Do massive neutron stars end as invisible dark energy objects?

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- **Theoretical and observational properties of neutron stars**
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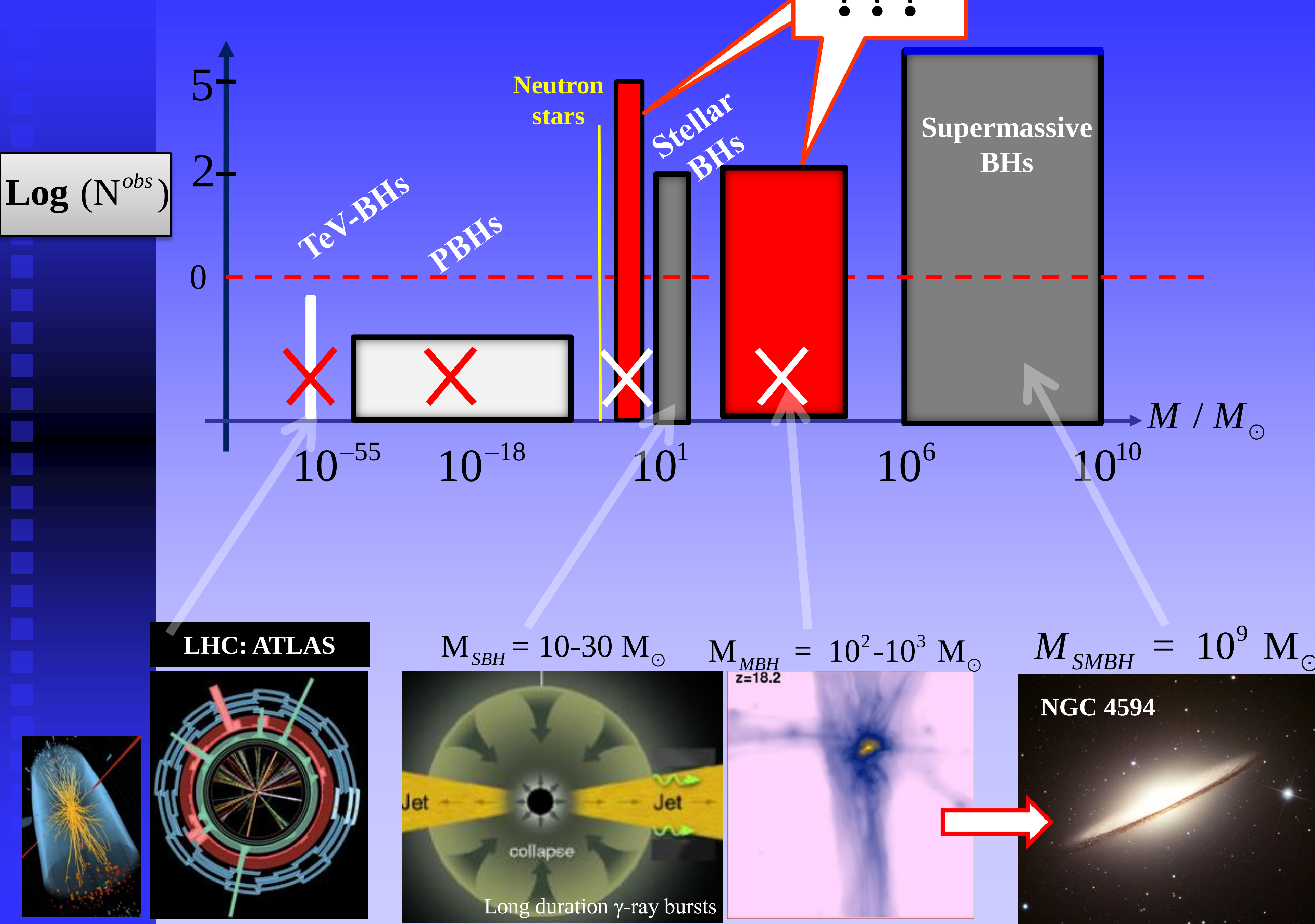
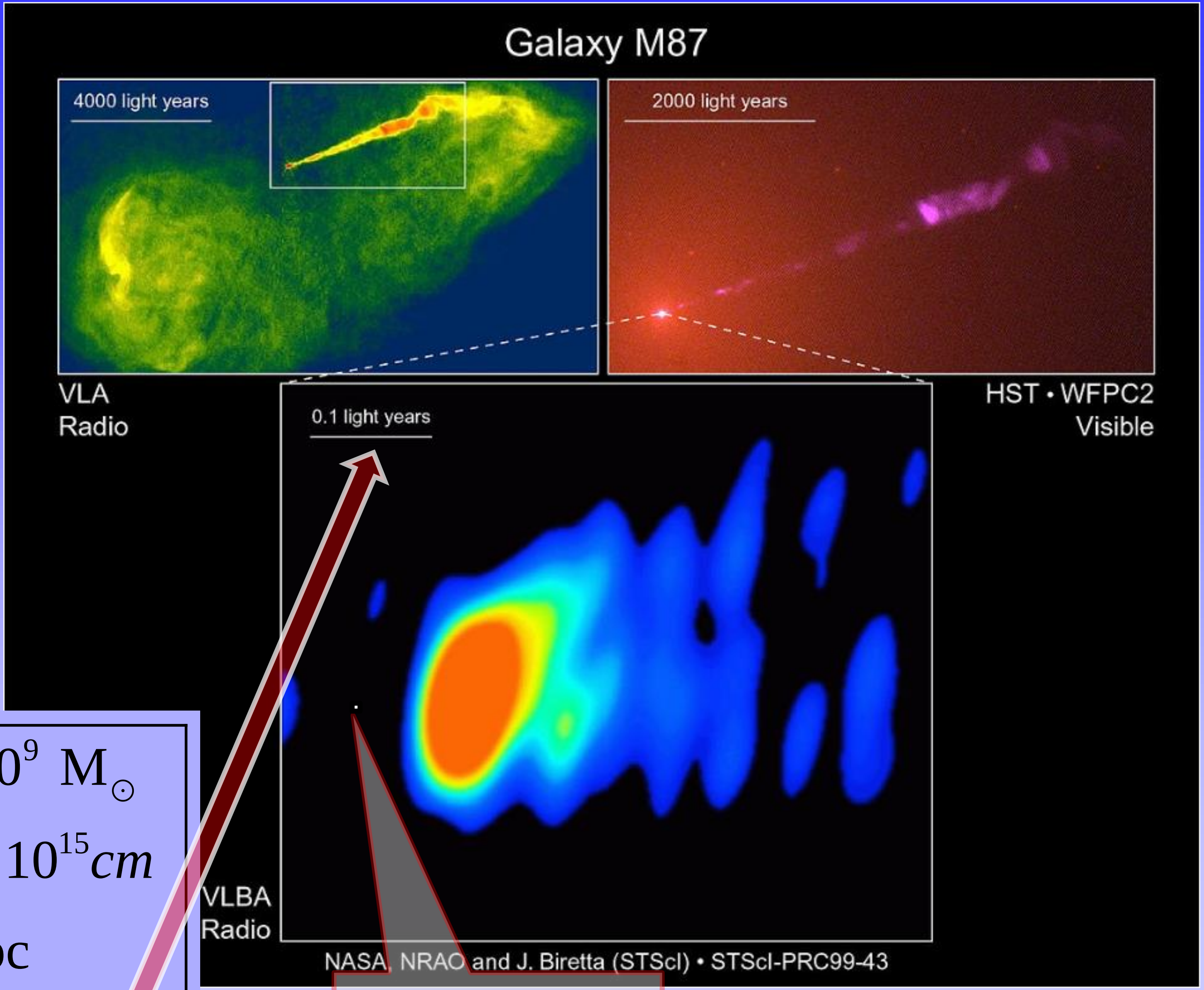
The mass spectrum of black holes



Are BH-horizons observable?



$M_{BH} \approx 5 \times 10^9 \; M_\odot$

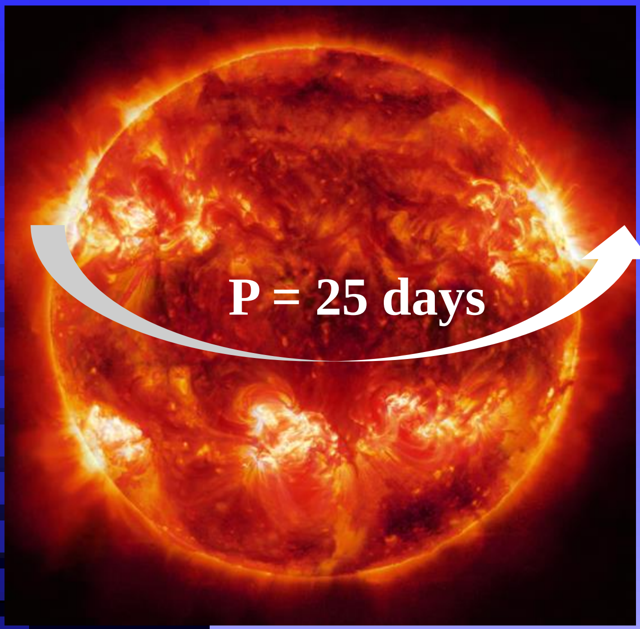
$R_H \approx 1.5 \times 10^{15} cm$

$\approx 10^{-4}$ pc

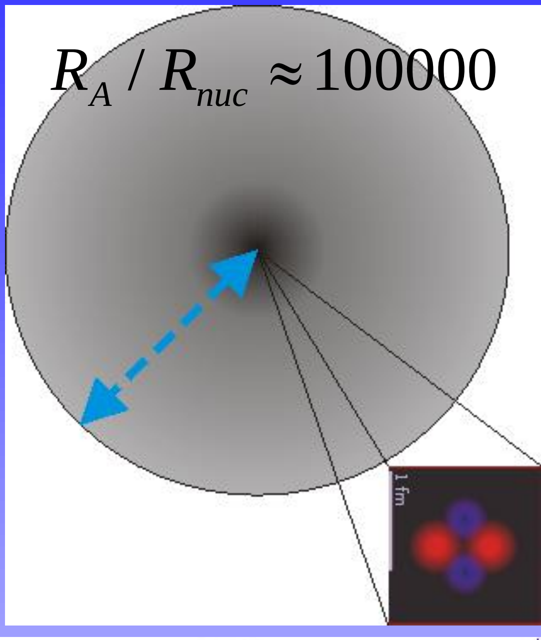
$\approx 10^{-3}$ -10^{-2} OR

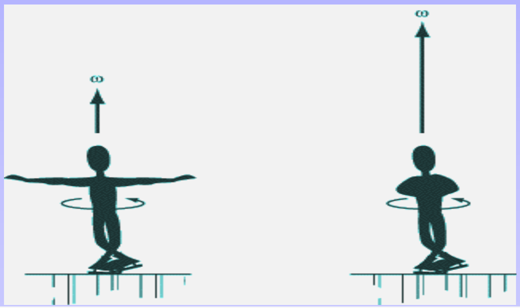
The size of the solar system

Pulsars and NSs: The amazing objects

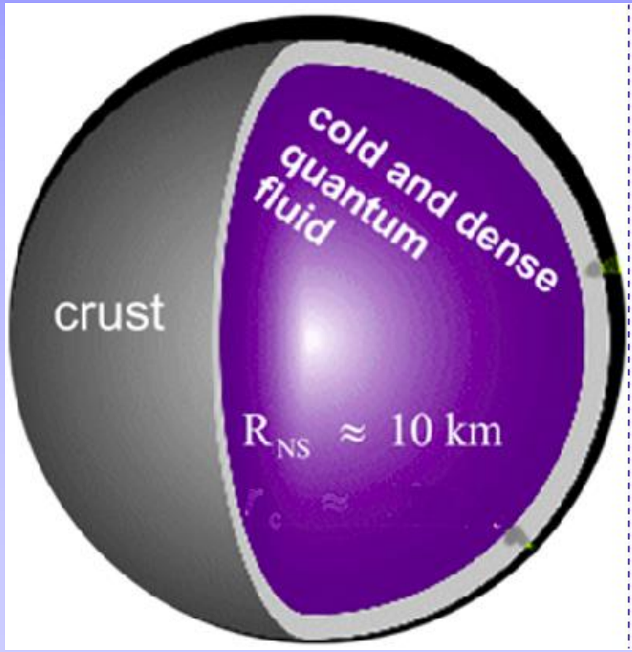


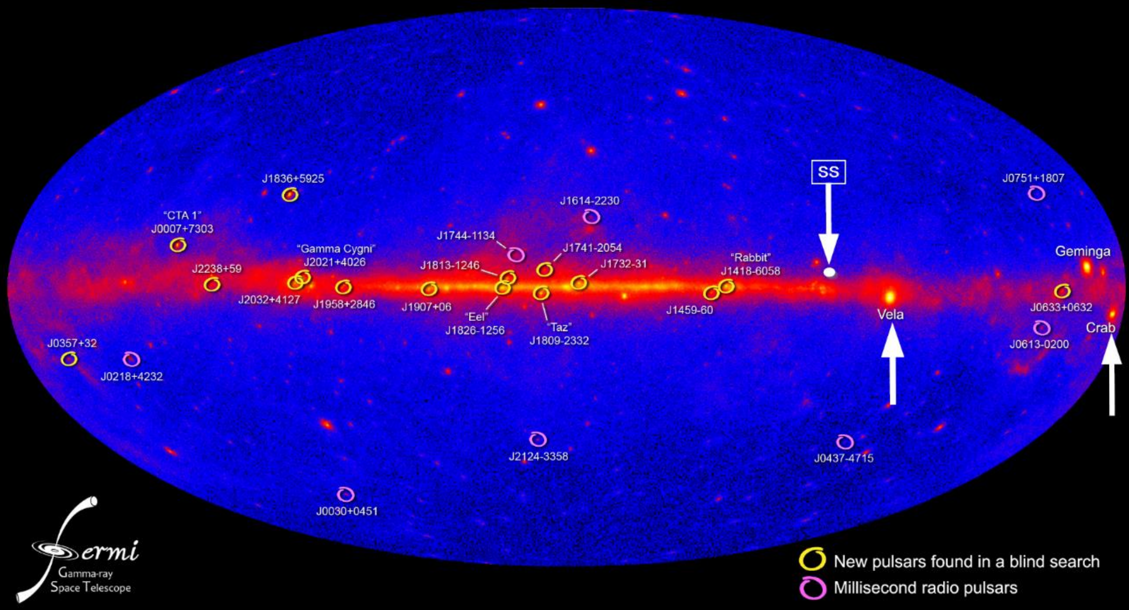
$R_\odot / R_{NS} \approx 100000$

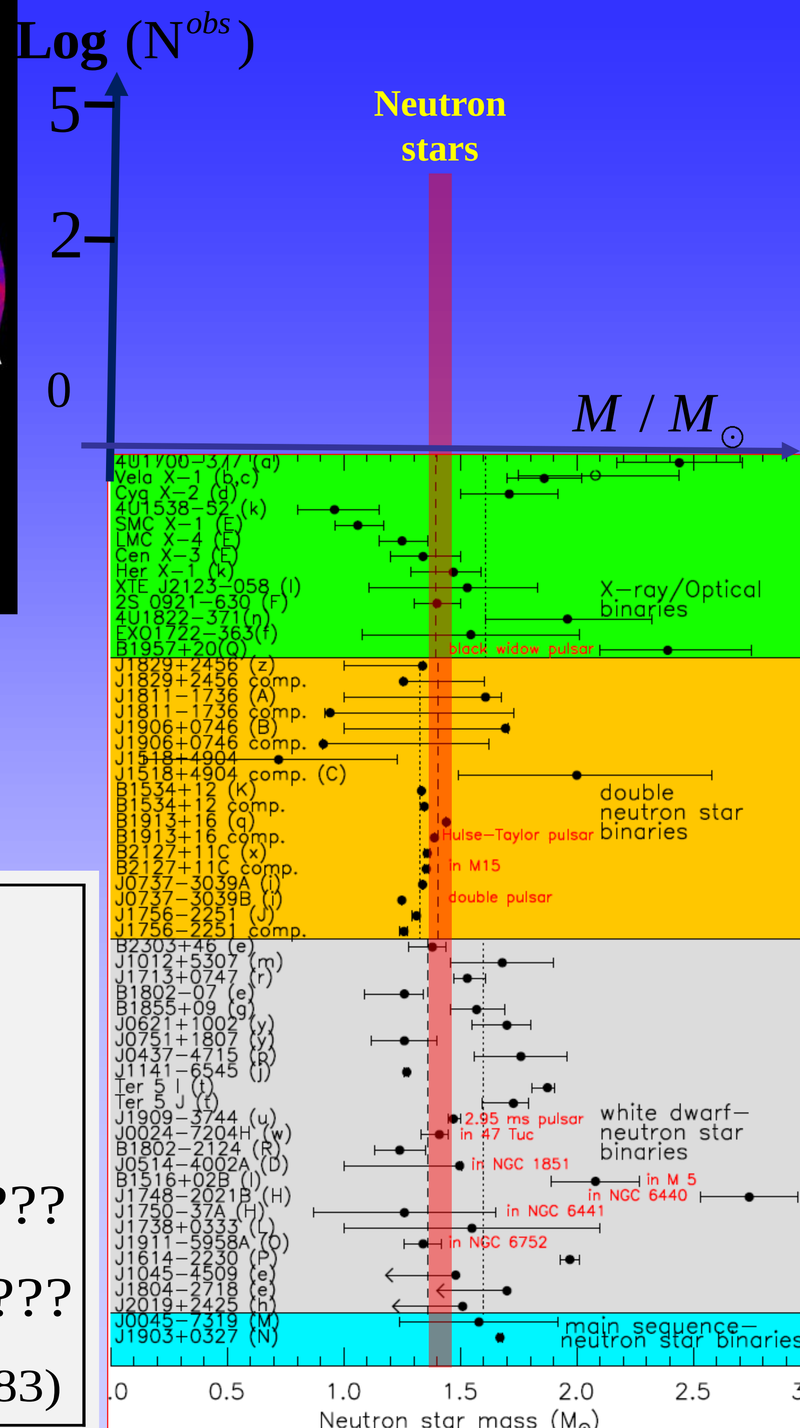




$$\frac{\Omega_{NS}}{\Omega_\odot} \sim \left(\frac{r_\odot}{r_{NS}}\right)^2 \approx \text{one billion} \sim \frac{B_{NS}}{B_\odot}$$







$$N_{Stars} \approx 10^{11}$$

$$N_{NSs} \approx 2\,\%\,N_{Stars} \approx 10^{9}$$

$$N_{BHs} \approx 0.1\,\%\,N_{Stars} \approx 10^{8}$$

$$N_{NSs}^{Observed} \approx 10^{3} \approx 10^{-6}\,N_{NSs} \;????$$

$$N_{SBHs}^{Observed} < 10^{2} \approx 10^{-6}\,N_{BHs} \;????$$

(Witten 1984, Shapiro et al. 1983)

The internal structure of NSs

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

$$T^0_{\mu\nu} = -P^0 g_{\mu\nu} + (P^0 + \mathcal{E}^0) U_\mu U_\nu$$

$$g_{\mu\nu} = e^{2\nu} dt^2 - e^{2\lambda} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d^2\varphi^2$$

$$\frac{dP}{dr} = -\frac{G[\varepsilon + p][m + 4\pi r^3 p]/c^4}{r^2\ (1 - r_s/r)} \xrightarrow{p<<\varepsilon} \frac{dP}{dr} = -\frac{G[\varepsilon][m]/c^4}{r^2}$$

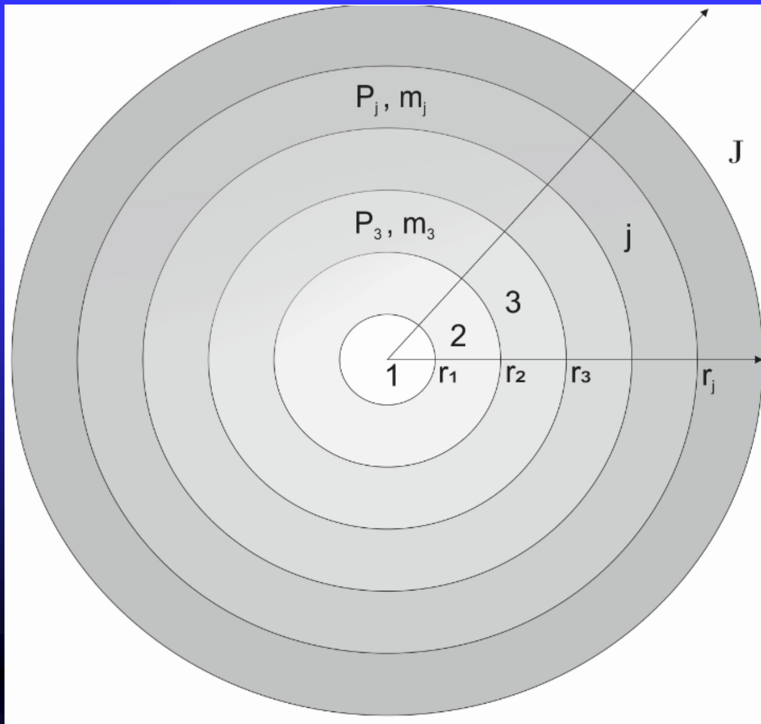
$$\frac{dm}{dr} = 4\pi \int_0^r \varepsilon r^2 dr$$

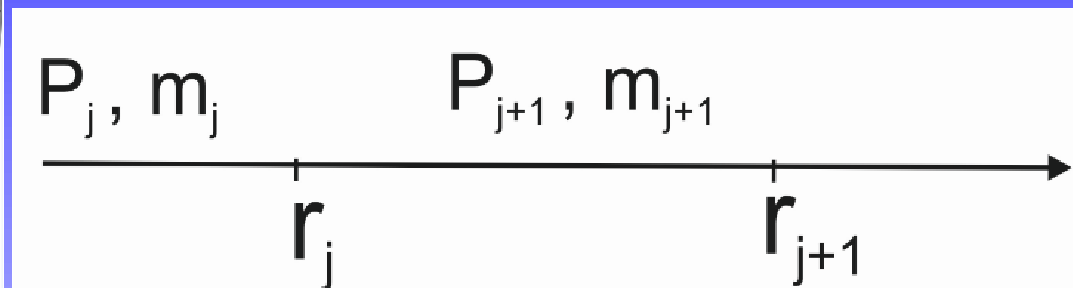
$$\frac{d\nu}{dr} = \frac{[m + 4\pi r^3 p]/c^4}{r^2\ (1 - r_s/r)}$$

$$r_s = 2Gm/c^2$$

$$P = P(\varepsilon) \quad \text{(EOS)}$$

TOV in the finite space:



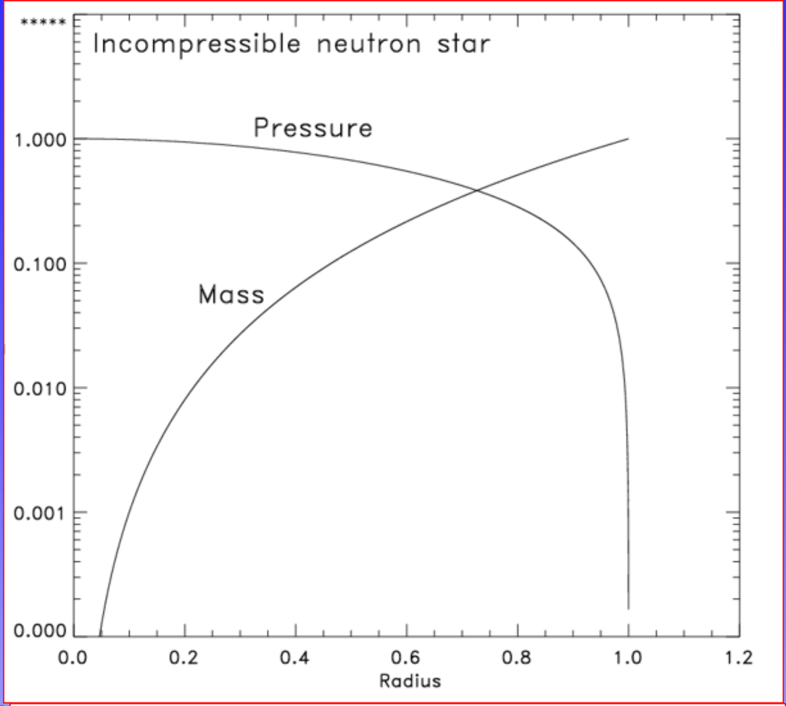


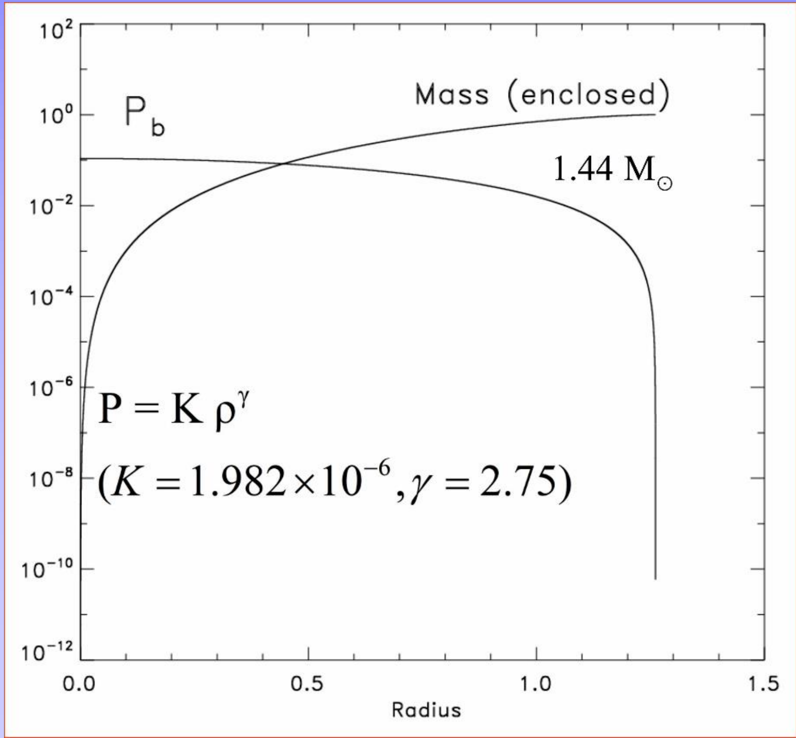
$$\frac{dP}{dr} = -\frac{G[\varepsilon+\mathrm{p}]\left[m+4\pi r^3 p\right]/c^4}{r^2\ (1-r_s/r)} \xrightarrow[\text{EOS/}P=P(\varepsilon)]{\textit{finite}\ \text{space}} \mathrm{P}_{j+1} = \mathrm{P}_j - \mathrm{dr}_j \times \mathrm{RHS}_\mathrm{P}$$

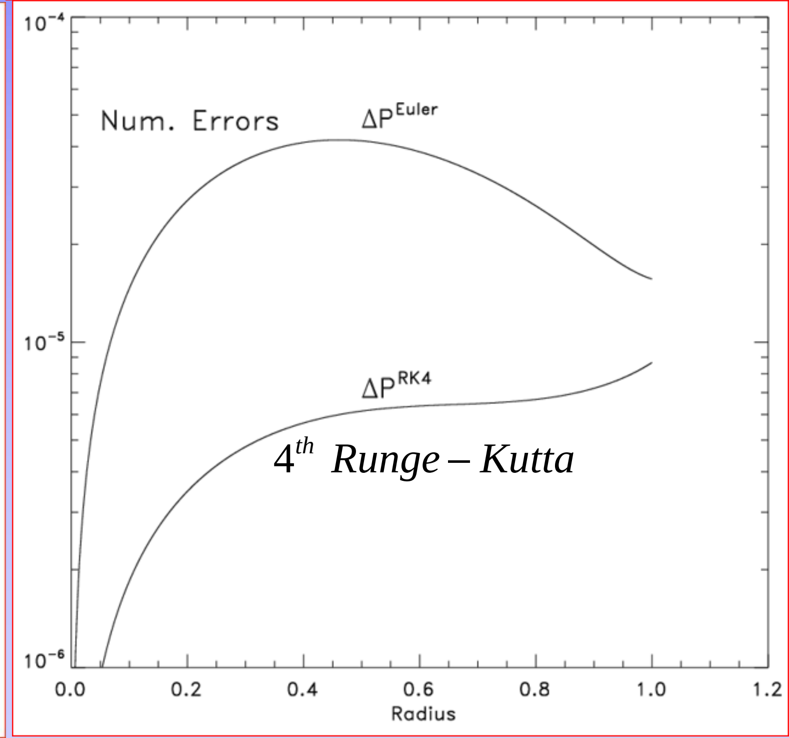
$$\frac{dm}{dr} = 4\pi\int_0^r \varepsilon r^2 dr \xrightarrow{\textit{finite}\ \text{space}} m_{j+1} = \mathrm{m}_j - \frac{4\pi}{3}\sum_{j=1}^{J}\varepsilon_j \Delta r_j^3$$

$$\frac{d\nu}{dr} = \frac{\left[m+4\pi r^3 p\right]/c^4}{r^2\ (1-r_s/r)} \xrightarrow{\textit{finite}\ \text{space}} \nu_{j+1} = \nu_j - \mathrm{dr}_j \times \mathrm{RHS}_\nu$$

1ord Euler & 4ord Runge-Kutta integration of the TOV-equation

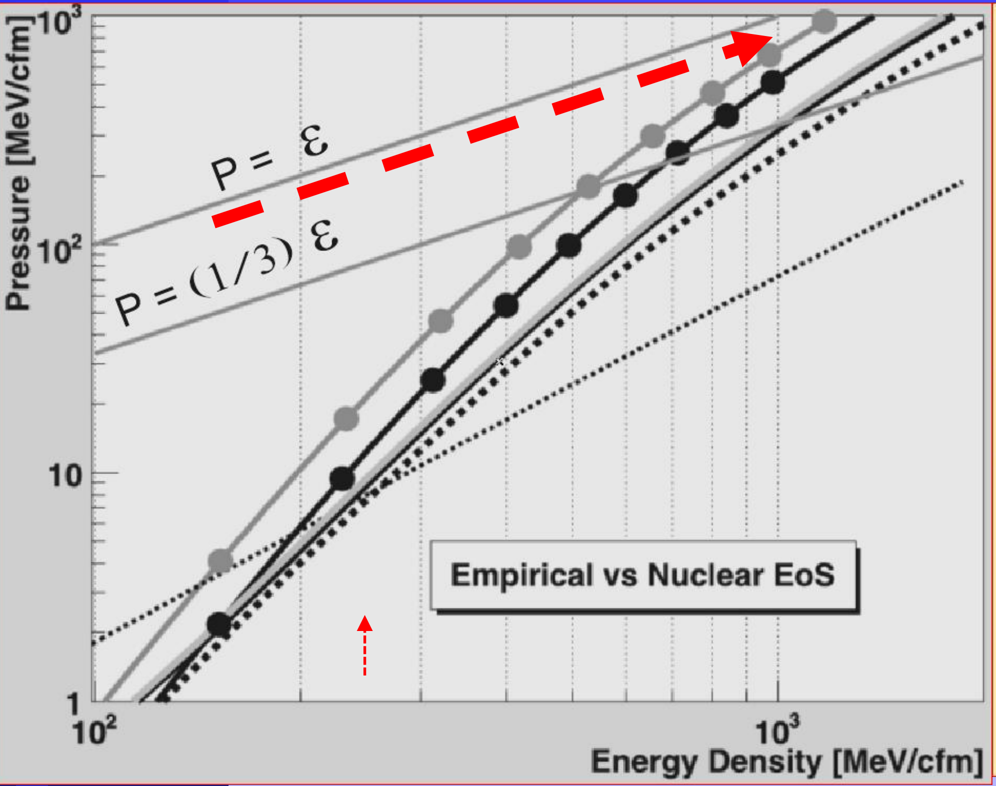




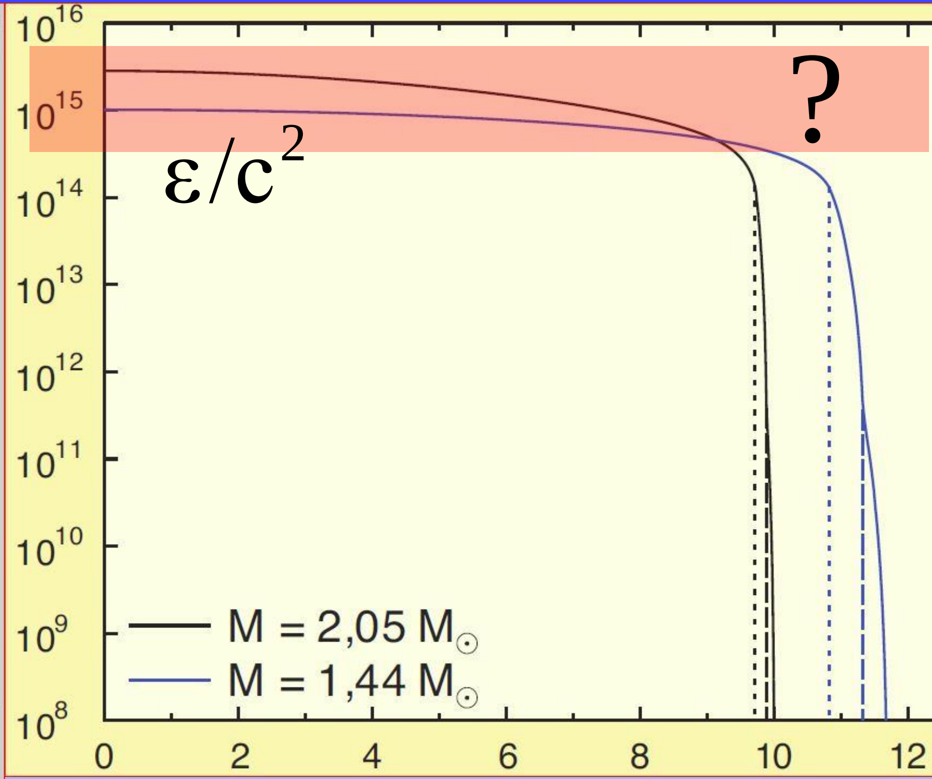


EOS-uncertainties:

Camenzind 2009



Sly4/Hampel et al 2012



Issues & difficulties:

- $\rho_c >> \rho_{nuc}$: Physics & causality problem
- Weak-compressibility problem

$$P \xrightarrow[V_s \to c]{EOS} \varepsilon: \quad \text{EOS: } P = \varepsilon \Leftrightarrow incompressible.$$

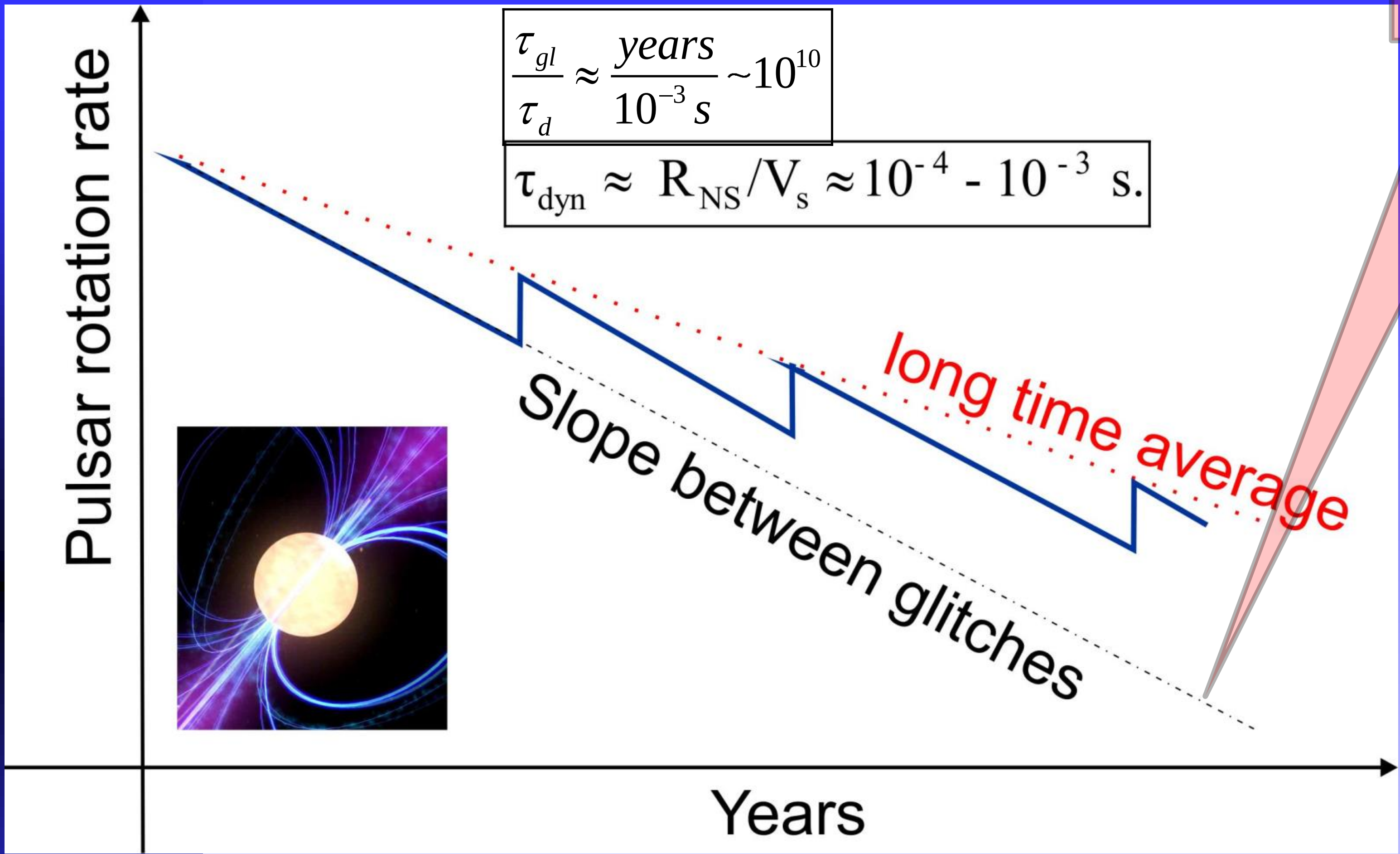
- $\nexists$ $M_{NS} > 2\, M_{\odot}$

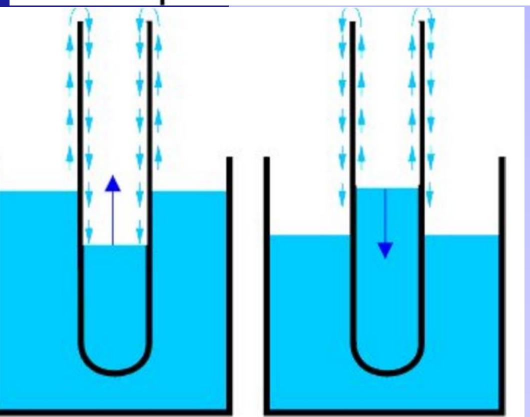
The glitch phenomena in pulsars and NSs

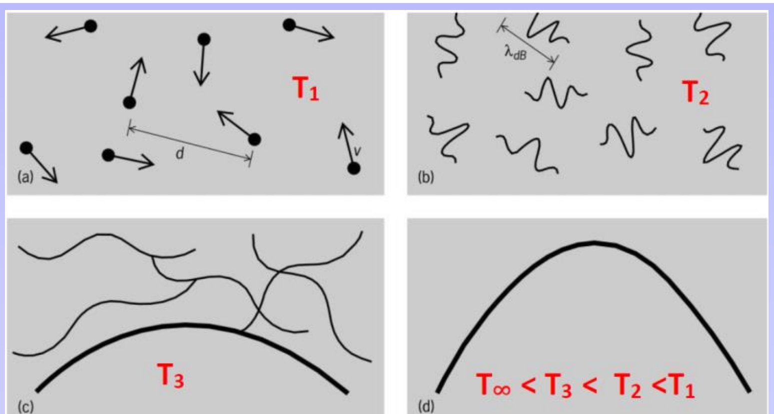
$$\dot{E} = -\frac{B_p^2 R^6 \Omega^4 \sin^2\alpha}{6c^3}$$

$$\frac{\tau_{gl}}{\tau_d} \approx \frac{years}{10^{-3}\,s} \sim 10^{10}$$

$$\tau_{dyn} \approx R_{NS}/V_s \approx 10^{-4} - 10^{-3}\ \text{s.}$$







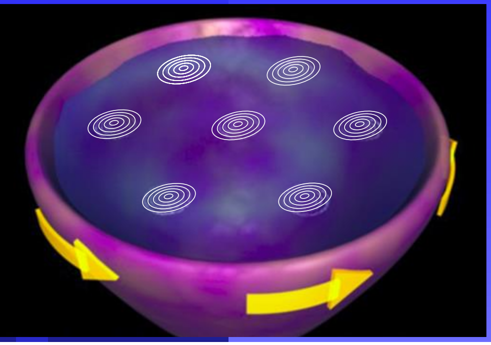
$T \approx 10^{-2} T_{STP} \approx 2\ ^{o}\text{K}$

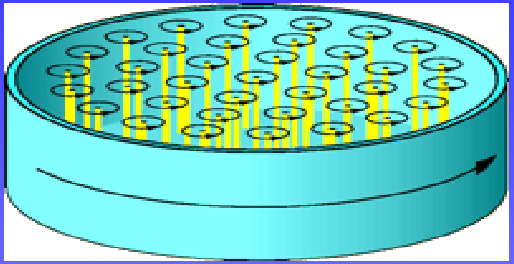
$\exists$ HII is superfluid

$T_{NS} \approx 10^{-4} T_F \Rightarrow$

Cores are SFluid?

Superfluidity & storage of rotational energy

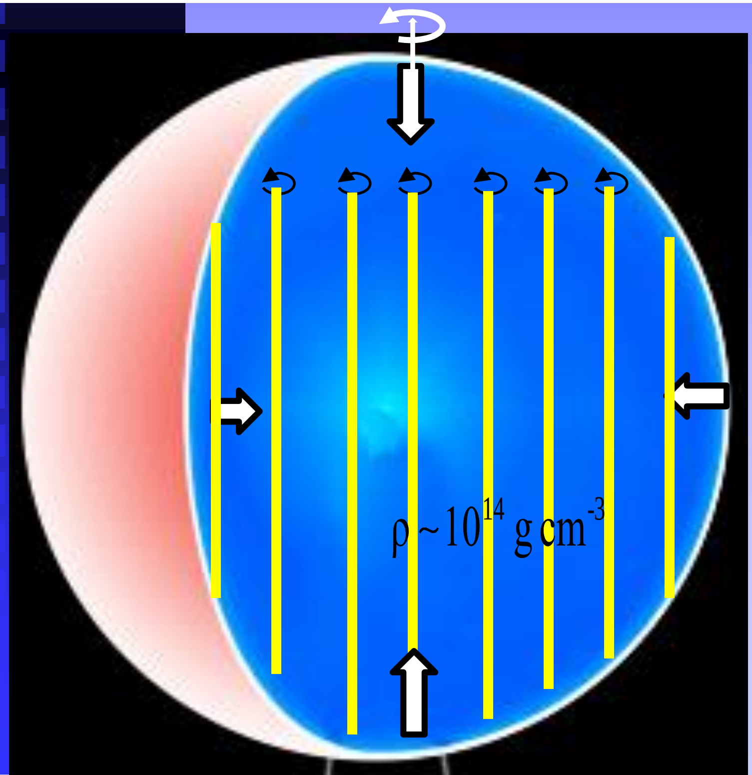




Onsager-Feynmann equation

$$\oint V \cdot dl = \frac{h}{2m} n$$

$$\int \nabla \times V ds \approx \int \nabla \times \langle V \rangle ds = 2\Omega \cdot 4\pi r_s^2 = \frac{h}{2m} n \Rightarrow n = n(\Omega)$$



$$n = n(\Omega) \xrightarrow{Crab} \mathrm{n} \approx 5.3 \times 10^{18} \text{ vortex lines}$$

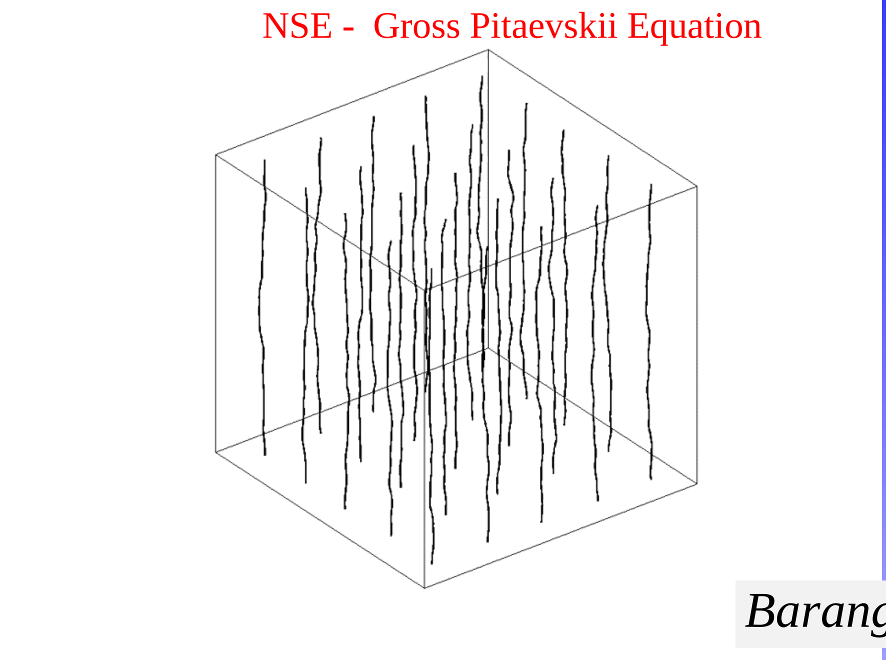
$$\dot{n} = n(\dot{\Omega}) \xrightarrow{Crab} \dot{\mathrm{n}} \approx 10^{6} \text{ s}^{-1}$$

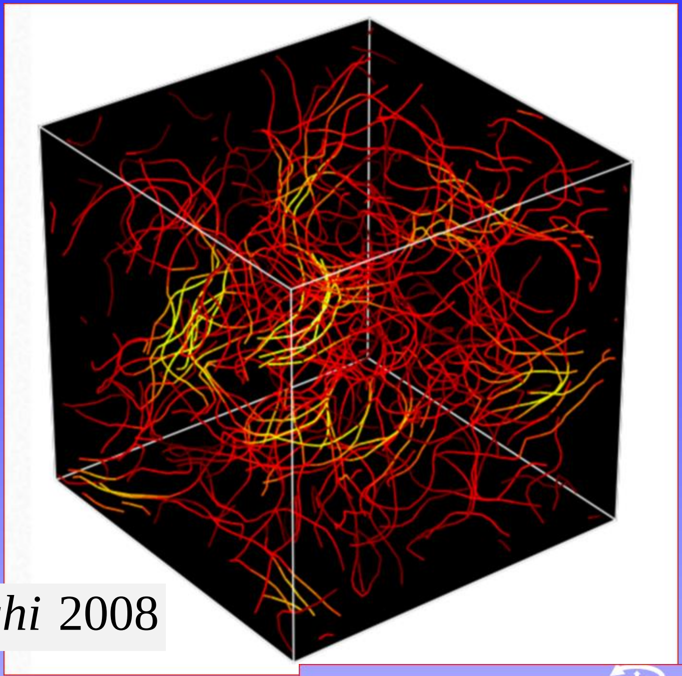
$$E_{rot} = \frac{1}{2} I \Omega^2 \approx 10^{49} \text{ ergs}$$

$$\dot{E}_L = \dot{N} \mathcal{E}^d = 1.35\ 10^{37} \text{ erg/s.}$$

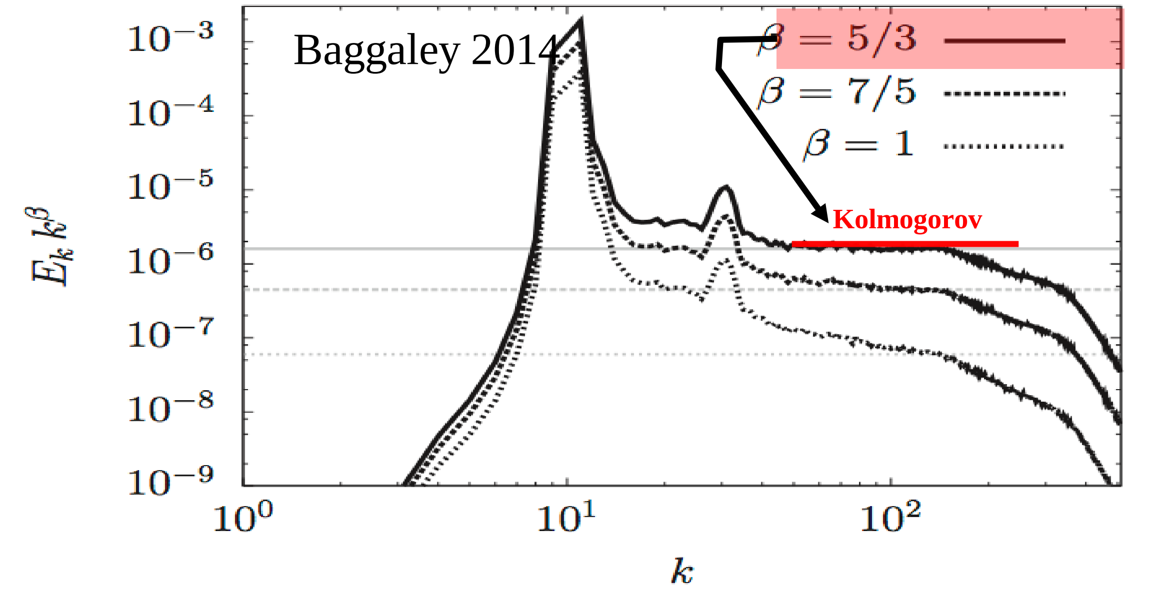
$$\tau_{LT} \approx 10^{12} \text{ s} \approx 10^{5} \text{ years.}$$

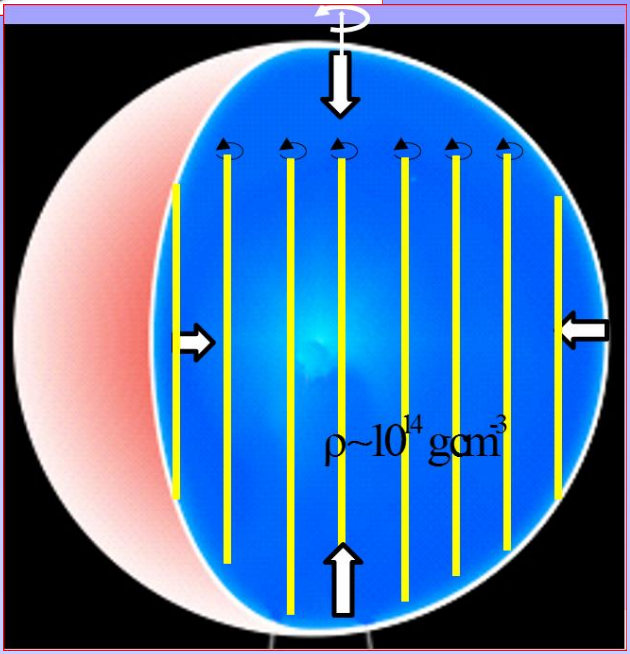
Superfluid turbulence





Baranghi 2008





Turbulence in NS-superfluids

$$\frac{\partial n}{\partial t} + \nabla \times n u_f = v_{tur} \Delta n$$

$$\begin{cases} \tau_{diff} = \dfrac{R_{NS}^2}{v_{tur}} \\ v_{tur} = \ell_{tur} \times u_{tur} \end{cases}$$

$$v_{tur} = \ell_{tur} \times u_{tur}$$

$$\ell_{tur} = \begin{cases} = 10^{-3} \text{ cm} & \text{t=0} \\ \ell_\infty = 10^{6} \text{ cm} & \text{t} = \infty \end{cases} \Bigg| \Rightarrow \langle \ell \rangle_{tur} = \sqrt{\ell_0 \times \ell_\infty}$$

$$|u_{tur}| \leq u_f^{\max} = -\left(\frac{\dot{\Omega}}{\Omega} \right) \times r = \begin{Bmatrix} 1.28 \times 10^{-11}/\text{s} \;\; \text{Crab} \\ 1.44 \times 10^{-12}/\text{s} \;\; \text{Vela} \end{Bmatrix} \times 10^{6} \text{cm} \ll \; 10^{-6} \text{ cm/s}$$

$$\Rightarrow \boxed{\tau_{diff} = \frac{R_{NS}^2}{v_{tur}} \approx \frac{10^{12} \text{cm}^2}{10^{-4 \to -3} cm^2 / s} = 10^{8} \to 10^{9}\, yr}$$

- **Isolated NSs that fromed from the first generation of stars should be by now invisible**
- **Isolated and old NSs (> 100 Myr) must be invisible too**
- **→ invisible yes, but they may still interact with their surrounding and so detectable.** } $\nexists$

These objects need to be repulsive.

Could the combination of incompressibiliy, superfluidity and scalar fileds do the job?

The answer is yes.

But why incompressibility:

Incompressible Navier-Stokes equations in the Newtonian regime

$$\left.\begin{aligned} &\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\vec{V} = 0 \\ &\frac{\partial \mathrm{E}^{\mathrm{d}}_{\mathrm{int}}}{\partial t} + \nabla\cdot(\mathrm{E}^{\mathrm{d}}_{\mathrm{int}}\vec{\mathrm{V}}) = -\mathrm{P}\nabla\cdot\mathrm{V} + \Gamma^{+} - \Lambda^{-} \\ &\frac{\partial \rho\mathrm{V}}{\partial t} + \nabla\cdot(\rho\mathrm{V}\otimes\mathrm{V}) = -\nabla P + L^{vis}_{2} \\ &\mathrm{P}=\mathrm{P}(\rho,\mathrm{E}^{\mathrm{d}}_{\mathrm{int}}) \Leftrightarrow \ \mathrm{EOS} \end{aligned}\right\} \xrightarrow{V^2 \ll \frac{P}{\rho}} \left\{\begin{aligned} &\nabla\cdot\vec{V} = 0 \\ &\mathrm{E}^{\mathrm{d}}_{\mathrm{int}} = \mathrm{const.} \\ &\frac{\partial \mathrm{V}}{\partial t} + (\mathrm{V}\cdot\nabla)\mathrm{V} = -\frac{1}{\rho}\nabla\mathrm{P}_{\mathrm{Lagrangian}} + \mathrm{f}_{\mathrm{ext}} + \cancel{\frac{1}{\rho}\mathrm{L}^{\mathrm{vis}}_{2}} \\ &\Delta\mathrm{P} = -\xi_0\nabla\cdot\vec{V} \rightarrow \boxed{P^{acausal}_{P:\,<0\,/\,>0} : P \doteq \overset{scalar\ field}{\Phi}} \end{aligned}\right.$$

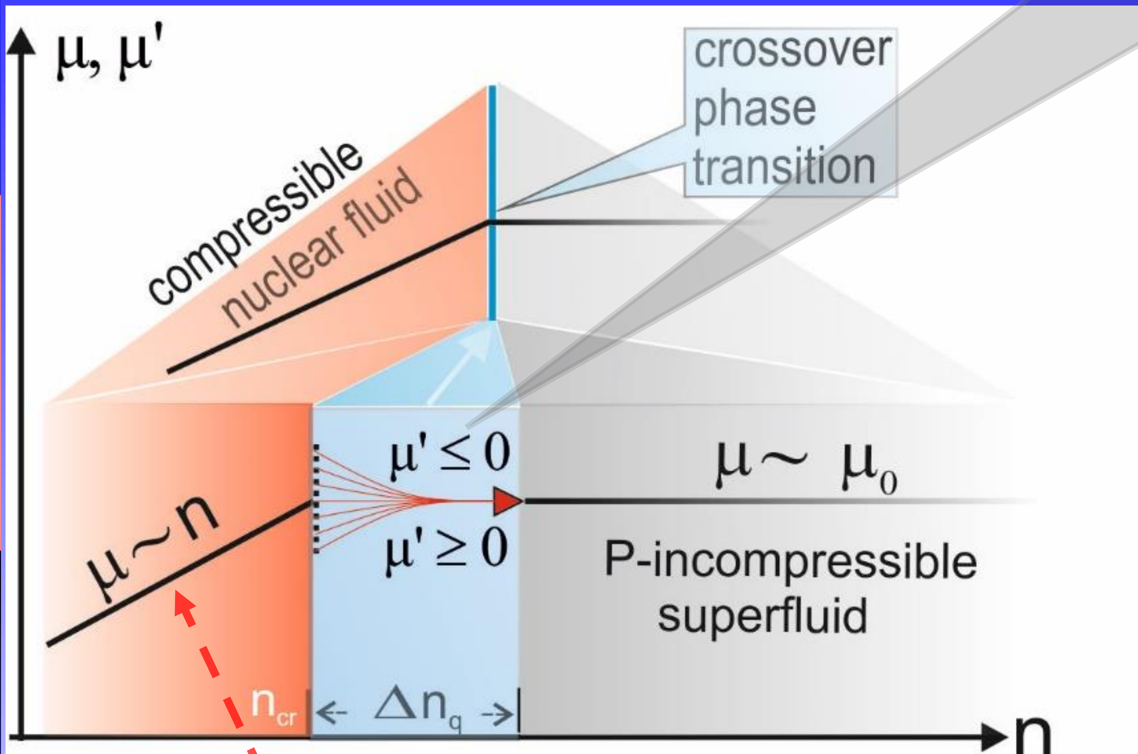
In NSs:

$$\left.\begin{aligned} &\mathrm{V} = 0, \\ &0 = -\frac{1}{\rho}\nabla\mathrm{P}_{\mathrm{Lagrangian}} + \mathrm{f}_{\mathrm{ext}} \end{aligned}\right] \quad \text{Is} \quad \mathrm{P} = \mathrm{P}_{\mathrm{L}},\ \mathrm{P}_{\mathrm{NL}} \ \text{or}\ \mathrm{P}_{\mathrm{L\&NL}}\ ?$$

The incompressibility phase in NSs:

$$\frac{1}{\mu}\frac{d\mu}{dr} = -\frac{G}{c^2}\frac{n}{r^2}\left(\frac{d\mu}{dn}\right)\left(\frac{dn}{dP}\right)\left(\frac{m+4\pi r^3 P}{1-\frac{r_s}{r}}\right)$$

$\mu \left(\doteq (\mathcal{E}+p)/n_b\right)$



$$\mu = \frac{\partial \varepsilon}{\partial n} = \frac{\varepsilon + p}{n} \xrightarrow{\varepsilon = P} \frac{2\varepsilon}{n} \Rightarrow \varepsilon = a n^2$$

But if $\exists\, n_{max} \Rightarrow \varepsilon,\ p\ = const. \Rightarrow \nabla P_L = 0$

$\Rightarrow$ we need a P_{NL} to avoid formation of

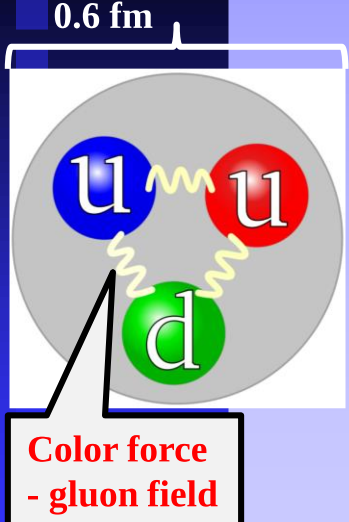
BHs with $M < 2M_{\odot}$

– generates a non-local pressure P_{NL}

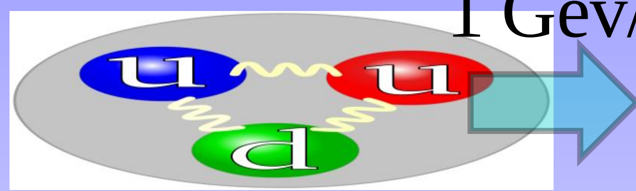
– injects non-dissipative energy, E_ϕ, to maintain superfluidity as eternal stable state of matter in an ever expanding universe

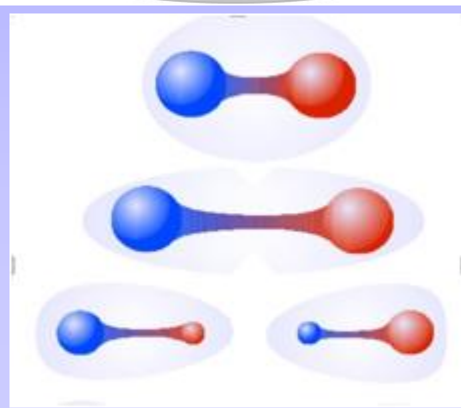
– interacts with matter when $\rho \geq \rho_{crit}$ $\cap$ A ↘ (A= baryonic number).

<u>Under these circumstances: the gluon-like field embedding a sea of quarks could do the job:</u>



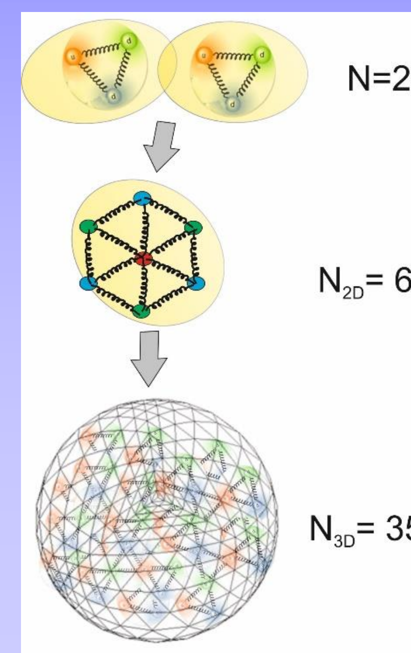
1 Gev/fm





$\rho \geq \rho_{crit}$

$$\sum_i B_i = \bigcup B + B_\phi$$



Bag energy = vacuum pressure
B ~ 200 MeV/fm^3

The GR TOV-Equations modified to include scalar fields

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

$$T_{\mu\nu}^{mod} = T_{\mu\nu}^{0} + T_{\mu\nu}^{\phi}.$$

$$T_{\mu\nu}^{0} = -P^{0} g_{\mu\nu} + (P^{0} + \mathcal{E}^{0}) U_{\mu} U_{\nu} \quad \text{and}$$

$$T_{\mu\nu}^{\phi} = (\partial_{\mu}\phi)(\partial_{\nu}\phi) - g_{\mu\nu}[\frac{1}{2}(\partial_{\sigma}\phi)(\partial^{\sigma}\phi) - V(\phi)]$$

$$g_{\mu\nu} = e^{2\nu} dt^2 - e^{2\lambda} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d^2\varphi^2$$

$$\varepsilon^{tot} = \varepsilon^{0} + \varepsilon^{\Phi}, \text{ where } \varepsilon^{\Phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}(\nabla\phi)^2$$

$$P^{tot} = P^{0} + P^{\Phi}, \; P^{\Phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{1}{6}(\nabla\phi)^2$$

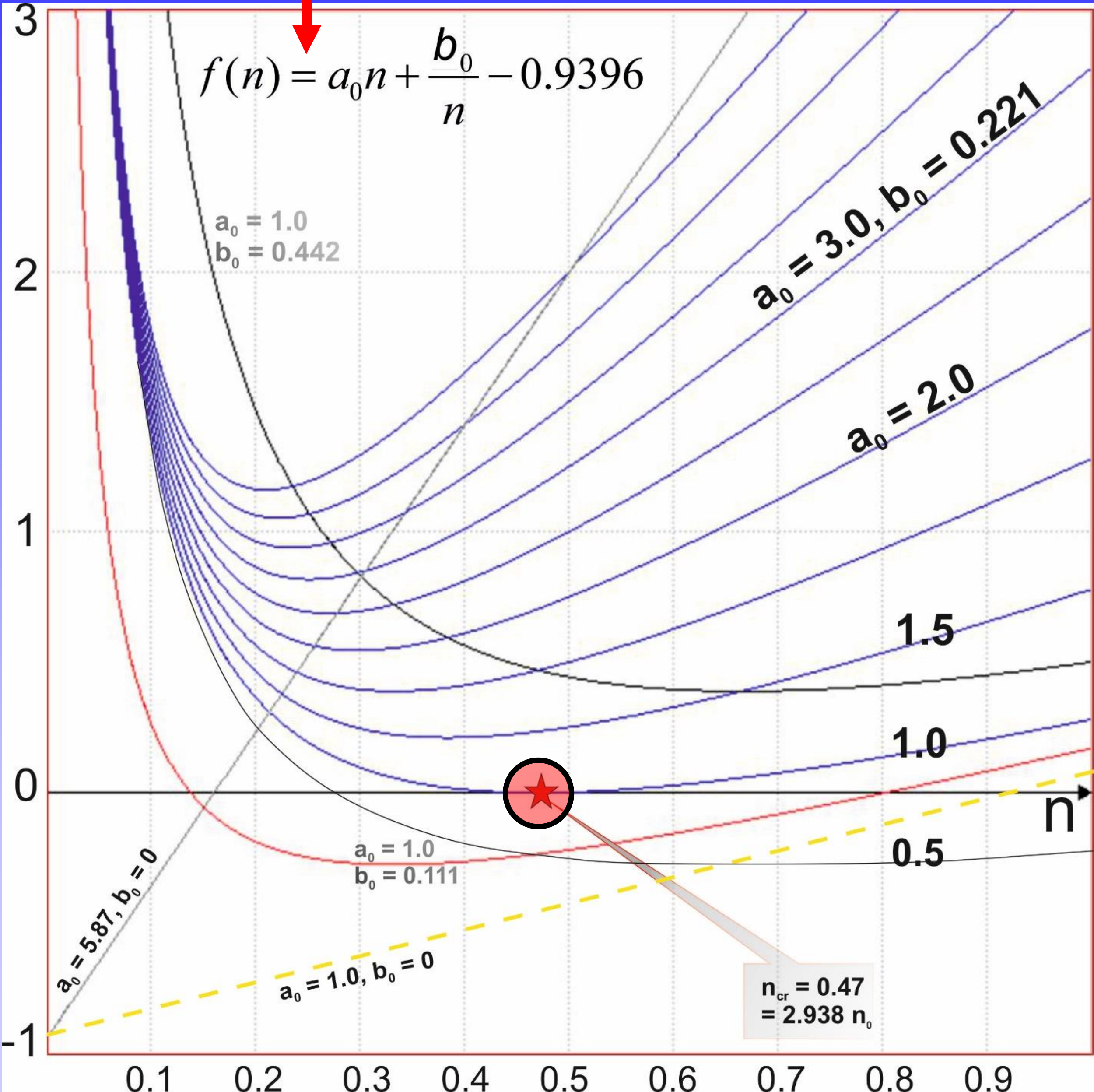
Here we assume: $\dot{\phi} = \nabla\phi = 0$ and $V(\phi) = \bar{\alpha} A_q^{\Gamma} = a_0 r^2 + b_0$

The critical density for the scalar field baryon matter interaction and phase transition into quark-superfluid

$$\frac{\varepsilon_b + \varepsilon_\phi}{n} \geq 0.9396 \text{ GeV}$$

$$P = \mathcal{E} = a_0 n^2$$

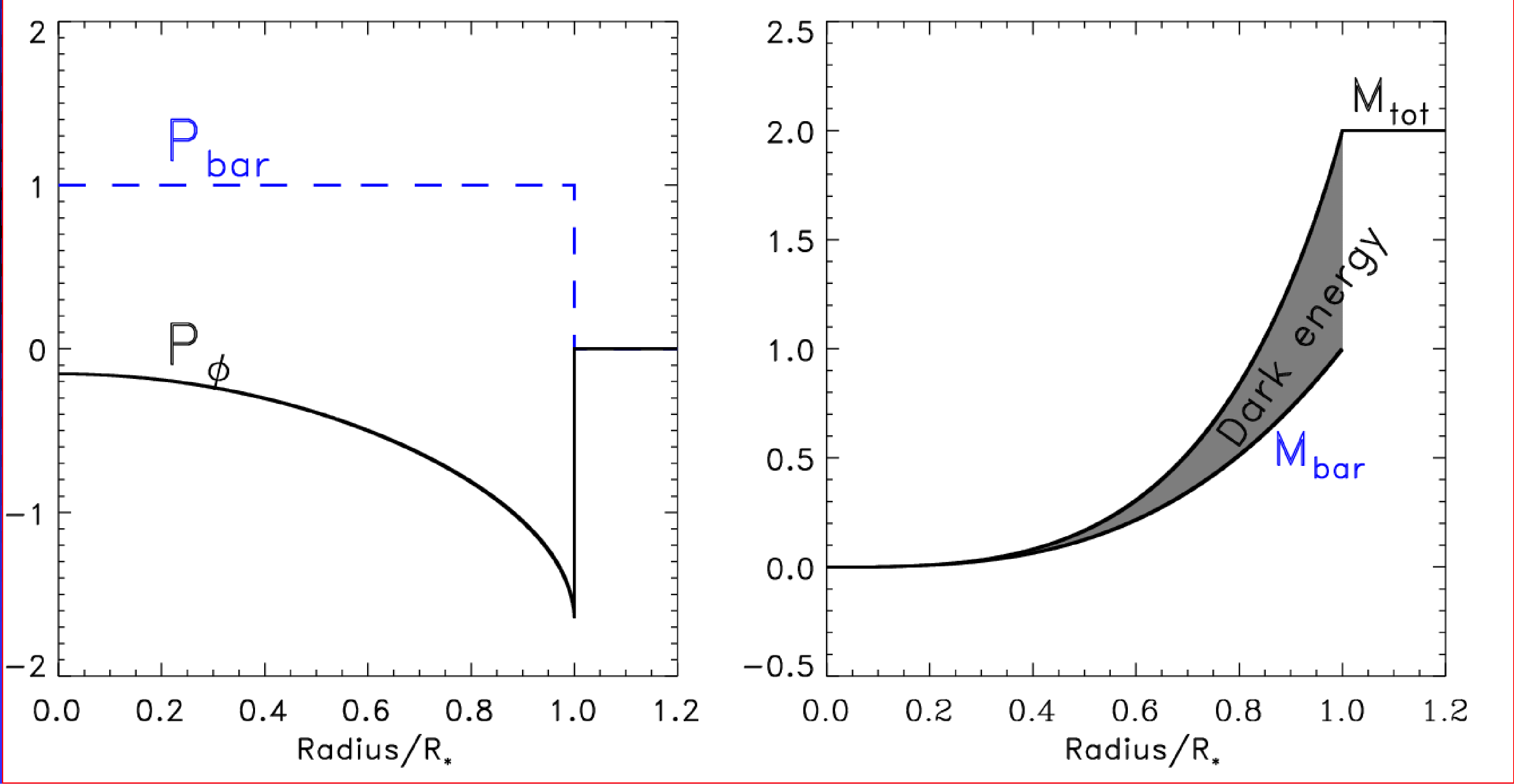
$$\mathcal{E}_\phi(r) = \bar{a}_0 r^2 + b_0$$



$$\min_n f(n) = 0$$

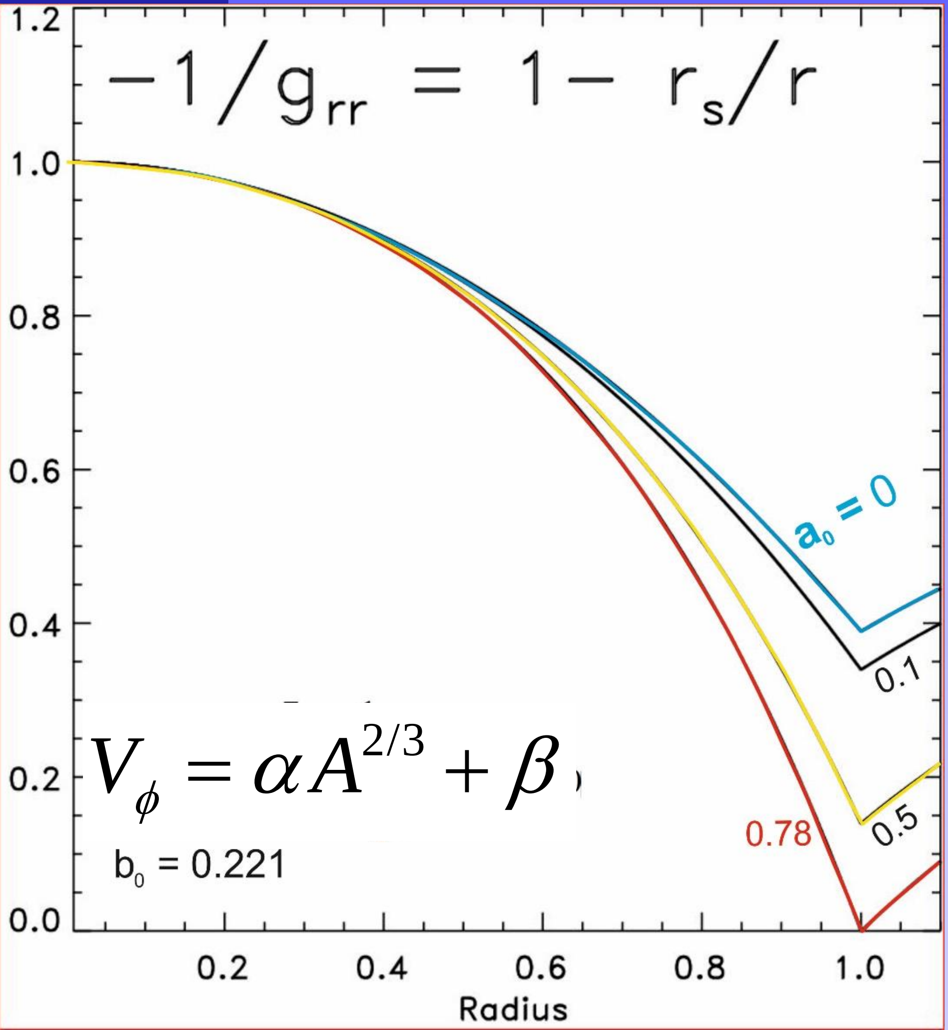
$$\Rightarrow n_{cr} \approx 3 \times n_0$$

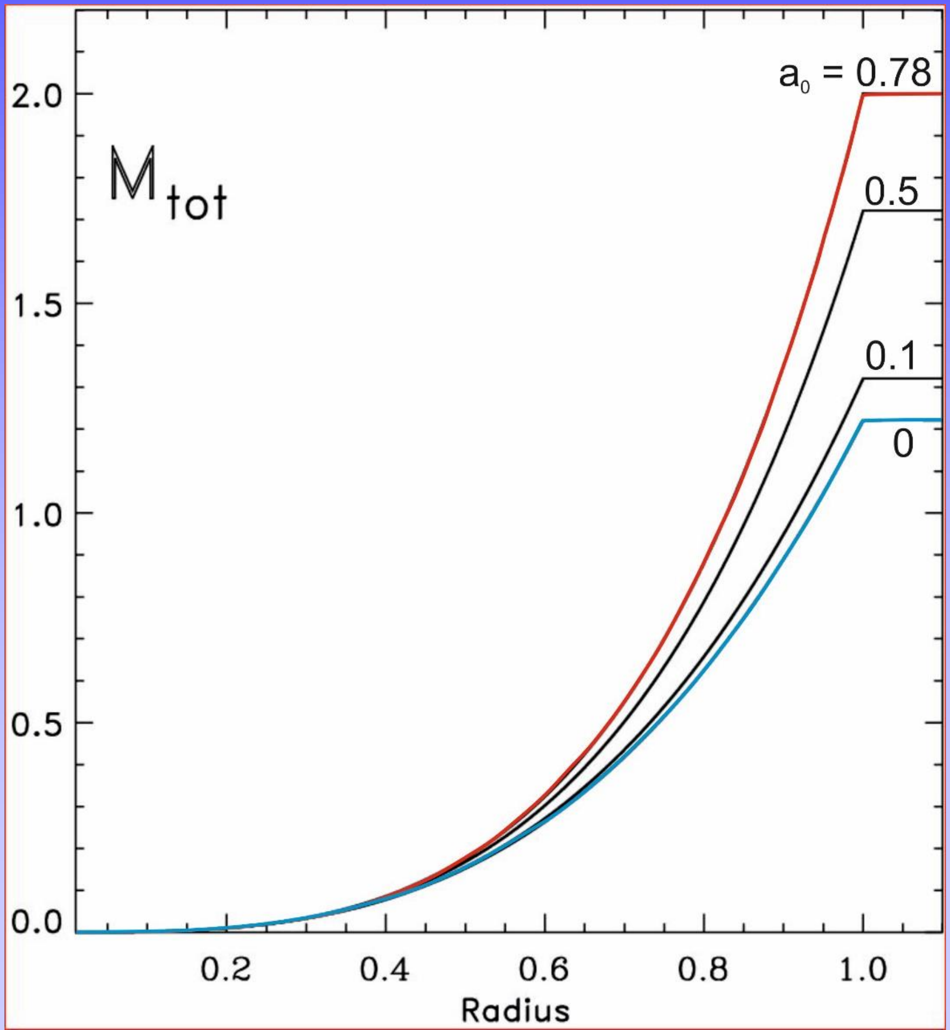
Formation of dark energy objects - DEOs



Formation of dark energy objects - DEOs

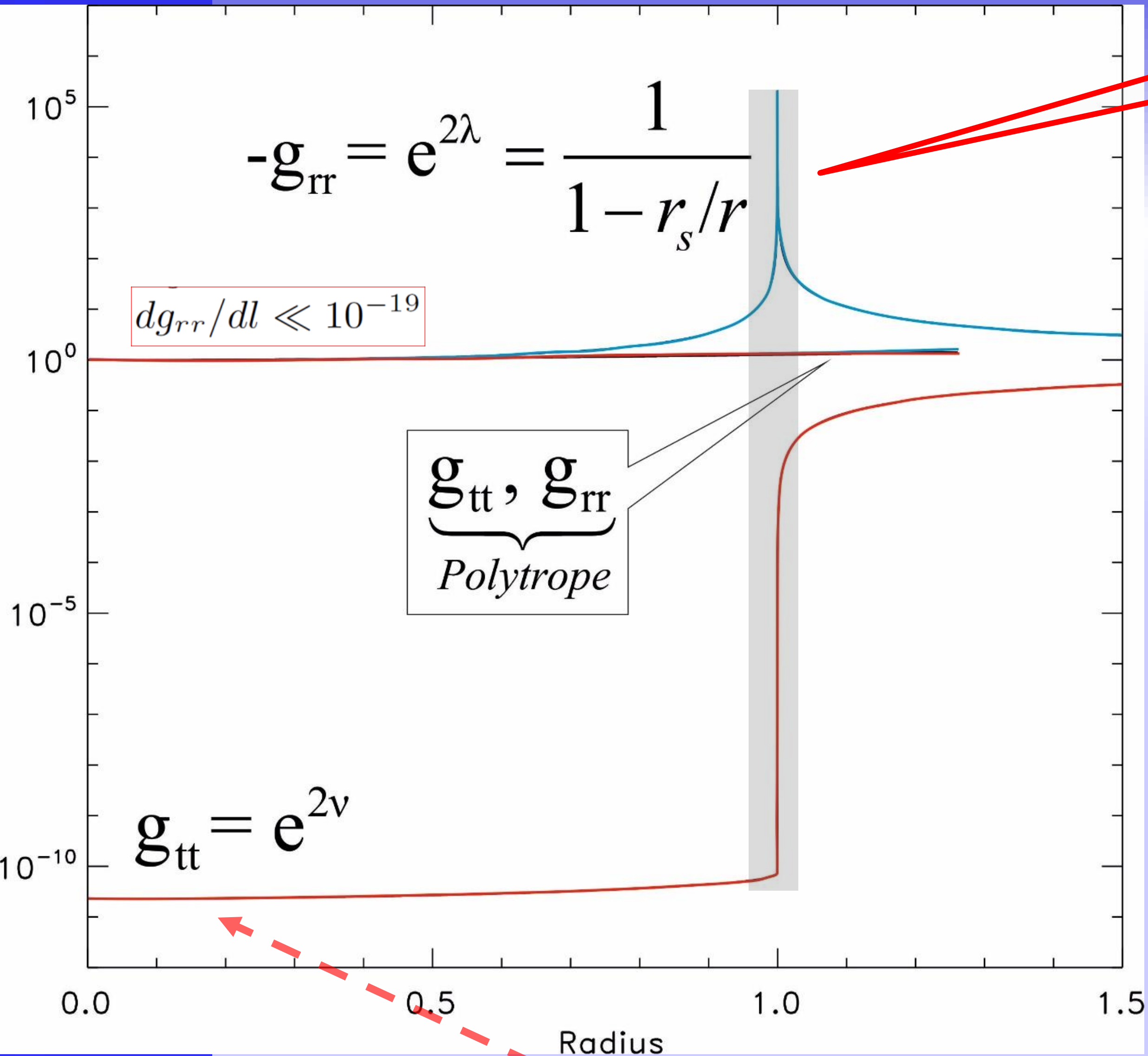
$$z = \frac{1}{\sqrt{1 - r_s / R_*}} + 1 = g_{rr} - 1 \quad \Rightarrow \quad \frac{-1}{g_{rr}} = \frac{1}{(z+1)^2}$$

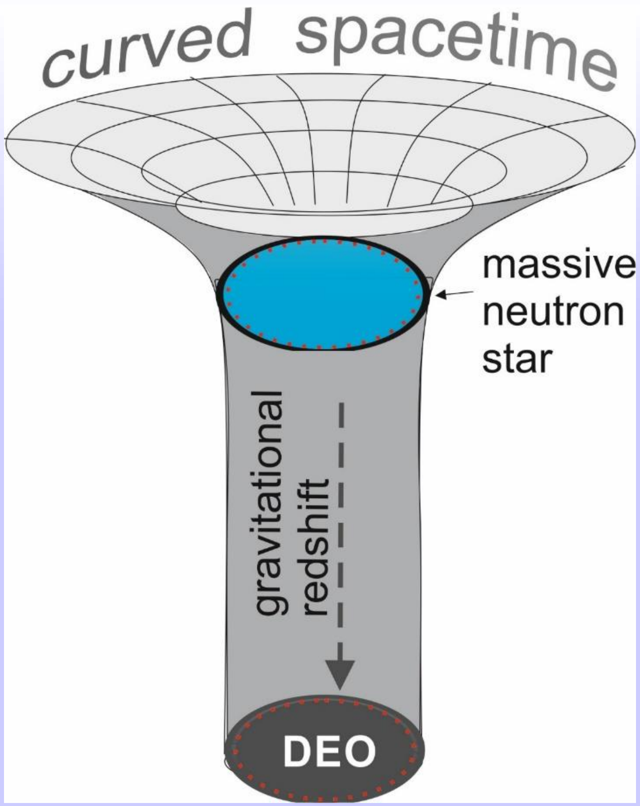




DEOs: are objects that live in flat spacetime, but surrounden by strongly curved spcetime

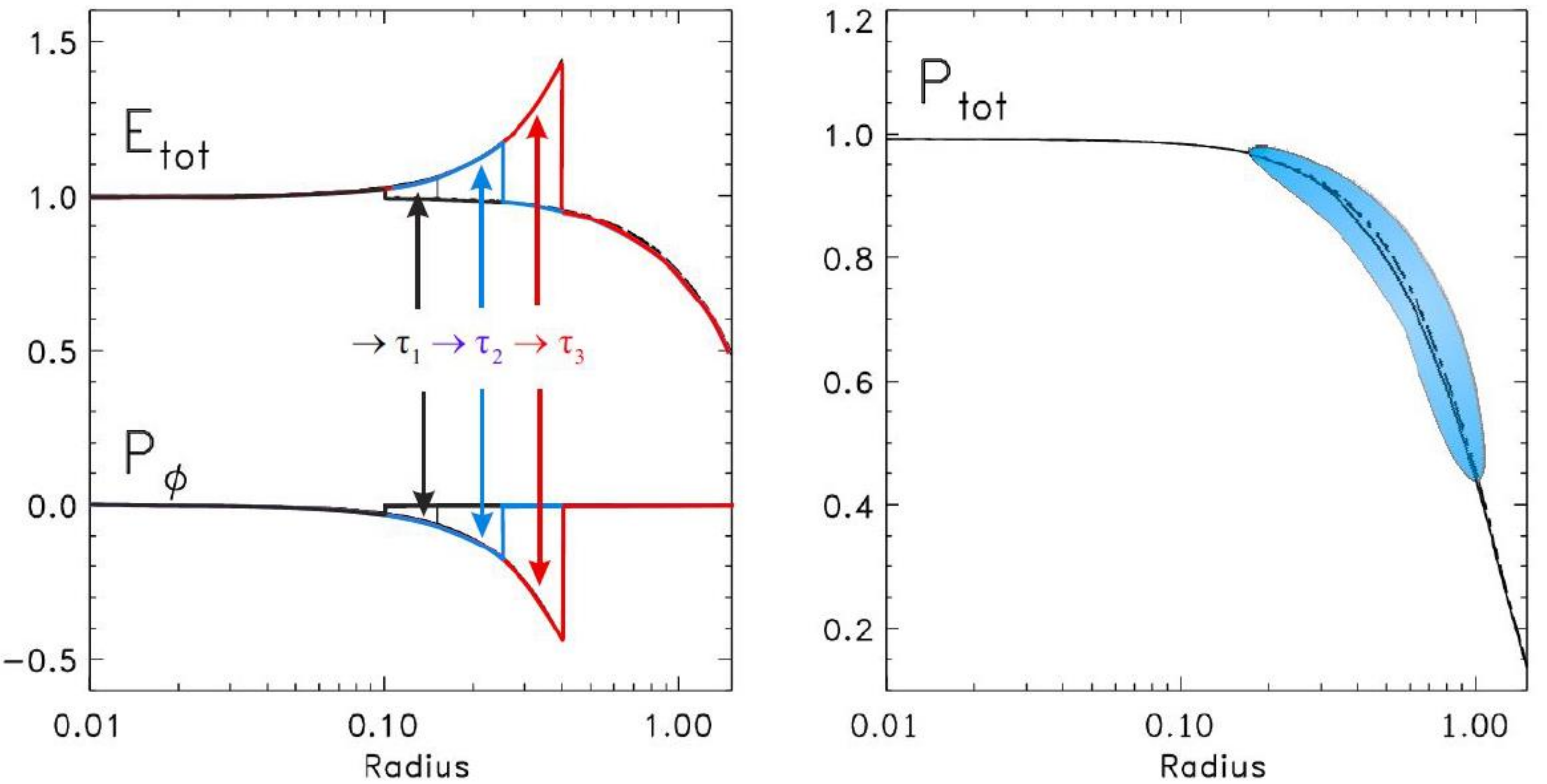
$$-g_{rr} = (z+1)^2$$



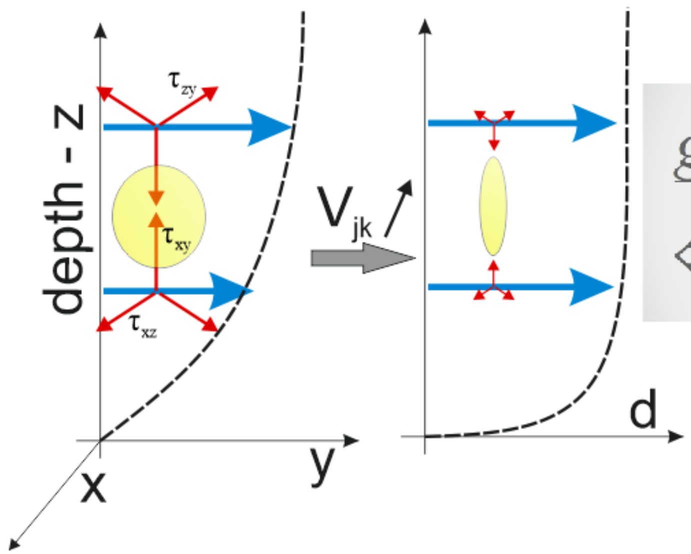


$$\mu(r)e^{V(r)} = const. \xrightarrow{e^{V(r)}=const.} \mu(r) = const.$$

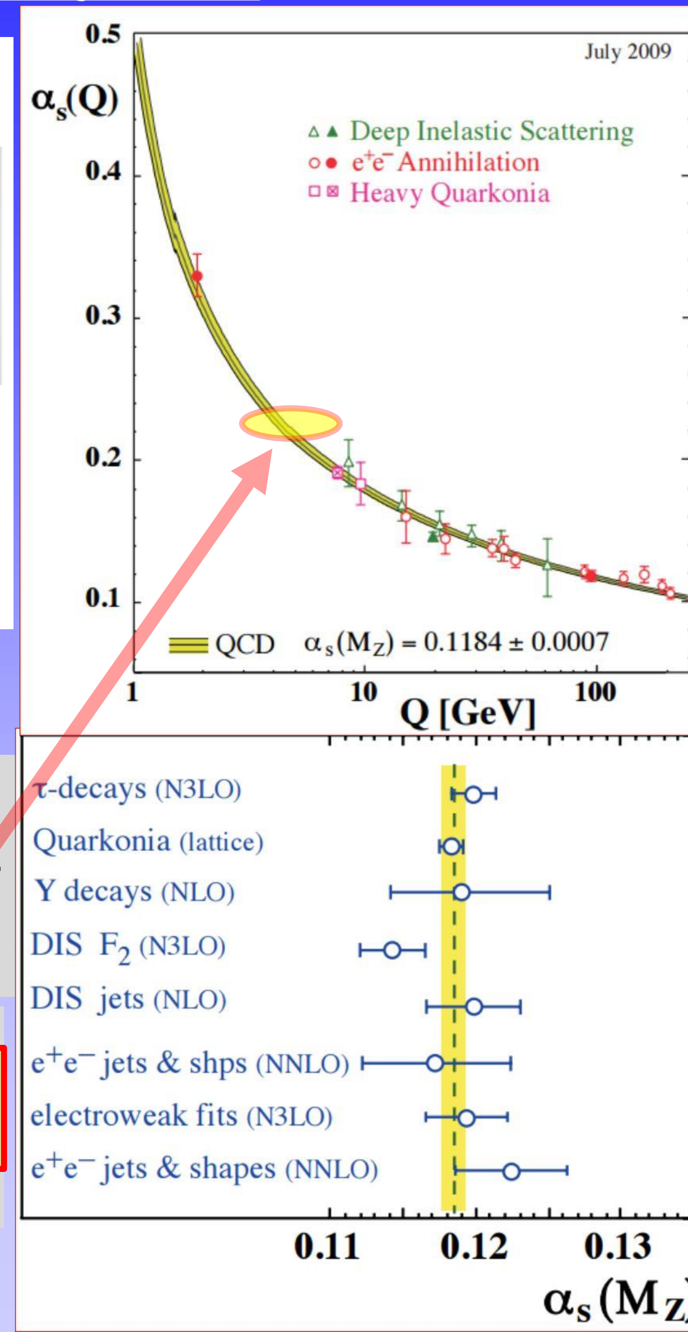
INQS-core & shell



Do the quarks inside DEOs move asymptotically free?



$$\text{gluons}: Q \nearrow \; \Leftrightarrow Q_{\mu\bar{\nu}} \nearrow \& \, Q_{\bar{\mu}\nu} \searrow$$

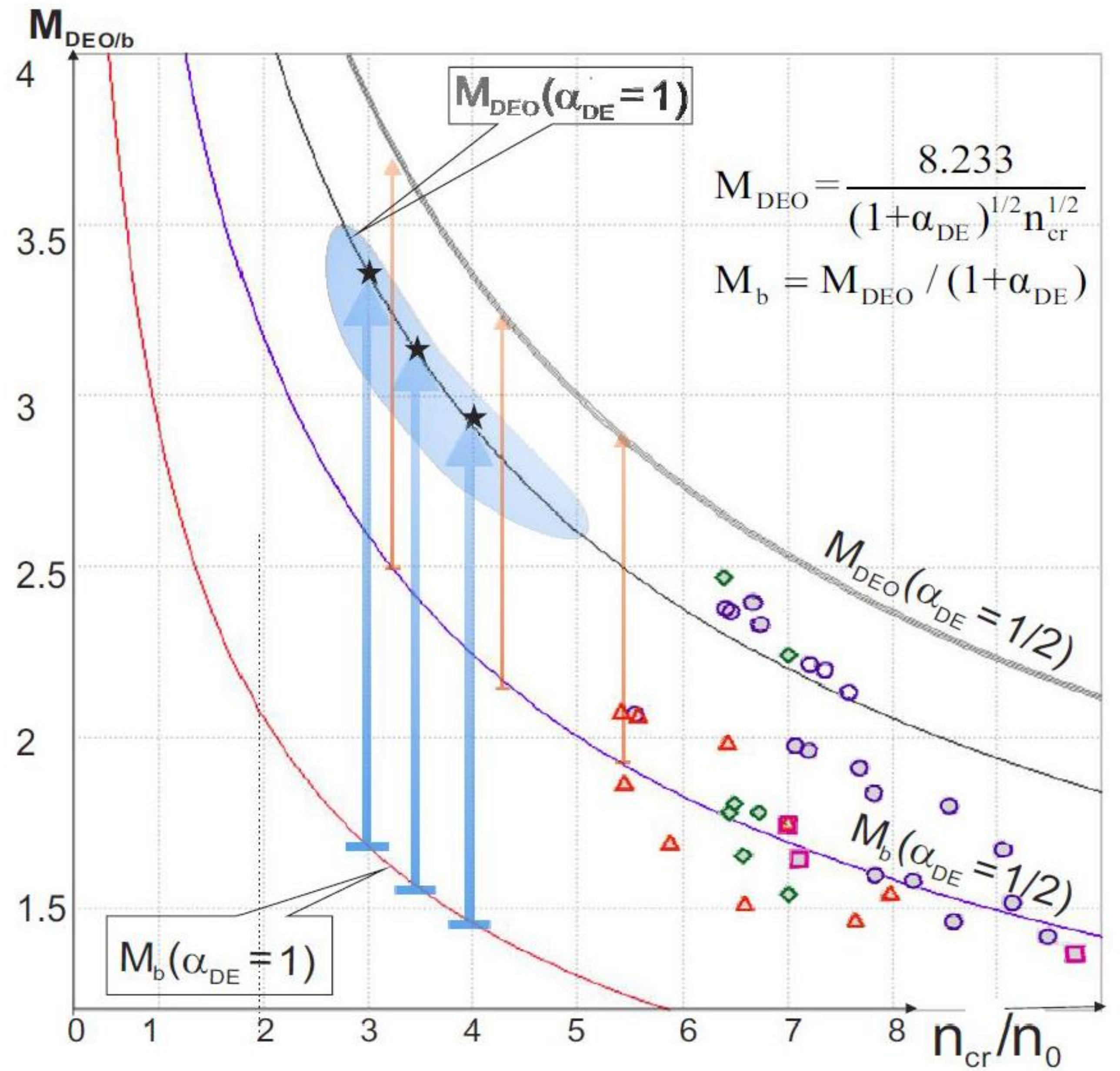


$$\alpha_s = \frac{\pi}{9}\frac{1}{\ln(Q^2/\Lambda^2)} \rightarrow \frac{\pi}{9}\frac{1}{\ln(k_F^2/\Lambda_F^2)}$$

$$\left.\begin{array}{l} k_F = (3\pi^2 n)^{1/3} \xrightarrow{n=n_{cr}} 2.4/\ fm \\ \Lambda = 197.33\ \text{MeV}_{(\Lambda_{can}=217\pm 23\ \text{MeV})} \end{array}\right\} \Rightarrow \boxed{\alpha_s = 0.199}$$

Observed NSs & migration towards invisibility

$$M_{DEO} = M_{\phi} + M_b = (1+\alpha_D) M_b$$



Summary:

A model for converting old and isolated NSs into DEOs.

A DEOs is general relativistic giant hadron, filled with a Fermi-sea of quarks.

- **Embedded in a fairly flate spacetime**
- **globally confined by the surrounding, but strongly curved spacetime**
- **Maximally compressed (incompressible)**
- **in a quark-superfluid state**
- **and**
- **moving freely in line with the asymptotic freedom of QCD.**