Horizon, homogeneity and flatness problems -- do their resolutions really depend upon inflation?

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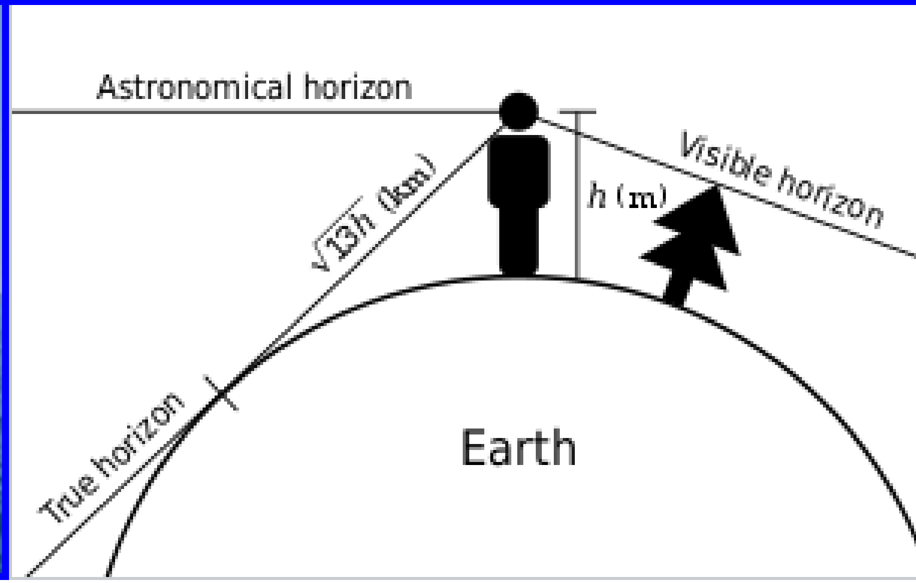
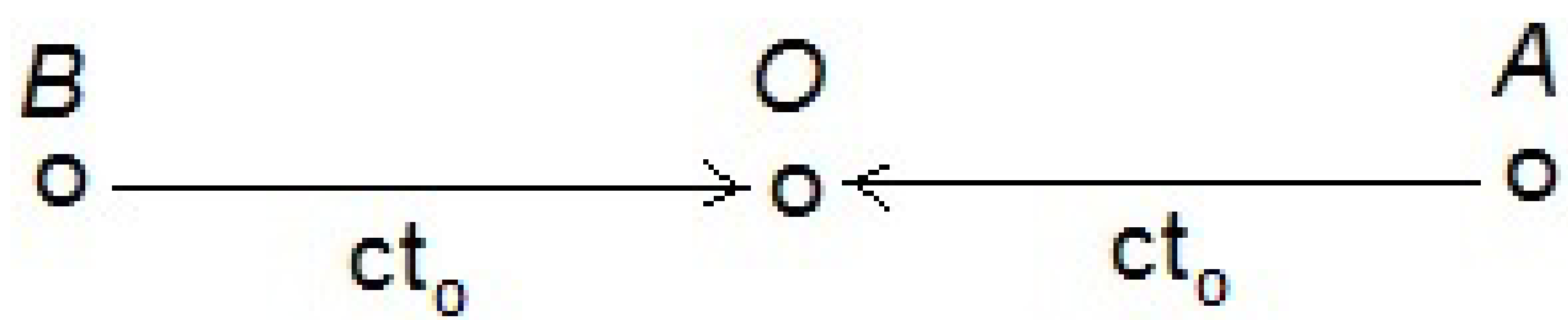
A naive concept of Horizon





Robertson-Walker metric

$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{(1 - k\,r^2)^{1/2}} + r^2 d\omega^2\right],$$

where the only time dependent function is the scale factor $R(t)$. Here $r d\omega = r\sqrt{d\theta^2 + \sin^2\theta d\phi^2}$ represents the angular line element. The constant k is the curvature index that can take one of the three possible values $+1, 0$ or -1 and (r, θ, ϕ) are the time-independent comoving coordinates.

Density parameter

$$\frac{k}{R_o^2} = (\Omega_o - 1)\frac{H_o^2}{c^2},$$

where $\Omega_o = \Omega_m + \Omega_\Lambda$ with Ω_m, Ω_Λ as matter density and vacuum energy (dark energy) density parameters respectively.

Horizon

In general it is not possible to express the comoving coordinate distance $rR_{\circ}$ in terms of the cosmological redshift z of the source in a close-form analytical expression and one may have to evaluate it numerically. For example, in the $\Omega_{\Lambda} \neq 0, \Omega_{\circ} = 1$ world-models, $rR_{\circ}$ is given by

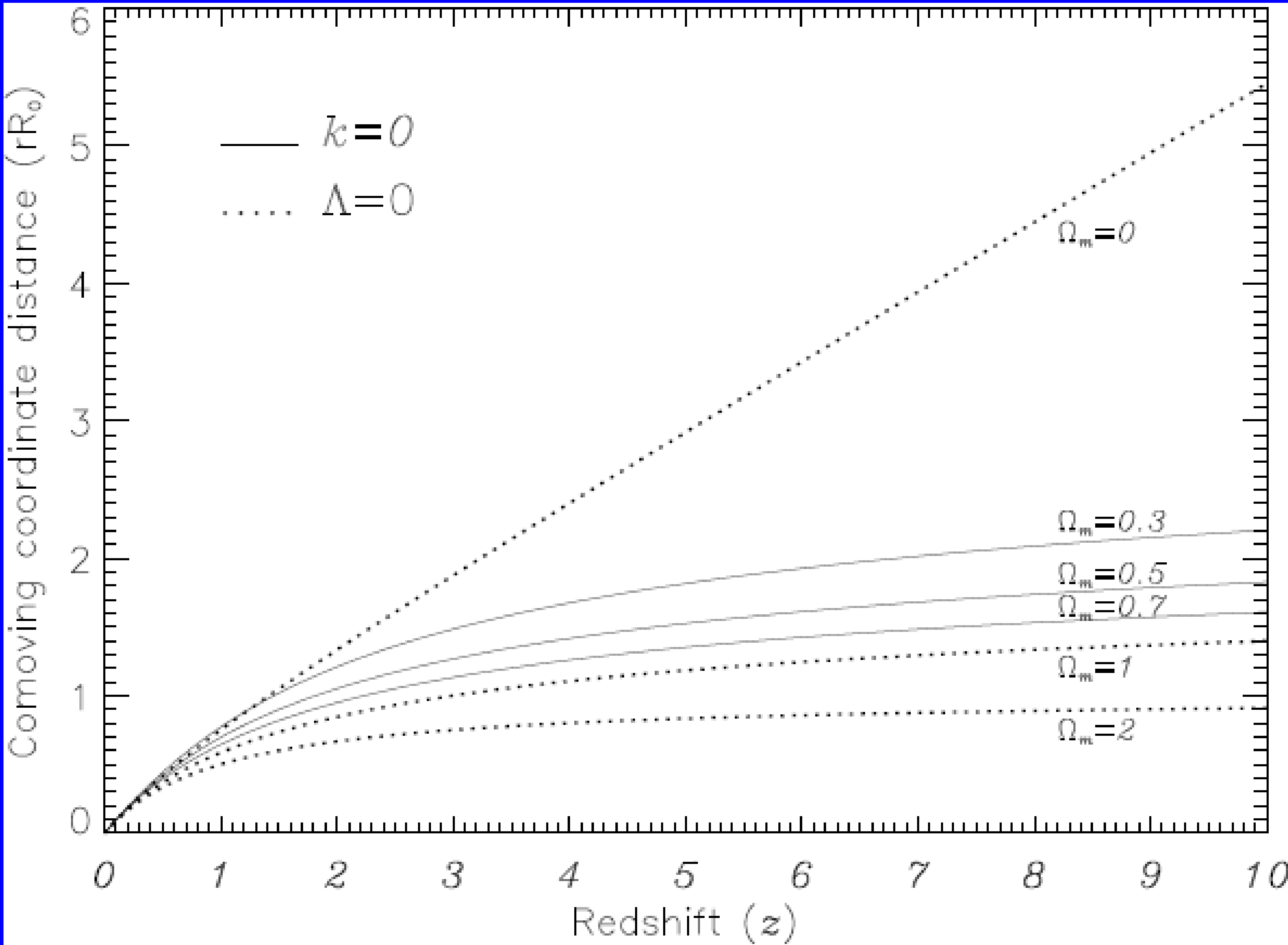
$$rR_{\circ} = \frac{c}{H_{\circ}} \int_0^z \frac{\mathrm{d}z}{[\Omega_{\Lambda} + \Omega_{\mathrm{m}}(1+z)^3]^{1/2}} \quad ,$$

For a given finite Ω_{Λ}, one can evaluate $rR_{\circ}$ by a numerical integration.

However, for $\Omega_\Lambda = 0, \Omega_o = \Omega_m$ cosmologies, it is possible to express rR_o as an analytical function of redshift

$$rR_o = \frac{c}{H_o} \frac{z}{(1+z)} \frac{\left[1 + z + \sqrt{1 + z\Omega_o}\right]}{\left[1 + z\Omega_o/2 + \sqrt{1 + z\Omega_o}\right]}.$$

From the expression one finds that as $z \to \infty$, rR_o converges to a finite value $2c/(H_o\Omega_o)$, though the range of possible values of coordinate distance rR_o extends up to infinity.



k=0
Λ=0
Ωm=0
Ωm=0.3
Ωm=0.5
Ωm=0.7
Ωm=1
Ωm=2
Comoving coordinate distance (rR₀)
Redshift (z)
0
1
2
3
4
5
6
7
8
9
10

Object Horizons

Table 1. Object horizon for various FRW world models

k	Ω_Λ	Ω_m	$rR_o H_o / c(z \to \infty)$
-1	0	0	∞
0	0.7	0.3	3.2
0	0.5	0.5	2.6
0	0.3	0.7	2.3
0	0	1	2
$+1$	0	2	1

It turns out that *all* finite density FRW world-models, starting with a big bang, have an object horizon

It is thought that a finite horizon exists because there is only a finite amount of time since the big bang singularity (corresponding to $z \to \infty$), and that photons could have travelled only a finite distance within the finite age of the universe. However, it is interesting to note that for Milne's universe ($\Omega_\circ = 0$), there is no finite horizon limit in this case and the whole infinite universe is visible to any observer at any time.

Therefore the argument that a "finite horizon" arises in cosmological models because photons could have travelled only a finite distance since the big bang singularity, does not hold good in a general case.

Homogeneity

Appearance of object horizon in a world model is generally interpreted as that different parts of the universe in that model did not get sufficient time to interact with each other and thus may have yet no causal relations and therefore could not have achieved uniformity everywhere.

Therefore inflation is invoked in which an exponential expansion of space takes place at time $t \sim 10^{-35}$ sec by a factor of $\sim 10^{28}$ or larger and the space-points now far apart (and thus apparently not in touch with each other so they appear to be causally unrelated) were actually much nearer before $t \sim 10^{-35}$ sec or so and could have had time to interact with each other before inflation.

A crucial point that somehow seems to have been overlooked (or ignored) in these deliberations is that the question of horizon comes up only when the cosmological principle of homogeneity and isotropy at all times holds good to begin with as then only we could apply Robertson-Walker element where we separate the time co-ordinate from the 3-d space (which may or may not be flat) and has the time-dependence only through a single scale parameter $R(t)$.

And it is only then that horizon makes an appearance which in turn has given rise to the oft-discussed question of the uniformity and homogeneity of the universe at large.

Homogeneity begets Horizon

The Robertson-Walker line element ensues from the assumption of homogeneity and isotropy, where the presence of a horizon is inferred. However that in itself may not imply a non-existence or lack of homogeneity as horizon itself makes an appearance in models where to begin with homogeneity is presumed. All we find from calculations is that in such an isotropic and homogeneous universe, the light signals in a finite value of the the parameter time do not, in general, cover the whole available range of space coordinate r in the universe.

Cause and effect reversed!

Cause and effect seem to have been reversed in their roles in this particular problem. It is not that because horizon exists so uniformity is not possible, ironically it is where a uniformity is present to begin with that we seem to end up with a horizon problem.

In these models we assign only a single parameter t, and all other parameters describing the universe like the scale factor $R(t)$, density parameter $\Omega(t)$, Hubble parameter $H(t)$, deceleration parameter $q(t)$ etc. at any given time t to be the same *everywhere*

Flatness Problem

An equation valid at any epoch is

$$H^2 R^2 (\Omega - 1) = kc^2.$$

We can rewrite it as

$$(\Omega - 1) = \frac{kc^2}{H^2 R^2} = \frac{kc^2}{\dot{R}^2}.$$

For all world models with a big bang origin, $R \propto t^{1/2}$ near the big bang event , implying $\dot{R}^2 \propto t^{-1}$
or $\Omega \to 1$ as $t \to 0$. This of course is the reason why in all such models, $(\Omega - 1)$ can be extremely small in the early universe.

Comparing the density parameter Ω at an earlier epoch to the present epoch value Ω_o, for the open universe models ($k = -1$ and $\Omega < 1$) we can write

$$(1-\Omega) = \frac{\dot{R}_o^2}{\dot{R}^2}(1-\Omega_o),$$

which can be written as

$$(1-\Omega) = \epsilon\,(1-\Omega_o),$$

where ϵ could be an extremely small number depending upon the earlier epoch of reference. Now for a given world model and for the chosen epoch, ϵ may be a definite number, though we may know it only very approximately, perhaps only to an order of magnitude. For instance, for the epoch of inflation ($t \sim 10^{-35}$ sec), $\epsilon \approx 10^{-53}$, while for the Planck epoch ($t \sim 10^{-43}$ sec), $\epsilon \approx 10^{-61}$

The standard argument prevalent in literature is that as the present density of the universe is very close (within an order of magnitude) to the critical density value, i.e., $0.1 < \Omega_\circ < 1$, the universe *must be flat* since otherwise in past at 10^{-35} second (near the epoch of inflation) there will be extremely low departures of density from the critical density value (i.e., they would differ from unity by a fraction of order $\sim 10^{-53}$), requiring a sort of fine tuning.

Further, this type of fine-tuning argument can be applied to almost any present value of the observed density of the Universe. What is implied here is that even in a hypothetical, almost empty universe where the density of universe is say, $\rho_o \sim 10^{-56}$ gm/cc) or so (with density parameter $\Omega_o \sim 10^{-28}$, having only a mass equivalent to that of Earth alone to fill the whole universe), the density parameter at the epoch of inflation would differ from unity by the same fraction, of order $\sim 10^{-53}$.

Is there really any substance in this type of arguments as even a mass equal to that of earth alone spread over the universe will lead to the same low departures from unity of 10^{-53}? In fact even the presence of a mere single observer would imply the same departures from unity of 10^{-53}.

That way, irrespective of the actual density, we could use any sufficiently early epoch and use the "extreme fine-tuning" arguments to reject all non-flat models. But that is not what one could call a falsifiable theory.

To understand that better, we write $(1-\Omega) = \eta$ and $(1 - \Omega_\circ) = \eta_\circ$ to get

$$\eta = \epsilon\, \eta_\circ.$$

Here both η and $\eta_\circ$ lie between 0 and 1 for our open-universe model. That immediately implies that $\eta/\epsilon < 1$. η cannot be larger than ϵ by even a tinniest amount. For instance, if we have $\eta = \epsilon(1+\epsilon)$, then $\eta_\circ = \eta/\epsilon = (1+\epsilon)$. which violates the condition that $\eta_\circ$ is between 0 and 1.

A use of fine tuning argument to promote $k = 0$ model amounts to *a priori* rejection of all models with $k \neq 0$, because inflation or no inflation, the density parameter in all FRW world models gets arbitrarily close to unity as we approach the epoch of the big bang. That is the property of all these FRW models.

Now if the universe is flat ($k = 0$) then inflation of course plays no part in this respect as it cannot make it any more flat. However in nearly flat universe scenario, Inflationary theories purportedly alleviate the problem of fine-tuning by proposing that the universe in an interval of $\sim 10^{-32}$ seconds expands exponentially by a factor of $\sim 10^{28}$ in its linear size, thereby decreasing the curvature to a value close to zero and thereby bringing the density parameter of the universe very close to the required value of unity.

However the huge expansion factor ($\sim 10^{28}$) in size then has to extremely fine-tuned so that the resulting density parameter is such that η does not exceed ϵ ($\sim 10^{-50}$) by even a tiniest amount. This assumption of inflation factor in a rather tight range, does it not imply replacing the erstwhile fine-tuning problem with another but more severe form of fine-tuning?

The so-called fine-tuning in non-inflationary models is not really a fine-tuning as that is the nature of the FRW cosmological models and it depends upon the epoch chosen for the investigation of the density parameter, but the fine-tuning implied in the inflationary models has to be just right at the end of the inflation. Does it really alleviate the fine-tuning problem in a fruitful manner.

In fact if

inflation brings the value of η down by a large factor so as to match the present conditions, it would mean that before the inflationary era, for η to be a moderate value (~ 1), the expansion rate near the big bang need to be also a more moderate value ($\dot{R} \sim c$), a condition that could be a problem in the FRW models to satisfy. Does not the remedy seem to be worse than the ailment, if any?

As Adler & Overduin (2005) have, however, pointed out, from the type of arguments used in literature, one might consider a flat universe to be infinitely fine-tuned, since it has $\Omega_\circ$ to be identically one, thereby making it the most unnatural choice.

Lake (2005) has shown, for $\Omega_\Lambda \neq 0$ models, there exist non-flat FRW models for which $\Omega_\circ \sim 1$ throughout the entire history of the universe, and that these really are not fine-tuned models.

From an examination of the flatness problem quantitatively for all cosmological models, Helbig (2012) has concluded that the flatness problem does not exist, not only for the cosmological models corresponding to the currently popular values of λ and $\Omega_\circ$ values but indeed for all FRW models with $\lambda \neq 0$.

In fact by assuming a flat model we are assuming the ultimate finest-ever tuning imaginable where even the least amount of perturbation on this unstable equilibrium model (in the form of an excess or deficiency of the smallest amount of matter from the critical density - a single particle or atom extra or missing!) can ultimately take the universe away from the flat-space model to a curved one.