Generalizing the Janis–Newman algorithm

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Outline

Introduction

Extension of JNA

Conclusion

Outline: 1. Introduction

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Plebański–Demiański solution ('76)

Most general black hole solution [Plebański–Demiański '76]

- Einstein–Maxwell theory with cosmological constant Λ (equivalently pure $N = 2$ gauged supergravity)
- 6 parameters
  - mass m
  - NUT charge n
  - electric charge q
  - magnetic charge p
  - rotation a
  - acceleration α

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  - acceleration α
- natural pairing as complex parameters

$$m + in, \qquad q + ip, \qquad a + i\alpha$$

Introduction

Demiański–Janis–Newman algorithm [Newman–Janis '65] [Demiański–Newman '66] [Demiański '72]

- idea: **complex** change of coordinates $\rightarrow$ new charges (rotation, NUT)
- off-shell (derived metric is **not** necessarily solution)
- two (equivalent) prescriptions: Newman–Penrose formalism (more rigorous), direct complexification (quicker) [Giampieri '90] [1410.2602, H.E.]

Introduction

Demiański–Janis–Newman algorithm [Newman–Janis '65] [Demiański–Newman '66] [Demiański '72]

- idea: **complex** change of coordinates → new charges (rotation, NUT)
- off-shell (derived metric is **not** necessarily solution)
- two (equivalent) prescriptions: Newman–Penrose formalism (more rigorous), direct complexification (quicker) [Giampieri '90] [1410.2602, H.E.]
- main achievement: discovery of Kerr–Newman solution [Newman et al. '65]
- before 2014: defined only for the metric, 3 examples fully known without fluid (and 2 partly) (Kerr, BTZ, singly-rotating Myers-Perry)

Needs for supergravity

- gauge fields
- complex scalar fields
- topological horizons
- dyonic charges
- NUT charge: understand the complexification

Needs for supergravity

- ✓ gauge fields [1410.2602, H.E.]
- ✓ complex scalar fields [1501.02188, H.E.–Heurtier]
- ✓ topological horizons [1411.2909, H.E.]
- ✓ dyonic charges [1501.02188, H.E.–Heurtier]
- ✓ NUT charge: understand the complexification [1411.2909, H.E.]
- ✓ *bonus:* higher dimensions [1411.2030, H.E.–Heurtier]

Janis–Newman algorithm

Giampieri's prescription

$$1)\quad \mathrm{d}t = \mathrm{d}u - k(r)\,\mathrm{d}r \quad \Longrightarrow \quad g_{rr} = 0$$

$$2)\quad u, r \in \mathbb{C}, \qquad f_i(r) \to \tilde{f}_i = \tilde{f}_i(r, \bar{r}) \in \mathbb{R}$$

$$3)\quad u = u' + i\,G(\theta), \qquad r = r' - i\,F(\theta)$$

$$4)\quad i\,\mathrm{d}\theta = \sin\theta\,\mathrm{d}\phi$$

$$5)\quad \left(\begin{cases} \mathrm{d}t' = \mathrm{d}u' - g(r)\mathrm{d}r \\ \mathrm{d}\phi' = \mathrm{d}\phi' - h(r)\mathrm{d}r \end{cases} \quad \Longrightarrow \quad \begin{cases} g_{tr} = 0 \\ g_{r\phi} = 0 \end{cases} \right)$$

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Complexification rules for $f \to \tilde{f}$

$$r \longrightarrow \frac{1}{2}\,(r + \bar{r}) = \mathrm{Re}\,r$$

$$\frac{1}{r} \longrightarrow \frac{1}{2}\left(\frac{1}{r} + \frac{1}{\bar{r}}\right) = \frac{\mathrm{Re}\,r}{|r|^2}$$

$$r^2 \longrightarrow |r|^2$$

Simple example (metric only)

Reissner–Nordström

$$\begin{aligned} \mathrm{d}s^2 &= -f\,\mathrm{d}t^2 + f^{-1}\,\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \\ &= -f\,\mathrm{d}u^2 - 2\,\mathrm{d}u\mathrm{d}r + r^2\mathrm{d}\Omega^2, \end{aligned} \qquad f = 1 - \frac{2m}{r} + \frac{q^2}{r^2}$$

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Kerr–Newman

$$\begin{aligned} \mathrm{d}s^2 = &- \tilde{f}\,(\mathrm{d}u' - a\sin^2\theta\,\mathrm{d}\phi)^2 + \rho^2\mathrm{d}\Omega^2 \\ &- 2\,(\mathrm{d}u' - a\sin^2\theta\,\mathrm{d}\phi)(\mathrm{d}r' + a\sin^2\theta\,\mathrm{d}\phi) \end{aligned}$$

$$\tilde{f} = 1 - \frac{2mr'}{\rho^2} + \frac{q^2}{\rho^2}, \qquad \rho^2 \equiv |r|^2 = r'^2 + a^2\cos^2\theta$$

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Outline: 2. Extension of JNA

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Reissner–Nordström

$$A = \frac{q}{r}\,\mathrm{d}t = \frac{q}{r}\left(\mathrm{d}u - f^{-1}\mathrm{d}r\right)$$

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Additional ingredient: gauge transformation to set

$$A_r = 0$$

$\rightarrow$ missing step in [Newman et al. 65']!
(other approach: [1407.4478, Keane])

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Additional ingredient: gauge transformation to set

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Kerr–Newman

$$A = \frac{qr'}{\rho^2}\,(\mathrm{d}u' - a\sin^2\theta\,\mathrm{d}\phi')$$

Scalar fields

Example: axion–dilaton pair

$$\tau = \mathrm{e}^{-2\phi} + i\sigma$$

Static

$$\mathrm{e}^{2\phi} = 1 + \frac{R}{r}, \qquad \sigma = 0$$

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Need to transform the complex field as a single entity

$$\tilde{\tau} = 1 + \frac{R}{r' - i\,a\cos\theta} = 1 + \frac{R\,(r' + i\,a\cos\theta)}{r'^2 + a^2\cos^2\theta}$$

Generates axion

$$\mathrm{e}^{2\phi} = 1 + \frac{R\,r'}{\rho^2}, \qquad \sigma = \frac{R\,a\cos\theta}{\rho^2}$$

Complex parameters

- presence of magnetic charge: use the central charge

$$Z = q + ip$$

example: dyonic Reissner–Nordström

$$A = \operatorname{Re} \frac{Z}{r} \, \mathrm{d}t + \operatorname{Im} Z \cos\theta \, \mathrm{d}\phi$$

Complex parameters

- presence of magnetic charge: use the central charge

$$Z = q + ip$$

example: dyonic Reissner–Nordström

$$A = \mathrm{Re}\,\frac{Z}{r}\,\mathrm{d}t + \mathrm{Im}\,Z\cos\theta\,\mathrm{d}\phi$$

- adding a NUT charge: complexify the mass, shift horizon curvature

$$m = m' + i\kappa n, \qquad \kappa = \kappa' - \frac{4\Lambda}{3}\,n^2$$

Full DJNA

Ansatz: Einstein–Maxwell plus scalar fields [1411.2909, H.E.]

$$\mathrm{d}s^2 = -f_t(r)\,\mathrm{d}t^2 + f_r(r)\,\mathrm{d}r^2 + f_\Omega(r)\,(\mathrm{d}\theta^2 + H'(\theta)^2\,\mathrm{d}\phi^2)$$
$$A = f_A(r)\,\mathrm{d}t, \qquad \chi = \chi(r)$$

DJN transformation

$$r = r' + i\,F(\theta), \qquad u = u' + i\,G(\theta), \qquad i\,\mathrm{d}\theta = H'(\theta)\,\mathrm{d}\phi$$

Resulting metric

$$\mathrm{d}s^2 = -\tilde{f}_t\big(\mathrm{d}t + \omega H\,\mathrm{d}\phi\big)^2 + \frac{\tilde{f}_\Omega}{\Delta}\,\mathrm{d}r^2 + \tilde{f}_\Omega\big(\mathrm{d}\theta^2 + \sigma^2 H^2 \mathrm{d}\phi^2\big)$$
$$\Delta = \frac{\tilde{f}_\Omega}{\tilde{f}_r}\,\sigma^2, \qquad \omega = G' + \sqrt{\frac{\tilde{f}_r}{\tilde{f}_t}}\,F', \qquad \sigma^2 = 1 + \frac{\tilde{f}_r}{\tilde{f}_\Omega}\,F'^2$$
$$A = \tilde{f}_A\left(\mathrm{d}t - \frac{\tilde{f}_\Omega}{\sqrt{\tilde{f}_t\tilde{f}_r}\,\Delta}\,\mathrm{d}r + G'H\,\mathrm{d}\phi\right), \qquad \tilde{\chi} = \tilde{\chi}(r,\theta)$$

Solutions for F and G

Strategy: find F and G by solving the equations of motion in one example, declare this transformation always valid [Demiański '72]

– $\Lambda = 0$

$$F(\theta) = n - a\,H(\theta) + c\left(1 + H(\theta)\,\ln\frac{H'(\theta/2)}{H(\theta/2)}\right)$$

$$G(\theta) = -2\kappa\,n\,\ln H'(\theta) + \kappa\,a\,H(\theta) - \kappa\,c\,H(\theta)\,\ln\frac{H'(\theta/2)}{H(\theta/2)}$$

– $\Lambda \neq 0$

$$F(\theta) = n, \qquad G(\theta) = -2\kappa\,n\,\ln H'(\theta)$$

Note: the solution is *not* unique

Parameters: n: NUT charge, a: rotation, c: ?

New examples

- Kerr–Newman–NUT
- dyonic Kerr–Newman
- Yang–Mills Kerr–Newman [Perry '77]
- adS–NUT Schwarzschild
- BPS solutions from $N = 2$ ungauged supergravity [hep-th/9705169, Behrndt–Lüst–Sabra]
- non-extremal rotating black hole in T^3 model [hep-th/9204046, Sen] [gr-qc/9907092, Yazadjiev]
- SWIP solutions [hep-th/9605059, Bergshoeff–Kallosh–Ortín]
- charged Taub–NUT–BBMB with Λ [1311.1192, Bardoux–Caldarelli–Charmousis]
- 5d Myers–Perry [Myers–Perry '86]
- BMPV [hep-th/9602065, Breckenridge–Myers–Peet–Vafa]

Outline: 3. Conclusion

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Achievements

- extend DJN algorithm to all fields with spin ≤ 2 and topological horizons
- define DJNA with m, n, p, q, a (a only for $\Lambda = 0$)

Outlook

- generalization to rotation when $\Lambda \neq 0$ and to acceleration (?)
- more $N = 2$ gauged supergravity solutions
- $d \geq 6$ Myers–Perry
- charged $d > 4$ black holes (in Einstein–Maxwell)

Thank you!