

# Effects of quantum gravity in MAGIC experiment

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& MAGIC Collaboration

- Manifestation of quantum gravity (QG) for a “low” energy radiation probe
- QG induced spectral time lags in emission from astrophysical sources
- MAGIC telescope observations of flaring by Mkn 501
- Optimisation of a dispersive broadened pulse by the energy cost function (ECF)
- Log-likelihood cross-check
- Conclusions

The existence of the lower bound at which space-time responses actively to the present of energy, may lead to violation of Lorentz invariance.

- Liouville strings (J. Ellis, N. Mavromatos, D. Nanopoulos, 1997, 1998, 1999)
- Effective field theory approach (R.C. Mayers, M. Pospelov 2003)
- Space-time foam (L.J. Garay 1998)
- Loop quantum gravity (R. Gambini, J. Pullin, 1999)
- Noncommutative geometry (G. Amelino-Camelia, 2001)

Modification of the propagation of energetic particles due to nontrivial refractive index induced by the QG fluctuations in the space-time foam.

In the approximation  $E \ll M$ , the distortion of the standard dispersion relations may be represented as an expansion in  $E/M$ :

$$E^2 = m^2 + p^2(1 + \xi_1(p/M) + \xi_2(p/M)^2 + \dots)$$

Linear deviation

$$\xi_1 < 0; \quad v = c\left(1 - \frac{E}{M_{QG1}}\right); \quad n(E) = 1 + \frac{E}{M_{QG1}}$$

Quadratic deviation

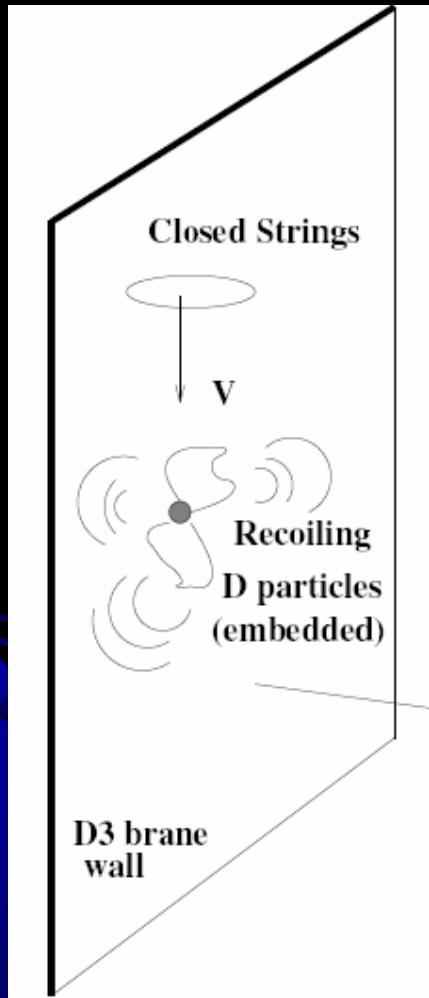
$$\xi_1 = 0; \quad \xi_2 < 0; \quad v = c\left(1 - \frac{E^2}{M_{QG2}^2}\right); \quad n(E) = 1 + \frac{E^2}{M_{QG2}^2}$$

# Effective field theory in a preferred reference frame

Dimension 5 operators (Lorentz symmetry is broken by a background 4 vector)

- Quadratic in the same field
- One more derivative than the usual kinetic term
- Gauge invariant
- Lorentz invariant, except for the appearance of the 4 vector
- Not reducible to lower dimension operators by the equations of motion
- Not reducible to a total derivative

## Example with a recoil



$$G_{ij} = \delta_{ij}, \quad G_{00} = -1, \quad G_{0i} \sim \bar{U}_i$$

$$h_{0i} \simeq \bar{U}_i \quad \bar{U}/c = O(E/M_D c^2)$$

$$G_{00} \equiv -h, \quad \mathcal{G}_i = -\frac{G_{0i}}{G_{00}}, \quad i = 1, 2, 3$$

$$\nabla \cdot B = 0, \quad \nabla \times H = \frac{1}{c} \frac{\partial}{\partial t} D = 0$$

$$\nabla \cdot D = 0, \quad \nabla \times E = -\frac{1}{c} \frac{\partial}{\partial t} B = 0$$

$$D = \frac{E}{\sqrt{h}} + H \times \mathcal{G}; \quad B = \frac{H}{\sqrt{h}} + \mathcal{G} \times E$$

$$\frac{1}{c^2} \frac{\partial^2}{\partial^2 t} B - \nabla^2 B - 2 (\bar{U} \cdot \nabla) \frac{1}{c} \frac{\partial}{\partial t} B = 0$$

$$\frac{1}{c^2} \frac{\partial^2}{\partial^2 t} E - \nabla^2 E - 2 (\bar{U} \cdot \nabla) \frac{1}{c} \frac{\partial}{\partial t} E = 0$$

$$E_y(x, t) = E_0 e^{ikx - \omega t}; \quad B_z(x, t) = B_0 e^{ikx - \omega t}$$

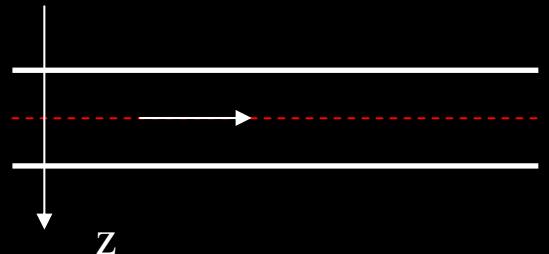
$$k^2 - \omega^2 - 2\bar{U}k\omega = 0$$

## Example with photon on the brane

$$\epsilon = M^{-1}$$

$$ds^2 = e^{-z^2/\epsilon^2} dl^2 - dz^2 \quad dl^2 = g_{\alpha\beta}(x^\nu) dx^\alpha dx^\beta$$

$\alpha, \beta, \dots = 0, 1, 2, 3$



$$g^{AB} P_A P_B = 0 \quad \frac{E^2}{v^2} - P^2 - e^{-z^2/\epsilon^2} P_z^2 = 0 \quad \text{- dispersion relation}$$

$$P \sim P_z \quad \text{- in Minkowski}$$

$$v^2 = \frac{c^2}{1 + e^{-z^2/\epsilon^2}} \approx \frac{c^2}{2} \left( 1 + \frac{z^2}{\epsilon^2} \right)$$

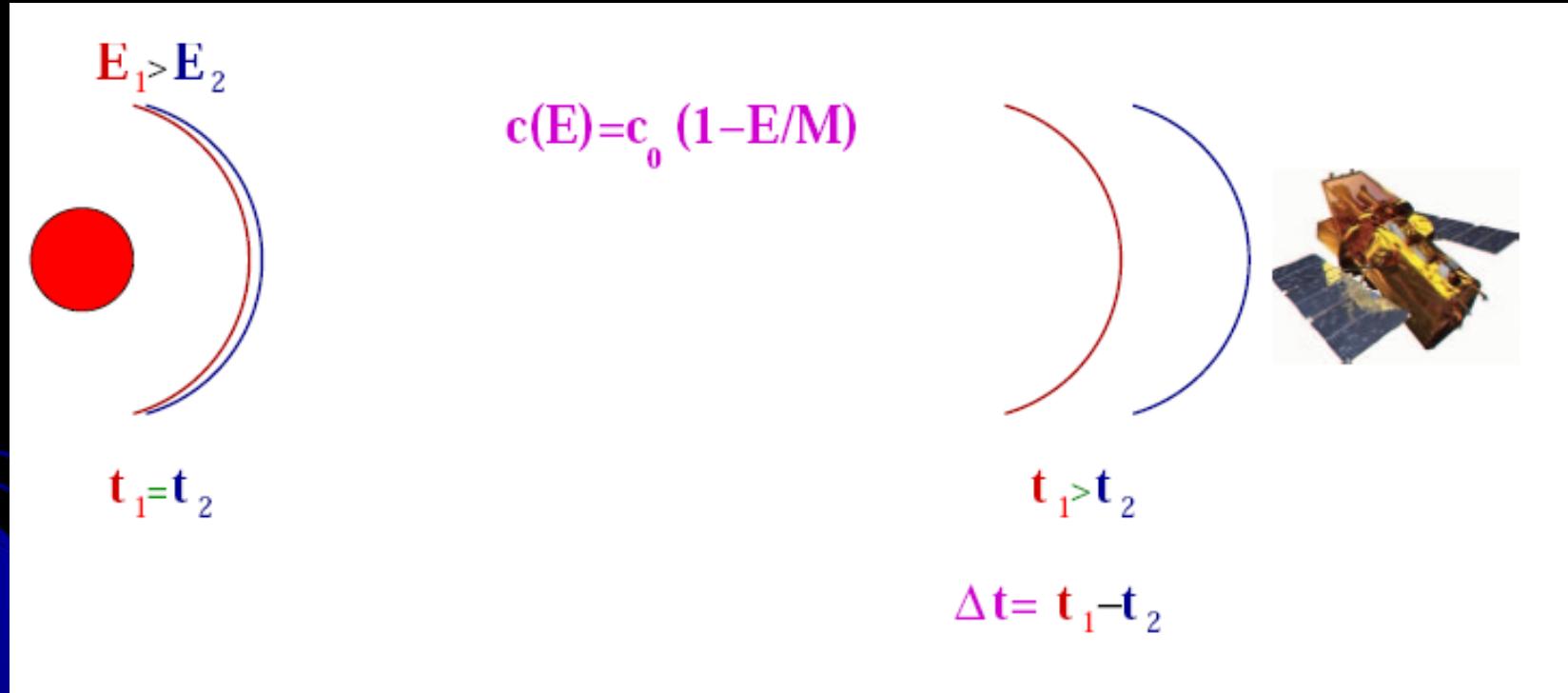
$$U \sim maz \quad \text{- potential energy of a particle} \quad a \sim \Gamma_{00}^z$$

$$z \sim E \quad \frac{z^2}{\epsilon^2} \sim \frac{E^2}{E_\epsilon^2} \sim \frac{E^2}{M^2 c^4} \quad v = \frac{c}{\sqrt{2}} \left( 1 + \frac{E^2}{M^2 c^4} \right)^{1/2}$$

$$n(E) = 1 + \frac{E^2}{\tilde{M}^2}$$

(Gogberashvili, Sarkisyan, A.S., 2006)

The modification of the group velocity would affect the simultaneity of the arrival times of photons with different energies from remote sources



(G. Amelino-Camelia et al, 1998)

Light propagating from the remote object is affected by the expansion of the Universe

$$dt = -H_0^{-1} \frac{dz}{(1+z)h(z)}; \quad h(z) = \sqrt{\Omega_\Lambda + \Omega_M(1+z)^3}$$

$$\Delta L = H_0^{-1} \int_0^z \frac{\Delta u dz}{h(z)}$$

We consider two photons traveling with velocities very close to  $c$ , whose present day energies  $E1 > E2$ .

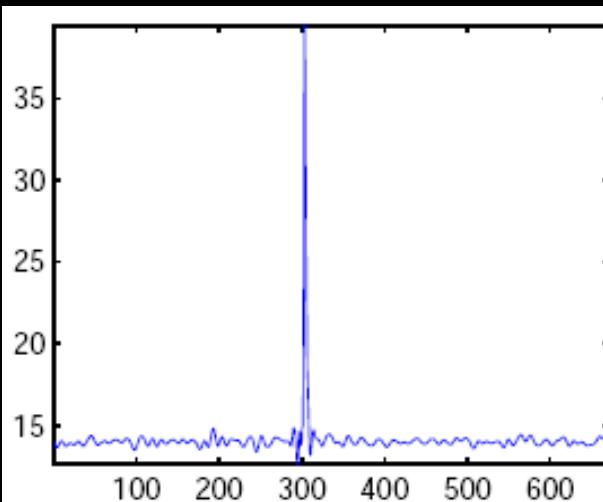
Linear

$$\Delta t = H_0^{-1} \frac{\Delta E}{M_{QG1}} \int_0^z \frac{(1+z)dz}{h(z)}$$

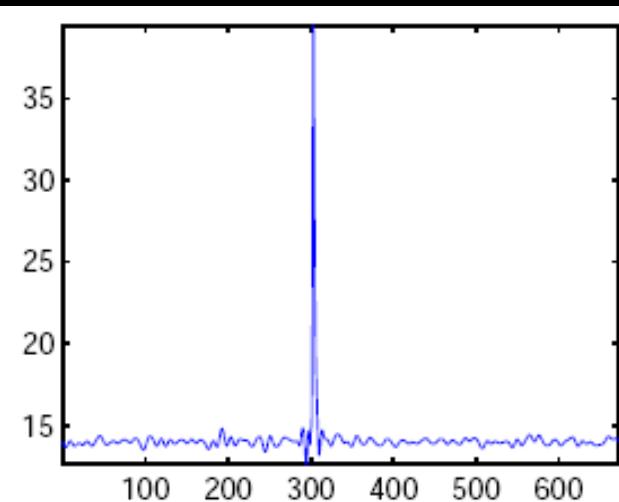
Quadratic

$$\Delta t = H_0^{-1} \frac{\Delta E(E1+E2)}{8\pi M_{QG2}^2} \int_0^z \frac{(1+z)^2 dz}{h(z)}$$

At the source



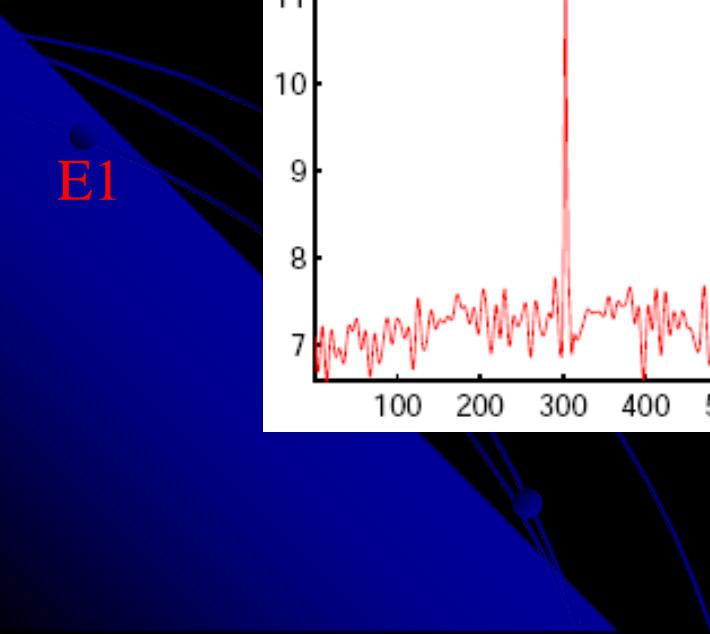
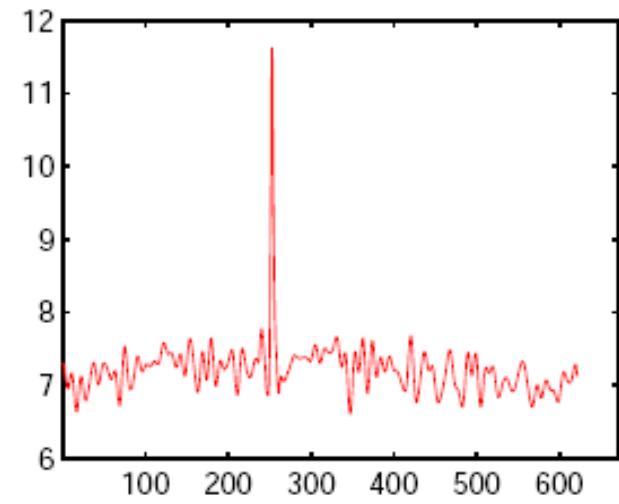
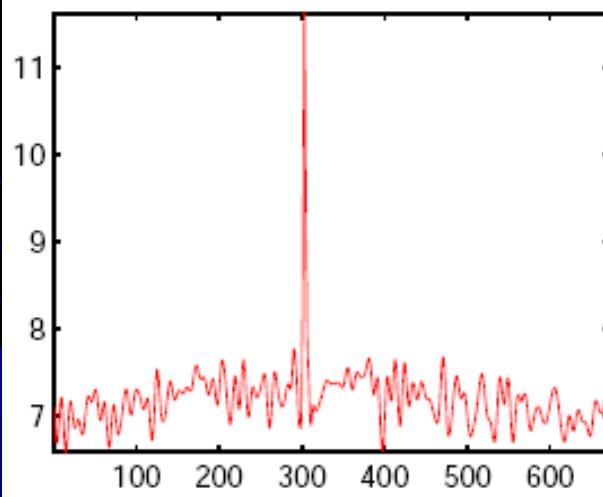
At the Earth



$E_2$

$E_2 < E_1$

$E_1$



- Gamma rays of high energies (**E**)
- Cosmological distances (**D**)
- Short duration transients in time profiles

Pulsars: **E** up to 2GeV, **D** about 10 kpc, (Kaaret, 1999)

$$M_{QG1} \geq 1.5 \times 10^{15} \text{ GeV} \quad \text{ms}$$

AGNs: **E** up to 10 TeV, **D** about 100s Mpc (Biller, et al, 1999)

$$M_{QG1} \geq 4 \times 10^{16} \text{ GeV} \quad M_{QG2} \geq 6 \times 10^9 \text{ GeV} \quad 10 \text{ minutes}$$

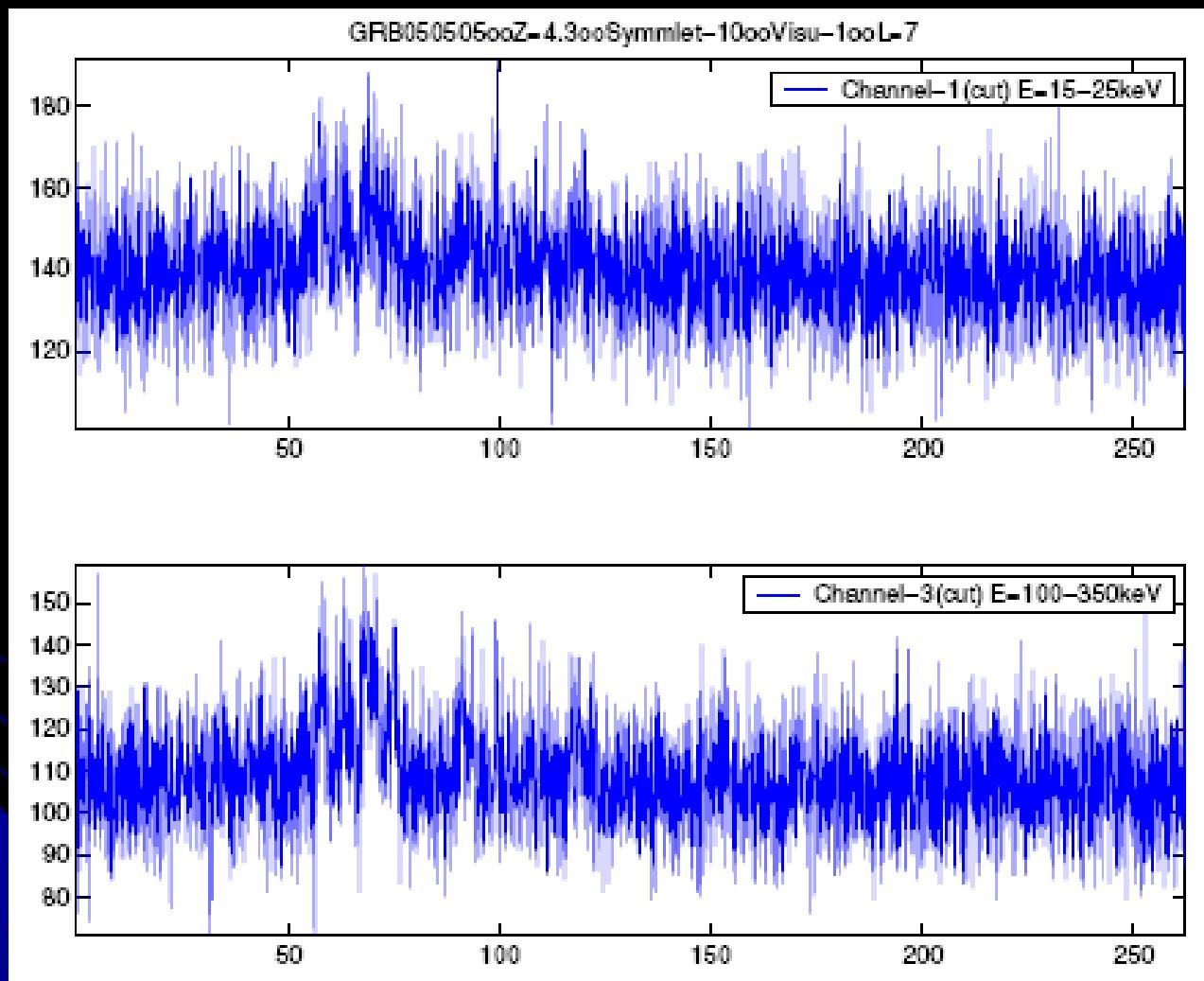
GRBs: **E** up to MeV, **D** beyond 7000 Mpc

$$M_{QG1} \geq 1.4 \times 10^{16} \text{ GeV} \quad M_{QG1} \geq 1.2 \times 10^6 \text{ GeV} \quad 100 \text{ ms}$$

(Ellis, et al, 2005,2007)

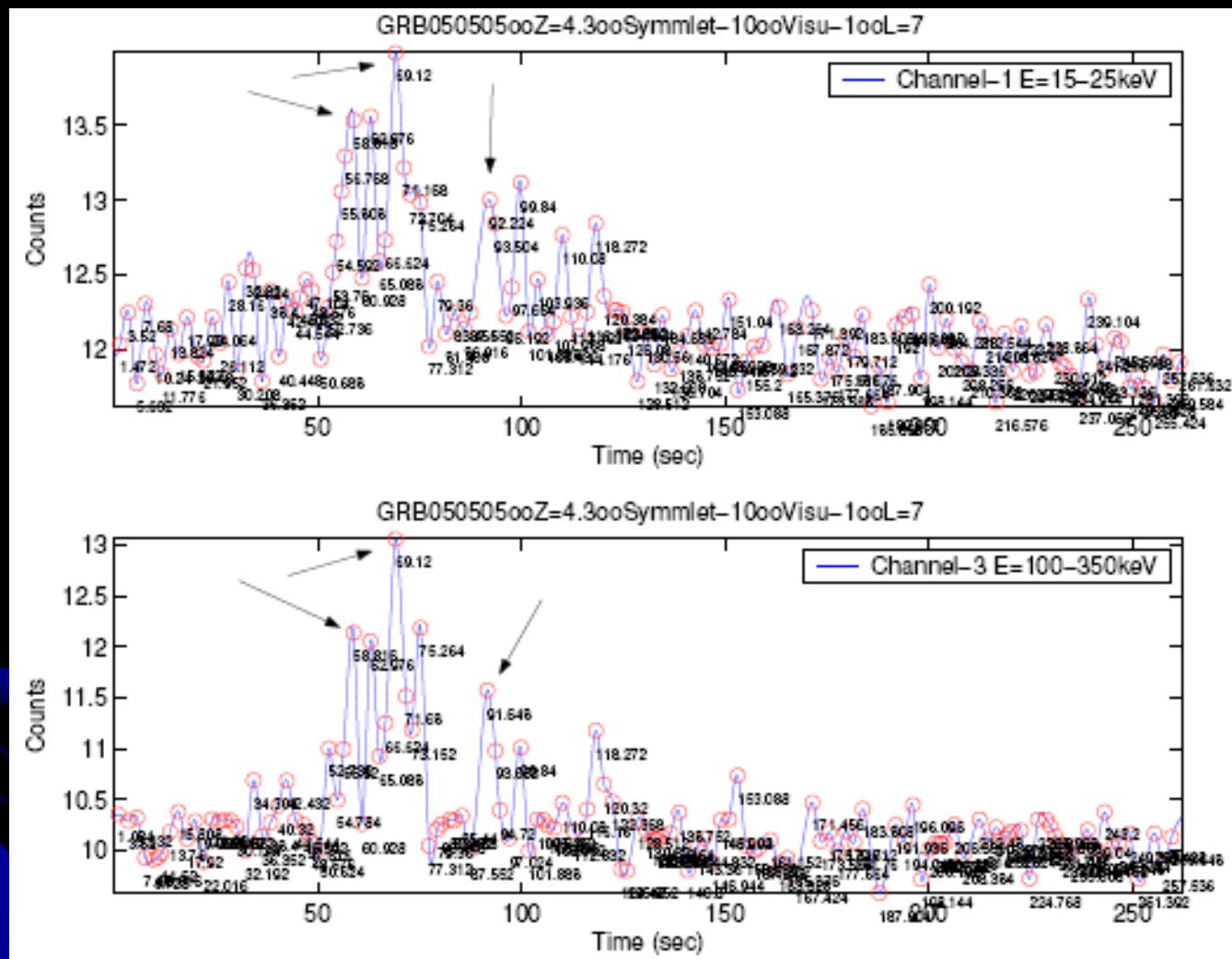
(Gogberashvili, Sarkisyan, A.S., 2006)

# GRBs



(Ellis, et al 2002, 2005, 2007; Bolmont, et al 2006; Lamon, et al, 2007)

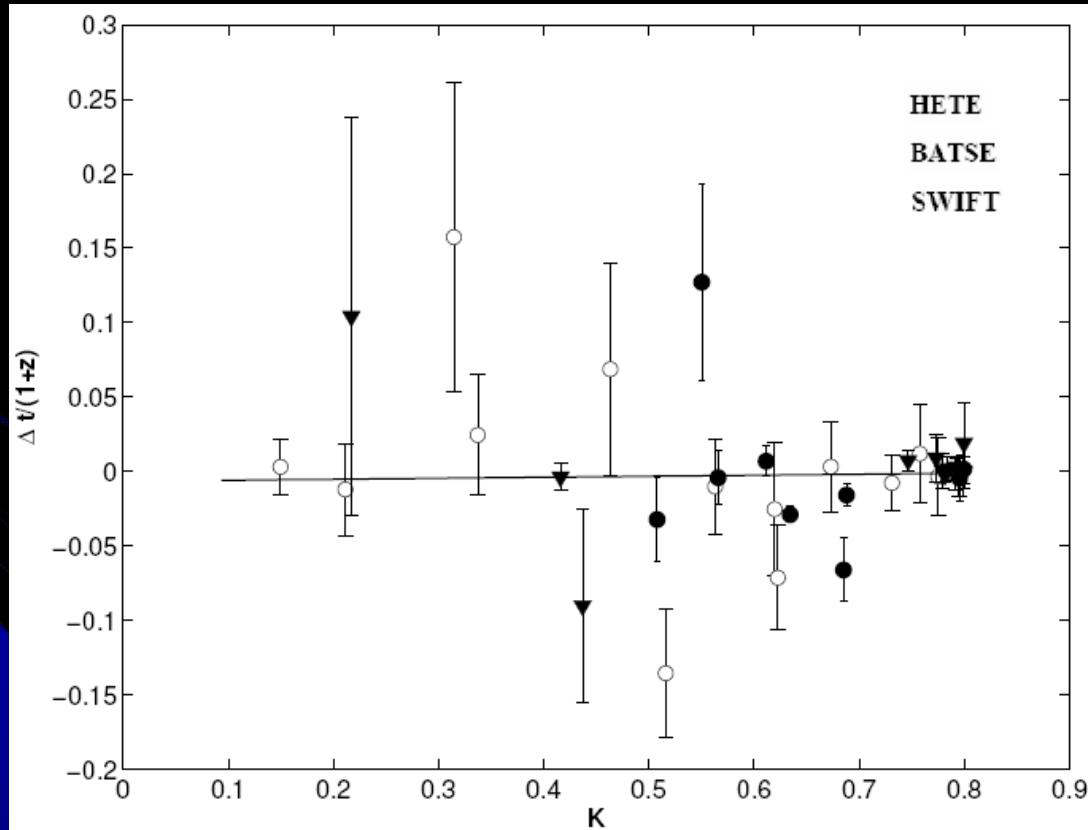
# GRBs



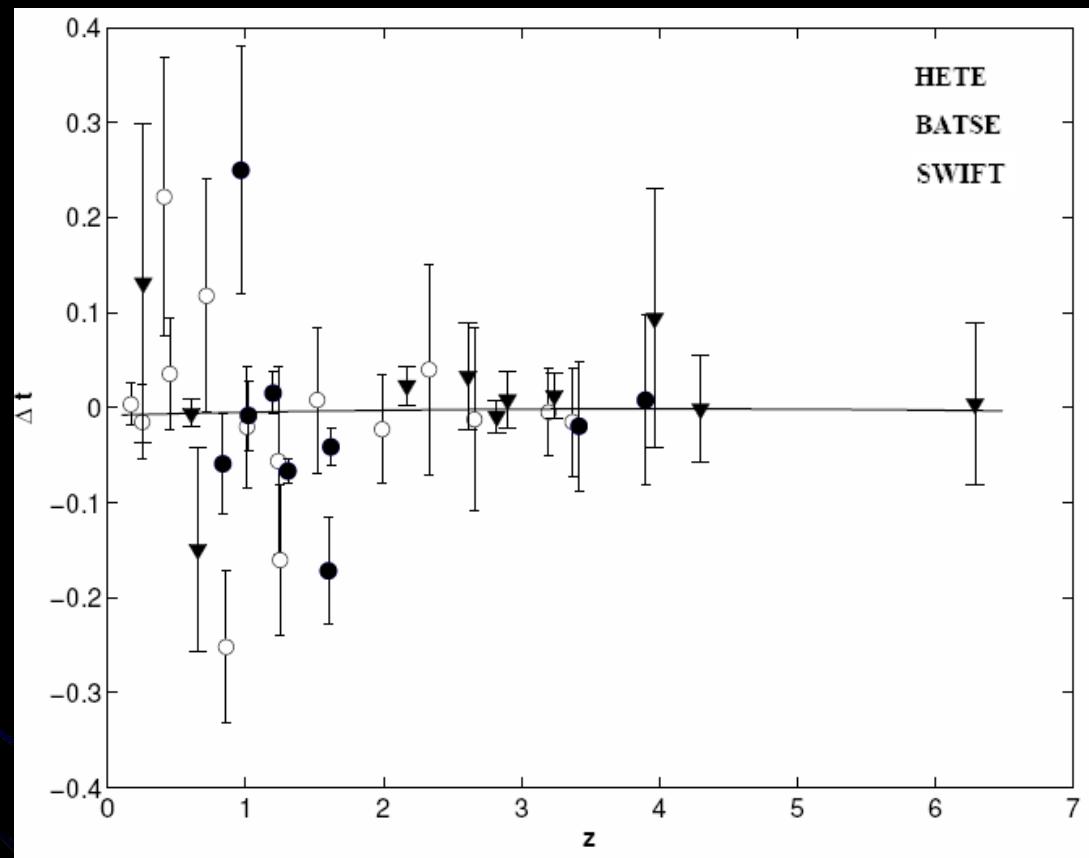
$$\Delta t_{\text{obs}} = \Delta t_{LV} + b_{\text{sf}}(1+z)$$

$$\frac{\Delta t_{\text{obs}}}{1+z} = a_{LV} K + b_{\text{sf}}$$

$$a_{LV} = H_0^{-1} \frac{\Delta E}{M} \quad K = \frac{1}{1+z} \int_0^z \frac{(1+z)dz}{h(z)}$$

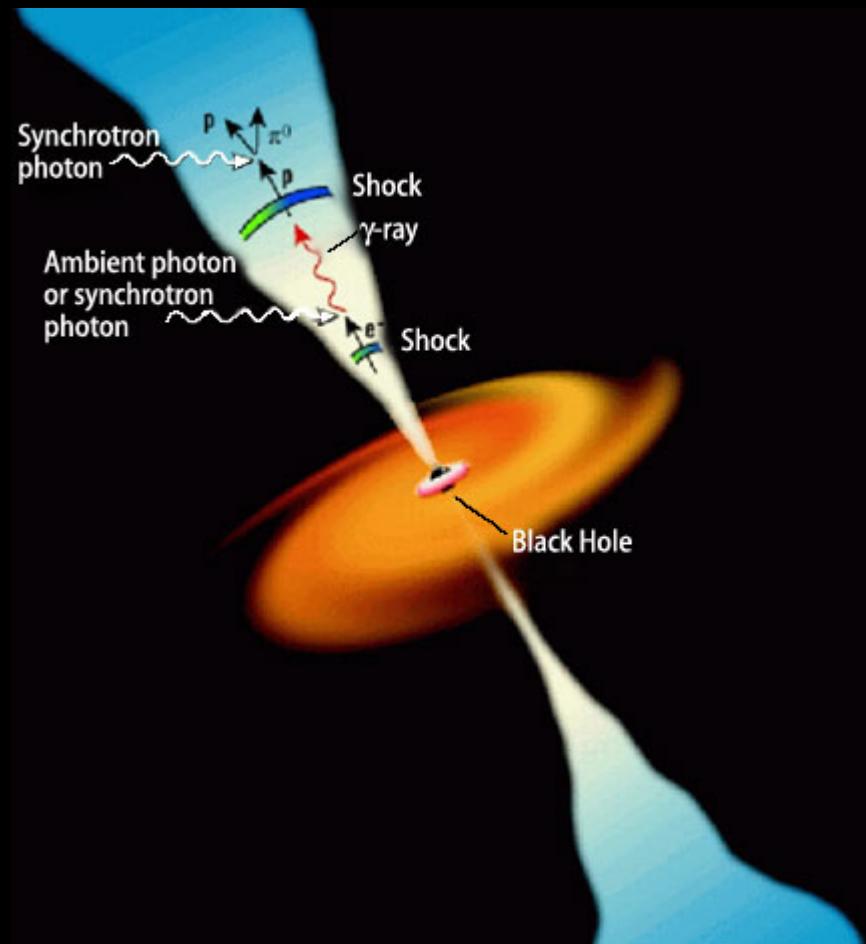
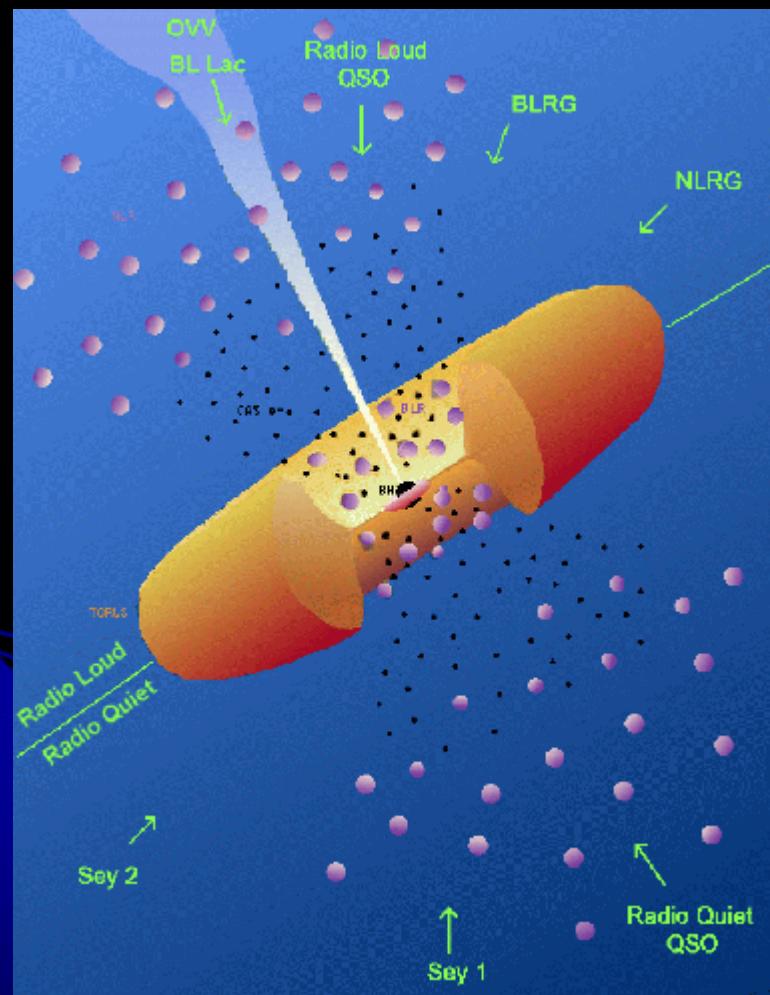


$$\Delta t_{\text{obs}} = a_{\text{LV}} I(z) + b_{\text{sf}}(1+z)$$

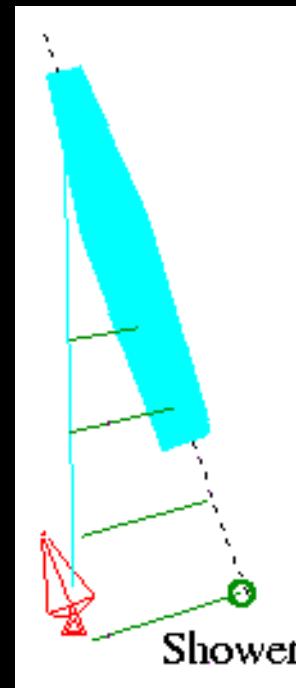
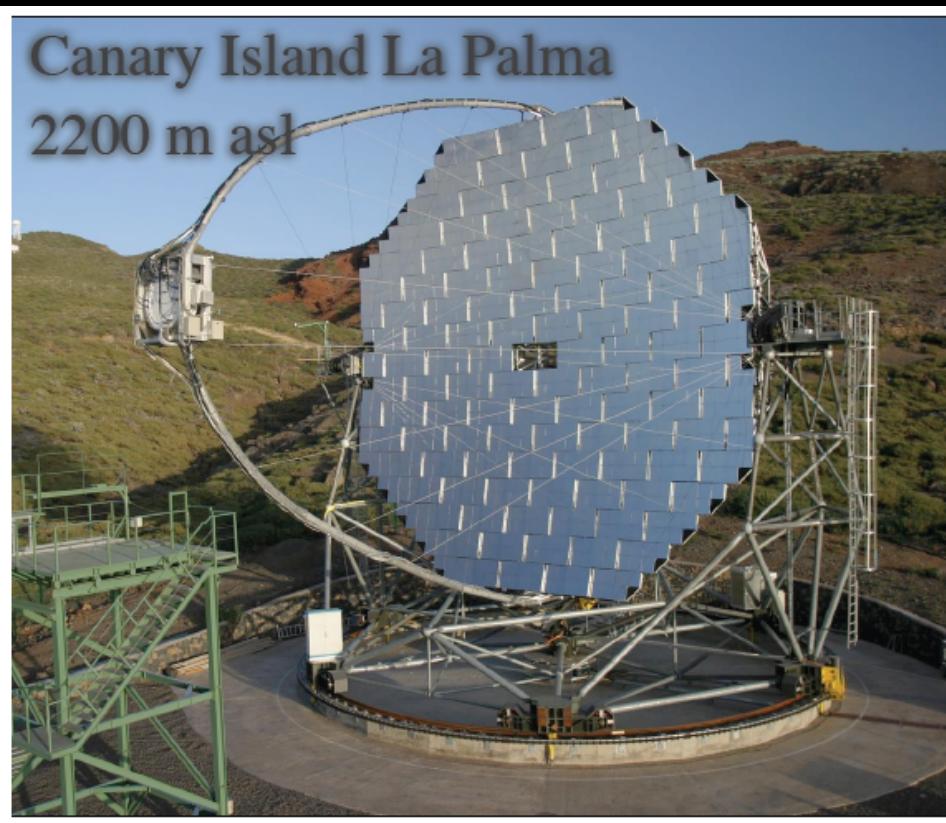


$$I(z) = \int_0^z \frac{(1+z)dz}{h(z)} = 0.001817z^3 - 0.05956z^2 + 1.207z$$

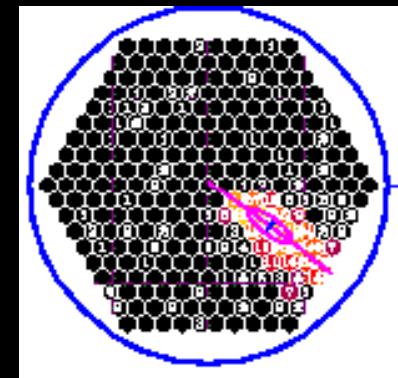
# AGNs



Canary Island La Palma  
2200 m asl



MAGIC

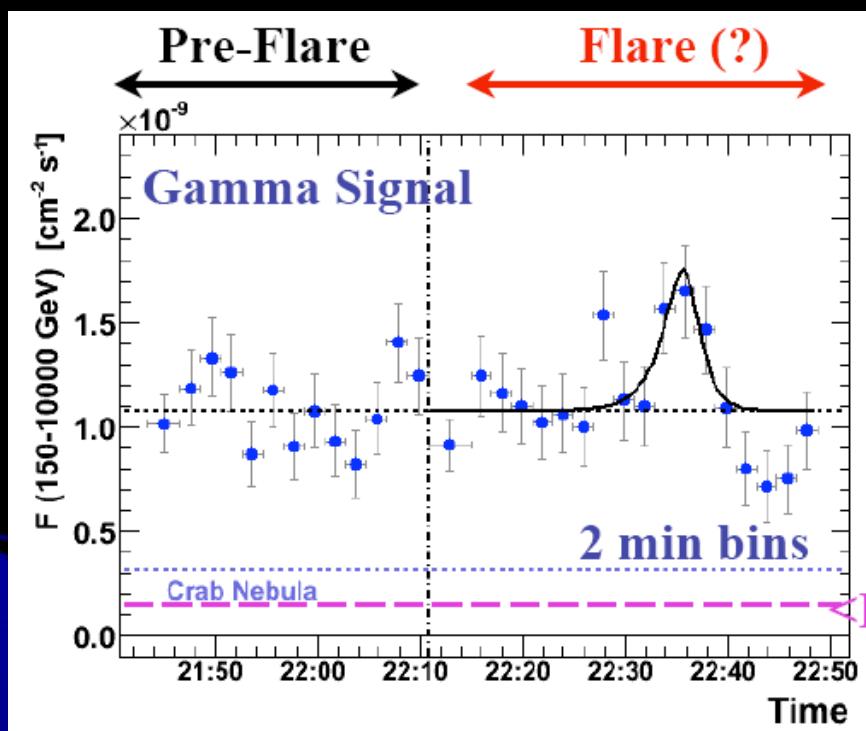


17 m  $\varnothing$  mirror dish ( $239 \text{ m}^2$ )  
 $3.5^\circ$  Field of View camera with **576**  
high-QE PMTs  
Fast repositioning  $t_R < 40 \text{ s}$

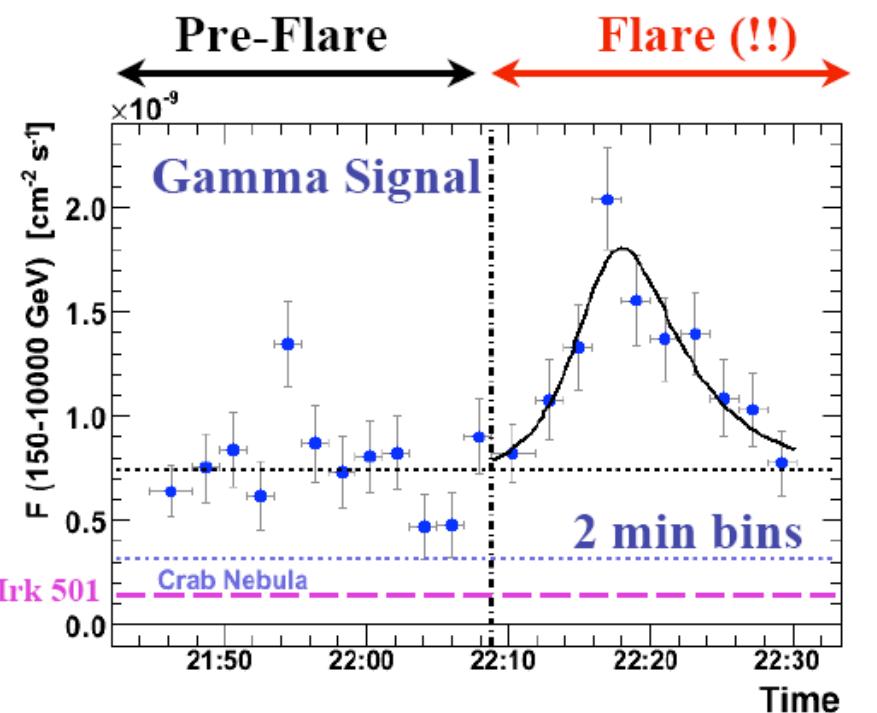
Trigger threshold energy: **~50 GeV**  
Minimum energy for spectral analysis :  
**100 GeV**  
Angular resolution per incoming photon:  **$0.1^\circ$ - $0.15^\circ$**   
Energy resolution : **20% - 30%**  
Point source sensitivity:  
**2.5% Crab / 50 hours**

# Mkn 501 Flares z=0.034 (D=146 Mpc)

June 30th

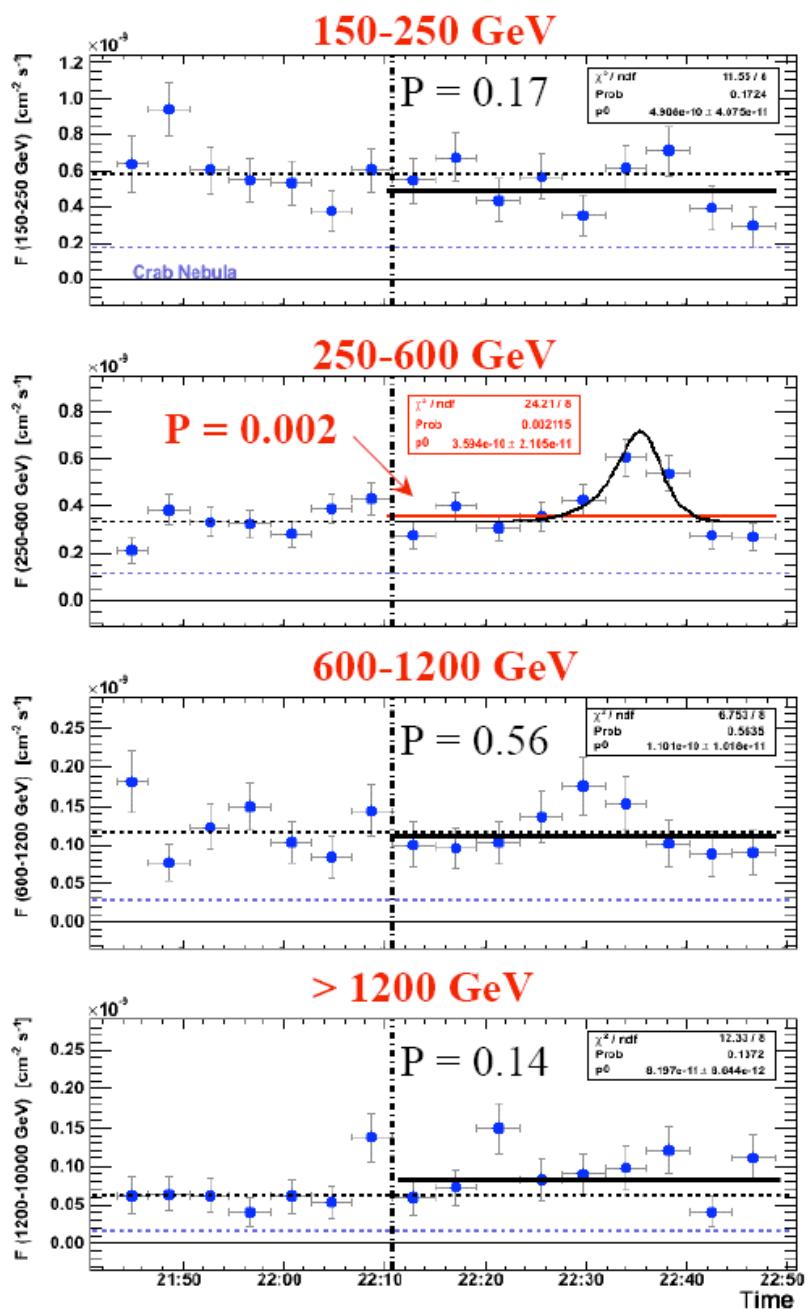


July 9th

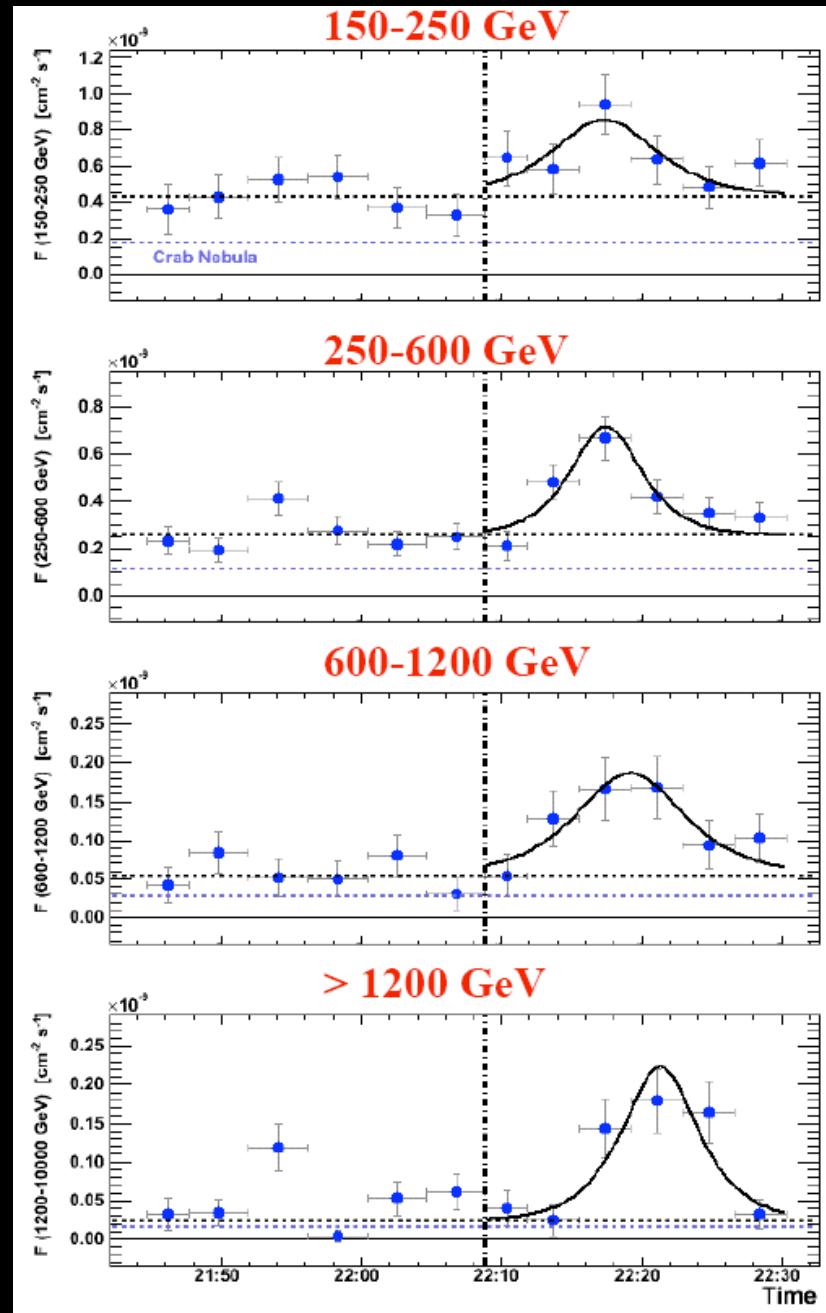


Fastest variability observed in Mkn 501

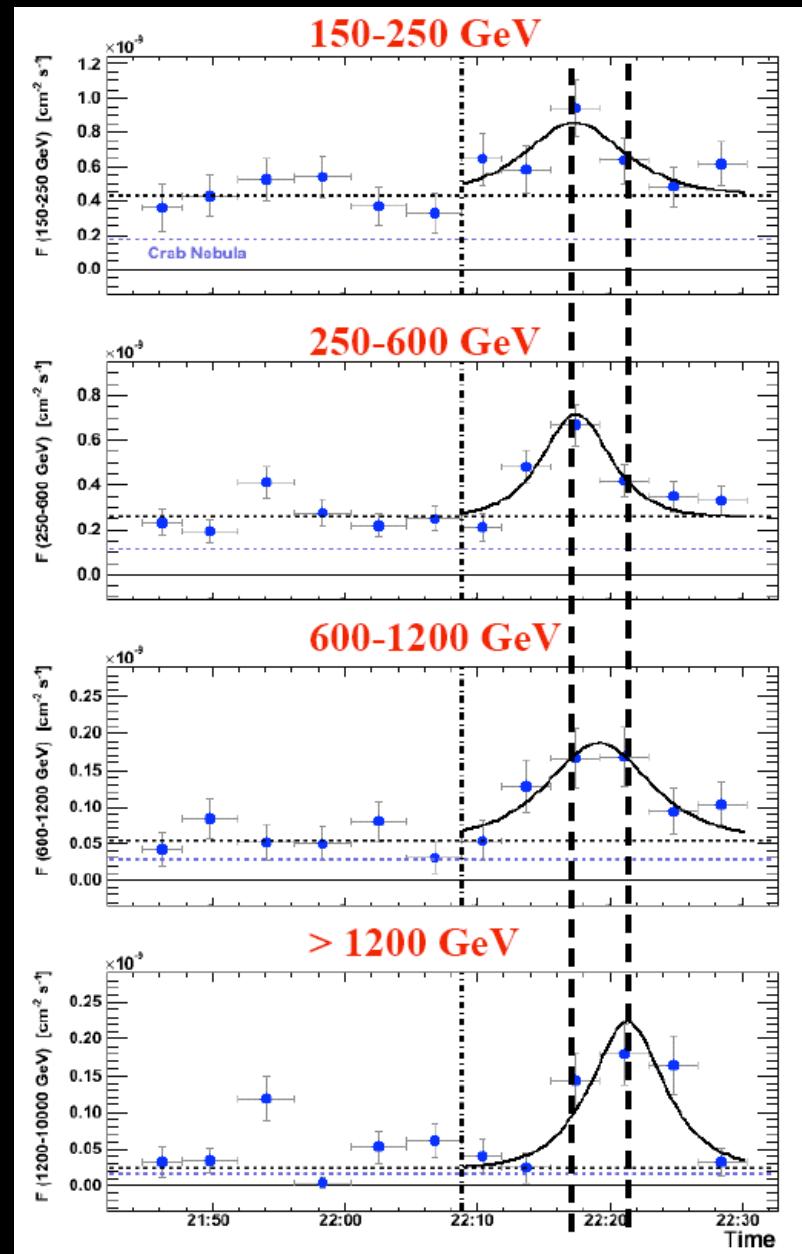
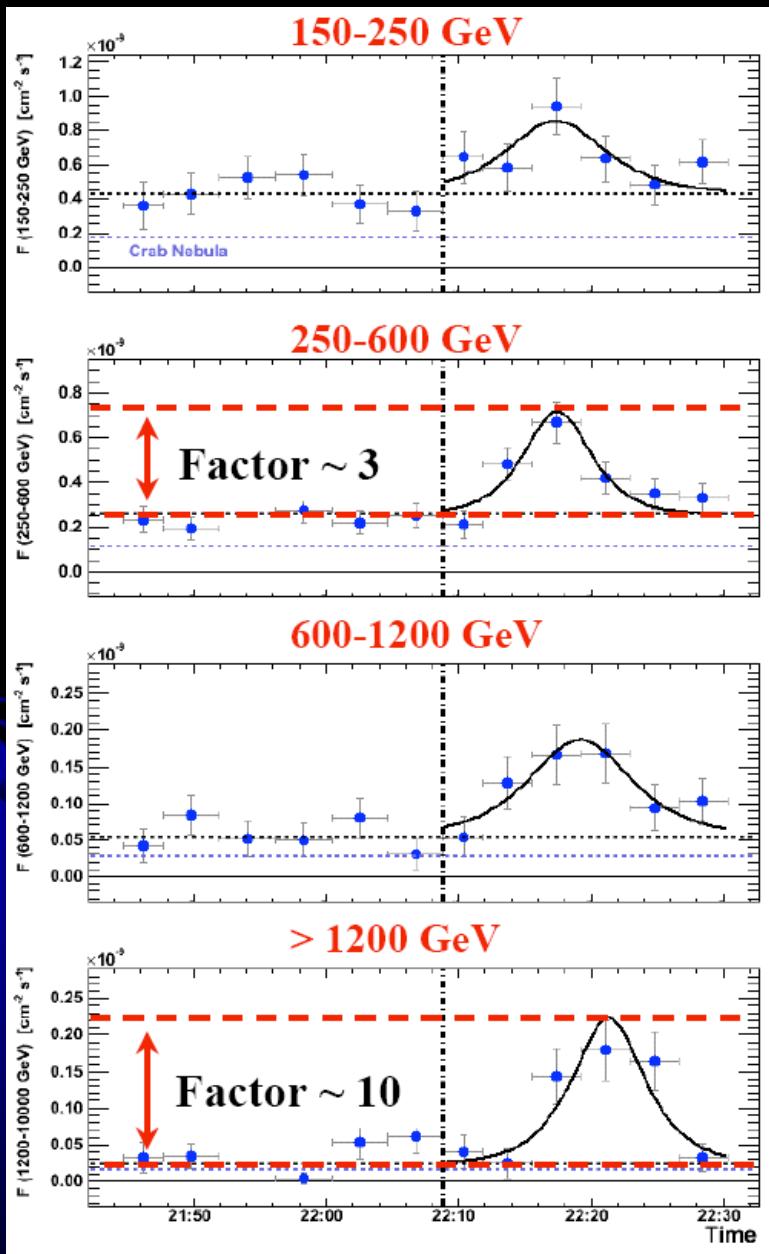
June 30th



July 9th



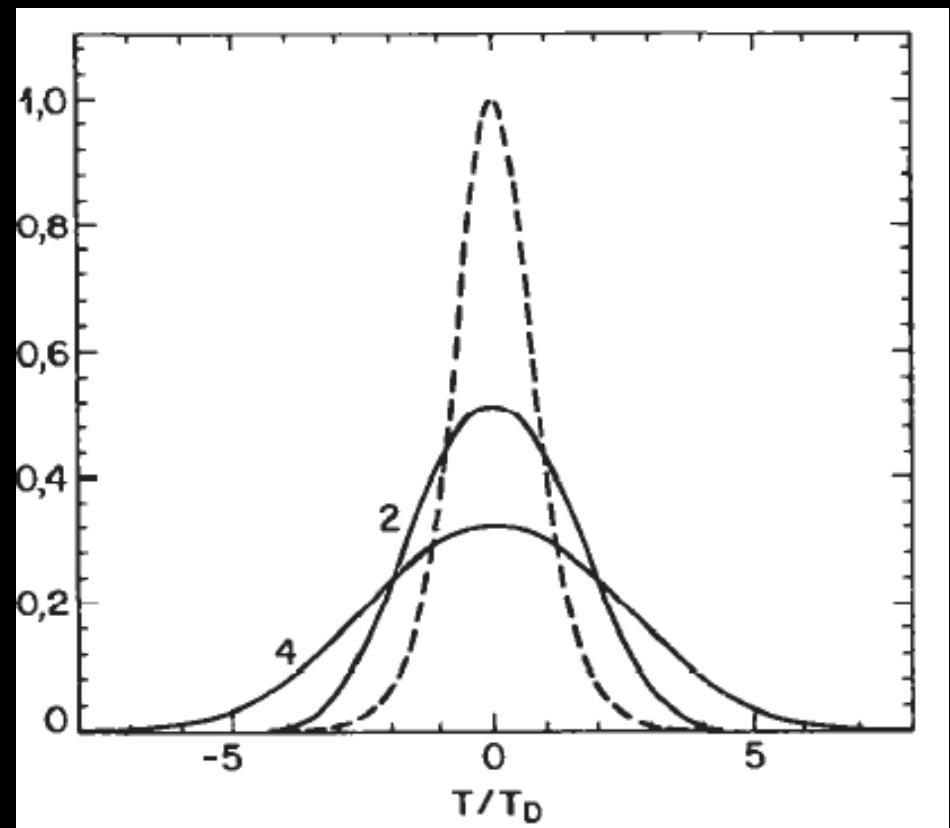
July 9th



The apparent duration of a pulse is only going to be increased by dispersion.

The energy per unit time decreases with the distance from the source.

The dispersion can be figured out by "undoing" the dispersion such that as much energy as possible is emitted at the source.

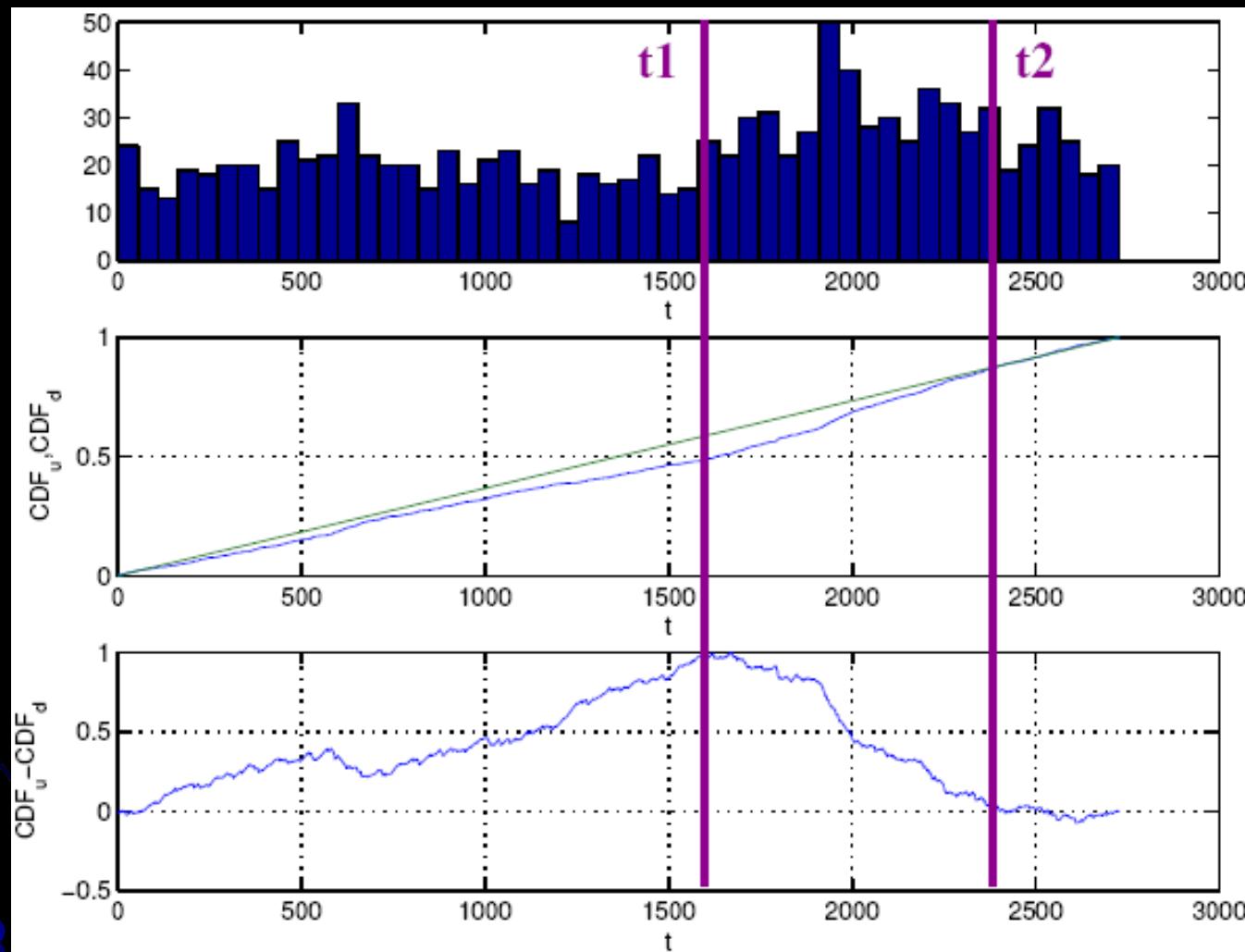


$t1, t2$  contains the most active part of the flare, as determined using KS statistics

One corrects for given model of photon dispersion, linear and quadratic, by applying to each photon of energy  $E$  the time shifts.

$$t_{sl} = t \pm \tau_l E$$

$$t_{sq} = t \pm \tau_q E^2$$

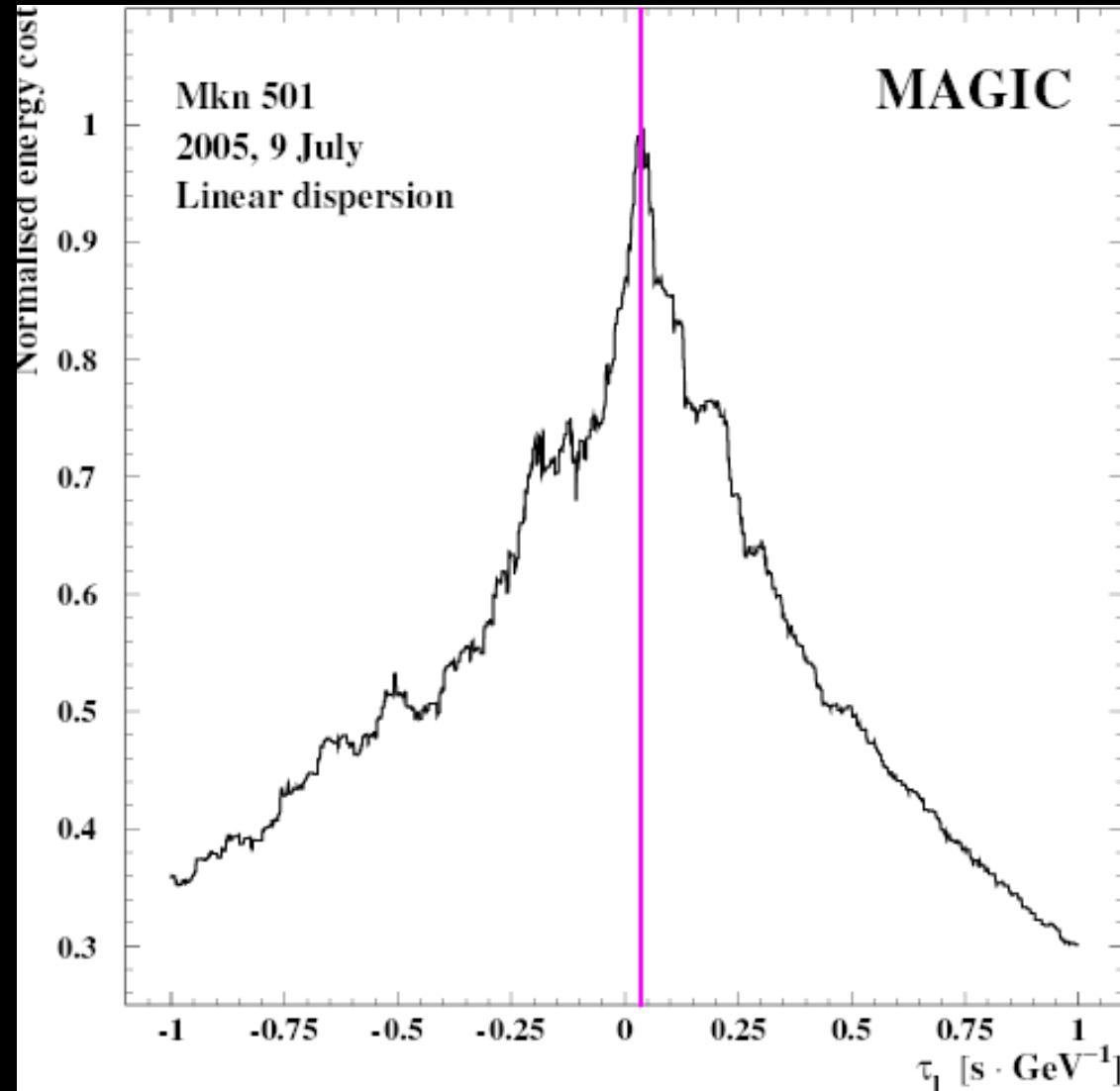


## The energy cost function (ECF)

The transformation is repeated for many value of  $\tau$ .

$$\text{ECF} = \sum_{t_1 \leq i \leq t_2} E(i)$$

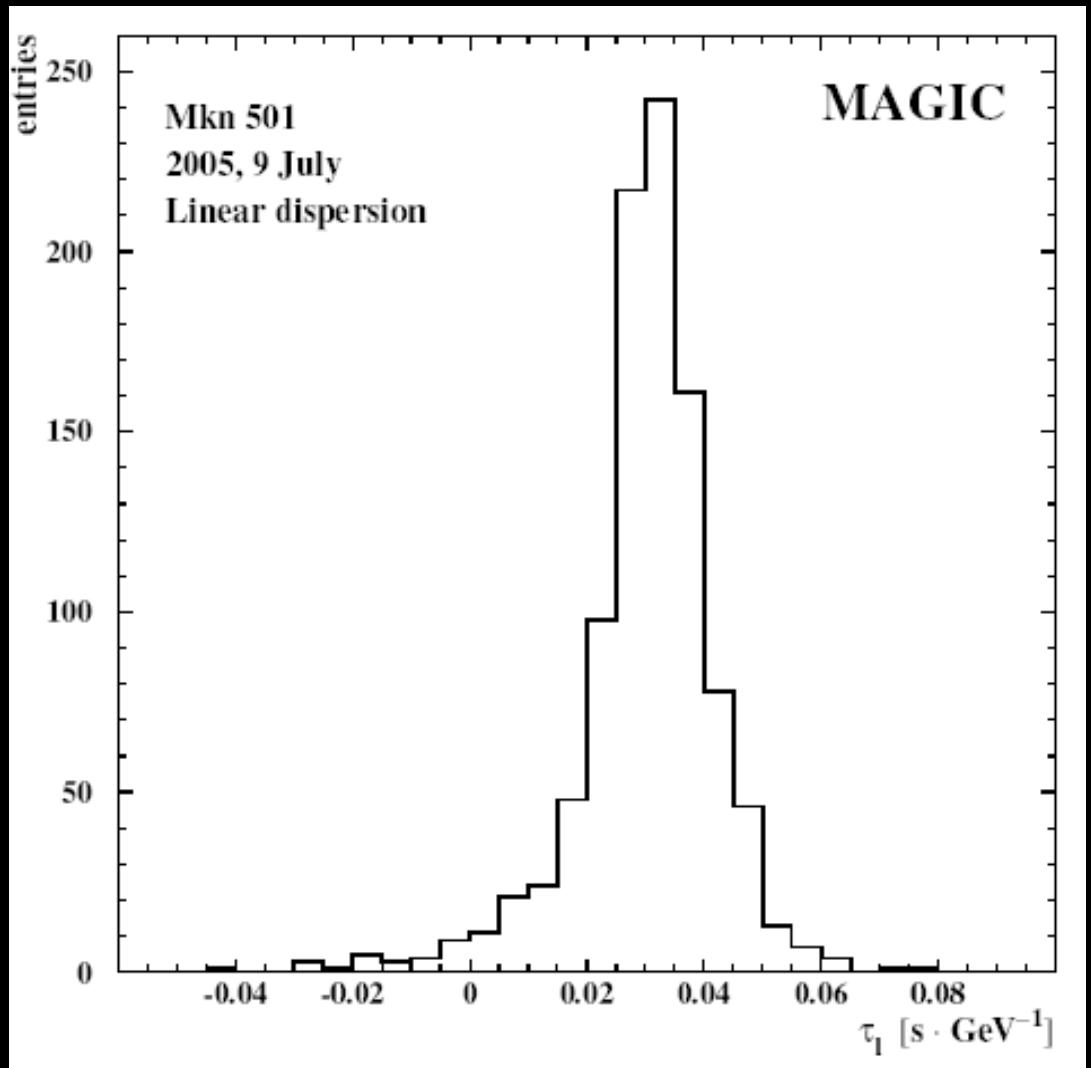
The position of the maximum of ECF indicates the value of  $\tau$  which recovers the signal in the sense of maximizing its power.



1000 realizations of the flare with photon energy smeared by MC.

ECF applied blindly to a 5 sets of 1000 artificial MC samples resembling the July 9 flare, but with different types of dispersion encoded.

MC sets with no dispersive signal have been analysed.



## ECF optimization results

Linear

$$\tau_l = (0.030 \pm 0.012) \text{ s/GeV}$$

$$M_{\text{QG1}} = 1.445 \times 10^{16} (1 \text{ s}/\tau_l)$$

$$M_{\text{QG1}} = (0.48^{+0.32}_{-0.14}) \times 10^{18} \text{ GeV}$$

$$M_{\text{QG1}} > 0.26 \times 10^{18} \text{ GeV}$$

Quadratic

$$\tau_q = (3.71 \pm 2.57) \times 10^{-6} \text{ s/GeV}^2$$

$$M_{\text{QG2}} = 1.222 \times 10^8 (1 \text{ s}/\tau_q)^{1/2}$$

$$M_{\text{QG2}} = (0.63^{+0.44}_{-0.19}) \times 10^{11} \text{ GeV}$$

95% CL

$$M_{\text{QG2}} > 0.33 \times 10^{11} \text{ GeV}$$

$z=0.034$  ( $D=146$  Mpc)

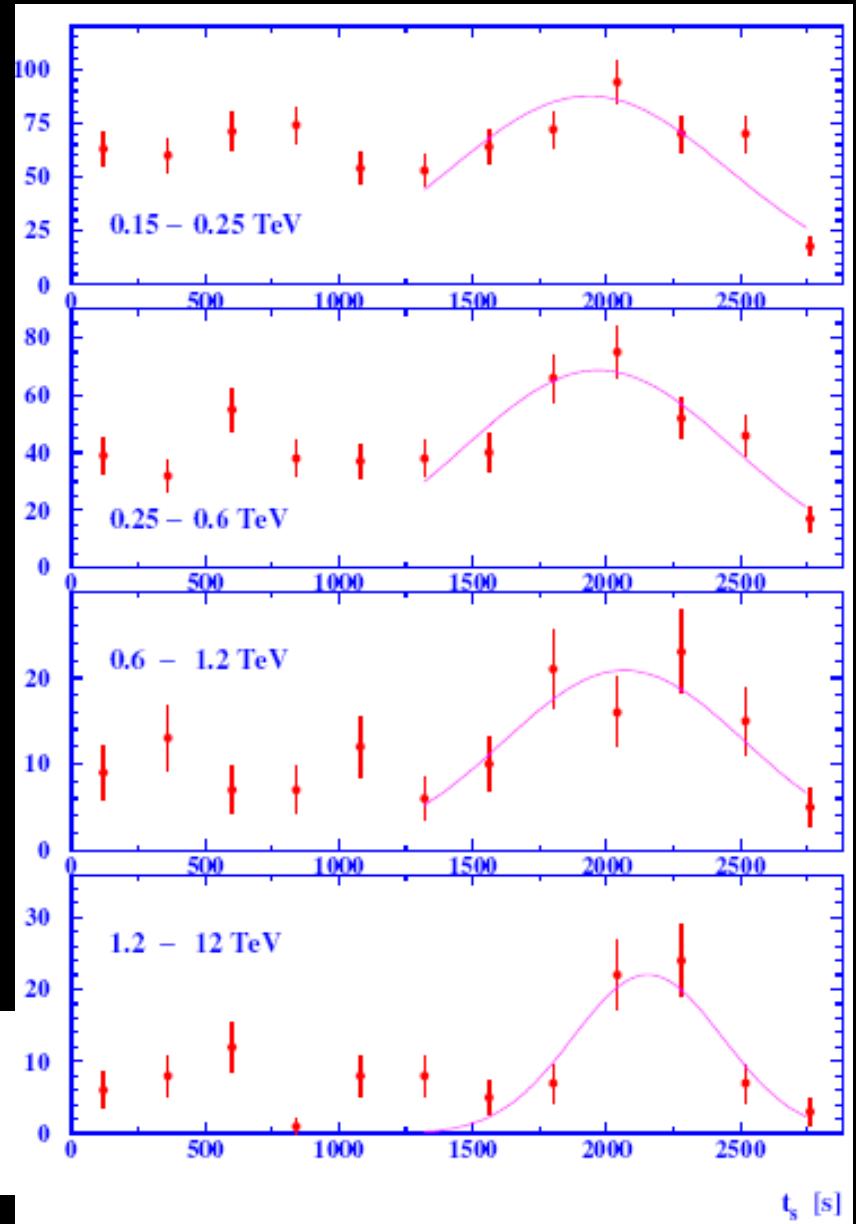
$E=0.15-12$  TeV

Energy resolution 23%

Angle parameter  $\alpha < 6^0$

Spectral distribution

$$\frac{dF}{dE} \sim E^{-\beta}; \quad \beta_{bg} \approx 2.7; \quad \beta_{tr} \approx 2.4$$

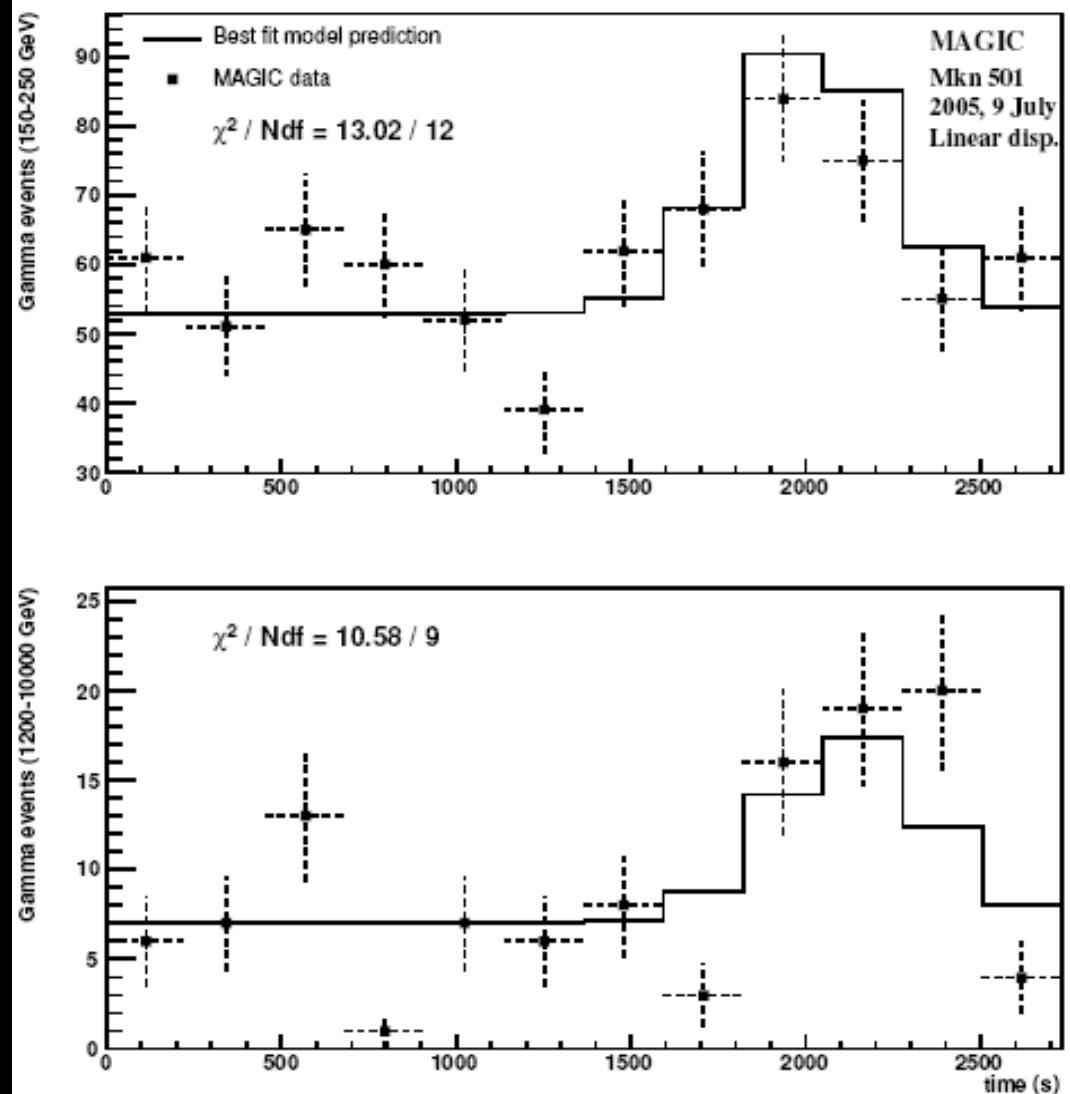


## Log-Likelihood cross check

Energy binned analysis showed that the high energy light-curve is described well by a Gaussian envelope superimposed on a uniform background.

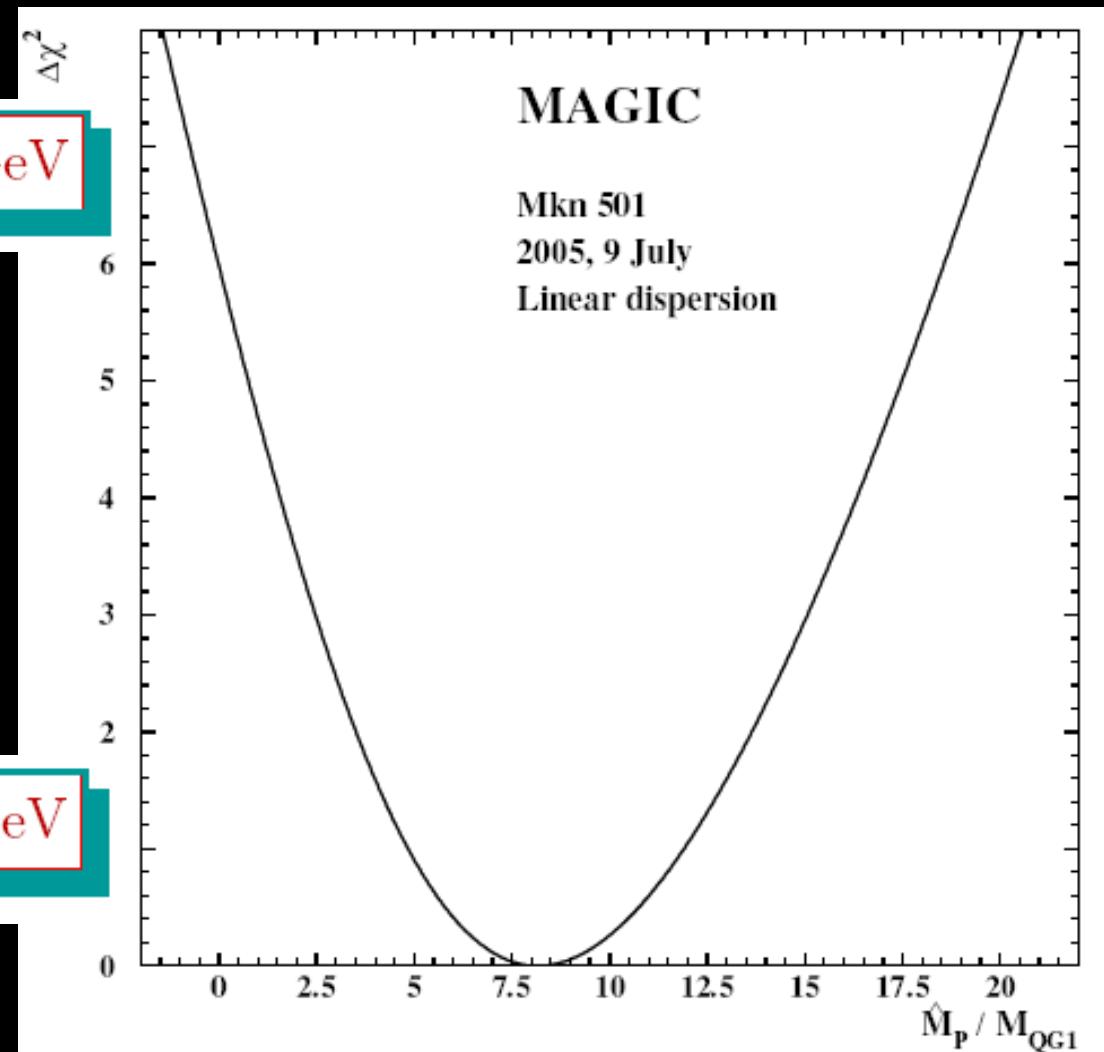
The likelihood function  $L$  is fitted to the July 9 flare minimizing  $-\log(L)$  as a function of four parameters.

- the width
- position of the maximum
- the level of uniform bkg.
- QG dispersion



$$M_{\text{QG1}} = (0.30^{+0.24}_{-0.10}) \times 10^{18} \text{ GeV}$$

$$M_{\text{QG2}} = (0.57^{+0.75}_{-0.19}) \times 10^{11} \text{ GeV}$$



QED plasma refraction induced as photons propagate through the source

$$\Delta t_{QED}(E) = cD \frac{\alpha^2 T^2}{6E^2} \ln^2 \left( \frac{ET}{m_e^2} \right)$$

$$T \sim 10^{-2} \text{ MeV}$$

$$D \sim 10^9 \text{ km}$$

$$\Delta t_{QED}(1 \text{ TeV}) \sim 10^{-5} \text{ s}$$

ECF sensitivity

$$t_{sinvq} = t \pm \frac{\tau_{qi}}{E^2}$$

$$\tau_{qi} \simeq 10^6 \text{ s} \cdot \text{GeV}^2$$

Plasma propagation source effect negligible

(J.I. Latorre, et al, 1995)

## Conclusions

The peaking of the July 9 flare from Mkn 501 observed by MAGIC is found to maximize for QG mass scales:

$$M_{QG1} \simeq 0.4 \times 10^{18} \text{ GeV}$$

$$M_{QG2} \simeq 0.6 \times 10^{11} \text{ GeV}$$

We establish lower limits:

$$M_{QG1} > 0.26 \times 10^{18} \text{ GeV}$$

$$M_{QG2} > 0.33 \times 10^{11} \text{ GeV}$$

Thermal plasma propagation effects in the source are negligible.