Gravitational lensing by black holes and their alternatives

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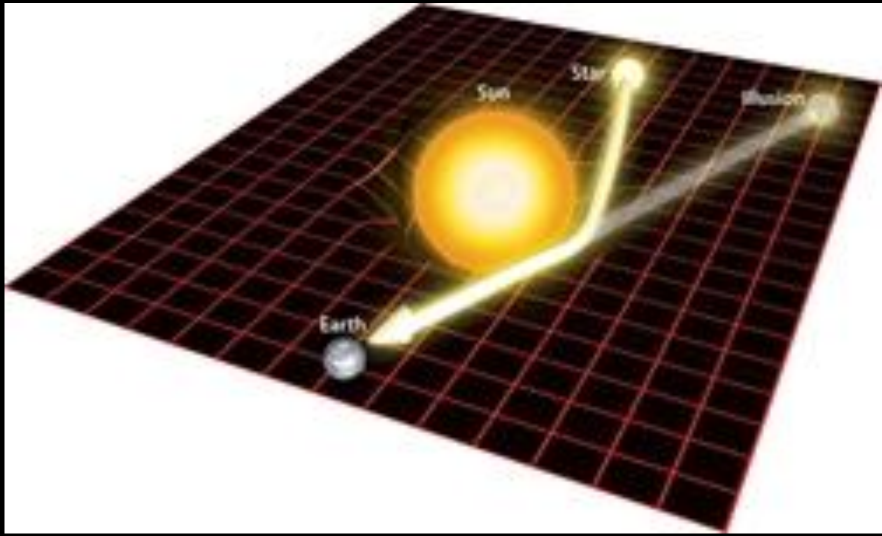
Summary

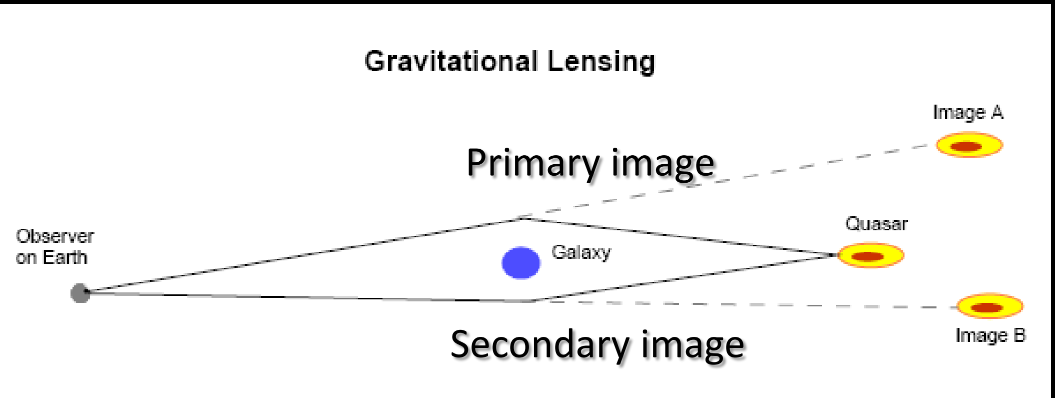
- Strong deflection limit of gravitational lensing
    - Caustics and number of images in Kerr lensing
        - Observing gravitational lensing by Sgr A* with VLTI
            - Alternatives to Schwarzschild in the weak field limit

Gravitational lensing

The gravitational field bends the trajectories of light rays:



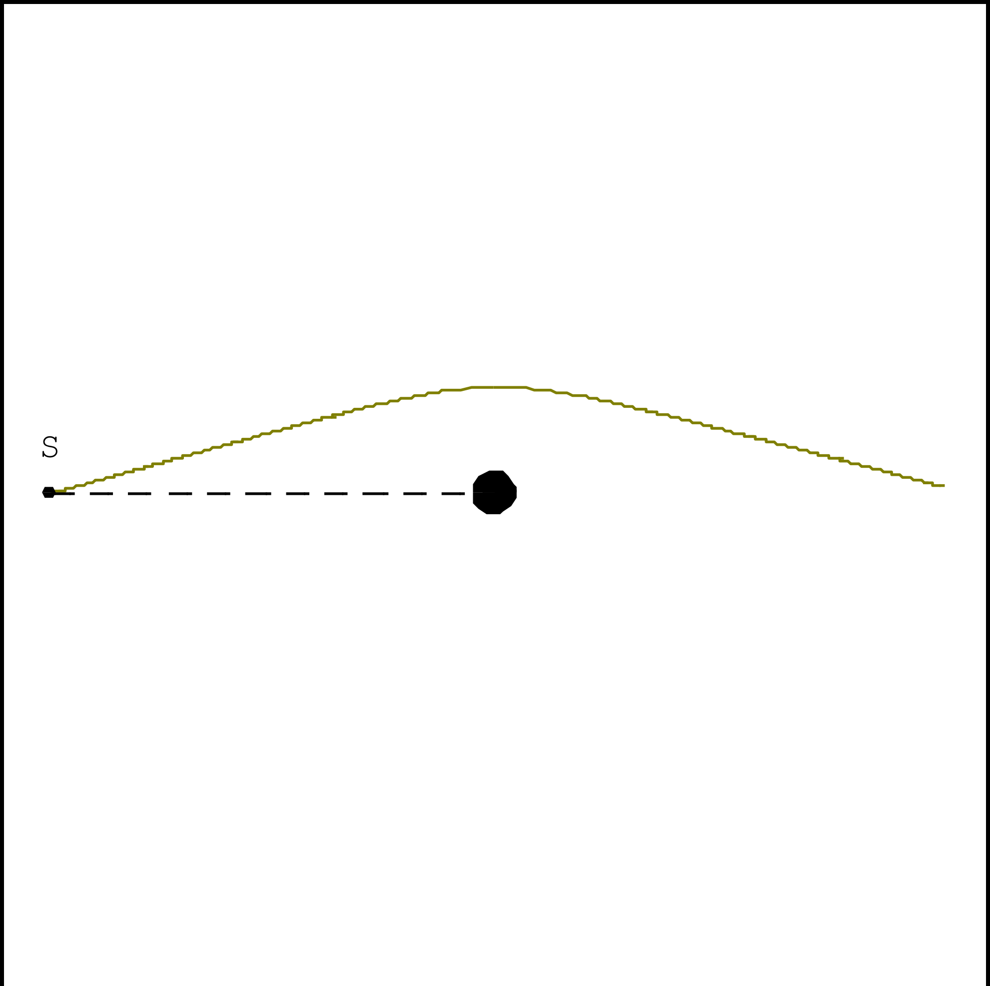
With a good alignment between source, lens and observer, two images can be observed.



Einstein calculated the deflection by integrating null geodesics in Schwarzschild metric and taking the weak field limit.

Which distortions would be induced by a black hole?

Gravitational lensing by Black Holes

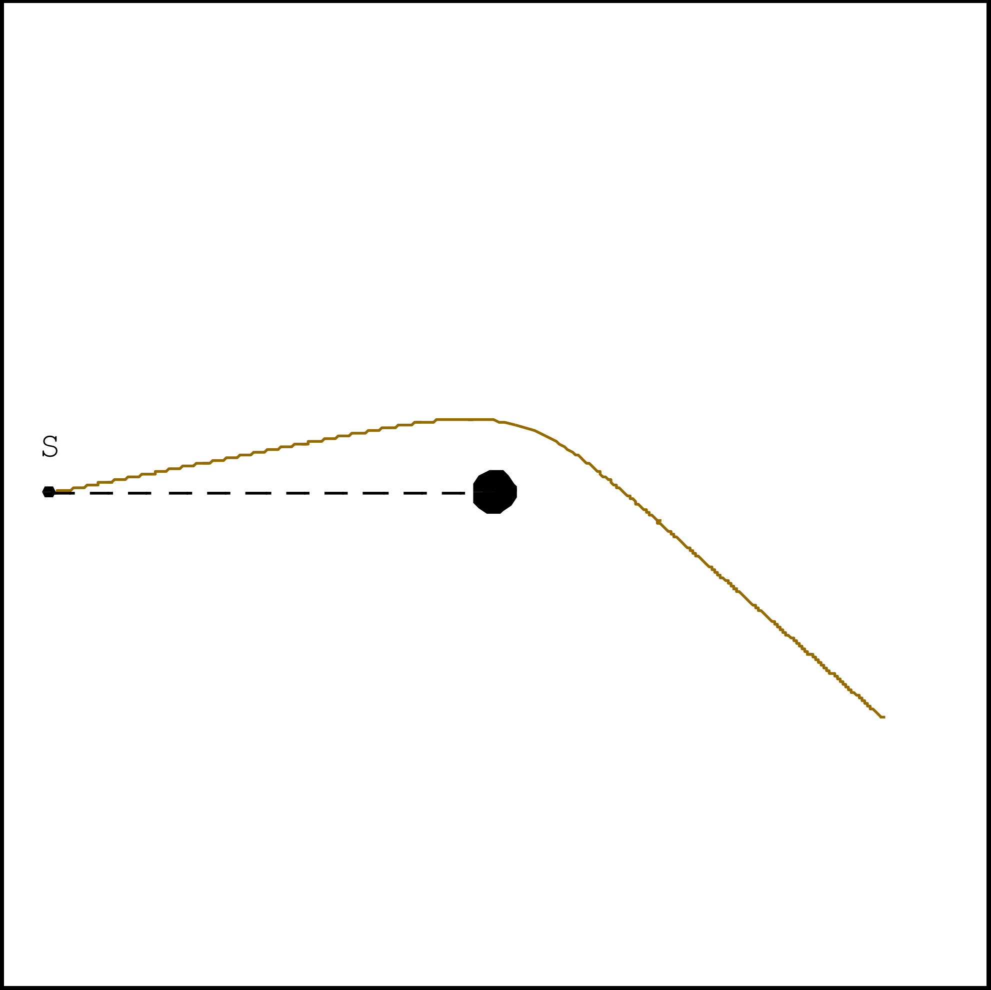


Light rays passing far away from the black hole are only very slightly deflected.

$$\alpha(\theta) \sim \frac{4MG}{D_{OL}\theta}$$

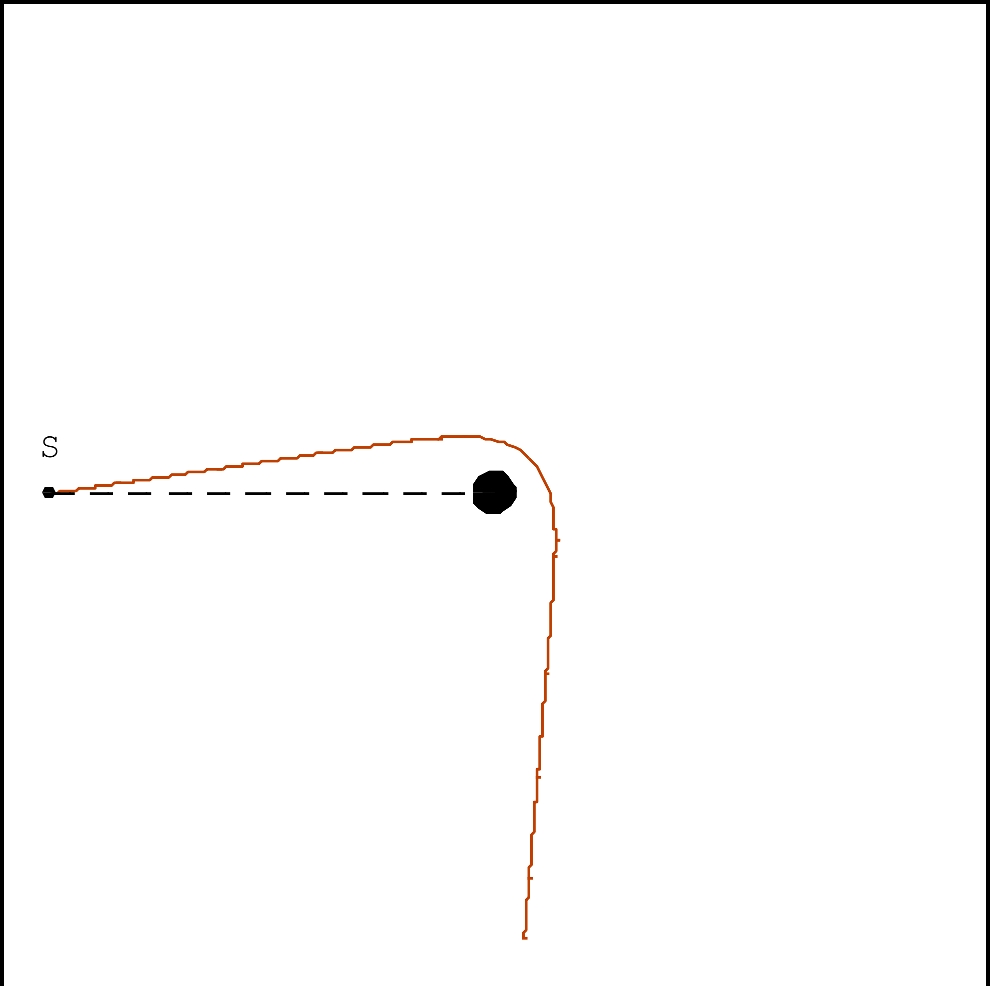
Einstein's weak deflection formula holds.

Gravitational lensing by Black Holes



As the impact parameter decreases, the deflection grows more and more.

Gravitational lensing by Black Holes



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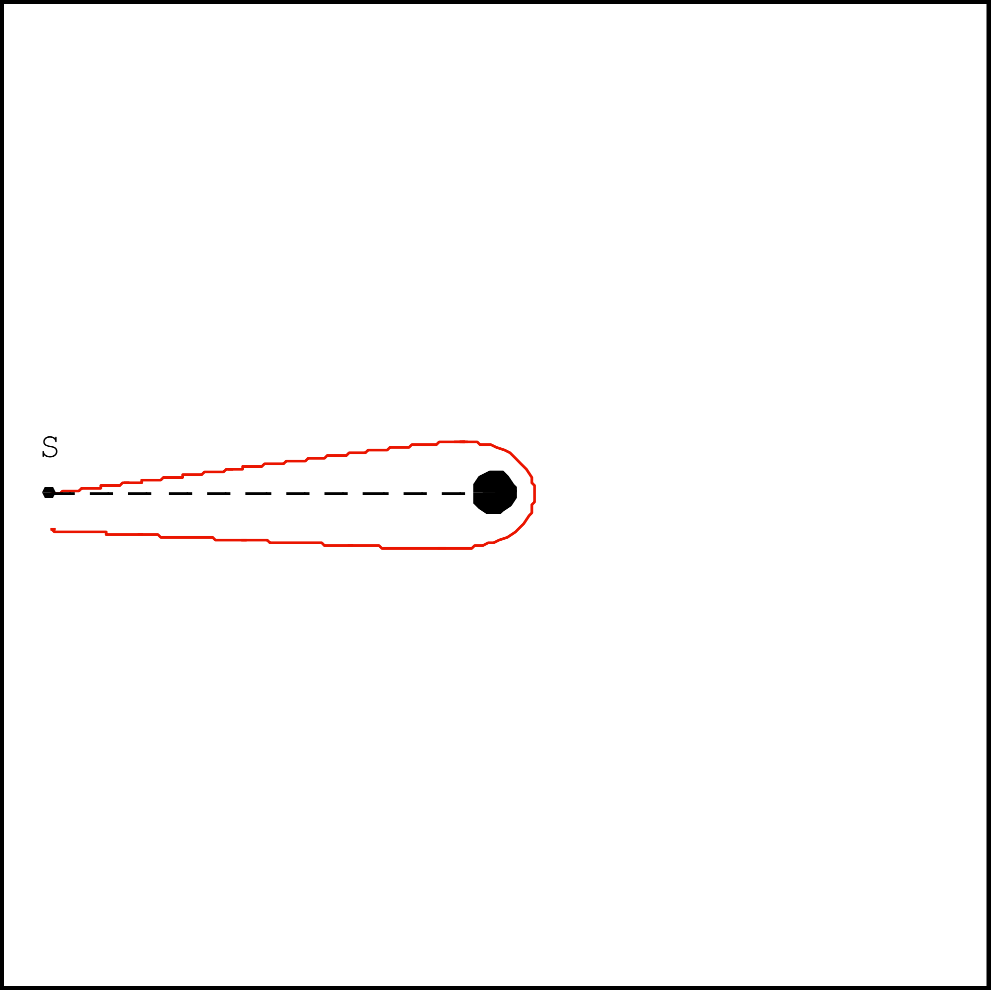
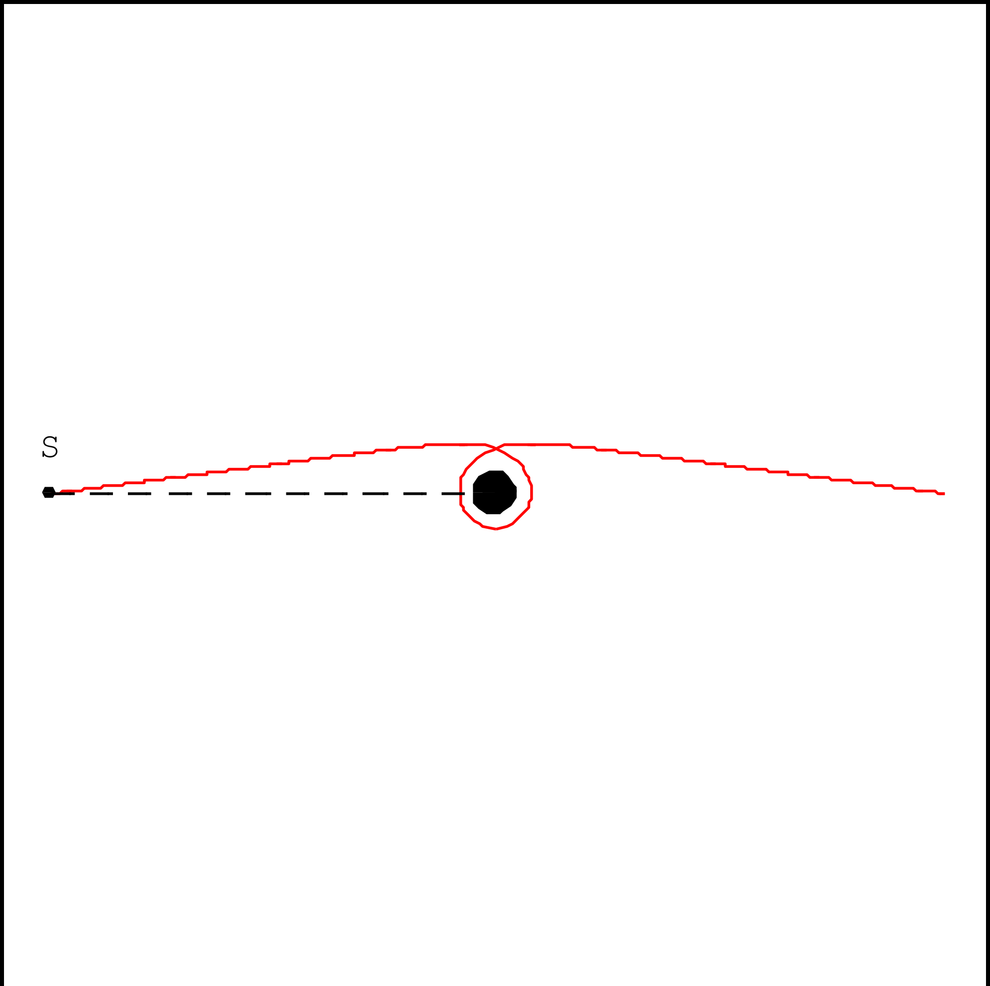
Gravitational lensing by Black Holes



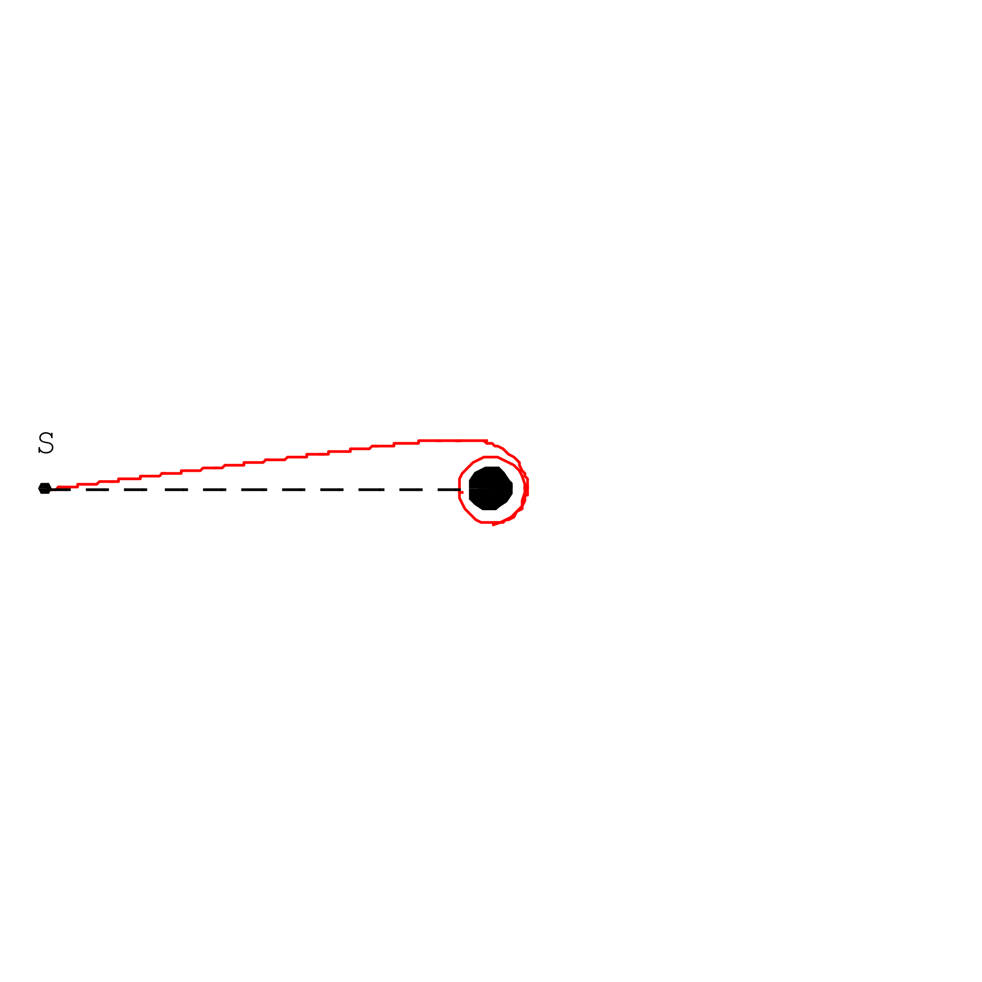
A light ray may even be back-scattered *(retro-lensing)*…

Gravitational lensing by Black Holes



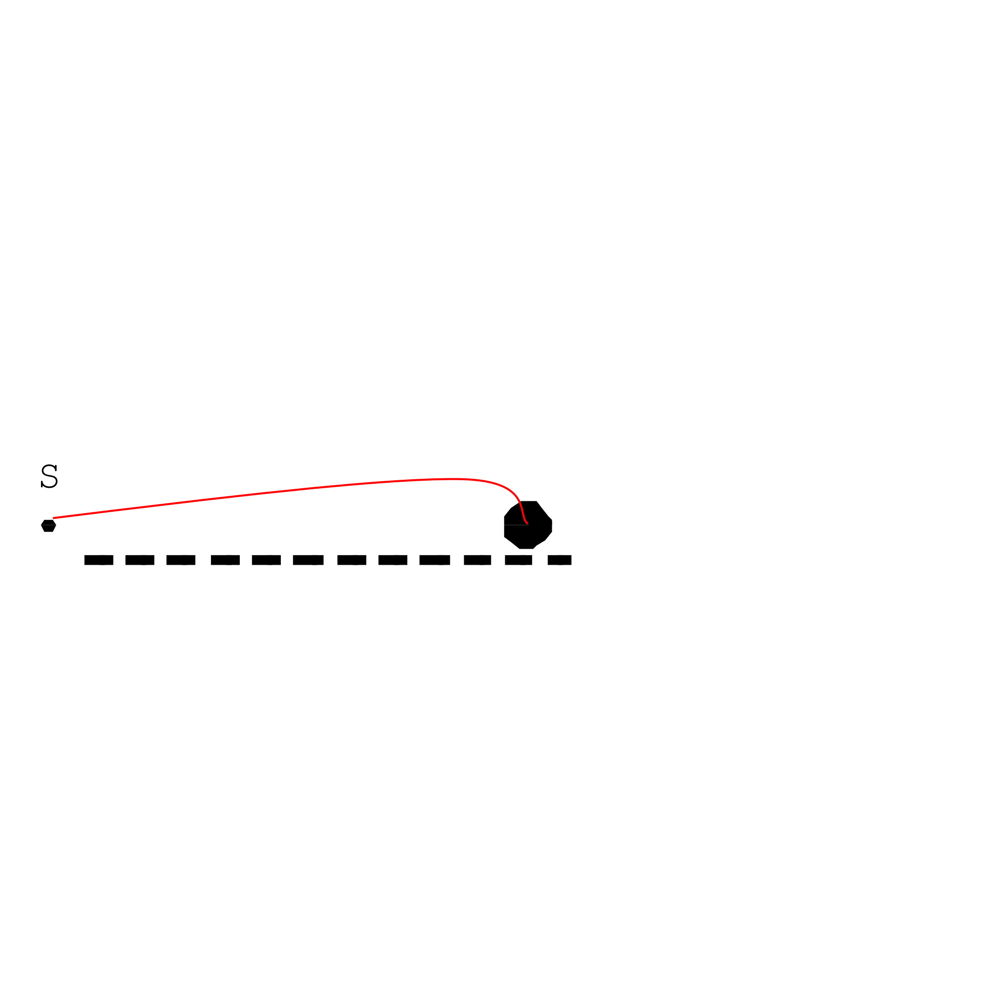
… or perform one or more loops around the black hole!

Gravitational lensing by Black Holes



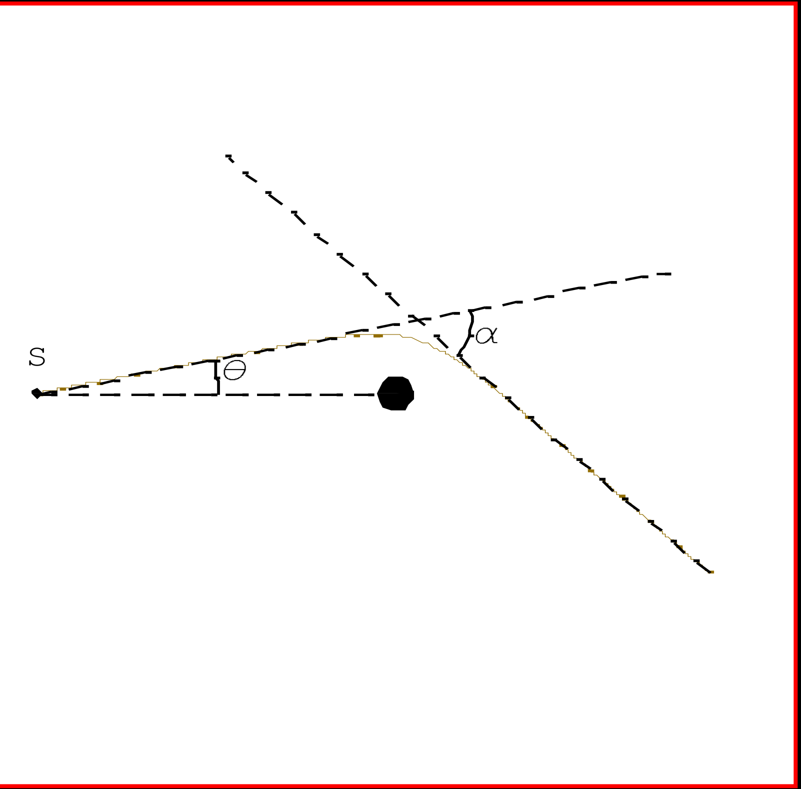
When the impact angle reaches a minimum value θ_m, the photon is injected on a circular orbit around the black hole.

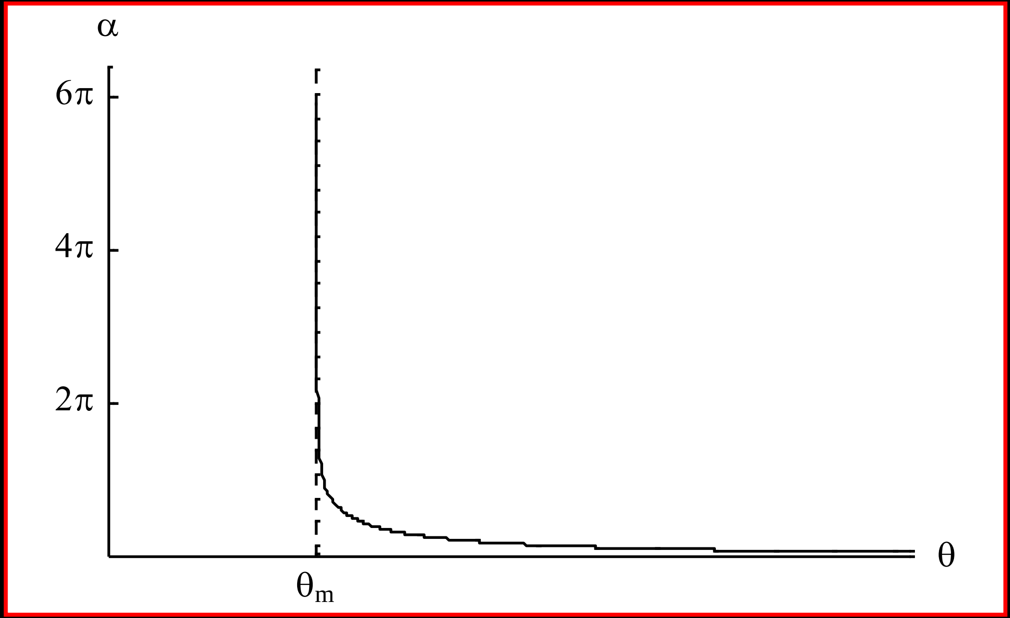
Gravitational lensing by Black Holes



For impact angles smaller than θ_m, the light crosses the event horizon and is eaten by the black hole.

Deflection Angle

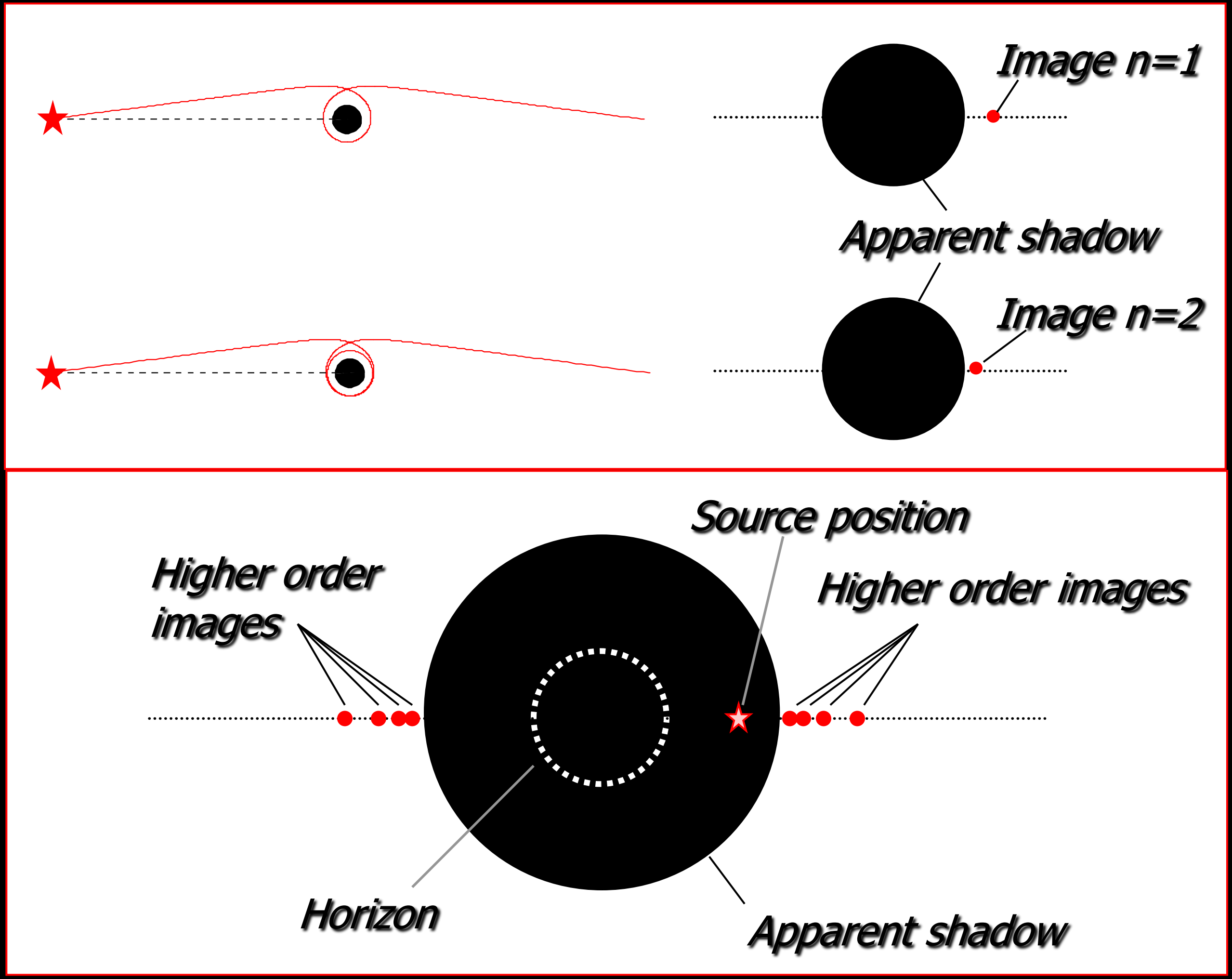




The deflection angle diverges as the impact angle θ tends to θ_m.

No gravitational lensing images can be found within θ_m (Shadow of the black hole)

Higher order images



Light rays winding once form a first order image.
Light rays winding twice form a second order image.

Two infinite sequences of images are formed for any given source.

(Darwin 1959)

Exact Deflection Angle

Schwarzschild deflection angle can be expressed as an elliptic integral *(Darwin 1959)*

$$\alpha(x_0) = -\pi + F(\phi_0,\lambda)G(x_0)$$

$$F(\phi_0,\lambda) = \int_0^{\phi_0}\left(1-\lambda\sin^2\phi\right)^{-1/2}\mathrm{d}\phi$$

$$G(x_0) = \sqrt{\frac{8x_0\left(-3+x_0+\sqrt{x_0^2+2x_0-3}\right)}{2x_0-3}}$$

$$\lambda(x_0) = \frac{3-x_0-\sqrt{x_0^2+2x_0-3}}{3-x_0+\sqrt{x_0^2+2x_0-3}}$$

$$\phi_0(x_0) = \sqrt{\frac{-3+x_0-\sqrt{x_0^2+2x_0-3}}{2(2x_0-3)}}$$

For large values of the impact angle θ, we recover the weak deflection limit calculated by Einstein

$$\alpha(\theta) \rightarrow \frac{4MG}{D_{OL}\theta}$$

Strong Deflection Limit

For small values of the impact angle, Darwin (1959) proposed the approximate form

$$\alpha(\theta) = -a \log\left(\frac{\theta}{\theta_m} - 1\right) + b + O(\theta - \theta_m)$$

with $$D_{OL}\theta_m = 3\sqrt{3}M, \quad a = 1, \quad b = \log\left[216\left(7 - 4\sqrt{3}\right)\right] - \pi$$

Is the Strong Deflection Limit specific of Schwarzschild or universal?

Strong Deflection Limit

- It can be proved that the divergence in the deflection angle for $\theta \to \theta_m$ is always logarithmic for any kinds of black hole metrics *(V. Bozza, Phys. Rev. D 66 (2002) 103001)*.
- Therefore, one can write a universal formula for gravitational lensing by spherically symmetric black holes in the Strong Deflection Limit:

$$\alpha(\theta) = -a \log\left(\frac{\theta}{\theta_m} - 1\right) + b + O(\theta - \theta_m)$$

- The coefficients θ_m, *a*, *b* are functions of the black hole metric.
- The position and the magnification of the images depend on these coefficients.

$$\theta = \theta_m \left(1 + e^{\frac{b-\gamma}{a}}\right)$$

- Through the observation of the higher order images, it is possible to identify the class of the black hole and its properties.

Applications

Black holes investigated by the SDL method (updated to 2013)

- Reissner - Nordström BH *(Eiroa, Romero & Torres 2002, Bozza 2002)*
- Janis, Newman, Winicour BH *(Bozza 2002)*
- GMGHS charged BH in string theory (*Bhadra 2003)*
- Braneworld BH *(Eiroa 2004; Whisker 2005; Majumdar & Mukherjee 2005; Bin-Nun 2010; Eiroa & Sendra 2012)*
- Wormholes *(Tejeiro & Larranaga 2005; Nandi et al. 2006; Rahaman et al. 2007)*
- Brans-Dicke BH *(Sarkar & Bhadra 2006)*
- Born-Infeld BH *(Eiroa 2006)*
- Horava-Lifshitz *(Chen & Jing 2009)*
- Dilaton Anti-de-Sitter *(Ghosh & Sengupta 2010)*
- Chern-Simons BH *(Chen et al. 2011)*
- Global monopole *(Cheng et al. 2011)*
- Regularized BH *(Eiroa & Sendra 2011)*
- Non-Commutative BH *(Ding et al. 2011)*
- Kaluza-Klein BH *(Liu et al. 2010; Chen et al. 2011; Sadeghi et al. 2013)*
- Phantom BH *(Gyulchev & Stefanov 2013)*

Strong Deflection Limit

- Contribution to Sgr A* microlensing *(Petters 2003)*
- Retrolensing by Sgr A* *(Eiroa & Torres 2004)*
- Time delay *(Bozza & Mancini 2004)*
- Extension to arbitrary distances *(Bozza & Scarpetta 2007)*
- Second order expansion *(Iyer & Petters 2007)*

From Schwarzschild to Kerr

The Kerr geodesics equations can be put in integral form *(Carter 1968)*

$$\pm\int\frac{dx}{\sqrt{R}}=\pm\int\frac{d\vartheta}{\sqrt{\Theta}}$$

$$\phi_f-\phi_0=a\int\frac{x^2+a^2-aJ}{\Delta\sqrt{R}}dx-a\int\frac{dx}{\sqrt{R}}+J\int\frac{\csc^2\vartheta}{\sqrt{\Theta}}d\vartheta$$

They can be analytically solved in terms of elliptic integrals *(Rauch & Blandford 1994)*

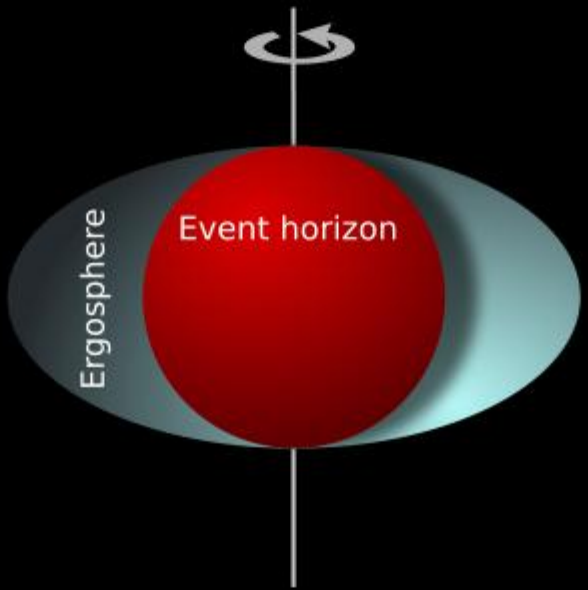
$$\Theta=Q+a^2\cos^2\vartheta-J^2\cot^2\vartheta$$

$$R=x^4+\left(a^2-J^2-Q\right)x^2+\left[Q+(J-a)^2\right]x-a^2Q$$

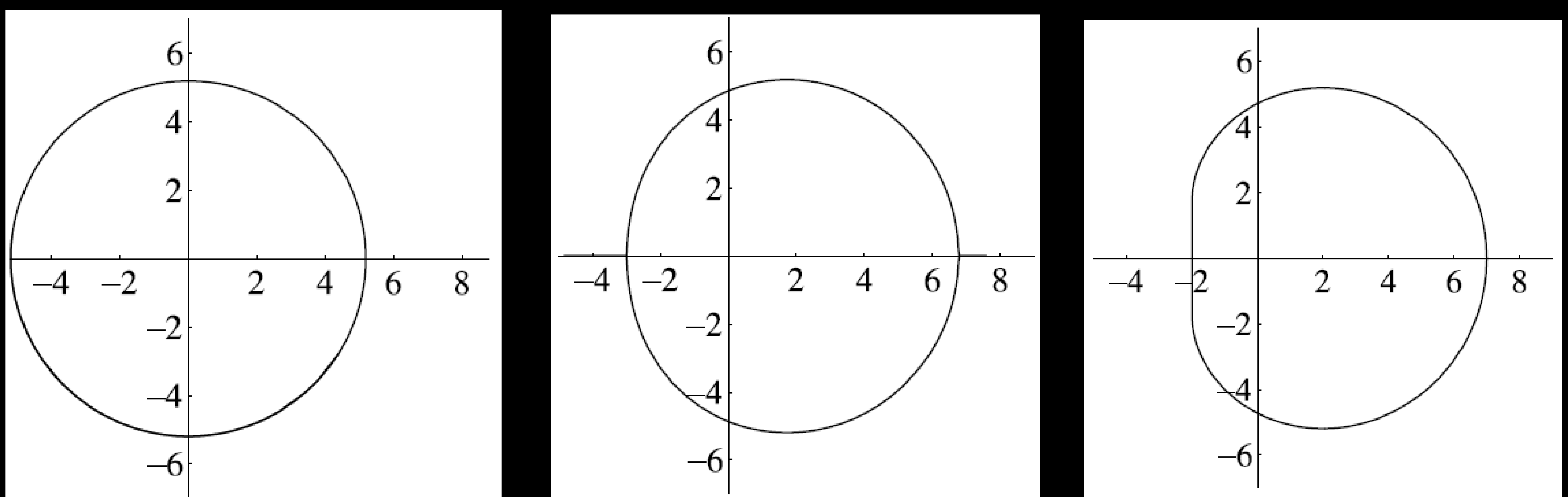
- Many many numerical and semianalytical codes solving Kerr geodesics! *(Dovčiak et al. 2004)*

- What is the phenomenology of higher order images and caustics?
- What information can we obtain by analytical approximations?

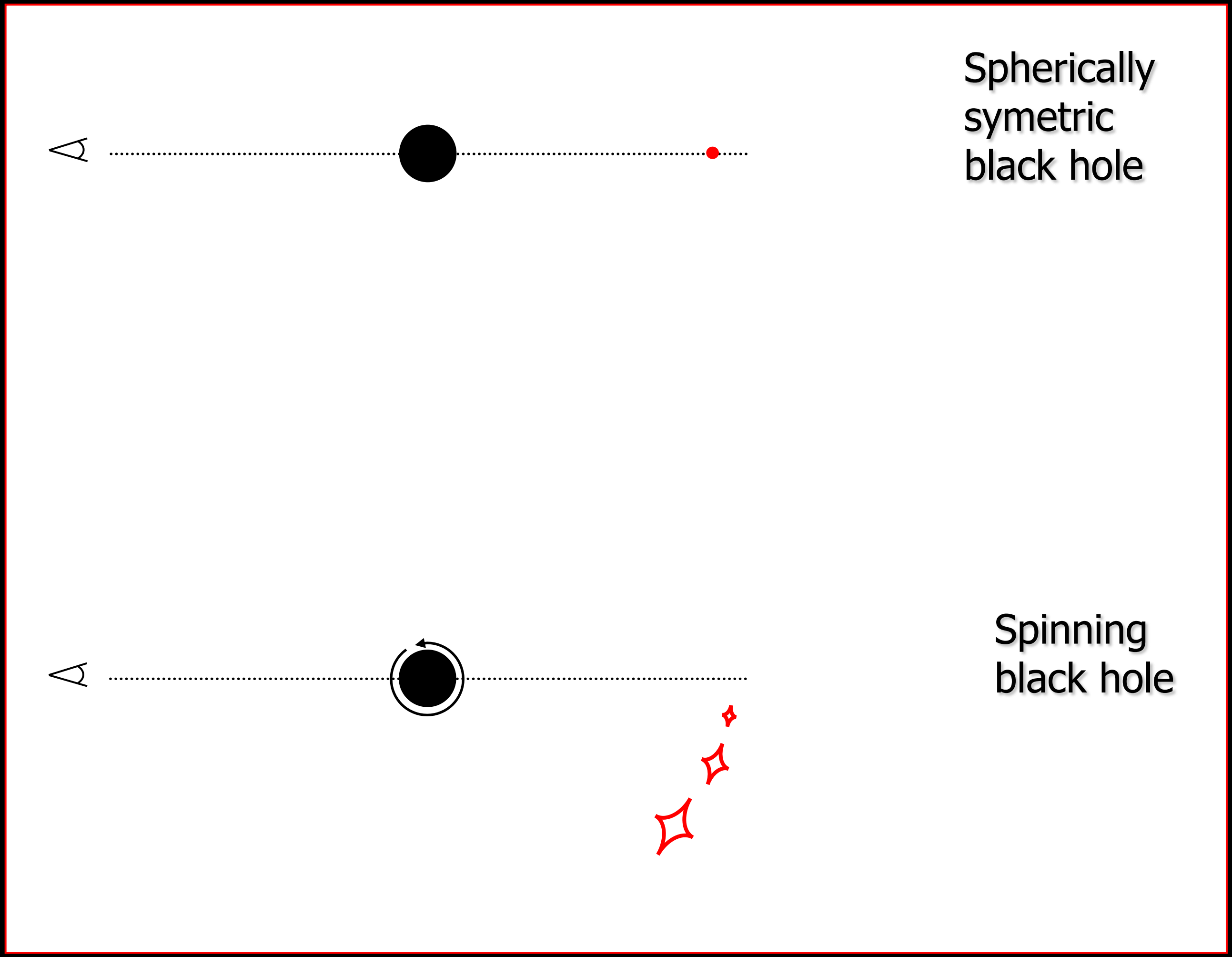
Shadow of a spinning black hole



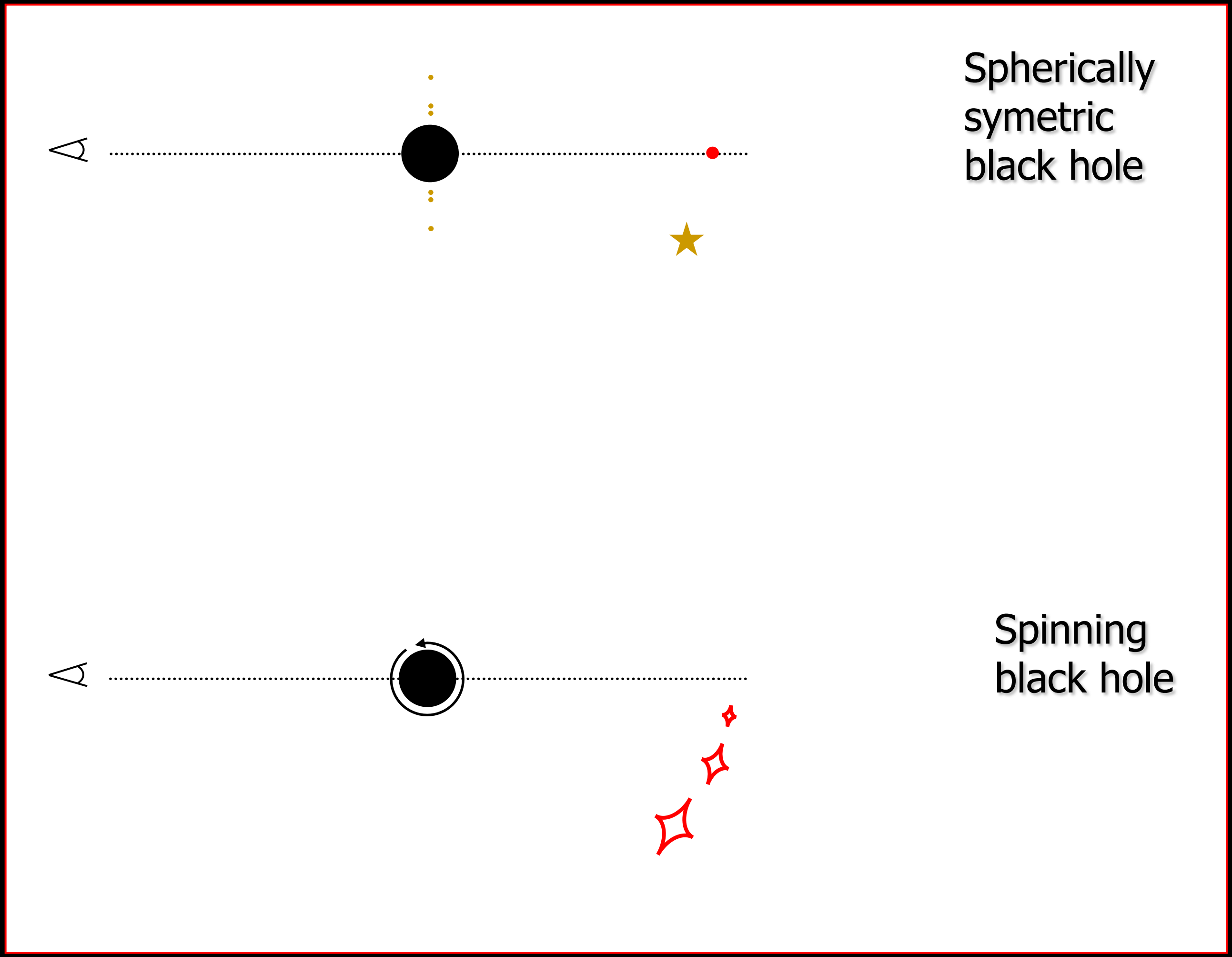
Spinning black holes are distinguished by an asymmetric shape of the shadow.
(Bardeen 1972)



From Schwarzschild to Kerr



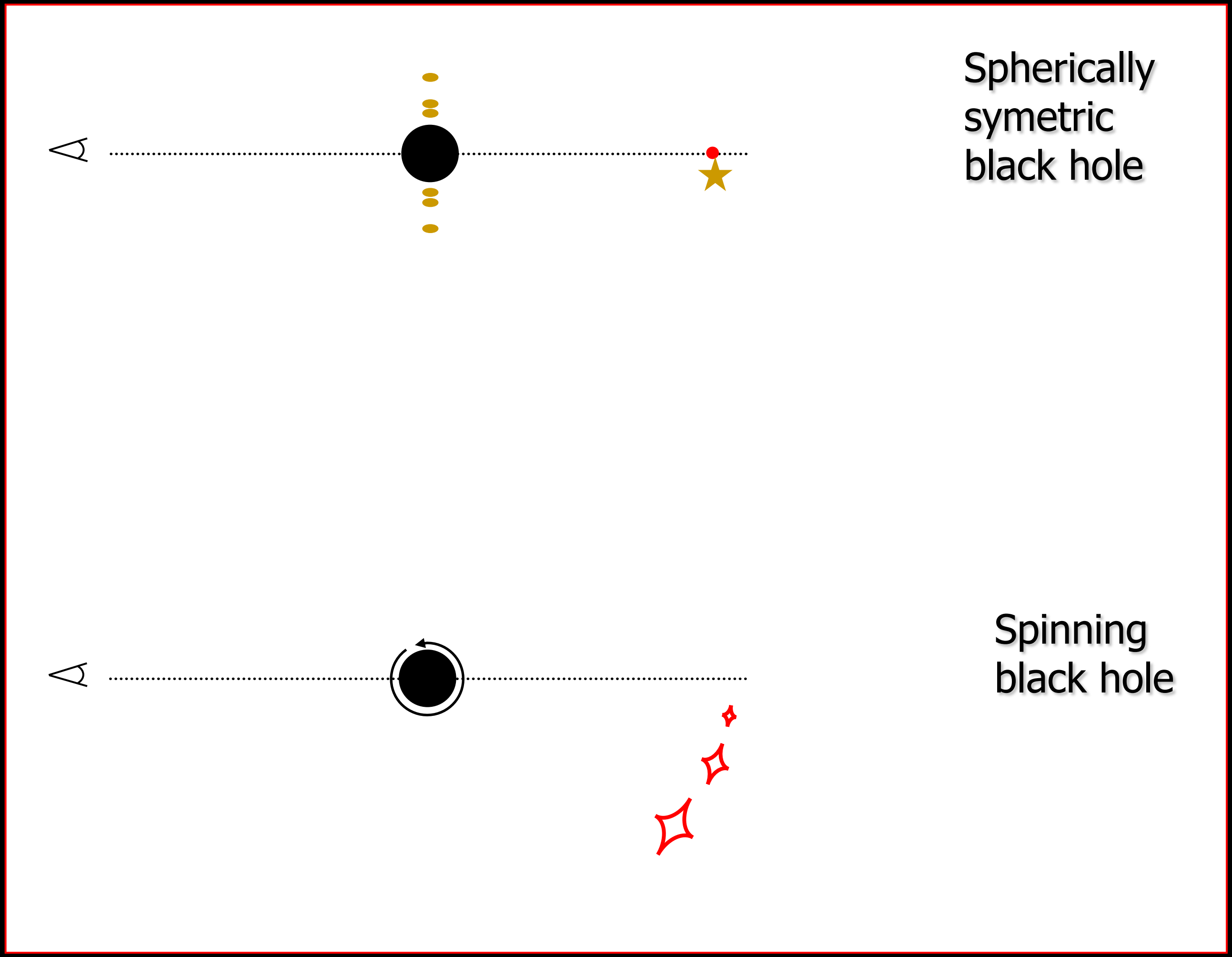
From Schwarzschild to Kerr



Schwarzschild:

When the source is far from the optical axis, all images are very faint.

From Schwarzschild to Kerr

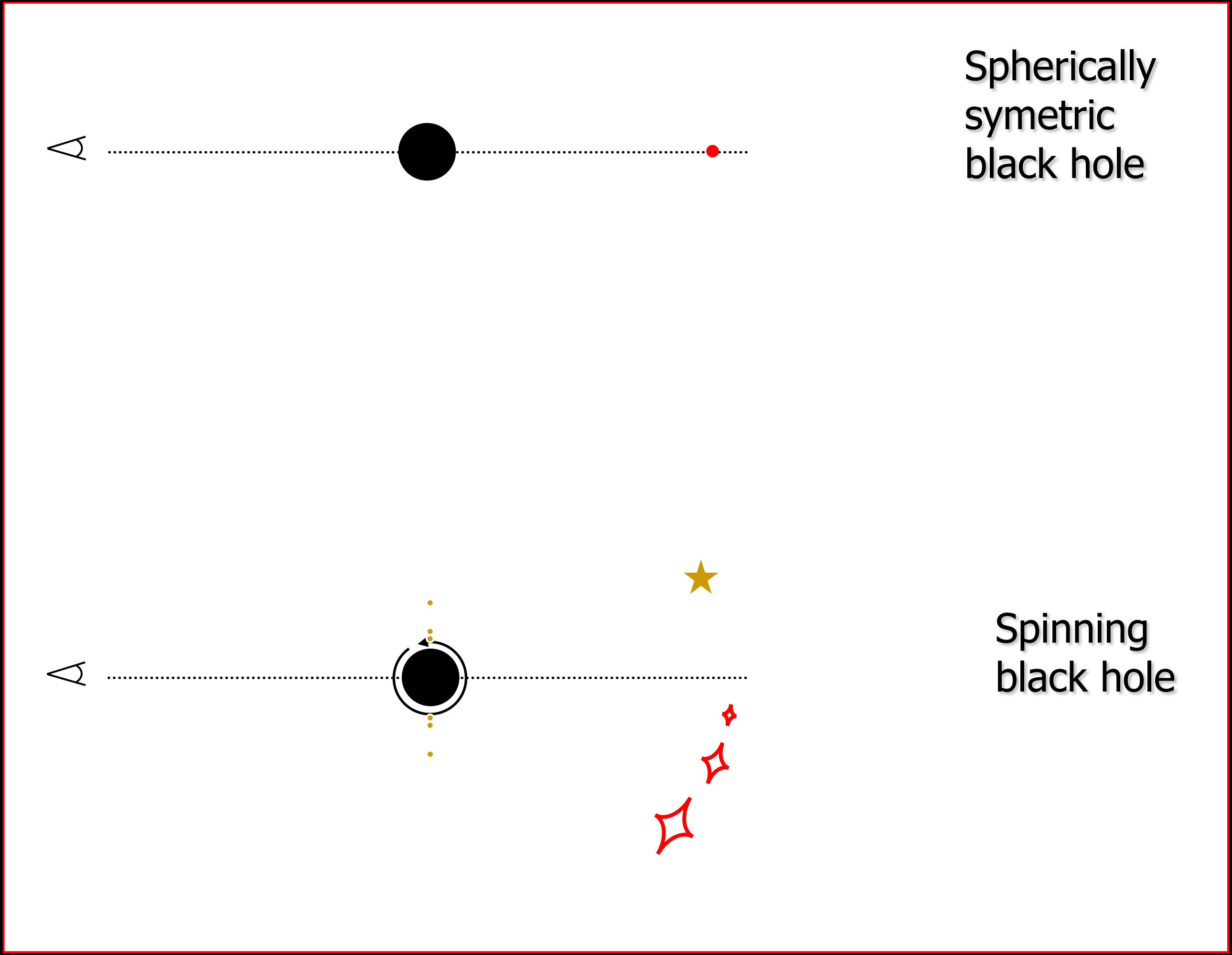


Schwarzschild:

When the source is close to the optical axis, all images get simultaneously bright.

Caustics for all images coincide.

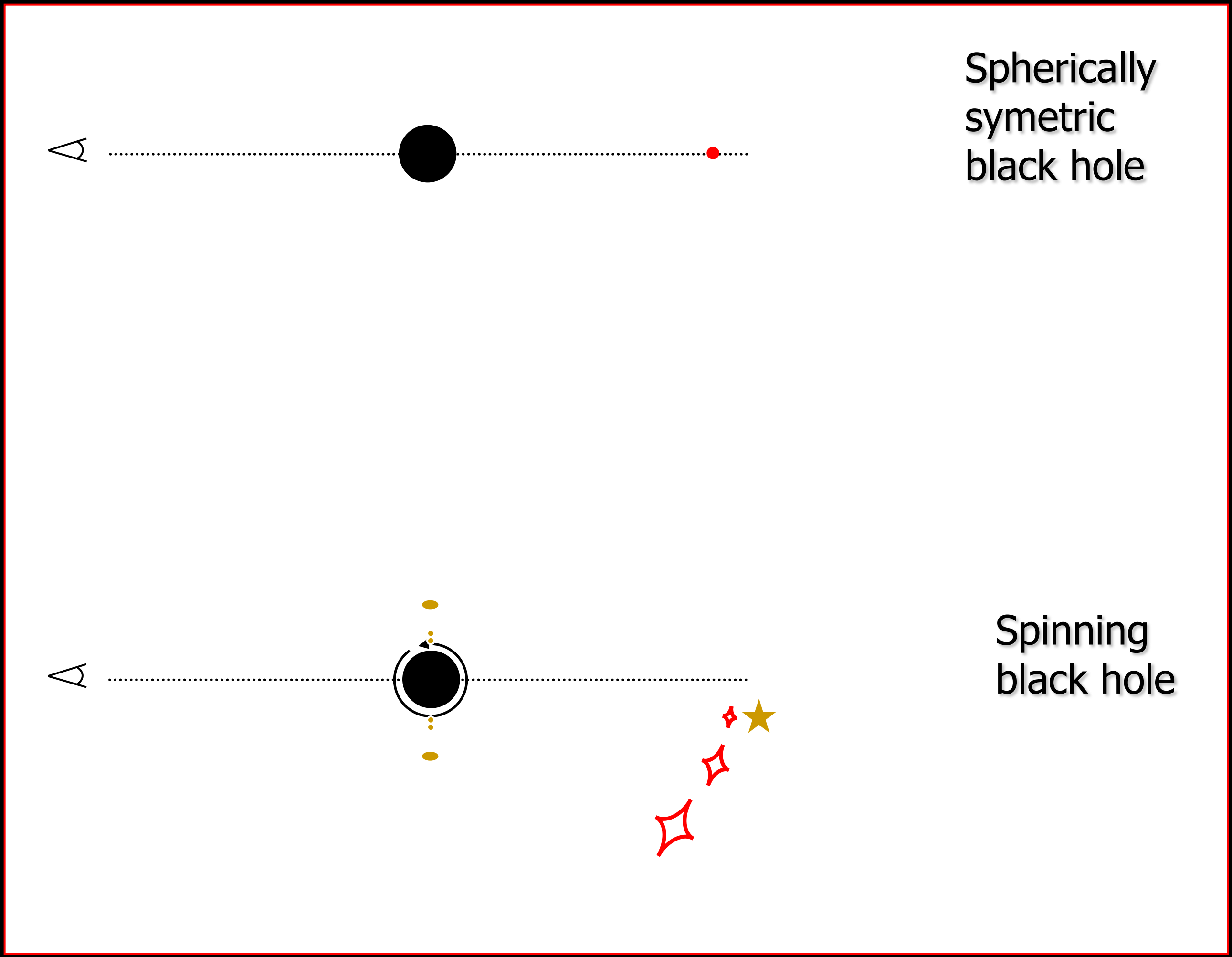
From Schwarzschild to Kerr



Kerr:

Every image pair has its own caustic. When the source is far from all caustics, all images are faint.

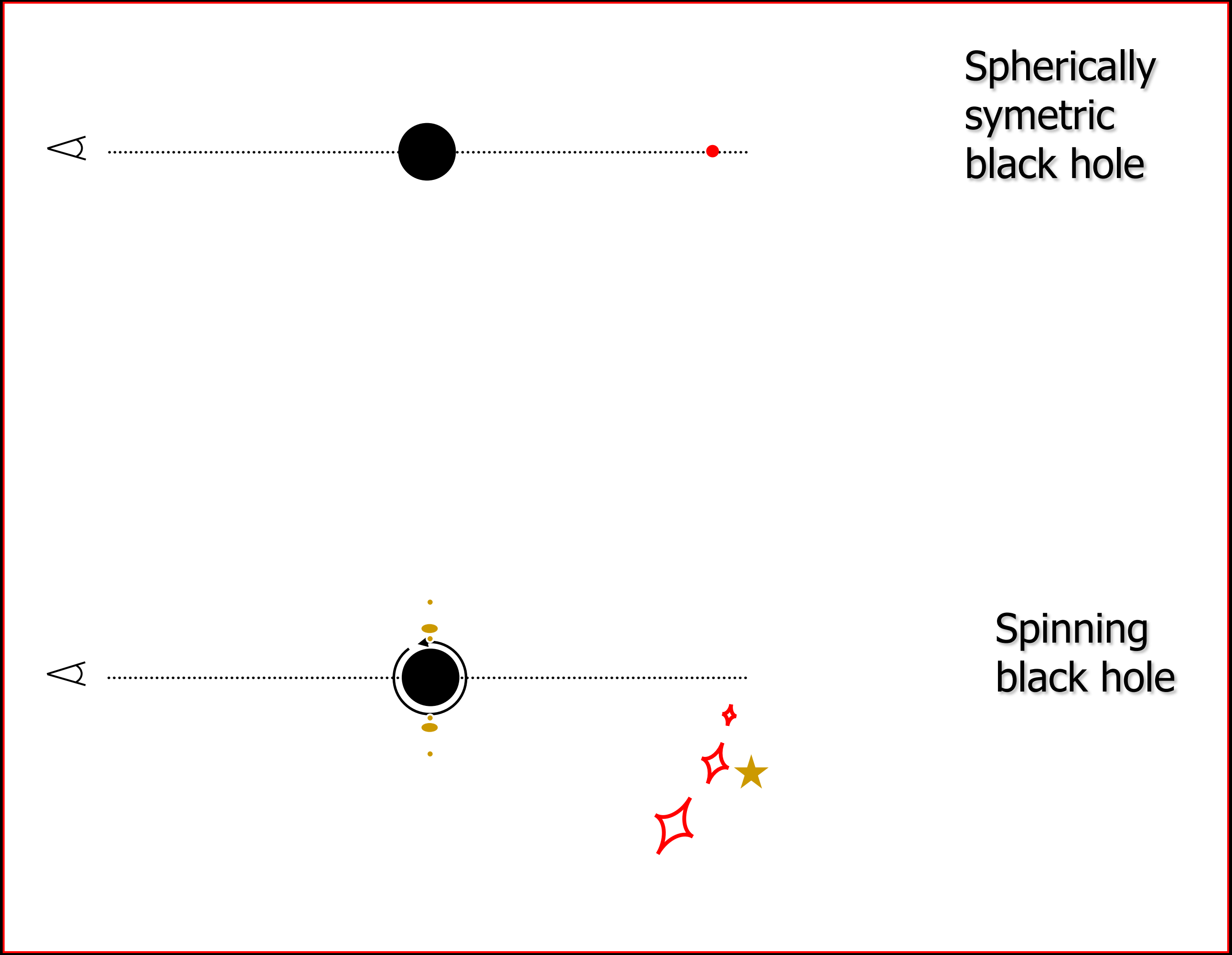
From Schwarzschild to Kerr



Kerr:

When the source is close to the first caustic, the first pair of images gets bright.

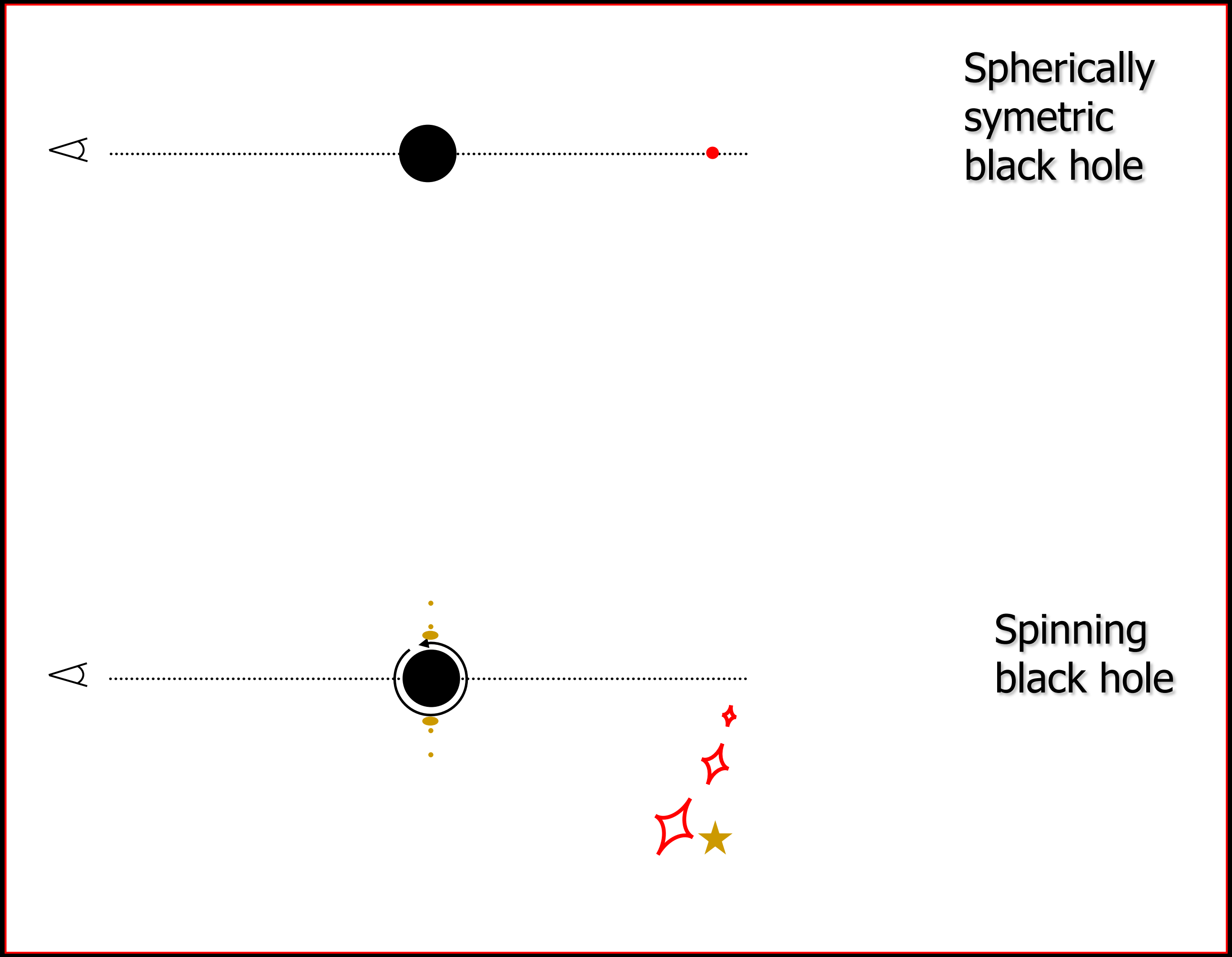
From Schwarzschild to Kerr



Kerr:

When the source is close to the second caustic, the second pair of images gets bright.

From Schwarzschild to Kerr

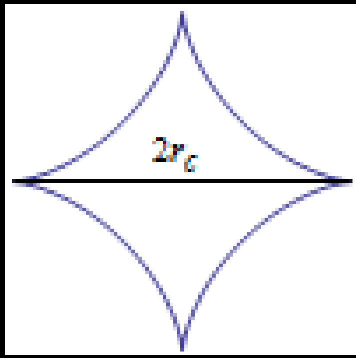


Kerr:

When the source is close to the third caustic, the third pair of images gets bright and so on.

Kerr caustics

- Analytical approximations for the Kerr caustics can be found in the weak deflection limit (Sereno & De Luca 2006, 2008)



$$r_c = \frac{15\pi}{16}\left(\frac{\theta_E D_{OS}}{4D_{LS}}\right)^4 a^2 \sin^2 i$$

- … and in the strong deflection limit (Bozza et al. 2005, 2006)

$$r_c = \frac{1}{18}\left(5m\pi + 8\sqrt{3} - 36\right)a^2 \sin^2 i$$

- When the source is inside a caustic of order *m*, two additional images of order *m* appear.

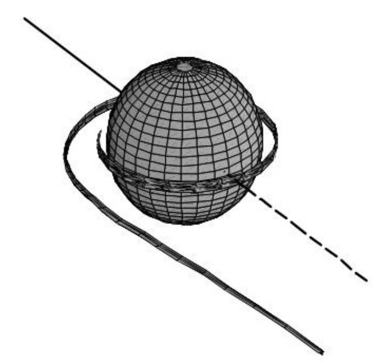
- The degeneracy between the spin and the inclination is lifted at high spin values.

- Extended caustics are also obtained in presence of external shear *(Bozza & Sereno 2006)*

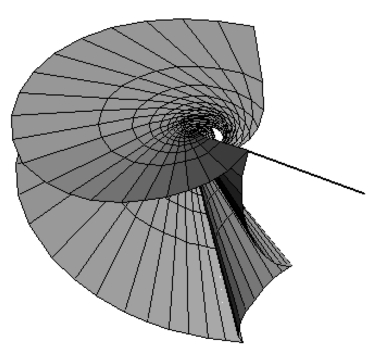
$$r_c = 2\gamma_{ext}\theta_m\left(1 + e^{\frac{b-2k\pi}{a}}\right)$$

Kerr caustics

- The primary caustic has been studied by *Rauch & Blandford (1994)*.
- At small source distances it winds around the BH.
- Higher order caustics become very extended for highly spinning BHs *(Bozza 2008)*



(c)



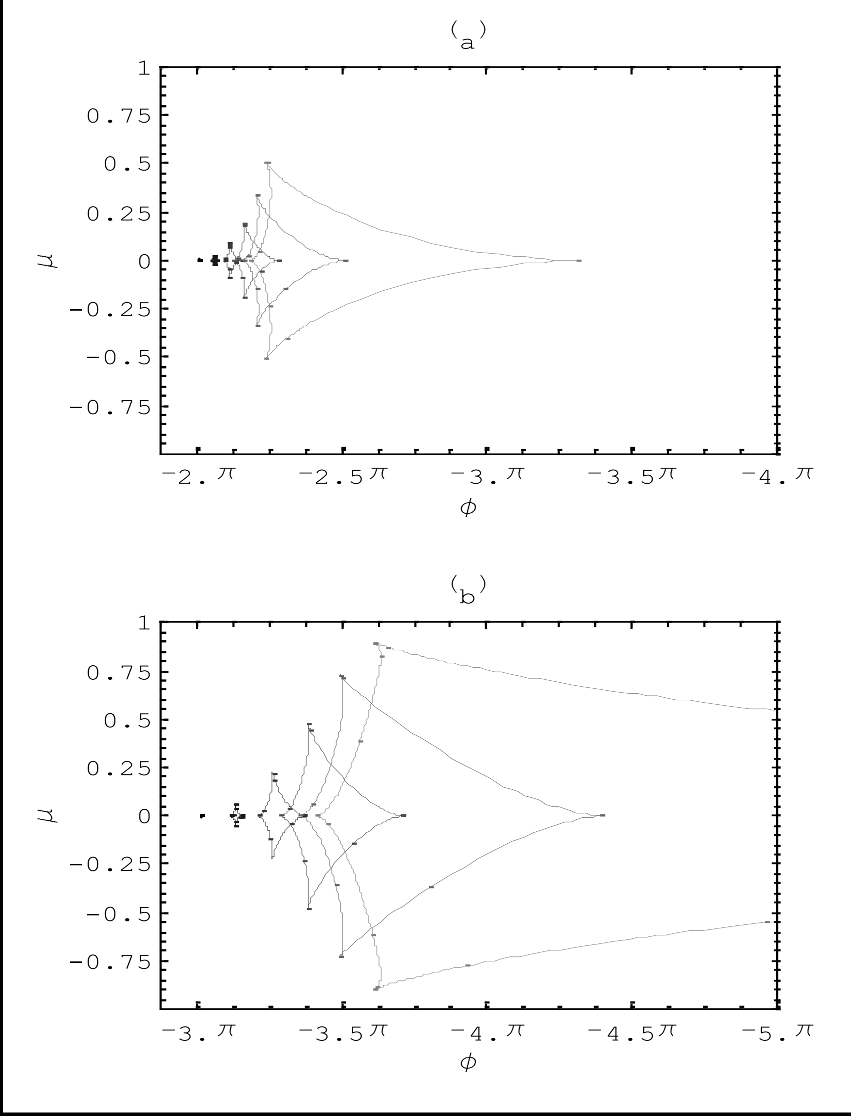
- Higher order caustics may self-intersect on the equatorial plane many times.
- The number of higher order images for an equatorial source grows exponentially large with the order *m*

Kerr caustics

As the spin of the black hole grows the caustics are shifted more and more clockwise.

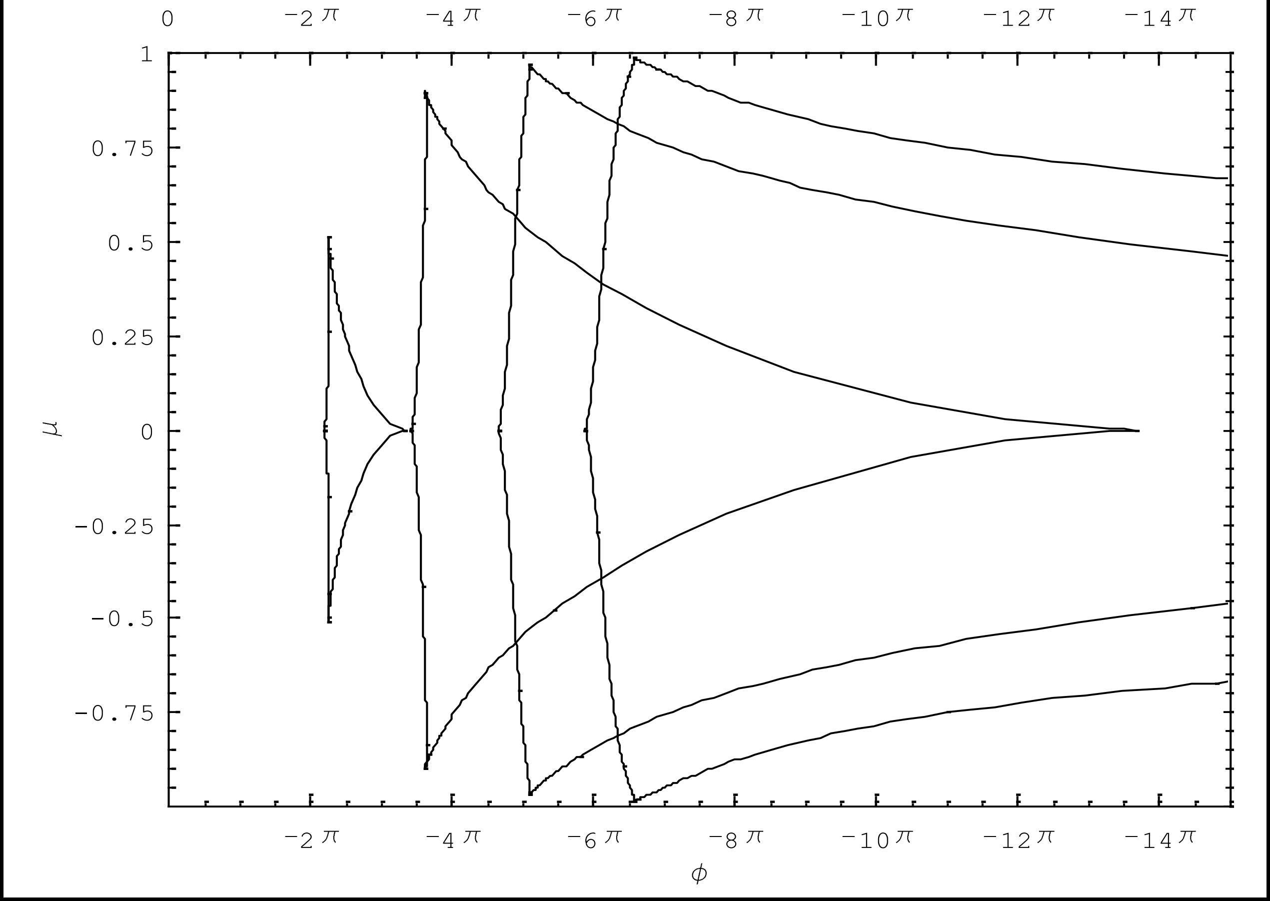
They become larger and larger until they reach the pole of the black hole.

The side corresponding to the creation of images by co-rotating photons is very elongated.



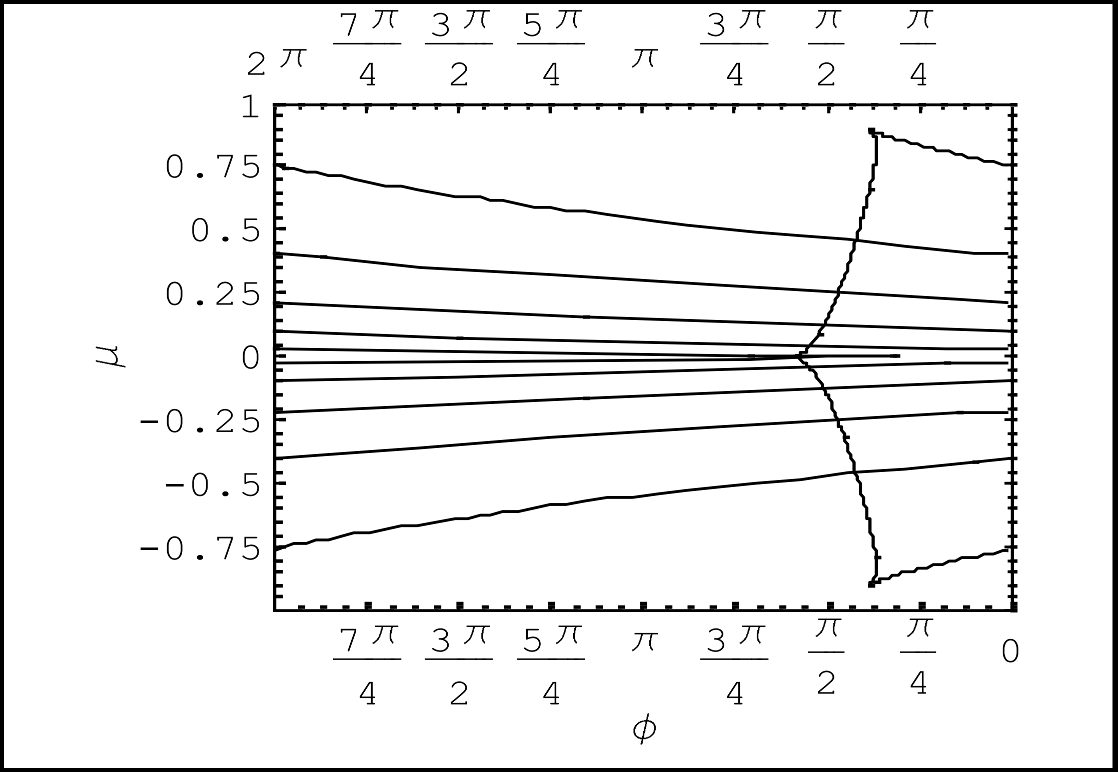
Extremal caustics

Cross-sections of caustics of order 2,3,4,5 for an extremal Kerr black hole a=M.



Caustics larger than 2π

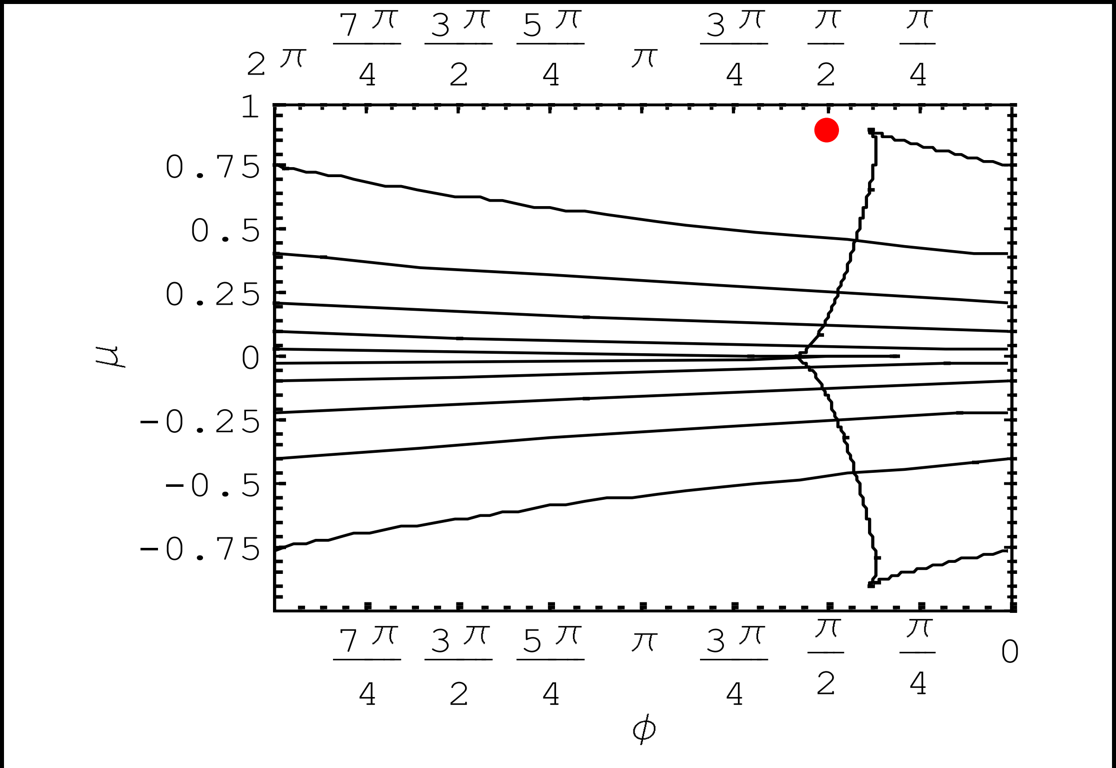
Third order caustic projected on the physical azimuth interval [0,2π].



Caustics larger than 2π

How many images with three inversion points in the polar motion do we have?

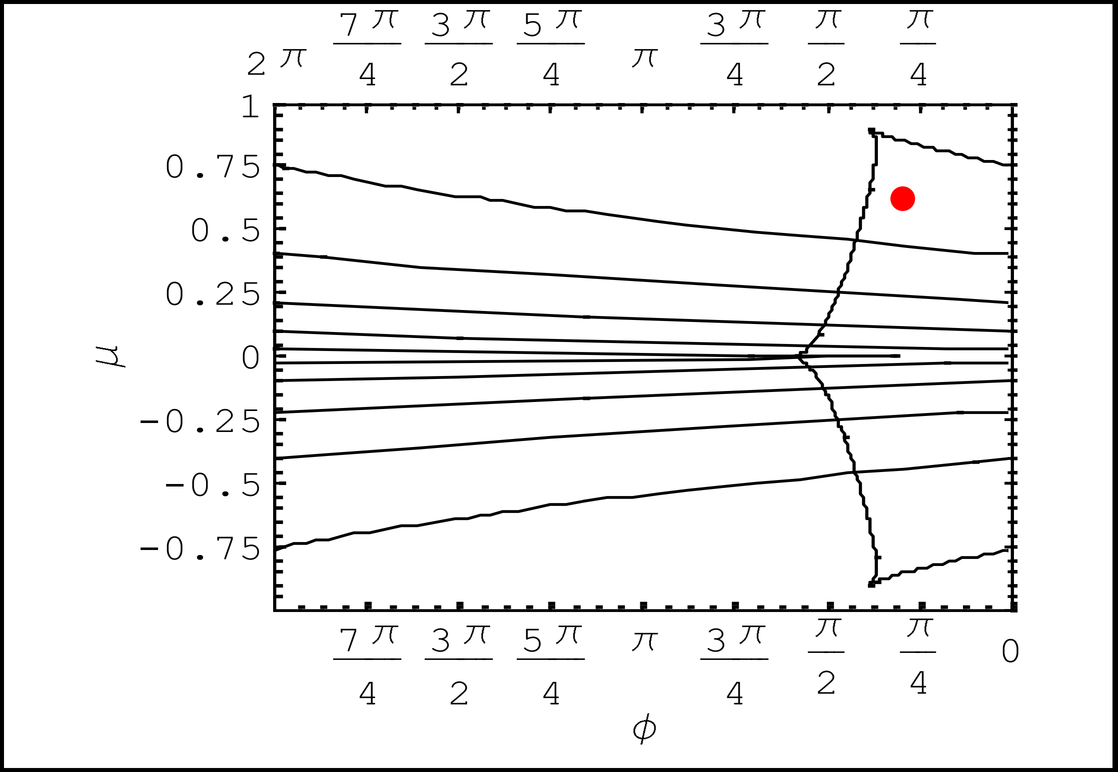
2 images
(source outside caustic)



Caustics larger than 2π

How many images with three inversion points in the polar motion do we have?

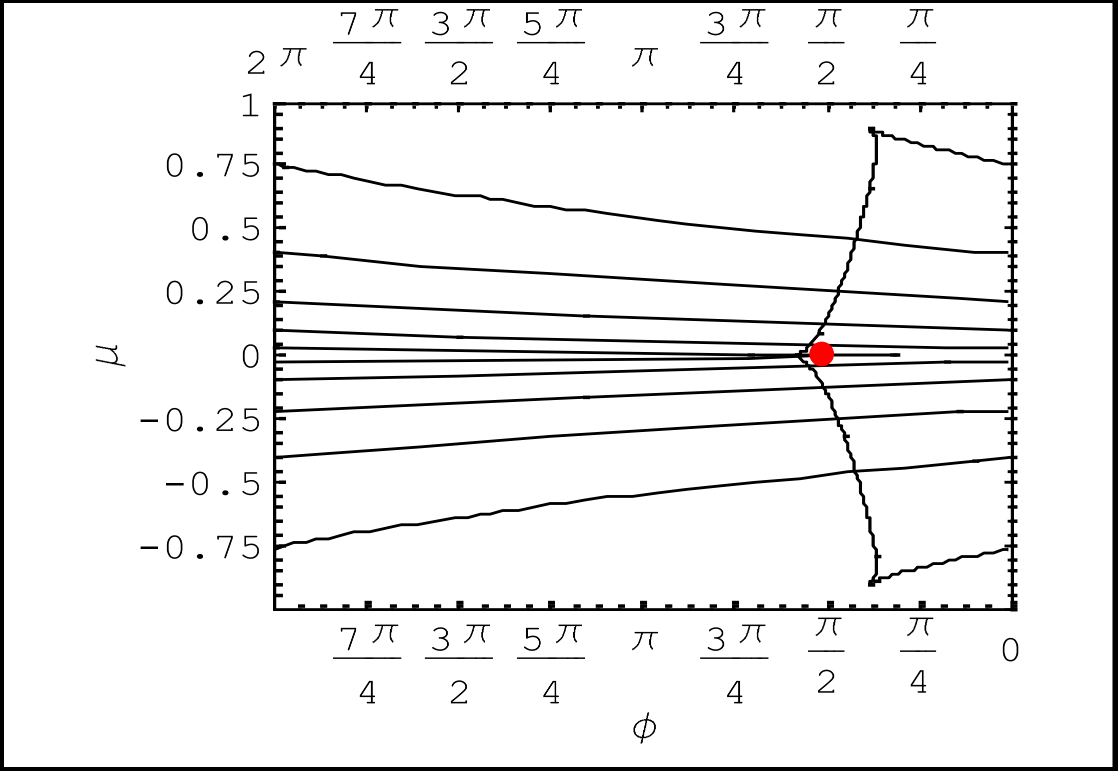
4 images



Caustics larger than 2π

How many images with three inversion points in the polar motion do we have?

14 images



Number of images

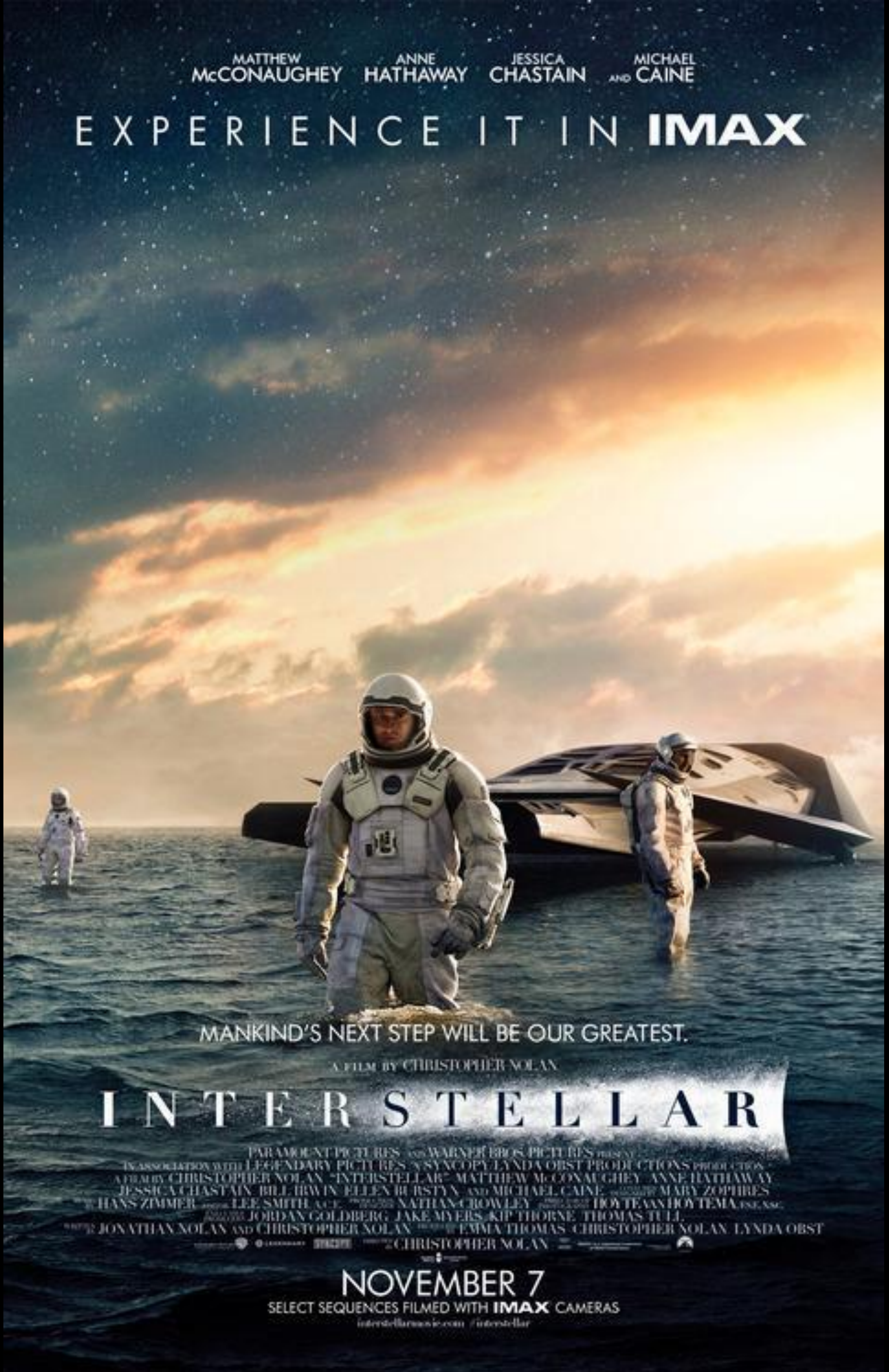
For any given caustic order k, the number of images with k inversions in the polar motion ranges from 2 (source outside the caustic) to

$$n = 0.36 \exp(1.57k) - 2.82k + 0.87$$

The number of images of an equatorial source grows exponentially large!

(Bozza 2008)

What about industrial applications of these studies?


MATTHEW McCONAUGHEY
ANNE HATHAWAY
JESSICA CHASTAIN
AND MICHAEL CAINE
EXPERIENCE IT IN IMAX
MANKIND'S NEXT STEP WILL BE OUR GREATEST.
A FILM BY CHRISTOPHER NOLAN
INTERSTELLAR
NOVEMBER 7
SELECT SEQUENCES FILMED WITH IMAX CAMERAS

The movie “Interstellar”

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Class. Quantum Grav. 32 (2015) 065001 (41pp) | doi:10.1088/0264-9381/32/6/065001

Gravitational lensing by spinning black holes in astrophysics, and in the movie *Interstellar*

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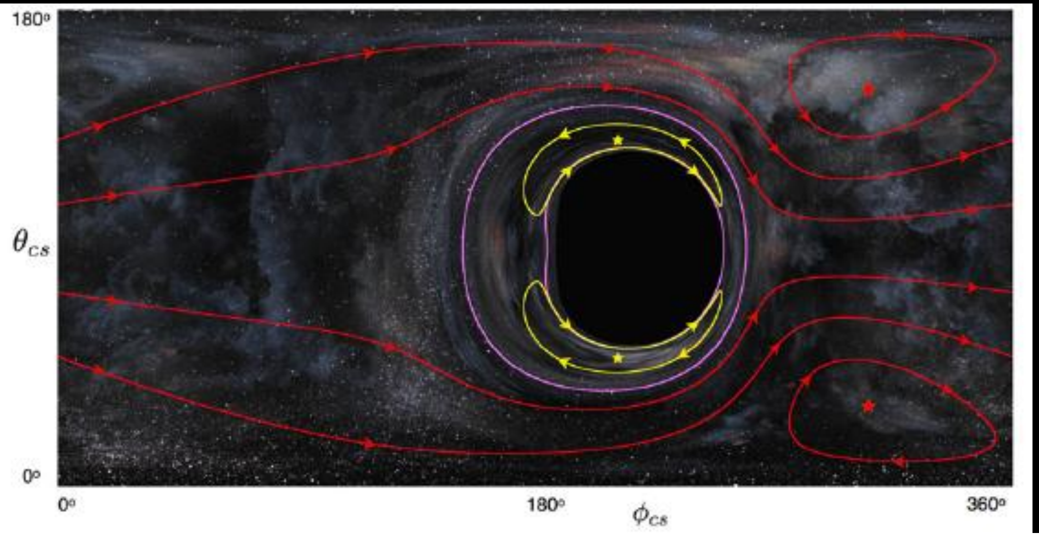
CrossMark

Abstract

Interstellar is the first Hollywood movie to attempt depicting a black hole as it

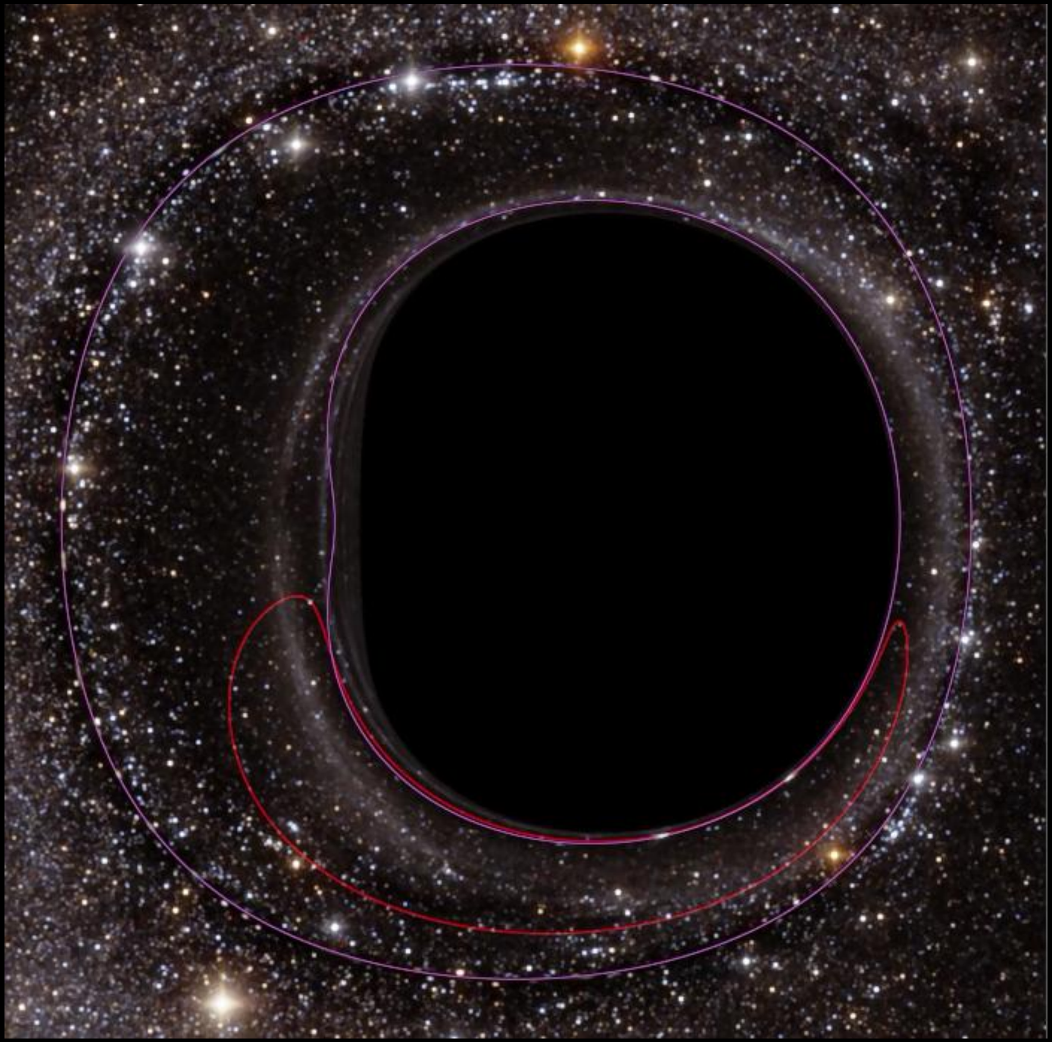
The movie “Interstellar”



3.3.1. Primary and secondary critical curves and their caustics. After seeing these stellar-image motions in our simulations, we explored the nature of the critical curves and caustics for a camera near a fast-spinning black hole, and their influence. Our exploration, conceptually, is a rather straightforward generalization of ideas laid out by Rauch and Blandford [9] and by Bozza [10]. They studied a camera or observer on the celestial sphere and light sources orbiting a black hole; our case is the inverse: a camera orbiting the hole and light sources on the celestial sphere.

Just as the Einstein ring, for a nonspinning black hole, is the image of a caustic point on the celestial sphere—the intersection of the celestial sphere with a caustic line on the camera's past light cone—so the critical curves for our spinning black hole are also images of the intersection of the celestial sphere with light-cone caustics. But the spinning hole's light-cone caustics generically are two-dimensional (2D) surfaces (folds) in the three-dimensional light cone, so

The movie “Interstellar”



http://download.iop.org/aas/cqg/CQG508751movie3.mp4

Black hole lensing: where?

- By integrating geodesics in General Relativity we have found fascinating phenomena created by photons winding closely around black holes.

- Do we have any chances to observe such phenomena in an astrophysical system?

- We need a massive black hole, as close as possible to us.

- We need small sources close to the black hole.

The best case is Sgr A*, the black hole in the center of our Galaxy!

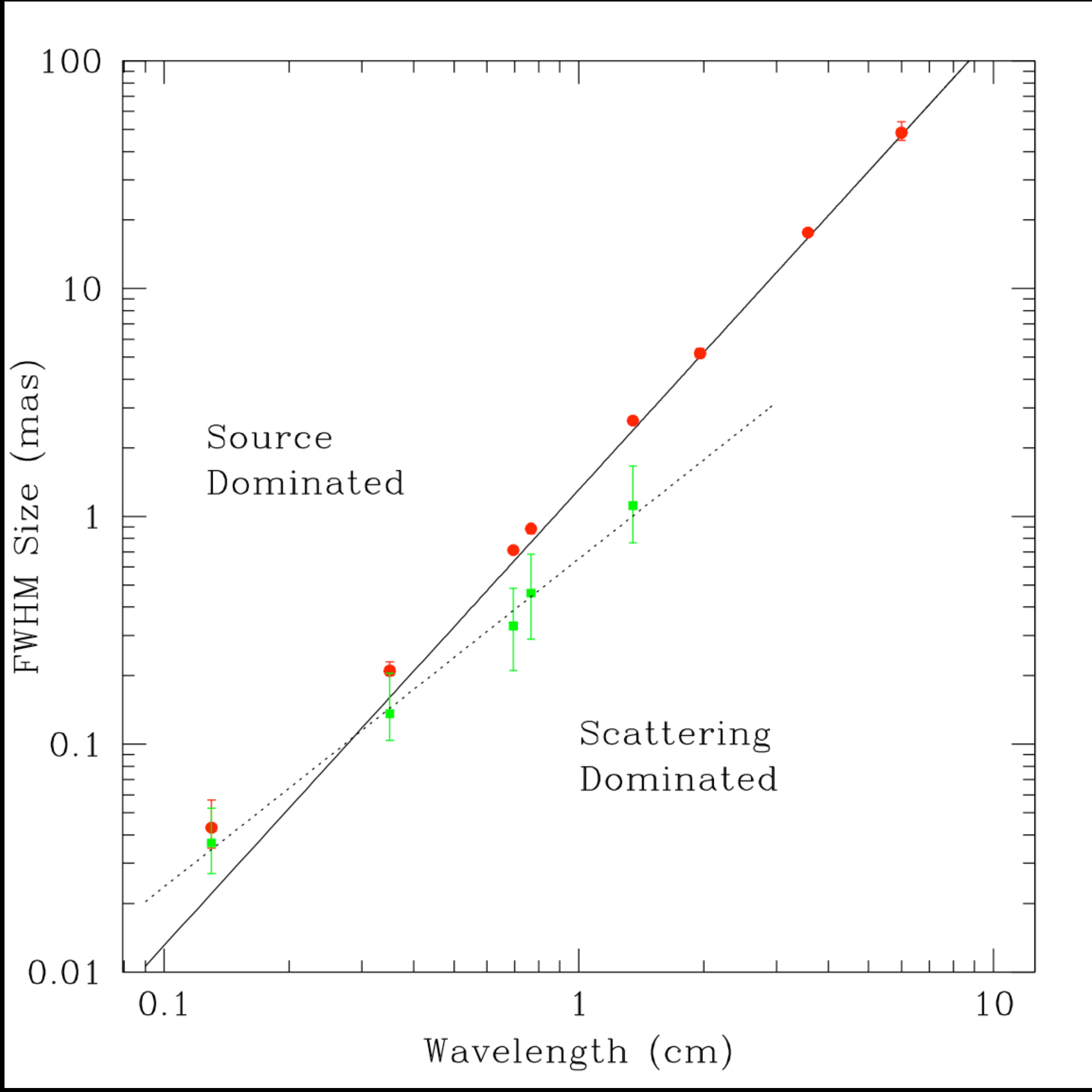
Sgr A*: an almost point-source

The apparent size of Sgr A* decreases as λ^2.

This is just an effect of scattering by interstellar electrons!

Only at λ=1.3mm the intrinsic size of Sgr A* has been appreciated:
37^{+16}_{-10} μas
(Doeleman et al. Nature 2008)

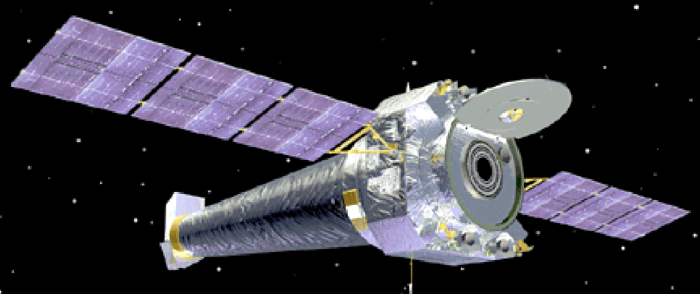
This corresponds to an intrinsic diameter of 45 million km.

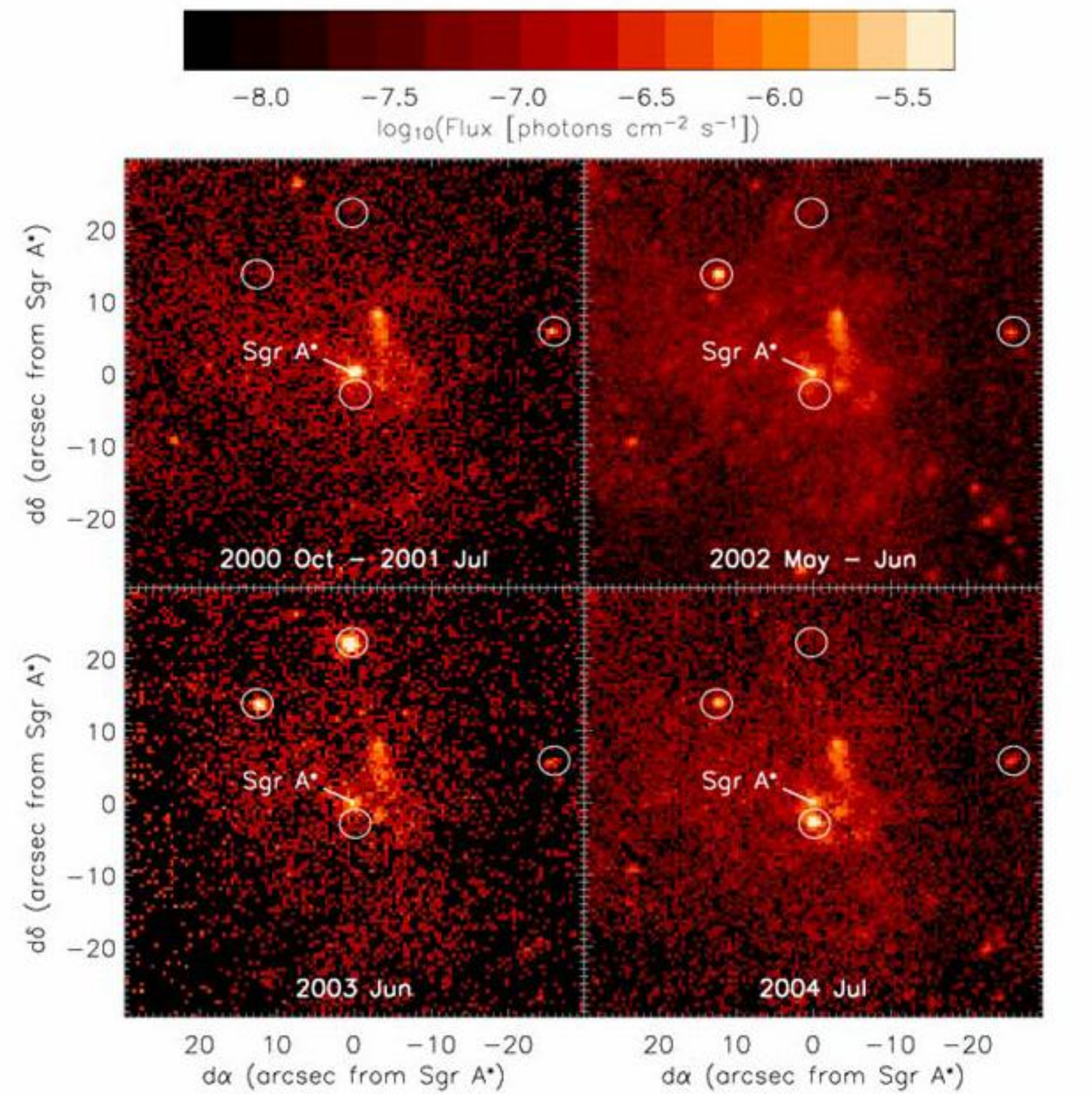


Sgr A* in the X-rays

Since 2001, Sgr A* has been observed in the X-rays thanks to the Chandra satellite *(Baganoff et al. 2001)*.

Its quiescent state is daily interrupted by violent flares.



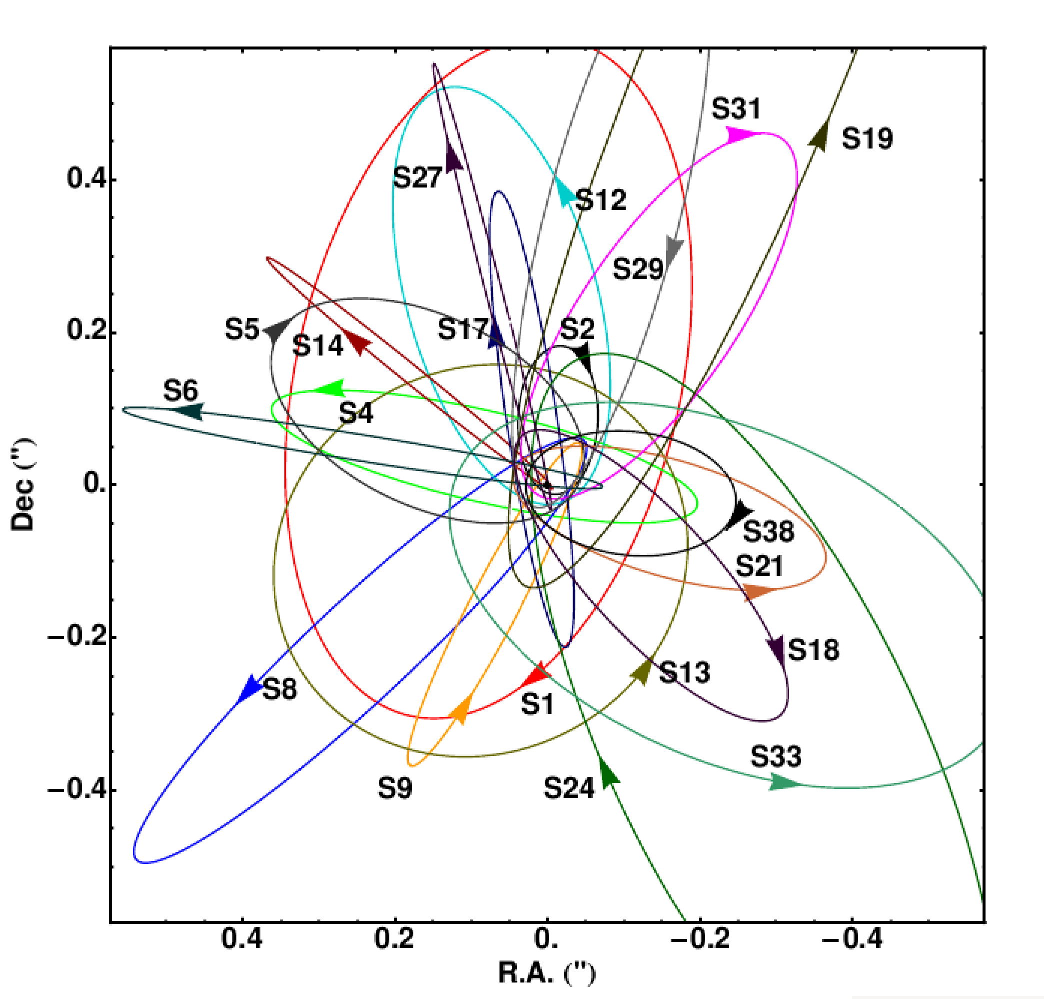


S-Stars in the IR

More than 100 stars have been identified within 0.1 light years from Sgr A*.

For 27 stars the orbit has been determined with good accuracy.

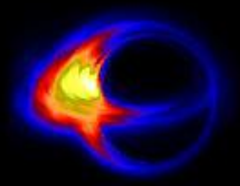
S2 has a period of 15.8 years, getting as close as 120 AU.



From the Kepler law we derive the most accurate measurement of the mass of Sgr A*: $(4.31 \pm 0.36) \times 10^6\ M_\odot$ *(Gillessen et al. 2009)*

Observations at high resolution

Will we ever observe the "shadow" of Sgr A*?



The best resolution we can reach today is 10 μas at λ=1mm.

We need to go at shorter wavelengths to overcome interstellar electron scattering.

JCMT, Hawaii

Very Long Baseline (intercontinental) Interferometry is under development within the Event Horizon Telescope project.

GRAVITY

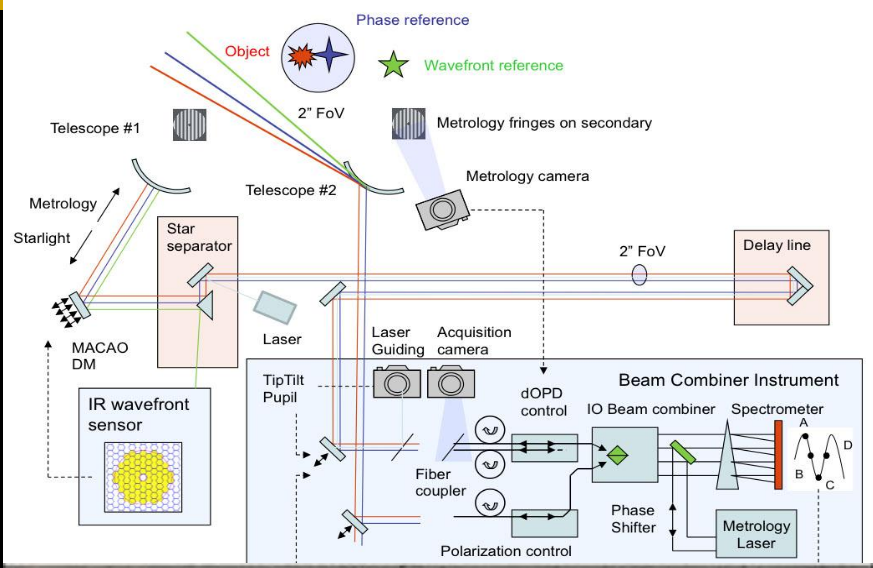
- Interferometry at the VLT will be performed by **GRAVITY** in the near infrared K- band (λ = 2.2 μm).

Performance:

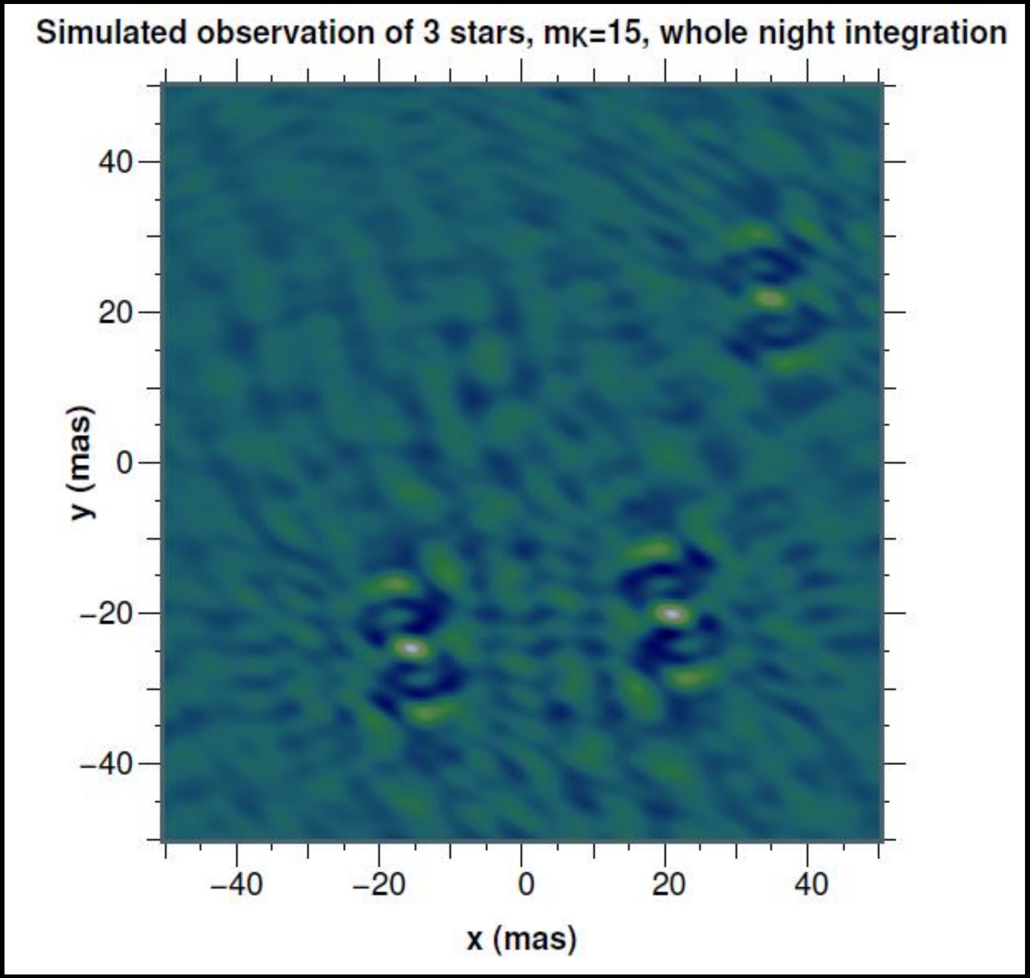
- The resolving power should be **3 mas**
- Astrometry precision should reach **10 μas**
- Limiting magnitude is **K < 19**

Science:

- Motion of hot spots on the accretion disk.
- Discovery and distribution of many S-stars.
- Accurate orbit determination: precession.



GRAVITY



(Vincent et al. 2010)

Will we observe any gravitational lensing effects and test the black hole nature of Sgr A*?

Sgr A* as a gravitational lens

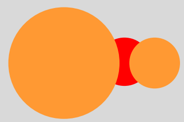
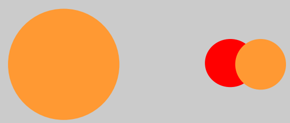
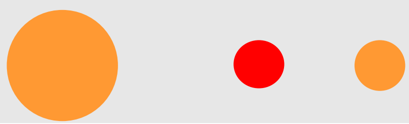
- **Wardle & Yusuf-Zadeh (1992):** Gravitational lensing by the central black hole would slightly **deplete** the inner 10 mas region.

- **Jaroszynski (1998), Alexander & Sternberg (1999), Chaname´ et al. (2001):** At least 10 bulge stars should have a **secondary image** with magnitude K<23.

- **Nusser & Broadhurst 2004:** The primary image should be **shifted** by ~ 30 μas

- **De Paolis et al. (2004), Bozza & Mancini (2004, 2005, 2009, 2012), Bin-Nun (2010, 2011): S-stars** are ideal candidate sources for gravitational lensing.

Measurable effects

Regime I		• **Amplification** • **Astrometric shift of the total centroid**
Regime II		• **Amplification of the primary** • **Astrometric shift of the primary** • **Amplification of the secondary+Sgr A*** • **Astrometric shift of the secondary+Sgr A***
Regime III		• **Amplification and positions of both images (full system solved)**

- All effects require a sufficient alignment of the source star behind the black hole.

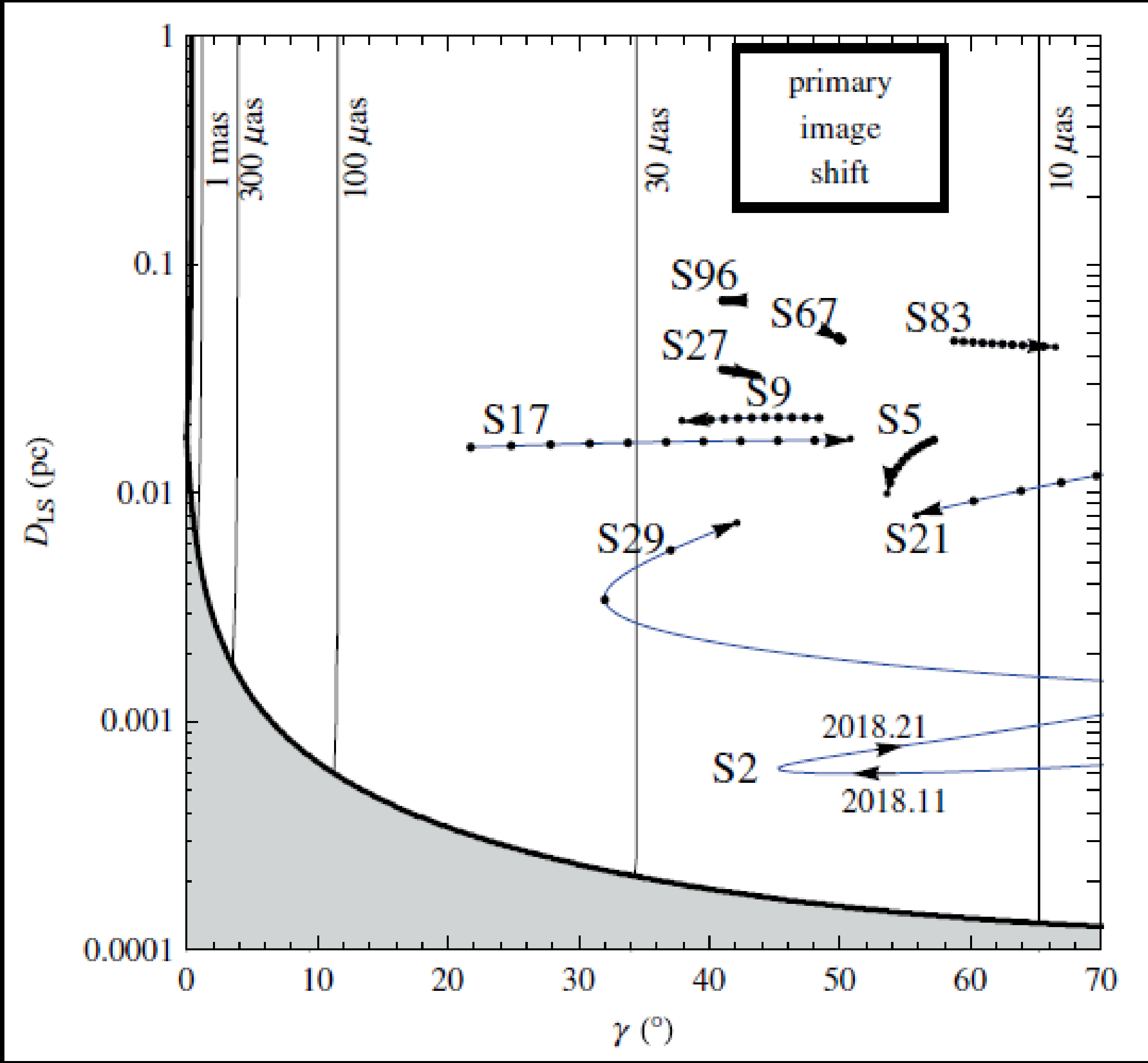
(Bozza & Mancini, ApJ 2012)

Measuring astrometric shift

- Astrometric shift will be measurable for at least 10 known S-stars.

Science:

- Orbit reconstruction must take gravitational lensing into account.
- Gravitational lensing will help constrain the mass and distance to the black hole.
- The mass distribution in the inner parsec will be better known.



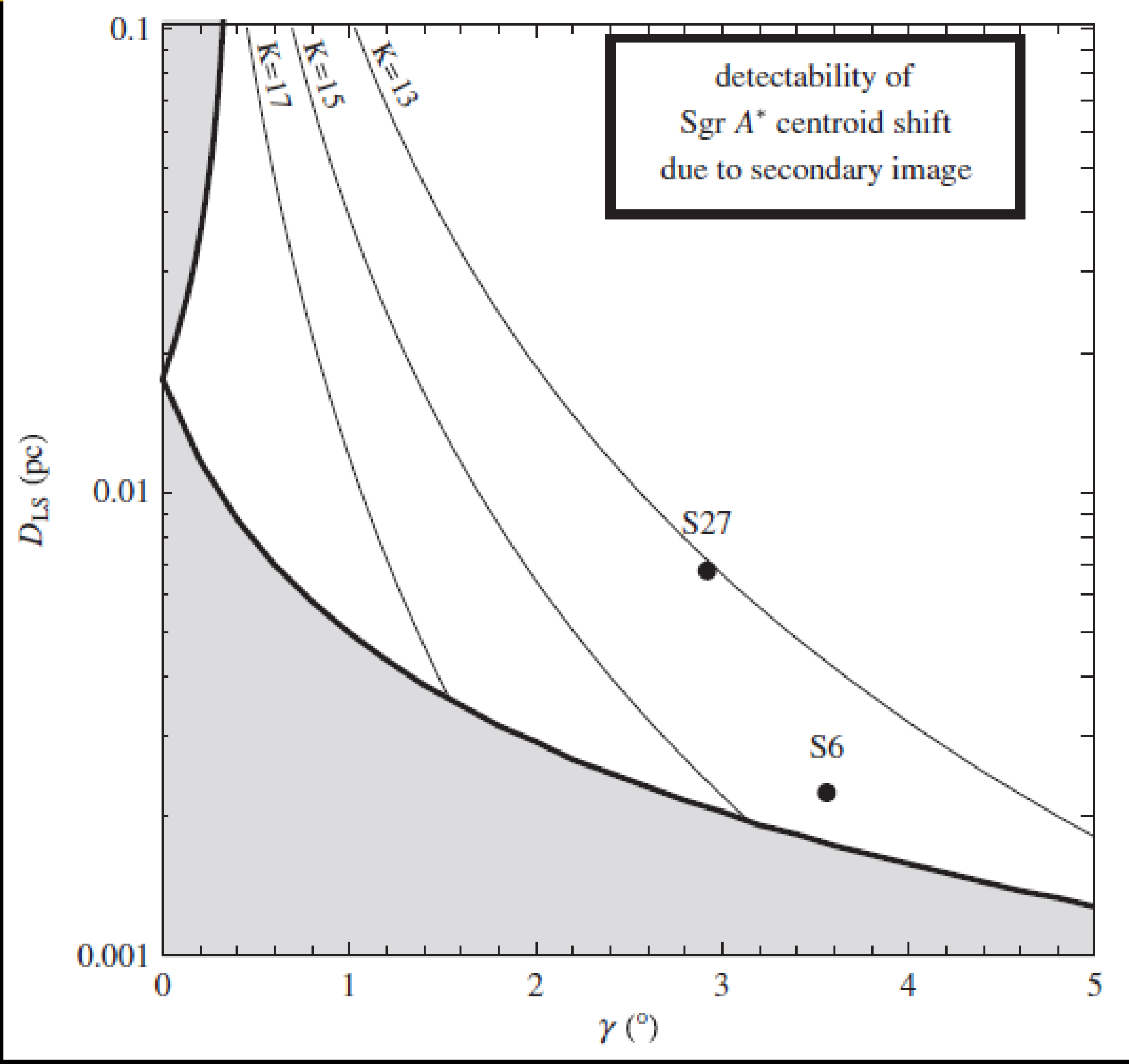
(Bozza & Mancini, ApJ 2012)

Post-Newtonian gravity

- Astrometric shift of the centroid of Sgr A*+secondary image is marginally detectable for some S-stars.

Science:

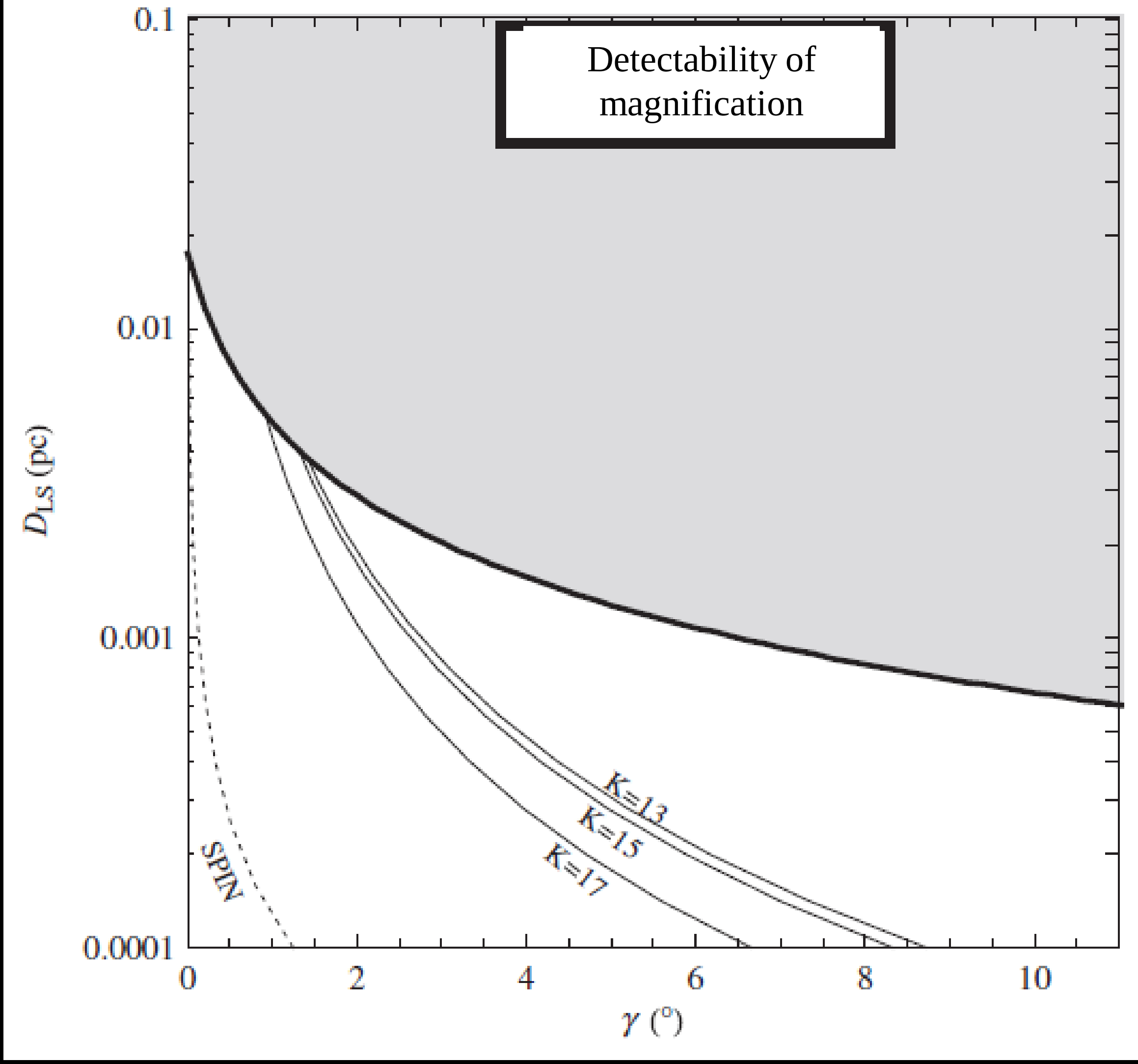
- Marginal sensitivity to **Post-Newtonian effects**.



$\Delta\theta$=25 μas $K_{Sgr\ A*}$ = 17

Spin of the black hole

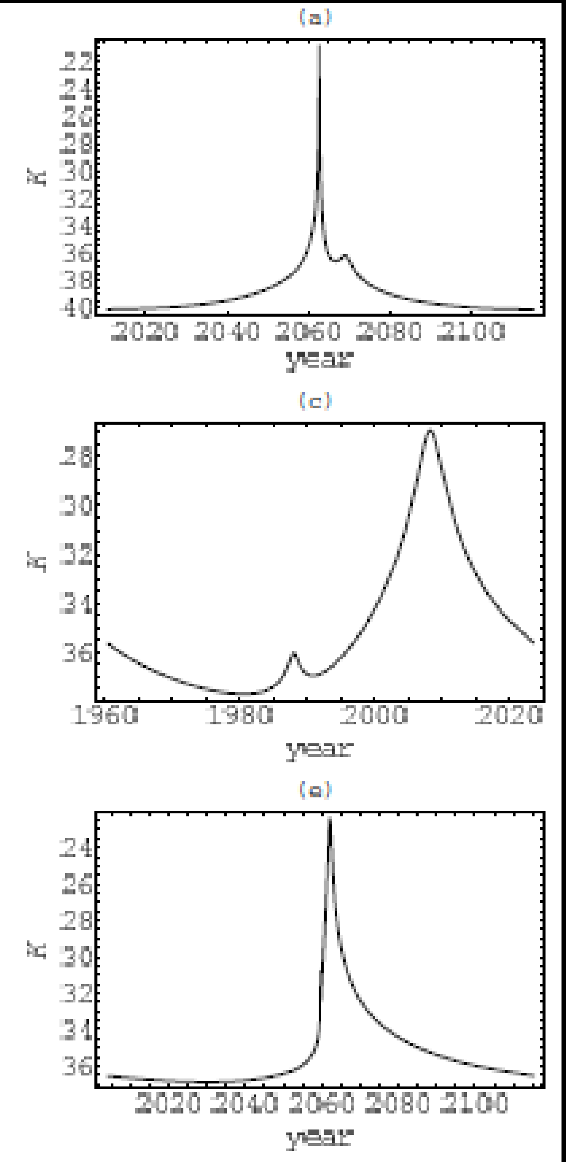
- Stars orbiting very close to Sgr A* are periodically amplified by gravitational lensing.
- Gravitational lensing is sensitive to the **intrinsic angular momentum** of the black hole.
- Sources very close to the inner accretion disc (0.00005 pc) are required.



$\Delta K=0.1$ $K_{Sgr\ A*} = 17$

Imaging of secondary images

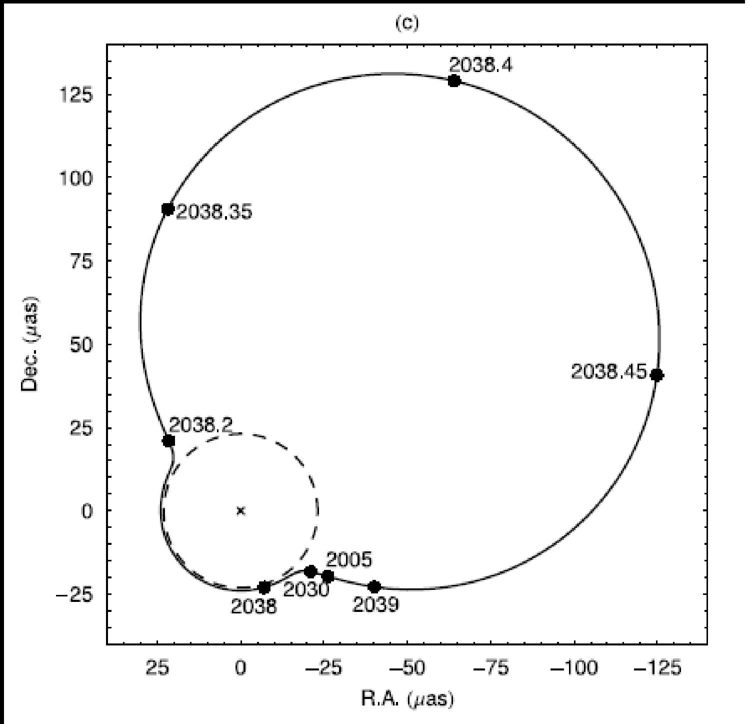
- With 100 μas resolution, secondary images of S-stars would be routinely observed.
- A detailed study of the gravitational field with controlled probes is possible!



S6

S17

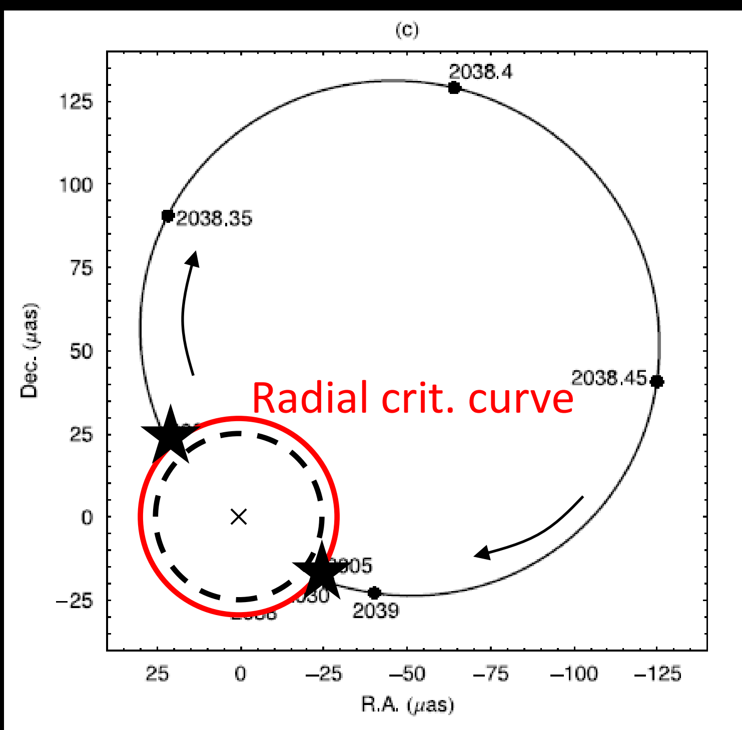
S27

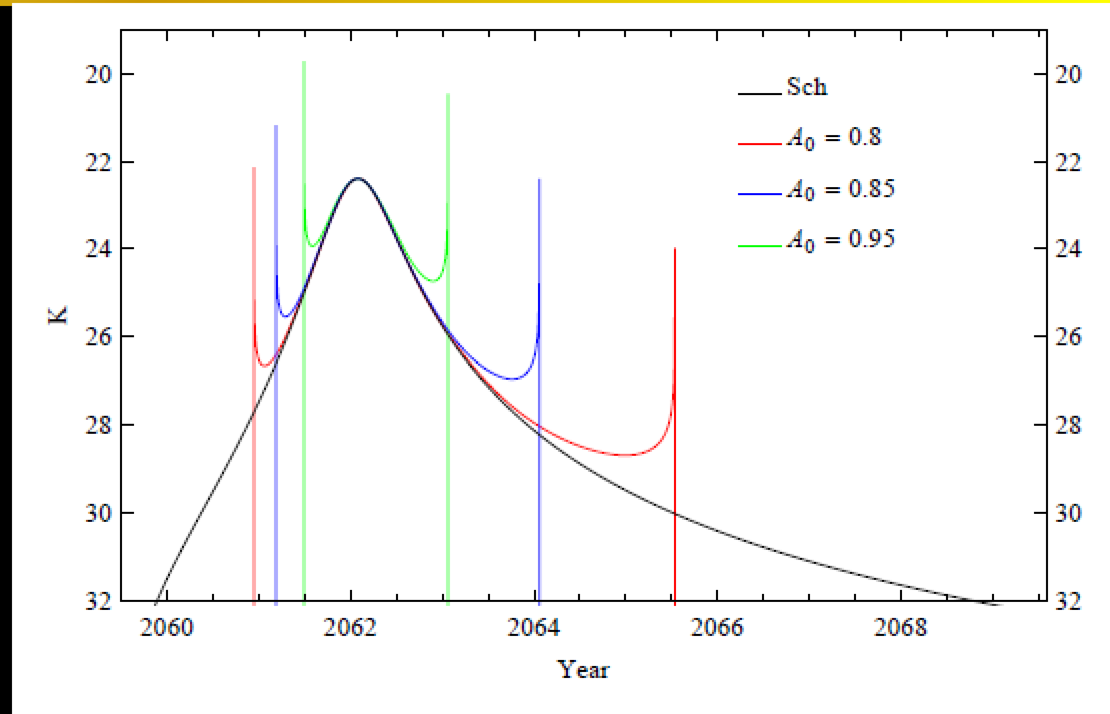


(Bozza & Mancini 2004, 2005, 2009)

Testing the existence of the horizon

- If there is no event horizon, a radial caustic must exist.
- Secondary images would appear and disappear with a flash.

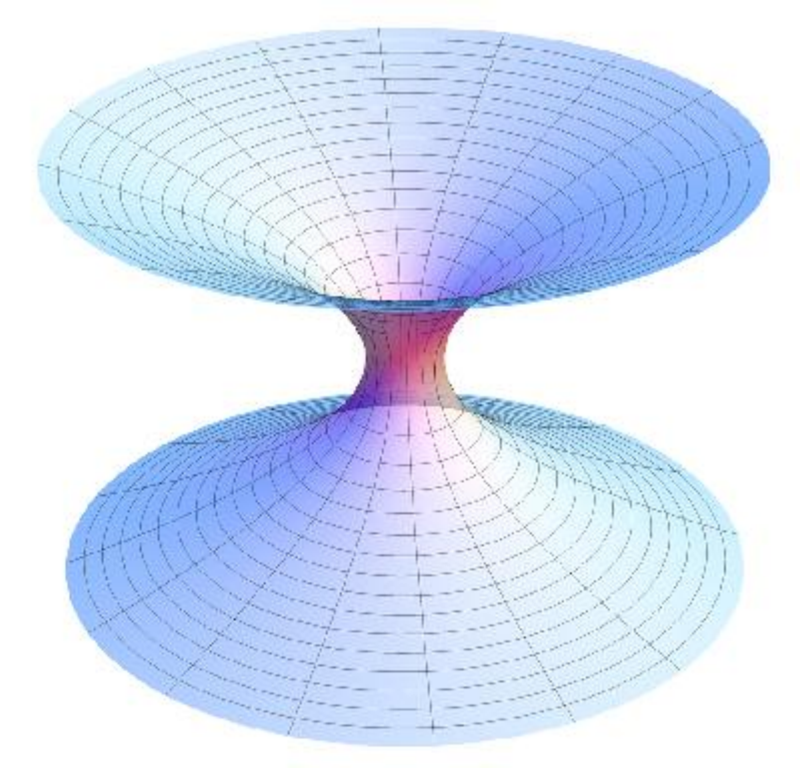


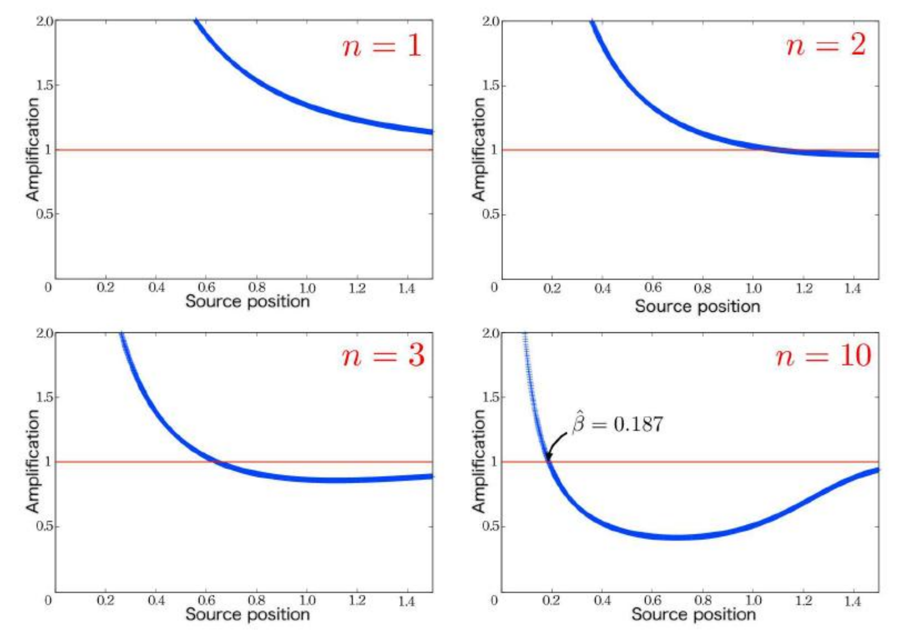


Light curve of the secondary image in the case of a boson star *(Bin-Nun 2013)*

Alternatives to Schwarzschild

- Wormholes have been introduced in the 70's *(Ellis 1973, Morris-Thorne 1988)* using matter with negative energy density.
- The metric asymptotically falls as $1/r^2$ in the Ellis wormhole.
- Gravitational lensing goes as $1/b^2$ *(Chetouani & Clément 1984, Abe 2010).*





- *Kitamura et al. (2013)* have studied the general case of a metric falling as $1/r^n$ showing that the defocusing effect is generic for n>1.

Sources of alternative metrics

- Let us consider a metric asymptotically falling as $1/r^q$ *(Bozza & Postiglione 2015)*.
- The generic energy-momentum tensor compatible with this metric is diagonal.

$$ds^2 = \left(1 - \frac{\alpha}{r^q}\right)dt^2 - \left(1 + \frac{\gamma}{r^q}\right)d\vec{x}^2$$

$$T_\mu^\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p_r & 0 & 0 \\ 0 & 0 & -p_t & 0 \\ 0 & 0 & 0 & -p_t \end{pmatrix}$$

$$\frac{q(1-q)\gamma}{r^{q+2}} = \kappa\rho; \quad \frac{q(\gamma-\alpha)}{r^{q+2}} = -\kappa p_r; \quad \frac{q^2(\alpha-\gamma)}{r^{q+2}} = -\kappa p_t$$

- The Einstein equations allow us to connect the form of the metric to the form of the exotic matter required to support it.

Equations of state

- All components of the energy-momentum tensor are in linear relation

$$p_r = w_r \rho; \qquad p_t = w_t \rho$$

$$w_r = \frac{\gamma - \alpha}{\gamma(q-1)} \qquad w_t = \frac{q(\alpha - \gamma)}{2\gamma(q-1)}$$

- The exponent *q* regulating the fall-off the metric only depends on the ratio of tangential to radial pressure

$$\frac{w_t}{w_r} = -\frac{q}{2}$$

(Bozza & Postiglione 2015).

Energy conditions

$$ds^2 = \left(1 - \frac{\alpha}{r^q}\right)dt^2 - \left(1 + \frac{\gamma}{r^q}\right)d\vec{x}^2$$

- Positive energy density

$$\rho \geq 0 \quad \Leftrightarrow \quad q(1-q)\gamma \geq 0$$

- Weak energy condition

$$\rho + p_{r,t} \geq 0 \quad \Leftrightarrow \quad q(\alpha - q\gamma) \geq 0$$

- Strong energy condition

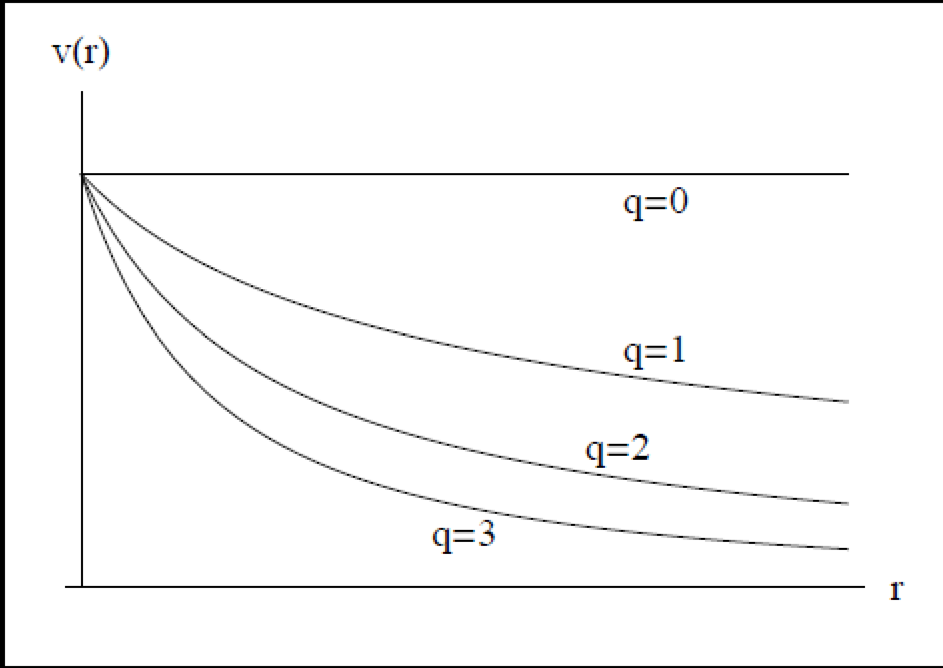
$$\rho + p_r + 2p_t \geq 0 \quad \Leftrightarrow \quad q(1-q)(\alpha + \gamma) \geq 0$$

(Bozza & Postiglione 2015).

Observational consequences

- The rotation curve is

$$v(r) = \sqrt{\frac{q\alpha}{2r^q}}$$



- The gravitational redshift goes as $z = \dfrac{\alpha}{2r^q}$ with heavy consequences on models of the appearance and observables from the accretion disk.

(Bozza & Postiglione 2015).

Gravitational lensing

- The bending angle is

$$\hat{\alpha} = \frac{\sqrt{\pi}\,\Gamma\left[(1+q)/2\right]}{\Gamma\left[q/2\right]} \frac{\alpha+\gamma}{b^{q}}$$

Kitamura et al. (2013)

- The gravitational time delay is

$$\Delta t = \frac{\sqrt{\pi}\,\Gamma\left[(q-1)/2\right]}{2\Gamma\left[q/2\right]} \frac{\alpha+\gamma}{b^{q}}$$

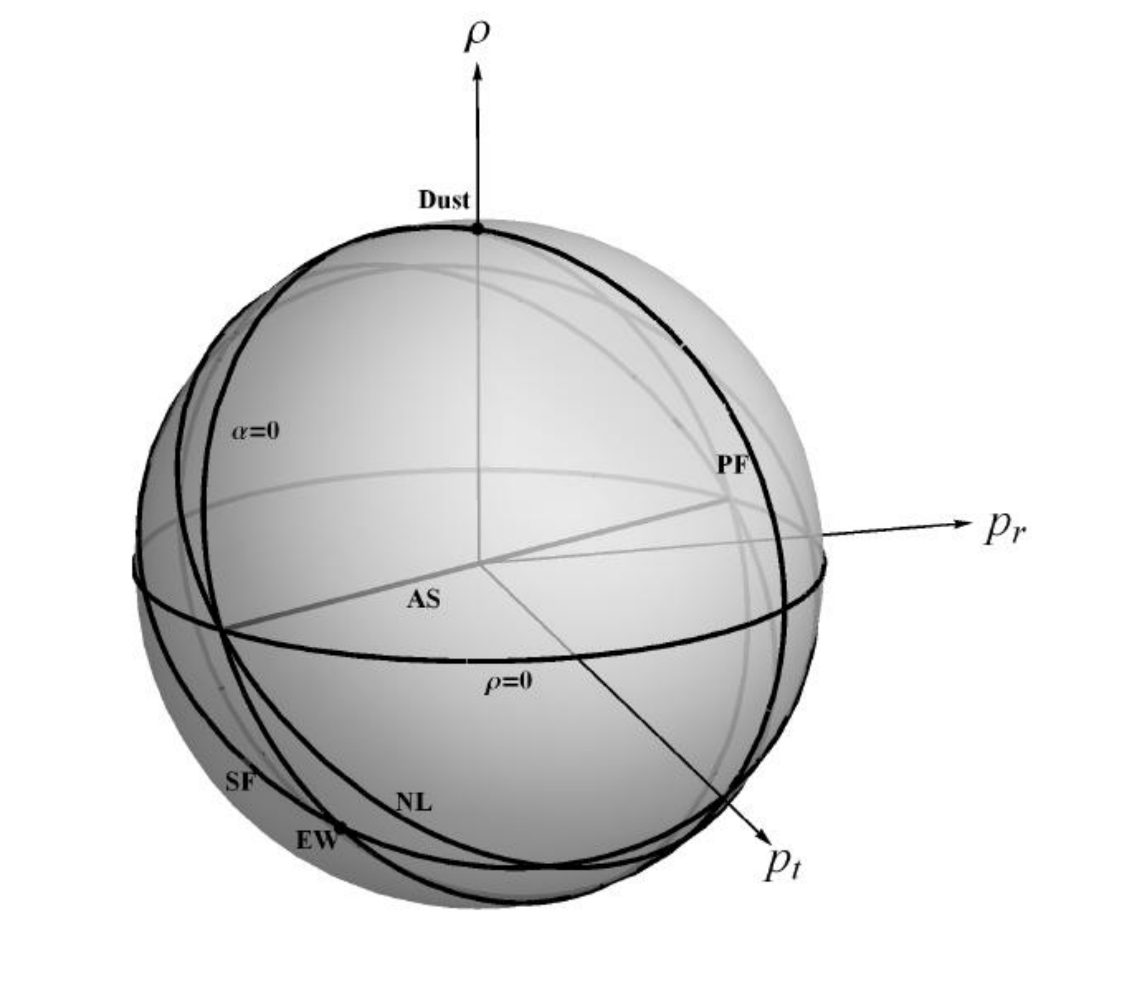
- The lensing convergence is

$$\kappa = \frac{1}{2}\int dz \left(T_t^{\;t} - T_z^{\;z}\right) = \frac{\sqrt{\pi}\,\Gamma\left[(1+q)/2\right]}{\Gamma\left[q/2\right]} \frac{(1-q)(\alpha+\gamma)}{b^{q+1}}$$

- For q>1 and positive deflection, the convergence is negative, implying a violation of the weak energy condition along the line of sight.
- The expression of the convergence also implies a violation of the strong energy condition.

Remarkable families

- Every model source is identified by the relative ratios of ρ, p_r, p_t.
- We can thus represent all models on a unit sphere.
- The perfect fluid slice (PF) cannot give asymptotically flat metrics.
- Interesting families include:
  AS: pure anisotropic stress
  ρ=0: no energy density
  α=0: no force
  NL: no lensing
  SF: scalar field with power-law potential.



(Bozza & Postiglione 2015).

Scalar field

- Consider a minimally coupled scalar field with a power-law potential

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{2\kappa}+\varepsilon\left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-V_0\varphi^n\right)\right]$$

- The ansatz $\varphi(r)=\frac{\varphi_0}{r^m}$ solves the equations with $V_0=\frac{2(4-n)\varphi_0^{2-n}}{n(n-2)^2};\quad m=\frac{1}{n/2-1}$

- We thus have a metric with

$$q=\frac{2}{n/2-1};\quad \alpha=\varepsilon\kappa\frac{(n-4)\varphi_0^2}{n(n-6)};\quad \gamma=\varepsilon\kappa\frac{2\varphi_0^2}{n(n-6)};$$

- This simple model shows how “exotic” matter supporting these metrics can be much more familiar than expected.

(Bozza & Postiglione 2015).

Conclusions

- Black holes are able to form an infinite number of images of a given source.
  - The mathematical structure of the caustics and the images has been clarified.
    - Sgr A* is a natural laboratory to observe gravitational lensing effects beyond the weak deflection regime.
      - Alternatives to Schwarzschild have very interesting phenomenological and theoretical implications.
        - With the progress of observational techniques, in the next few years the secrets of the event horizon will be probably disclosed!

(V. Bozza, "Gravitational Lensing by Black Holes", Gen. Rel. Grav. 2010)