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Cosmological constant, q–theory, and TeV-scale physics

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Introduction

The main **Cosmological Constant Problem** (CCP1) can be phrased as follows (see, e.g., [1] for a review):

why does the zero-point energy of the vacuum not produce naturally a large cosmological constant Λ *in the Einstein field equations?*

The magnitude of the problem is enormous:

$$|\Lambda^{\text{naive theory}}|/|\Lambda^{\text{experiment}}| \geq 10^{42} \,.$$

[1] S. Weinberg, RMP 61, 1 (1989).

Introduction

Indeed, it is known that QCD in the laboratory involves a vacuum energy density (e.g., gluon condensate) of order

$$\left|\epsilon_V^{\text{(QCD)}}\right| \sim \big(100\ \text{MeV}\big)^4 \sim 10^{32}\ \text{eV}^4\,.$$

Moreover, this energy density can be expected to change as the temperature T of the Universe drops,

$$\epsilon_V^{\text{(QCD)}} = \epsilon_V^{\text{(QCD)}}(T)\,.$$

How can it be that the Universe ends up with a vacuum energy density

$$\left|\Lambda^{\text{(obs)}}\right| \equiv \left|\epsilon_{\text{present}}\right| < 10^{-28}\ \text{g cm}^{-3} \sim 10^{-10}\ \text{eV}^4\,?$$

Here, there are 42 orders of magnitude to explain:

$$\left|\Lambda^{\text{(obs)}}/\epsilon_V^{\text{(QCD)}}\right| \leq 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,001\,.$$

Introduction

Even more CCPs after the discovery of the "accelerating Universe":

CCP1 – why $|\Lambda| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{UV}})^4$?

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\big|_{\text{present}} \sim 10^{-11}\ \text{eV}^4$?

Hundreds of papers have been published on CCP2.

But CCP1 needs to be <u>solved first</u> before CCP2 can even be <u>addressed</u>.

Introduction

Here, a brief review of a particular approach to CCP1, which goes under the name of **q–theory** [2,3].

Then, turn to CCP2, describe a possible mechanism, and discuss a hint for new **TeV–scale** physics [4–6].

Outline talk:

1.1 Basics of q–theory $\longleftarrow$ most important part of the talk

2.1 *Coup d'envoi*

2.2 Electroweak kick

2.3 Effective Λ and E_{ew}

2.4 Recap mechanism

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[2] F.R. Klinkhamer & G.E. Volovik, PRD 77, 085015 (2008), arXiv:0711.3170.

[3] F.R. Klinkhamer & G.E. Volovik, JETPL 91, 259 (2010), arXiv:0907.4887.

[4] F.R. Klinkhamer and G.E. Volovik, PRD 80, 083001 (2009), arXiv:0905.1919.

[5] F.R. Klinkhamer, PRD 82, 083006 (2010), arXiv:1001.1939.

[6] F.R. Klinkhamer, arXiv:1101.1281.

1.1 Basics of q–theory

Crucial insight [2]: *there is vacuum energy and vacuum energy.*

More specifically and introducing an appropriate notation:

the vacuum energy density $\boxed{\epsilon}$ appearing in the action need not be the same as the vacuum energy density $\boxed{\rho_V}$ in the Einstein field equations.

How can this happen concretely . . .

1.1 Basics of q–theory

Consider the full quantum vacuum to be a **self-sustained medium** (as is a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Then, consider **macroscopic** equations of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

An analogy: the mass density in liquids, which describes a microscopic quantity – the number density of atoms – but obeys the macroscopic equations of hydrodynamics, because of particle-number conservation.

However, is the quantum vacuum just like a normal fluid?

1.1 Basics of q–theory

No, as the vacuum is known to be **Lorentz invariant**
(cf. experimental limits at the 10^{-15} level in the photon sector [7–9]).

The Lorentz invariance of the vacuum rules out the standard type of charge density which arises from the time component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material fluids.

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[7] A. Kostelecký and M. Mewes, PRD 66, 056005 (2002), arXiv:hep-ph/0205211.

[8] F.R. Klinkhamer and M. Risse, PRD 77, 117901 (2008), arXiv:0709.2502

[9] F.R. Klinkhamer and M. Schreck, PRD 78, 085026 (2008), arXiv:0809.3217.

1.1 Basics of q–theory

With such a variable $q(x)$, the vacuum energy density of the effective action can be a generic function

$$\epsilon = \epsilon(q)\,, \tag{1}$$

which may include a constant term due to the zero-point energies of the fields of the Standard Model (SM), $\epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{var}}(q)$.

From ① thermodynamics and ② Lorentz invariance, it then follows that

$$P_V \overset{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \overset{②}{=} -\rho_V\,, \tag{2}$$

with the first equality corresponding to an integrated form of the Gibbs–Duhem equation (for chemical potential $\mu \equiv d\epsilon/dq$).

Recall GD-eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

1.1 Basics of q–theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{UV}})^4$, but they can cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence, for a generic function $\epsilon(q)$,

$$\exists\, q_0 \;:\quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\,\frac{d\,\epsilon(q)}{d\,q}\right]_{q=q_0} = 0\,, \tag{3}$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved, in principle ...

But, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q (see later).

1.1 Basics of q–theory

To summarize, the q–theory approach to the main Cosmological Constant Problem (CCP1) provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the “beyond-the-Standard-Model” physics harbors an appropriate q–type variable.

Still, better to have one possible solution than none.

(Two remarks in Appendix A.)

2.1 *Coup d'envoi*

Now, the remaining problems (or puzzles, rather):

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\big|_{\text{now}} \sim 10^{-29}$ g cm^{-3} $\sim 10^{-11}$ eV^4 ?

CCP2b also goes under the name of 'cosmic coincidence puzzle' (ccp).

Here, consider a possible realization of q operative at an UV (Planckian) energy scale.

In the very early Universe, the vacuum energy density $\rho_V(t)$ rapidly drops to zero and stays there, but small effects may occur at cosmic temperatures T of the order of the TeV scale . . .

2.2 Electroweak kick

Explicit realization of vacuum variable q via a 3–form gauge field A [10,11].

Effective action of GR+SM,

$$S^{\text{eff}}[g,\psi] = \int_{\mathbb{R}^4} d^4x\,\sqrt{-\det g}\,\Big(K_N\,R[g] + \Lambda_{\text{SM}} + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \tag{4}$$

with $K_N \equiv 1/(16\pi G_N)$ and $\hbar = c = 1$, is replaced by [3]

$$\widetilde{S}^{\text{eff}}[A,g,\psi] \;=\; \int_{\mathbb{R}^4} d^4x\,\sqrt{-\det g}\,\Big(K(q)\,R[g] + \widetilde{\epsilon}(q) + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \tag{5a}$$

$$q^2 \;\equiv\; -\frac{1}{24}\,F_{\alpha\beta\gamma\delta}\,F^{\alpha\beta\gamma\delta}\,. \tag{5b}$$

$$F_{\alpha\beta\gamma\delta} \;=\; \nabla_{[\alpha}A_{\beta\gamma\delta]}\,, \tag{5c}$$

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[10] M.J. Duff and P. van Nieuwenhuizen, PLB 94, 179 (1980).

[11] A. Aurilia, H. Nicolai, and P.K. Townsend, NPB 176, 509 (1980).

2.2 Electroweak kick

Then, variational principle produces generalized Einstein equations with a vacuum energy density term

$$\rho_V = \widetilde{\epsilon} - q\,\frac{d\,\widetilde{\epsilon}}{d\,q}\,, \tag{6}$$

which is precisely of the Gibbs–Duhem form (2). Technically, the extra term on the RHS of (6) appears because of the fact that $q = q(A,\, g)$.

Specifically, the generalized Einstein and Maxwell equations give:

$$\begin{aligned} 2K(q)\left(R_{\alpha\beta} - g_{\alpha\beta}\,R/2\right) &= -2\left(\nabla_\alpha\nabla_\beta - g_{\alpha\beta}\,\Box\right)K(q) \\ &\quad +\rho_V(q)\,g_{\alpha\beta} - T^M_{\alpha\beta}\,, \end{aligned} \tag{7a}$$

$$\frac{d\rho_V(q)}{dq} + R\,\frac{dK(q)}{dq} = 0\,. \tag{7b}$$

Eqs. (6)–(7) are generic, i.e., independent of scale and dimension of q.

2.2 Electroweak kick

Spatially-flat (F)RW universe with two types of matter, massive ('type 1') and massless ('type 2') particles. Resulting ODEs:

$$6\left(H\,\frac{dK}{dq}\,\frac{dq}{dt}+K\,H^2\right) = \rho_V+\rho_{M1}+\rho_{M2}\,, \tag{8a}$$

$$6\,\frac{dK}{dq}\left(\frac{dH}{dt}+2H^2\right) = \frac{d\rho_V}{dq}\,, \tag{8b}$$

$$\frac{d\rho_{M1}}{dt}+\left[4-\kappa_{M1}(t/t_{\text{ew}})\right]H\,\rho_{M1} = 0\,, \tag{8c}$$

$$\frac{d\rho_{M2}}{dt}+4\,H\,\rho_{M2} = 0\,, \tag{8d}$$

with prescribed equation-of-state (EOS) function $\kappa_{M1}(x)$ peaking at $x=1$.

2.2 Electroweak kick

Analytically, it has been shown [4] that there exists a solution which

- starts from a standard radiation-dominated FRW universe with $\rho_V = 0$,
- is perturbed around $t = t_{\text{ew}} \sim E_{\text{Planck}}/(E_{\text{ew}})^2$ with $\rho_V \neq 0$,
- resumes the standard radiation-dominated expansion with $\rho_V = 0$.

Specifically, the vacuum energy density for $t \sim t_{\text{ew}}$ is given by

$$\rho_V(t) \sim \kappa_{M1}^2(t)\, H(t)^4 \,, \tag{9}$$

which has a peak value of order $(t_{\text{ew}})^{-4} \sim \left((E_{\text{ew}})^2 / E_{\text{Planck}}\right)^4$ but vanishes as $t \to \infty$.

$\Rightarrow$ standard (nondissipative) dynamic equations of q–theory do not produce a constant $\rho_{V,\,\text{remnant}} > 0$ from the electroweak kick.

2.3 Effective Λ and E_{ew}

As argued in [4], quantum-dissipative effects of the vacuum energy density may lead to a finite remnant value of order

$$\Lambda \equiv \rho_{V,\,\text{remnant}} \sim \left((E_{\text{ew}})^2 / E_{\text{Planck}}\right)^4 \sim (10^{-3}\ \text{eV})^4\,, \tag{10}$$

for $E_{\text{ew}} \sim 1$ TeV and $E_{\text{Planck}} \sim 10^{15}$ TeV. In fact, expression (10) was already suggested by Arkani-Hamed, Hall, Kolda, and Murayama [12].

It is possible [5] to modify the “classical” q–theory equations (8) in such a way as to recover (10).
Even better, a simple field-theory model has been presented in [6].

Details for simple model [6] in Appendix C (skip Appendix B for [5]).

Here, focus on the physics implications.

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[12] N. Arkani-Hamed et al., PRL 85, 4434 (2000), arXiv:astro-ph/0005111.

2.3 Effective Λ and E_{ew}

Theoretical value of the effective cosmological constant given by

$$\Lambda^{\text{theory}} \equiv \lim_{t\to\infty} \rho_V^{\text{theory}}(t) = r_V^{\text{num}} \, (E_{\text{ew}})^8 \,/ (E_{\text{Planck}})^4 \,, \tag{11}$$

with a number $r_V^{\text{num}} \equiv r_V(\tau_{\text{freeze}})$ from the solution of the ODEs.

Equating this to the experimental value $\Lambda^{\text{exp}} \approx (2 \text{ meV})^4$ gives

$$E_{\text{ew}} = \left(\frac{\Lambda^{\text{exp}}}{r_V^{\text{num}}}\right)^{1/8} (E_{\text{Planck}})^{1/2} \approx 3.8 \text{ TeV} \left(\frac{0.013}{r_V^{\text{num}}}\right)^{1/8} . \tag{12}$$

Analytic bound: $r_V^{\text{num}} \lesssim 1 \Rightarrow E_{\text{ew}} \gtrsim 2$ TeV.

Numerical results for r_V^{num} give E_{ew} estimates of Table 1.

2.3 Effective Λ and E_{ew}

Table 1: Preliminary estimates [5] of the energy scale E_{ew} for hierarchy parameter $\xi \equiv (E_{\text{Planck}}/E_{\text{ew}})^4 \gg 1$. Both massive type–1 and massless type–2 particles are assumed to have been in thermal equilibrium before the "kick" and the number of type–2 particles is taken as $N_{\text{eff},2} = 10^2$.
Left: Prescribed kick with type–1 particles of equal mass $M = E_{\text{ew}}$ and, for dissipative coupling constant $\zeta = 2$, E_{ew} shown as a function of the effective number of d.o.f. $N_{\text{eff},1}$.
Right: Dynamic kick with case–A type–1 mass spectrum $(N_{1a},\, M_{1a}\,;\, N_{1b},\, M_{1b}) = (40,\, 2\times E_{\text{ew}};\, 60,\, 1/3\times E_{\text{ew}})$ and $E_{\text{ew}} =< M_{1i} >$ shown as a function of ζ.

ζ	$N_{\text{eff},1}$	E_{ew} [TeV]
2	1	8.5
2	10^1	4.9
2	10^2	$\boxed{3.2}$
2	10^3	2.8
2	10^4	2.7

ζ	$N_{\text{eff},1}$	E_{ew} [TeV]
0.2	10^2	14.8
2	10^2	$\boxed{3.8}$
20	10^2	5.6

2.4 Recap mechanism

- Presence of massive particles with electroweak interactions [average mass $< M > = E_{\text{ew}} \sim$ TeV] changes the expansion rate $H(t)$ of the Universe compared to the radiation-dominated case.
- Change of the expansion rate kicks $\rho_V(t)$ away from zero.
- Quantum-dissipative effects operating at cosmic time t_{ew} set by E_{ew} may result in a finite remnant value of ρ_V.
- Phenomenological description of this process with a simple field theoretic model.
- Required E_{ew} value ranges from 2 to 20 TeV, depending on the effective number of new particles and details of the model.

Conclusions

CCP1: self-adjustment of a special type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0$.

CCP2: finite remnant value of $\rho_V(t)$ may result from quantum-dissipative effects operating at a cosmic time t_{ew} set by the scale $E_{\text{ew}} \sim$ TeV of massive particles with $M \sim E_{\text{ew}}$ and electroweak interactions.

Hint: required E_{ew} value ranges from 2 to 20 TeV, which, if correct, implies new TeV–scale physics beyond the SM.

(+ Appendices for technical details.)

Appendix A: Two remarks

Two remarks [3]:

1. The adjustment-type solution (3) of the CCP1 circumvents Weinberg's no–go "theorem" [1].

   Crux: q is a non-fundamental scalar field (cf. theory of Sec. 2.2).

2. Next question is how the Universe got the right value q_0?

   Possible answer via a generalization of q–theory, for which the correct value q_0 arises dynamically (cf. brief summary below).

Appendix A: Two remarks

Realization of vacuum variable q by aether-type velocity field u_β [13,14], setting $E_{\text{UV}} = E_{\text{Planck}}$. For a flat FRW metric with cosmic time t, there is an asymptotic solution for $u_\beta = (u_0, u_b)$ and Hubble parameter $H(t)$:

$$u_0(t) \to q_0\, t\,, \quad u_b(t) = 0\,, \quad H(t) \to 1/t\,. \tag{A.1}$$

Define $v \equiv u_0/E_{\text{Planck}}$, $\tau \equiv t\, E_{\text{Planck}}$, $h \equiv H/E_{\text{Planck}}$, and $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$. Then, the field equations are [13]:

$$\ddot{v} + 3\,h\,\dot{v} - 3\,h^2\,v \;=\; 0\,, \tag{A.2a}$$

$$2\,\lambda - (\dot{v})^2 - 3\,(h\,v)^2 \;=\; 6\,h^2\,, \tag{A.2b}$$

with the overdot standing for differentiation with respect to τ.
Starting from a de-Sitter universe with $\lambda > 0$, there is a unique value of $\widehat{q}_0 \equiv q_0/(E_{\text{Planck}})^2$ to end up with a static Minkowski spacetime, $\widehat{q}_0 = \sqrt{\lambda/2}$.

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[13] A.D. Dolgov, PRD 55, 5881 (1997), arXiv:astro-ph/9608175.

[14] T. Jacobson, PoS QG-PH, 020 (2007), arXiv:0801.1547.

Appendix A: Two remarks

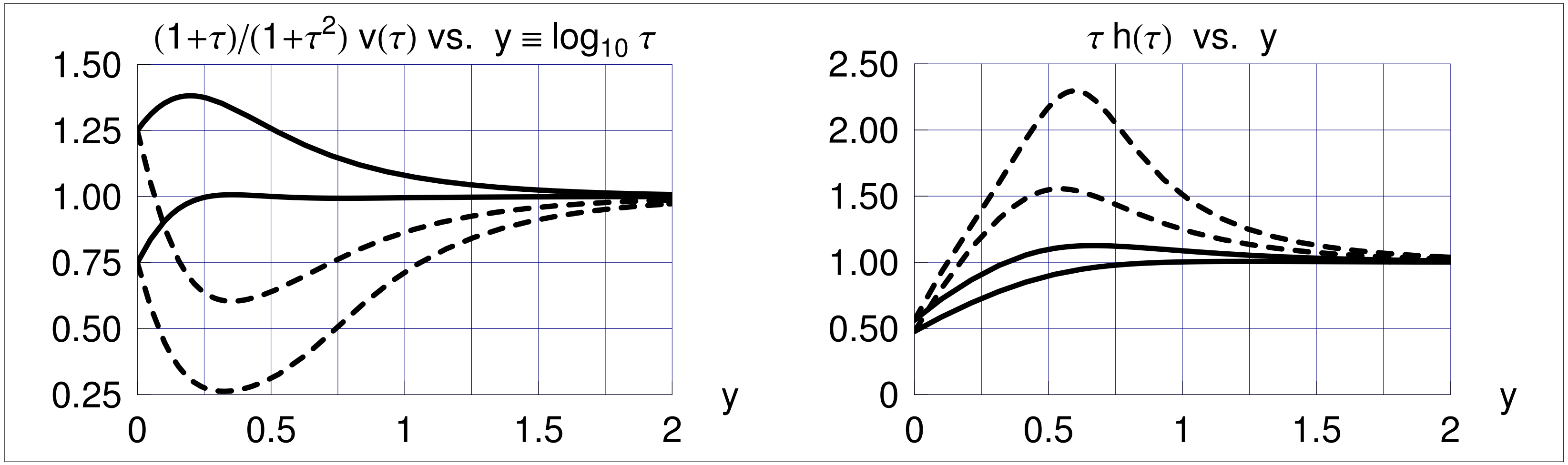

Fig. A1: Four numerical solutions of ODEs (A.2ab) for $\lambda = 2$ and boundary conditions $v(1) = 1 \pm 0.25$ and $\dot{v}(1) = \pm 1.25$.

$\Rightarrow$ Minkowski value $\widehat{q}_0 = \sqrt{\lambda/2} = 1$ arises <u>dynamically</u> [see left panel].

$\Rightarrow$ Minkowski spacetime is an <u>attractor</u>.

Appendix B: Model universe

Model universe with three components (see Appendix A of [5]):

0. Vacuum variable q entering the gravitational coupling $K(q)$.
1. Massive 'type 1' particles (subspecies $i = a, b, c, \ldots$) with masses M_i of order $E_{\text{ew}} \sim 1$ TeV and electroweak interactions.
2. Massless 'type 2' particles with electroweak interactions.

Now, proceed as follows:

- Consider a flat FRW universe with Hubble parameter $H(t)$.
- Allow for energy exchange between the two matter components, so that total type–1 energy density peaks around $t_{\text{ew}} \equiv E_{\text{Planck}}/(E_{\text{ew}})^2$.
- Get function $\overline{\kappa}_{M1i}(t)$ from EOS parameter $w_{M1i}(t)$, with $\overline{\kappa}_{M1i}(t) \sim 0$ for $t \ll t_{\text{ew}}$ in the ultrarelativistic regime.
- Introduce a dissipative coupling constant $\zeta = \text{O}(1)$ and a function $\gamma(t)$ which equals 1 for $t \ll t_{\text{ew}}$ and drops to zero for $t > t_{\text{ew}}$.

Appendix B: Model universe

Modified q–theory ODEs (standard ODEs recovered for $\zeta = 0$ and $\gamma = 1$):

$$6\,\left(H\,K'\,\dot{q} + K\,H^2\right) = \rho_V + \sum_{i=a,b,c,\ldots} \rho_{M1i} + \rho_{M2}\,, \tag{B.1a}$$

$$6\,K'\left(\dot{H} + 2H^2\right) = \gamma\,\rho_V' + (1-\gamma)\,\frac{K'}{K}\left[2\rho_V + \sum_i \tfrac{1}{2}\,\overline{\kappa}_{M1i}\,\rho_{M1i}\right], \tag{B.1b}$$

$$\dot{\rho}_{M1i} + (4-\overline{\kappa}_{M1i})H\rho_{M1i} = \frac{N_{1i}}{N_1}\left[\frac{\lambda_{21}}{t_{\text{ew}}}\widehat{\omega}\,\rho_{M2} - \frac{\zeta}{\gamma}q\,\dot{\rho}_V'\right] - \frac{\lambda_{12}}{t_{\text{ew}}}\widehat{\nu}\,\rho_{M1i}, \tag{B.1c}$$

$$\dot{\rho}_{M2} + 4\,H\,\rho_{M2} = -\frac{\lambda_{21}}{t_{\text{ew}}}\,\widehat{\omega}\,\rho_{M2} + \frac{\lambda_{12}}{t_{\text{ew}}}\,\widehat{\nu}\,\sum_i \rho_{M1i}\,, \tag{B.1d}$$

where the overdot [prime] stands for differentiation with respect to t [q]. Functions γ, $\widehat{\omega}$, and $\widehat{\nu}$ shown in Figs. B1–B4 below.

Appendix B: Model universe

Use simple *Ansätze*: $\rho_V(q) \propto (q - q_0)^2$ and $K(q) \propto q$.

With t_{ew} and $\xi \equiv (E_{\text{Planck}}/E_{\text{ew}})^4 \gg 1$, define dimensionless variables:

$$\tau \equiv (t_{\text{ew}})^{-1}\, t\,, \qquad h \equiv t_{\text{ew}}\, H\,, \tag{B.2a}$$

$$r_V \equiv (t_{\text{ew}})^4\, \rho_V\,, \qquad r_{Mn} \equiv \xi^{-1}\,(t_{\text{ew}})^4\, \rho_{Mn}\,, \tag{B.2b}$$

$$x \equiv \xi\,\big(q/q_0 - 1\big)\,. \tag{B.2c}$$

Figures B1–B3 and B4 show numerical results for $\xi = 10^2$ and $\xi = \infty$.

Appendix B: Model universe

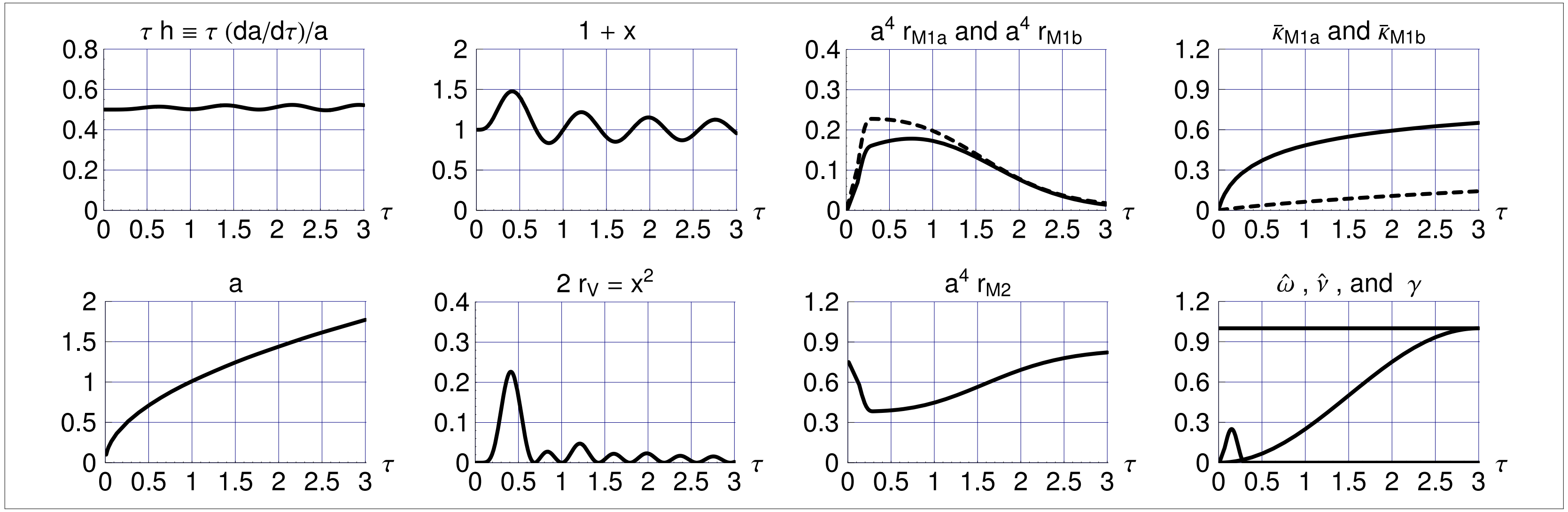

Fig. B1: Numerical solution [5] of standard (nondissipative) q–theory ODEs (B.1) for $\zeta = 0$ and $\gamma = 1$. The hierarchy parameter is $\xi = 10^2$ [oscillatory effects suppressed for larger values of ξ, recovering the smooth behavior of (9)]. Further coupling constants $\{\lambda_{21},\, \lambda_{12}\} = \{18,\, 2\}$ and case–A type–1 mass spectrum $(N_{1a},\, M_{1a}\,;\, N_{1b},\, M_{1b}) = (40,\, 2\, E_{\text{ew}};\, 60,\, 1/3\, E_{\text{ew}})$.

Appendix B: Model universe

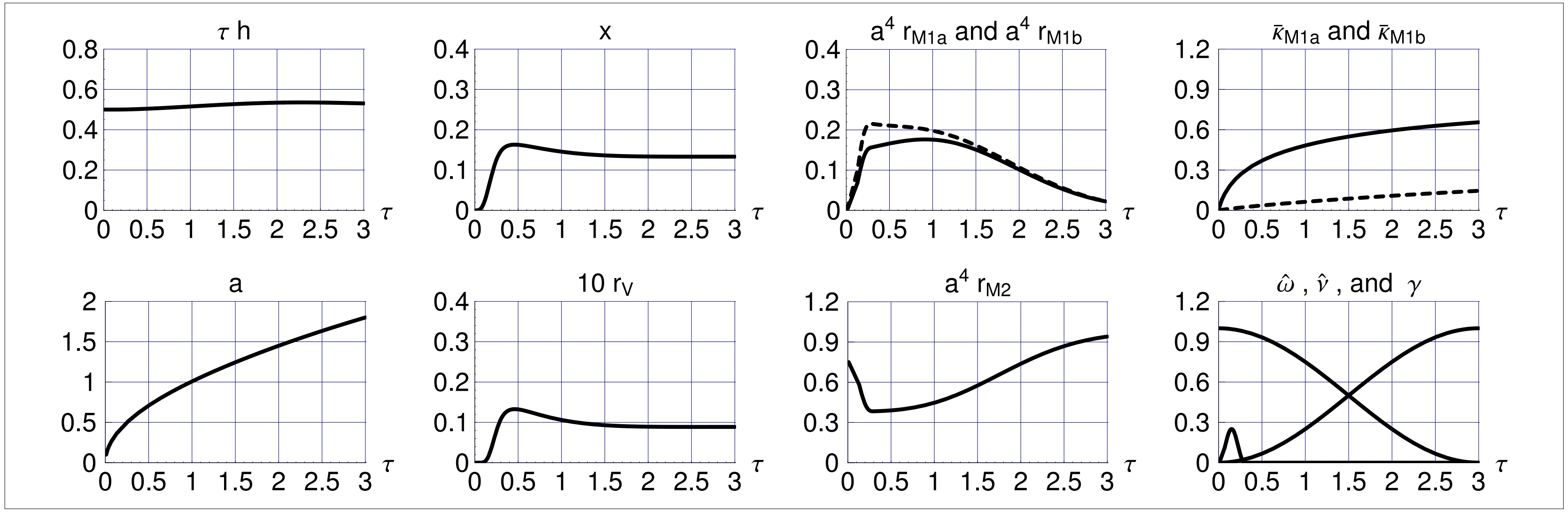

Fig. B2: Same as Fig. B1, but now for the <u>modified</u> q–theory ODEs (B.1) with dissipative coupling constant $\zeta = 2$ and $\gamma(\tau) = 0$ for $\tau \geq \tau_{\text{freeze}} = 3$.

Appendix B: Model universe

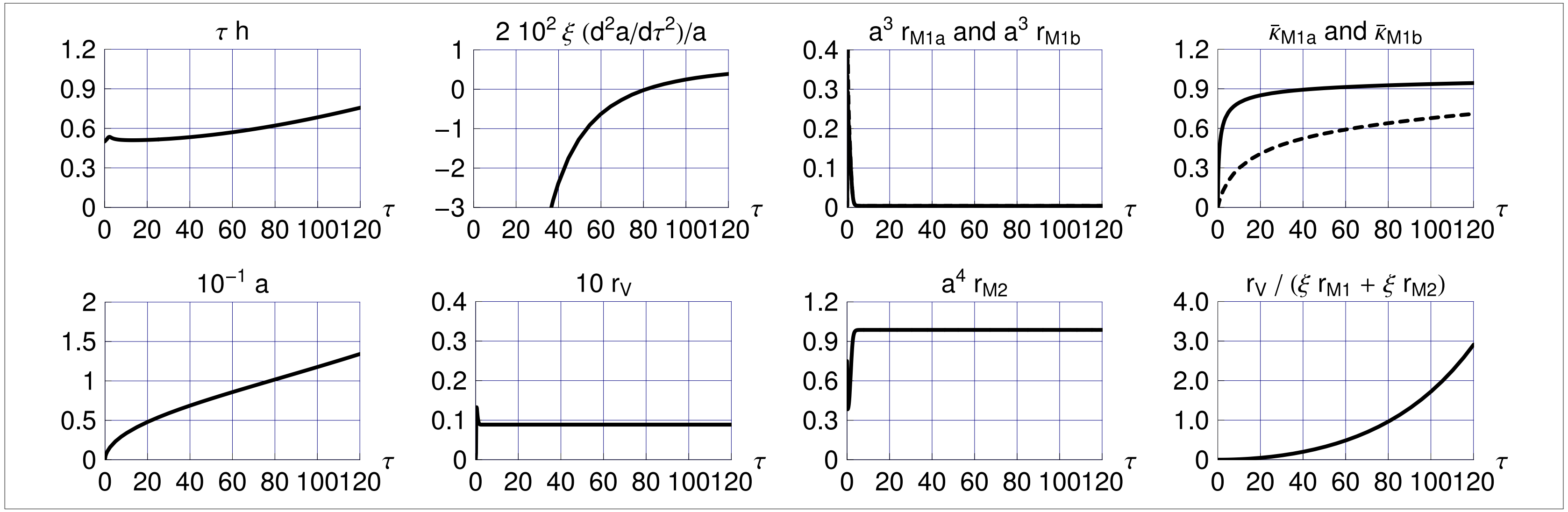

Fig. B3: Same as Fig. B2, but evolved further.

Appendix B: Model universe

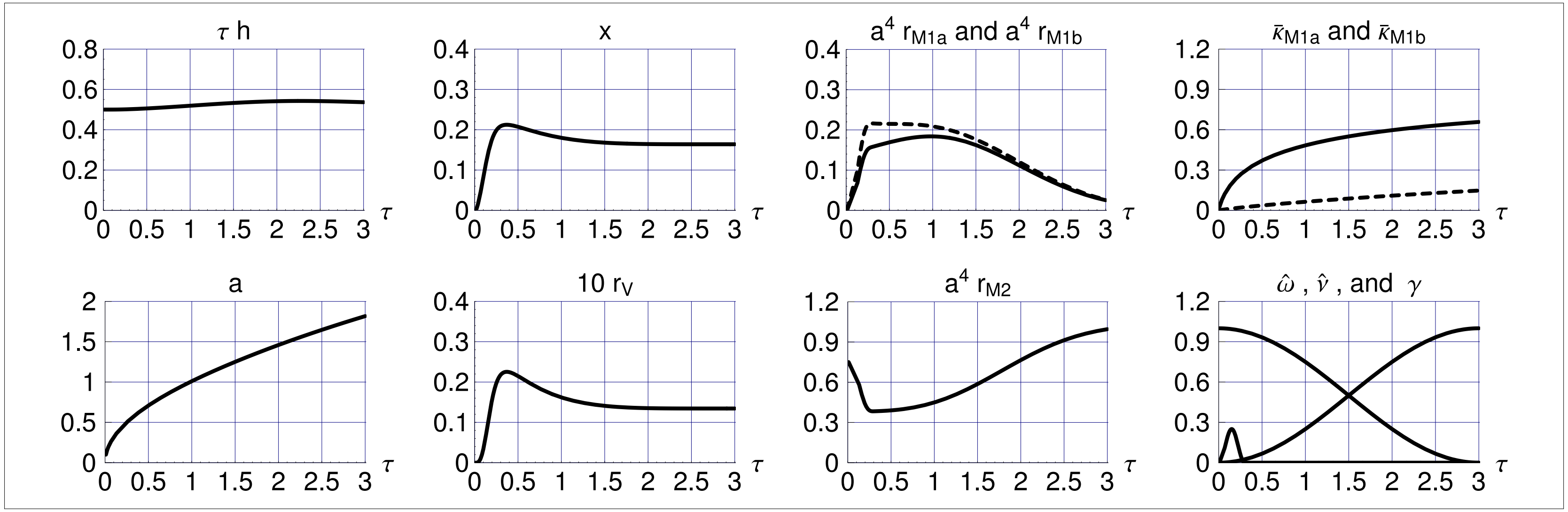

Fig. B4: Same as Fig. B2, but now for $\xi = \infty$.

Appendix C: Field-theoretic model

Simple field-theoretic model can generate an effective cosmological constant (remnant vacuum energy density) of order $\Lambda_{\text{eff}} \sim (\text{meV})^4$, from **new TeV–scale ultramassive particles with electroweak interactions**.

The model is **simple** in the sense that it involves only a few types of fields and two energy scales, E_{Planck} and E_{ew}.

Specifically, two types of scalars:

- ultramassive (type–1) fields ϕ_a for $a = 1, \ldots, N_1$;
- massless (type–2) fields ψ_b for $b = 1, \ldots, N_2$;
- take $N_1 \overset{①}{=} N_2 \overset{②}{=} 10^2$ from ② SM and ① SUSY?.

Basic model equations are ($\hbar = c = k = 1$; signature $-, +, +, +$):

Appendix C: Field-theoretic model

$$S_{\text{eff},\,T} = \int_{\mathbb{R}^4} d^4x\,\sqrt{-g}\,\Big(K_T(q)\,R[g] + \epsilon_V(q) + \mathcal{L}^M_{\text{eff},\,T}[\phi,\psi,g]\Big)\,, \tag{C.1a}$$

$$q \equiv -\frac{1}{24}\,\epsilon^{\alpha\beta\gamma\delta}\,\nabla_{[\alpha}A_{\beta\gamma\delta]}\,/\,\sqrt{-g}\,, \tag{C.1b}$$

$$\rho_V(q) \equiv \epsilon_V(q) - \mu_0\,q = \frac{1}{2}\,(q-q_0)^2\,, \tag{C.1c}$$

$$K_T(q) = \begin{cases} q/2 & \text{for} \quad T > T^{(+)}_{c,\,K}\,, \\ q_0/2 & \text{for} \quad T \leq T^{(+)}_{c,\,K}\,, \end{cases} \tag{C.1d}$$

$$q_0 = 1/(8\pi\,G_N) \equiv (E_{\text{Planck}})^2 \approx (2.44\times 10^{18}\ \text{GeV})^2\,. \tag{C.1e}$$

Appendix C: Field-theoretic model

$$\mathcal{L}^{M}_{\text{eff},\,T} = \frac{1}{2}\,\partial_\alpha\psi\,\cdot\,\partial^\alpha\psi + \frac{1}{2}\,\partial_\alpha\phi\,\cdot\,\partial^\alpha\phi + \frac{1}{2}\,M^2\,(\phi\cdot\phi) + g_T\,(\psi\cdot\psi)\,(\phi\cdot\phi)\,, \tag{C.2a}$$

$$g_T = \begin{cases} g_0\left(1-(T/T_{c,\,g})^2\right) & \text{for} \quad T \leq T_{c,\,g}\,, \\ 0 & \text{for} \quad T > T_{c,\,g}\,, \end{cases} \tag{C.2b}$$

$$M = E_{\text{ew}}\,, \tag{C.2c}$$

$$T_{c,\,g} = \mathsf{O}(E_{\text{ew}})\,. \tag{C.2d}$$

$$T_{c,\,g} > T^{(+)}_{c,\,K} = \mathsf{O}(E_{\text{ew}})\,. \tag{C.2e}$$

$$\xi \equiv (E_{\text{Planck}}/E_{\text{ew}})^4\,. \tag{C.3}$$

Appendix C: Field-theoretic model

Spatially flat, homogeneous, and isotropic (F)RW universe.

Timescale set by

$$t_{\text{ew}} \equiv E_{\text{Planck}}/(E_{\text{ew}})^2 = \left(1/\text{meV}\right)\,\left(\text{TeV}/E_{\text{ew}}\right)^2 . \tag{C.4}$$

Dimensionless variables:

$$\tau \equiv (t_{\text{ew}})^{-1}\, t\,, \qquad h \equiv t_{\text{ew}}\, H\,, \tag{C.5a}$$

$$r_{Mn} \equiv \xi^{-1}\, (t_{\text{ew}})^4\, \rho_{Mn}\,, \qquad r_V \equiv (t_{\text{ew}})^4\, \rho_V = x^2/2\,, \tag{C.5b}$$

$$x \equiv \xi\,\left(q/q_0 - 1\right). \tag{C.5c}$$

Appendix C: Field-theoretic model

Dimensionless ODEs:

$$
\begin{aligned}
\left(\dot{h}+2h^2\right)\left(x^2/2+\xi\left(r_{M1}+r_{M2}-3\,h^2\right)\right)\;-\;h\,x\,\dot{x} &= 0\,, && \text{(C.6a)}\\
\dot{r}_{M1}+(4-\overline{\kappa}_{M1})\,h\,r_{M1}-\lambda_{21}\,r_{M2}+\lambda_{12}\,r_{M1} &= 0\,, && \text{(C.6b)}\\
\dot{r}_{M2}+4\,h\,r_{M2}+\lambda_{21}\,r_{M2}-\lambda_{12}\,r_{M1} &= 0\,, && \text{(C.6c)}\\
3\,h\,\dot{x}\,\theta\left[r_{M2}(\tau)-r_{c,\,K}\right]-\left(x^2/2+\xi\left(r_{M1}+r_{M2}-3\,h^2\right)\right) &= 0\,, && \text{(C.6d)}
\end{aligned}
$$

with EOS function $\overline{\kappa}_{M1}$ from [5] and coupling parameters $\left[\lambda \propto (g_0)^2\right]$:

$$
\begin{aligned}
\lambda_{12}(\tau) &= \lambda\,\theta[r_{c,\,g}-r_{M2}]\left(1-\sqrt{r_{M2}/r_{c,\,g}}\right)^2, && \text{(C.6e)}\\
\lambda_{21}(\tau) &= \lambda_{12}(\tau)\;\exp\left[-\left(\frac{\pi N_2}{30\,r_{M2}(\tau_{\text{min}})}\right)^{1/4}\frac{a(\tau)}{a(\tau_{\text{min}})}\;\frac{M}{E_{\text{ew}}}\right]. && \text{(C.6f)}
\end{aligned}
$$

Appendix C: Field-theoretic model

Model universe has early phase given by a standard radiation-dominated FRW universe $\Rightarrow$ fully determined boundary conditions of ODEs.

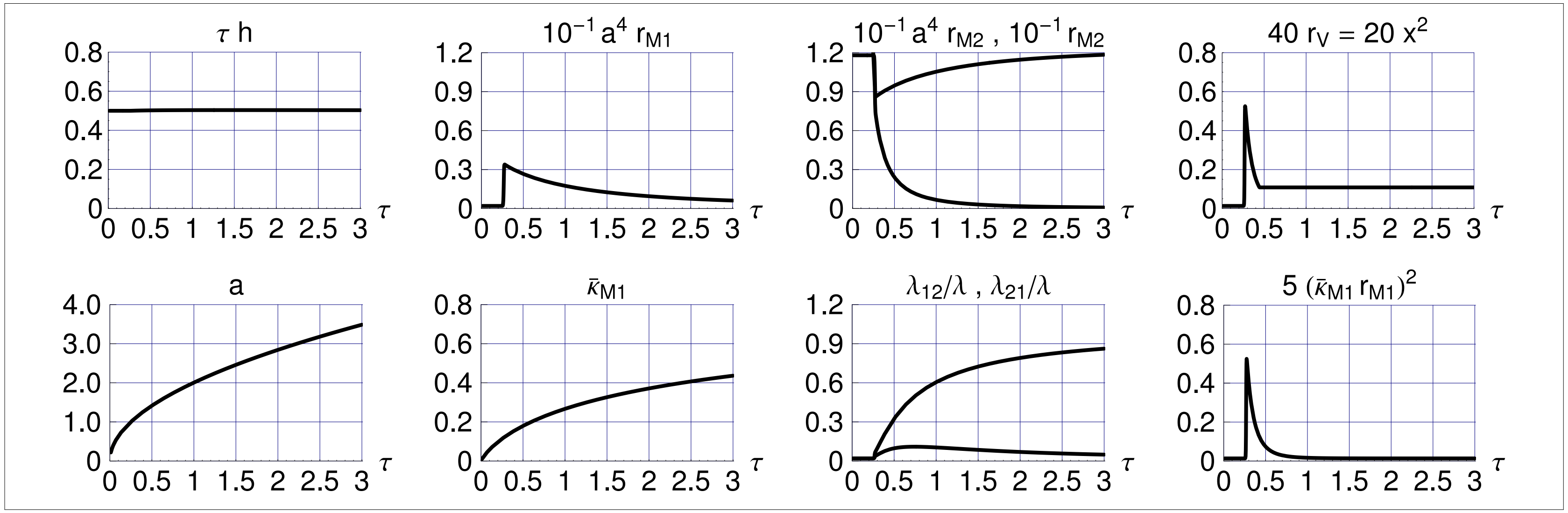

Fig. C1: Numerical solution of the dimensionless ODEs (C.6). Model parameters are $\{\xi,\, \lambda,\, r_{c,\,g},\, r_{c,\,K}\} = \{10^7,\, 10^4,\, 12,\, 3\}$. The ODEs are solved over the interval $[\tau_{\text{min}},\, \tau_{\text{max}}] = [0.01,\, 3]$ with the boundary conditions at $\tau = \tau_{\text{bcs}} = 0.25$: $\{x,\, h,\, a,\, r_{M1},\, r_{M2}\} = \{0,\, 2,\, 1,\, 0,\, 12\}$. Essentially the same results for $\xi = 10^{60}$.

Appendix C: Field-theoretic model

The calculated value $r_{V,\,\text{remnant}} \approx 2.4 \times 10^{-3}$ gives $E_{\text{ew}} \approx 4.7$ TeV, according to (12).

But, here, main focus on the physical content of a theory capable of generating the observed cosmological "constant" of our Universe.

Hence, analytic result of interest:

$$\lim_{\tau \to \infty} r_V(\tau)\,\Big|^{\xi=\infty} = \frac{1}{8}\Big(\overline{\kappa}_{M1}(\tau_{\text{freeze}})\; r_{M1}(\tau_{\text{freeze}})\Big)^2\,\Big|_{r_{M2}(\tau_{\text{freeze}})=r_{c,\,K}}\,. \tag{C.7}$$