History of the mass limit of compact stars

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Outline

• Introduction
• The discovery of the companion of Sirius.
• Eddington's puzzle concerning the huge density of white dwarfs.
• Anderson,Stoner, Frenkel,Landau, Fowler and the Physics behind the discovery of the white dwarf mass limit: Quantum Mechanics and Special Relativity.
Chandrasekhar and the legend of his discovery of the white dwarf mass limit.
• Stoner's calculation of the mass-radius relation of a white dwarf.
• The 1935 confrontation between Eddington and Chandrasekhar.
• Correspondence of Chandrasekhar, Milne, Eddington and Stoner.

Quantum mechanics, relativity, and the mass limit of compact stars

One of the most striking early applications of quantum mechanics and special relativity was the solution of a major puzzle in astrophysics in the early 1920's on the origin of the enormous density of white dwarfs which could not be explained by classical physics. The discovery of a limiting mass for these stars is attributed solely to S. Chandrasekhar, but as is often the case, the history of this discovery is more complex. A correct solution was first given by J. Frenkel, and independently by E. C. Stoner, who in a brilliant paper now generally forgotten, anticipated Chandrasekhar's result. Stoner also had played an important role in the eventual formulation by Pauli of the exclusion principle, which has a central role in the solution of the dense star puzzle. This episode gave rise to modern astrophysics by explaining the occurrence of supernova, the formation of neutron stars and the collapse of stars into black holes

Journal for the History of Astronomy 39 (2008) 297-312

Physics Today 64 (2011) 8

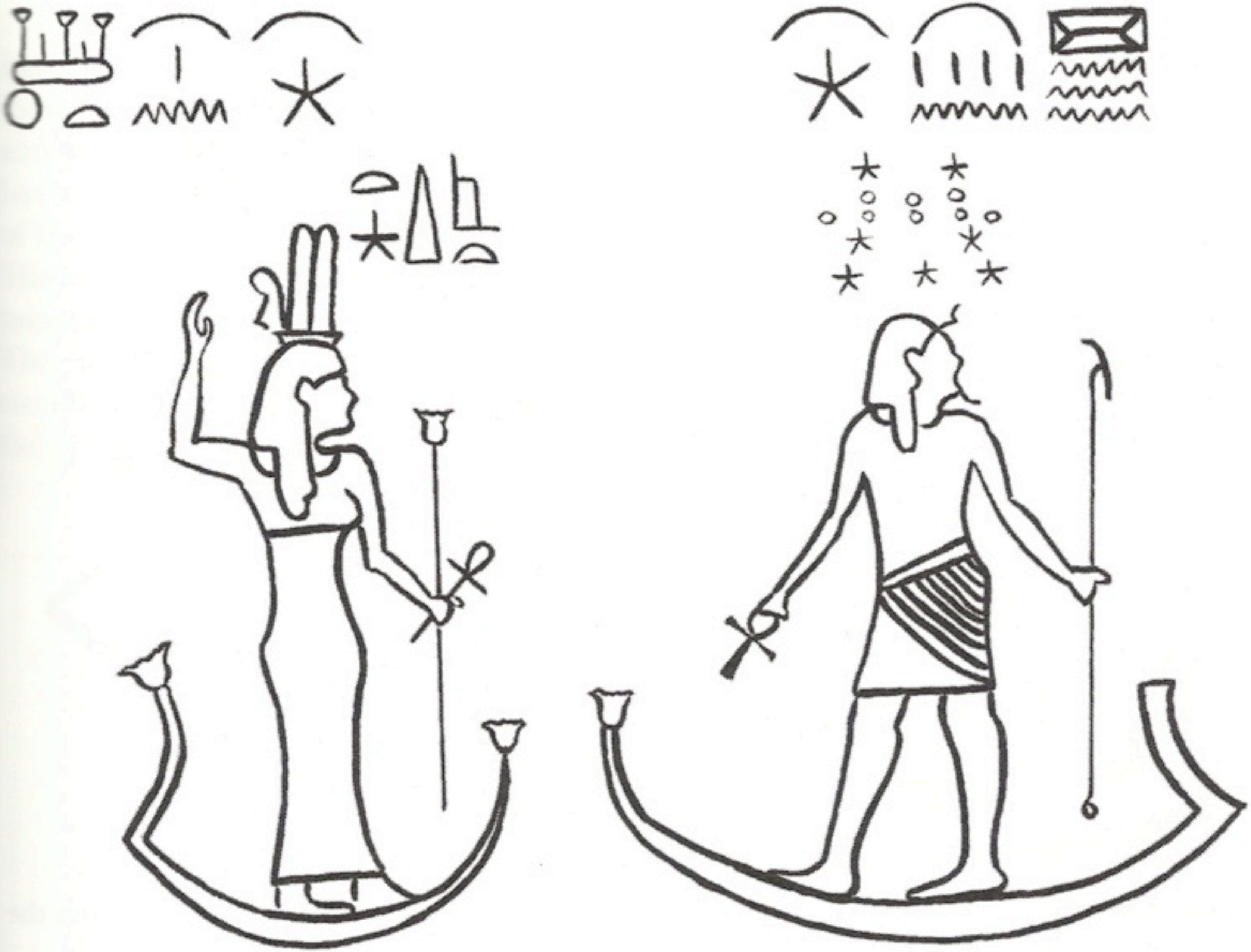


Figure 1.1. Isis and Osiris in their sacred barges and below their stellar associations. From the Ramesseum, the mortuary temple of Rameses II at Thebes (Michael Soroka).

Santa Cruz
Feb. 8, 2008,
12 p.m.

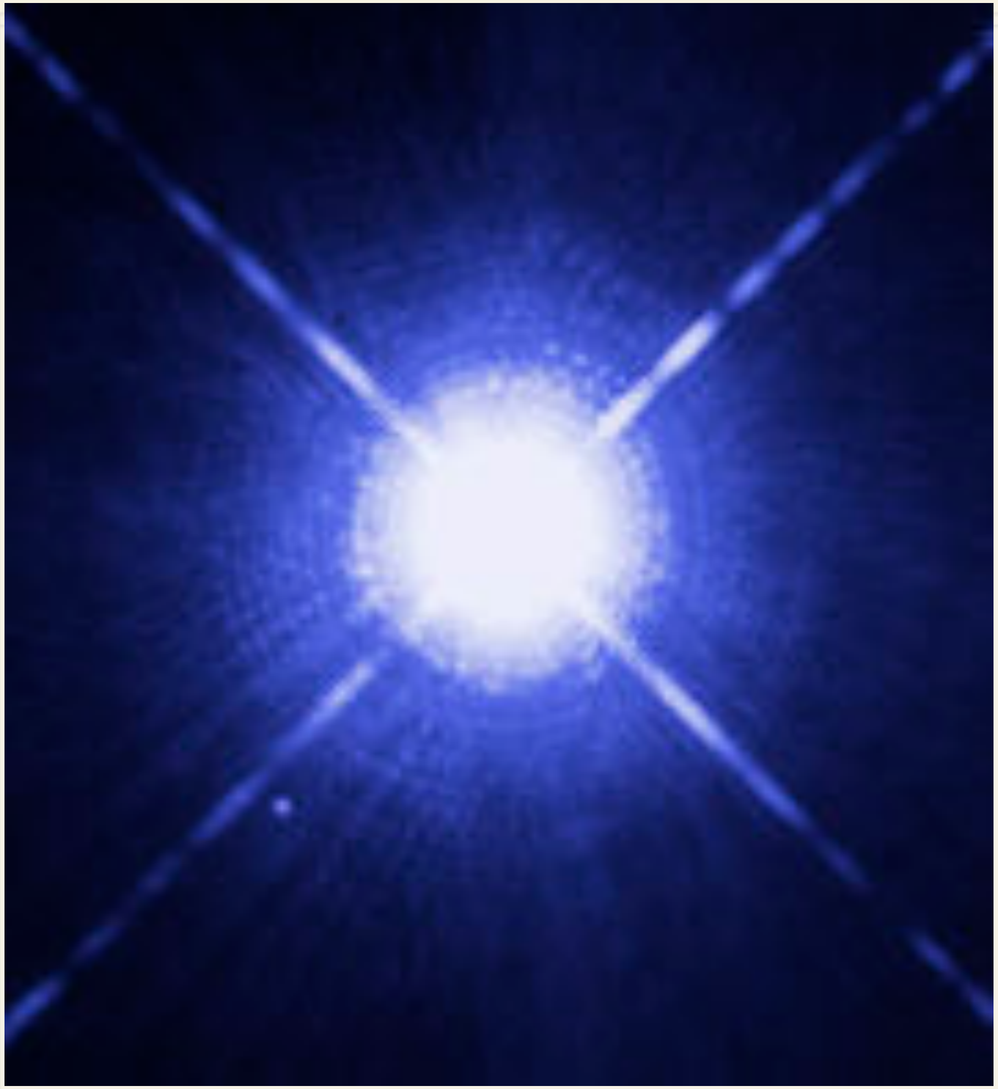
South

Friedrich Wilhelm Bessel

1784- 1846

Director of the Konigsberg Observatory

Sirius dark companion 1844



Hubble space telescope image of Sirius and white dwarf companion

density of Sirius= .57 gms/cc
density of companion= 2 million gms/cc
=40 tons/ cubic inch

The Feynman Lectures on Physics, vol 1, pg. 74

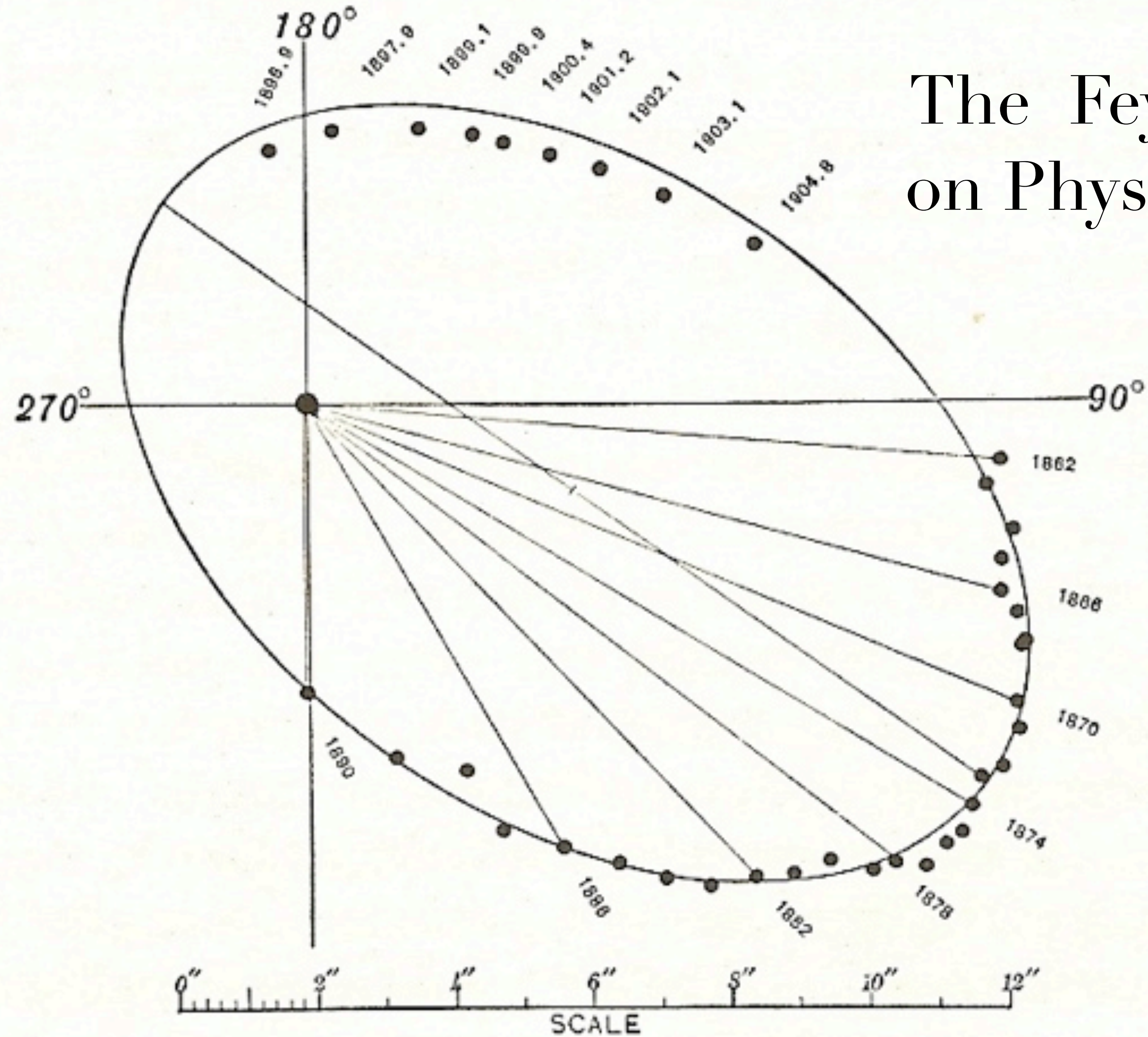
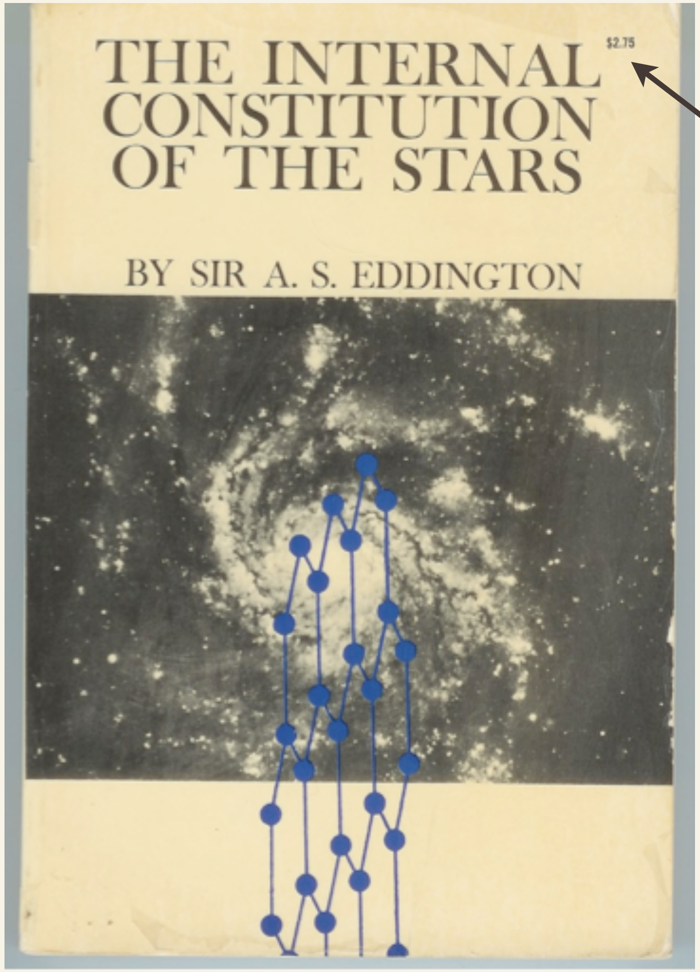


Fig. 7–7. Orbit of Sirius *B* with respect to Sirius *A*.

Period = 50.09 years

Arthur S. Eddington
1882-1944



In 1920, Eddington pointed to the fusion of 4 hydrogen atoms into a Helium atom as the likely energy source of stars.

“I do not see how a star which has once got into this compressed state is ever going to go out of it…
The star will need energy in order to cool…
It would seem that the star will be in an awkward predicament when its supply of subatomic energy fails
Imagine a body continually losing heat but with insufficient energy to grow cold.
We leave here the difficulty as not being necessarily fatal”

A. S. Eddington , “The Internal Constitution of the Stars” (Cambridge Univ. Press 1926) section 117


Writing, April 1924 (CERN)

Wolfgang Pauli
1900-1968

On the connection of the completion of the electronic group in the atom with the complex structure of the spectrum"

Zeitschrift fur Physik 31 765-783 (1925)

. . an essential advance by the reflections of E.C. Stoner p. 773

Wolfgang Pauli

Pauli's formulation of the exclusion principle
(. . . folgende allgemeine Regel)
as a generalization of Stoner's result:

In an atom there cannot be more equivalent electrons for which the value of all quantum numbers, n,k,m_1,m_2 in a strong magnetic field, coincide. If an electron exist in an atom for which all these quantum numbers have definite values, then this state is occupied

Zeitschrift fur Physik 31 (1925) p. 776

The Pauli principle does not forbid "divorces" when an electron goes to another free or partly occupied "room", but absolutely rules out a "menage a trois" J.I. Frenkel

Yakov Ilich Frenkel
1894-1952

``Application of the Pauli-Fermi electron gas theory to the problem of cohesive forces”

Zeitschrift fur Physik
47, 819 (1928)

Zeldovich and Novikov wrote: “Frenkel and Landau made a large contribution to the theory of white dwarfs” Pravda, March 9, 1975

In the preface of the fourth edition of his classic book
“ Atomic Structure and Spectral lines”,
Sommerfeld gave special mention to “einen grossen Fortschritt” [a great advance]
brought about by Stoner’s analysis in this paper.

Through Sommerfeld’s book, Stoner’s work came to the attention of Pauli, and played an important role in his formulation of the exclusion principle in quantum mechanics

Enrico Fermi

"Quantization of the ideal atomic gas"
Z. Physik 36, 902-912 (1926)

. . . Pauli, in connection with the work of E.C. Stoner, establish the rule that when an atom has fixed quantum numbers, there cannot be any further electrons in the shell that is characterized by these quantum numbers.

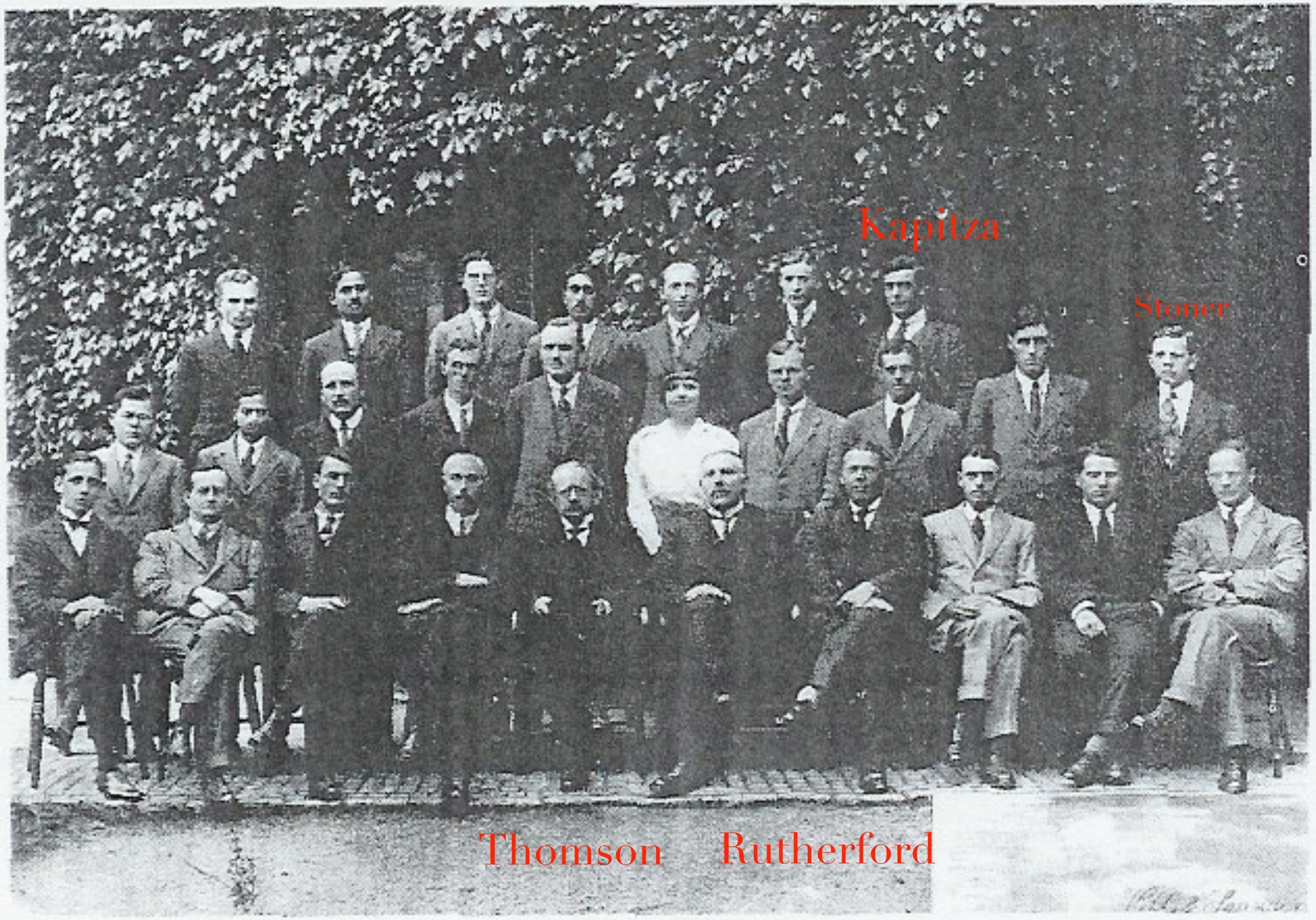
Edmund C. Stoner

1899 -1966

The distribution of electrons among atomic levels Philosophical Magazine 48 (1924) 719-736

Communicated by R.H Fowler

"If electrons in the atom are distributed according to the present scheme . . . the interesting point is suggested there is then one electron in each possible equally probably state."



F.Madgwick. B.N.Banerji. H.D.Smyth. N.Ahmad. W.T.Gibson. L.L.Whyte. P.Kapitza.

Y.Nishina. A.C.Chakravarti. P.Mercier. J.Crackston. H.Robinson. Miss Taylor. E.S.Bieler. J.K.Roberts. P.M.S.Blackett. E.C.Stoner.

.Belz. G.Henderson. D.A.Keys. F.W.Aston. Prof. Sir J.J.Thomson. Prof. Sir E.Rutherford. J.A.Crowther. G.Stead. E.V.Appleton. A.Muller.

One night in May 1924 a distribution scheme occurred to me in which the numbers in full levels were simply related to the quantum numbers specifying them...
I wrote a brief note about the scheme for Rutherford and ... left it on his desk. He must have passed to R, H. Fowler (with whom at this period I had several most helpful discussions ...) for soon afterwards Fowler asked me to call on him to discuss it. He was favourably impressed, and suggested that I should write a full and detailed paper about it...

Stoner's recollections in L.F. Bates "Biographical Memoirs of Fellows of the Royal Society 15 (1969) 201-237

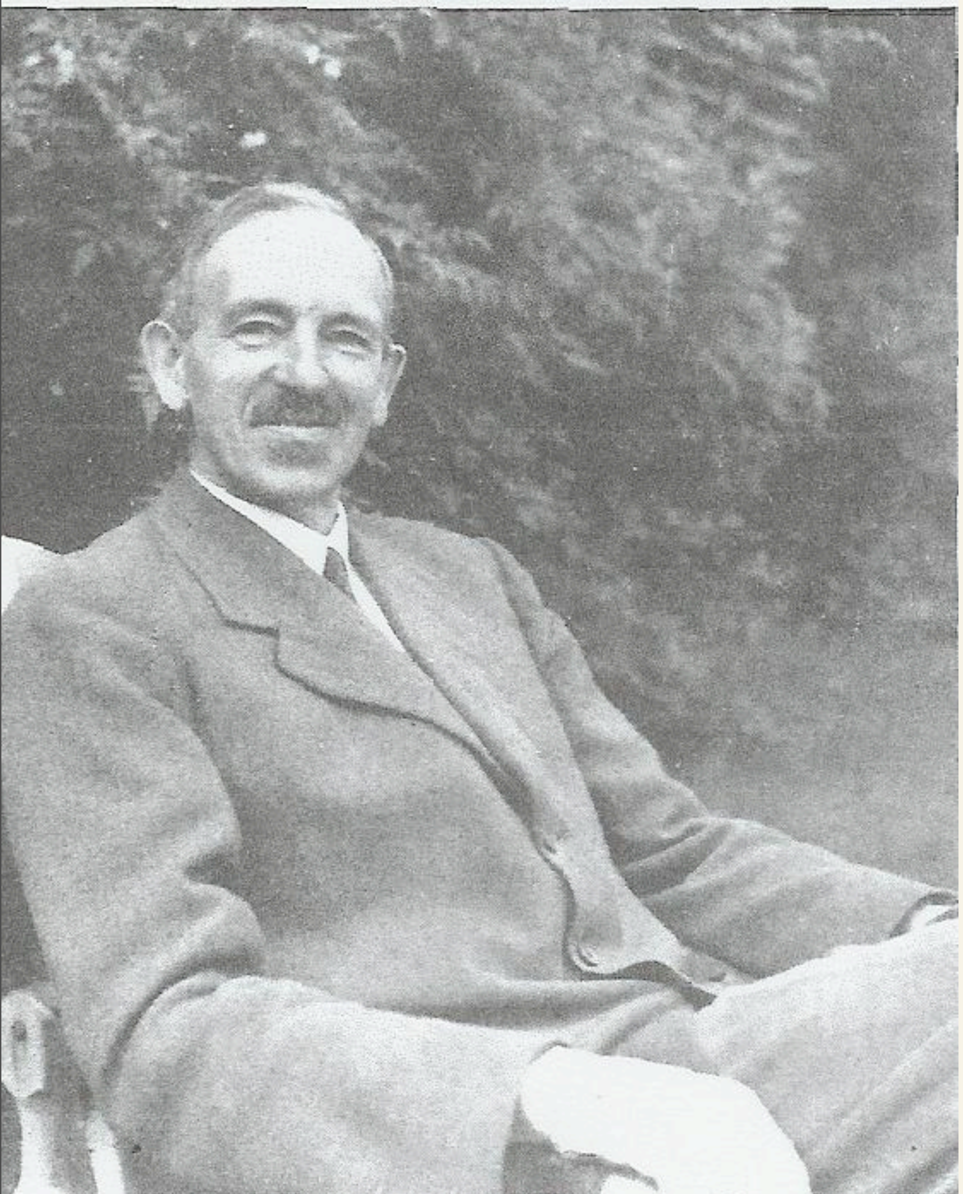
Wilhelm Anderson

1880-1940

``About the limiting density of matter and energy”
Zeitschrift fur Physik 56 (1929) 851-856

Wilhem Anderson, a privatdozent at Tartu University, Estonia, who read Stoner's paper noticed that for the mass of a white dwarf comparable to the mass of the Sun the density calculated from Stoner's non-relativistic relation implied that the electrons are actually moving with velocities comparable to the velocity of light

Zeitschrift fur Physik 56 (1929) 851-87



RALPH HOWARD FOWLER

Ralph Fowler

1889-1947

``On dense Matter”

Monthly Notices of the Royal Astronomical Societ

87, 115-122 (1926)

``It may be accepted now as certain that classical statistical mechanics is not applicable at extreme densities, even to ideal material composed of extensionless mass points and that the form used here is fairly certain the correct substitute”

Landau in 1936

Lev Landau

1908-1968

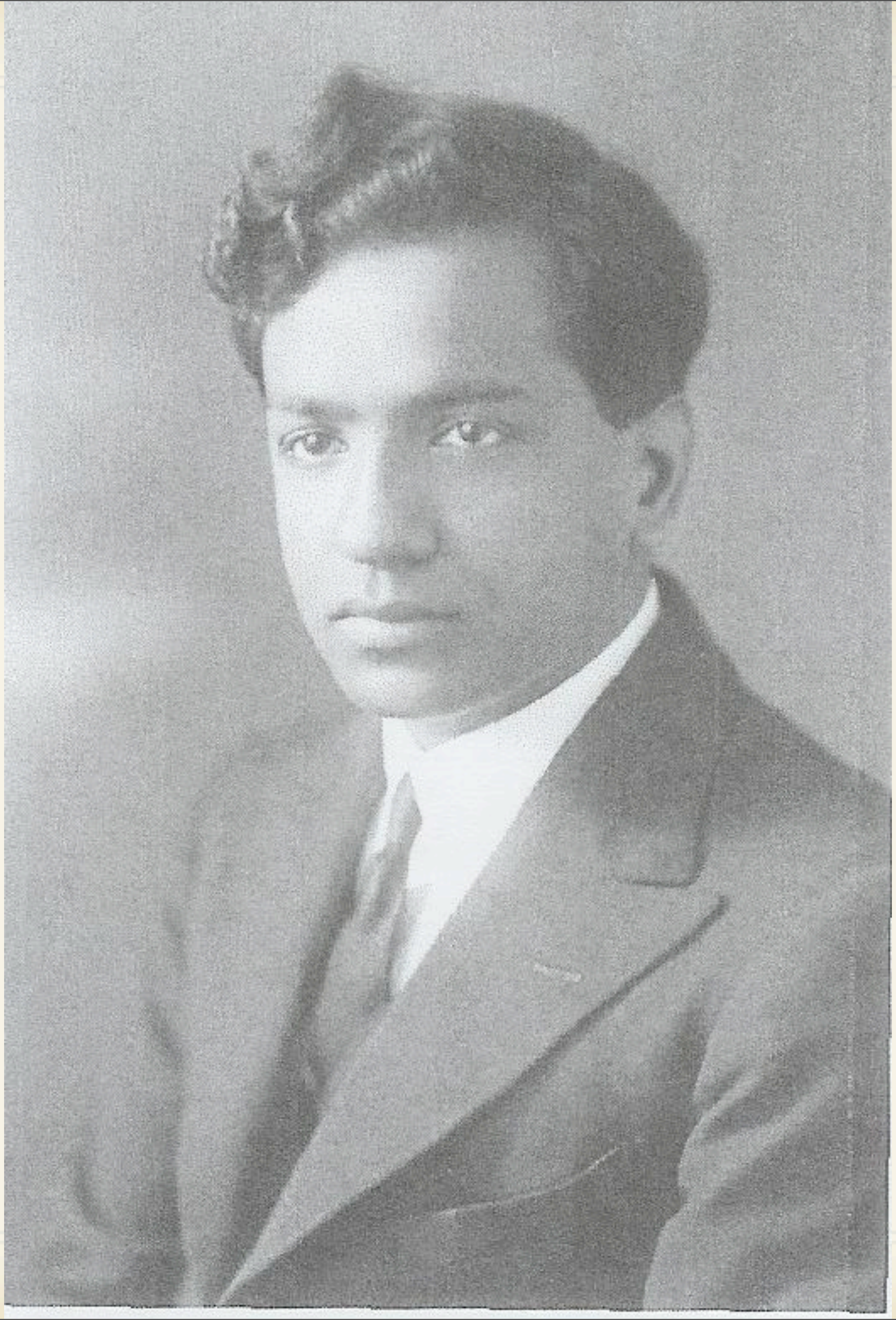
“On the theory of stars”, Physikalische Zeitschrift der Sowjetunion**, 1 (1932), 285–288.**

admitted in Milne's calculations) would never make such condensations possible.

As the velocities of electrons in the Fermi-distribution rise with the density we have to apply, for sufficiently great densities, the relativistic theory. In the extreme-relativistic case the inner free energy per unit volume varies as $\rho^{1/3}$, i. e. the same power of density as the gravitational energy. The free energy F is therefore of the form $F = a\rho^{1/3}$. If a is positive the system will expand in order to have F minimum, until the density becomes too small for the extreme-relativistic relation $F = a\rho^{1/3}$ to be valid. If a is negative the system will have a tendency to collapse to a point. In order to find the criterion separating these two cases we have to investigate the solution of the general equation

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{d\mu}{dr}\right] = -4\pi G\rho \quad (1)$$

L. Landau, Physikalische Zeitschrift der Sowjetunuion Feb . 1932



S. Chandrasekhar

1910-1995

“ The density of white dwarfs”, Philosophical magazine, 11 (1931), 592–597.

``T\he maximum mass of ideal white dwarfs’’, Astrophysical journal 74 (1931) 81–82.

Chandrasekhar, “Philosophical Magazine 11, 592-596 (1931)

Summary.

The density of the white dwarf stars is reconsidered from the point of view of the theory of the polytropic gas spheres, and gives for the *mean density* of a white dwarf (under ideal conditions) the formula

$$\rho = 2{\cdot}162 \times 10^{6} \times (M/\odot)^{2}.$$

The above formula is derived on considerations which are a much nearer approximation to the conditions *actually existent* in a white dwarf than the previous calculations of Stoner based on uniform density distribution in the star and which gave for the limiting density the formula

$$\rho = 3{\cdot}977 \times 10^{6} \times (M/\odot)^{2}.$$

As far my paper I had nearly completed, writing it out, a paper by a German, Wilhem Anderson, appeared discussing the same problem. Even mathematically his treatment was identical to mine. So the satisfaction is that I was able to do it independently. I do not intend sending it for publication.

Chandrasekhar letter to his father August 30, 1929

“… at first I didn’t understand what this limit meant and I didn’t know how it would end, and how it related to the 3/2 low mass polytropes. But all that I did when I was in England and wrote my second paper on it “

Chandrasekhar interview by S. Weart in 1977

31st March 1931

Dear Dr. Stoner,

I am at present engaged in doing some work on Prof. Milne's new theory of Stellar Structure and your two papers on the density of white-dwarfs in The Phil. Mag are therefore of great value to me. I am not sure if you have reprints of them still left with you. If you have, I should be very much obliged to you if I can have a copy of each

Thanking you very much in anticipation

Yours Sincerely

S Chandrasekhar

Chandrasekhar, " The Maximum Mass of Ideal White Dwarfs",Astrophysical Journal 74 , (1931) 81-82

Eddington, Eq.57.3

$$\left(\frac{GM}{M'}\right)^2 = \frac{(4K)^3}{4\pi G},$$

$p = K\varrho^{4/3}$, $K \sim hc/m^{4/3}$

$M \sim m(hc/Gm^2)^{3/2}$

or

$$\boxed{\begin{aligned} M &= 1.822 \times 10^{33}, \\ &= .91 \odot \text{(nearly)}. \end{aligned}$$

As we have derived this mass for the star under ideal conditions of extreme degeneracy, we can regard 1.822×10^{33} as the maximum mass of an ideal white dwarf. This can be compared with the earlier estimate of Stoner[2]

$$\boxed{M_{max} = 2.2 \times 10^{33},} \tag{6}$$

based again on uniform density distribution. The "agreement" between the accurate working out, based on the theory of the polytropes, and the cruder form of the theory is rather surprising in view of the fact that in the corresponding non-relativistic case the deviations were rather serious.

TRINITY COLLEGE
CAMBRIDGE
November 12, 1930

[1] A. S. Eddington, *Internal Constitution of Stars*, p. 83, eq. (57.3.)

[2] *Philosophical Magazine*, 9, 944, 1930.

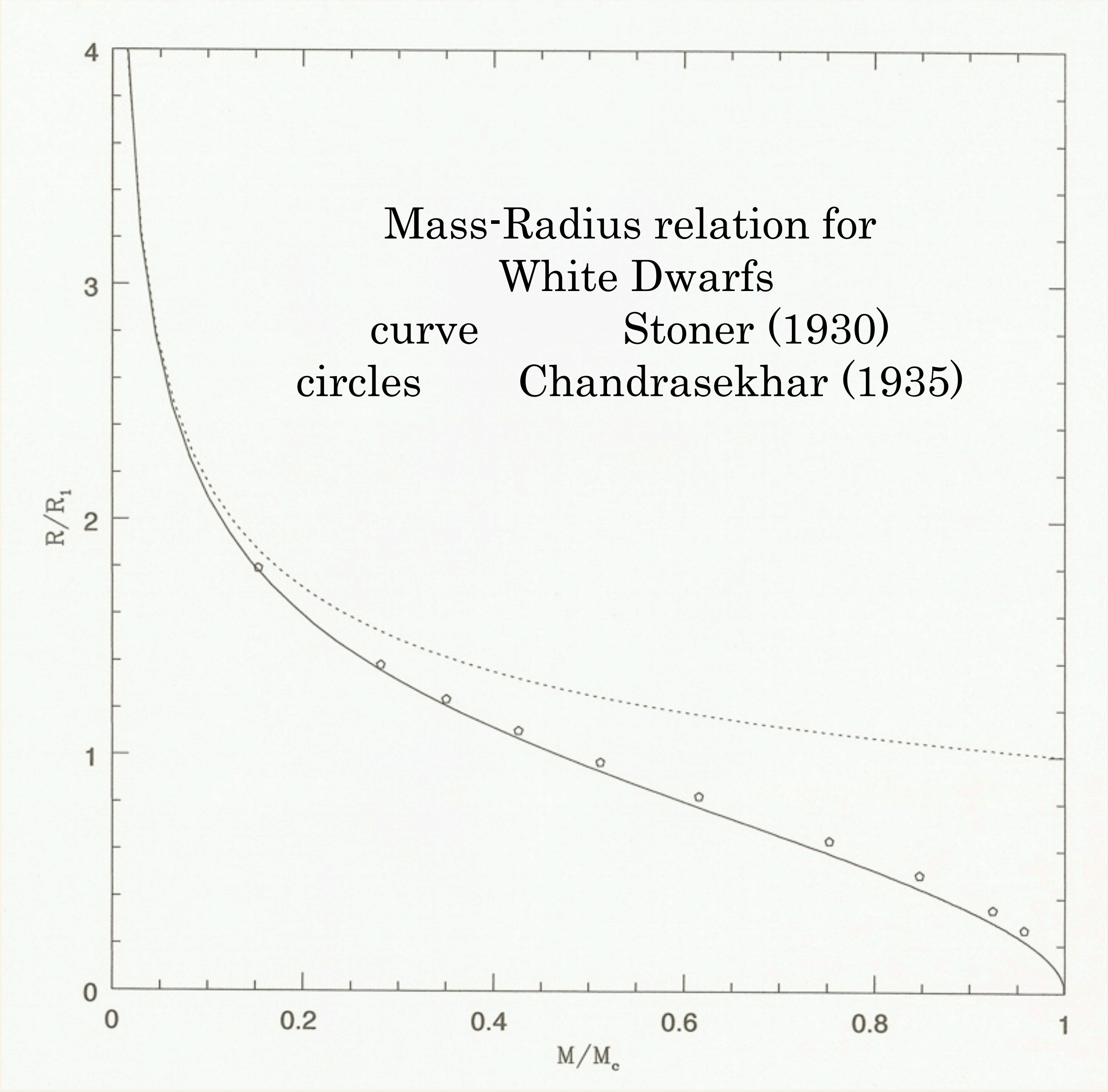
Edward A. Milne

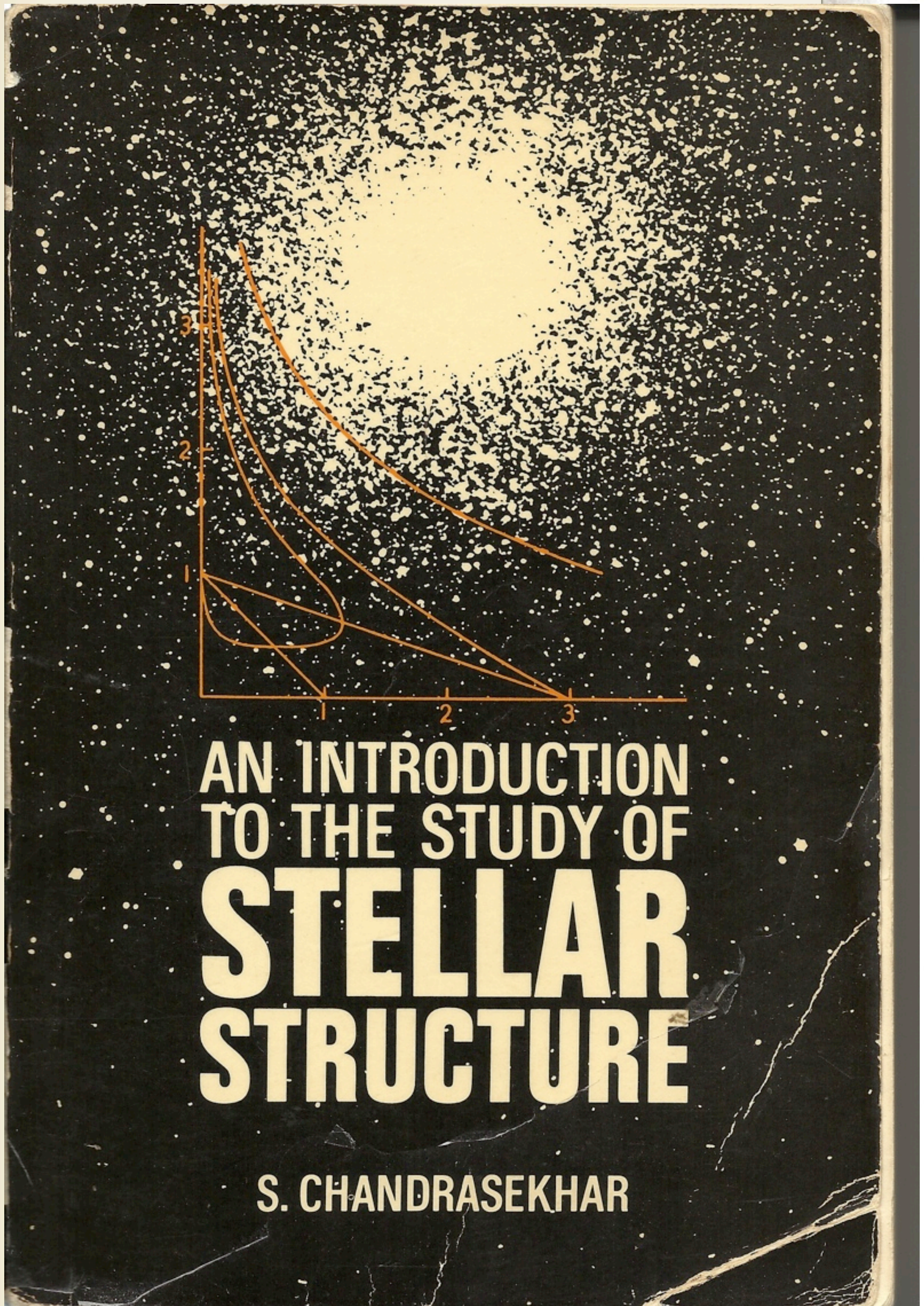
1896-1950

I notice of course that dimensionally our limiting formulas are identical. Any physical principle I suppose yields density proportional to (G/K) cubed times M squared [where the degenerate electron pressure is K times the density to the power (5/3), G is Newton's constant and M is the mass of the star]

But I tried in vain [to understand the relation between] the principle behind your formula with that behind mine

Letter from Milne to Stoner, Jan 24, 1931


Mass-Radius relation for
White Dwarfs
curve Stoner (1930)
circles Chandrasekhar (1935)
R/R1
M/Mc
0
1
2
3
4
0
0.2
0.4
0.6
0.8
1



slightly larger than the radius of the earth, while the mass $0.957M_3$ has a radius considerably less than the radius of the earth. In Fig-

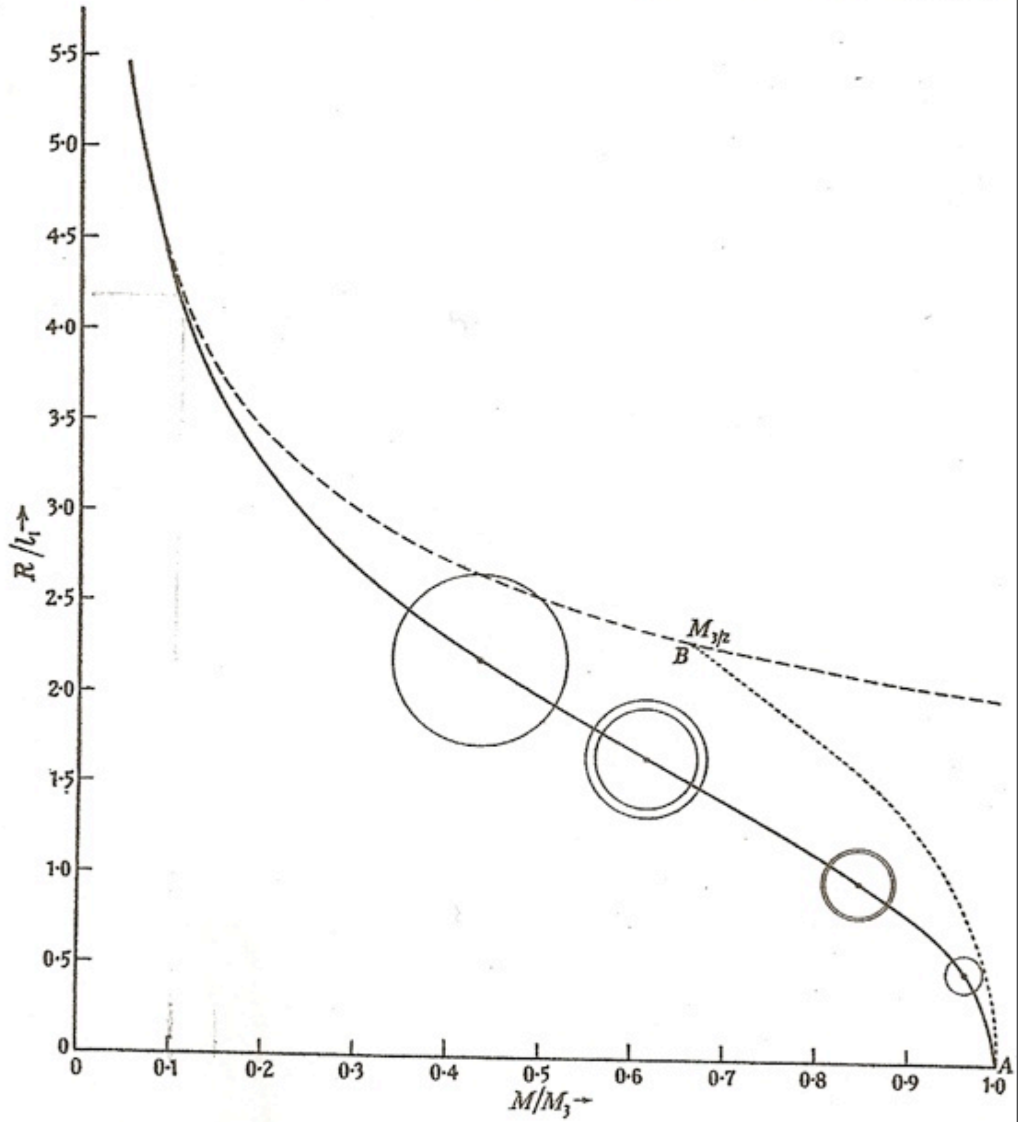


FIG. 31.—The solid-line curve represents the exact (mass, radius) relation for the completely degenerate configurations. This curve tends asymptotically to the dotted curve as $M \to 0$.

ures 31 and 32 we have illustrated the mass-radius and the mass-central density relationships. The dotted curves in the two cases are the corresponding relations based on the Lane-Emden polytrope

THE ASTROPHYSICAL JOURNAL, 175:417–430, 1972 July 15

ANALYTIC APPROXIMATIONS TO THE MASS-RADIUS RELATION AND ENERGY OF ZERO-TEMPERATURE STARS

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Received 1971 November 5

ABSTRACT

The variational principle is applied to two approximate forms of the energy integral of a star at zero temperature in order to obtain analytic expressions for the mass-radius relation and for the energy. Several well-known equations of state are considered, and the effects of rotation and of stored magnetic fields are included. Comparison of the analytic approximations with published numerical solutions of the exact equations show good agreement.

I. INTRODUCTION

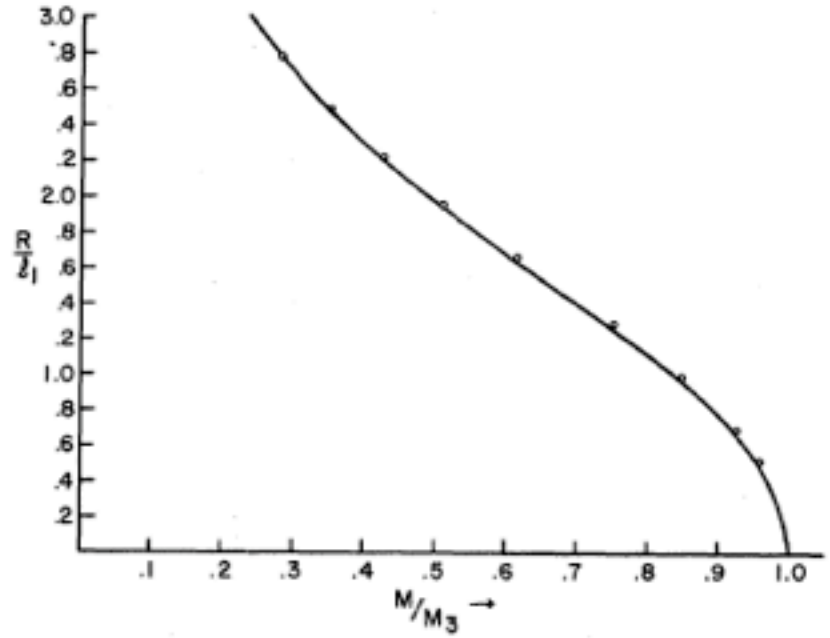

FIG. 1.—Mass radius relation obtained from equations (27) and (28). The circles correspond to the numerical results obtained by Chandrasekhar (1938): $l_1/R_\odot = .0112/\mu$, $M_3/M_\odot = 5.82/\mu^2$.

where $\gamma = y_1[-(d\theta_3/dy_1)/d\psi/dy)^{1/2}$, ψ is the perturbation function introduced by Reiz (1949) evaluated at the first zero of θ_3, $y_1 = 6.897$, and $l_1 = (\frac{3}{4}\pi)^{1/2}(\hbar/m_e c)(\hbar c/Gm^2)^{1/2} = (7.769/\mu) \times 10^8$ cm. The first term in equation (29) has the same analytic form as our

THE ASTROPHYSICAL JOURNAL, 179:277–287, 1973 January 1

DETERMINATION OF PROPERTIES OF COLD STARS IN GENERAL RELATIVITY BY A VARIATIONAL METHOD

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AND
GEORGE CHAPLINE, JR.
Lawrence Livermore Laboratory, California
Received 1972 April 19

ABSTRACT

Approximate analytic formulae are obtained for the mass, the number of baryons, and the radius of a cold star by applying the energy variational principle in general relativity to a uniform-density sphere of baryons. Conditions for equilibrium and for stability are expressed by transcendental equations which generalize familiar results of Newtonian theory. These formulae allow one to understand readily how various assumptions concerning the equation of state affect the properties of a neutron star. Comparison with some numerical calculations in the literature based on the differential equation for hydrostatic equilibrium in general relativity indicates that our analytic approximations are in reasonable quantitative agreement with exact results. By making use of the condition that the speed of sound is less than the speed of light, and of properties of neutron matter near nuclear densities, we derive an upper limit to the maximum mass of a stable neutron star.

Subject headings: interiors, stellar — neutron stars — relativity

Reproduced in Shapiro and Teukolsky, "Black Holes, White Dwarfs and Neutron Stars" pp, 259-264

Chandrasekhar. Nobel speech 1983

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17. For an account of these matters pertaining to the classical ellipsoids see Chandrasekhar, S *Ellipsoidal Figures of Equilibrium* (Yale University Press, New Haven, 1968).
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22. The author's investigations on the mathematical theory of black holes, continued over tl years 1974- 1983, are summarized in his last book *The Mathematica/ Theory of Black Hol* (Clarendon Press, Oxford, 1983).

The reader may wish to consult the following additional references:

1. Chandrasekhar, S., 'Edward Arthur Milnc: his part in the development of modern astroph sics,' *Quuart.J. Roy. Astr. Soc., 21,* 93-107 (1980).
2. Chandrasekhar, *S., Eddington; The Most Distinguished Astrophysicist of His Time* (Cambridg University Press, 1983).

If you want to find anything from the theoretical physicist about the method they use, I advise you not to listen to their words, fix your attention on their deeds

Albert Einstein