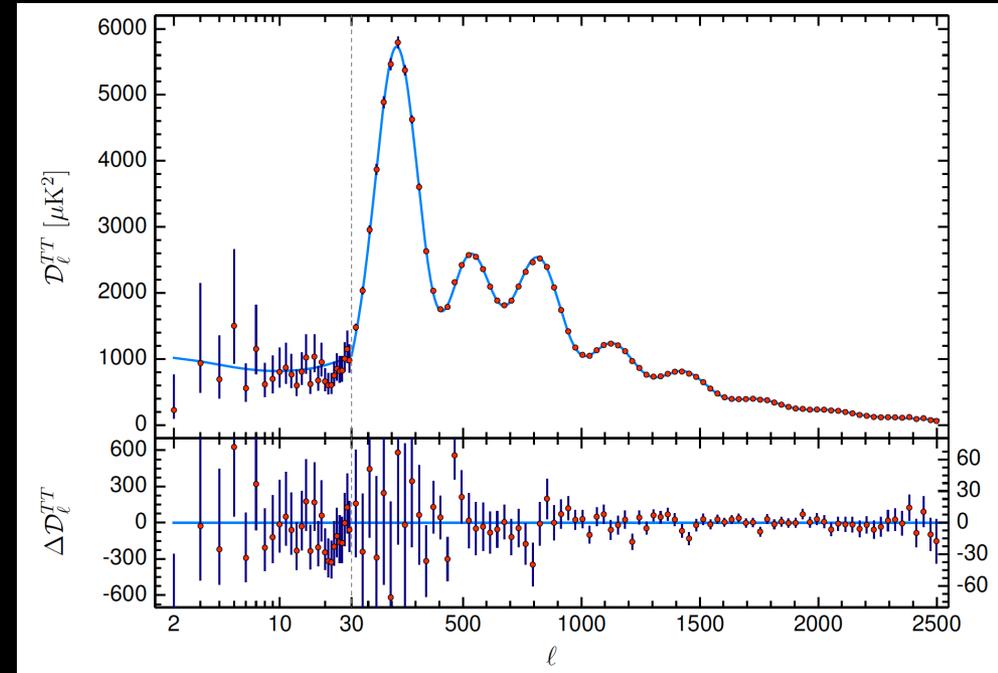
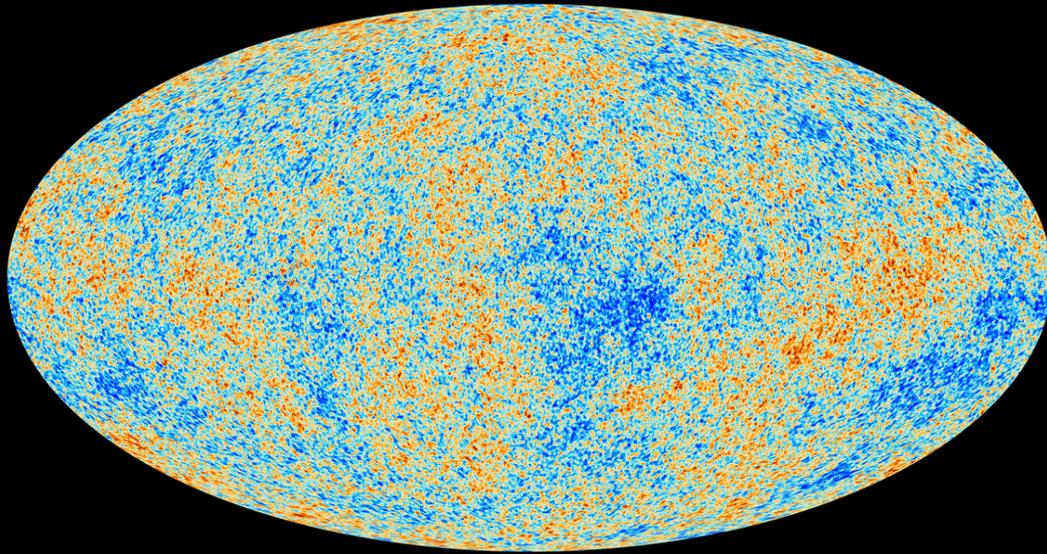


# Cosmology in Tension

December 13th, 2019

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University of Manchester

# Introduction to CMB



Planck collaboration, 2018

An important tool of research in cosmology is the angular power spectrum of CMB temperature anisotropies.

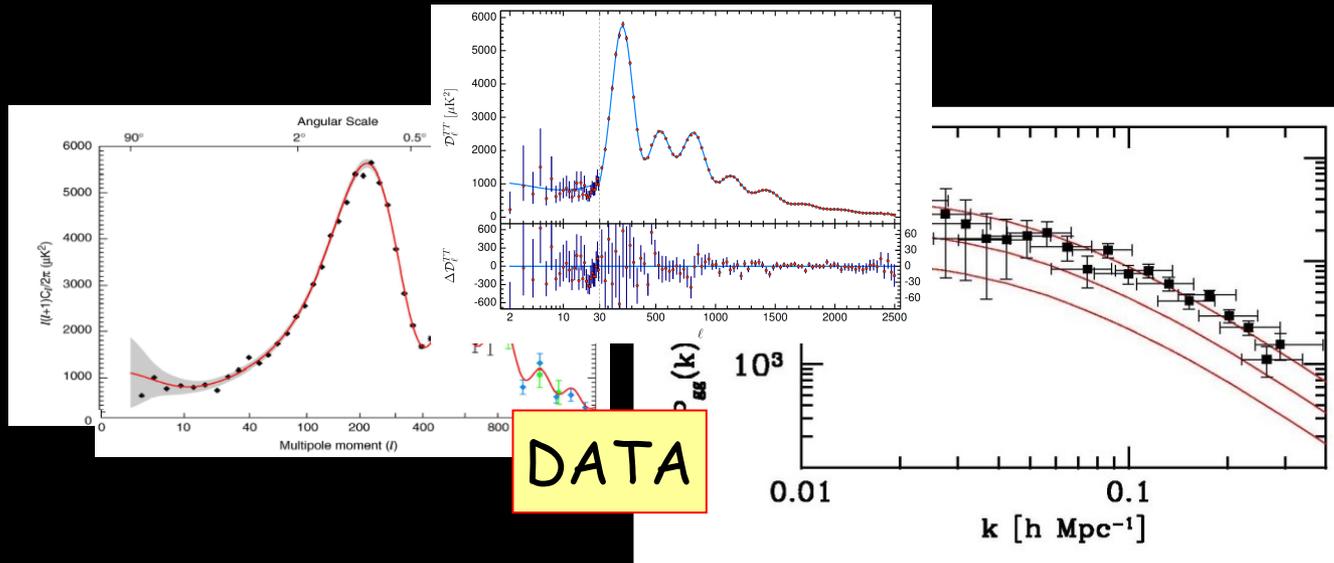
$$\left\langle \frac{\Delta T}{T}(\vec{\gamma}_1) \frac{\Delta T}{T}(\vec{\gamma}_2) \right\rangle = \frac{1}{2\pi} \sum_{\ell} (2\ell + 1) C_{\ell} P_{\ell}(\vec{\gamma}_1 \cdot \vec{\gamma}_2)$$

# Introduction to CMB

Cosmological parameters:  
( $\Omega_b h^2$ ,  $\Omega_m h^2$ ,  $h$ ,  $n_s$ ,  $\tau$ ,  $\Sigma m_\nu$ )



Theoretical model



PARAMETER  
CONSTRAINTS



# Introduction to CMB

From one side we have **very accurate theoretical predictions** on their angular power spectra while on the other side we have **extremely precise measurements**, culminated with the recent 2018 legacy release from the Planck satellite experiment.

# Planck satellite experiment

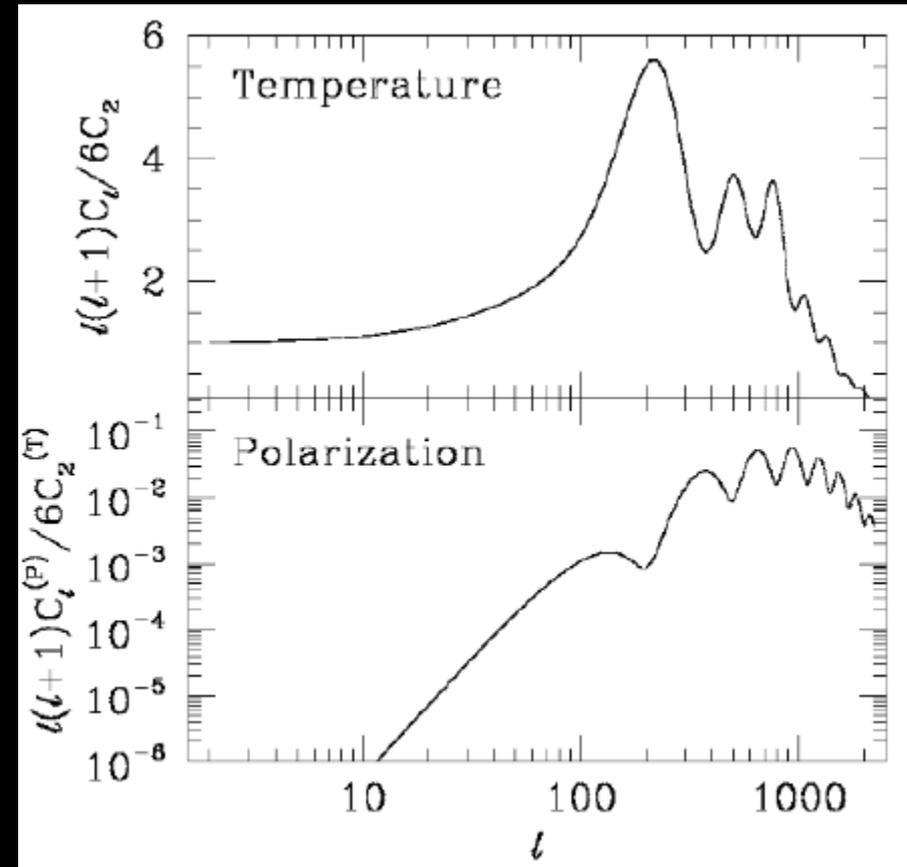


- Frequency range of 30GHz to 857GHz;
- Orbit around L2;
- Composed by 2 instruments:
  - LFI → 1.5 meters telescope; array of 22 differential receivers that measure the signal from the sky comparing with a black body at 4.5K.
  - HFI → array of 52 bolometers cooled to 0.1K.

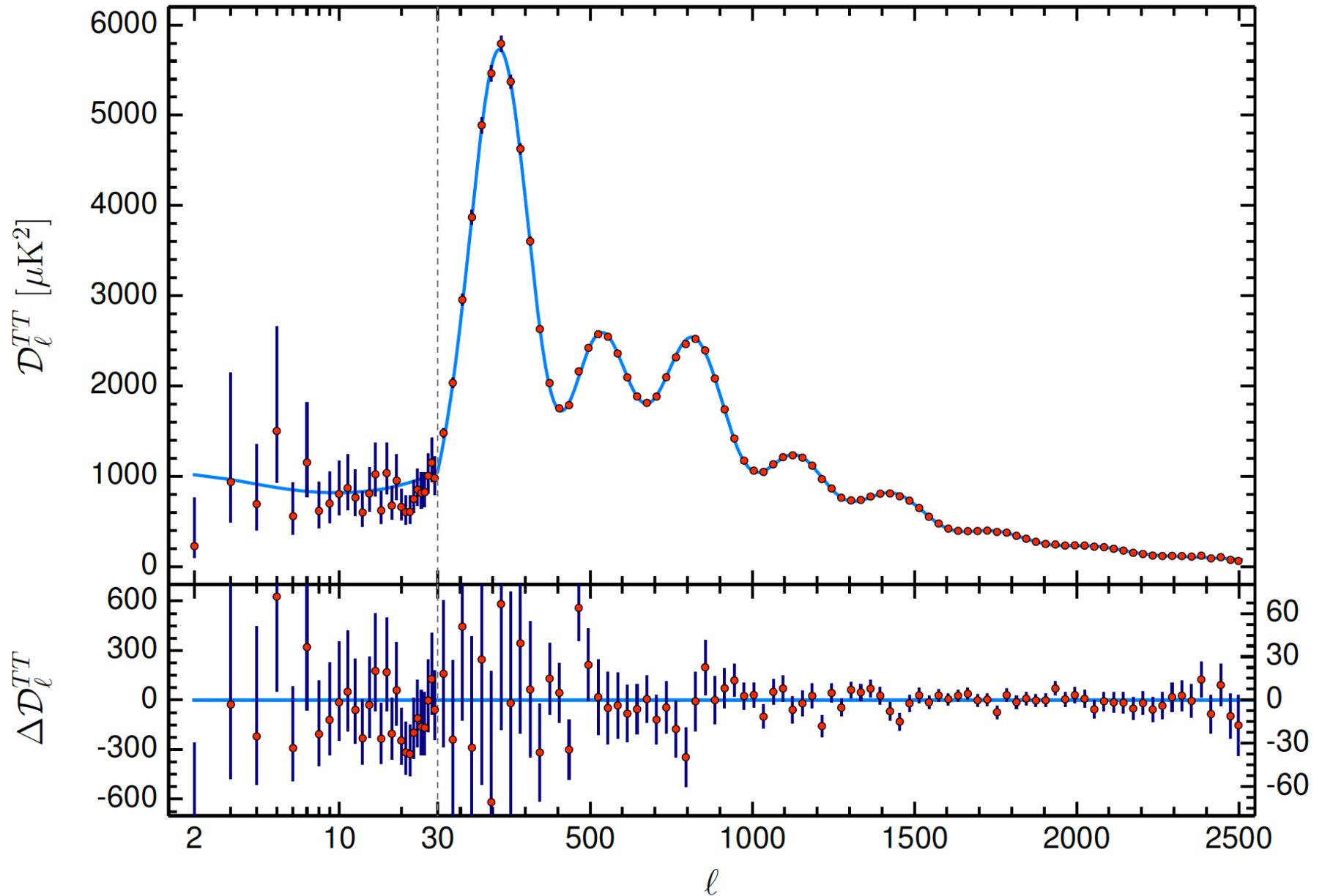
# Introduction to CMB

We can extract 4 independent angular spectra from the CMB:

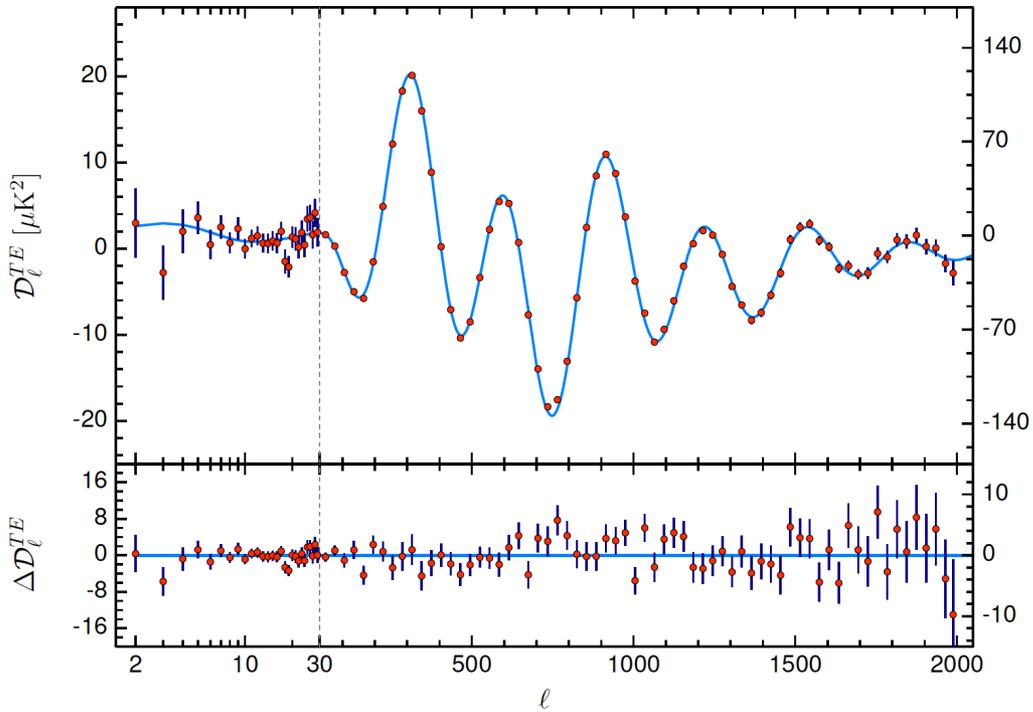
- Temperature
- Cross Temperature Polarization E
- Polarization type E (density fluctuations)
- Polarization type B (gravitational waves)



# Planck satellite experiment



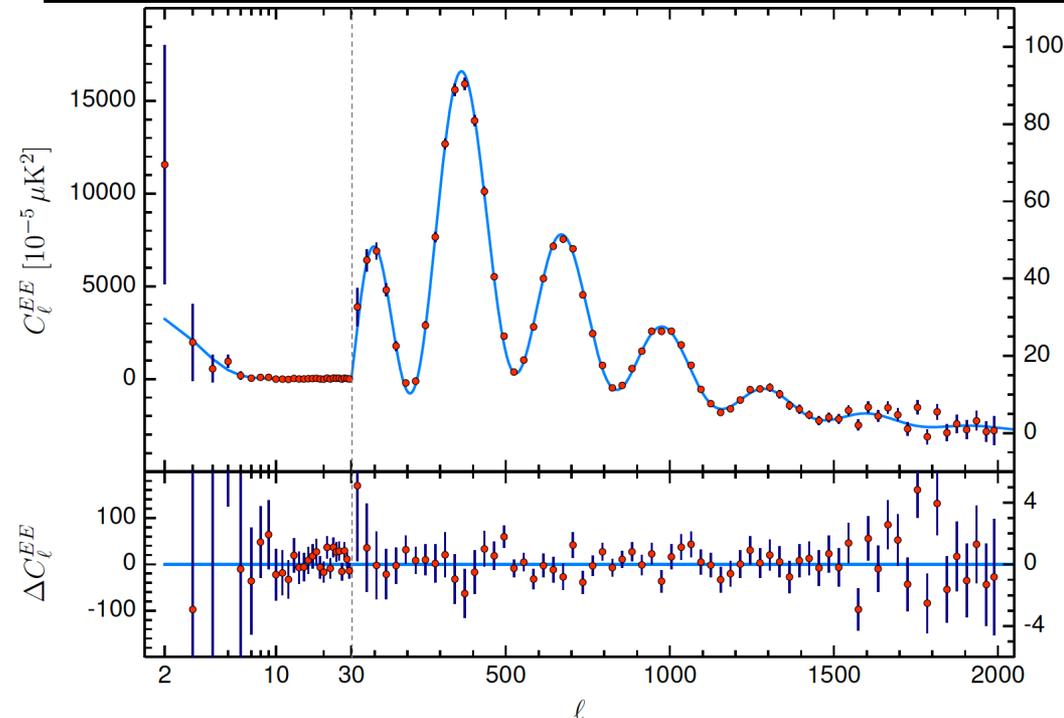
# Planck satellite experiment



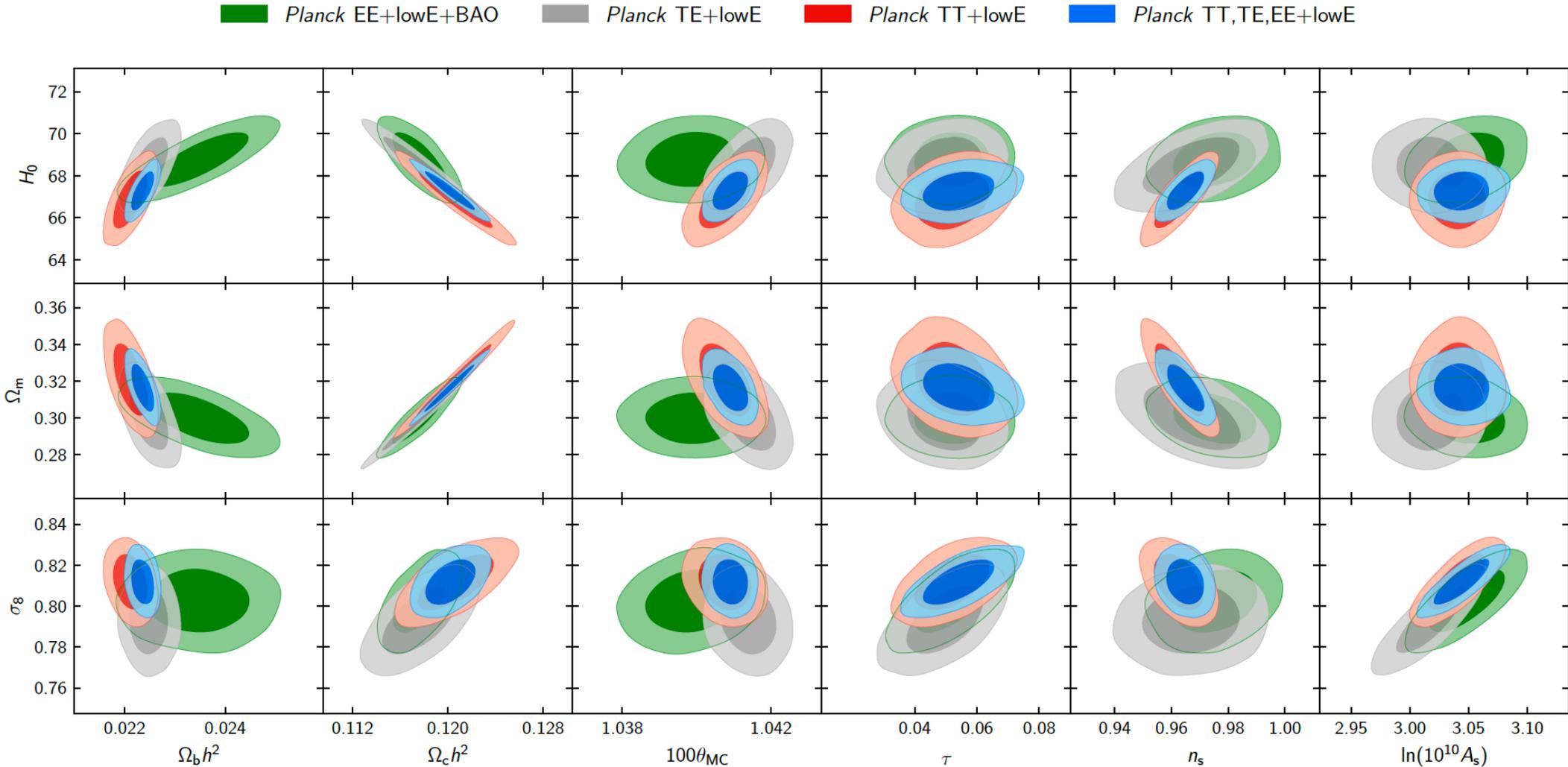
The theoretical spectra in light blues are computed from the best-fit base- $\Lambda$ CDM theoretical spectrum fit to the Planck TT,TE,EE+lowE+lensing likelihood.

Residuals with respect to this theoretical model are shown in the lower panel in each plot.

## Polarization spectra



# CMB constraints



Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

Constraints on parameters of the base- $\Lambda$ CDM model from the separate Planck EE, TE, and TT high- $l$  spectra combined with low- $l$  polarization (lowE), and, in the case of EE also with BAO, compared to the joint result using Planck TT,TE,EE+lowE.

# CMB constraints

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	$0.02212 \pm 0.00022$	$0.02249 \pm 0.00025$	$0.0240 \pm 0.0012$	$0.02236 \pm 0.00015$	$0.02237 \pm 0.00015$	$0.02242 \pm 0.00014$
$\Omega_c h^2$	$0.1206 \pm 0.0021$	$0.1177 \pm 0.0020$	$0.1158 \pm 0.0046$	$0.1202 \pm 0.0014$	$0.1200 \pm 0.0012$	$0.11933 \pm 0.00091$
$100\theta_{MC}$	$1.04077 \pm 0.00047$	$1.04139 \pm 0.00049$	$1.03999 \pm 0.00089$	$1.04090 \pm 0.00031$	$1.04092 \pm 0.00031$	$1.04101 \pm 0.00029$
$\tau$	$0.0522 \pm 0.0080$	$0.0496 \pm 0.0085$	$0.0527 \pm 0.0090$	$0.0544^{+0.0070}_{-0.0081}$	$0.0544 \pm 0.0073$	$0.0561 \pm 0.0071$
$\ln(10^{10} A_s)$	$3.040 \pm 0.016$	$3.018^{+0.020}_{-0.018}$	$3.052 \pm 0.022$	$3.045 \pm 0.016$	$3.044 \pm 0.014$	$3.047 \pm 0.014$
$n_s$	$0.9626 \pm 0.0057$	$0.967 \pm 0.011$	$0.980 \pm 0.015$	$0.9649 \pm 0.0044$	$0.9649 \pm 0.0042$	$0.9665 \pm 0.0038$
$H_0$ [km s <sup>-1</sup> Mpc <sup>-1</sup> ]	$66.88 \pm 0.92$	$68.44 \pm 0.91$	$69.9 \pm 2.7$	$67.27 \pm 0.60$	$67.36 \pm 0.54$	$67.66 \pm 0.42$
$\Omega_\Lambda$	$0.679 \pm 0.013$	$0.699 \pm 0.012$	$0.711^{+0.033}_{-0.026}$	$0.6834 \pm 0.0084$	$0.6847 \pm 0.0073$	$0.6889 \pm 0.0056$
$\Omega_m$	$0.321 \pm 0.013$	$0.301 \pm 0.012$	$0.289^{+0.026}_{-0.033}$	$0.3166 \pm 0.0084$	$0.3153 \pm 0.0073$	$0.3111 \pm 0.0056$
$\Omega_m h^2$	$0.1434 \pm 0.0020$	$0.1408 \pm 0.0019$	$0.1404^{+0.0034}_{-0.0039}$	$0.1432 \pm 0.0013$	$0.1430 \pm 0.0011$	$0.14240 \pm 0.00087$
$\Omega_m h^3$	$0.09589 \pm 0.00046$	$0.09635 \pm 0.00051$	$0.0981^{+0.0016}_{-0.0018}$	$0.09633 \pm 0.00029$	$0.09633 \pm 0.00030$	$0.09635 \pm 0.00030$
$\sigma_8$	$0.8118 \pm 0.0089$	$0.793 \pm 0.011$	$0.796 \pm 0.018$	$0.8120 \pm 0.0073$	$0.8111 \pm 0.0060$	$0.8102 \pm 0.0060$
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	$0.840 \pm 0.024$	$0.794 \pm 0.024$	$0.781^{+0.052}_{-0.060}$	$0.834 \pm 0.016$	$0.832 \pm 0.013$	$0.825 \pm 0.011$

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

The precision measurements of the CMB polarization spectra have the potential to constrain cosmological parameters to higher accuracy than measurements of the temperature spectra because the acoustic peaks are narrower in polarization and unresolved foreground contributions at high multipoles are much lower in polarization than in temperature.

2018 Planck results are perfectly in agreement with the standard  $\Lambda$ CDM cosmological model.

However, **anomalies and tensions between Planck and other cosmological probes are present well above the 3 standard deviations.** These discrepancies, already hinted in previous Planck data releases, have **persisted and strengthened** despite several years of accurate analyses.

Last year, the Royal Astronomical Society awarded Planck their Group Achievement Award with the citation "**(Planck) has now ushered in an era of tension cosmology.**", clearly indicating that these tensions have reached such a level of statistical significance that the understanding of their physical nature is of utmost importance for modern cosmology.

If not due to systematics, the current anomalies could represent a **crisis for the standard cosmological model** and their experimental confirmation can bring a **revolution** in our current ideas of the structure and evolution of the Universe.

The most famous and persisting **anomalies and tensions** of the CMB are:

- $H_0$  with local measurements
- $S_8$  with cosmic shear data
- $A_L$  internal anomaly
- $\Omega_k$  different from zero

The most famous and persisting anomalies and tensions of the CMB are:

- $H_0$  with local measurements
- $S_8$  with cosmic shear data
- $A_L$  internal anomaly
- $\Omega_k$  different from zero

# The H0 tension at more than $3\sigma$

**CMB:** in this case the cosmological constraints are obtained by assuming a cosmological model and are therefore **model dependent**. Moreover these bounds are also affected by the degeneracy between the parameters that induce similar effects on the observables. Therefore the Planck constraints can change when modifying the assumptions of the underlying cosmological model.

$$H_0 = 67.27 \pm 0.60 \text{ km/s/Mpc in } \Lambda\text{CDM}$$

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

**Direct local distance ladder measurements:** the 2016 estimate of the Hubble constant is based on Supernovae type-Ia measurements, obtained combining four different geometric distance calibrations of Cepheids,

$$H_0 = 73.24 \pm 1.74 \text{ km/s/Mpc}$$

Riess et al. *Astrophys.J.* 826, no. 1, 56 (2016)

The 2018 estimate include parallax measurements of 7 long-period ( $> 10$  days) Milky Way Cepheids using astrometry from spatial scanning of WFC3 on HST.

$$H_0 = 73.48 \pm 1.66 \text{ km/s/Mpc}$$

Riess et al. *Astrophys.J.* 855, 136 (2018)

# The H<sub>0</sub> tension at more than 4σ

**Table 5.** Best Estimates of H<sub>0</sub> Including Systematics

Anchor(s)	Value [km s <sup>-1</sup> Mpc <sup>-1</sup> ]	Δ Planck* + ΛCDM (σ)
LMC	74.22 ± 1.82	3.6
Two anchors		
LMC + NGC 4258	73.40 ± 1.55	3.7
LMC + MW	74.47 ± 1.45	4.6
NGC 4258 + MW	73.94 ± 1.58	3.9
<b>Three anchors (preferred)</b>		
<b>NGC 4258 + MW + LMC</b>	<b>74.03 ± 1.42</b>	4.4

NOTE—\* :  $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$   
(Planck Collaboration et al. 2018)

Riess et al. arXiv:1903.07603 [astro-ph.CO]

Recently, the H<sub>0</sub> measurement has been improved using Hubble Space Telescope observations of 70 long-period Cepheids in the Large Magellanic Cloud.

The tension becomes of 4.4σ between the local measurement of H<sub>0</sub> and<sub>15</sub> the value predicted from Planck in ΛCDM.

# The $H_0$ tension at more than $5\sigma$

**CMB:**  $H_0 = 67.27 \pm 0.60$  km/s/Mpc in  $\Lambda$ CDM

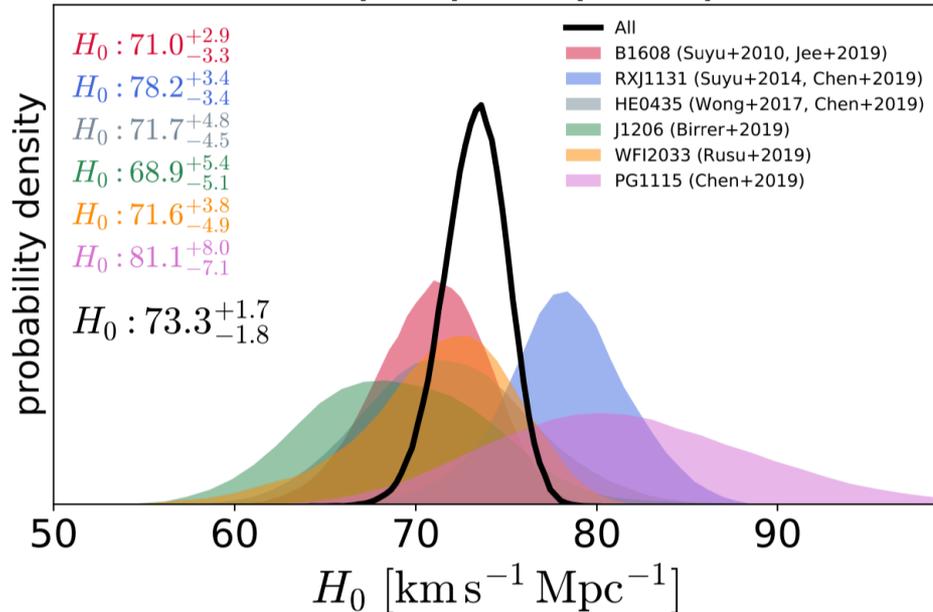
**BAO+Pantheon+BBN+ $\theta_{MC}$ , Planck:**  $H_0 = 67.9 \pm 0.8$  km/s/Mpc

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

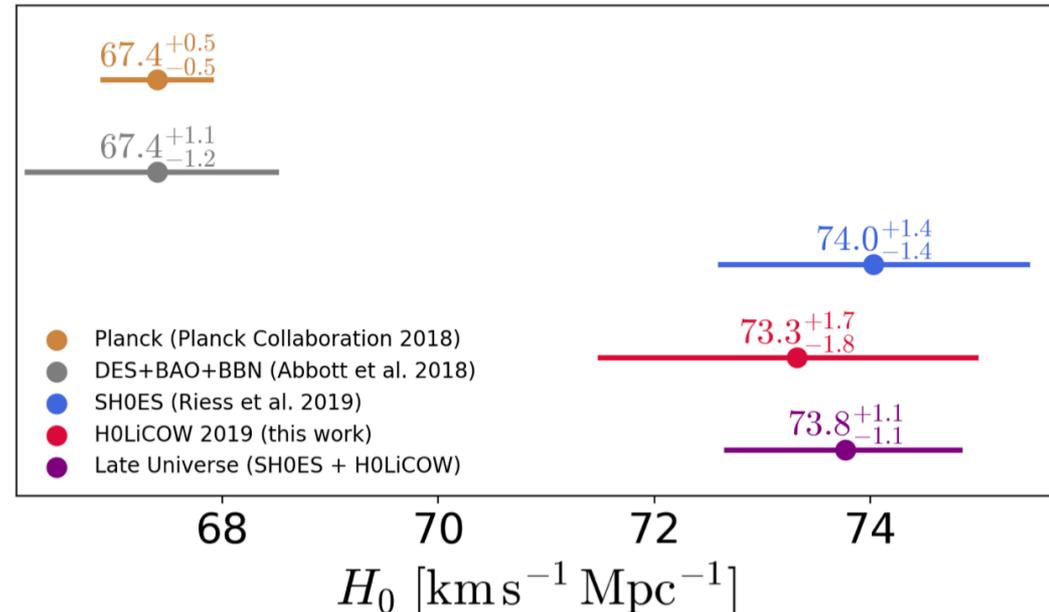
**SH0ES:**  $H_0 = 74.03 \pm 1.42$  km/s/Mpc

Riess et al. arXiv:1903.07603 [astro-ph.CO]

$H_0 \in [0, 150]$   $\Omega_m \in [0.05, 0.5]$



flat  $\Lambda$ CDM



**Strong Lensing:** Multiply-imaged quasar systems through strong gravitational lensing made by the H0LiCOW collaboration  $H_0 = 73.3^{+1.7}_{-1.8}$  km/s/Mpc

Wong et al. arXiv:1907.04869v1

Since the Planck constraints are **model dependent**, we can try to expand the cosmological scenario and see which extensions work in solving the tensions between the cosmological probes.

For example, the most **famous extensions** for solving the H0 tension are:



the neutrino effective number



the dark energy equation of state

# The Neutrino effective number

When the rate of the weak interaction reactions, which keep neutrinos in equilibrium with the primordial plasma, becomes smaller than the expansion rate of the Universe, neutrinos decouple at a temperature of about:

$$T_{dec} \approx 1MeV$$

After neutrinos decoupling, photons are heated by electrons-positrons annihilation. After the end of this process, the ratio between the temperatures of photons and neutrinos will be fixed, despite the temperature decreases with the expansion of the Universe. We expect today a Cosmic Neutrino Background (CNB) at a temperature:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.945K \rightarrow kT_\nu \approx 1.68 \cdot 10^{-4} eV$$

With a number density of:

$$n_f = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_f T_f^3 \rightarrow n_{\nu_k, \bar{\nu}_k} \approx 0.1827 \cdot T_\nu^3 \approx 112 cm^{-3}$$

# The Neutrino effective number

The relativistic neutrinos contribute to the present energy density of the Universe:

$$\rho_{rad} = \rho_{\gamma} + \rho_{\nu} = g_{\gamma} \left( \frac{\pi^2}{30} \right) T_{\gamma}^4 + g_{\nu} \left( \frac{\pi^2}{30} \right) \left( \frac{7}{8} \right) T_{\nu}^4$$

$$\rho_{rad} = \left( 1 + \left( \frac{7}{8} \right) \left( \frac{4}{11} \right)^{\frac{4}{3}} \left( \frac{g_{\nu}}{g_{\gamma}} \right) \right) \rho_{\gamma}$$

We can introduce the effective number of relativistic degrees of freedom:

$$\rho_{rad} = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_{\gamma}$$

The expected value is  $N_{\text{eff}} = 3.046$ , if we assume standard electroweak interactions and three active massless neutrinos. The 0.046 takes into account effects for the non-instantaneous neutrino decoupling and neutrino flavour oscillations (Mangano et al. hep-ph/0506164, de Salas and Pastor JCAP 2016).

# The Neutrino effective number

If we measure a  $N_{\text{eff}} > 3.046$ , we are in presence of extra radiation. This extra radiation, essentially, increases the expansion rate  $H$ :

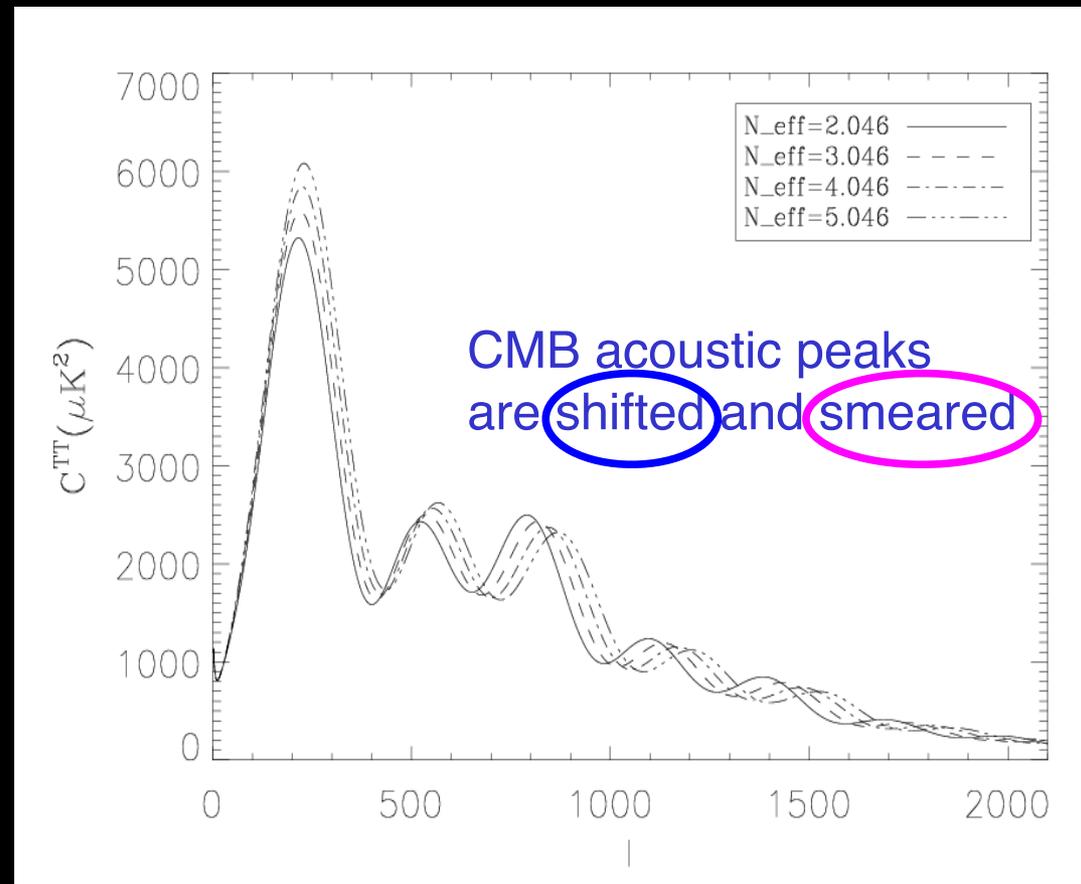
$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = H_0^2 \left( \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda \right)$$

and it decreases the sound horizon at recombination,

$$r_s = \int_0^{t_*} c_s dt/a = \int_0^{a_*} \frac{c_s da}{a^2 \bar{H}}$$

and the diffusion distance (damping scale):

$$r_d^2 = (2\pi)^2 \int_0^{a_*} \frac{da}{a^3 \sigma_T n_e \bar{H}} \left[ \frac{R^2 + \frac{16}{15}(1+R)}{6(1+R^2)} \right]$$



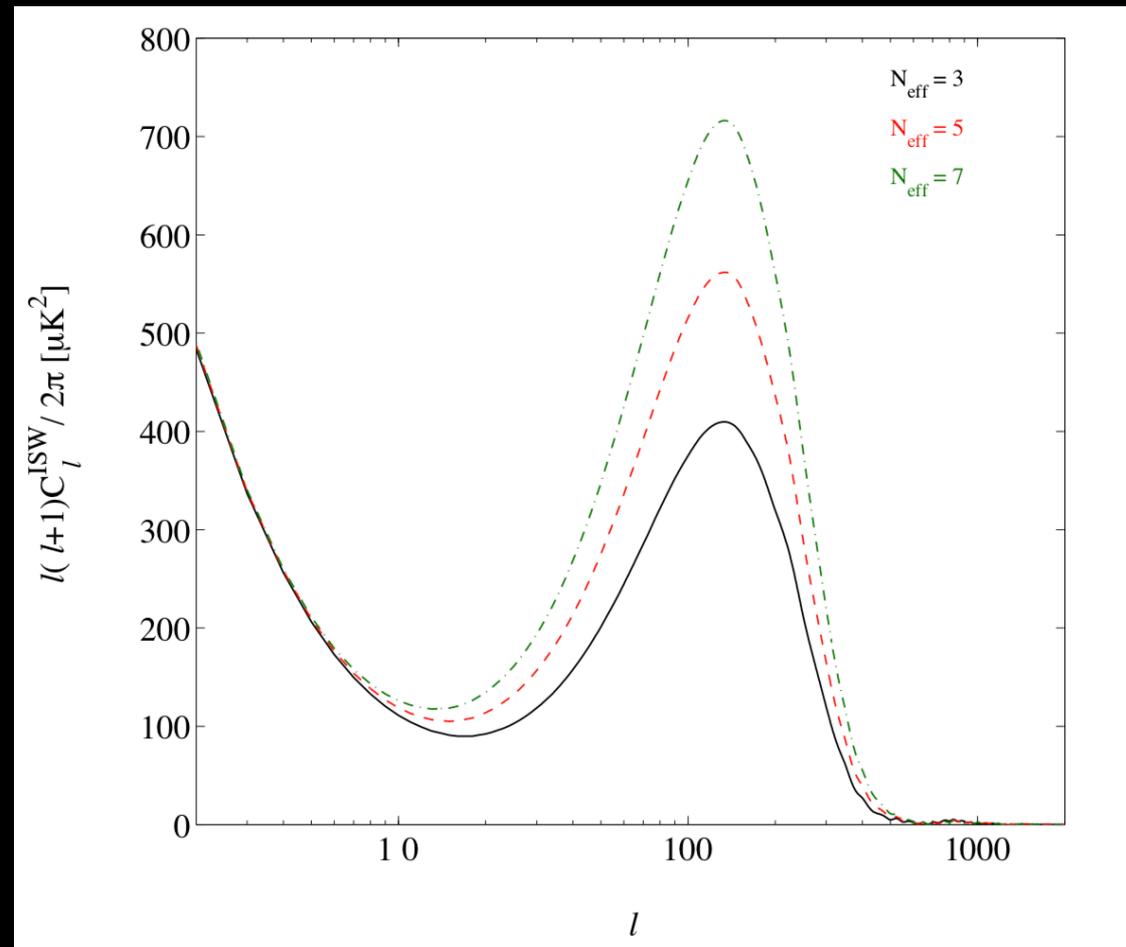
# The Neutrino effective number

Varying  $N_{\text{eff}}$  changes the time of the matter radiation equivalence: a higher radiation content due to the presence of additional relativistic species leads to a delay in  $z_{\text{eq}}$ :

$$1 + z_{\text{eq}} = \frac{\Omega_m}{\Omega_r} = \frac{\Omega_m h^2}{\Omega_\gamma h^2} \frac{1}{(1 + 0.2271 N_{\text{eff}})}$$

This implies that at the time of decoupling the radiation is still a subdominant component and the gravitational potential is still slowly decreasing.

This shows up as an **enhancement of the early Integrated Sachs Wolfe (ISW) effect** that increases the CMB perturbation peaks at  $l \sim 200$ .



# The Neutrino effective number

If we compare the Planck 2015 constraint on  $N_{\text{eff}}$  at 68% cl

$$N_{\text{eff}} = 3.13 \pm 0.32 \quad \text{Planck TT+lowP,}$$

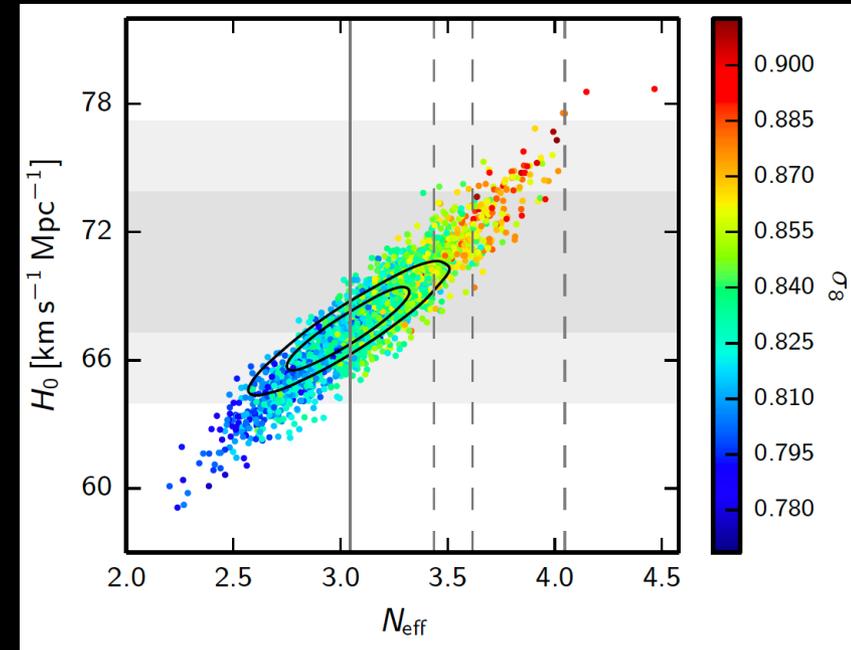
$$N_{\text{eff}} = 3.15 \pm 0.23 \quad \text{Planck TT+lowP+BAO,}$$

with the new Planck 2018 bound,

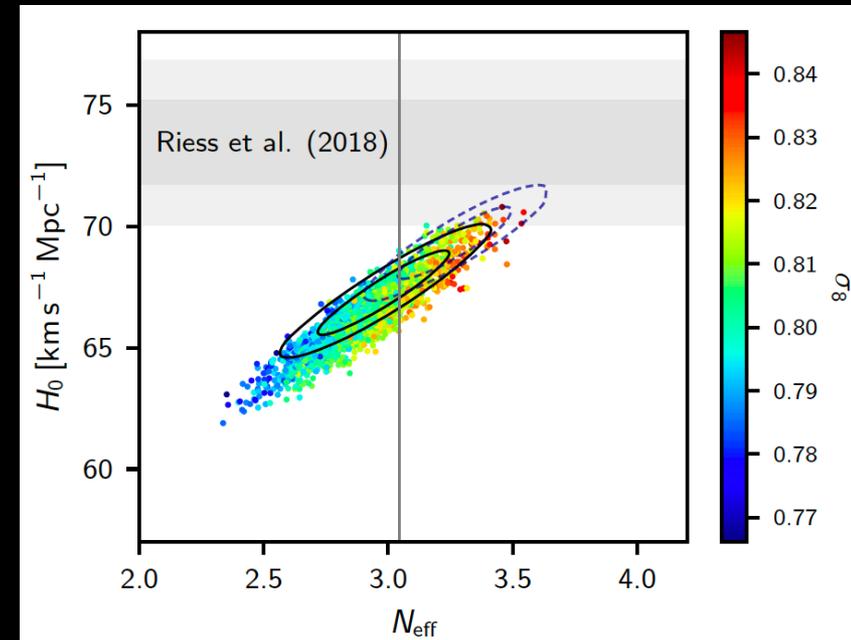
$$N_{\text{eff}} = 2.92^{+0.36}_{-0.37} \quad (95\%, \text{Planck TT,TE,EE+lowE}),$$

we see that the neutrino effective number is now very well constrained.

The main reason for this good accuracy is due to the lack of the early integrated Sachs Wolfe effect in polarization data. The inclusion of polarization helps in determining the amplitude of the eISW and  $N_{\text{eff}}$ .  $H_0$  passes from  $68.0 \pm 2.8$  km/s/Mpc (2015) to  $66.4 \pm 1.4$  km/s/Mpc (2018), and the tension with Riess+19 increases from  $2.1\sigma$  to  $3.8\sigma$  also varying  $N_{\text{eff}}$ .



Planck collaboration, 2015



Planck collaboration, 2018

# The Dark energy equation of state

Changing the dark energy equation of state  $w$ , we are changing the expansion rate of the Universe:

$$H^2 = H_0^2 \left[ \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_{\text{de}} (1+z)^{3(1+w)} + \Omega_k (1+z)^2 \right]$$

$w$  introduces a geometrical degeneracy with the Hubble constant that will be unconstrained using the CMB data only, resulting in agreement with Riess+19.

We have in 2018  $w = -1.58^{+0.52}_{-0.41}$  with  $H_0 > 69.9$  km/s/Mpc at 95% c.l.

Planck data prefer a **phantom dark energy**, with an energy component with  $w < -1$ , for which the density increases with time in an expanding universe that will **end in a Big Rip**. A phantom dark energy violates the energy condition  $\rho \geq |\rho|$ , that means that the matter could move faster than light and a comoving observer measure a negative energy density, and the Hamiltonian could have vacuum instabilities due to a negative kinetic energy.

Anyway, there exist models that expect an effective energy density with a phantom equation of state without showing the problems before, as for example the Parker Vacuum Metamorphosis [Di Valentino et al., Phys.Rev. D97 \(2018\) no.4, 043528](#).

## More specific extensions for solving the $H_0$ tension are:

- **Interacting dark sector** (Di Valentino et al. arXiv:1704.08342, Kumar and Nunes arXiv:1702.02143, Yang et al. arXiv:1805.08252, Yang et al. arXiv:1809.06883, Yang et al. arXiv:1906.11697, Martinelli et al. arXiv:1902.10694, Di Valentino et al. 2019, etc...)
- **Parker Vacuum Metamorphosis** (Di Valentino et al. 2018)
- **Vacuum Dynamics** (Sola Peracaula et al. arXiv:1705.06723)
- **Early dark Energy** (Poulin et al. arXiv:1811.04083)
- **Uber-gravity** (Khosravi et al. arXiv:1710.09366)
- **Bulk viscosity** (Yang et al. arXiv:1906.04162)
- **Decaying dark matter** (Pandey et al. arXiv:1902.10636, Vattis et al. arXiv:1903.06220, etc..)
- **Metastable Dark Energy** (Li et al. arXiv:1904.03790)
- **Many many others...** (Colgain et al. arXiv:1807.07451, Nunes arXiv:1802.02281, Agrawal et al. arXiv:1904.01016, Yang et al. arXiv:1907.05344, Martinelli and Tutusaus arXiv:1906.09189, Adhikari and Huterer arXiv:1905.02278, Gelmini et al. arXiv:1906.10136, Colgain et al. arXiv:1905.02555, Pan et al. 1907.12551, Knox and Millea arXiv:1908.03663, etc..)

# IDE can solve the $H_0$ tension

In the standard cosmological framework, the dark matter is assumed to be collisionless. In practice this means that one arbitrarily sets the dark matter interactions to zero when predicting the angular power spectrum of the CMB.

In particular, dark matter and dark energy are described as separate fluids not sharing interactions beyond gravitational ones. However, from a microphysical perspective it is hard to imagine how non-gravitational DM-DE interactions can be avoided, unless forbidden by a fundamental symmetry. This has motivated a large number of studies based on models where DM and DE share interactions other than gravitational.

# IDE can solve the H0 tension

If we consider the interacting dark energy scenario characterised by a modification to the usual conservation equations, with the introduction of an interaction:

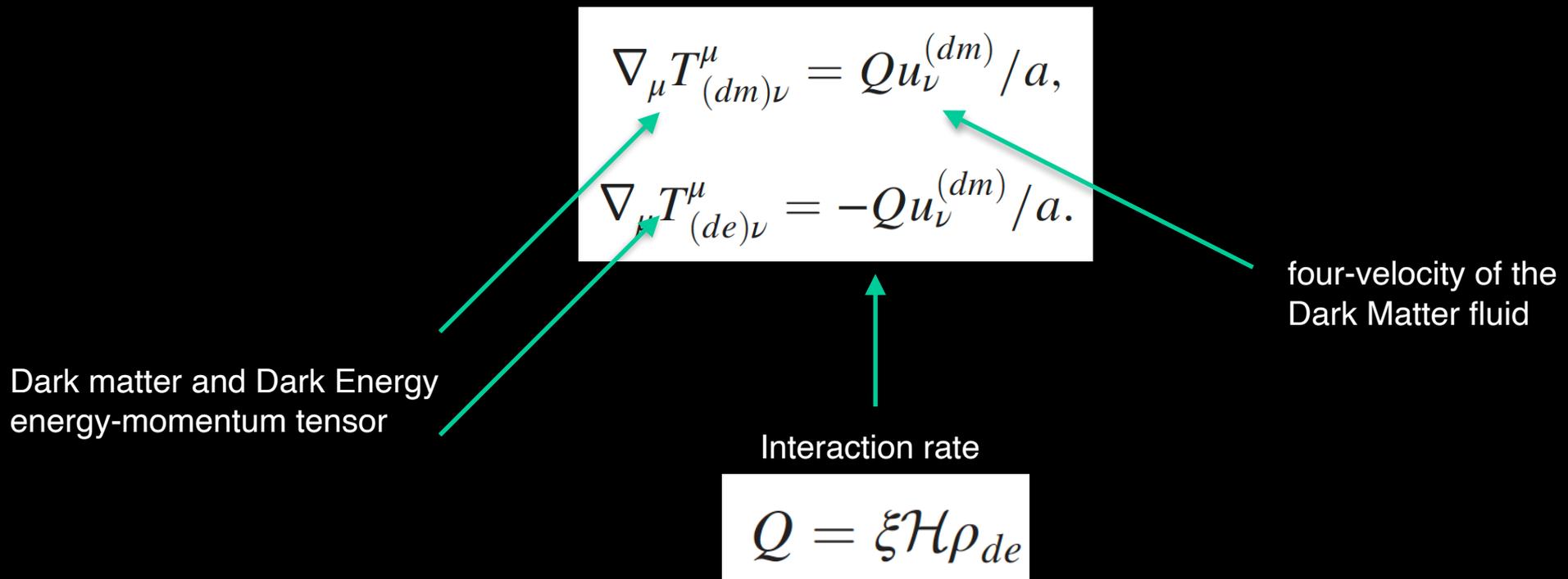
$$\begin{aligned}\nabla_{\mu} T^{\mu}_{(dm)\nu} &= Qu_{\nu}^{(dm)} / a, \\ \nabla_{\mu} T^{\mu}_{(de)\nu} &= -Qu_{\nu}^{(dm)} / a.\end{aligned}$$

Dark matter and Dark Energy energy-momentum tensor

Interaction rate

$$Q = \xi \mathcal{H} \rho_{de}$$

four-velocity of the Dark Matter fluid



With the interaction rate proportional to the dark energy density  $\rho_{de}$  via a negative dimensionless parameter  $\xi$  quantifying the strength of the coupling, to avoid early-time instabilities.

# Planck 2018

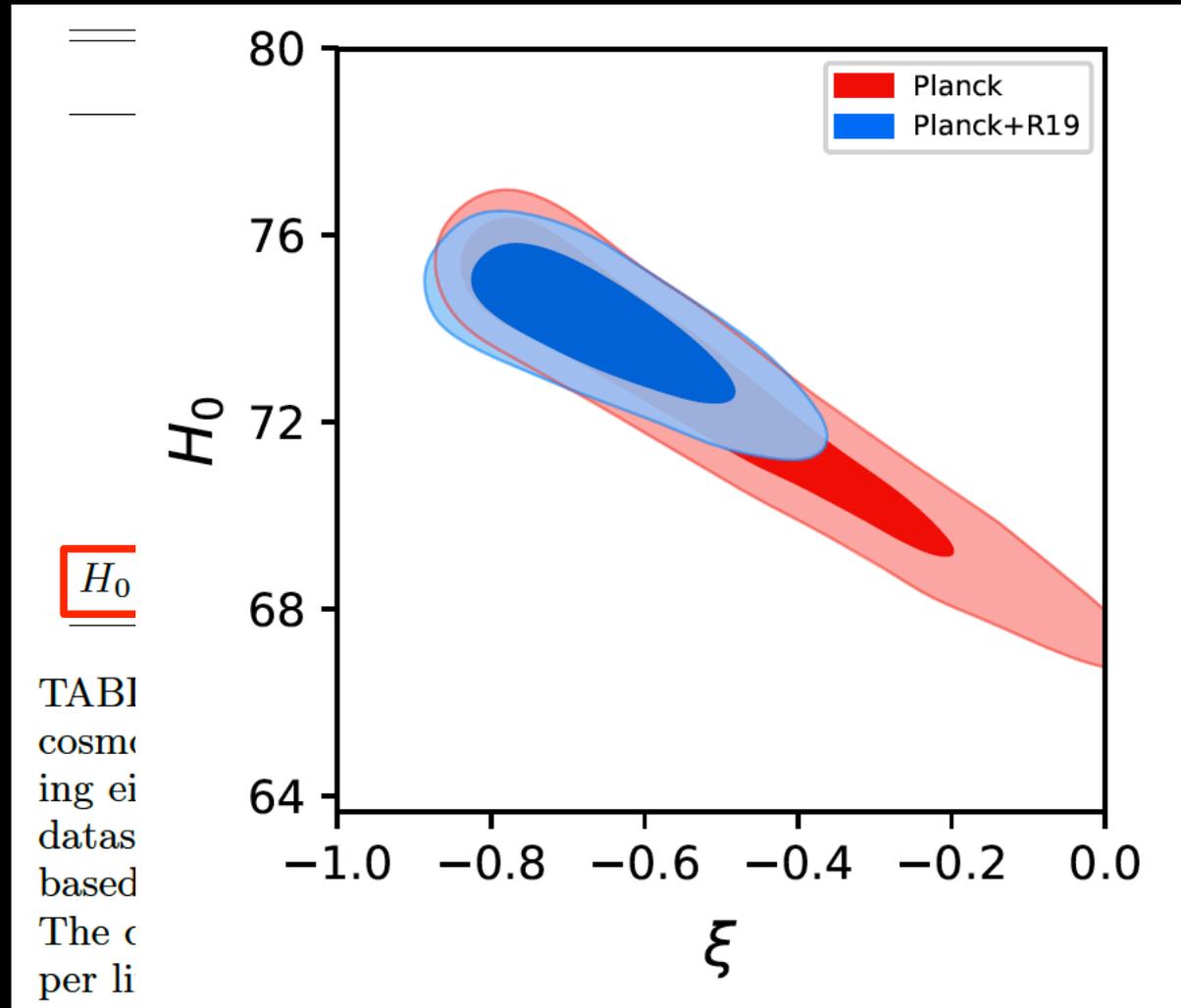
In this scenario of IDE the tension on  $H_0$  between the Planck satellite and R19 is completely solved. The coupling could affect the value of the present matter energy density  $\Omega_m$ . Therefore, if within an interacting model  $\Omega_m$  is smaller (because for negative  $\xi$  the dark matter density will decay into the dark energy one), a larger value of  $H_0$  would be required in order to satisfy the peaks structure of CMB observations, which accurately determine the value of  $\Omega_m h^2$ .

Parameter	<i>Planck</i>	<i>Planck</i> + <i>R19</i>
$\Omega_b h^2$	$0.02239 \pm 0.00015$	$0.02239 \pm 0.00015$
$\Omega_c h^2$	$< 0.105$	$< 0.0615$
$n_s$	$0.9655 \pm 0.0043$	$0.9656 \pm 0.0044$
$100\theta_s$	$1.0458^{+0.0033}_{-0.0021}$	$1.0470 \pm 0.0015$
$\tau$	$0.0541 \pm 0.0076$	$0.0534 \pm 0.0080$
$\xi$	$-0.54^{+0.12}_{-0.28}$	$-0.66^{+0.09}_{-0.13}$
$H_0$ [km s <sup>-1</sup> Mpc <sup>-1</sup> ]	$72.8^{+3.0}_{-1.5}$	$74.0^{+1.2}_{-1.0}$

TABLE I. Mean values with their 68% C.L. errors on selected cosmological parameters within the  $\xi\Lambda$ CDM model, considering either the *Planck* 2018 legacy dataset alone, or the same dataset in combination with the *R19* Gaussian prior on  $H_0$  based on the latest local distance measurement from *HST*. The quantity quoted in the case of  $\Omega_c h^2$  is the 95% C.L. upper limit.

# Planck 2018

Therefore we can safely combine the two datasets together, and we obtain a **non-zero dark matter-dark energy coupling  $\xi$  at more than FIVE standard deviations.**



# Bayes factor

Anyway it is clearly interesting to quantify the better **accordance of a model with the data respect to another by using the marginal likelihood also known as the Bayesian evidence.**

Given a vector of parameters  $\theta$  of a model  $M$  and a set of data  $x$ , the parameters posterior distribution is given by

$$p(\theta|x, M) = \frac{p(x|\theta, M) \pi(\theta|M)}{p(x|M)}$$

Prior

Likelihood

The marginal likelihood (or evidence) given by

$$E \equiv p(x|M) = \int d\theta p(x|\theta, M) \pi(\theta|M)$$

Given two competing models  $M_0$  and  $M_1$  it is useful to consider the ratio of the likelihood probability **(the Bayes factor):**

$$\ln \mathcal{B} = p(x|M_0) / p(x|M_1)$$

According to the revised Jeffrey's scale by **Kass and Raftery 1995**, the evidence for  $M_0$  (against  $M_1$ ) is considered as **"positive"** if  $|\ln \mathcal{B}| > 1.0$ , **"strong"** if  $|\ln \mathcal{B}| > 3.0$ , and **"very strong"** if  $|\ln \mathcal{B}| > 5.0$ .

# Planck 2018

Computing the Bayes factor for the IDE model with respect to LCDM for the **Planck** dataset we find  $\ln B = 1.2$ , i.e. a **positive preference** for the IDE model. If we consider **Planck + R19** we find the extremely high value  $\ln B = 10.0$ , indicating a **very strong preference for the IDE model**.

Parameter	<i>Planck</i>	<i>Planck</i> + <i>R19</i>
$\Omega_b h^2$	$0.02239 \pm 0.00015$	$0.02239 \pm 0.00015$
$\Omega_c h^2$	$< 0.105$	$< 0.0615$
$n_s$	$0.9655 \pm 0.0043$	$0.9656 \pm 0.0044$
$100\theta_s$	$1.0458^{+0.0033}_{-0.0021}$	$1.0470 \pm 0.0015$
$\tau$	$0.0541 \pm 0.0076$	$0.0534 \pm 0.0080$
$\xi$	$-0.54^{+0.12}_{-0.28}$	$-0.66^{+0.09}_{-0.13}$
$H_0$ [km s <sup>-1</sup> Mpc <sup>-1</sup> ]	$72.8^{+3.0}_{-1.5}$	$74.0^{+1.2}_{-1.0}$

TABLE I. Mean values with their 68% C.L. errors on selected cosmological parameters within the  $\xi\Lambda$ CDM model, considering either the *Planck* 2018 legacy dataset alone, or the same dataset in combination with the *R19* Gaussian prior on  $H_0$  based on the latest local distance measurement from *HST*. The quantity quoted in the case of  $\Omega_c h^2$  is the 95% C.L. upper limit.

# Planck 2018

Di Valentino et al. arXiv:1910.09853

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	$0.02239 \pm 0.00015$	$0.02239 \pm 0.00015$	$0.02241 \pm 0.00014$	$0.02236 \pm 0.00014$	$0.02235 \pm 0.00015$
$\Omega_c h^2$	$< 0.0634$	$0.031^{+0.013}_{-0.023}$	$< 0.0675$	$0.095^{+0.022}_{-0.008}$	$0.103^{+0.013}_{-0.007}$
$100\theta_{MC}$	$1.0458^{+0.0033}_{-0.0021}$	$1.0470 \pm 0.0015$	$1.0456^{+0.0031}_{-0.0024}$	$1.0424^{+0.0006}_{-0.0013}$	$1.04185^{+0.00049}_{-0.00078}$
$\tau$	$0.0541 \pm 0.0076$	$0.0534 \pm 0.0080$	$0.0526 \pm 0.0074$	$0.0540 \pm 0.0076$	$0.0540 \pm 0.0076$
$n_s$	$0.9655 \pm 0.0043$	$0.9656 \pm 0.0044$	$0.9663 \pm 0.0040$	$0.9647 \pm 0.0040$	$0.9643 \pm 0.0042$
$\ln(10^{10} A_s)$	$3.044 \pm 0.016$	$3.042 \pm 0.017$	$3.039^{+0.013}_{-0.015}$	$3.044 \pm 0.016$	$3.044 \pm 0.016$
$\xi$	$-0.54^{+0.12}_{-0.28}$	$-0.66^{+0.09}_{-0.13}$	$-0.51^{+0.12}_{-0.29}$	$-0.22^{+0.21}_{-0.05}$	$-0.15^{+0.12}_{-0.06}$
$H_0$ [km/s/Mpc]	$72.8^{+3.0}_{+1.5}$	$74.0^{+1.2}_{-1.0}$	$72.8^{+3.0}_{+1.6}$	$69.4^{+0.9}_{-1.5}$	$68.6^{+0.8}_{-1.0}$
$\sigma_8$	$2.3^{+0.4}_{-1.4}$	$2.71^{+0.05}_{-1.3}$	$2.2^{+0.4}_{-1.4}$	$1.05^{+0.03}_{-0.24}$	$0.95^{+0.04}_{-0.12}$
$S_8$	$1.30^{+0.17}_{-0.44}$	$1.44^{+0.17}_{-0.34}$	$1.30^{+0.15}_{-0.42}$	$0.93^{+0.03}_{-0.10}$	$0.892^{+0.028}_{-0.054}$

The addition of low-redshift measurements, as BAO data, still hints to the **presence of a coupling**, albeit at a lower statistical significance.

Also for this data sets the **Hubble constant values is larger** than that obtained in the case of a pure LCDM scenario, enough to bring the **H0 tension well below the 3 $\sigma$  from 4.4 $\sigma$** .

In other words, **the tension between Planck+BAO and R19 could be due to a statistical fluctuation** in this case.

Moreover, **BAO data is extracted under the assumption of LCDM**, and the modified scenario of interacting dark energy could affect the result. In fact, the **full procedure which leads to the BAO constraints** carried out by the different collaborations **might be not necessarily valid in extended DE models**.

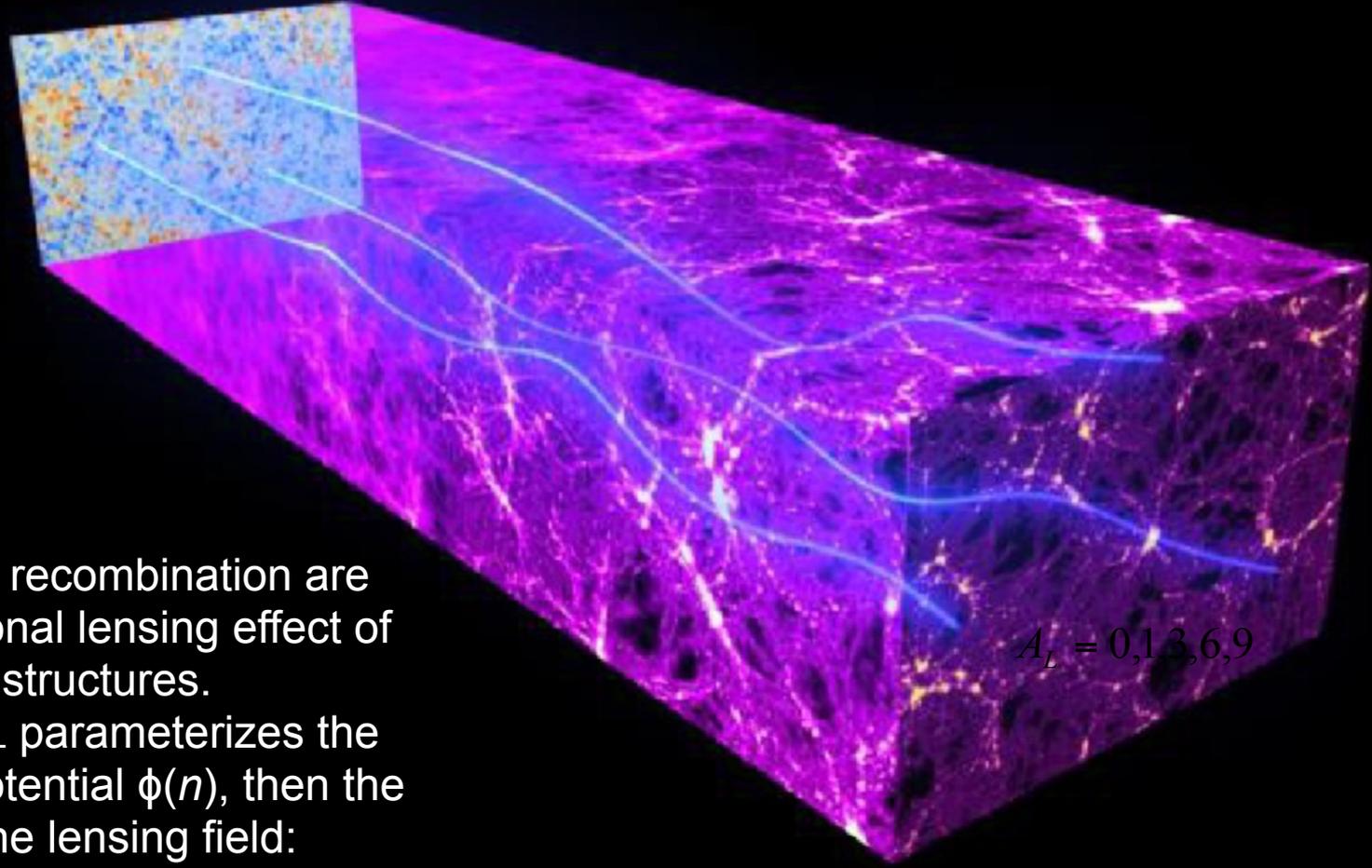
For instance, the BOSS collaboration advises caution when using their BAO measurements (both the pre- and post reconstruction measurements) in more exotic dark energy cosmologies.

**BAO constraints themselves might need to be revised in a non-trivial manner when applied to constrain extended dark energy cosmologies.**

The most famous and persisting anomalies and tensions of the CMB are:

- $H_0$  with local measurements
- $S_8$  with cosmic shear data
- $A_L$  internal anomaly
- $\Omega_k$  different from zero

# $A_L$ internal anomaly



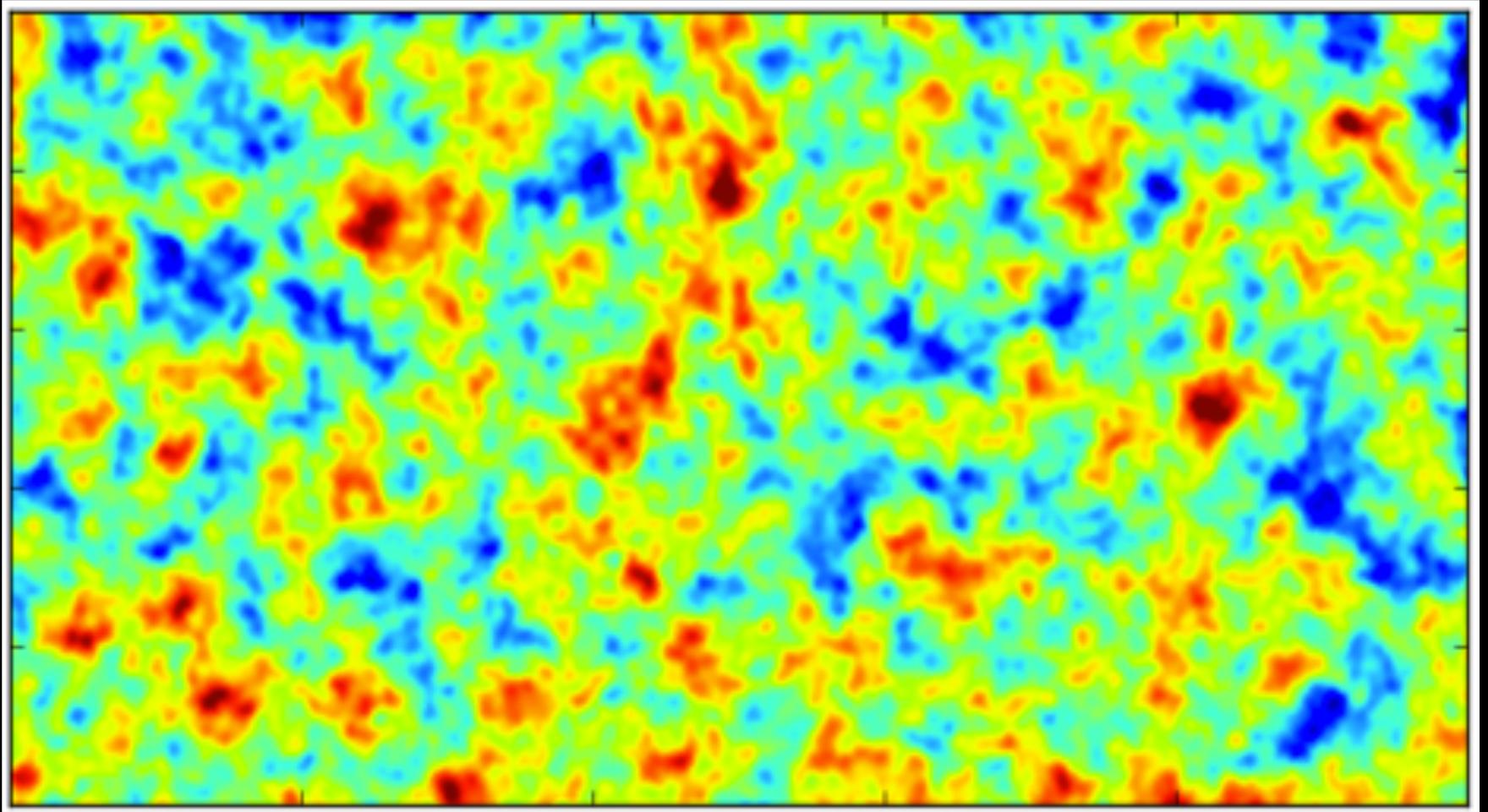
CMB photons emitted at recombination are deflected by the gravitational lensing effect of massive cosmic structures.

The lensing amplitude  $A_L$  parameterizes the rescaling of the lensing potential  $\phi(n)$ , then the power spectrum of the lensing field:

$$C_{\ell}^{\phi\phi} \rightarrow A_L C_{\ell}^{\phi\phi}$$

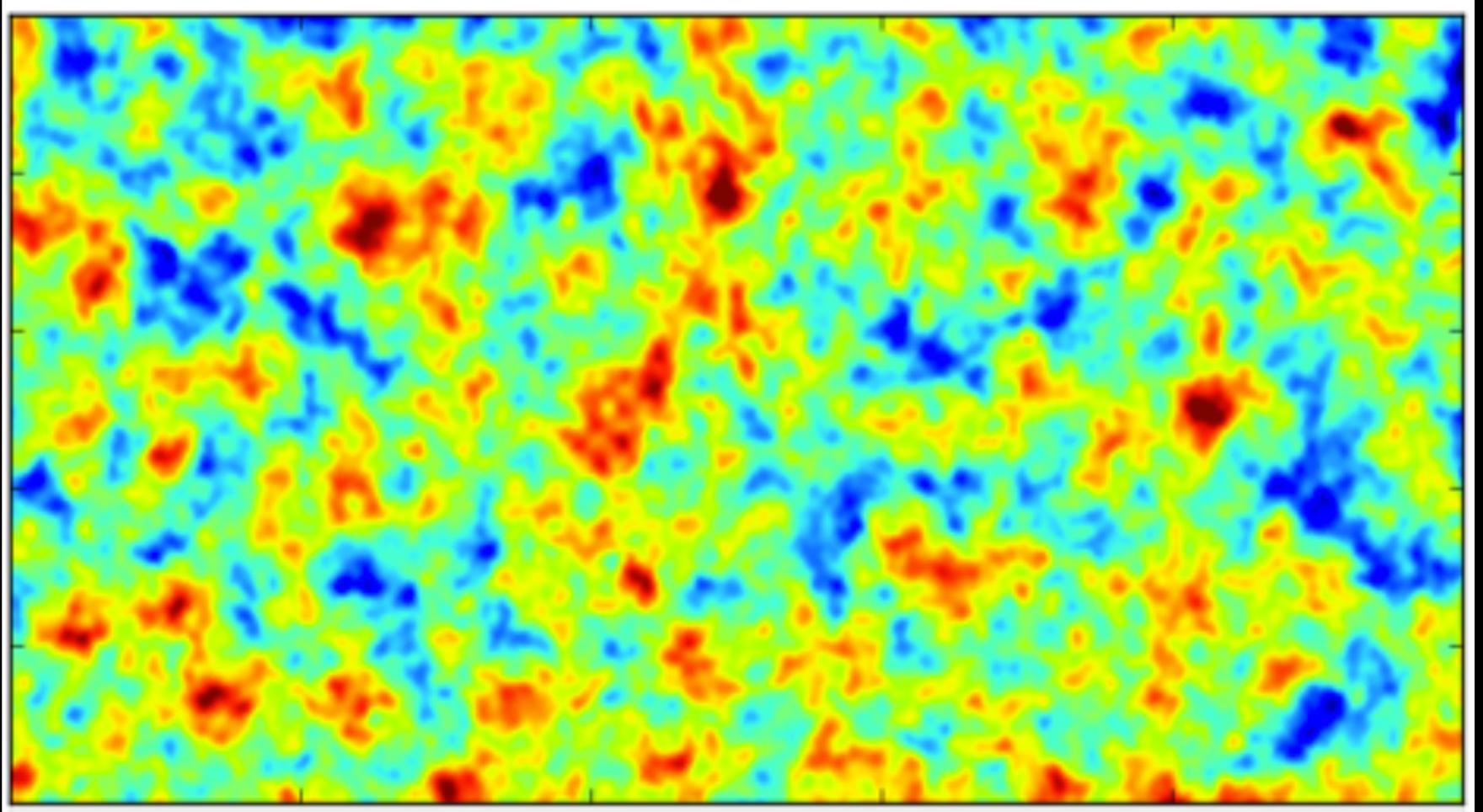
The gravitational lensing deflects the photon path by a quantity defined by the gradient of the lensing potential  $\phi(n)$ , integrated along the line of sight  $n$ , remapping the temperature field.

# The CMB lensing



A simulated patch of CMB sky – **before dark matter lensing**

# The CMB lensing



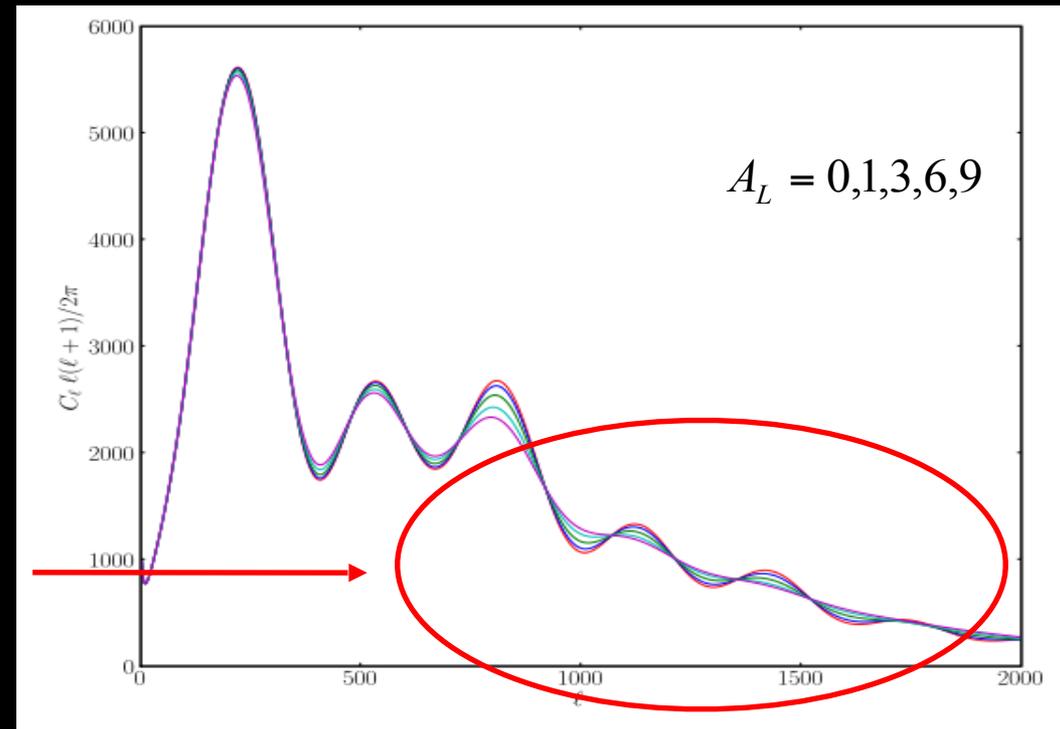
A simulated patch of CMB sky – **after dark matter lensing**

# $A_L$ internal anomaly

Its effect on the power spectrum is the smoothing of the acoustic peaks, increasing  $A_L$ .

Interesting consistency checks is if the amplitude of the smoothing effect in the CMB power spectra matches the theoretical expectation  $A_L = 1$  and whether the amplitude of the smoothing is consistent with that measured by the lensing reconstruction.

If  $A_L = 1$  then the theory is correct, otherwise we have a new physics or systematics.



Calabrese et al., Phys. Rev. D, 77, 123531

# $A_L$ internal anomaly

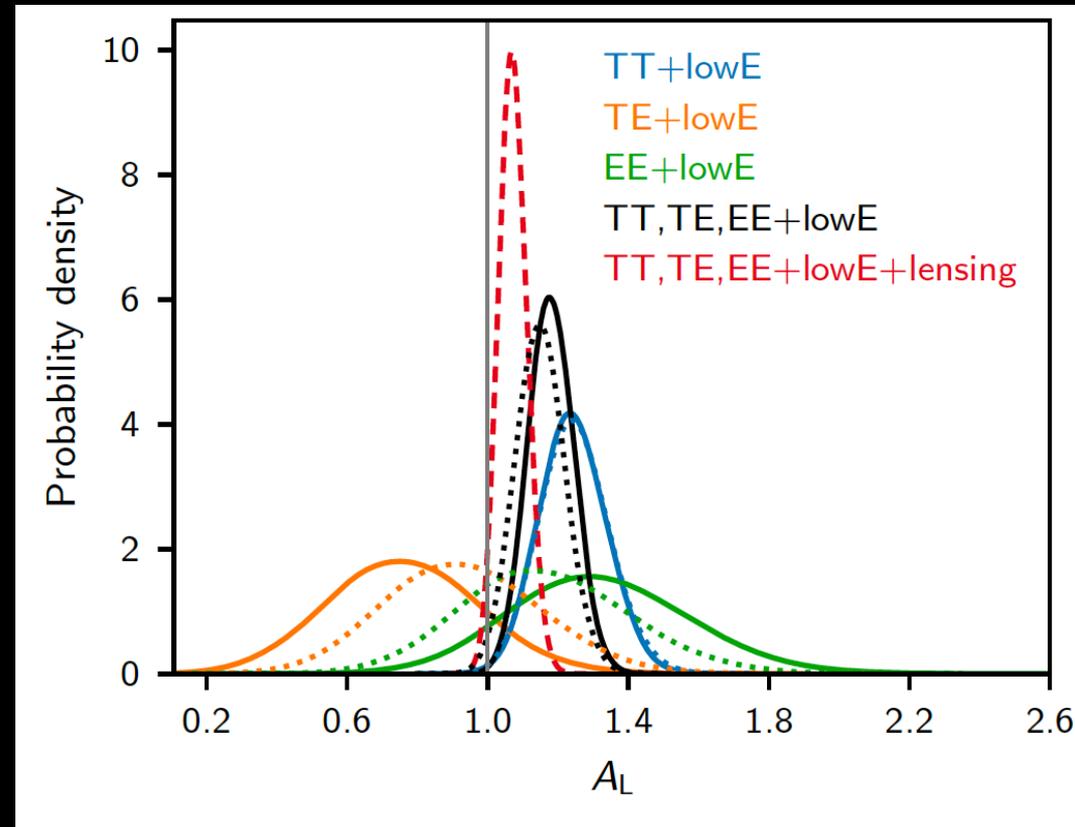
The Planck lensing-reconstruction power spectrum is consistent with the amplitude expected for  $\Lambda$ CDM models that fit the CMB spectra, so the Planck lensing measurement is compatible with  $A_L = 1$ .

However, the distributions of  $A_L$  inferred from the CMB power spectra alone indicate a preference for  $A_L > 1$ .

The joint combined likelihood shifts the value preferred by the TT data downwards towards  $A_L = 1$ , but the error also shrinks, increasing the significance of  $A_L > 1$  to  $2.8\sigma$ .

The preference for high  $A_L$  is not just a volume effect in the full parameter space, with the best fit improved by  $\Delta\chi^2 \sim 9$  when adding  $A_L$  for TT+lowE and 10 for TTTEEE+lowE.

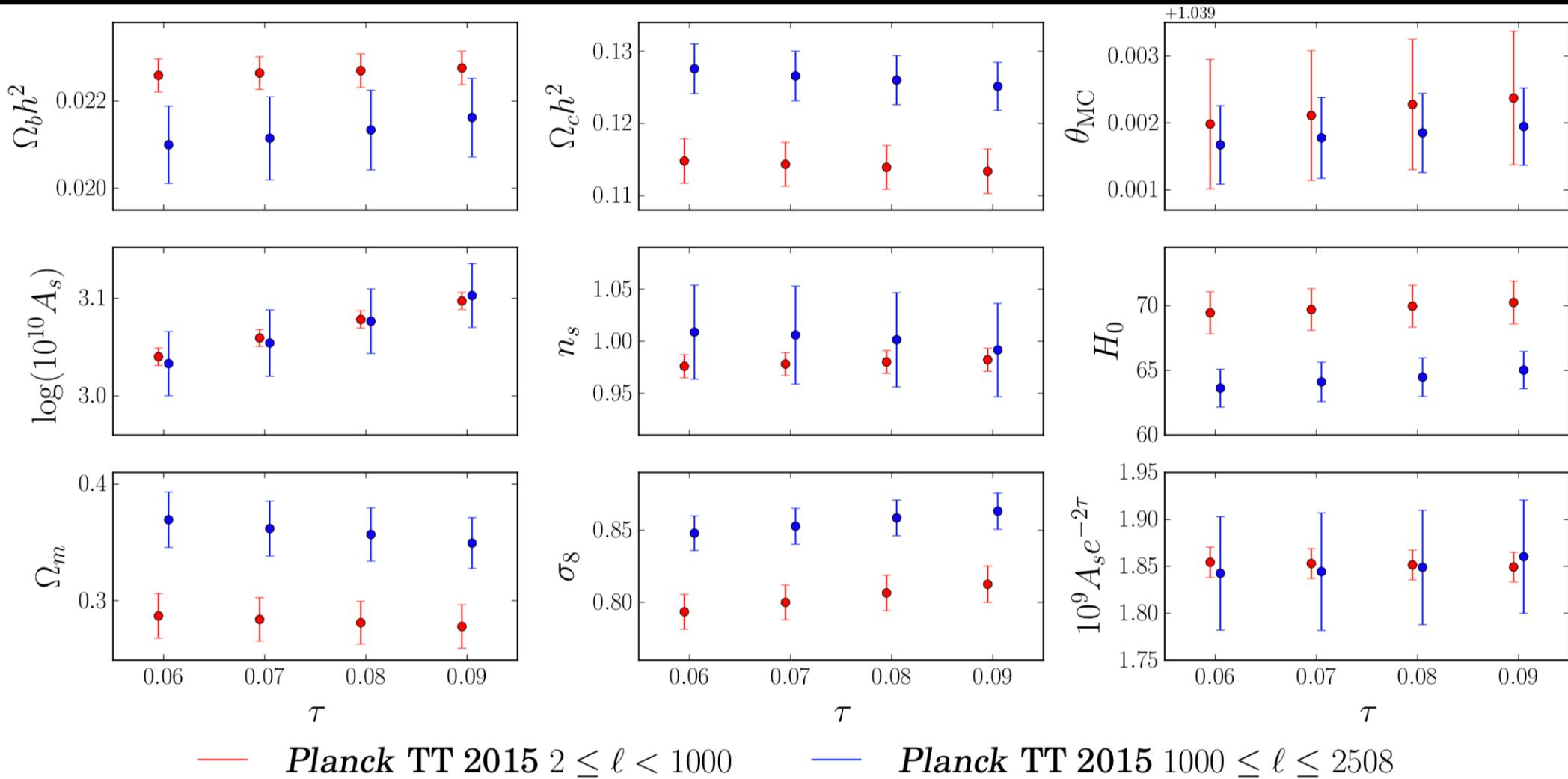
Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]



$$A_L = 1.243 \pm 0.096 \quad (68\%, \text{ Planck TT+lowE}),$$

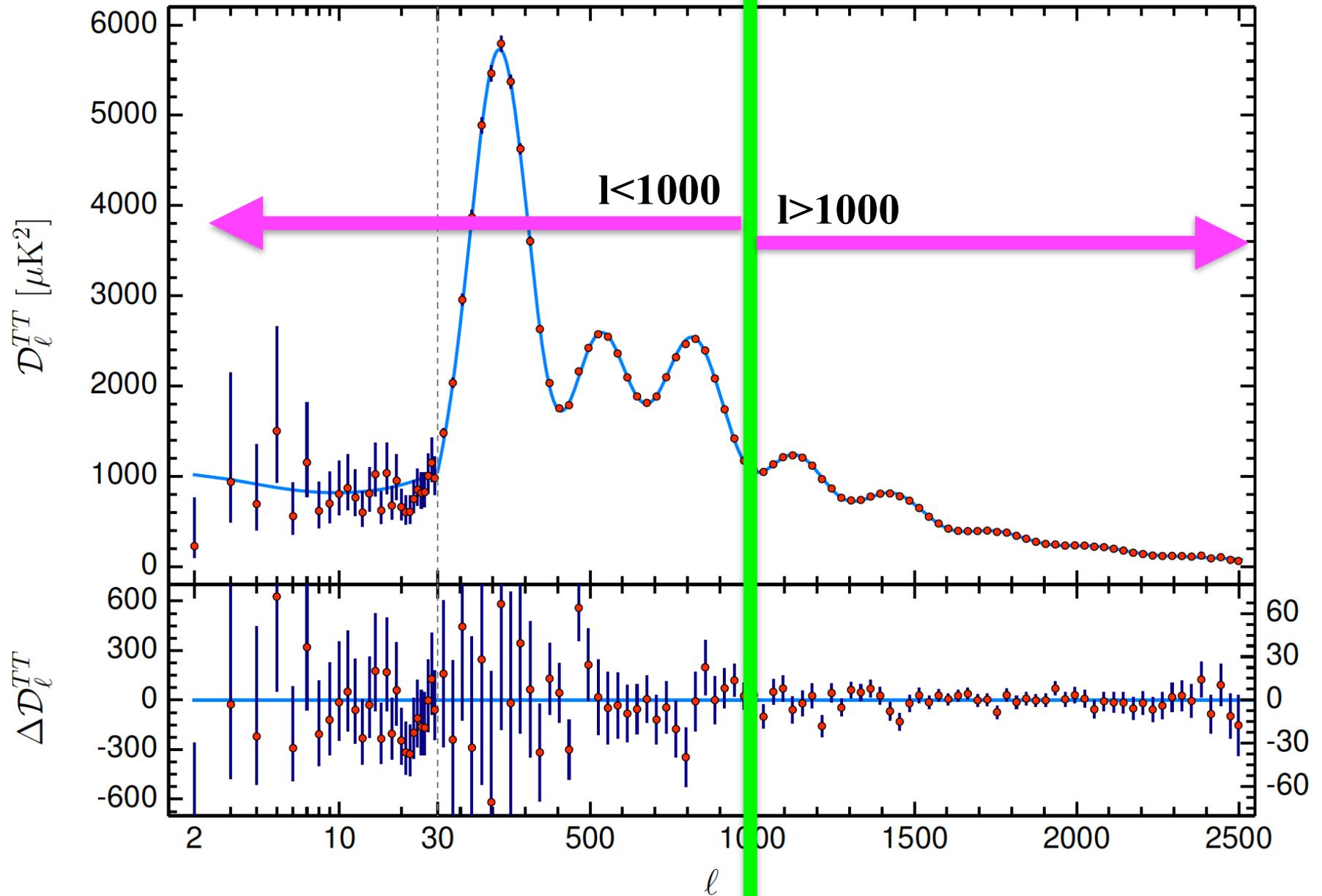
$$A_L = 1.180 \pm 0.065 \quad (68\%, \text{ Planck TT,TE,EE+lowE}),$$

# Internal inconsistency

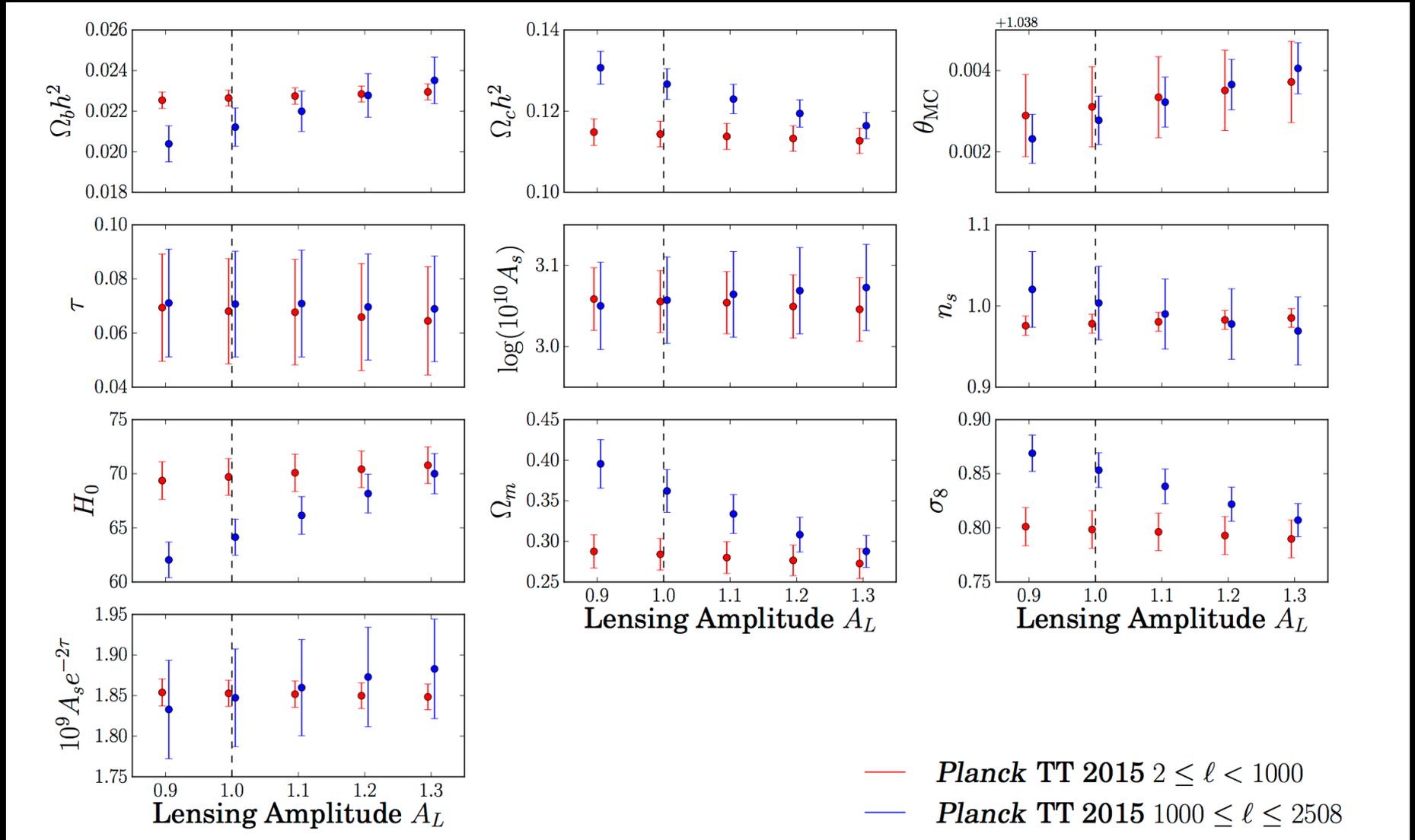


Marginalized 68.3% confidence  $\Lambda$ CDM parameter constraints from fits to the  $l < 1000$  and  $l \geq 1000$  *Planck* TT 2015 spectra, fixing AL at different values. Tension at more than  $2\sigma$  level is apparent in  $\Omega_c h^2$  and derived parameters, including  $H_0$ ,  $\Omega_m$ , and  $\sigma_8$ .

# $A_L$ internal anomaly



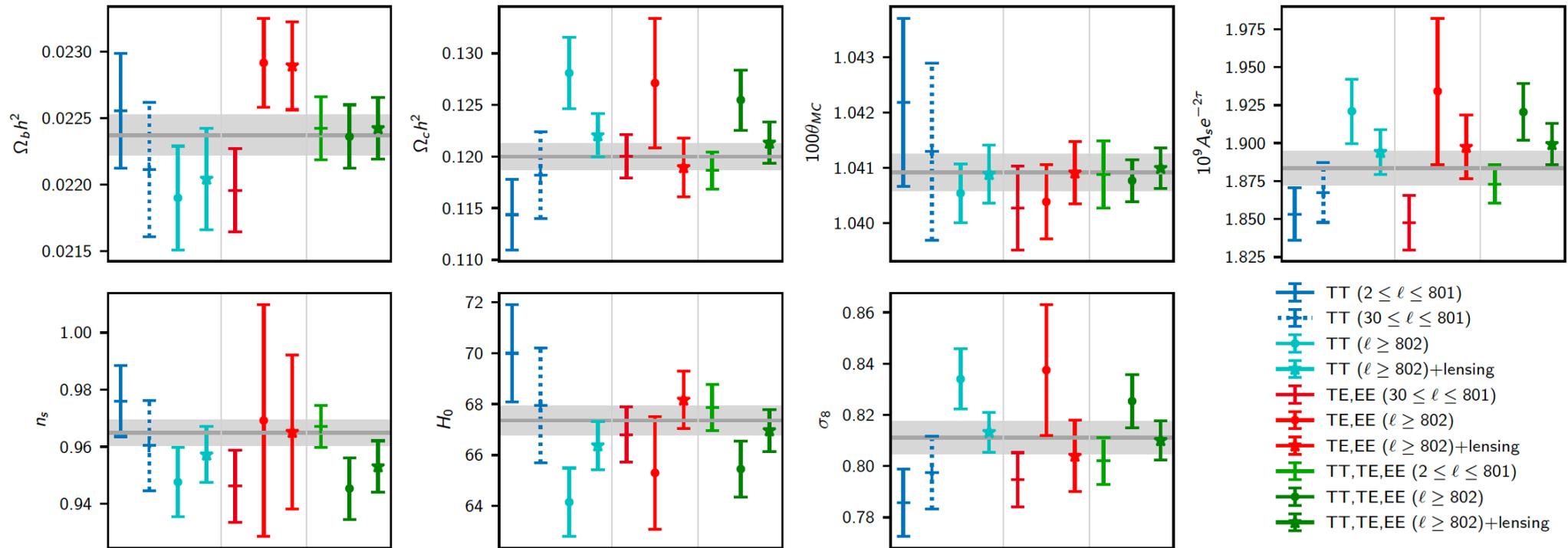
# $A_L$ internal anomaly



Increasing  $A_L$  smooths out the high order acoustic peaks, improving the agreement between the two multipole ranges.

# $A_L$ internal anomaly

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]



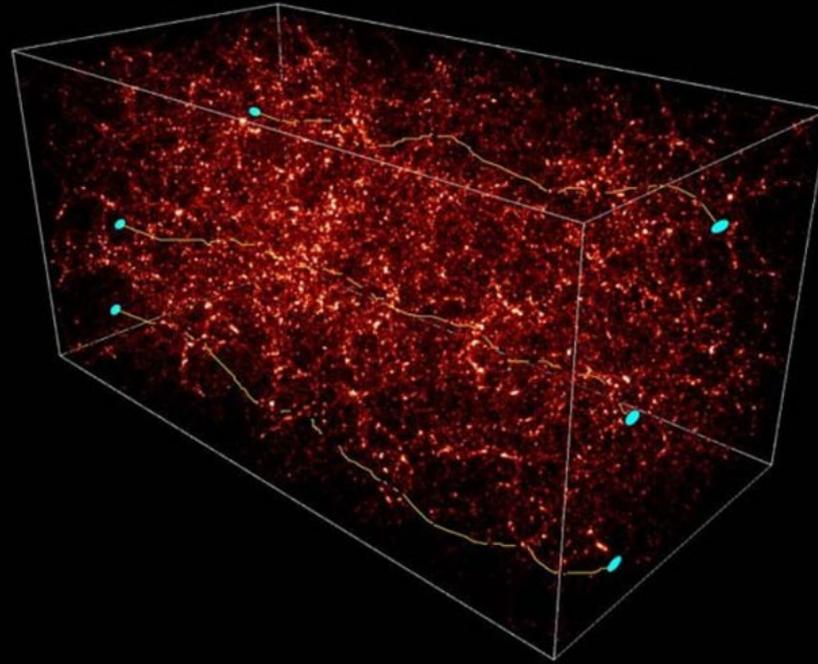
LCDM 68% marginalized parameter constraints for  $l=[2-801]$  (points marked with a cross),  $l>802$  (points marked with a circle), and  $l>802 +$  lensing (points marked with a star). Correcting for the lensing, all the results from high multipoles are in better consistency with the results from lower multipoles.

Dotted error bars are the results from  $l=[30-801]$ , without the large-scale TT likelihood, showing that  $l < 30$  pulls the low-multipole parameters further from the joint result.

The most famous and persisting anomalies and tensions of the CMB are:

- $H_0$  with local measurements
- **S8 with cosmic shear data**
- $A_L$  internal anomaly
- $\Omega_k$  different from zero

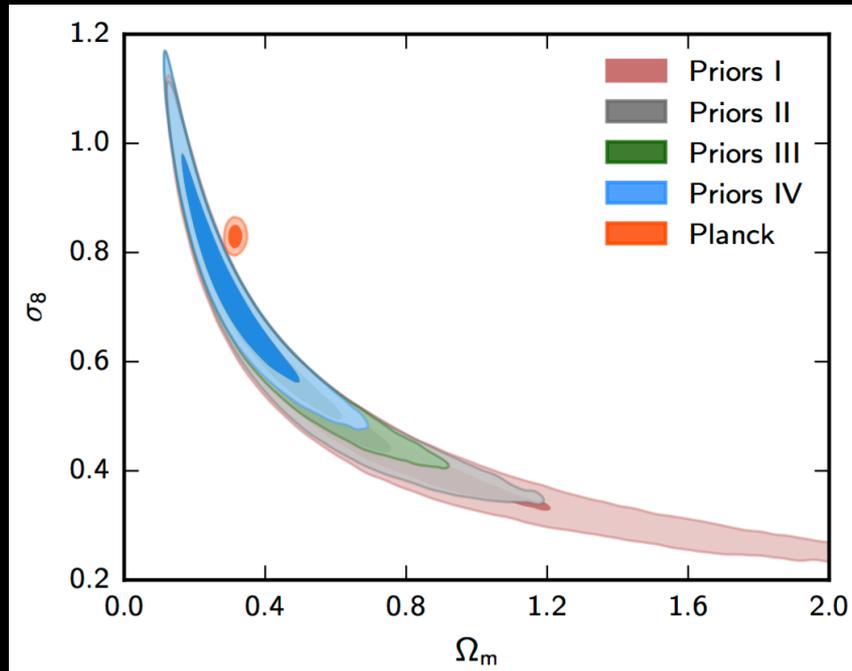
# S8 tension



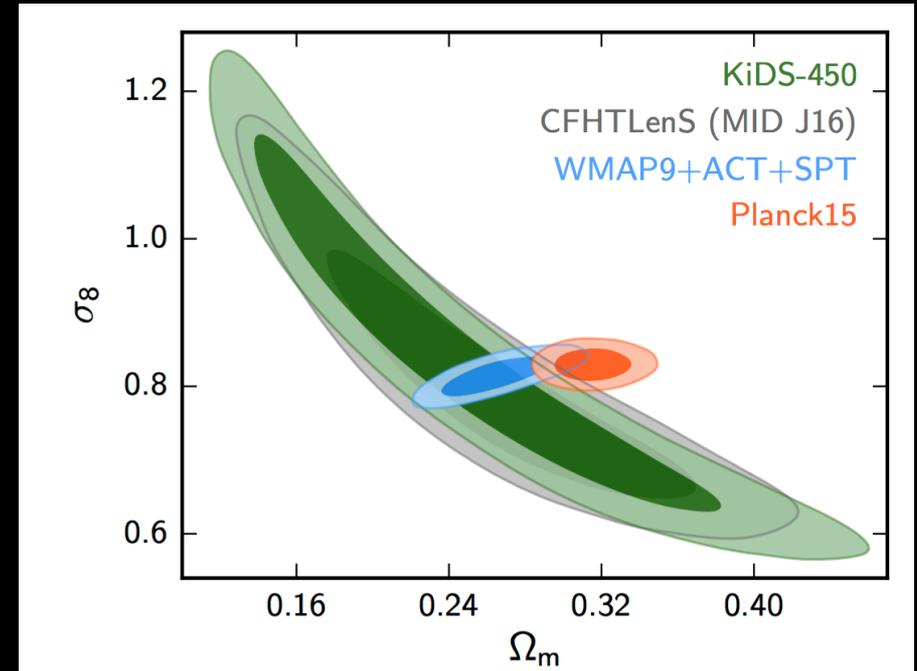
$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$

A tension on **S8** is present between the Planck data in the  $\Lambda$ CDM scenario and the cosmic shear data.

# S8 tension



Joudaki et al, arXiv:1601.05786



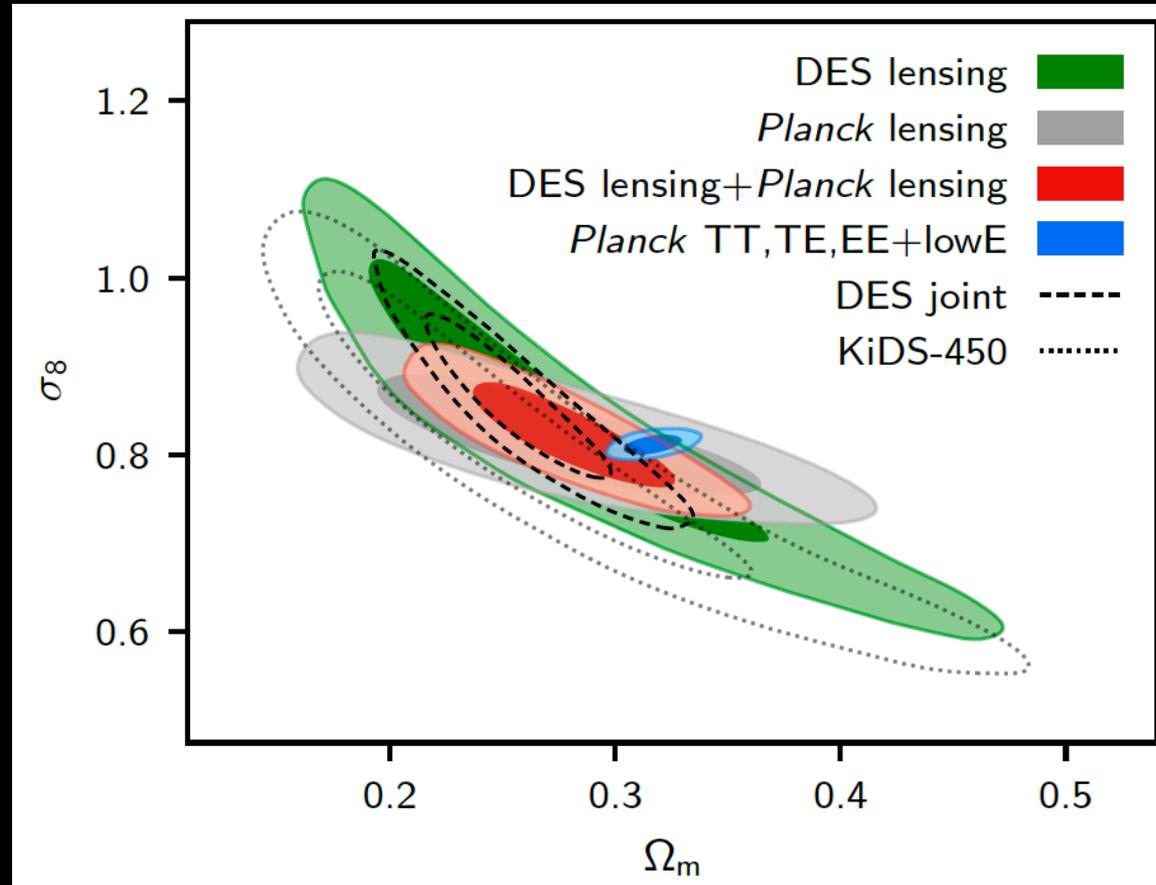
Hildebrandt et al., arXiv:1606.05338.

$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$

The **S8 tension** is at about **2.6 sigma** level between the Planck data in the  $\Lambda$ CDM scenario and CFHTLenS survey and KiDS-450.

# S8 tension

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]



While there is no tension with DES galaxy lensing, a **tension at about 2.5 sigma** level is present for the DES results that include galaxy clustering.

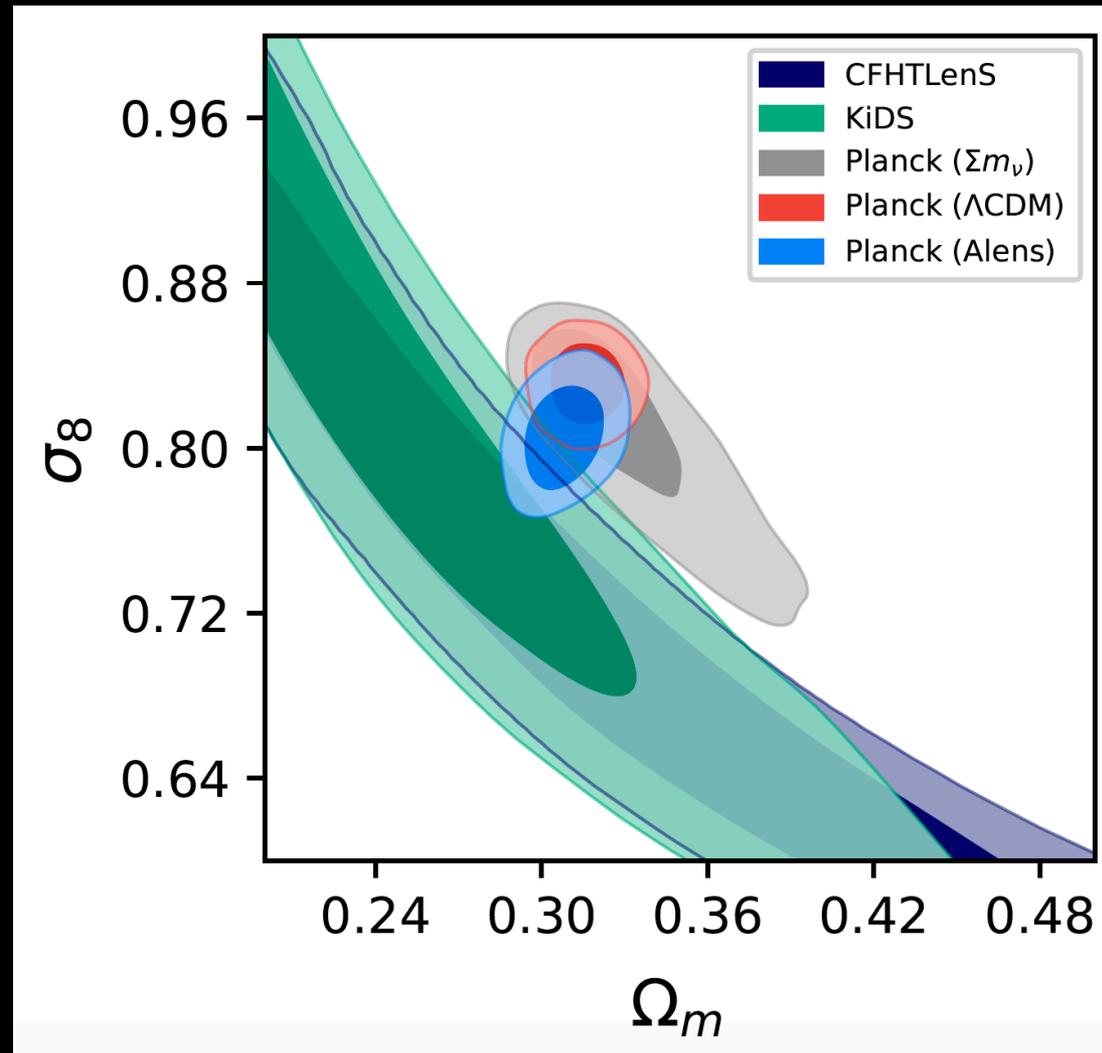
# The S8 tension

This is mainly due to the anomalous value of  $A_L$ .

We find that the CMB and cosmic shear datasets, in tension in the standard  $\Lambda$ CDM model, are still in tension adding massive neutrinos.

However, if we include the additional scaling parameter on the CMB lensing amplitude  $A_L$ , we find that this can put in agreement the Planck 2015 with the cosmic shear data.

$A_L$  is a phenomenological parameter that is found to be more than  $2\sigma$  higher than the expected value in the Planck 2015 data, suggesting a higher amount of lensing in the power spectra, not supported by the trispectrum analysis.



Di Valentino and Bridle, *Symmetry* 10 (2018) no.11, 585

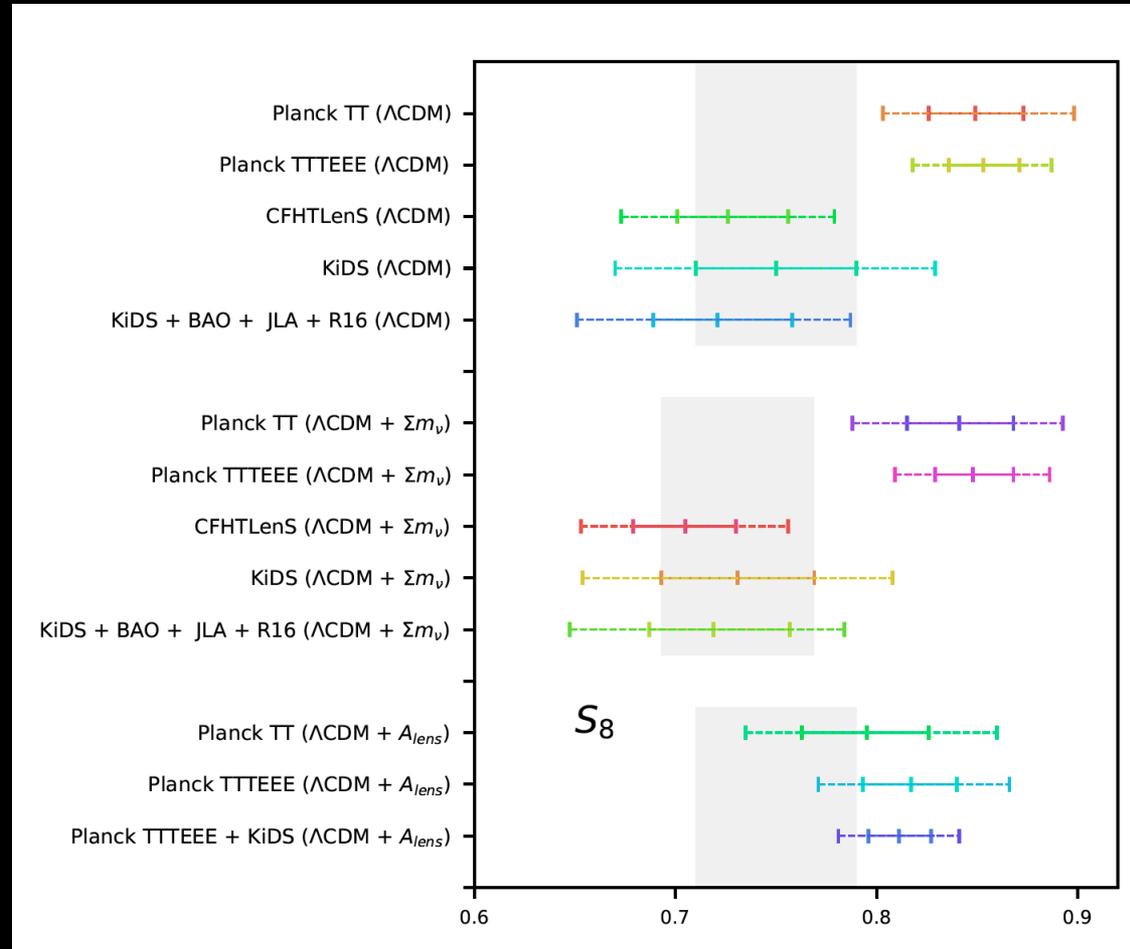
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Di Valentino and Bridle, *Symmetry* 10 (2018) no.11, 585

The most famous and persisting anomalies and tensions of the CMB are:

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- $\Omega_k$  different from zero

# Curvature of the universe

The  $\Lambda$ CDM model assumes that the universe is specially flat. The combination of the Planck temperature and polarization power spectra gives

$$\Omega_K = -0.044^{+0.018}_{-0.015} \quad (68\%, \text{Planck TT,TE,EE+lowE}),$$

a detection of curvature at about  $3.4\sigma$ .

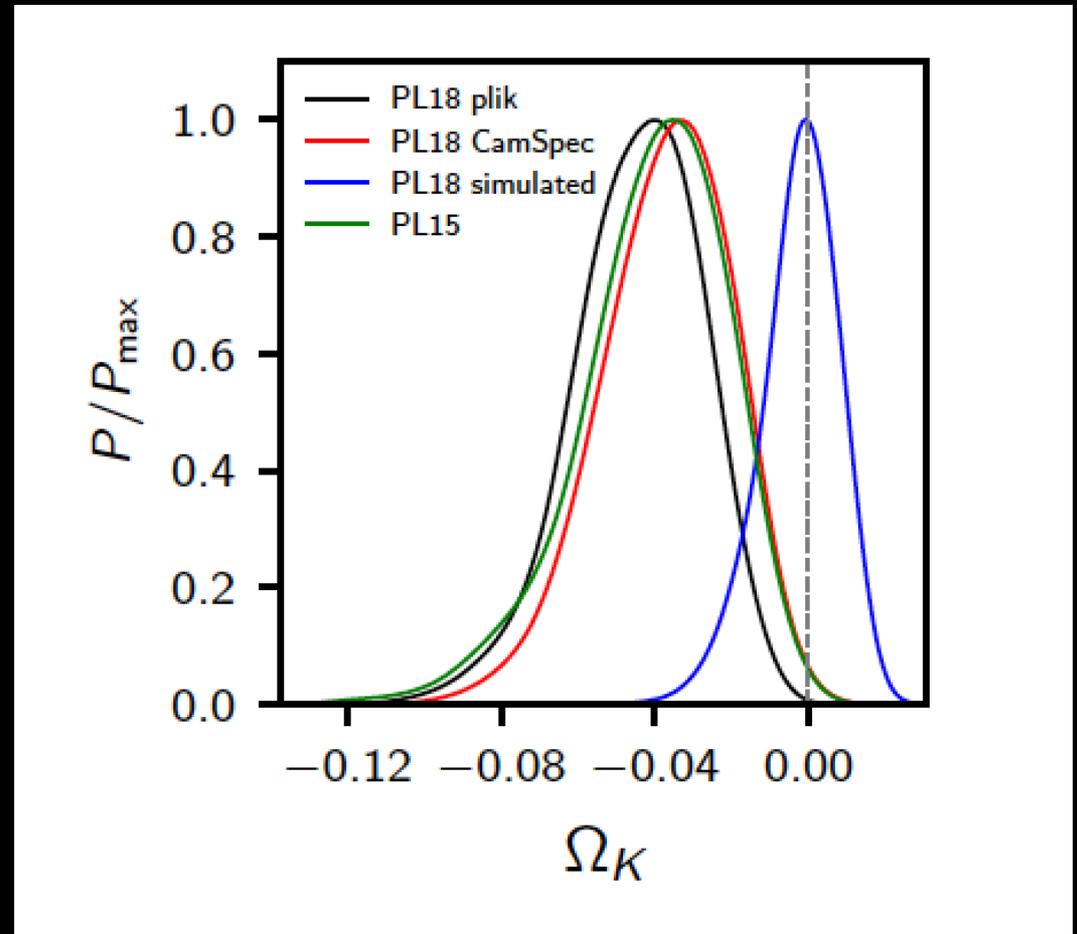
# Curvature of the universe

Can Planck provide an **unbiased and reliable estimate** of the curvature of the Universe?

This may not be the case since a "geometrical degeneracy" is present with  $\Omega_m$ .

When precise CMB measurements at arc-minute angular scales are included, since **gravitational lensing** depends on the matter density, its detection **breaks the geometrical degeneracy**. The Planck experiment with its improved angular resolution offers the unique opportunity of a precise measurement of curvature from a single CMB experiment.

**We simulated Planck, finding that such experiment could constrain curvature with a 2% uncertainty, without any significant bias towards closed models.**



Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

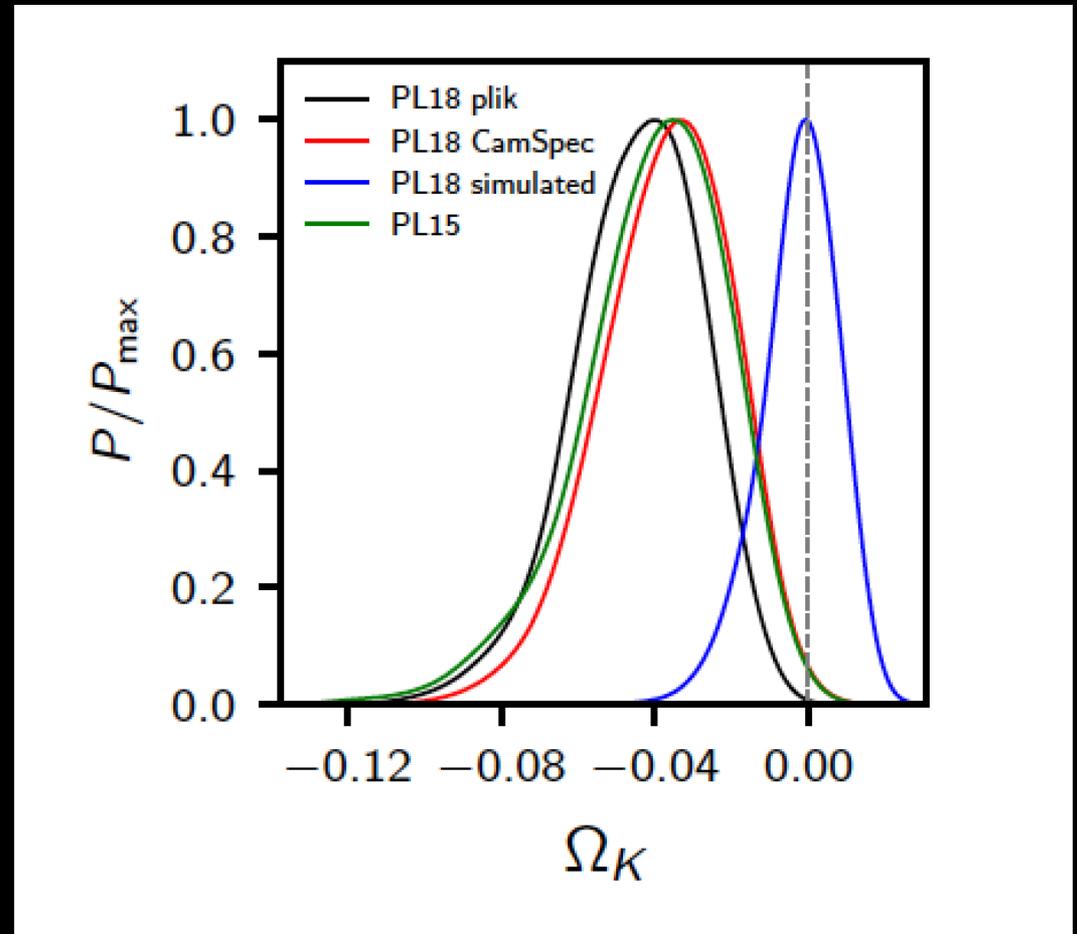
# Curvature of the universe

Planck favours a closed Universe ( $\Omega_K < 0$ ) with 99.985% probability.

A closed Universe with  $\Omega_K = -0.0438$  provides a better fit to PL18 with respect to a flat model.

This is not entirely a volume effect, since the best-fit  $\Delta\chi^2$  changes by -11 compared to base  $\Lambda$ CDM when adding the one additional curvature parameter.

The improvement is due also to the fact that closed models could also lead to a large-scale cut-off in the primordial density fluctuations in agreement with the observed low CMB anisotropy quadrupole.



Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

# Curvature of the universe

To better quantify the preference for a closed model, we adopt the deviance information criterion (DIC), which takes into account the Bayesian complexity, that is, the effective number of parameters, of the extended model and is defined as

$$\text{DIC} = 2\overline{\chi_{\text{eff}}^2} - \chi_{\text{eff}}^2$$

where the bar denotes a mean over the posterior distribution. We find that the Planck data yield  $\Delta\text{DIC} = -7.4$ ; that is, a closed Universe with  $\Omega_k = -0.0438$  is preferred, with a probability ratio of about 1/41, with respect to a flat model.

# Curvature of the universe

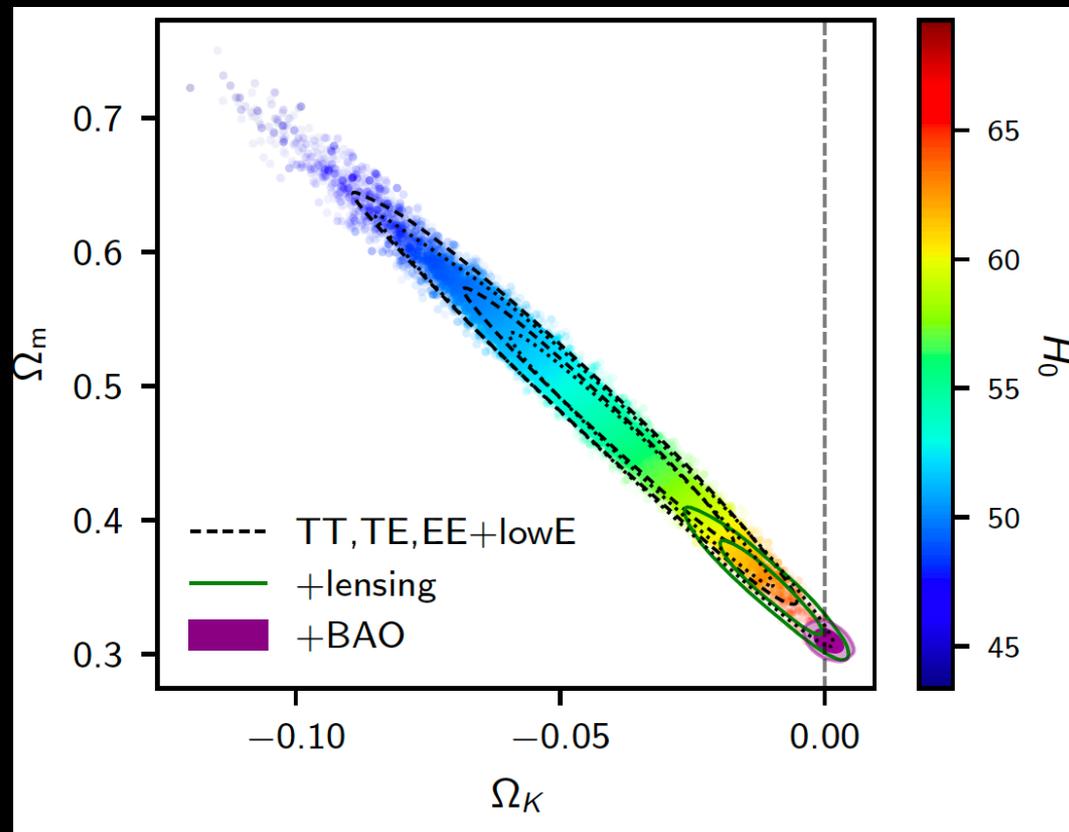
We also compute the **Bayesian evidence ratio by making use of the Savage–Dickey density ratio**. In this case the Bayes factor can be written as

$$B_{01} = \frac{p(\Omega_K | d, M_1)}{\pi(\Omega_K | M_1)} \Big|_{\Omega_K=0}$$

where  $M_1$  denotes the model with curvature,  $p(\Omega_K | d, M_1)$  is the posterior for  $\Omega_K$  in this theoretical framework, computed from a specific dataset  $d$ , and  $\pi(\Omega_K | M_1)$  is the prior on  $\Omega_K$  that we assume to be flat in the range  $-0.2 \leq \Omega_K \leq 0$ .

**For Planck we obtain a Bayes ratio of  $|\ln B_{01}| = 3.3$ , i.e. a strong evidence for a closed universe with respect to a flat one.**

# Curvature of the universe



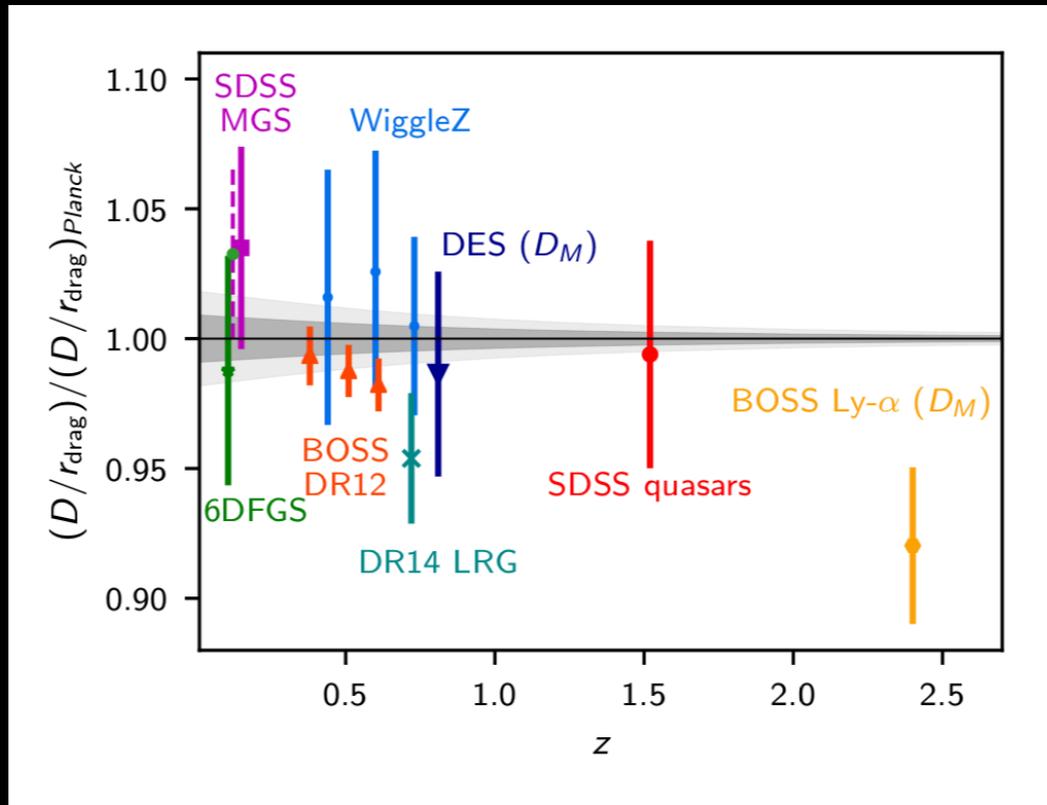
Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

Adding BAO data, a joint constraint is very consistent with a flat universe.

$$\Omega_K = 0.0007 \pm 0.0019 \quad (68\%, \text{TT, TE, EE+lowE} \\ \text{+lensing+BAO}).$$

Given the significant change in the conclusions from Planck alone, it is reasonable to **investigate whether they are actually consistent**. In fact, a basic assumption for combining complementary datasets is that these ones must be consistent, ie they must plausibly arise from the same cosmological model.

# Curvature of the universe

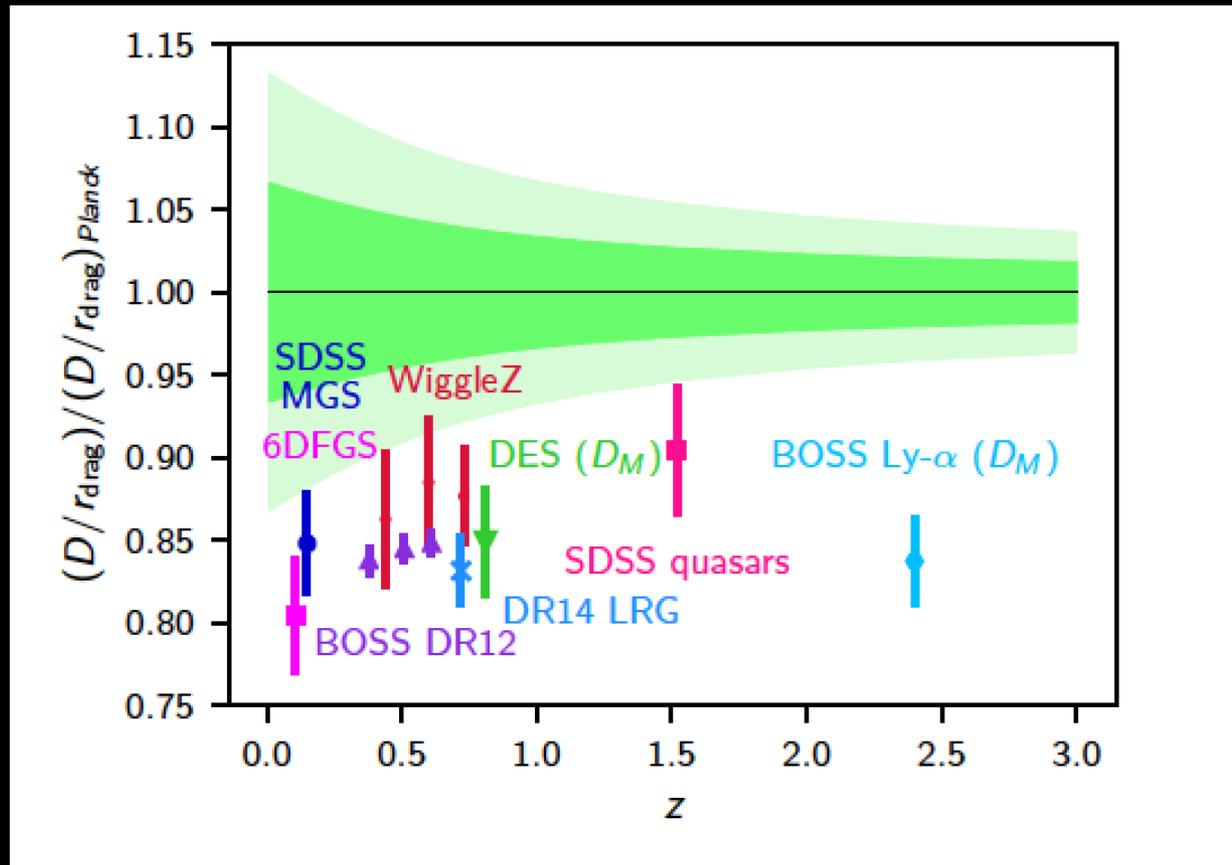


Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]

This is a **plot of the acoustic-scale distance ratio**,  $DV(z)/r_{\text{drag}}$ , as a function of redshift, taken from several recent BAO surveys, and divided by the mean acoustic-scale ratio obtained by Planck adopting a model.  $r_{\text{drag}}$  is the comoving size of the sound horizon at the baryon drag epoch, and  $DV$ , the dilation scale, is a combination of the Hubble parameter  $H(z)$  and the comoving angular diameter distance  $DM(z)$ .

In a  $\Lambda$ CDM model the BAO data agree really well with the Planck measurements...

# Curvature of the universe



Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

... but when we let curvature to vary  
there is a striking disagreement between Planck spectra and BAO measurements!

# Curvature of the universe

Observable	Redshift	BAO (68% CL)	Planck (68% CL)	Tension
$D_M(r_{d,\text{fid}}/r_d)$ (Mpc)	0.38	$1,518 \pm 22.8$	$1,843 \pm 100$	$2.9\sigma$
$D_M(r_{d,\text{fid}}/r_d)$ (Mpc)	0.51	$1,977 \pm 26.9$	$2,361 \pm 115$	$3.0\sigma$
$D_M(r_{d,\text{fid}}/r_d)$ (Mpc)	0.61	$2,283 \pm 32.3$	$2,726 \pm 130$	$3.3\sigma$
$H(r_{d,\text{fid}}/r_d)$ ( $\text{km s}^{-1} \text{Mpc}^{-1}$ )	0.38	$81.5 \pm 1.9$	$71.6 \pm 3.3$	$2.6\sigma$
$H(r_{d,\text{fid}}/r_d)$ ( $\text{km s}^{-1} \text{Mpc}^{-1}$ )	0.51	$90.5 \pm 1.97$	$78.9 \pm 3.1$	$3.1\sigma$
$H(r_{d,\text{fid}}/r_d)$ ( $\text{km s}^{-1} \text{Mpc}^{-1}$ )	0.61	$97.3 \pm 2.1$	$85.0 \pm 3.0$	$3.3\sigma$

Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

In the Table we have the constraints on DM and H(z) from the recent analysis of BOSS DR12 data and the corresponding constraints obtained indirectly from Planck, assuming a  $\Lambda$ CDM model with curvature.

**Planck is inconsistent with each of the BAO measurements at more than  $3\sigma$ !**

The assumption of a flat universe could therefore mask a cosmological crisis where disparate observed properties of the Universe appear to be mutually inconsistent.

# Curvature of the universe

Additional dataset	$\Delta\chi^2_{\text{eff}}$	$\Delta N_{\text{data}}$	$\log_{10}\mathcal{I}$
flat $\Lambda$ CDM			
+ BAO	+6.15	8	0.2
+ CMB lensing	+8.9	9	0.6
$\Lambda$ CDM + $\Omega_k$			
+ BAO	+16.9	8	-1.8
+ CMB lensing	+16.9	9	-0.84

Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

As we can see from the Table, the Planck  $\chi^2$  best fit is worse by  $\Delta\chi^2 \approx 16.9$  when the BAO data are included under the assumption of curvature. This is a significantly larger  $\Delta\chi^2$  than obtained for the case of  $\Lambda$ CDM ( $\Delta\chi^2 \approx 6.15$ ).

The BAO dataset that we adopted consists of two independent measurements (6dFGS36 and SDSS-MGS37) with relatively large error bars, and six correlated measurements from BOSS DR12.

# Curvature of the universe

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Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

To quantify the discrepancy between two cosmological datasets,  $D_1$  and  $D_2$ , we use the following quantity based on the DIC approach:

$$\mathcal{I}(D_1, D_2) \equiv \exp\{-\mathcal{F}(D_1, D_2)/2\}$$

where

$$\mathcal{F}(D_1, D_2) = \text{DIC}(D_1 \cup D_2) - \text{DIC}(D_1) - \text{DIC}(D_2)$$

Following the Jeffreys's scale the agreement/disagreement is considered 'substantial' if  $|\log_{10} \mathcal{I}| > 0.5$ , 'strong' if  $|\log_{10} \mathcal{I}| > 1.0$  and 'decisive' if  $|\log_{10} \mathcal{I}| > 2.0$ . When is positive, then two datasets are in agreement, whereas they are in tension if this parameter is negative. We find a **strong disagreement between Planck and BAO**.

# Curvature of the universe

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Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

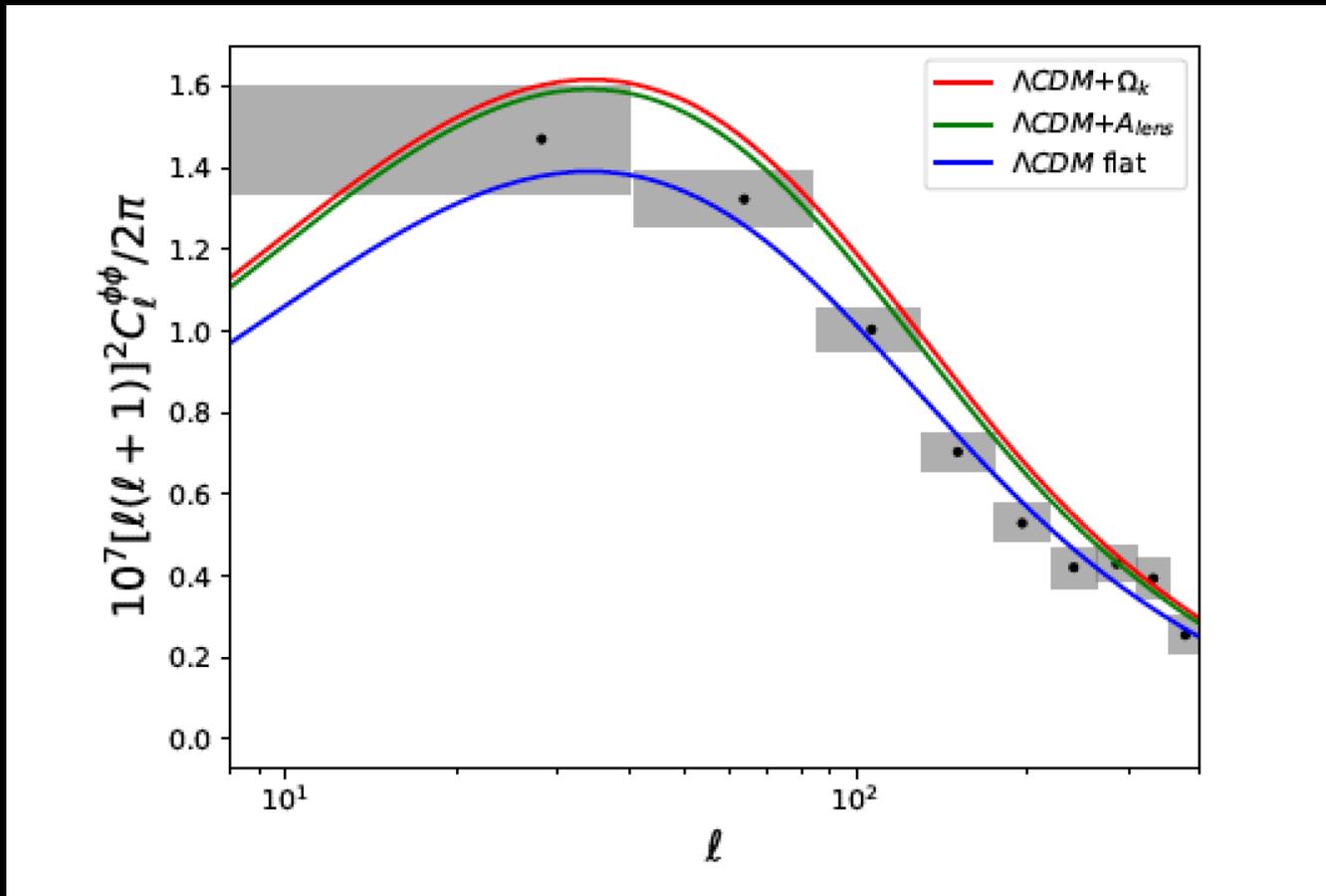
A second tension is present between Planck power spectra and the constraints on the lensing potential derived from the four-point correlation function of Planck CMB maps.

The inclusion of CMB lensing in Planck increases the best-fit  $\Delta\chi^2 = 16.9$  in the case of  $\Lambda$ CDM +  $\Omega_K$  (while in the case of the  $\Lambda$ CDM model, we have  $\Delta\chi^2 = 8.9$ ). The CMB lensing dataset consists of nine correlated data points.

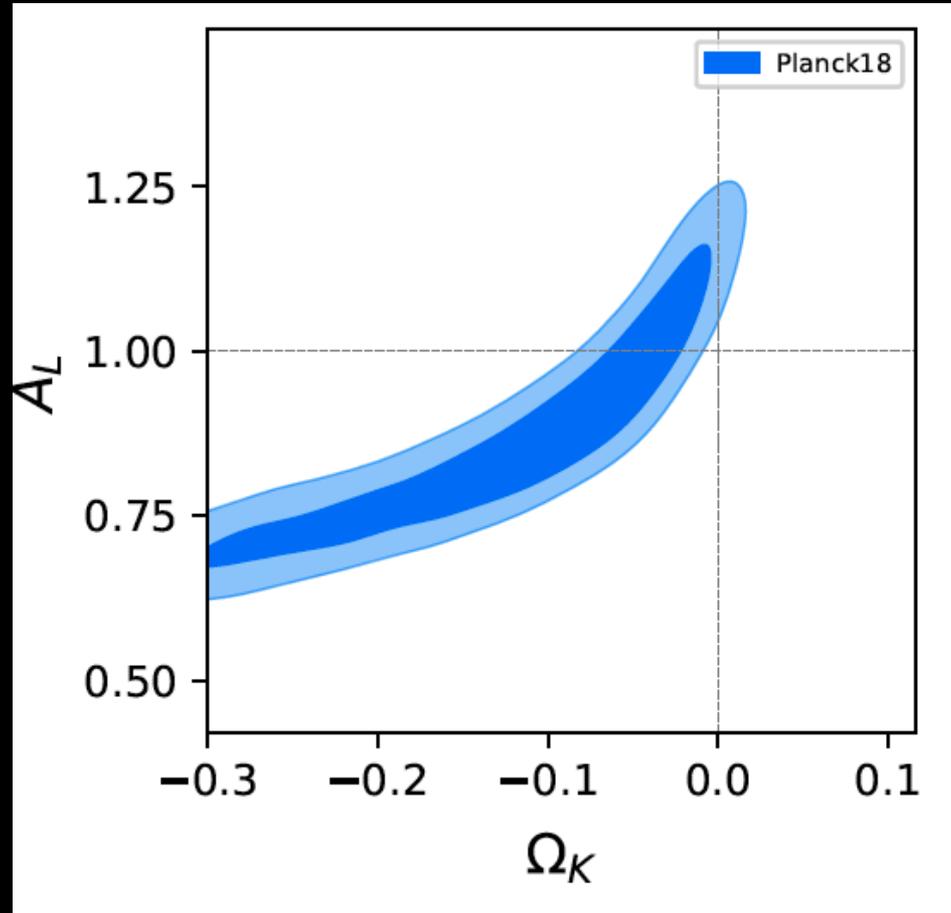
We identify **substantial discordance between Planck and CMB lensing.**

# Curvature of the universe

Closed models predict substantially higher lensing amplitudes than in  $\Lambda$ CDM, because the dark matter content can be greater, leading to a larger lensing signal. The reasons for the pull towards negative values of  $\Omega_K$  are essentially the same as those that lead to the preference for  $AL > 1$ .



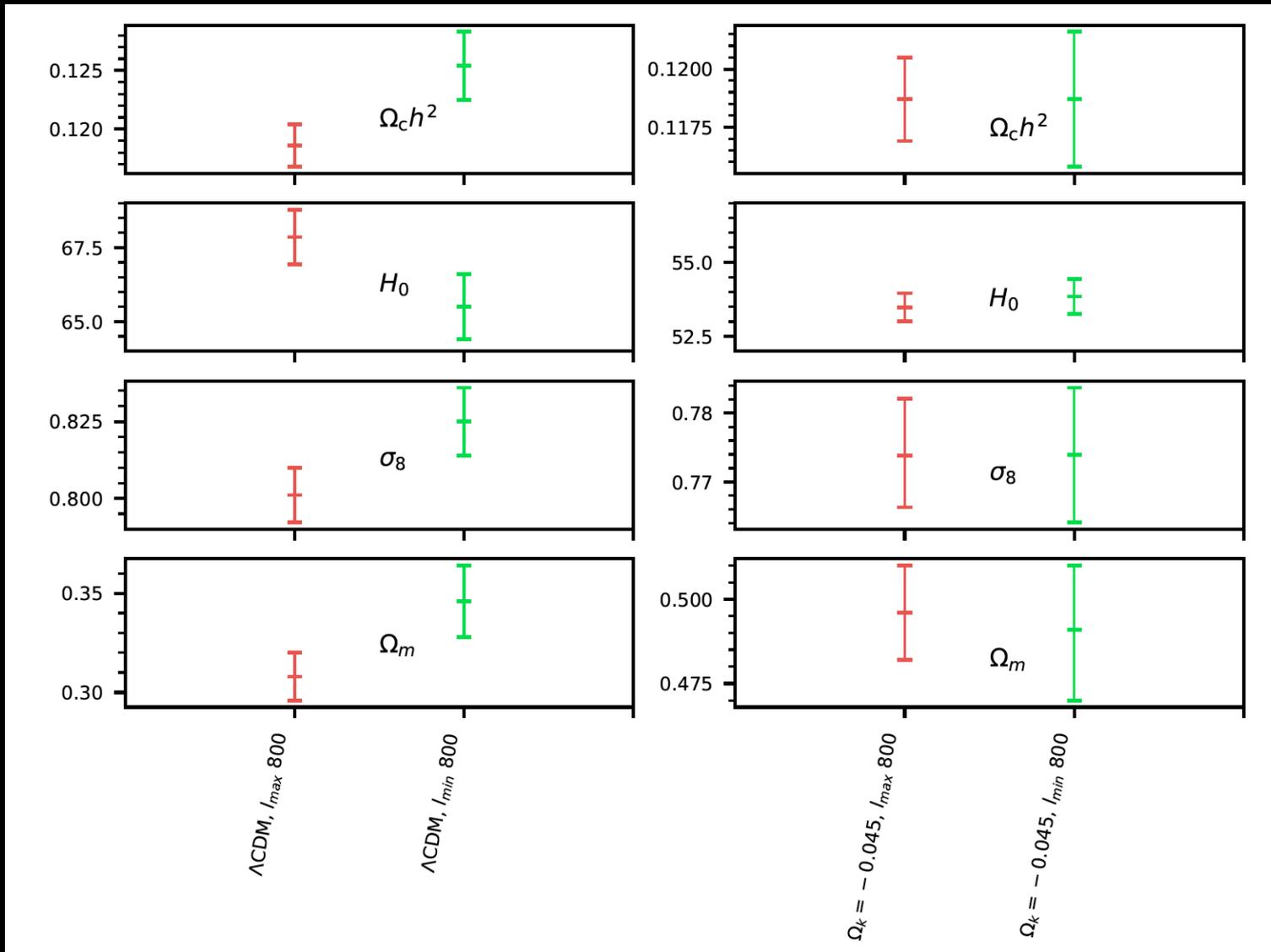
# Curvature can explain $A_L$



Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

A degeneracy between curvature and the  $A_L$  parameter is clearly present. **A closed universe can provide a robust physical explanation** to the enhancement of the lensing amplitude. Note that a model with  $\Omega_K < 0$  is slightly preferred with respect to a flat model with  $A_L > 1$ , because closed models better fit the low-multipole data.

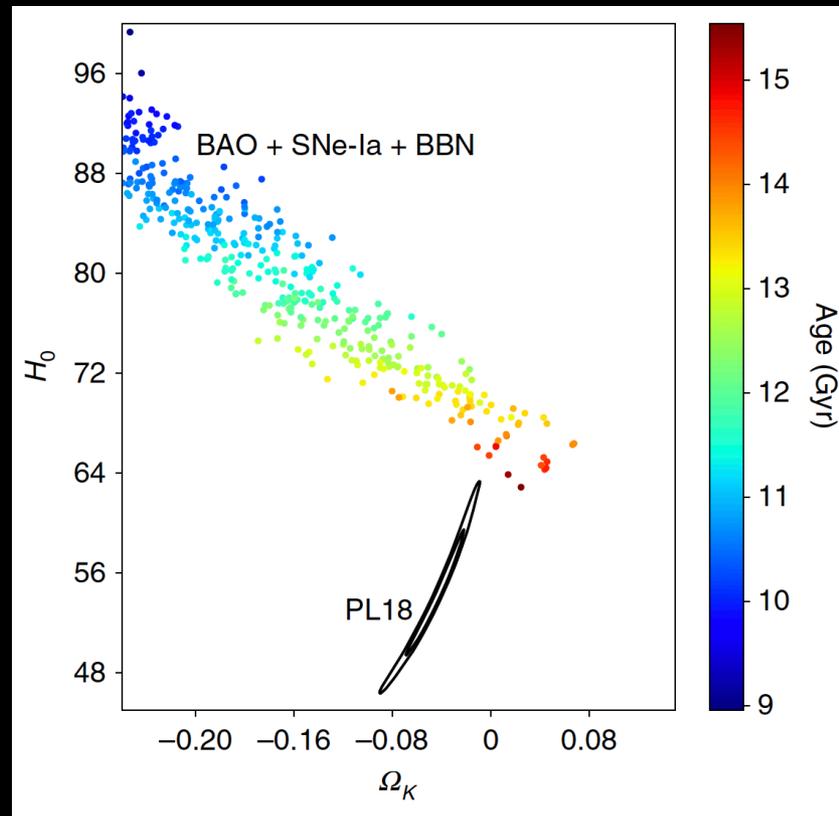
# Curvature can explain internal tension



Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

In a closed Universe with  $\Omega_K = -0.045$ , the cosmological parameters derived in the two different multipole ranges are now fully compatible.

# Curvature of the universe



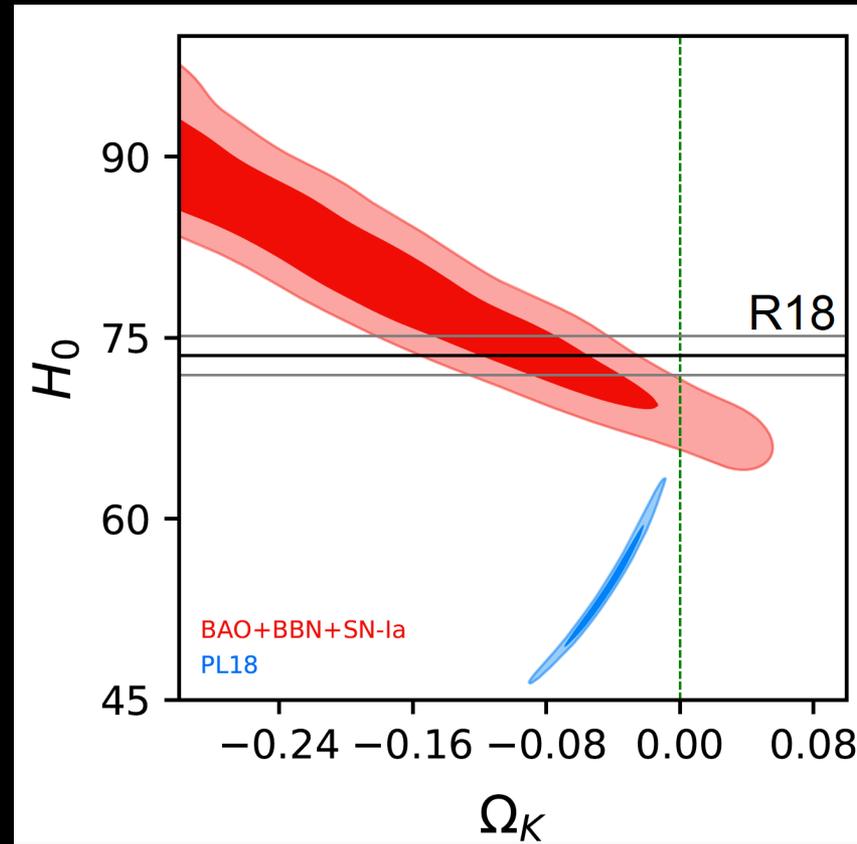
Di Valentino, Melchiorri and Silk, *Nature Astronomy* (2019)

It is now interesting to address the **compatibility of Planck with combined datasets**, like BAO + type-Ia supernovae + big bang nucleosynthesis data.

In principle, **each dataset prefers a closed universe**,

but **BAO+SN-Ia+BBN gives  $H_0 = 79.6 \pm 6.8$  km/s/Mpc** at 68%cl, perfectly consistent with R19, but at  $3.4\sigma$  tension with Planck.

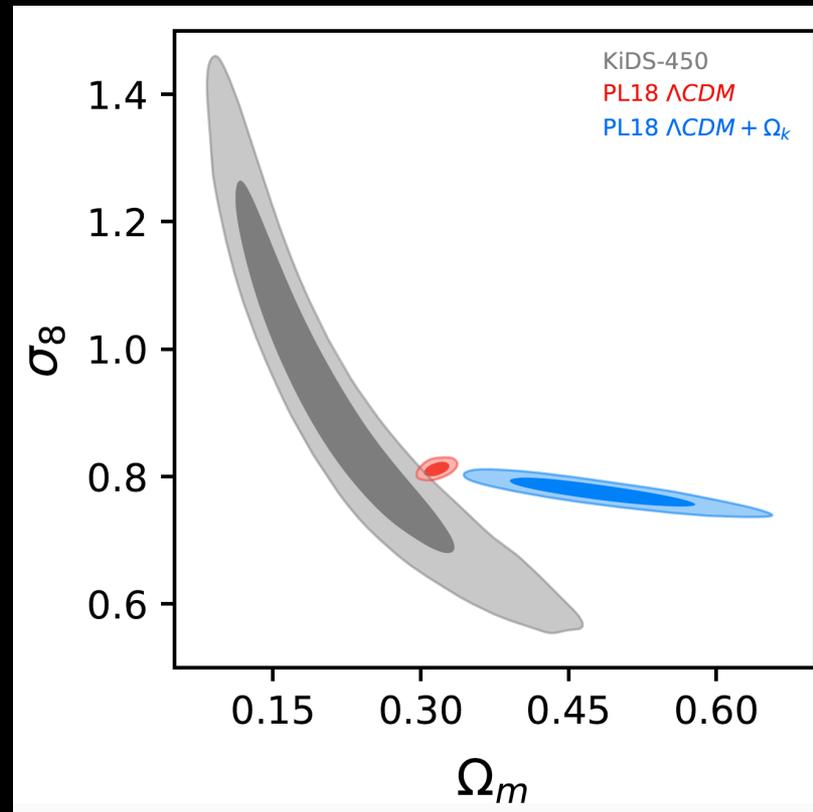
# Curvature can't explain external tensions



Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

Varying  $\Omega_K$ , both the well know tensions on  $H_0$  and  $S_8$  are exacerbated.  
In a  $\Lambda$ CDM +  $\Omega_K$  model, Planck gives  $H_0 = 54.4^{+3.3}_{-4.0}$  km/s/Mpc at 68% cl., increasing the tension with R19 at  $5.4\sigma$ .

# Curvature can't explain external tensions



Di Valentino, Melchiorri and Silk, Nature Astronomy (2019)

Varying  $\Omega_k$ , both the well know tensions on  $H_0$  and  $S_8$  are exacerbated.

In a  $\Lambda$ CDM +  $\Omega_k$  model, Planck gives  $S_8$  in disagreement at about  $3.8\sigma$  with KiDS-450, and more than  $3.5\sigma$  with DES.

What happens if we vary all the parameters together?

In practice **we do not try to solve any single tension** with a specific theoretical mechanism, but we allow for a significant number of motivated extensions of  $\Lambda$ CDM, looking for a **possible combination of parameters** that could solve or at least ameliorate, the current discordances.

# Measuring the CMB..

In the past twenty years, measurements of the CMB anisotropy angular power spectrum have witnessed one of the most **impressive technological advances** in experimental physics.

Following the first detection of CMB temperature anisotropies at large angular scales by the **COBE satellite in 1992**, passing through balloon-borne experiments such as **BOOMERanG, MAXIMA**, the **WMAP satellite**, and ground-based experiments as **DASI, ACT and SPT**, we have now a **cosmic-variance limited measurements made by the Planck experiment**.

Despite this **impressive progress on the experimental side**, the constraints on cosmological parameters are still presented under the **assumption of a simple  $\Lambda$ CDM model, based on the variation of just 6 cosmological parameters**.

While this model still provides a good fit to the data, it is **the same model used**, for example, in the analysis of the BOOMERanG 1998 data, more than **twenty years ago**.

While this "minimal" approach is justified by the good fit to the data that the  $\Lambda$ CDM provides we believe that it does not do adequate justice to the high quality of the most recent datasets. In light of the new precise data, some of **the assumptions or simplifications made in the 6 parameters approach are indeed not anymore fully justified and risk an oversimplification** of the physics that drives the evolution of the Universe.

# Beyond six parameters: extending $\Lambda$ CDM

- The **total neutrino mass** is fixed arbitrary to  $0.06\text{eV}$ . However, we know that neutrinos are massive and that current cosmological datasets are sensitive to variations in the absolute neutrino mass scale of order  $\sim 100\text{ meV}$ .
- The **cosmological constant** offers difficulties in any theoretical interpretation: fixing the dark energy equation of state to  $-1$  is not favoured by any theoretical argument. Moreover, while both matter and radiation evolve rapidly,  $\Lambda$  is assumed not to change with time, so its recent appearance in the standard cosmological model implies an extreme fine-tuning of initial conditions. This fine-tuning is known as the coincidence problem. Therefore it seems reasonable to incorporate in the analysis a possible dynamical dark energy component.
- Most inflationary models predict a sizable contribution of **gravitational waves**. Given the progress made in the search for B-mode polarization, it is an opportune moment to allow any such contribution to be directly constrained by the data, without assuming a null contribution as in the 6-parameter model.
- A similar argument can be made for the **running of the scalar spectral index**, expected for slow rolling inflation at the level of  $(1-n_s)^2 \sim 10^{-3}$ .
- The **effective number of relativistic degrees of freedom** could be easily different from the standard expected value of  $3.046$ , for example for the presence of sterile neutrinos or thermal axions.
- We need to take into account the anomalous value for the **lensing amplitude  $A_{\text{lens}}$** . While this parameter is purely phenomenological, one should clearly consider it and check if the cosmology obtained is consistent with other datasets.

# Beyond six parameters: extending $\Lambda$ CDM

Cosmological constraints are usually derived under the assumption of a 6 parameters  $\Lambda$ CDM theoretical framework or simple one-parameter extensions.

In [Di Valentino, Melchiorri and Silk, Phys.Rev. D92 \(2015\) no.12, 121302, arXiv:1507.06646](#) we present, for the first time, cosmological constraints in a significantly extended scenario, varying up to 12 cosmological parameters simultaneously, including:

- the sum of neutrino masses,
- the dark energy equation of state,
- the gravitational waves background,
- the running of the spectral index of primordial perturbations,
- the neutrino effective number,
- the angular power spectrum lensing amplitude,  $A_{\text{lens}}$ .

# Beyond six parameters: extending $\Lambda$ CDM

Model Dataset	$\Omega_b h^2$	$\Omega_c h^2$	$H_0$ [km/s/Mpc]	$\tau$	$n_s$	$\frac{dn_s}{dn_k}$	$r$	$w$	$\Sigma m_\nu$ [eV]	$N_{\text{eff}}$	$A_{\text{lens}}$	
$\Lambda$ CDM Planck TT+LowP	$0.02222^{+0.00046}_{-0.00044}$	$0.1198^{+0.0042}_{-0.0043}$	$67.3^{+2.0}$	-	-	-	-	-	-	-	-	
$\Lambda$ CDM Planck	$0.02226^{+0.00031}_{-0.00029}$	$0.1198^{+0.0028}_{-0.0028}$	$67.3^{+2.0}$	$0.079^{+0.034}_{-0.035}$	$0.9646^{+0.0092}_{-0.0092}$	$0.831^{+0.026}_{-0.026}$	-	-	-	-	-	
$\Lambda$ CDM Planck+ BAO	$0.02229^{+0.00028}_{-0.00027}$	$0.1193^{+0.0021}_{-0.0020}$	$67.52^{+0.93}_{-0.93}$	$0.082^{+0.031}_{-0.032}$	$0.9662^{+0.0078}_{-0.0079}$	$0.832^{+0.025}_{-0.025}$	-	-	-	-	-	
$e$ CDM Planck TT+LowP	$0.0245^{+0.0024}_{-0.0022}$	$0.127^{+0.017}_{-0.016}$	$> 43$	$0.073^{+0.051}_{-0.051}$	$1.06^{+0.10}_{-0.098}$	$0.56^{+0.35}_{-0.27}$	$-0.004^{+0.042}_{-0.041}$	$< 0.383$	$-0.53^{+0.61}_{-0.96}$	$< 1.30$	$4.66^{+2.3}_{-2.1}$	$2.50^{+2.3}_{-1.7}$
$e$ CDM Planck	$0.02239^{+0.00060}_{-0.00056}$	$0.1186^{+0.0071}_{-0.0068}$	$> 51.2$	$0.058^{+0.040}_{-0.043}$	$0.967^{+0.025}_{-0.025}$	$0.81^{+0.24}_{-0.26}$	$-0.003^{+0.020}_{-0.019}$	$< 0.183$	$-1.32^{+0.98}_{-0.85}$	$< 0.959$	$3.08^{+0.57}_{-0.51}$	$1.21^{+0.27}_{-0.24}$
$e$ CDM Planck+BAO	$0.02251^{+0.00056}_{-0.00052}$	$0.1185^{+0.0069}_{-0.0069}$	$68.4^{+4.3}_{-4.1}$	$0.058^{+0.041}_{-0.043}$	$0.972^{+0.024}_{-0.024}$	$0.781^{+0.065}_{-0.063}$	$-0.004^{+0.018}_{-0.018}$	$< 0.187$	$-1.04^{+0.20}_{-0.21}$	$< 0.534$	$3.11^{+0.52}_{-0.48}$	$1.20^{+0.19}_{-0.19}$
$e$ CDM Planck+lensing	$0.02214^{+0.00053}_{-0.00052}$	$0.1176^{+0.0069}_{-0.0066}$	$> 54.5$	$0.058^{+0.040}_{-0.042}$	$0.972^{+0.024}_{-0.024}$	$0.81^{+0.21}_{-0.24}$	$-0.005^{+0.018}_{-0.018}$	$< 0.178$	$-1.45^{+0.96}_{-0.83}$	$< 0.661$	$2.93^{+0.51}_{-0.48}$	$1.04^{+0.16}_{-0.15}$
$e$ CDM Planck+HST	$0.02239^{+0.00059}_{-0.00057}$	$0.1187^{+0.0072}_{-0.0070}$	$74.4^{+4.1}_{-4.1}$	$0.058^{+0.040}_{-0.042}$	$0.967^{+0.025}_{-0.025}$	$0.81^{+0.10}_{-0.11}$	$-0.003^{+0.020}_{-0.019}$	$< 0.186$	$-1.32^{+0.29}_{-0.31}$	$< 0.957$	$3.09^{+0.58}_{-0.55}$	$1.18^{+0.19}_{-0.18}$
$e$ CDM Planck+JLA	$0.02242^{+0.00058}_{-0.00056}$	$0.1188^{+0.0071}_{-0.0067}$	$67.4^{+4.1}_{-4.2}$	$0.058^{+0.040}_{-0.043}$	$0.968^{+0.025}_{-0.025}$	$0.759^{+0.088}_{-0.089}$	$-0.004^{+0.020}_{-0.019}$	$< 0.183$	$-1.06^{+0.13}_{-0.14}$	$< 0.854$	$3.10^{+0.57}_{-0.54}$	$1.20^{+0.19}_{-0.17}$
$e$ CDM Planck+WL	$0.02251^{+0.00056}_{-0.00055}$	$0.1188^{+0.0073}_{-0.0069}$	$> 54.2$	$< 0.0835$	$0.972^{+0.024}_{-0.024}$	$0.82^{+0.22}_{-0.25}$	$0.000^{+0.020}_{-0.019}$	$< 0.197$	$-1.41^{+0.98}_{-0.79}$	$< 0.974$	$3.16^{+0.58}_{-0.56}$	$1.24^{+0.23}_{-0.22}$
$e$ CDM Planck+BAO-RSD	$0.02253^{+0.00052}_{-0.00050}$	$0.1184^{+0.0069}_{-0.0067}$	$68.6^{+4.2}_{-3.9}$	$0.056^{+0.038}_{-0.042}$	$0.972^{+0.023}_{-0.023}$	$0.774^{+0.055}_{-0.058}$	$-0.004^{+0.018}_{-0.018}$	$< 0.188$	$-1.05^{+0.17}_{-0.19}$	$< 0.626$	$3.12^{+0.51}_{-0.48}$	$1.22^{+0.18}_{-0.17}$
$e$ CDM Planck+BKP	$0.02237^{+0.00057}_{-0.00056}$	$0.1186^{+0.0072}_{-0.0069}$	$> 52.3$	$0.058^{+0.039}_{-0.044}$	$0.966^{+0.026}_{-0.026}$	$0.81^{+0.23}_{-0.25}$	$-0.003^{+0.019}_{-0.018}$	$< 0.101$	$-1.31^{+0.96}_{-0.89}$	$< 0.876$	$3.07^{+0.57}_{-0.55}$	$1.20^{+0.24}_{-0.22}$

6 parameters in  $\Lambda$ CDM

12 parameters space

In this Table we show for comparison the constraints obtained assuming the standard, **6 parameters in  $\Lambda$ CDM**, and in **our extended 12 parameters space**.

# Beyond six parameters: extending $\Lambda$ CDM

Model Dataset	$\Omega_b h^2$	$\Omega_c h^2$	$H_0$ [km/s/Mpc]	$\tau$	$n_s$	$\sigma_8$	$\frac{dn_s}{dn_k}$	$r$	$w$	$\Sigma m_\nu$ [eV]	$N_{\text{eff}}$	$A_{\text{lens}}$
$\Lambda$ CDM Planck TT+LowP	$0.02222^{+0.00046}_{-0.00044}$	$0.1198^{+0.0042}_{-0.0043}$	$67.3^{+2.0}_{-1.8}$	$0.077^{+0.038}_{-0.036}$	$0.966^{+0.012}_{-0.012}$	$0.829^{+0.028}_{-0.028}$	-	-	-	-	-	-
$\Lambda$ CDM Planck	$0.02226^{+0.00031}_{-0.00029}$	$0.1198^{+0.0028}_{-0.0028}$	$67.3^{+1.3}_{-1.3}$	$0.079^{+0.034}_{-0.035}$	$0.9646^{+0.0092}_{-0.0092}$	$0.831^{+0.026}_{-0.026}$	-	-	-	-	-	-
$\Lambda$ CDM Planck+ BAO	$0.02229^{+0.00028}_{-0.00027}$	$0.1193^{+0.0021}_{-0.0020}$	$67.52^{+0.93}_{-0.93}$	$0.082^{+0.031}_{-0.032}$	$0.9662^{+0.0078}_{-0.0079}$	$0.832^{+0.025}_{-0.025}$	-	-	-	-	-	-
$e$ CDM Planck TT+LowP	$0.0245^{+0.0024}_{-0.0022}$	$0.127^{+0.017}_{-0.016}$	$> 43$	$0.073^{+0.051}_{-0.051}$	$1.06^{+0.10}_{-0.098}$	$0.56^{+0.35}_{-0.27}$	$-0.004^{+0.042}_{-0.041}$	$< 0.383$	$-0.53^{+0.61}_{-0.96}$	$< 1.30$	$4.66^{+2.3}_{-2.1}$	$2.50^{+2.3}_{-1.7}$
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$e$ CDM Planck+BAO	$0.02251^{+0.00056}_{-0.00052}$	$0.1185^{+0.0069}_{-0.0069}$	$68.4^{+4.3}_{-4.1}$	$0.058^{+0.041}_{-0.043}$	$0.972^{+0.024}_{-0.024}$	$0.781^{+0.065}_{-0.063}$	$-0.004^{+0.018}_{-0.018}$	$< 0.187$	$-1.04^{+0.20}_{-0.21}$	$< 0.957$	$3.09^{+0.58}_{-0.55}$	$1.20^{+0.19}_{-0.19}$
$e$ CDM Planck+lensing	$0.02214^{+0.00053}_{-0.00052}$	$0.1176^{+0.0069}_{-0.0066}$	$> 54.5$	$0.058^{+0.040}_{-0.043}$	$0.959^{+0.024}_{-0.024}$	$0.85^{+0.21}_{-0.24}$	$-0.005^{+0.018}_{-0.018}$	$< 0.1$	$-1.06^{+0.13}_{-0.14}$	$< 0.854$	$3.10^{+0.57}_{-0.54}$	$1.04^{+0.16}_{-0.15}$
$e$ CDM Planck+HST	$0.02239^{+0.00059}_{-0.00057}$	$0.1187^{+0.0072}_{-0.0070}$	$74.4^{+5.1}_{-5.1}$	$0.057^{+0.040}_{-0.045}$	$0.966^{+0.025}_{-0.025}$	$0.81^{+0.10}_{-0.11}$	$-0.003^{+0.020}_{-0.019}$	$< 0.1$	$-1.06^{+0.13}_{-0.14}$	$< 0.854$	$3.09^{+0.58}_{-0.55}$	$1.18^{+0.19}_{-0.18}$
$e$ CDM Planck+JLA	$0.02242^{+0.00058}_{-0.00056}$	$0.1188^{+0.0071}_{-0.0067}$	$67.4^{+4.4}_{-4.2}$	$0.058^{+0.040}_{-0.043}$	$0.968^{+0.025}_{-0.025}$	$0.759^{+0.088}_{-0.089}$	$-0.004^{+0.020}_{-0.019}$	$< 0.183$	$-1.06^{+0.13}_{-0.14}$	$< 0.854$	$3.10^{+0.57}_{-0.54}$	$1.20^{+0.19}_{-0.17}$
$e$ CDM Planck+WL	$0.02251^{+0.00056}_{-0.00055}$	$0.1188^{+0.0073}_{-0.0069}$	$> 54.2$	$< 0.0835$	$0.972^{+0.024}_{-0.024}$	$0.82^{+0.22}_{-0.25}$	$0.000^{+0.020}_{-0.019}$	$< 0.197$	$-1.41^{+0.98}_{-0.79}$	$< 0.974$	$3.16^{+0.58}_{-0.56}$	$1.24^{+0.23}_{-0.22}$
$e$ CDM Planck+BAO-RSD	$0.02253^{+0.00052}_{-0.00050}$	$0.1184^{+0.0069}_{-0.0067}$	$68.6^{+4.2}_{-3.9}$	$0.056^{+0.038}_{-0.042}$	$0.972^{+0.023}_{-0.023}$	$0.774^{+0.055}_{-0.058}$	$-0.004^{+0.018}_{-0.018}$	$< 0.188$	$-1.05^{+0.17}_{-0.19}$	$< 0.626$	$3.12^{+0.51}_{-0.48}$	$1.22^{+0.18}_{-0.17}$
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Extensions

The significant increase in the number of parameters produces, as expected, a **relaxation in the constraints on the 6  $\Lambda$ CDM parameters**. It is impressive that despite the increase in the number of the parameters, some of the constraints on key parameters are relaxed **but not significantly altered**. The cold dark matter ansatz remains robust and the baryon density is compatible with BBN predictions.

# Beyond six parameters: extending $\Lambda$ CDM

Model Dataset	$\Omega_b h^2$	$\Omega_c h^2$	$H_0$ [km/s/Mpc]	$\tau$	$n_s$	$\sigma_8$	$\frac{dn_s}{dn_k}$	$r$	$w$	$\Sigma m_\nu$ [eV]	$N_{\text{eff}}$	$A_{\text{lens}}$
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$\Lambda$ CDM Planck	$0.02226^{+0.00031}_{-0.00029}$	$0.1198^{+0.0028}_{-0.0028}$	$67.3^{+1.3}_{-1.3}$	$0.079^{+0.034}_{-0.035}$	$0.9646^{+0.0092}_{-0.0092}$	$0.831^{+0.026}_{-0.026}$	-	-	-	-	-	-
$\Lambda$ CDM Planck+ BAO	$0.02229^{+0.00028}_{-0.00027}$	$0.1193^{+0.0021}_{-0.0020}$	$67.52^{+0.93}_{-0.93}$	$0.082^{+0.031}_{-0.032}$	$0.9662^{+0.0078}_{-0.0079}$	$0.832^{+0.025}_{-0.025}$	-	-	-	-	-	-
$e$ CDM Planck TT+LowP	$0.0245^{+0.0024}_{-0.0022}$	$0.127^{+0.017}_{-0.016}$	> 43	$0.073^{+0.051}_{-0.051}$	$1.06^{+0.10}_{-0.098}$	$0.56^{+0.35}_{-0.27}$	$-0.004^{+0.042}_{-0.041}$	< 0.383	$-0.53^{+0.61}_{-0.91}$	< 1.30	$4.66^{+2.3}_{-2.1}$	$2.50^{+2.3}_{-1.7}$
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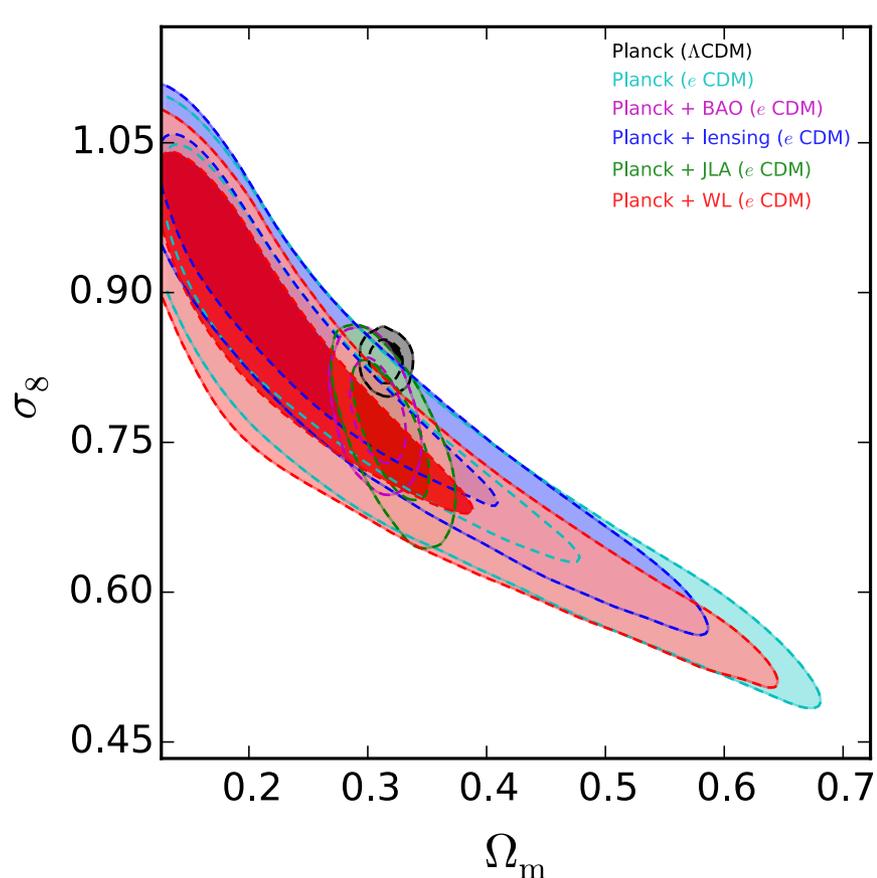
We see no evidence for "new physics": we just have (weaker) upper limits on the neutrino mass, the running of the spectral index is compatible with zero, the dark energy equation of state is compatible with  $w = -1$ , and the neutrino effective number is remarkably close to the standard value  $N_{\text{eff}} = 3.046$ .

# Beyond six parameters: extending $\Lambda$ CDM

Model Dataset	$\Omega_b h^2$	$\Omega_c h^2$	$H_0$ [km/s/Mpc]	$\tau$	$n_s$	$\sigma_8$	$\frac{dn_s}{dlnk}$	$r$	$w$	$\Sigma m_\nu$ [eV]	$N_{\text{eff}}$	$A_{\text{lens}}$
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$\Lambda$ CDM Planck	$0.02226^{+0.00031}_{-0.00029}$	$0.1198^{+0.0028}_{-0.0028}$	$67.3^{+1.3}_{-1.3}$	$0.079^{+0.034}_{-0.035}$	$0.9646^{+0.0092}_{-0.0092}$	$0.831^{+0.026}_{-0.026}$	-	-	-	-	-	-
$\Lambda$ CDM Planck+ BAO	$0.02229^{+0.00028}_{-0.00027}$	$0.1193^{+0.0021}_{-0.0020}$	$67.52^{+0.93}_{-0.93}$	$0.082^{+0.031}_{-0.032}$	$0.9662^{+0.0078}_{-0.0079}$	$0.832^{+0.025}_{-0.025}$	-	-	-	-	-	-
$e$ CDM Planck TT+LowP	$0.0245^{+0.0024}_{-0.0022}$	$0.127^{+0.017}_{-0.016}$	$> 43$	$0.073^{+0.051}_{-0.051}$	$1.06^{+0.10}_{-0.098}$	$0.56^{+0.35}_{-0.27}$	$-0.004^{+0.042}_{-0.041}$	$< 0.383$	$-0.53^{+0.61}_{-0.96}$	$< 1.30$	$4.66^{+2.3}_{-2.1}$	$2.50^{+2.3}_{-1.7}$
$e$ CDM Planck	$0.02239^{+0.00060}_{-0.00056}$	$0.1186^{+0.0071}_{-0.0068}$	$> 51.2$	$0.058^{+0.040}_{-0.043}$	$0.967^{+0.025}_{-0.025}$	$0.81^{+0.24}_{-0.26}$	$-0.003^{+0.020}_{-0.019}$	$< 0.183$	$-1.32^{+0.98}_{-0.85}$	$< 0.959$	$3.08^{+0.57}_{-0.51}$	$1.21^{+0.27}_{-0.24}$
$e$ CDM Planck+BAO	$0.02251^{+0.00056}_{-0.00052}$	$0.1185^{+0.0069}_{-0.0069}$	$68.4^{+4.3}_{-4.1}$	$0.058^{+0.041}_{-0.043}$	$0.972^{+0.024}_{-0.024}$	$0.781^{+0.065}_{-0.063}$	$-0.004^{+0.018}_{-0.018}$	$< 0.187$	$-1.04^{+0.20}_{-0.21}$	$< 0.534$	$3.11^{+0.52}_{-0.48}$	$1.20^{+0.19}_{-0.19}$
$e$ CDM Planck+lensing	$0.02214^{+0.00053}_{-0.00052}$	$0.1176^{+0.0069}_{-0.0066}$	$> 54.5$	$0.058^{+0.040}_{-0.043}$	$0.959^{+0.024}_{-0.024}$	$0.85^{+0.21}_{-0.24}$	$-0.005^{+0.018}_{-0.018}$	$< 0.178$	$-1.45^{+0.96}_{-0.83}$	$< 0.661$	$2.93^{+0.51}_{-0.48}$	$1.04^{+0.16}_{-0.15}$
$e$ CDM Planck+HST	$0.02239^{+0.00059}_{-0.00057}$	$0.1187^{+0.0072}_{-0.0070}$	$74.4^{+5.1}_{-5.1}$	$0.057^{+0.040}_{-0.045}$	$0.966^{+0.025}_{-0.025}$	$0.81^{+0.10}_{-0.11}$	$-0.003^{+0.020}_{-0.019}$	$< 0.186$	$-1.32^{+0.29}_{-0.31}$	$< 0.957$	$3.09^{+0.58}_{-0.55}$	$1.18^{+0.19}_{-0.18}$
$e$ CDM Planck+JLA	$0.02242^{+0.00058}_{-0.00056}$	$0.1188^{+0.0071}_{-0.0067}$	$67.4^{+4.4}_{-4.2}$	$0.058^{+0.040}_{-0.043}$	$0.968^{+0.025}_{-0.025}$	$0.759^{+0.088}_{-0.089}$	$-0.004^{+0.020}_{-0.019}$	$< 0.183$	$-1.06^{+0.13}_{-0.14}$	$< 0.854$	$3.10^{+0.57}_{-0.54}$	$1.20^{+0.19}_{-0.17}$
$e$ CDM Planck+WL	$0.02251^{+0.00056}_{-0.00055}$	$0.1188^{+0.0073}_{-0.0069}$	$> 54.2$	$< 0.0835$	$0.972^{+0.024}_{-0.024}$	$0.82^{+0.22}_{-0.25}$	$0.000^{+0.020}_{-0.019}$	$< 0.197$	$-1.41^{+0.98}_{-0.79}$	$< 0.974$	$3.16^{+0.58}_{-0.56}$	$1.24^{+0.23}_{-0.22}$
$e$ CDM Planck+BAO-RSD	$0.02253^{+0.00052}_{-0.00050}$	$0.1184^{+0.0069}_{-0.0067}$	$68.6^{+4.2}_{-3.9}$	$0.056^{+0.038}_{-0.042}$	$0.972^{+0.023}_{-0.023}$	$0.774^{+0.055}_{-0.058}$	$-0.004^{+0.018}_{-0.018}$	$< 0.188$	$-1.05^{+0.17}_{-0.19}$	$< 0.626$	$3.12^{+0.51}_{-0.48}$	$1.22^{+0.18}_{-0.17}$
$e$ CDM Planck+BKP	$0.02237^{+0.00057}_{-0.00056}$	$0.1186^{+0.0072}_{-0.0069}$	$> 52.3$	$0.058^{+0.039}_{-0.044}$	$0.966^{+0.026}_{-0.026}$	$0.81^{+0.23}_{-0.25}$	$-0.003^{+0.019}_{-0.018}$	$< 0.101$	$-1.31^{+0.96}_{-0.89}$	$< 0.876$	$3.07^{+0.57}_{-0.55}$	$1.20^{+0.24}_{-0.22}$

We find a **relaxed value for the Hubble constant**, with respect to the one derived under the assumption of  $\Lambda$ CDM. The main reason for this relaxation is the inclusion in the analysis of the dark energy equation of state  $w$ , that introduces a geometrical degeneracy with the matter density and the Hubble constant. In this way, we can solve the existing tensions with the direct measurements.

# Beyond six parameters: extending $\Lambda$ CDM

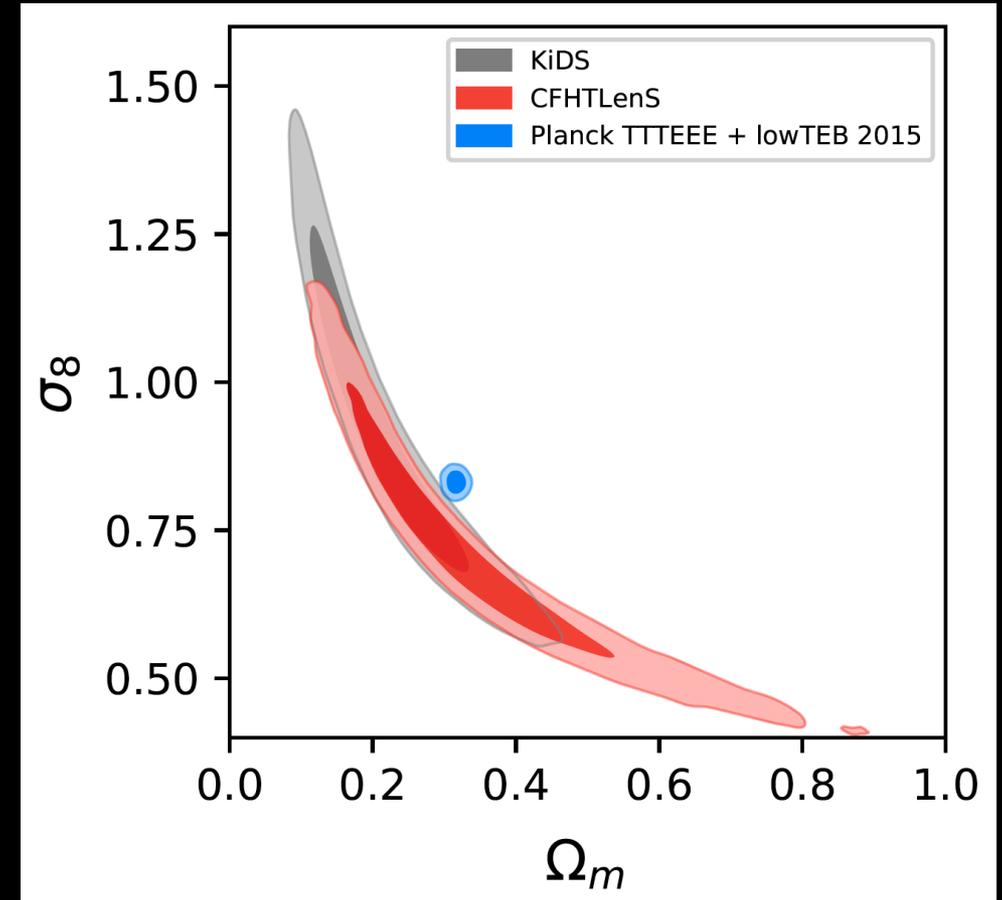
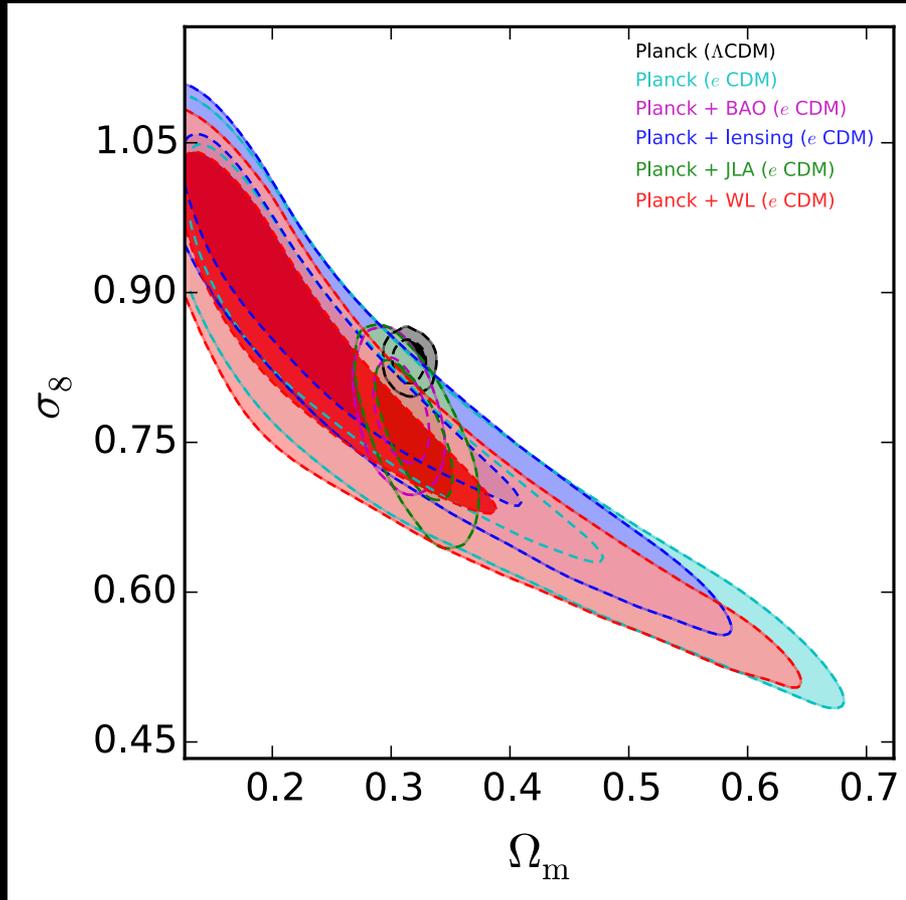


	$n_s$	$\sigma_8$	$\frac{dn_s}{dnk}$	$r$	$w$	$\Sigma m_\nu [eV]$	$N_{\text{eff}}$	$A_{\text{lens}}$
Planck ( $\Lambda$ CDM)	$0.966^{+0.012}_{-0.012}$	$0.829^{+0.028}_{-0.028}$	-	-	-	-	-	-
Planck (e CDM)	$0.9646^{+0.0092}_{-0.0092}$	$0.831^{+0.026}_{-0.026}$	-	-	-	-	-	-
Planck + BAO (e CDM)	$0.9662^{+0.0078}_{-0.0079}$	$0.832^{+0.025}_{-0.025}$	-	-	-	-	-	-
Planck + lensing (e CDM)	$1.06^{+0.10}_{-0.098}$	$0.56^{+0.35}_{-0.27}$	$-0.004^{+0.042}_{-0.041}$	$< 0.383$	$-0.53^{+0.61}_{-0.96}$	$< 1.30$	$4.66^{+2.3}_{-2.1}$	$2.50^{+2.3}_{-1.7}$
Planck + JLA (e CDM)	$0.967^{+0.025}_{-0.025}$	$0.81^{+0.24}_{-0.26}$	$-0.003^{+0.020}_{-0.019}$	$< 0.183$	$-1.32^{+0.98}_{-0.85}$	$< 0.959$	$3.08^{+0.57}_{-0.51}$	$1.21^{+0.27}_{-0.24}$
Planck + WL (e CDM)	$0.972^{+0.024}_{-0.024}$	$0.781^{+0.065}_{-0.063}$	$-0.004^{+0.018}_{-0.018}$	$< 0.187$	$-1.04^{+0.20}_{-0.21}$	$< 0.534$	$3.11^{+0.52}_{-0.48}$	$1.20^{+0.19}_{-0.19}$
Planck + JLA + WL (e CDM)	$0.959^{+0.024}_{-0.024}$	$0.85^{+0.21}_{-0.24}$	$-0.005^{+0.018}_{-0.018}$	$< 0.178$	$-1.45^{+0.96}_{-0.83}$	$< 0.661$	$2.93^{+0.51}_{-0.48}$	$1.04^{+0.16}_{-0.15}$
Planck + BAO + lensing (e CDM)	$0.966^{+0.025}_{-0.025}$	$0.81^{+0.10}_{-0.11}$	$-0.003^{+0.020}_{-0.019}$	$< 0.186$	$-1.32^{+0.29}_{-0.31}$	$< 0.957$	$3.09^{+0.58}_{-0.55}$	$1.18^{+0.19}_{-0.18}$
Planck + JLA + lensing (e CDM)	$0.968^{+0.025}_{-0.025}$	$0.759^{+0.088}_{-0.089}$	$-0.004^{+0.020}_{-0.019}$	$< 0.183$	$-1.06^{+0.13}_{-0.14}$	$< 0.854$	$3.10^{+0.57}_{-0.54}$	$1.20^{+0.19}_{-0.17}$
Planck + BAO + JLA (e CDM)	$0.972^{+0.024}_{-0.024}$	$0.82^{+0.22}_{-0.25}$	$0.000^{+0.020}_{-0.019}$	$< 0.197$	$-1.41^{+0.98}_{-0.79}$	$< 0.974$	$3.16^{+0.58}_{-0.56}$	$1.24^{+0.23}_{-0.22}$
Planck + BAO + lensing + JLA (e CDM)	$0.972^{+0.023}_{-0.023}$	$0.774^{+0.055}_{-0.058}$	$-0.004^{+0.018}_{-0.018}$	$< 0.188$	$-1.05^{+0.17}_{-0.19}$	$< 0.626$	$3.12^{+0.51}_{-0.48}$	$1.22^{+0.18}_{-0.17}$
Planck + BAO + lensing + JLA + WL (e CDM)	$0.966^{+0.026}_{-0.026}$	$0.81^{+0.23}_{-0.25}$	$-0.003^{+0.019}_{-0.018}$	$< 0.101$	$-1.31^{+0.96}_{-0.89}$	$< 0.876$	$3.07^{+0.57}_{-0.55}$	$1.20^{+0.24}_{-0.22}$

We find a relaxed and **lower value for the clustering parameter**, respect to the one derived under the assumption of  $\Lambda$ CDM.

# Beyond six parameters: extending $\Lambda$ CDM

Di Valentino et al. in preparation



$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$

In this way, we can solve the existing  $S_8$  tensions with the CFHTLenS and KiDS-450 cosmic shear surveys.

# Beyond six parameters: extending $\Lambda$ CDM

Model Dataset	$\Omega_b h^2$	$\Omega_c h^2$	$H_0$ [km/s/Mpc]	$\tau$	$n_s$	$\sigma_8$	$\frac{dn_s}{dn_k}$	$r$	$w$	$\Sigma m_\nu$ [eV]	$N_{\text{eff}}$	$A_{\text{lens}}$
$\Lambda$ CDM Planck TT+LowP	$0.02222^{+0.00046}_{-0.00044}$	$0.1198^{+0.0042}_{-0.0043}$	$67.3^{+2.0}_{-1.8}$	$0.077^{+0.038}_{-0.036}$	$0.966^{+0.012}_{-0.012}$	$0.829^{+0.028}_{-0.028}$	-	-	-	-	-	-
$\Lambda$ CDM Planck	$0.02226^{+0.00031}_{-0.00029}$	$0.1198^{+0.0028}_{-0.0028}$	$67.3^{+1.3}_{-1.3}$	$0.079^{+0.034}_{-0.035}$	$0.9646^{+0.0092}_{-0.0092}$	$0.831^{+0.026}_{-0.026}$	-	-	-	-	-	-
$\Lambda$ CDM Planck+ BAO	$0.02229^{+0.00028}_{-0.00027}$	$0.1193^{+0.0021}_{-0.0020}$	$67.52^{+0.93}_{-0.93}$	$0.082^{+0.031}_{-0.032}$	$0.9662^{+0.0078}_{-0.0079}$	$0.832^{+0.025}_{-0.025}$	-	-	-	-	-	-
$e$ CDM Planck TT+LowP	$0.0245^{+0.0024}_{-0.0022}$	$0.127^{+0.017}_{-0.016}$	> 43	$0.073^{+0.051}_{-0.051}$	$1.06^{+0.10}_{-0.098}$	$0.56^{+0.35}_{-0.27}$	$-0.004^{+0.042}_{-0.041}$	< 0.383	$-0.53^{+0.61}_{-0.96}$	< 1.30	$4.66^{+2.3}_{-2.1}$	$2.50^{+2.3}_{-1.7}$
$e$ CDM Planck	$0.02239^{+0.00060}_{-0.00056}$	$0.1186^{+0.0071}_{-0.0068}$	> 51.2	$0.058^{+0.040}_{-0.043}$	$0.967^{+0.025}_{-0.025}$	$0.81^{+0.24}_{-0.26}$	$-0.003^{+0.020}_{-0.019}$	< 0.183	$-1.32^{+0.98}_{-0.85}$	< 0.959	$3.08^{+0.57}_{-0.51}$	$1.21^{+0.27}_{-0.19}$
$e$ CDM Planck+BAO	$0.02251^{+0.00056}_{-0.00052}$	$0.1185^{+0.0069}_{-0.0069}$	$68.4^{+4.3}_{-4.1}$	$0.058^{+0.041}_{-0.043}$	$0.972^{+0.024}_{-0.024}$	$0.781^{+0.065}_{-0.063}$	$-0.004^{+0.018}_{-0.018}$	< 0.187	$-1.04^{+0.20}_{-0.21}$	< 0.534	$3.11^{+0.52}_{-0.48}$	$1.20^{+0.19}_{-0.19}$
$e$ CDM Planck+lensing	$0.02214^{+0.00053}_{-0.00052}$	$0.1176^{+0.0069}_{-0.0066}$	> 54.5	$0.058^{+0.040}_{-0.043}$	$0.959^{+0.024}_{-0.024}$	$0.85^{+0.21}_{-0.24}$	$-0.005^{+0.018}_{-0.018}$	< 0.178	$-1.45^{+0.96}_{-0.83}$	< 0.661	$2.93^{+0.51}_{-0.48}$	$1.04^{+0.16}_{-0.15}$
$e$ CDM Planck+HST	$0.02239^{+0.00059}_{-0.00057}$	$0.1187^{+0.0072}_{-0.0070}$	$74.4^{+5.1}_{-5.1}$	$0.057^{+0.040}_{-0.045}$	$0.966^{+0.025}_{-0.025}$	$0.81^{+0.10}_{-0.11}$	$-0.003^{+0.020}_{-0.019}$	< 0.186	$-1.32^{+0.29}_{-0.31}$	< 0.957	$3.09^{+0.58}_{-0.55}$	$1.18^{+0.19}_{-0.18}$
$e$ CDM Planck+JLA	$0.02242^{+0.00058}_{-0.00056}$	$0.1188^{+0.0071}_{-0.0067}$	$67.4^{+4.4}_{-4.2}$	$0.058^{+0.040}_{-0.043}$	$0.968^{+0.025}_{-0.025}$	$0.759^{+0.088}_{-0.089}$	$-0.004^{+0.020}_{-0.019}$	< 0.183	$-1.06^{+0.13}_{-0.14}$	< 0.854	$3.10^{+0.57}_{-0.54}$	$1.20^{+0.19}_{-0.17}$
$e$ CDM Planck+WL	$0.02251^{+0.00056}_{-0.00055}$	$0.1188^{+0.0073}_{-0.0069}$	> 54.2	< 0.0835	$0.972^{+0.024}_{-0.024}$	$0.82^{+0.22}_{-0.25}$	$0.000^{+0.020}_{-0.019}$	< 0.197	$-1.41^{+0.98}_{-0.79}$	< 0.974	$3.16^{+0.58}_{-0.56}$	$1.24^{+0.23}_{-0.22}$
$e$ CDM Planck+BAO-RSD	$0.02253^{+0.00052}_{-0.00050}$	$0.1184^{+0.0069}_{-0.0067}$	$68.6^{+4.2}_{-3.9}$	$0.056^{+0.038}_{-0.042}$	$0.972^{+0.023}_{-0.023}$	$0.774^{+0.055}_{-0.058}$	$-0.004^{+0.018}_{-0.018}$	< 0.188	$-1.05^{+0.17}_{-0.19}$	< 0.626	$3.12^{+0.51}_{-0.48}$	$1.22^{+0.18}_{-0.17}$
$e$ CDM Planck+BKP	$0.02237^{+0.00057}_{-0.00056}$	$0.1186^{+0.0072}_{-0.0069}$	> 52.3	$0.058^{+0.039}_{-0.044}$	$0.966^{+0.026}_{-0.026}$	$0.81^{+0.23}_{-0.25}$	$-0.003^{+0.019}_{-0.018}$	< 0.101	$-1.31^{+0.96}_{-0.89}$	< 0.876	$3.07^{+0.57}_{-0.55}$	$1.20^{+0.24}_{-0.22}$

And in fact, the only notable exception is the angular power spectrum lensing amplitude,  $A_L$  that is larger than the expected value at more than two standard deviations even when combining the Planck data with BAO and supernovae type Ia external datasets.

## Planck 2018

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	$0.02246 \pm 0.00028$	$0.02248^{+0.00028}_{-0.00032}$	$0.02228 \pm 0.00026$	$0.02264 \pm 0.00026$	$0.02250 \pm 0.00028$
$\Omega_c h^2$	$0.1172 \pm 0.0033$	$0.1174 \pm 0.0035$	$0.1164 \pm 0.0033$	$0.1175 \pm 0.0033$	$0.1174^{+0.0031}_{-0.0035}$
$100\theta_{MC}$	$1.04112 \pm 0.00051$	$1.04111 \pm 0.00052$	$1.04119 \pm 0.00050$	$1.04120 \pm 0.00049$	$1.04111 \pm 0.00050$
$\tau$	$0.0496 \pm 0.0086$	$0.0508 \pm 0.0091$	$0.0494^{+0.0086}_{-0.0076}$	$0.0502 \pm 0.0087$	$0.0499^{+0.0086}_{-0.0078}$
$\Sigma m_\nu$ [eV]	$< 0.863$	$< 0.821$	$< 0.714$	$< 0.352$	$< 0.822$
$w$	$-1.27 \pm 0.53$	$-1.33^{+0.17}_{-0.11}$	$-1.33 \pm 0.52$	$-1.009^{+0.092}_{-0.070}$	$-1.071^{+0.073}_{-0.050}$
$N_{\text{eff}}$	$2.95 \pm 0.24$	$2.97 \pm 0.26$	$2.85 \pm 0.23$	$3.04 \pm 0.23$	$2.98^{+0.23}_{-0.25}$
$A_L$	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	$1.116^{+0.061}_{-0.096}$	$1.213^{+0.076}_{-0.088}$	$1.232 \pm 0.090$
$\ln(10^{10} A_s)$	$3.027 \pm 0.020$	$3.030 \pm 0.022$	$3.024 \pm 0.020$	$3.030 \pm 0.020$	$3.028^{+0.020}_{-0.018}$
$n_s$	$0.964 \pm 0.012$	$0.965 \pm 0.013$	$0.958 \pm 0.012$	$0.971 \pm 0.012$	$0.965 \pm 0.012$
$\alpha_S$	$-0.0053 \pm 0.0085$	$-0.0047 \pm 0.0082$	$-0.0066 \pm 0.0082$	$-0.0041 \pm 0.0081$	$-0.0049 \pm 0.0086$
$H_0$ [km/s/Mpc]	$73^{+10}_{-20}$	$74.0 \pm 1.4$	$74^{+10}_{-20}$	$67.9 \pm 1.7$	$66.9 \pm 2.0$
$\sigma_8$	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	$0.80^{+0.15}_{-0.13}$	$0.782 \pm 0.025$	$0.750^{+0.055}_{-0.034}$
$S_8$	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	$0.757^{+0.047}_{-0.038}$	$0.791^{+0.025}_{-0.019}$	$0.775^{+0.036}_{-0.026}$

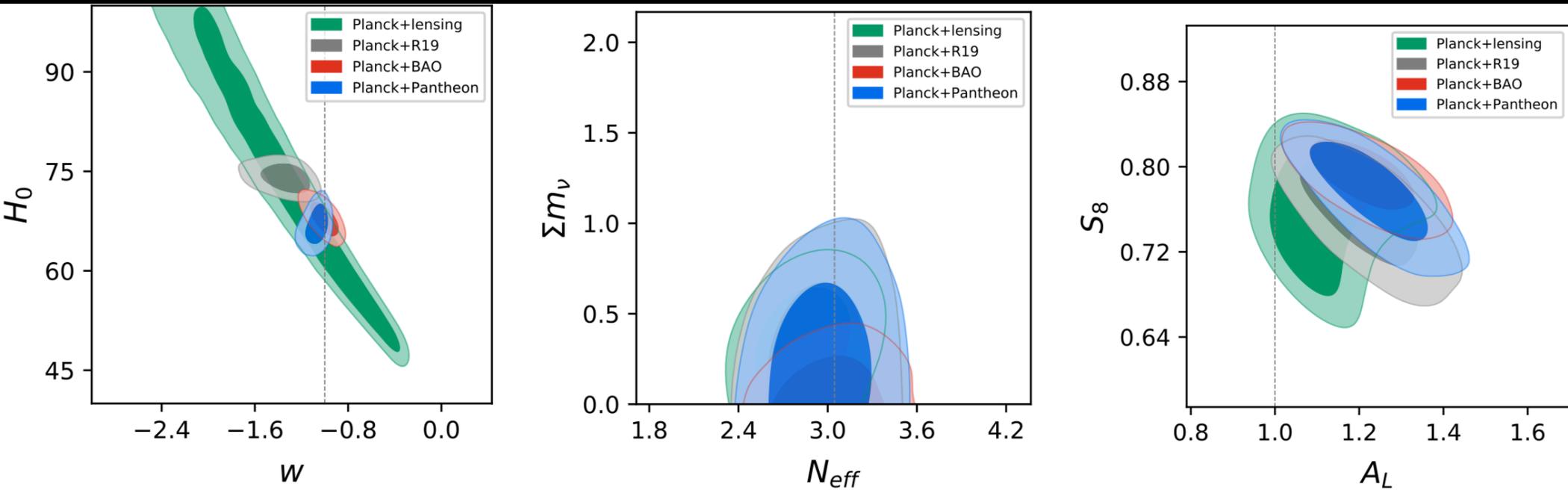
These results are completely confirmed by the Planck 2018 data: there is not evidence for new physics,  $H_0$  is almost unconstrained,  $\sigma_8$  and  $S_8$  shift towards lower values,  $A_L > 1$  at more than 2 standard deviations.

## Planck 2018

Parameters	Planck	Planck +R19	Planck +lensing	Planck +BAO	Planck + Pantheon
$\Omega_b h^2$	$0.02246 \pm 0.00028$	$0.02248^{+0.00028}_{-0.00032}$	$0.02228 \pm 0.00026$	$0.02264 \pm 0.00026$	$0.02250 \pm 0.00028$
$\Omega_c h^2$	$0.1172 \pm 0.0033$	$0.1174 \pm 0.0035$	$0.1164 \pm 0.0033$	$0.1175 \pm 0.0033$	$0.1174^{+0.0031}_{-0.0035}$
$100\theta_{MC}$	$1.04112 \pm 0.00051$	$1.04111 \pm 0.00052$	$1.04119 \pm 0.00050$	$1.04120 \pm 0.00049$	$1.04111 \pm 0.00050$
$\tau$	$0.0496 \pm 0.0086$	$0.0508 \pm 0.0091$	$0.0494^{+0.0086}_{-0.0076}$	$0.0502 \pm 0.0087$	$0.0499^{+0.0086}_{-0.0078}$
$\Sigma m_\nu$ [eV]	$< 0.863$	$< 0.821$	$< 0.714$	$< 0.352$	$< 0.822$
$w$	$-1.27 \pm 0.53$	$-1.33^{+0.17}_{-0.11}$	$-1.33 \pm 0.52$	$-1.009^{+0.092}_{-0.070}$	$-1.071^{+0.073}_{-0.050}$
$N_{\text{eff}}$	$2.95 \pm 0.24$	$2.97 \pm 0.26$	$2.85 \pm 0.23$	$3.04 \pm 0.23$	$2.98^{+0.23}_{-0.25}$
$A_L$	$1.25^{+0.09}_{-0.14}$	$1.21^{+0.09}_{-0.10}$	$1.116^{+0.061}_{-0.096}$	$1.213^{+0.076}_{-0.088}$	$1.232 \pm 0.090$
$\ln(10^{10} A_s)$	$3.027 \pm 0.020$	$3.030 \pm 0.022$	$3.024 \pm 0.020$	$3.030 \pm 0.020$	$3.028^{+0.020}_{-0.018}$
$n_s$	$0.964 \pm 0.012$	$0.965 \pm 0.013$	$0.958 \pm 0.012$	$0.971 \pm 0.012$	$0.965 \pm 0.012$
$\alpha_S$	$-0.0053 \pm 0.0085$	$-0.0047 \pm 0.0082$	$-0.0066 \pm 0.0082$	$-0.0041 \pm 0.0081$	$-0.0049 \pm 0.0086$
$H_0$ [km/s/Mpc]	$73^{+10}_{-20}$	$74.0 \pm 1.4$	$74^{+10}_{-20}$	$67.9 \pm 1.7$	$66.9 \pm 2.0$
$\sigma_8$	$0.79^{+0.15}_{-0.13}$	$0.811^{+0.051}_{-0.035}$	$0.80^{+0.15}_{-0.13}$	$0.782 \pm 0.025$	$0.750^{+0.055}_{-0.034}$
$S_8$	$0.754^{+0.053}_{-0.041}$	$0.758^{+0.039}_{-0.027}$	$0.757^{+0.047}_{-0.038}$	$0.791^{+0.025}_{-0.019}$	$0.775^{+0.036}_{-0.026}$

Since now datasets are fully compatible, we combine Planck 2018 with R19 ( $H_0=74.03 \pm 1.42$  km/s/Mpc), in order to see which parameter is preferred by the data to solve the tension. **We find a phantom-like dark energy component with an equation of state  $w < -1$  at more than three standard deviations, while the neutrino effective number is fully compatible with standard expectations.**

# Planck 2018



However BAO and Pantheon are still in tension with R19, preferring a  $\Lambda$ CDM model.

Thanks to the anti correlation present between  $S_8$  and  $A_L$ , a value of  $A_L > 1$  shifts the  $S_8$  parameter to values more consistent with those recently determined by the KiDS-450 cosmic shear survey under  $\Lambda$ CDM.

# Summarising

Extended neutrino scenarios seem no more suitable for solving the  $H_0$  tension when the Planck polarisation is considered, but a phantom like dark energy equation of state is still OK.

Varying simultaneously 12 cosmological parameters, a higher value of  $H_0$  is naturally allowed, so we find that the tension is reduced with  $N_{\text{eff}}$  in very good agreement with the standard expectations, and  $w < -1$  at about  $2\sigma$ . Moreover, this extended scenario is fully compatible with cosmic shear data, but not with BAO and Supernovae.

We studied a simple IDE model that relieve the  $H_0$  tension hinting for an interaction different from zero at more than  $5\sigma$ . Even when BAO data are added in the analysis the Hubble constant tension is reduced at less than  $3\sigma$ .

We have an indication for a closed universe by Planck at more than 3 standard deviations, but this increases all the other tensions.

In order to have a new concordance model, next decade of experiments will be decisive.

Thank you!

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