Can we detect quantum gravity with compact binary inspirals?

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KING'S College LONDON

based on
arXiv:1809.06275 [gr-qc] (Phys. Rev. D 98, 104032)

Virtual Institute of Astroparticle Physics
15/02/2019

Plan of the talk:

-Part 1-
General motivation for a theory of quantum gravity

-Part 2-
Impact of quantum gravity corrections on gravitational-wave emission

General motivation for a theory of quantum gravity

Quantum mechanics and general relativity have revolutionised our understanding of space, time and matter.

- **Quantum theories**/quantum field theories: description of phenomena on a microscopic scale

  —> e.g. QFT applied to Standard Model of particle physics gives the most precise scientific theory available [Ellis, Sanz, You (2015)]

- **GR** describes fundamental interaction between matter and spacetime geometry: depict structure of our Universe on a large scale

  —> e.g. recent measurements of gravitational waves from inspiralling binary systems confirm GR to extremely high degree [Abbott et al. (2016)]

Pragmatic point of view:

Does an overlapping domain of quantum gravitational phenomena exist? How to describe and observe it?

- It is typically argued that an interface of both frameworks is needed for a satisfying description of the microstructure

  of spacetime together with matter at the **Planck scale,** as defined by the Planck length: $\ell_p = \left(G\hbar/c^3\right)^{1/2}.$

  This is the natural scale where quantum gravity effects are expected to occur.

- Furthermore, the analysis of their underlying elementary and universal assumptions shows that both frameworks are conceptually **incompatible**.

Conceptual problems:

- QFT assumes existence of a fixed, non-dynamical background metric living on a fixed, non-dynamical differentiable manifold.

  —> This breaks down when the gravitational field and the manifold structure become dynamical.

  —> In particular: it neglects the backreaction/interwoven co-evolution of the dynamical background geometry and the matter fields, as expressed via Einstein's field equations:

$$G_{\mu\nu}(g) = 8\pi G_{\mathrm{N}} T_{\mu\nu}(g)$$

  —> It can this be expected to become fully inadequate to describe extreme astrophysical and cosmological situations where the metric wildly fluctuates.

- Even worse, a backreaction between geometry and quantum matter cannot be consistently described using the field equations.

  —> Geometric content is classical and deterministic, while the quantum fields obey probabilistic laws.

  —> Even when ignoring this and thus promoting the energy-momentum tensor to an operator, its expectation value will depend on a fixed background metric and we run to the above-given problem.

  —> As long as backreaction can be neglected, QFT on curved spacetime provides revealing insights for quantum gravity (e.g. Hawking radiation & black hole thermodynamics).

Taking the universal coupling of gravity to all energy forms and the universality of quantum physics seriously, the quantisation of the gravitational field is naturally suggested.

Further problems:

- QFTs encounter divergences at large momenta rendering them ill-defined.

  —> It is expected that if gravity was consistently taken into account (providing a natural UV cutoff through the Planck length), this limitation could be cured.

- Penrose and Hawking have proven that there are inevitable spacetime singularities under reasonable conditions on causality and energy in the context of cosmology and gravitational collapse.

  —> Singular spacetimes are characterised by geodesics of finite affine length which are thus inextendible. Such geodesics are called incomplete.

  —> One then may classify a singularity according to whether it is a curvature or non-curvature singularity.

  —> The singular states of geometry with infinite curvature for the FLRW and Schwarzschild solutions are not artefacts of their high degree of symmetry.

  —> In domains of strong gravitational fields encountered near singularities, the classical description of gravitation and spacetime in terms of GR loses its validity and breaks down.

  —> Such singularities are unphysical and it is expected that quantum effects lead to their resolution [DeWitt (1967)].

Path integral for quantum gravity in the continuum

- Starting with the **quantum gravitational path integral** ("sum over spacetime histories") [Misner (1957)]

$$Z = \int \mathcal{D}g \; \mathrm{e}^{iS_{\mathrm{GR}}[g]}$$

which (in principle) provides a background independent covariant quantisation of spacetime geometry.

- It is difficult to make sense of this expression for several reasons:

1) topology change hard to implement in 4d, as 4-manifolds are not classifiable.

2) hard to define a probability measure on the space of metrics modulo diffeomorphisms

3) transition amplitudes hard to define, since notion of scalar product could so far not be found
—> lack of a clear interpretation of expectation values of observables in a phys. Hilbert space

4) Euclidean path integral (obtained after Wick rotation) is not bounded from below

- In the absence of a full understanding of Z, one can try to make progress and extract information regarding the quantum nature of gravity by **studying quantum disturbances around a fixed classical background metric**.

—> Given the success of the perturbative quantisation recipe for non-gravitational theories at small coupling this seems a viable avenue to follow.

Perturbative quantisation (1)

- To this aim, use the **background field method**: Expand the metric in terms of a (flat) classical background with small perturbations thereon, to be quantised later: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

- With this ansatz one **linearises** the Einstein-Hilbert action. When working in the harmonic gauge, schematically this leads to:

$$S_{\text{lin}} \sim \int \mathrm{d}^4x \; h_{\mu\nu} \Box h^{\mu\nu} + o(h^3)$$

- Using the background metric, we may perform a Wick rotation and then continue with standard path integral quantisation. (The Gaussian integrals of the weak field expansion liberate us from understanding the measure better. Notice that only by summing up the whole perturbation series, background independence could be restored.)

- In this way, one obtains that the low energy quanta of the gravitational field are massless particle-like excitations of spin 2, the **gravitions**.

- In analogy with other QFTs, scattering amplitudes of gravitons with themselves and matter can be computed.

—> At 1-loop order the **classical Newtonian potential receives quantum gravitational corrections**:

$$V(r) = -G_{\mathrm{N}} \frac{m_1 m_2}{r} \left(1 + \frac{3}{2} \frac{r_{\mathrm{S}}}{r} + \frac{41}{10\pi} \frac{\ell_p^2}{r^2} \right)$$

Observable?

with Schwarzschild radius $r_{\mathrm{S}} = 2G_{\mathrm{N}}M, \; M = m_1 + m_2$ [Donoghue (1994); Bjerrum-Bohr, Donoghue, Holstein (2003)]

Perturbative quantisation (2)

- However, notice that this background-dependent perturbative approach only makes sense as a **low energy effective description**:

—> It has to break down since at higher energies the backreaction of the perturbations onto the background will invalidate the linearisation.

—> Towards higher energies quantum fluctuations are not under control in the sense that the theory is **non-renormalisable**:

The mass dimensionality of the coupling constant determines a theory's renormalisability.
Newton's constant is dimensionful and of negative mass dimension in d=4.

- As a consequence, an endless number of counterterms must be introduced to cancel divergences at arbitrary loop orders:

—> Their couplings are free parameters and so an unlimited number of coupling constants has to be fixed by experiment.

—> The theory loses its predictivity in the UV [Goroff, Sagnotti (1986); van den Ven (1992)].

—> Anticipated: "Without a deep revision of classical notions, it seems hardly possible to extend the (perturbative) quantum theory of gravity also to the short-distance domain." [Bronstein (1936)]

Possible roads to follow

- UV divergences could disappear when gravity is “properly“ quantised.

—> To bypass the problems of the perturbative quantisation ansatz, in the **asymptotic safety programme** one assumes the existence of a non-perturbative (i.e. interacting) fixed point for gravity in the UV.

- It could also be taken as a hint that the degrees of freedom adequate for describing gravity at low energies are dissimilar to those encountered at high energies:

—> If we **venerate perturbative renormalisability** as guidance for a meaningful theory (and thus working background-dependently), one should increase the symmetries of the theory:

—> It is hoped that **supersymmetric extensions** of gravity (or embeddings of these in string theory) could enhance the UV behaviour by providing a mechanism to cancel perturbative divergences and thus regain perturbative renormalisability.

—> This is inspired by the example of replacing Fermi’s non-renormalisable model for the weak interaction through the electroweak theory by increasing the symmetry content of the theory.

—> Inspected from a different angle, one might expect the situation to be improved when considering that the continuum is replaced at small scales in favour of some **discretum**:

—> Hints for this may be provided by the phenomenon of black hole thermodynamics, since the Bekenstein-Hawking entropy calls for an explanation through more fundamental degrees of freedom behind the macroscopic description of the gravitational field in terms of the metric.

—> If we **venerate background independence** as guidance and do not add any additional structure:

—> **Quantise canonically**: quantum geometrodynamics & loop quantum gravity

—> **Quantise covariantly** through a discrete version of the path integral: spin foam approach, group field theory, tensor models, simplicity quantum gravity (quantum Regge calculus, Euclidean & causal dynamical triangulations)

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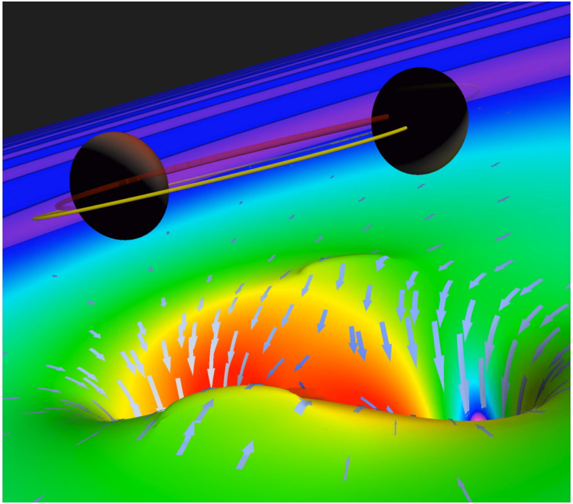
VIA Seminar
15 February 2019

Based on ACJ, Pithis, & Sakellariadou, *PRD* 2018, arXiv:1809.06275

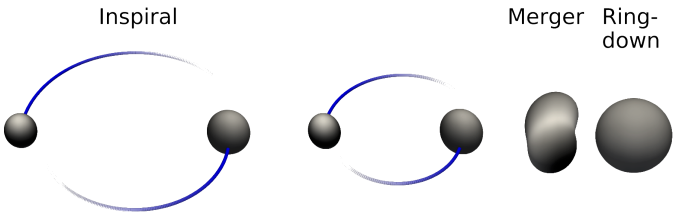
BBHs as nature's HEP laboratories



Credit: SXS collaboration

- Typical LIGO BBH has KE of
  - $\sim 100 M_\odot c^2$
  - $\sim 10^{40} M_P$
  - $\sim 10^{56}$ TeV
- Radiated energy is $\sim 5\%$ of this
- LHC aims for just ~ 10 TeV!
  (coupling much weaker here of course...)

What might QG modify?



- Source dynamics, $T_{\mu\nu}(t, \vec{x})$
- Quadrupole formula, $P_{\mathrm{rad}} = \frac{G}{5c^5} \left\langle \dddot{Q}_{ij} \dddot{Q}_{ij} \right\rangle$
- GW propagation (e.g. dispersion)

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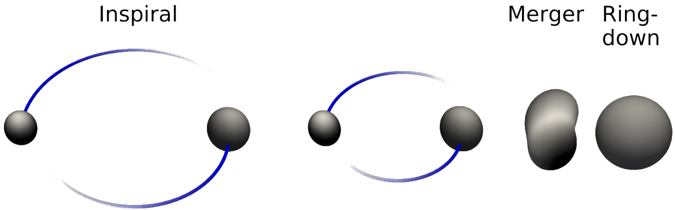

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The effective potential

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Leading Quantum Correction to the Newtonian Potential

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(Received 21 October 1993; revised manuscript received 23 March 1994)

I argue that the leading quantum corrections, in powers of the energy or inverse powers of the distance, may be computed in quantum gravity through knowledge of only the low-energy structure of the theory. As an example, I calculate the leading quantum corrections to the Newtonian gravitational potential.

$$V_{\text{1-loop}}(r) = -\frac{Gm_1 m_2}{r}\left(1 + \frac{3}{2}\frac{r_{\mathrm{S}}}{r} + k\frac{\ell_{\mathrm{P}}^2}{r^2}\right)$$

- *Definite* prediction of QG!
- Two scales: Planck ($\ell_{\mathrm{P}} \sim G^{1/2}$) and Schwarzschild ($r_{\mathrm{S}} \sim G$)

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What happens at higher loop order?

$$V_{\text{2-loop}}(r) = V_{\text{1-loop}}(r) - \frac{Gm_1 m_2}{r}\left(c_1 \frac{r_S^2}{r^2} + c_2 \frac{r_S \ell_P^2}{r^3} + c_3 \frac{\ell_P^4}{r^4}\right)$$

- $(r_S/r)^n$ is just n^{th} order classical PN

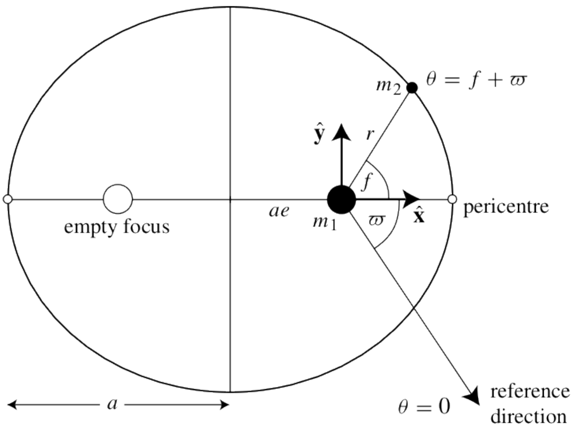
What we study

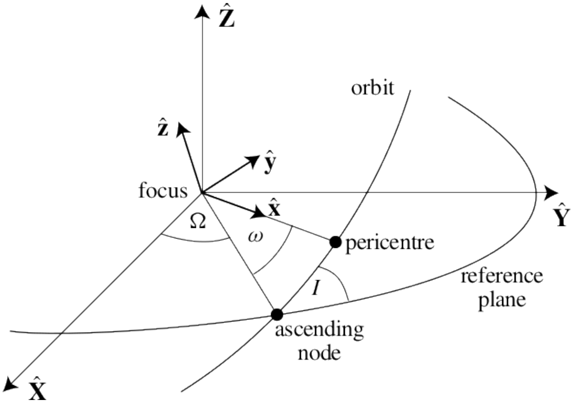
- Analytical $\implies r \gg r_S \gg \ell_P$
- PN already known up to $n = 4$
- So, to capture leading order QG:

$$V_{\text{eff}}(r) = -\frac{Gm_1 m_2}{r}\left(1 + k\frac{\ell_P^2}{r^2}\right)$$

- Equivalent to PN order $n \gtrsim 44$ at LIGO frequencies

(Pseudo-)Keplerian orbits





QG-corrected dynamics

- "Secular" $\iff$ orbit-averaged
- Mean motion

$$\langle\dot{\theta}\rangle = \sqrt{\frac{GM}{a^3}}\left[1 + \frac{3k\ell_P^2}{a^2(1-e^2)^{3/2}}\right]$$

- Pericentre advance

$$\langle\dot{\omega}\rangle = \langle\dot{\theta}\rangle\frac{3k\ell_P^2}{a^2(1-e^2)^2}$$

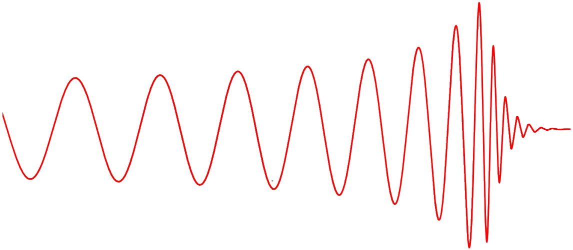
- $\sim 10^{-84}$ arcseconds per century for Mercury-Sun system (43 due to GR)

QG-corrected waveform

$$h(t) = \mathcal{A}(t)\exp[\mathrm{i}\Psi(t)]$$

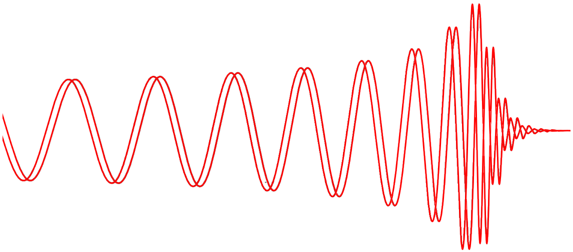
$$\frac{\delta\mathcal{A}_{\mathrm{QG}}}{\mathcal{A}} \sim \frac{\ell_{\mathrm{P}}^2}{r^2} \sim \frac{\ell_{\mathrm{P}}^2}{(GM)^{2/3}} f^{4/3}$$

$$\delta\Psi_{\mathrm{QG}} \sim \frac{1}{\nu}\left(\frac{\ell_{\mathrm{P}}}{r_{\mathrm{S}}}\right)^2\left(\frac{T_0}{t_{\mathrm{S}}}\right)^{1/3}$$



Can we detect this?

- Phase shift is our best bet
- Can measure $\delta\Psi \sim \frac{1}{\text{SNR}}$
- N signals $\implies \sqrt{\frac{1}{N}}$ improvement
- 100 BBH with SNR $= 100$
  $\implies \delta\Psi \sim 10^{-3}$rad
- But $\delta\Psi_{\text{QG}} \lesssim 10^{-74}$



But what about. . . ?

- Eccentricity?
  - LIGO binaries quasi-circular
- Quadrupole formula corrections? Propagation?
  - Calmet & Latosh, arXiv:1801.04698
  - Calmet, El-Menoufi, Latosh, & Mohapatra, arXiv:1809.07606
  - EFT gives new massive DoF. . . but only at $> 10^{13}$Hz

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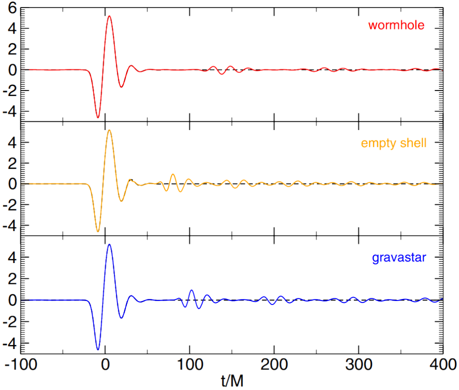
Non-perturbative phenomenology?

- Echoes



Cardoso+, arXiv:1608.08637

Warning!
Speculative

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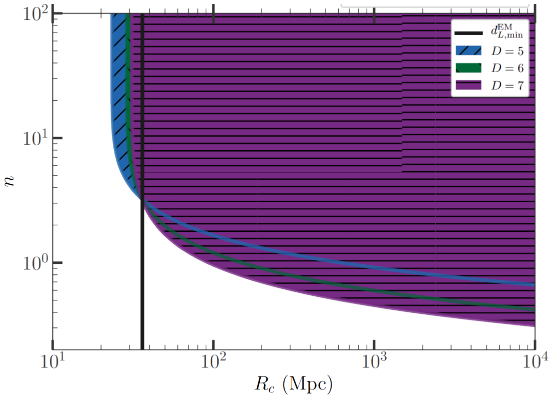
Non-perturbative phenomenology?

- Higher dimensions



Pardo+, arXiv:1801.08160

Warning!
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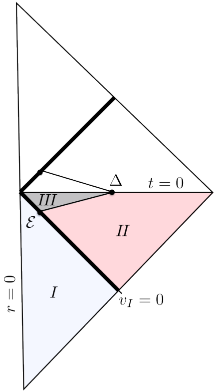
Non-perturbative phenomenology?

- Black hole $\rightarrow$ white hole tunnelling



Haggard & Rovelli, arXiv:1407.0989

Warning!
Speculative

Summary

- BBHs (and GWs more generally) are a new laboratory for HEP and QG
- Can compute *generic* perturbative QG correction to BBH
- . . . but it's tiny
- However, interesting non-perturbative prospects