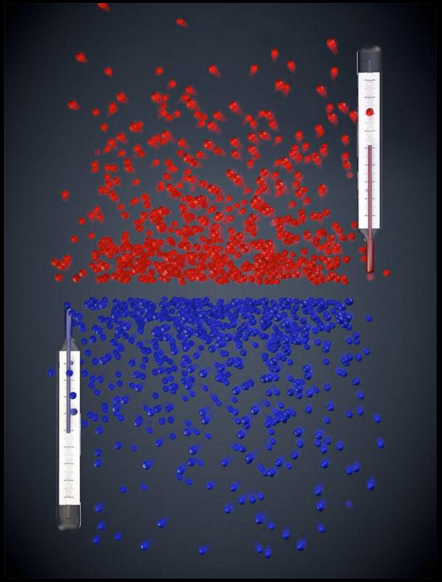
Cosmology With Negative Absolute Temperatures

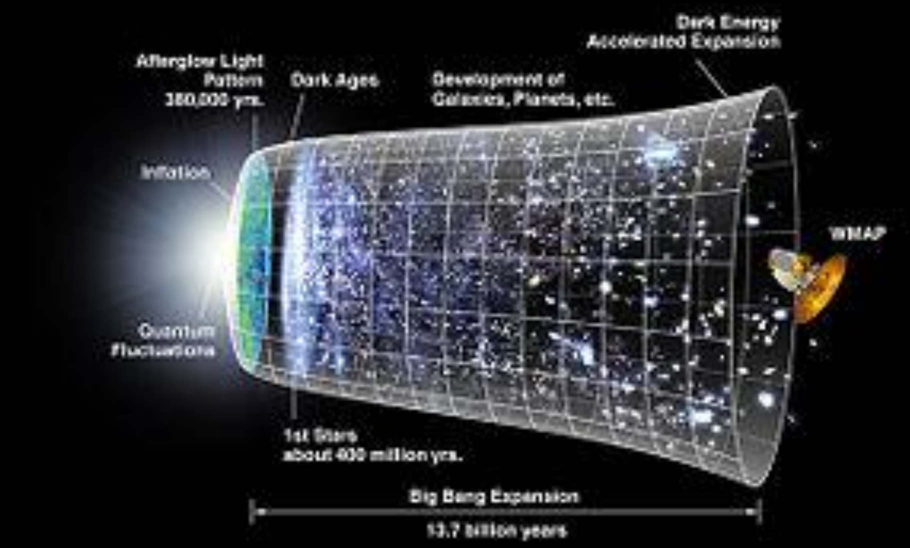
José P. P. Vieira

arXiv:1604.05099

VIA Lecture

17th June 2016





US University of Sussex

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What do you mean T<0?

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V$$

$$S = k_B \ln w$$

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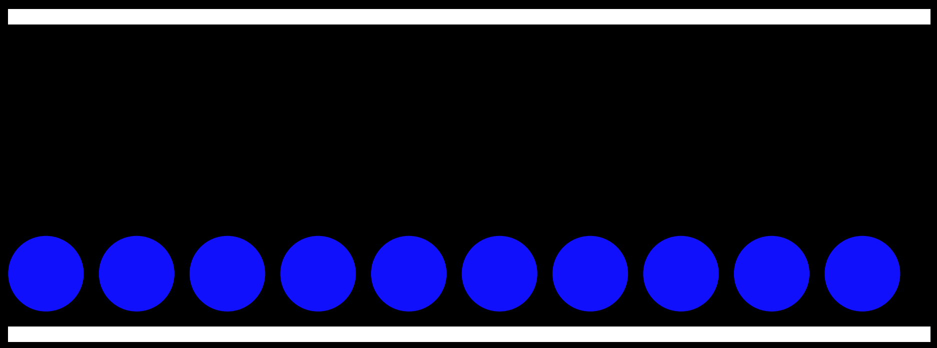

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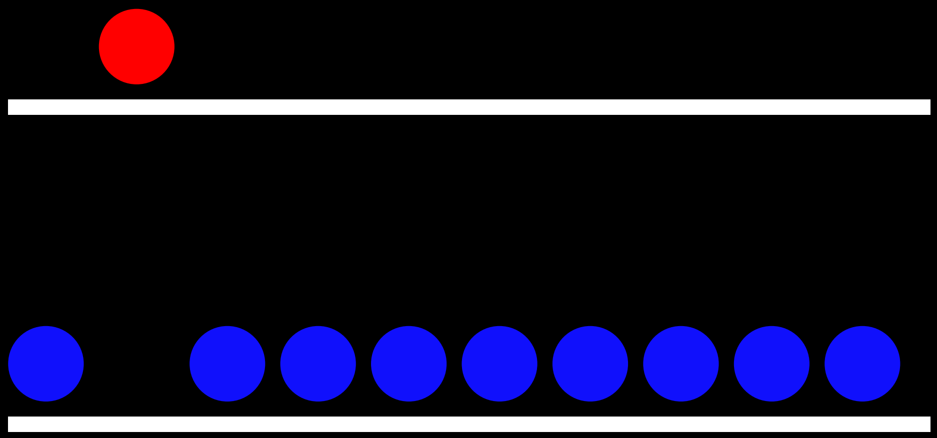

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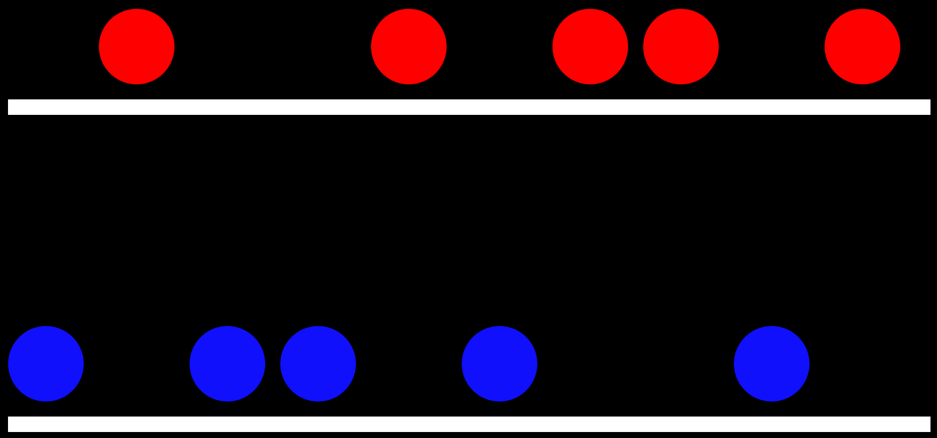

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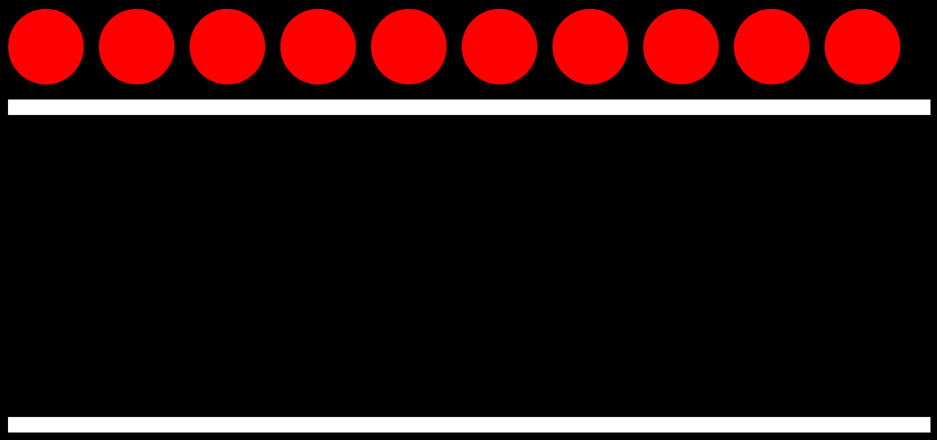

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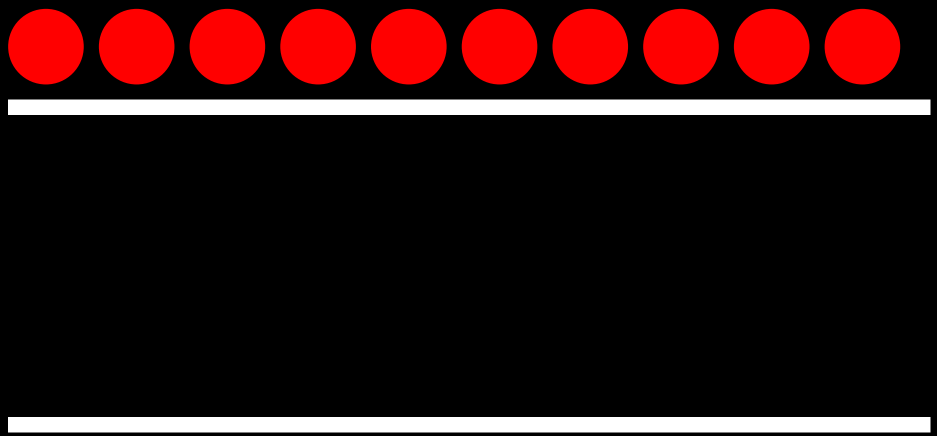

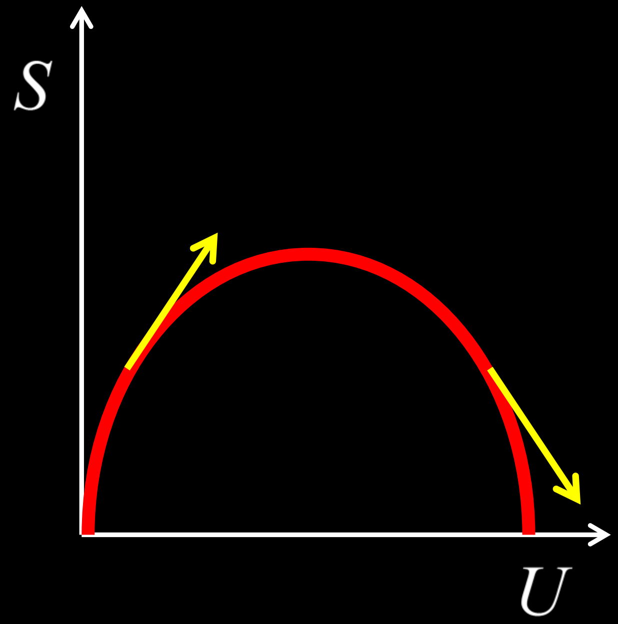

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  Ramsay, *Phys. Rev., 103:20-28*

- 2012 – T<0 with motional degrees of freedom

  Braun et al, *Science 339(6115)*

Why do we care?

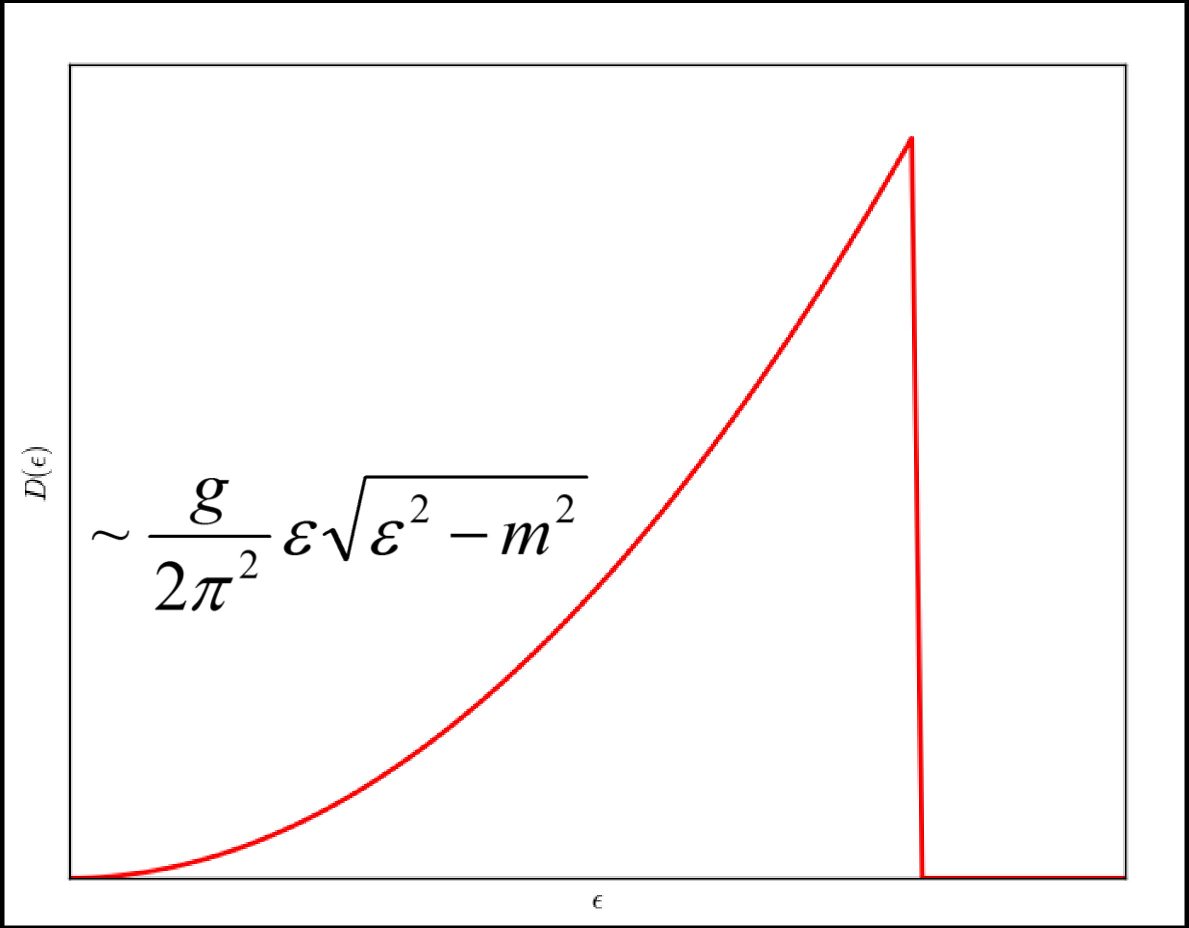
$$dU = TdS - PdV$$

$$\left(\frac{\partial S}{\partial V}\right)_U = \frac{P}{T} \geq 0$$

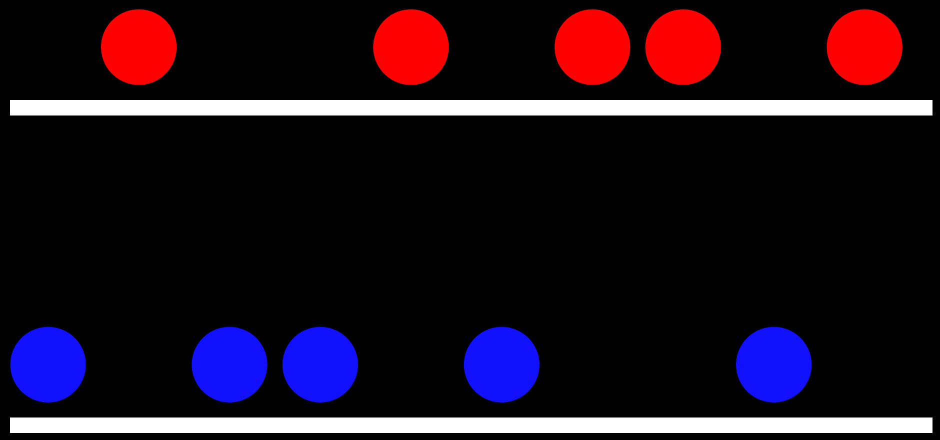
$$T < 0 \Rightarrow P < 0$$

Inflation?

Cosmology with an energy cut-off?



Making use of holes



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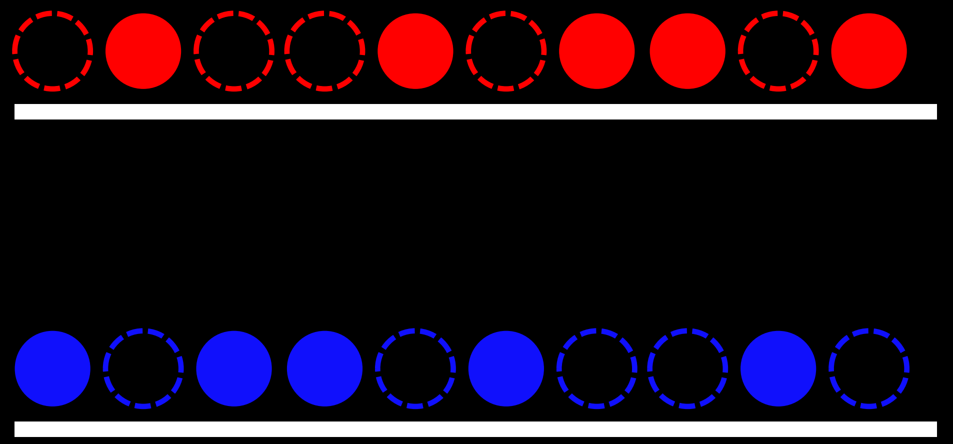

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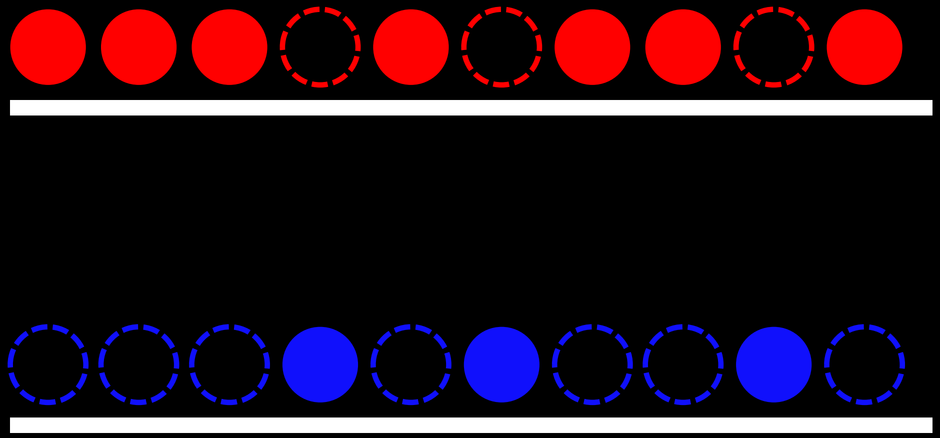

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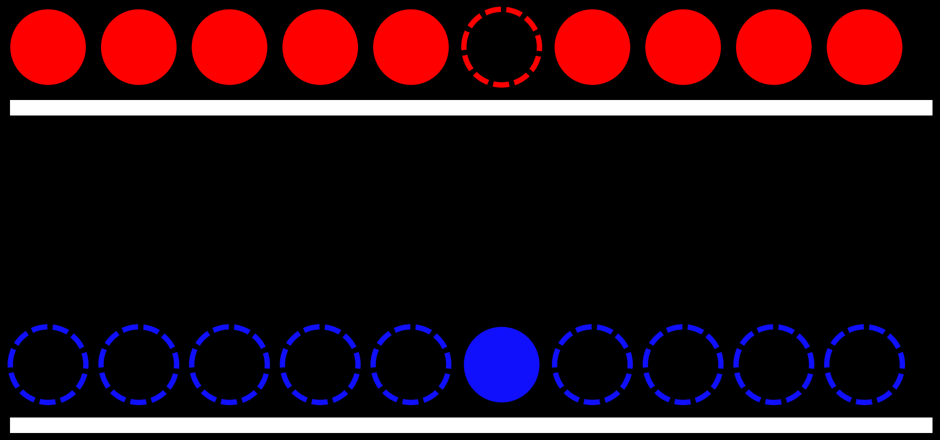
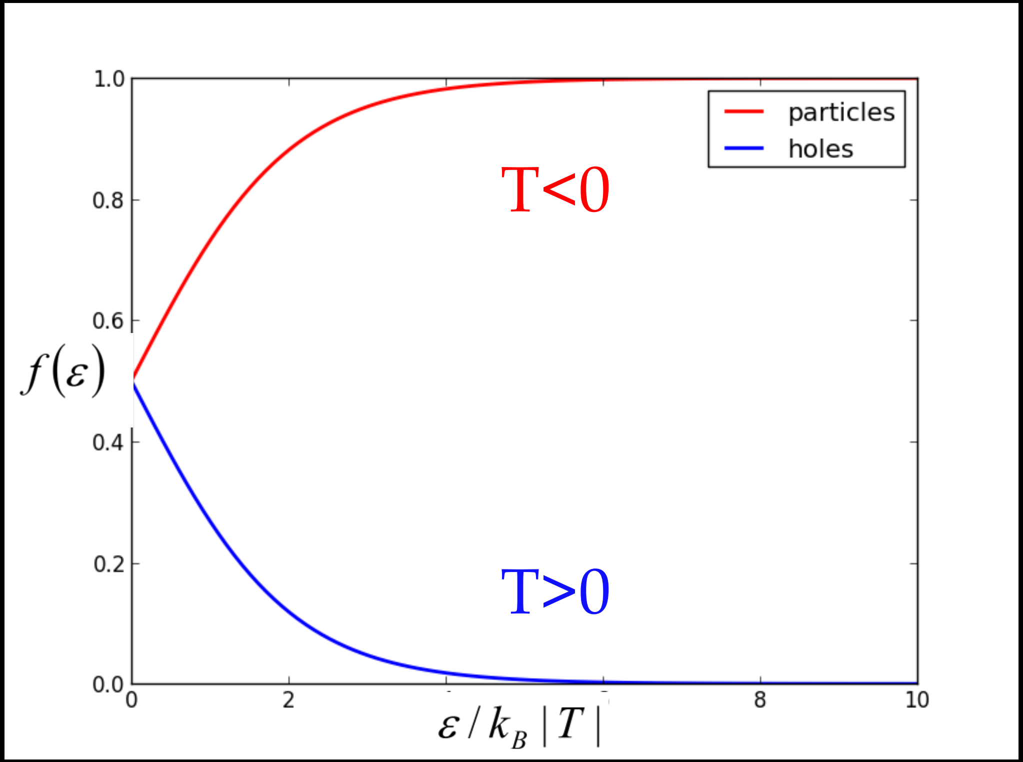


Making use of holes

$$f(\varepsilon;T) = 1 - f(\varepsilon;-T)$$
$$\rho(T) = \rho_{\max} - \rho(-T)$$
$$P(T) = -\rho_{\max} - P(-T)$$



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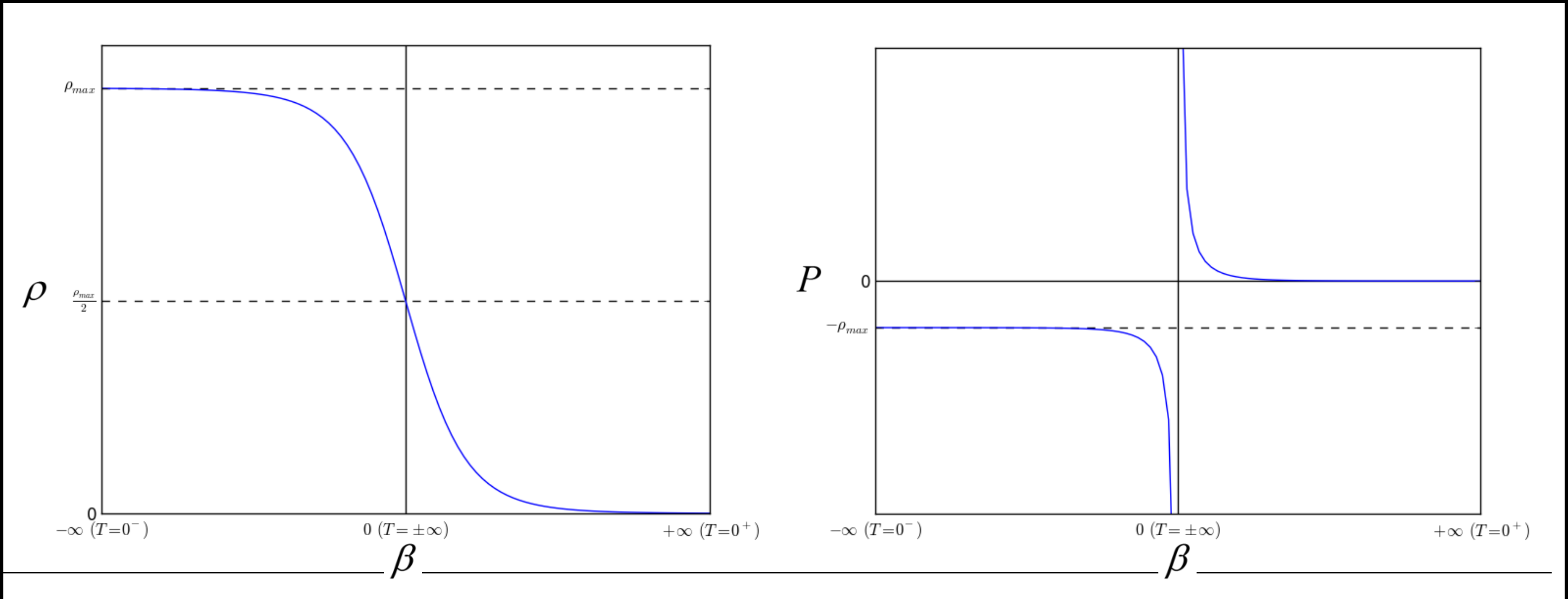


Expanding Universe

Low-temperature holes may behave like

radiation

$$w = -\frac{1}{3}\left(4\frac{\rho_{\max}}{\rho} - 1\right)$$

matter

$$w = -\frac{\rho_{\max}}{\rho}$$

$$\frac{d\ln\rho}{dt} = -3H(1+w) \geq 0$$

Contracting Universe

$$\frac{d\ln\rho}{dt} = -3H(1+w)$$

$$\rho \xrightarrow[H<0]{} \frac{1}{2}\rho_{\max}$$

$$\beta \xrightarrow[H<0]{} 0$$

Contracting Universe

$$\rho = \int \frac{g(\varepsilon)\varepsilon}{e^{\beta\varepsilon}+1} d\varepsilon$$

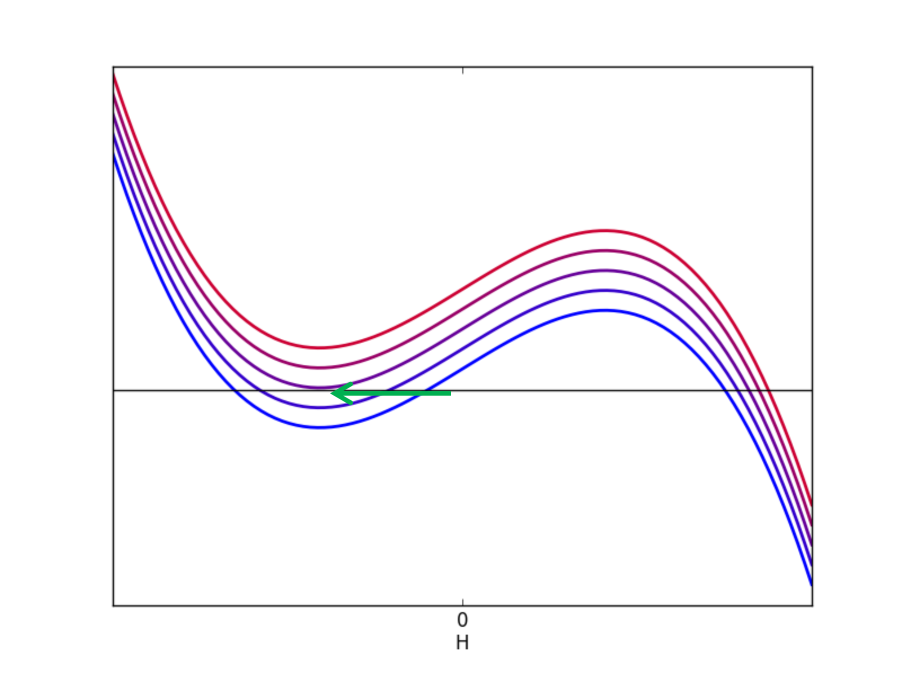
$$P = \frac{1}{\beta}\int g(\varepsilon)\ln\left[1+e^{-\beta\varepsilon}\right] d\varepsilon$$

$$\rho = \frac{1}{2}\rho_{\max} - \frac{\langle \varepsilon^2 \rangle_0}{4}\beta + \ldots$$

$$P = \frac{\ln 2}{\beta} n_{\max} - \frac{1}{2}\rho_{\max} + \frac{\langle \varepsilon^2 \rangle_0}{8}\beta + \ldots$$

Contracting Universe

$$-2\left(H^3 - H_i^{\,3}\right) + \rho_{\max}\left(H - H_i\right) + \frac{\ln 2}{4} n_{\max} \left\langle \varepsilon^2 \right\rangle_0 \left(t - t_i\right) = 0$$



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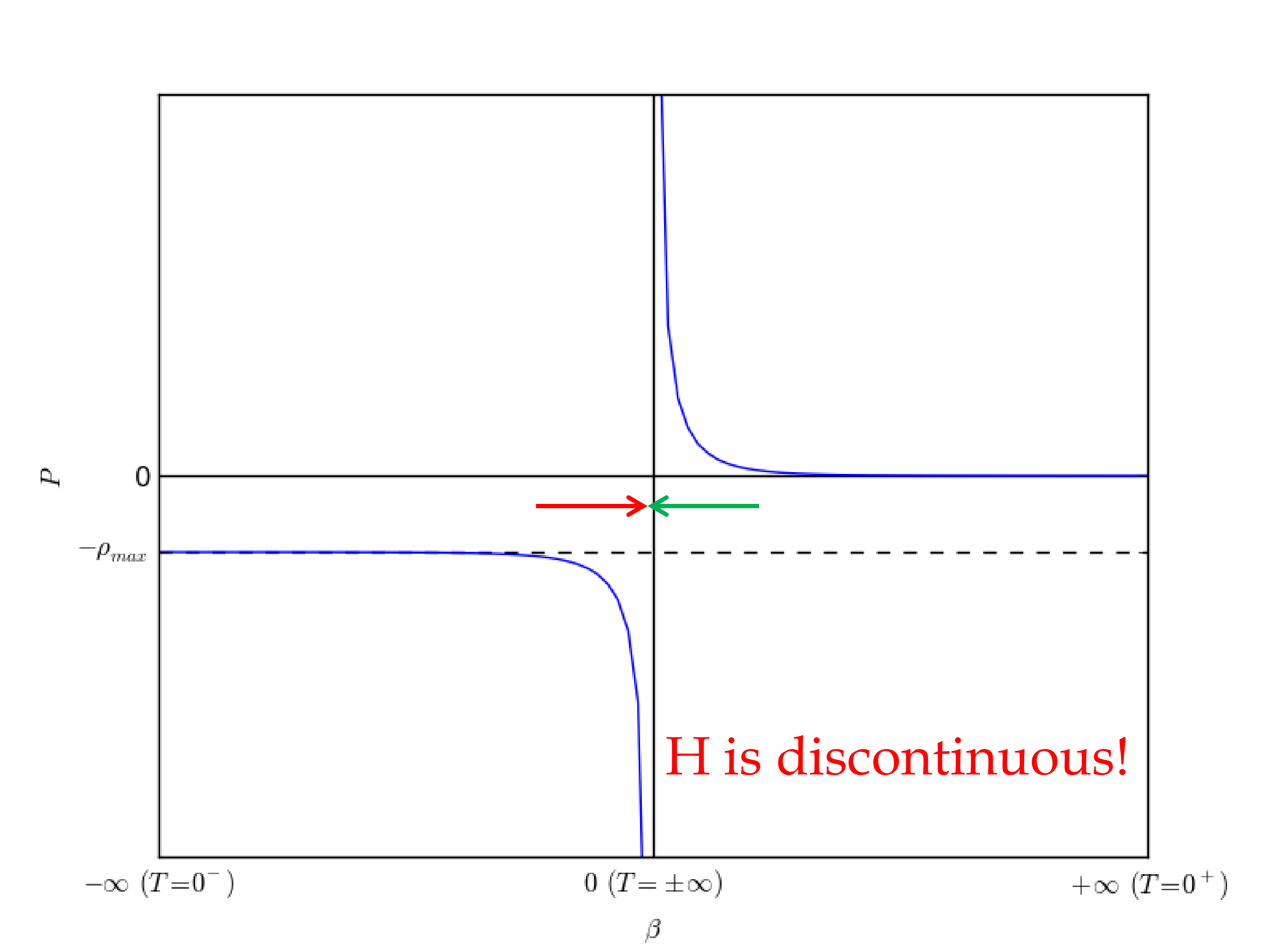

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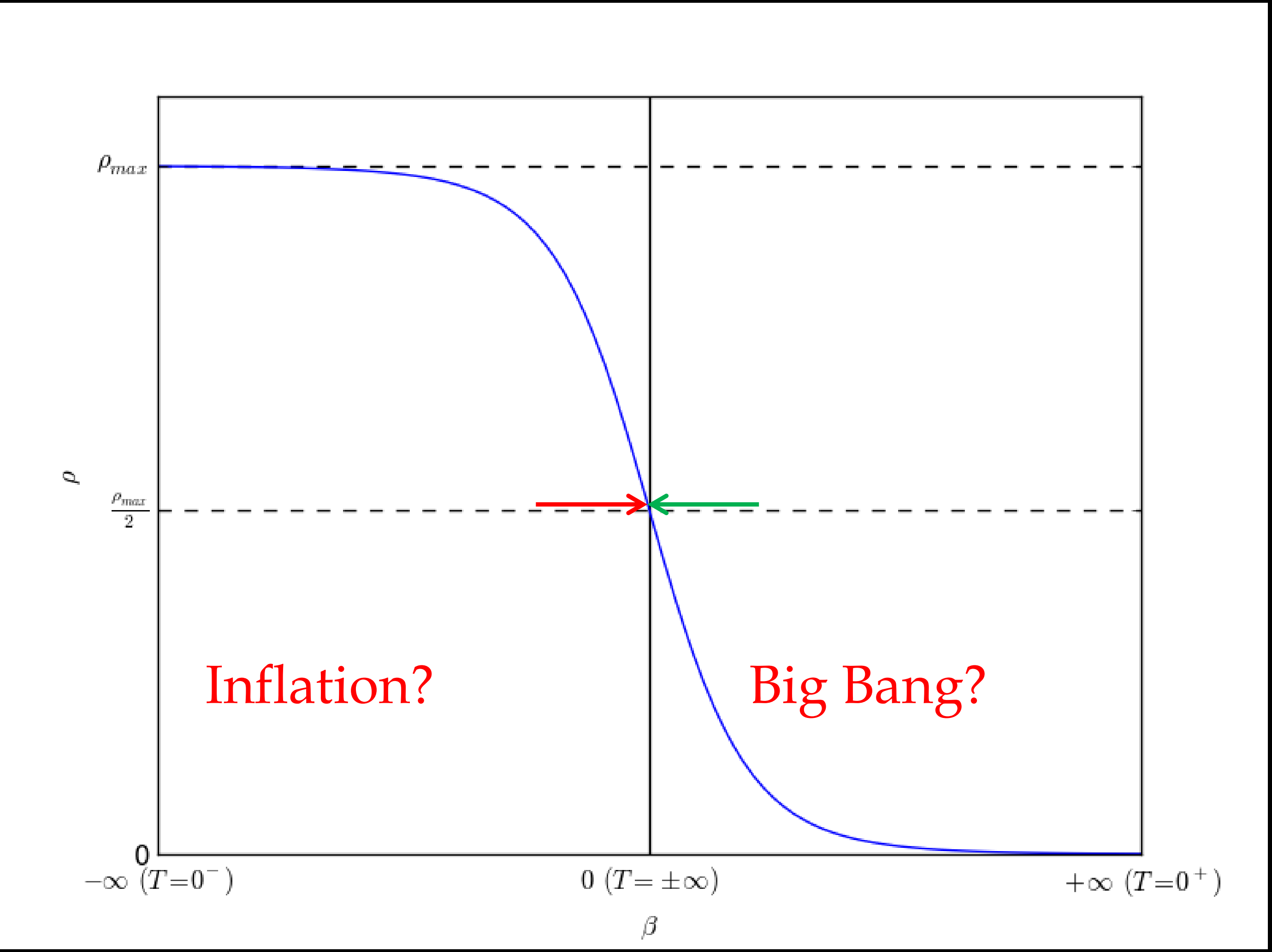
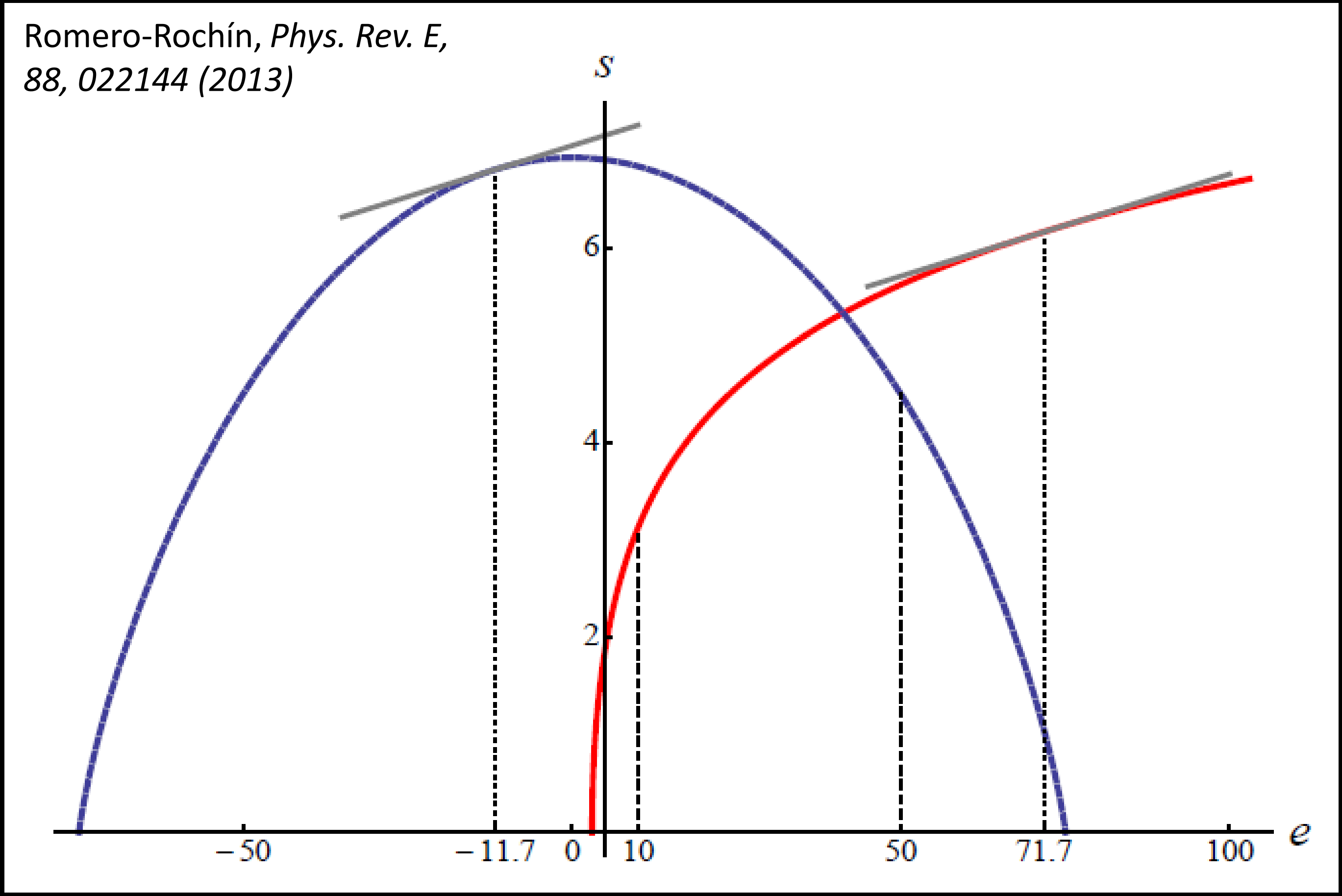


Recooling!


Romero-Rochín, *Phys. Rev. E, 88, 022144 (2013)*
s
6
4
2
e
−50
−11.7
0
10
50
71.7
100

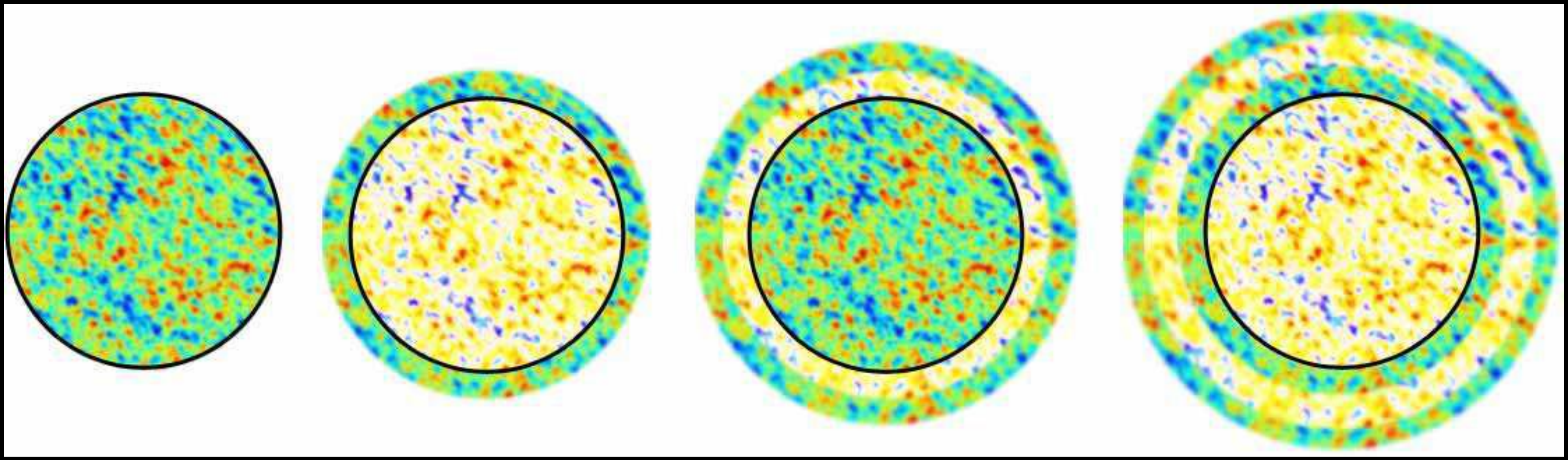
Full of holes?

$$N = \ln\left|\frac{a_f}{a_i}\right| = \frac{1}{3(1+w_h)}\ln\left|\frac{\rho_{hi}}{\rho_{hf}}\right|$$

$$\Downarrow$$

$$N \approx 60 \Rightarrow \frac{\rho_{\max} - \rho_f}{\rho_{\max}} \approx 10^{-6}$$

Perturbations!



Chen et al (2007)
arXiv:0712.2345v3

Thermal fluctuations? $Z = \sum_s e^{-\beta E_s} \Rightarrow \langle \rho^n \rangle = \frac{1}{Z}\left(-\frac{1}{V}\frac{\partial}{\partial \beta}\right)^n Z$

$$\langle \delta\rho^{n+1} \rangle = -\frac{1}{V}\frac{\partial}{\partial\beta}\langle \delta\rho^n \rangle - \frac{n}{V}\frac{\partial \langle \rho \rangle}{\partial \beta}\langle \delta\rho^{n-1} \rangle$$

Exact gaussianity at the bounce!

Revenge of de Sitter

$$\left\langle \delta\rho^2 \right\rangle = -\frac{1}{V}\frac{\partial\langle\rho\rangle}{\partial\beta} = \frac{8}{V}\left(\frac{15}{7\pi^2 g}\right)^{1/4} \rho_h^{5/4}$$

$$\left\langle \zeta^2 \right\rangle = \frac{1}{9}\frac{\left\langle \delta\rho^2 \right\rangle}{\left(\rho + P\right)^2} \propto a^3$$

$$P_\xi(k) = const$$

Amplifying Quantum Perturbations?

$$\left\langle \zeta^2 \right\rangle = H^2 \frac{\left\langle \delta\rho^2 \right\rangle}{\dot{\rho}^2}$$

$$\left\langle \zeta^2 \right\rangle = H^2 \frac{\left\langle \delta\rho_T^2 + \delta\rho_\sigma^2 \right\rangle}{\left(\dot{\rho}_T + \dot{\rho}_\sigma\right)^2} \xrightarrow[T \to 0^-]{} H^2 \frac{\left\langle \delta\rho_\sigma^2 \right\rangle}{\dot{\rho}_\sigma^2}$$

Depends only on H!

In conclusion

- If a component of the Universe is allowed to be at $T<0$:

phantom regimes with $w \leq -1$ naturally arise

(inflation? dark energy? Big Crunch averted?)

BUT…

In Conclusion

- What is the microscopic origin of P<0?
  Modified Heisenberg uncertainty?
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In Conclusion

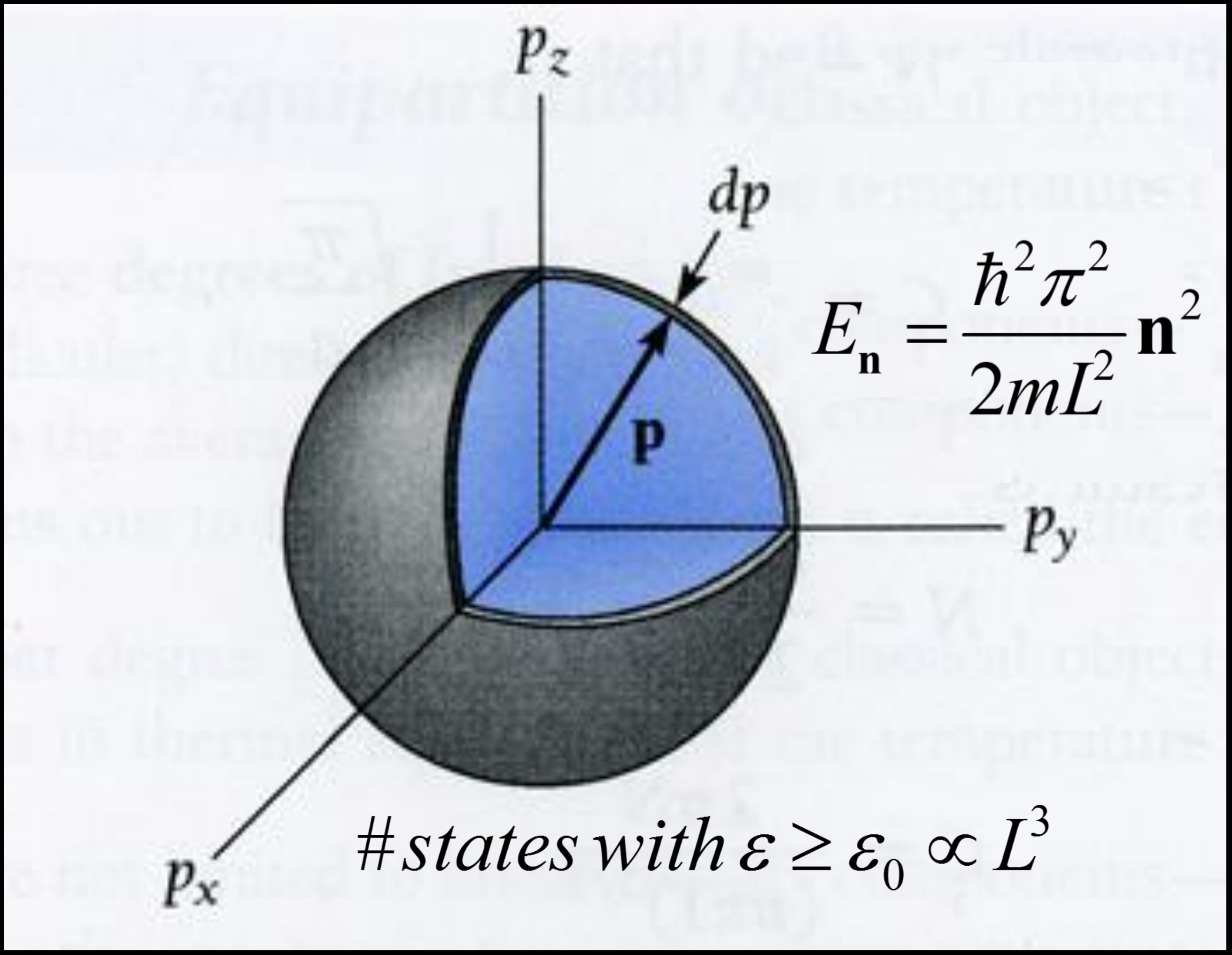
- What is the microscopic origin of P<0?
  Modified Heisenberg uncertainty?
  Could thermalisation at high temperatures be forbidden?

- What happens away from equilibrium?
  Thermalisation with bosons at end of inflation?
  What happens at the bounce?

- Do perturbations match CMB?
  Natural curvatons?
  Isocurvature modes?


THE
END

Cosmology with an energy cut-off?



Thermodynamic Formulas

$$f(\varepsilon;T,\mu) = 1 - f(\varepsilon;-T,\mu)$$

$$\rho(T,\mu) = \rho_{\max} - \rho(-T,\mu)$$

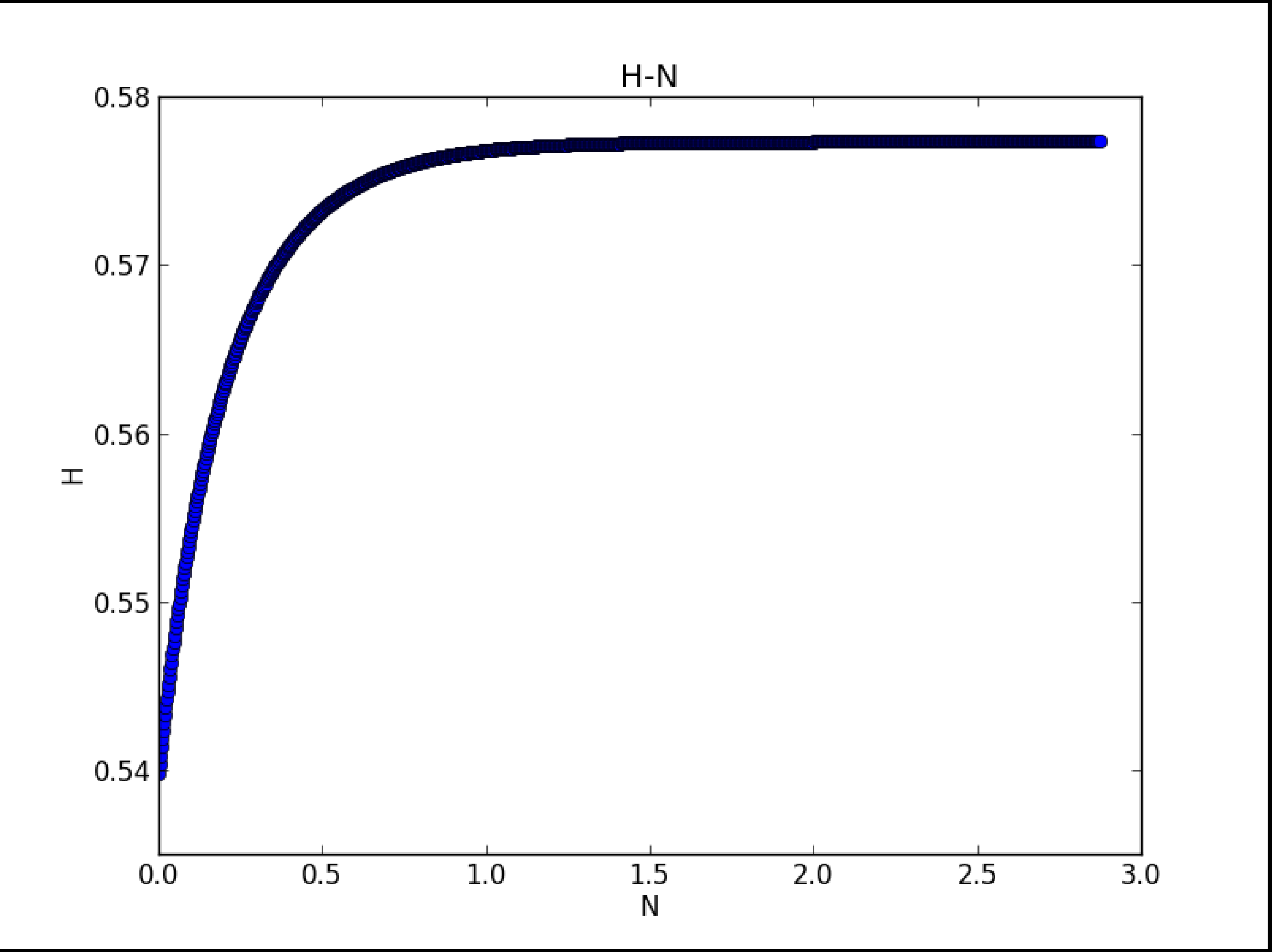
$$n(T,\mu) = n_{\max} - n(-T,\mu)$$

$$P(T,\mu) = 2\mu n_{\max} - \rho_{\max} - P(-T,\mu)$$

$$P(T,\mu) = -\left(\frac{\partial \Phi}{\partial V}\right)_{T,n}$$

$$P = k_B T \int f(\varepsilon;T,\mu) \ln\left(1 + e^{-\frac{(\varepsilon-\mu)}{k_B T}}\right) d\varepsilon$$

Thermodynamic Formulas



Gibbs-Hertz Entropy

Loris Ferrari (2015)
arXiv:1501.04566v3

$$S_B(E) = \ln\left[\sum_u \delta_{H(u),E}\right]$$

$$S_G(E) = \ln\left[\sum_u \Theta\left(E - H\{u\}\right)\right]$$

A Phantom Menace

Low-temperature holes may behave like

radiation

$$\left(\rho_{\max} - \rho\right) \propto a^{-4}$$

matter

$$\left(\rho_{\max} - \rho\right) \propto a^{-3}$$

$$H^2 = \frac{\rho_{\max}}{3} - \frac{\rho_{hi}}{3} a^{-3(1+w_h)}$$

No Big Rip!