Common misconceptions about LIGO detectors of gravitational waves

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Outline

Introduction

12 February 2016: *LIGO* reports detection of gravitational waves on Sept. 14, 2015 (GW150914)

Selected for a Viewpoint in *Physics*
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Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

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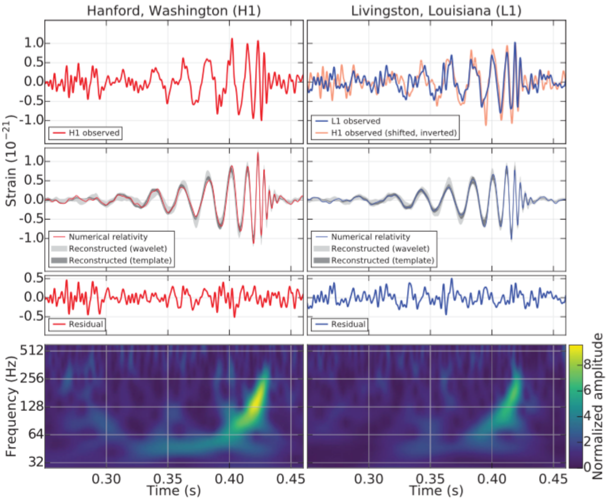
I. INTRODUCTION

In 1916, the year after the final formulation of the field equations of general relativity, Albert Einstein predicted the existence of gravitational waves. He found that the linearized weak-field equations had wave solutions: transverse waves of spatial strain that travel at the speed of light, generated by time variations of the mass quadrupole moment of the source [1,2]. Einstein understood that gravitational-wave amplitudes would be remarkably small; moreover, until the Chapel Hill conference in 1957 there was significant debate about the physical

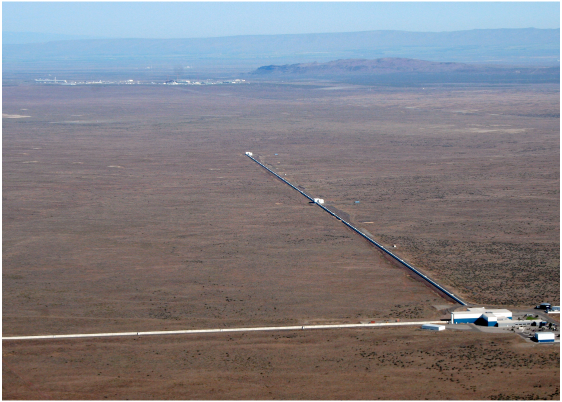
The discovery of the binary pulsar system PSR B1913+16 by Hulse and Taylor [20] and subsequent observations of its energy loss by Taylor and Weisberg [21] demonstrated the existence of gravitational waves. This discovery, along with emerging astrophysical understanding [22], led to the recognition that direct observations of the amplitude and phase of gravitational waves would enable studies of additional relativistic systems and provide new tests of general relativity, especially in the dynamic strong-field regime.

Experiments to detect gravitational waves began with Weber and his resonant mass detectors in the 1960s [23]



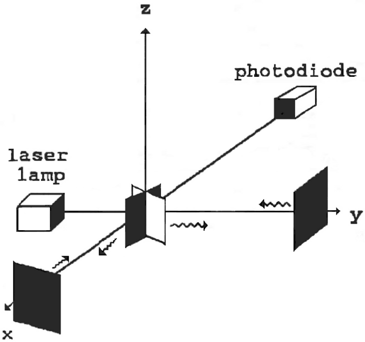
GW150914 event (B.F. Abbott *et al.* 2016, *Phys. Rev. Lett.* 116, 061102)

How does a laser interferometer work? *Interpretation* is gauge-dependent; *prediction* of the phase shift is not.



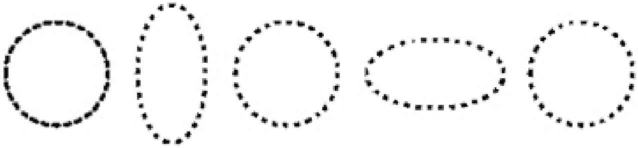
Hanford detector (from *LIGO* website)

REVIEW OF LASER INTERFEROMETERS



- beam splitter at the origin $(x, y) = (0, 0)$
- mirrors at $(L, 0)$ and $(0, L)$
- laser beams starting in phase propagate in both arms (of equal length L), reflect off mirrors, travel back to beam splitter where they are collected and analyzed.

Effect of a grav. wave on a ring of particles $\perp$ direction of propagation:



If a grav. wave impinges, it causes the lengths of the two arms to vary by different amounts. The lengths travelled by the two beams are different $\rightarrow$ phase shift $\Delta\phi = 2\pi\delta l/\lambda$

Grav. waves are small perturbations of Minkowski spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

in asymptotically Cartesian coords., with

$$|h_{\mu\nu}| \ll 1$$

(really! $h \sim 10^{-21}$ for *LIGO*)
Transverse-traceless (TT) gauge:

$$h_{0\mu} = h^{\mu}{}_{\mu} = 0$$

For a grav. wave propagating along the z-axis,

$$
\begin{aligned}
(h_{\mu\nu}) &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xx} & 0 & 0 \\ 0 & 0 & -h_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & h_{xy} & 0 \\ 0 & h_{xy} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\
&\equiv h_{+} + h_{\times}
\end{aligned}
$$

2 distinct polarizations

Effect of the $+$ mode on a ring of particles as time goes by:

Effect of the $\times$ mode on a ring of particles:

Consider, for simplicity, a grav. wave with single polarization h_+ propagating along the z-axis normal to the interferometer's plane, to first order O(h). Treat beam splitter and mirrors as test particles. Their separation x^i obeys the geodesic deviation eq.

$$\ddot{x}^i = R^i{}_{00j}\, x^j$$

where $x^i = x^i_{(0)} + \delta x^i$. In TT gauge,

$$\ddot{x}^i = \frac{1}{2}\, \ddot{h}^{(\mathrm{TT})}_{ij} x^j_{(0)}$$

Assumption: $L \ll \lambda_g$

grav. wave wavelength $\lambda_g \neq \lambda$ wavelength of laser light

for $\nu = 1$ kHz, $L/\lambda_g \sim 10^{-2}$ (not true for *LISA*)

Integrate $\ddot{x}^i = \frac{1}{2}\,\ddot{h}_{ij}^{(\mathrm{TT})} x_{(0)}^j \rightarrow$

$$\begin{cases} \delta x = \frac{1}{2} h_{xx} x \simeq h_+(t) x \\ \\ \delta y = \frac{1}{2} h_{yy} y \simeq h_+(t) y \end{cases}$$

tidal effect (effect of curvature = gradient of acceleration of gravity)

$$\boxed{\frac{\delta L}{L} \sim h}$$

for *LIGO* $L \sim 4$ km, $h \sim 10^{-21}$, $\delta L \sim 4 \cdot 10^{-3}$ fm

The variation in length of the interferometer's arms is

$$\delta L(t) = \delta x(t) - \delta y(t) = L h_+(t)$$

the phase difference at output is

$$\Delta\phi = 2\pi \frac{\delta L}{\lambda} = 2\pi \frac{L}{\lambda} h_+(t)$$

TWO OBJECTIONS ...

1) Given that the gravitational field stretches both the interferometer arm L and the wavelength λ of the laser light propagating through it, why is the grav. wave detectable? Analogy with cosmology (popular with astronomers): expansion of space stretches *all distances and wavelengths alike*, causing cosmological redshift.

2) (more technical): gravity deflects light and deflection is first order, O(h), so laser beams don't propagate along x- and y-axes. Then interferometers shouldn't work when grav. wave hits.

Only pedagogical interest? Not really, effects are tiny and everything 1st order should be scrutinized. One *LIGO* spokeperson could not answer 1) in a seminar. Answer is not trivial.

- Qualitative answer in P.R. Saulson, *Am. J. Phys.* 65, 501 (1997)
- Qualitative answer by Kip Thorne in Caltech lectures online: "spacetime curvature influences light in a different manner that it influences the mirror separations ... the influence on the light is negligible and it is only the mirrors that get moved back and forth and the light's wavelenght does not get changed at all ..."–but no calculations.
- D. Garfinkle, *Am. J. Phys.* 74, 196 (2006): analogy between gauge freedom of GR and Aharonov-Bohm effect of QM - not explicit.

Will provide a *quantitative* answer based on calculations in TT gauge universally used to describe *LIGO* interferometers.

V.F., *Gen. Rel. Grav.* 39, 677 (2007)

Re-iterated in

S. Hughes 2009, *Ann. Rev. Astr. Astrophys.* 47, 107; arXiv:1002.2591

Result is gauge-independent (but interpretations are not).

... AND THE ANSWER

Assume: TT gauge, single polarization grav. wave propagating along z-axis, $L \ll \lambda_g$.
Laser beams follow null geodesics

$$\frac{dk^\mu}{d\tau} + \Gamma^\mu_{\rho\sigma} k^\rho k^\sigma = 0\,, \qquad k^\mu \equiv \frac{dx^\mu}{d\tau}$$

Now

$$k^\mu = \underbrace{k^\mu_{(0)}}_{O(1)} + \underbrace{\delta k^\mu}_{O(h)} = \delta^{\mu 0} + \delta^{\mu 1} + \underbrace{\delta k^\mu}_{O(h)}$$

To 1st order,

$$\frac{d(\delta k^\mu)}{d\tau} = -\frac{1}{2}\eta^{\mu\alpha}\underbrace{\left(h_{\alpha\rho,\sigma} + h_{\alpha\sigma,\rho} - h_{\rho\sigma,\alpha}\right)}_{O(h)}\underbrace{k^\rho_{(0)} k^\sigma_{(0)}}_{O(1)}$$

with

$$k^{\rho}_{(0)}k^{\sigma}_{(0)} = \delta^{\rho 0}\delta^{\sigma 0} + 2\delta^{0(\rho}\delta^{\sigma)1} + \delta^{\rho 1}\delta^{\sigma 1} + \mathrm{O}(h)$$

Integrate along *unperturbed* path with error $\mathrm{O}(h^2) \;\rightarrow$

$$\begin{aligned}\delta k^{\mu} &= -\int_0^L dx \left(\cancel{h^{\mu}_{0,0}} + \cancel{h^{\mu}_{0,1}} + \underbrace{h^{\mu}_{1,0} + h^{\mu}_{1,1}}_{x=t\text{ along path}} \right) \\ &\quad + \frac{1}{2}\int_0^L dx \left(\cancel{h_{00}} + \cancel{2h_{01}} + h_{11}\right)^{,\mu} + \mathrm{O}(h^2) \\ &\simeq \frac{1}{2}\int_0^L dx\, h_{11}{}^{,\mu} + \mathrm{O}(h^2)\end{aligned}$$

so that

$$\delta k^\mu = \frac{\delta^{\mu 0}}{2}\left[h_{11}(t=2L) - h_{11}(t=0)\right] + O(h^2)$$

no spatial deflection to 1st order

Angular frequency measured by an observer u^μ is $\omega = -k_\mu u^\mu$. For the beam splitter,

$$u^\mu = u^\mu_{(0)} + \underbrace{\delta u^\mu}_{O(h)} = \delta^{\mu 0} + \delta u^\mu$$

$\omega = \omega_0 + \delta k^0$ and the percent variation is

$$\frac{\delta\omega}{\omega_0} = \frac{h_{11}(t=0) - h_{11}(t=2L)}{2} + O(h^2) = O(h^2)$$

For $L \ll \lambda_g$,

$$\frac{\delta\lambda}{\lambda} = \frac{\delta\omega}{\omega_0} = O(h^2)$$

so that, to 1st order,

$$\frac{\delta L}{L} = h_+(t)$$

$$\frac{\delta \lambda}{\lambda} = 0$$

(exactly as in Thorne's words)

and

$$\Delta\phi = 2\pi\,\frac{\delta L}{\lambda} = 2\pi\,\frac{L}{\lambda}\,h_+(t)$$

For *physical* lengths along, *e.g.*, the x-axis $l_{\mathrm{phys}} = \sqrt{g_{11}}\, l$, $\lambda_{\mathrm{phys}} = \sqrt{g_{11}}\, \lambda$ but

$$\frac{\delta\lambda_{\mathrm{phys}}}{\lambda_{\mathrm{phys}}} = \frac{\delta\lambda}{\lambda}\,, \qquad \frac{\delta L_{\mathrm{phys}}}{L_{\mathrm{phys}}} = \frac{\delta L}{L}$$

so calculation using *coordinate* lengths is correct.

CONCLUSIONS

- Interferometer arms and laser wavelength are *stretched differently* by grav. wave
- Spatial deflections of laser beams $\sim$ O(h^2) $\leq 10^{-42}$ for GW150914
- $\Delta\phi$ is gauge-independent, explanation (and objections) are not
- *LIGO* detectors work well, as demonstrated by the GW150914 event.

THANK YOU