh\left(\frac{\pi(\Omega_1+\Omega_2)}{2\kappa}\right)}$$
$$= F\left(\frac{\Omega_1}{\kappa}\right) F^*\left(\frac{\Omega_2}{\kappa}\right).$$

- The function $F(\Omega)$ can be obtained upto (an irrelevant) total phase.
- Field theoretic constructs can be obtained. (Semiclassical Hairs !)

Black Hole : Information Retrieval

- A similar exercise can be repeated with the diagonal element aka the spectrum operator.
- Spectrum operator has a thermal part with an overriding correction.
- Information encoded in $\delta\hat{N}_\Omega = \langle\Psi|\hat{b}^\dagger_\Omega\hat{b}_\Omega|\Psi\rangle - \langle 0|\hat{b}^\dagger_\Omega\hat{b}_\Omega|0\rangle$.
- Classify set of semiclassical hairs.
- However, a more restricted class of initial data encode the entire information in the distortion.

Black Hole : Information Retrieval

- Consult late time observers for **all the fields** in spacetime.
- A quantity $S_{\Omega_1 \Omega_2}$ constructed from the correlation operator will tell about the initial data.
- Field configurations in vacuum, $S_{\Omega_1 \Omega_2}$ will vanish.
- Field configurations in non-vacuum will encode information in $S_{\Omega_1 \Omega_2}$.
- Can it be done with back reaction taken into consideration ?

2-dimensional dilaton (CGHS) Black Hole

$$\mathcal{A} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right) - \frac{1}{2}\sum_{i=1}^{N}(\nabla\Phi_i)^2\right].$$

- Metric, Dilaton field, Matter field, Cosmological constant.
- Two dimensional metric is conformally flat.

$$ds^2 = -e^{2\rho}dx^+dx^-.$$

- Classical solutions : Black Hole of mass M

$$ds^2 = -\frac{dx^+dx^-}{\frac{M}{\lambda} - \lambda^2 x^+ x^-},$$

Linear Dilaton Vacuum

$$ds^2 = -\frac{dx^+dx^-}{-\lambda^2 x^+ x^-}.$$

2-dimensional dilaton (CGHS) Black Hole

- Equations of motion

$$-\partial_+\partial_- e^{-2\phi} - \lambda^2 e^{2\rho-2\phi} = 0,$$
$$2e^{-2\phi}\partial_+\partial_-(\rho-\phi) + \partial_+\partial_- e^{-2\phi} + \lambda^2 e^{2\rho-2\phi} = 0,$$

$$\partial_+^2 e^{-2\phi} + 4\partial_+\phi\partial_+\left(\rho-\phi\right) e^{-2\phi} + T_{++} = 0,$$
$$\partial_-^2 e^{-2\phi} + 4\partial_-\phi\partial_-\left(\rho-\phi\right) e^{-2\phi} + T_{--} = 0.$$

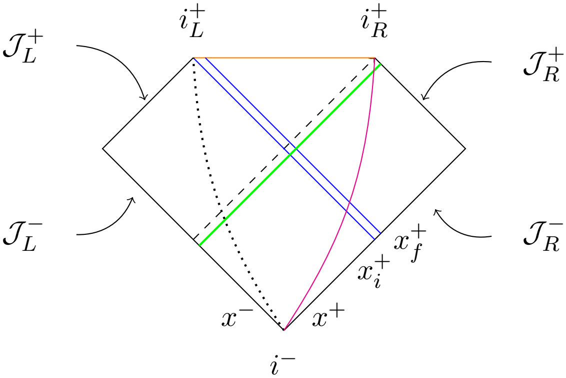
- Matter field $\Phi_i = \Phi_{i+}(x^+) + \Phi_{i-}(x^-)$.

$$\partial_\pm^2 e^{-2\phi} = -T_{\pm\pm}$$

Results in a black hole geometry

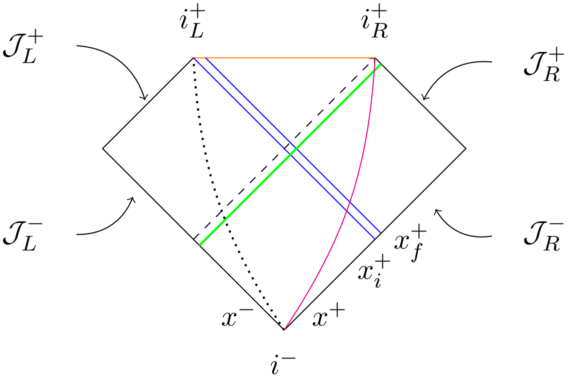
$$ds^2 = -\frac{dx^+dx^-}{\frac{M(x^+)}{\lambda} - \lambda^2 x^+ x^- - P^+(x^+)x^+},$$

2-dimensional dilaton (CGHS) Black Hole



- Let a matter shell collapse.
- Observers before the arrival of shell always find themselves in flat spacetime.
- Post shell, geometry is black hole region.

2-dimensional dilaton (CGHS) Black Hole



- Focus on left-moving observers
- Observers before the arrival of shell always find themselves in flat spacetime.

2 dimensional dilaton (CGHS) Black hole

- We require Bogoliubov coefficients between $\mathcal{J}_R^-$ and $\mathcal{J}_L^+$
- With $x^+ = -\frac{1}{\lambda y^+}; \qquad x^- = -\frac{1}{\lambda y^-}$

On $\mathcal{J}_R^-$:

$$\begin{aligned} e^{-\lambda \chi^+} &= -y^+, \\ e^{\lambda \chi^-} &= y^- \end{aligned}$$

with mode function

$$u_\omega^+(\chi^+) = \frac{1}{\sqrt{2\omega}} e^{-i\omega\chi^+}.$$

On $\mathcal{J}_L^+$:

$$\begin{aligned} e^{-\lambda \tilde{\chi}^+} &= -(y^+ - y_i^+), \\ e^{\lambda \tilde{\chi}^-} &= y^- \end{aligned}$$

with mode function

$$v_\omega^+(\tilde{\chi}^+) = \frac{1}{\sqrt{2\omega}} e^{-i\omega\tilde{\chi}^+}.$$

2 dimensional dilaton (CGHS) Black hole

- The Bogoliubov coefficients

$$\alpha_{\Omega\omega} = \frac{1}{2\pi\lambda}\sqrt{\frac{\omega}{\Omega}}|y_i^+|^{\frac{i(\Omega-\omega)}{\lambda}} \; B\left(-\frac{i\Omega}{\lambda}+\frac{i\omega}{\lambda}, 1+\frac{i\Omega}{\lambda}\right).$$

$$\beta_{\Omega\omega} = \frac{1}{2\pi\lambda}\sqrt{\frac{\omega}{\Omega}}|y_i^+|^{\frac{i(\Omega+\omega)}{\lambda}} \; B\left(-\frac{i\Omega}{\lambda}-\frac{i\omega}{\lambda}, 1+\frac{i\Omega}{\lambda}\right).$$

- Exact forms of the Bogoliubov coefficients are available, we can track the information content

$$f(\bar{\omega}') = \bar{\omega}'\tilde{f}(\bar{\omega}')e^{-\frac{\pi}{2}\bar{\omega}'}\Gamma[-i\bar{\omega}']\left|y_i^+\right|^{-i\bar{\omega}'}.$$

2 dimensional dilaton (CGHS) Black hole

- The Bogoliubov coefficients have the form exactly that for Schwarzschild case, for late time observers.
- All the mathematical steps carried over exactly.
- No test field approximation required. The field Φ itself formed the black hole, semi-classically.
- Presence of such semi-classical hairs is robust.
- Information is contained in these “new” hairs.

Conclusions

- The initial data of a collapsing geometry **includes some matter fields in non-vacuum state.**
- Black hole leaks energy in all field modes, vacuum or non-vacuum.
- Black hole leaks information **only** in the non-vacuum sector.
- **Such information can persist for late time observers, quantum mechanically.**
- **Black holes have semi-classical and (very likely) quantum hairs.**
- **This is not (yet) about restoring unitarity, but about recovering information**

Thank you !