End of Single Scalar Inflation Models: Time-dependent zeta Correlators

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arXiv: 1601.01106

Curvature Power Spectrum:

$$\Delta_R^2(k,t) = \frac{k^3}{2\pi^2} \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} \langle \Omega | R(t,\vec{x}) R(t,\vec{0}) | \Omega \rangle$$

Perturbation theory

$ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x}$ with $H \equiv \frac{\dot{a}}{a}$; $\epsilon \equiv -\frac{\dot{H}}{H^2}$

after horizon crossing $(k = H(t_k)\, a(t_k))$

$\Delta_R^2(t,k) \rightarrow$ almost constant $\rightarrow$ drop t;

- Prediction of a typical single-scalar inflaton

tree order $\Rightarrow \Delta_R^2(k) \simeq \frac{G H^2(t_k)}{\pi \epsilon(t_k)}$; and observed value reconstructed from CMB data $\Rightarrow 2.14 \times 10^{-9}$

1502.01589

Table 4. Parameter 68 % confidence limits for the base ΛCDM model from *Planck* CMB power spectra, in combination with lensing reconstruction ("lensing") and external data ("ext," BAO+JLA+H_0). Nuisance parameters are not listed for brevity (they can be found in the *Planck Legacy Archive* tables), but the last three parameters give a summary measure of the total foreground amplitude (in μK^2) at $\ell = 2000$ for the three high-ℓ temperature spectra used by the likelihood. In all cases the helium mass fraction used is predicted by BBN (posterior mean $Y_P \approx 0.2453$, with theoretical uncertainties in the BBN predictions dominating over the *Planck* error on $\Omega_b h^2$).

$$\Delta_R^2(k_0) = (2.142 \mp 0.049) \times 10^{-9} \left(\frac{k}{k_0}\right)^{0.0333 \mp 0.0040}$$

Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062
$\Omega_m h^2$	0.1426 ± 0.0020	0.1415 ± 0.0019	0.1413 ± 0.0011	0.1427 ± 0.0014	0.1422 ± 0.0013	0.14170 ± 0.00097
$\Omega_m h^3$	0.09597 ± 0.00045	0.09591 ± 0.00045	0.09593 ± 0.00045	0.09601 ± 0.00029	0.09596 ± 0.00030	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8 \Omega_m^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_8 \Omega_m^{0.25}$	0.621 ± 0.013	0.6069 ± 0.0076	0.6066 ± 0.0070	0.623 ± 0.011	0.6091 ± 0.0067	0.6083 ± 0.0066
z_{re}	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_s$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021

$$\Delta_R^2(k_0) = (2.142 \mp 0.049) \times 10^{-9} \left(\frac{k}{k_0}\right)^{0.0333 \mp 0.0040}$$

- This measurement $\Rightarrow$ Qua. fluc. $\leftrightarrow$ cosmology

Q_1: Is it worth to calculate the qua. effects beyond tree level?

$Q_{1.1}$: Which kind of interactions have the biggest effect?

- A_1: expanding universe increases the effect of virtual particles; especially massless ones.
- $A_{1.1}$: conformal non-invariance enhances:

$\Rightarrow$ MMCS, Gravitons interacting with any particle.

E. Mottola $\Rightarrow$ conformal anomaly $\Rightarrow$ same reason!

It is instructive to see this by E–t uncert. reln.

- Flat Space

Before		After
vacuum		
$\vec{p} = 0$	$\longrightarrow$	$\vec{p} = \vec{k} - \vec{k}$
$E = 0$		$E = 2\sqrt{m^2+k^2}$

- $\Delta E\, \Delta t \geq \hbar$ to Not resolve ΔE

$$\Delta t \lesssim \frac{1}{2\sqrt{m^2+k^2}}$$

$\therefore$ $m=0$ biggest effect.

- Curved $\Rightarrow a(t) \neq 1$ $\quad x \to x + \Delta x$ is ok $\Rightarrow k$ is a good qua #

but $\quad \cdot\; x_{phys} = a(t)\, x \quad \Rightarrow k = \frac{2\pi}{\lambda} \to \vec{k}_{phys} = \frac{\vec{k}}{a(t)}$

$$\therefore\; E = \sqrt{m^2 + \frac{k^2}{a^2(t)}}$$

$\cdot$ E–t reln $\Rightarrow$ $\displaystyle\int_t^{t+\Delta t} dt'\, E(t',k) \sim 1$

for $m=0$, $a = e^{Ht}$ $\Rightarrow$ $\displaystyle\int_t^{t+\Delta t} dt'\, E(t',k) = \left[1 - e^{-H\Delta t}\right] \frac{k}{Ha(t)} \sim 1$

- $k \lesssim Ha(t)$ survive forever ($\Delta t \to \infty$ still ~ 1).

$\Rightarrow$ massless virtual particle can survive forever $\Rightarrow$ qua. fluc $\Rightarrow$ seed

But... how many of these virt. particles come out?

Conf. inv. plays a crucial role $\quad \phi \to \dfrac{\phi}{\Omega}$

$\tilde{g}_{\mu\nu} = \Omega^2(x)\, g_{\mu\nu} \qquad A_\mu \to A_\mu \qquad \psi \to \dfrac{\psi}{\Omega^{3/2}}$

$$\mathcal{L}_{EM} = -\frac{1}{4} F_{\rho\sigma} F_{\mu\nu}\, g^{\rho\mu} g^{\sigma\nu} \sqrt{-g} \to -\frac{1}{4} F_{\rho\sigma} F_{\mu\nu}\, \cancel{\Omega^{-2}}\, g^{\rho\mu}\, \cancel{\Omega^{-2}}\, g^{\sigma\nu}\, \cancel{\Omega^{4}} \sqrt{-g}$$

$\gamma \to$ conf. Inv.

Remember FRW $\quad ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x} = a^2(t)\left[-d\eta^2 + d\vec{x}\cdot d\vec{x}\right]$

$d\eta = \dfrac{dt}{a(t)}$; $g_{\mu\nu} = a^2 \eta_{\mu\nu} \Rightarrow$ conf flat geo. ; $\Omega = \dfrac{1}{a}$

$$\mathcal{L}_{conf}(g_{\mu\nu}, A_\mu, \psi, \phi) = \mathcal{L}_{conf}(\eta_{\mu\nu}, A_\mu, a^{3/2}\psi, a\phi)$$

- local field redefn $\Rightarrow$ won't change physics $\Rightarrow$ conf inv $\Rightarrow$ same as flat

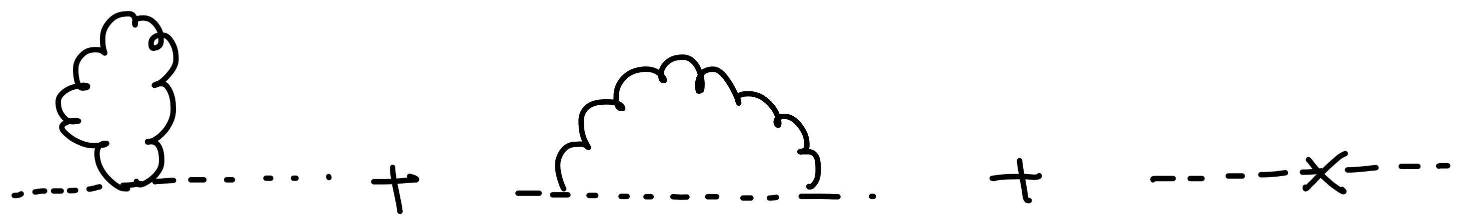
$\dfrac{dN}{d\eta} = \Gamma_{flat}$; $\dfrac{dN}{dt} = \dfrac{dN}{d\eta}\dfrac{d\eta}{dt} = \dfrac{\Gamma_{flat}}{a} \Rightarrow$ few will emerge

$\therefore$ Massless and Non-conf-inv $\Rightarrow$ Max effect $\qquad$ MMCS inter. others GR

* gr-qc/0508015 ; R.P. Woodard, EOK

MMCS + γ ; a Toy model

$-iM^2(x;x') =$



in x-space ; $i\Delta_{\mu\nu} \sim {}_2F_1\left(D-1, 2; \frac{D}{2}+1; 1-\frac{y}{4}\right) \frac{\partial^2 y}{\partial x^\mu \partial x'^\nu}$

+ ... and other hypers ; also we have $i\Delta(x;x')$.

$$\mathcal{L}_{SQED} = -(\partial_\mu - ie_0 A_\mu)\varphi^* (\partial_\nu + ie_0 A_\nu)\varphi\, g^{\mu\nu}\sqrt{-g}$$

$$- \xi_0 \varphi^* \varphi R \sqrt{-g} - \frac{1}{4} F_{\mu\nu} F_{\rho\sigma}\, g^{\mu\rho} g^{\nu\sigma} \sqrt{-g}$$

solve for φ ; $\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \varphi\right) - \int d^4x'\, M^2(x;x')\varphi(x') = 0$

Soln $\Rightarrow$ 0^{th} order: well known $u(\eta,k) = \frac{H}{\sqrt{2k^3}}\left[1-\frac{ik}{Ha}\right]\exp\left[\frac{ik}{Ha}\right]$

1-loop corrected mod. fnc. perturbatively;

$\Rightarrow$ $u_{1\text{-loop}} = u_0 \left[1 + \vartheta\left(\frac{\ln a}{a}\right)\right]$ • qr-qc/0508015

* Idea: Why don't we do it por inflaton?

• a much easier calculation with scalars •

$\Rightarrow$ not a good idea : small and irrelevant.

Just during those days:

• hep-th/0506236 $\Rightarrow$ • hep-th/0605244

Quantum contributions to cosmological correlations
Steven Weinberg (Texas U.). Aug 2005.
Published in **Phys.Rev. D72 (2005) 043514**
UTTG-01-05
DOI: 10.1103/PhysRevD.72.043514
e-Print: **hep-th/0506236** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; Phys. Rev. D Server
Detailed record - Cited by 404 records 250+

Quantum contributions to cosmological correlations. II. Can these corrections become large?
Steven Weinberg (Texas U.). May 2006. 10 pp.
Published in **Phys.Rev. D74 (2006) 023508**
UTTG-0306
DOI: 10.1103/PhysRevD.74.023508
e-Print: **hep-th/0605244** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; Phys. Rev. D Server
Detailed record - Cited by 168 records 100+

∴ Don't hesitate if you think you have a good idea!

- The question that Weinberg had was:
  Can qua. corrections become "large"?

VI. AFTERTHOUGHT

In generic theories the N integrals over time in N-th order perturbation theory will yield correlation functions at time t that grow as $(\ln a(t))^N$. Such a power series in $\ln a(t)$ can easily add up to a time dependence that grows like a power of $a(t)$, or even more dramatically. As everyone knows, the series of powers of the logarithm of energy encountered in various flat-space theories such as quantum chromodynamics can be summed by the method of the renormalization group. It will be interesting to see if the power series in $\ln a(t)$ encountered in calculating cosmological correlation functions at time t, though arising here in a very different way, can be summed by similar methods.

- arXiv: 0912.2734 (Senatore, Zaldarriaga)

  Claim: Weinberg is wrong, no t-dep for ζ.

* partially correct; Weinberg made an error in one example in his paper

* but general argument still true.

- arXiv: 1006.3999 ; EOK, V.K. Onemli, R.P. Woodard

Idea: Finding a counter-example for $\zeta-\zeta$ time dep.

$$\mathcal{L} = \left[\frac{R}{16\pi G} - \frac{1}{2}\varphi_{,\mu}\varphi_{,\nu}\, g^{\mu\nu} - V(\varphi) - \frac{1}{2}\sigma_{,\mu}\sigma_{,\nu}\, g^{\mu\nu} - U(\sigma)\right]\sqrt{-g}$$

use A.D.M 3+1 and Maldacena's procedure

$$g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt)$$

$$g_{ij} = a^2(t)\, e^{2\zeta(t,\vec{x})}\, \tilde{g}_{ij}(t,\vec{x}) \quad ; \quad \varphi(t,\vec{x}) - \varphi_0(t) = 0$$

$$R = \frac{e^{-2\zeta}}{a^2}\left[\tilde{R} - 2(D-2)\tilde{\nabla}^2\zeta - (D-2)(D-3)\,\zeta^{,k}\zeta_{,k}\right]$$

$$\Rightarrow \mathcal{R}(t,\vec{x}) \equiv -\frac{a^2(t)}{4\nabla^2} R = \left(\frac{D-2}{2}\right)\zeta(t,\vec{x}) + \mathcal{O}(\zeta^2, \zeta h, h^2)$$

$\therefore$ at lineer order $\langle \mathcal{R}\mathcal{R}\rangle \xrightarrow[D=4]{} \zeta-\zeta$ correlator

- Solving the constraints at quadratic order:

- $\mathcal{L}^{(2)}_{\zeta} = \frac{(D-2)\epsilon a^{D-1}}{16\pi G}\left\{\dot{\zeta}^2 - \frac{1}{a^2}\partial_k\zeta\,\partial_k\zeta\right\}$; • $\mathcal{L}^{(2)}_{h} = \frac{a^{D-1}}{64\pi G}\left\{\dot{h}_{ij}\dot{h}_{ij} - \frac{1}{a^2}\partial_k h_{ij}\,\partial_k h_{ij}\right\}$

- $\mathcal{L}^{(2)}_{\sigma} = \frac{a^{D-1}}{2}\left\{\dot{\sigma}^2 - \frac{1}{a^2}\partial_k\sigma\,\partial_k\sigma\right\}$; $\epsilon \sim \text{const} \Rightarrow i\Delta_\zeta(x;x') \approx \frac{8\pi G}{(D-2)\epsilon}\, i\Delta(x;x')$

Ford, Parker (77)
IR-problem $\Big\}$ • $i\Delta(x;x') = \int \frac{d^{D-1}k}{(2\pi)^{D-1}}\,\theta(k-L^{-1})\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$

$$\times\left\{\theta(t-t')u(t,k)u^*(t',k) + \theta(t'-t)u^*(t,k)u(t',k)\right\}$$

- $u(t,k)$ which I showed before in $(D=4)$

for const. $\epsilon \Rightarrow u(t,k) = \frac{\sqrt{\frac{\pi}{4(1-\epsilon)H}}}{a^{\frac{D-1}{2}}}\, H^{(1)}_{\nu}\left(\frac{k}{(1-\epsilon)Ha}\right)$; $\nu \equiv \frac{D-1-\epsilon}{2(1-\epsilon)}$

Ford - Vilenkin $\Rightarrow$ Inf . log (1982)

Let's look at $i\Delta \sim \int \frac{d^{D-1}k}{(2\pi)^{D-1}} u \cdot u^* \sim \int k^{D-2} dk \, |u|^2$

$u \sim H_\nu^{(1)}(-k\eta)$

remember $H_\nu^{(1)}(z) = J_\nu(z) + i N_\nu(z)$ or $\frac{1}{\sin(\nu\pi)}\left(\cos(\nu\pi) J_\nu(z) + i J_{-\nu}(z)\right)$

$$J_\nu(z) = \sum_{n=0}^{\infty} \frac{(z/2)^{\nu+2n}}{n!\,\Gamma(n+2\nu)} \Rightarrow J_{-\nu}(z) \sim z^{-\nu} \quad ; \nu = \frac{D-1}{2}$$

$$\therefore \quad i\Delta \sim \int dz \frac{z^{D-2}}{z^{D-1}} \Rightarrow \int \frac{dz}{z} \sim \log z$$

Can't integrate k integral all the way down (infrared div.)

This brings $\log a(t)$

$$S = \int d^4x\, a^3 \left[-\dot{H} M_{\rm Pl}^2 \left(\dot{\pi}^2 - \frac{1}{a^2}(\partial_i \pi)^2 \right) + \frac{2}{3} c_3 M^4 \left(2\dot{\pi}^3 + 3\dot{\pi}^4 - 3\frac{1}{a^2}\dot{\pi}^2 (\partial_i \pi)^2 \right) \right] \Rightarrow$$

all time dev

$\frac{d}{dt} \ln a(t) = H$

no time . dep.

Gettting $r \approx 16\epsilon$ reln:

From quadratic $\mathcal{L}^{(2)}$'s $\Rightarrow$ $i\Delta_\zeta(x;x') \approx \frac{8\pi G}{(D-2)\epsilon}\, i\Delta(x;x')$

$$i\left[{}_{ij}\Delta_{kl}\right](x;x') = 32\pi G\left[\Pi_{i(k}\Pi_{l)j} - \frac{\Pi_{ij}\Pi_{kl}}{(D-2)}\right] i\Delta(x;x')\,; \quad \Pi_{ij} \equiv \delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}$$

use the fact that $\lim_{t\to\infty} u(t,k) \simeq \frac{H(t_k)}{\sqrt{2k^3}}$

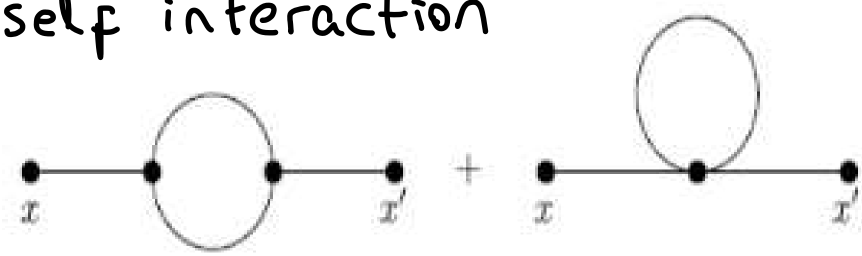
- $\left[\Delta_R^2(k,t)\right]_{\text{tree}} \simeq \frac{k^3}{2\pi^2} \times \frac{8\pi G}{2\epsilon} \times |u(t,k)|^2 \approx \frac{GH^2(t_k)}{\pi\epsilon}$
- $\left[\Delta_h^2(k,t)\right]_{\text{tree}} \simeq \frac{k^3}{2\pi^2} \times 32\pi G \times 2 \times |u(t,k)|^2 \approx \frac{16}{\pi} GH^2(t_k)$
- $r \equiv \frac{\Delta_h^2(k_0)}{\Delta_R^2(k_0)} \simeq 16\epsilon$ with $\Delta_h^2(t,k) \equiv \frac{k^3}{2\pi^2}\int d^3x\, e^{-i\vec{k}\cdot\vec{x}} \langle h_{ij}(t,\vec{x})\, h_{ij}(t,0)\rangle$

- Loop Contributions to $\Delta_{\mathcal{R}}^2$

Simplest generalization of $\mathcal{L}_\zeta$ and $\mathcal{L}_\sigma$

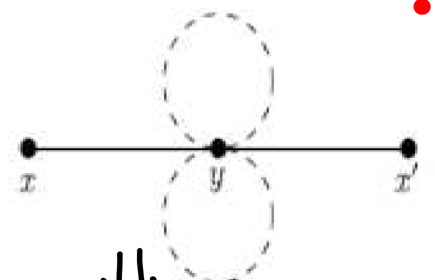
$$\mathcal{L}_\zeta \Rightarrow \frac{(D-2)\epsilon}{16\pi G} a^{D-1} e^{(D-1)\zeta} \left\{ \dot{\zeta}^2 - e^{-2\zeta} \partial_k\zeta \partial_k\zeta \right\} ; \mathcal{L}_\sigma = \frac{\epsilon}{D-1} a^{D-1} e^{(D-1)\zeta} U(\sigma)$$

- ζ self interaction



- spectator

$$U(\sigma) = \frac{\lambda\sigma^4}{4!}$$



$$\Downarrow \qquad\qquad \Downarrow$$

$$\left[\Delta_{\mathcal{R}}^2\right] \simeq \frac{GH^2}{\pi\epsilon}\left\{1 + \frac{27}{4\pi}\frac{GH^2}{\epsilon}\ln a + \mathcal{O}(G^2H^4)\right\} ; \quad \frac{GH^2}{\pi\epsilon}\left\{1 + \frac{\lambda GH^2}{48\pi^3}\ln^3 a\right\}$$

Remember $GH^2(t_{k_0}) \approx \frac{\pi}{16} \times r \times \Delta_{\mathcal{R}}^2(k_0) \sim 10^{-10}$

Planck 2015 $\Rightarrow r \lesssim 0.12$; $\epsilon(t_{k_0}) \simeq \frac{r}{16} \lesssim 0.0075$

$\Delta_r \approx 2 \times 10^{-9}$

On Loops in Inflation II: IR Effects in Single Clock Inflation
Leonardo Senatore (Stanford U., ITP & KIPAC, Menlo Park), Matias Zaldarriaga (Princeton, Inst. Advanced Study). Mar 2012. 13 pp.
Published in **JHEP 1301 (2013) 109**
SLAC-PUB-15860
DOI: 10.1007/JHEP01(2013)109
e-Print: **arXiv:1203.6354 [hep-th]** | PDF

intro

On Loops in Inflation III: Time Independence of zeta in Single Clock Inflation
Guilherme L. Pimentel (Princeton U.), Leonardo Senatore (Stanford U., ITP & KIPAC, Menlo Park & SLAC), Matias Zaldarriaga (Princeton, Inst. Advanced Study). Mar 2012. 48 pp.
Published in **JHEP 1207 (2012) 166**
DOI: 10.1007/JHEP07(2012)166
e-Print: **arXiv:1203.6651 [hep-th]** | PDF

a more serious calculation

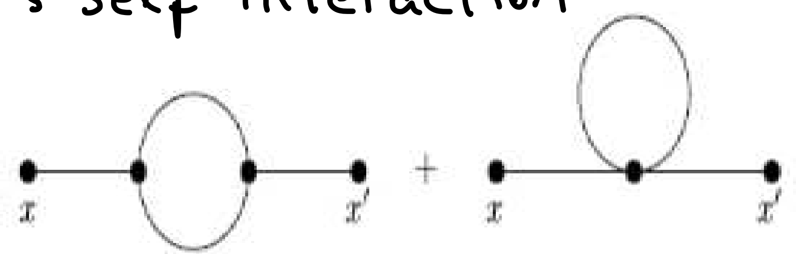
The constancy of ζ in single-clock Inflation at all loops
Leonardo Senatore (CERN & Stanford U., Phys. Dept. & Stanford U., ITP & SLAC & KIPAC, Menlo Park), Matias Zaldarriaga (Princeton, Inst. Advanced Study). Oct 2012. 15 pp.
Published in **JHEP 1309 (2013) 148**
DOI: 10.1007/JHEP09(2013)148
e-Print: **arXiv:1210.6048 [hep-th]** | PDF

???

- many flaws in their arguments:
  Pimentel accepts ϕ^4 giving t-dep but ...
  "Proof": taking $k \ll H$, $k \gg H$ but ignore $k \sim H$ which give logs
  all orders
- EFT of inflation ...

- 1-loop Correction to non-Gaussianity

- ζ self interaction



$$U_{tree} \simeq \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}} \left\{ 1 + \frac{k^2\eta^2}{2} + \cdots \right\} ; k\eta \ll 1$$

$$U_{1\text{-loop}} \simeq U_{tree} \left\{ 1 + \mathcal{O}(1)\, G H^2 \ln a + \cdots \right\}$$

$$U'_{tree} \Rightarrow \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}} \left\{ k^2\eta + \cdots \right\}$$

$$U'_{1\text{-loop}} \Rightarrow \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}}\, k^2\eta \left\{ 1 + \mathcal{O}(1)\, G H^2 \frac{1}{k^2\eta^2} \right\}$$

$$= U'_{tree} \left\{ 1 + \mathcal{O}(1)\, G H^2 \frac{1}{k^2\eta^2} \right\}$$

Power Spectrum $\sim \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle \sim \delta^3(\vec{k}_1 + \vec{k}_2)\, |U_k|^2$

Bispectrum $\sim \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \sim \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, U_{k_1}(\eta)\, U_{k_2}(\eta)\, U_{k_3}(\eta)$

$$* \left\{ \int_{\eta_0}^{\eta} \frac{d\eta}{\eta^3}\, U'^{*}_{k_1} U'^{*}_{k_2} U'^{*}_{k_3} + c.c. \right\}$$

$\therefore$ The loop correction to Power Spectrum is small

$$|U_k|^2 \sim |U_k|^2_{\text{tree}} \left(1 + \mathcal{O}(1)\, G H^2 \ln a\right)$$

But Bispectrum $\rightarrow f_{NL} \;\propto\; U^{*\prime}_{k_1}\, U^{*\prime}_{k_2}\, U^{*\prime}_{k_3}$

$$\therefore \text{loop corrected} \sim \left(U^{*\prime}_{k_1}\, U^{*\prime}_{k_2}\, U^{*\prime}_{k_3}\right)_{\text{tree}} \left[1 + \left(\frac{G H^2 \ln a}{k^2 \eta^2}\right)^3\right]$$

For $k\eta \ll 1$ (60-efolds) ; $GH^2 \sim 10^{-10}$

$\eta \sim -\dfrac{1}{aH}$ $\qquad$ loop correction $\sim$ tree $\left[1 + \left[\dfrac{10^{-10} \times 10^{2}}{e^{60}}\right]^3\right]$

Huge correction that would make all single scalar inflation models to be ruled out.

CONCLUSIONS

- Qua effects during Inflation are already observed. Why not go beyond tree-level ?
  - Massless and conformal non-inv. particles give rise to biggest effect.
  - Time dependent $[\log a(t)]$ corrections do occur for certain interactions:
- e.g. ζ - loops $\Rightarrow$ $U_0\left(1 + \# \frac{GH^2}{\epsilon} \ln a + \dots\right)$

  σ - loops $\Rightarrow$ $U_0\left(1 + \# \lambda G H^2 \ln^3 a + \mathcal{O}(\lambda^2)\right)$

  $GH^2(t_{ko}) \approx 10^{-10}$; $\ln a \sim 10^2$; $\frac{1}{\epsilon} \sim 10^2$

  $GH^2/_{\epsilon} \ln a \Rightarrow 10^{-6} \neq 0$; small but not zero.

- For power spectrum : correction is there but small

BUT for Bispectrum : 1-loop dominates the tree-level by 30-orders of magnitude

$\Rightarrow$ huge value for f_{NL} $$B_\zeta^{\mathrm{local}}(k_1,k_2,k_3)_{k_1\approx k_2\gg k_3} \approx \frac{12}{5} f_{\mathrm{NL}} P(k_1)P(k_3)$$

But *Planck* 2015 results. XVII. Constraints on primordial non-Gaussianity $f_{NL}^{local} = 2.5 \pm 5.7$

$$B_\zeta(k_3 \ll k_1) = (1-n_s)P(k_1)P(k_3)$$ $\Rightarrow$ Creminelli-Zaldarriaga consistency condition doesn't apply ($u_\sim$ not const.)

$\therefore$ Loop do matter after all.