The Schrödinger Equation and Axion DM: Small Scales, Solitons, and Dwarf Galaxies

David J. E. Marsh

1502.03456 w/ Ana-Roxana Pop

$$i\partial_t\psi+\nabla^2\psi/2m = mV\psi$$

$$\nabla^2 V = 4\pi G|\psi|^2$$

See also: DJEM & Silk, 2013, MNRAS, 437, 2652.
Bozek, DJEM, Silk & Wyse, 1409.3544, MNRAS in press

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Cold Dark Matter

CDM: "pressureless dust"

$$w_{\rm CDM} \approx c_{s,\rm CDM} \approx 0$$

$$w = \frac{\bar{P}}{\bar{\rho}} \qquad\qquad c_s^2 = \frac{\delta P}{\delta \rho}$$

Equation of state
→ Redshifts as matter

Sound speed
→ Clusters on "all" scales

(although, e.g. Loeb & Zaldarriaga 2005)

Candidates include WIMPs, heavy sterile ν's, QCD axion…

Structure formation

Power spectrum rises on small scales: hierarchical assembly. Verified by CMB and galaxy surveys for large scales.

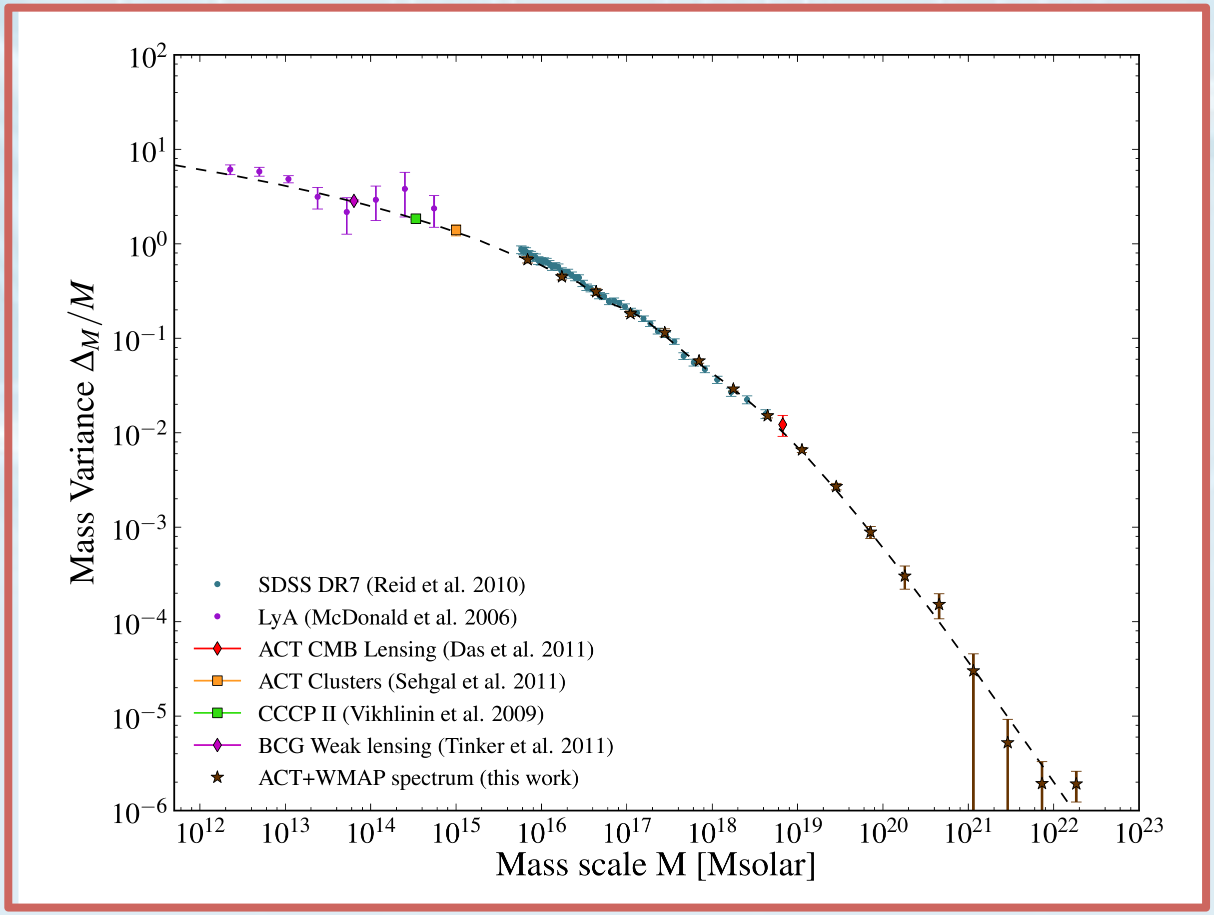
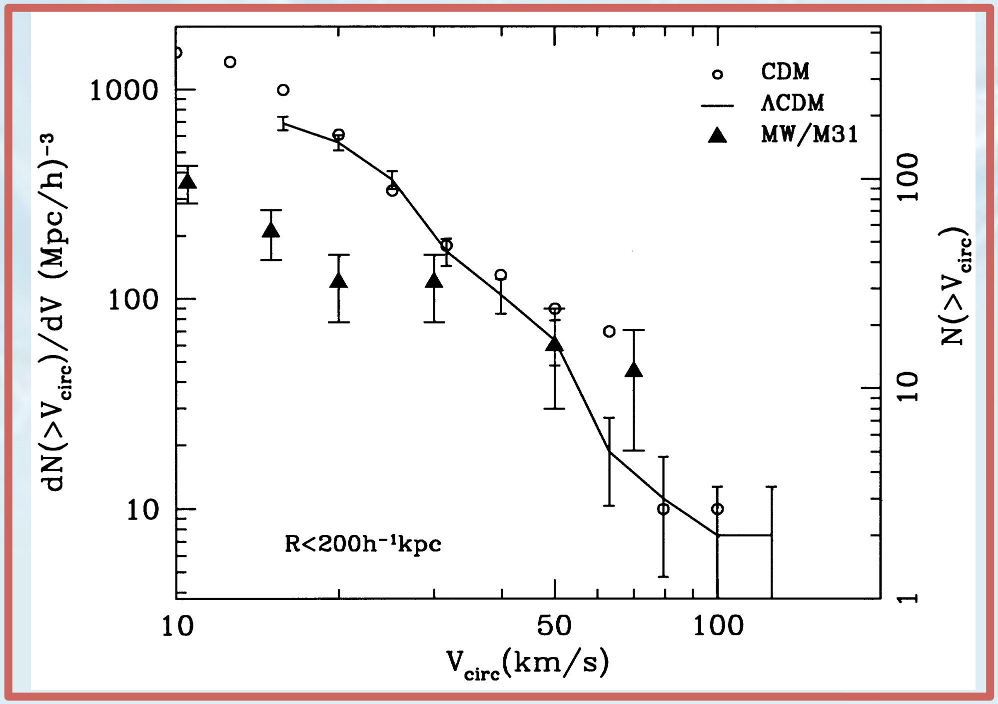


Fig: Hlozek et al (ACT, 2011)

Smallest objects and substructure

CDM predicts a large number of low mass objects.
→ "Missing Satellites Problem"



$$v_c^2 = \frac{GM(<r)}{r}$$

Klypin et al (1999):
"Models predict about 5 to 20 times more low mass satellites."

Strong and mililensing could resolve smaller DM substructure.

e.g. Hezaveh et al, 2014

The Milky Way Satellites

"Too Big To Fail:" CDM predicts larger number of massive satellites of order mass of the LMC.

Boylan-Kolchin et al (2011)

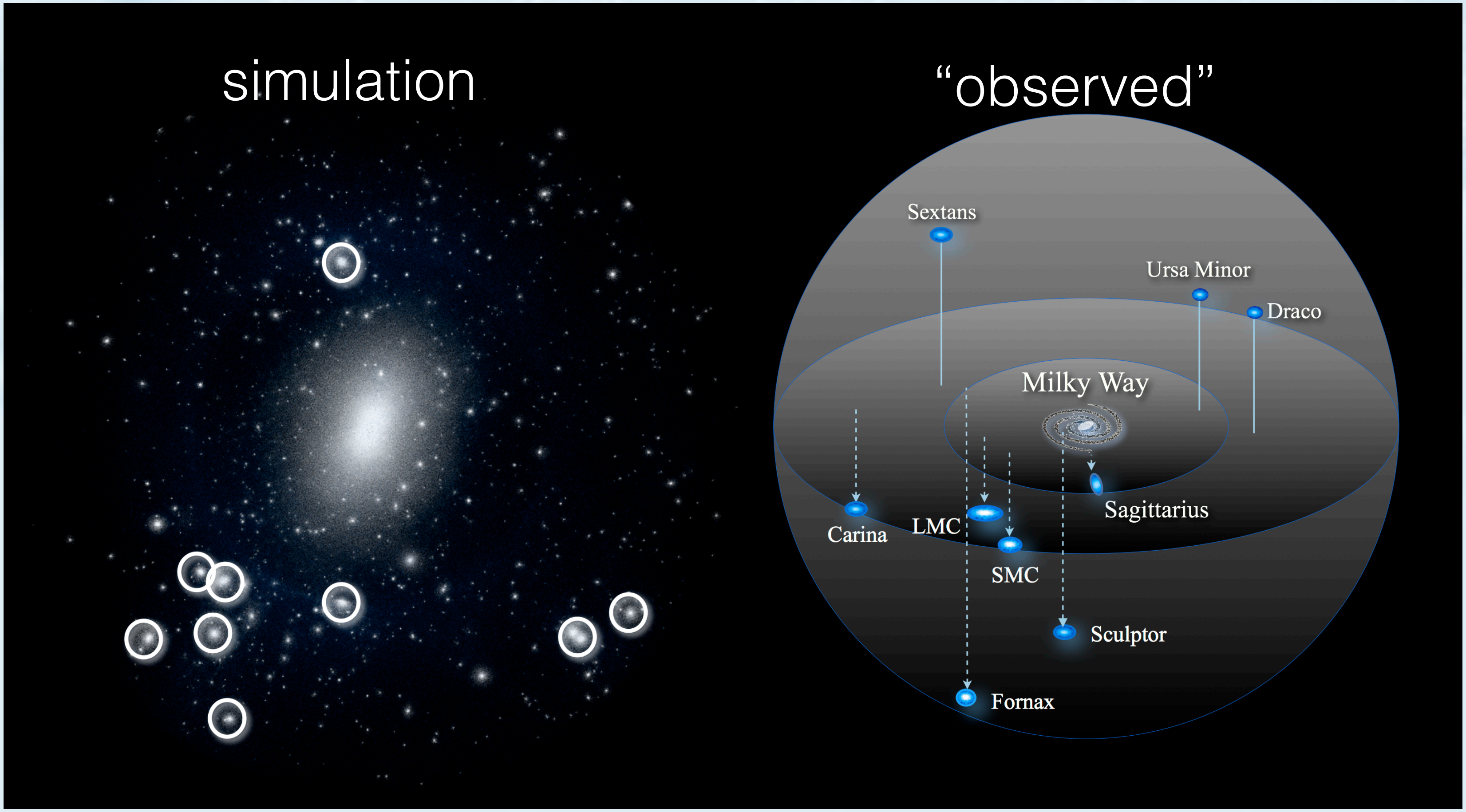


Fig: Weinberg et al 2013

Central densities

CDM density profiles are a slowly varying power law.

$$\frac{\rho_{\mathrm{NFW}}}{\rho_{\mathrm{crit}}} = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2}$$

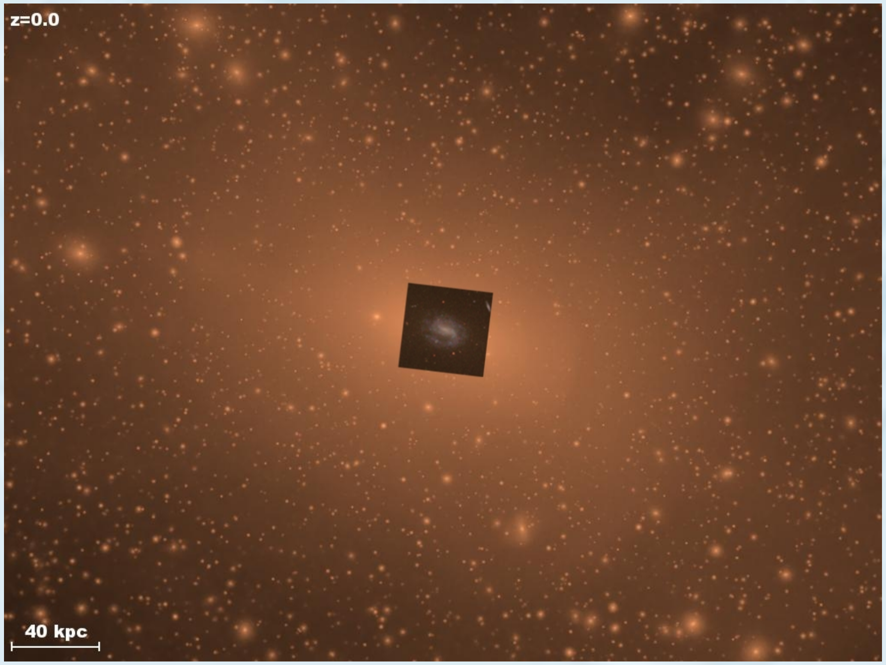
✧ Observed in simulation. Navarro et al, 1997

✧ Consequence of thermodynamics. e.g. Binney & Tremaine

✧ Consequence of phase space density? Taylor & Navarro, 2001

The “cusp-core” problem

Steep central densities in conflict w/ observation: LSBs



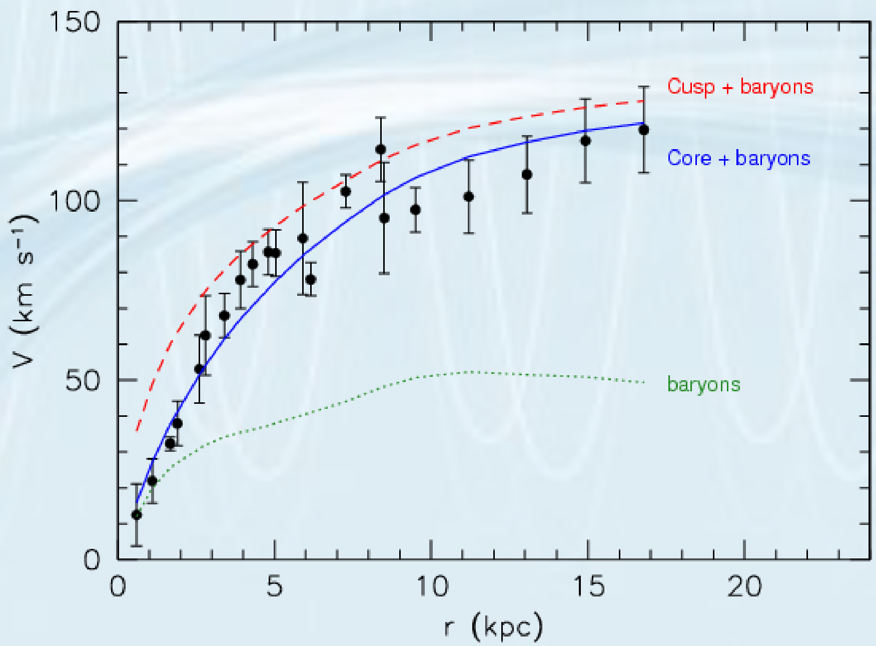
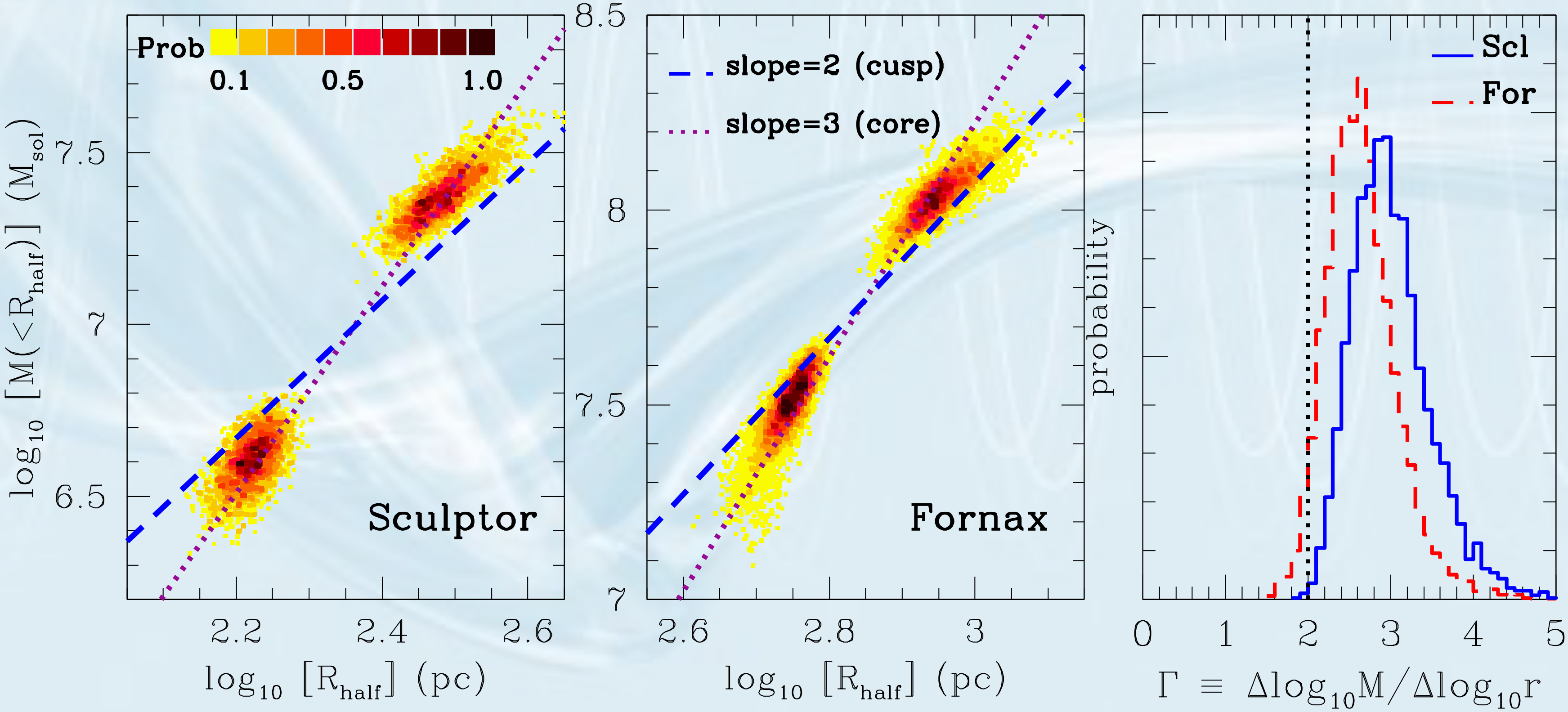


Fig: Weinberg et al 2013

The “cusp-core” problem

Steep central densities in conflict w/ observation: dSphs



Excludes NFW at >99% confidence.

Fig: Walker & Penarrubia, 2011

A foil for theory: DM and baryons

Unknown astrophysics could solve CDM problems.

✧ Supernova feedback — e.g. Pontzen & Governato (2014)

✧ Dynamical friction — e.g. Del Popo (2009)

Baryon solutions seem conservative. *But we also have no a priori reason to believe the DM is cold and collisionless!*

Modifying the theory of DM could solve problems, too.

✧ Thermal velocities: warm DM — e.g. Bond et al (1982), Bode et al (2001)

✧ Collisional: self interacting DM — e.g. Spergel & Steinhardt (2000), Boehm et al (2001)

✧ Non-thermal effects: fluid, fuzzy, axion DM

e.g. Peebles (2000), Hu et al (2000), Marsh & Silk (2013)

We can use small scales to test all these ideas.
We can use these ideas to formulate new tests of DM.
Learn something about DM either way: don't "oversolve".

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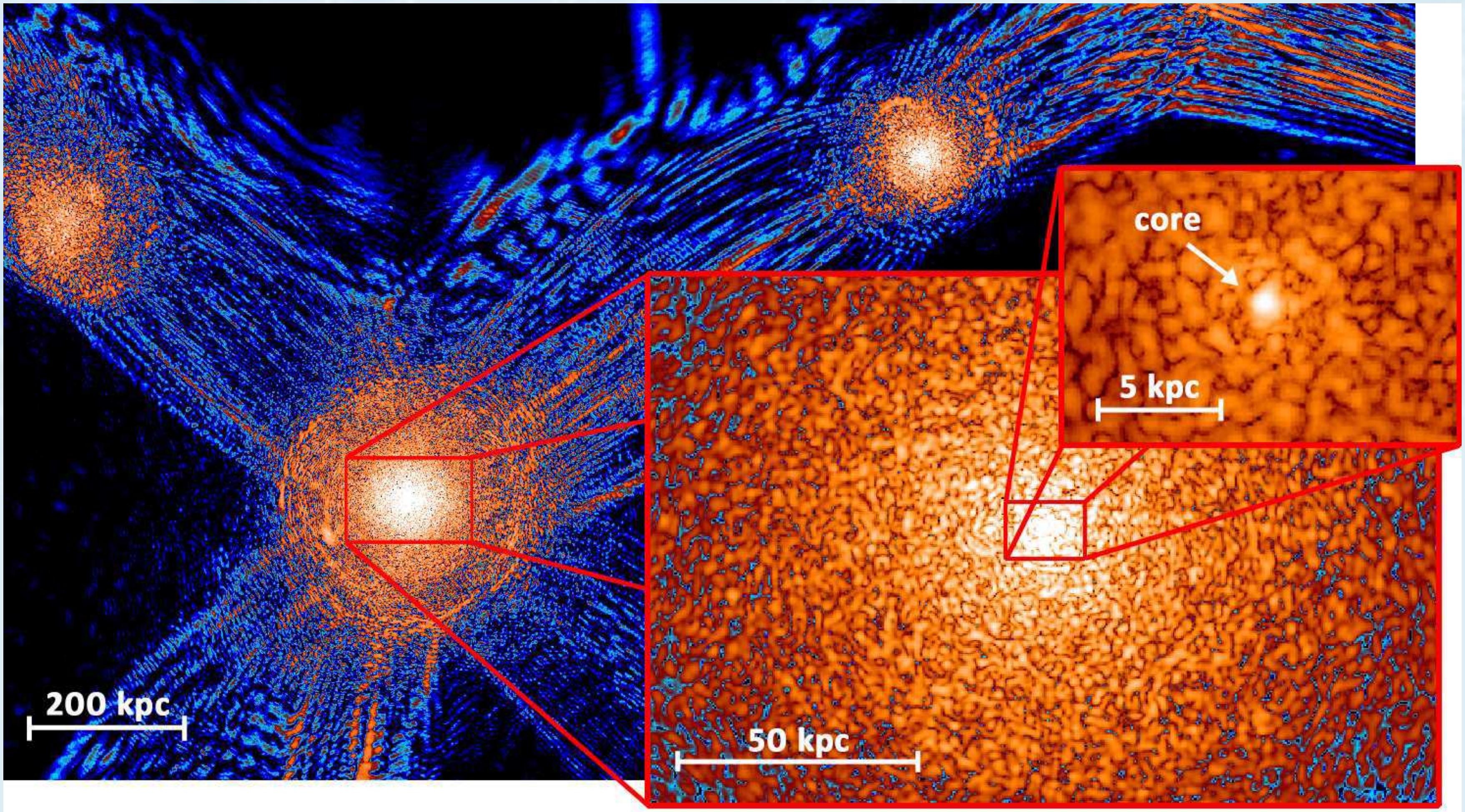
nature physics

Cosmic structure as the quantum interference of a coherent dark wave

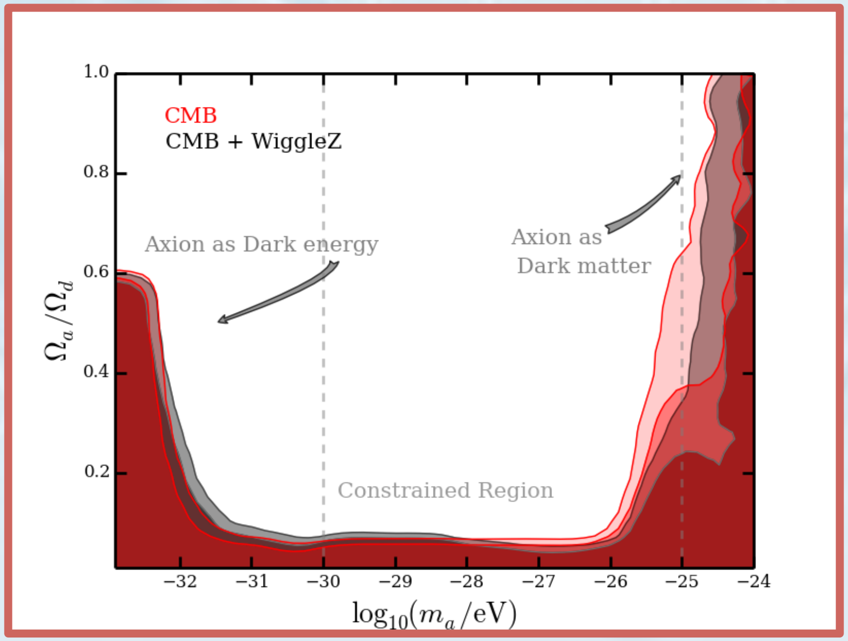
Hsi-Yu Schive[1], Tzihong Chiueh[1,2]* and Tom Broadhurst[3,4]

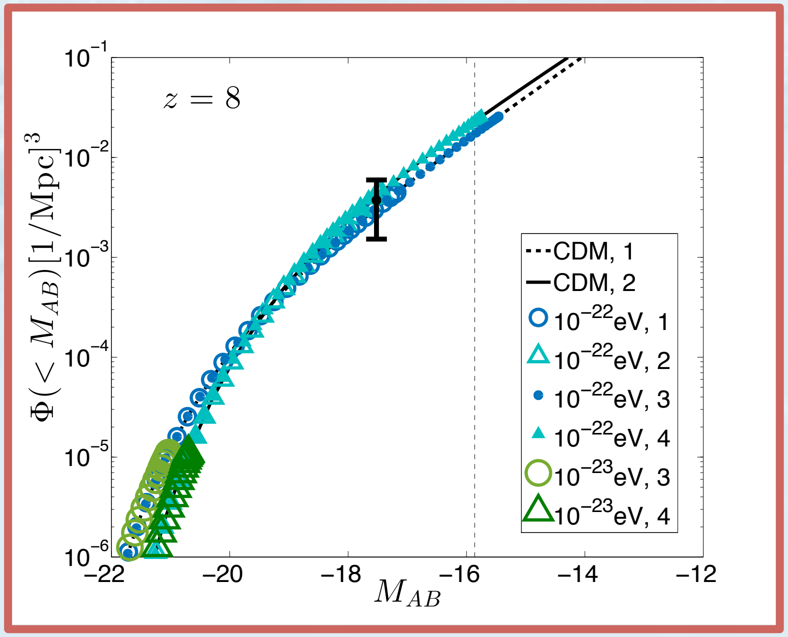


Ultra-light Axion DM

Computations with axionCAMB
(DJEM, Grin, Hlozek, to be released)

Massive scalar field minimally coupled to gravity.
Non-thermally produced by vacuum realignment.
Cosmological P.T. of relativistic Einstein-Klein-Gordon eqs.





Linear scales, CMB + LSS

$$m_a \gtrsim 10^{-25}\ \mathrm{eV}$$

Hlozek et al (2014)

Press-Schechter, HUDF:

$$m_a \gtrsim 10^{-23}\ \mathrm{eV}$$

Bozek et al (2014)

The non-relativistic limit

Take ansatz solution for oscillating (w=0) axion field:

$$\phi = \frac{1}{\sqrt{2}m_a}(\psi e^{imt} + \psi^* e^{-imt})$$

Plug into Einstein-Klein-Gordon with limits:

$H \ll m_a$ oscillating

$k \ll m_a$ low velocity

$k \ll H$ sub-horizon

$V \ll 1$ weak-field

Valid at late times on galactic scales

e.g. Seidel & Suen (1990)
Widrow & Kaiser (1993)

But: no perturbation theory on the density → valid inside halos

Schrödinger-Poisson equations

$$i\partial_t \psi = -\frac{1}{2m_a}\nabla^2\psi + m_a V\psi$$

$$\nabla^2 V = 4\pi G|\psi|^2$$

Classical field theory, but “very quantum” particles. e.g. Guth et al (2014)
Consistent limit for cosmology makes demands on solution.
Valid description for all axion DM, not just ultra-light.
Useful numerical model for CDM above “artificial” de Broglie.

e.g. Widrow & Kaiser (1993), Coles & Spencer (2003), Uhlemann et al (2014)

Soliton solutions

Consider a phase-coherent, localised energy eigenstate:

$$\psi(t,r) = \chi(r)e^{-i\gamma t}$$ (general phase → Madelung)

Pressure support and ground state → spherical symmetry.

$\gamma \ll m$ for consistent non-relativistic limit of EKG.

$$\underbrace{\chi'' + \frac{2\chi'}{r}}_{\text{Quantum potential → support}} = 2(V - \underset{\uparrow\ \text{Eigenvalue}}{\gamma})\chi$$

Quantum potential → support   Eigenvalue

$$V'' + \frac{2V'}{r} = \chi^2$$

Clarification note on "soliton"

With thanks to Mustafa Amin!

✧ Solutions are class of solitons called "oscillatons."
✧ Evade Derrik's theorem: time dep. + gravity.
✧ Complex field analogue is the "boson star." e.g. Liddle & Madsen (1992)
✧ Not protected by a charge → pseudo soliton.
→ Instability (relativistic) to black hole formation. Seidel & Suen (1990)

$$\phi_{\rm crit} \approx 0.3 M_{pl}$$

The axion soliton cores are technically "solitary waves."

Image credit: asianscientist.com

Numerical solution of BVP

Normalize central density and fix decays at infinity.

$$\chi(0) = 0; \quad \chi'(0) = 0; \quad V'(0) = 0$$

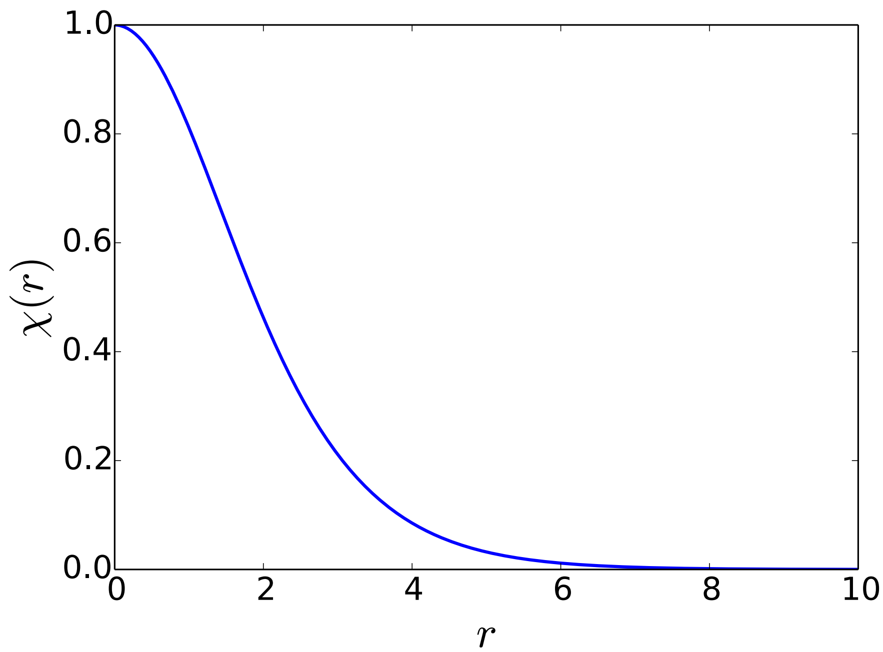
$$\chi(R) \propto e^{-\sqrt{2|\gamma|}R}; \quad V(R) \propto -\frac{1}{R}; \quad R \gg 1$$

Shooting method → Zero-node solution.

$$\gamma = -0.692$$

$$V(0) = -1.341$$

Relaxation to ground state via "gravitational cooling" (Seidel & Suen, 1994; Guzman & Ureña-Lopez,2006)



Scaling properties (I)

No explicit scale in the problem → scaling symmetry.

$$(r, \chi, V, \gamma, \rho_s) \to (r/\lambda, \lambda^2\chi, \lambda^2 V, \lambda^2\gamma, \lambda^4\rho_s)$$

Use this to rescale the BVP solution to astrophysical: $\lambda<1$
→ Rescale γ and k and check non-rel. limit + stability.

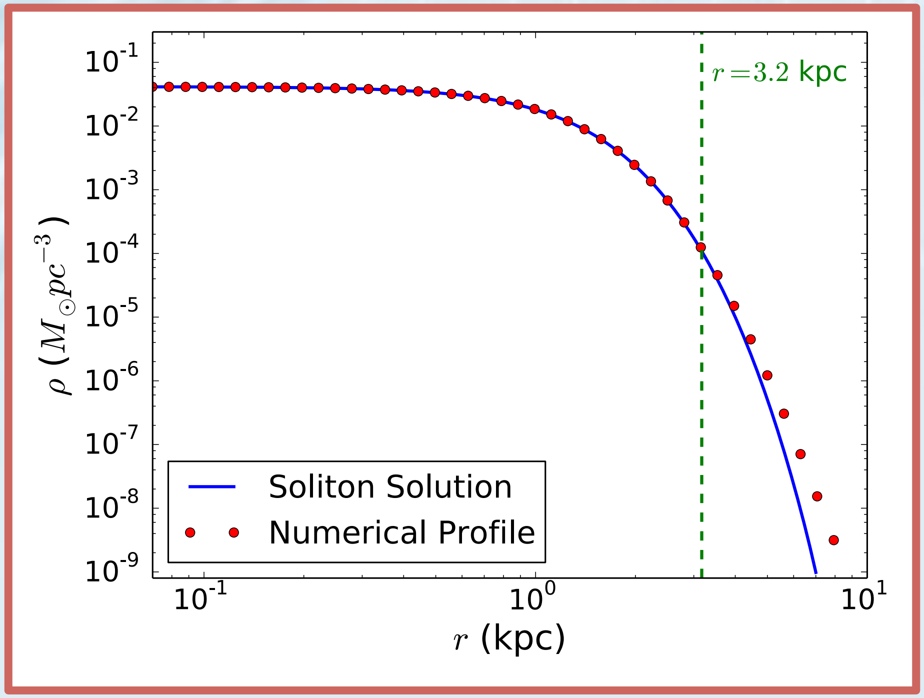
Fit for the soliton

Restoring units and using scale-free-ness:

$$\rho_{\mathrm{sol}} = 2m_a^2 M_{pl}^2 f(\alpha r); \quad r_{\mathrm{sol}} := (\alpha m_a)^{-1}$$

$$f(\alpha r) = (1+\alpha^2 r^2)^{-8}; \quad \alpha = 0.230$$

Better than Gaussian.
Gives analytic M and V.
Accurate beyond de Broglie.



Scaling properties (II)

Enforce correct scaling to astrophysical densities:

$$\rho_{\rm sol}(r) = \rho_{\rm crit}\delta_s f(r/r_{\rm sol})$$

→ Soliton radius fixed by central density and axion mass:

$$\delta_s = 1.8\times 10^7 \left(\frac{h}{0.7}\right)^{-2}\left(\frac{m_a}{10^{-22}\ {\rm eV}}\right)^{-2}\left(\frac{r_{\rm sol}}{\rm kpc}\right)^{-4}$$

Scaling relates soliton radius to linear Jeans scale:

$$r_{\rm sol} \propto \left(\frac{\rho_{\rm sol}(0)}{\rho_{\rm crit}}\right)^{-1/4} r_{J,{\rm lin}}$$

But, soliton shape → no match to NFW at common Jeans.

→ *cores more compact than expected from linear theory*

Hu et al (2000); Marsh & Silk (2013)

Beyond solitons

Solitons can't be the whole story:

✧ Thermo → need NFW on large scales.

✧ Lose phase coherence.

✧ Classical & CDM-like above r ~ de Broglie.

✧ Structure formation → no long range correlation

→ Characteristic size ~ Jeans

✧ View from simulation.

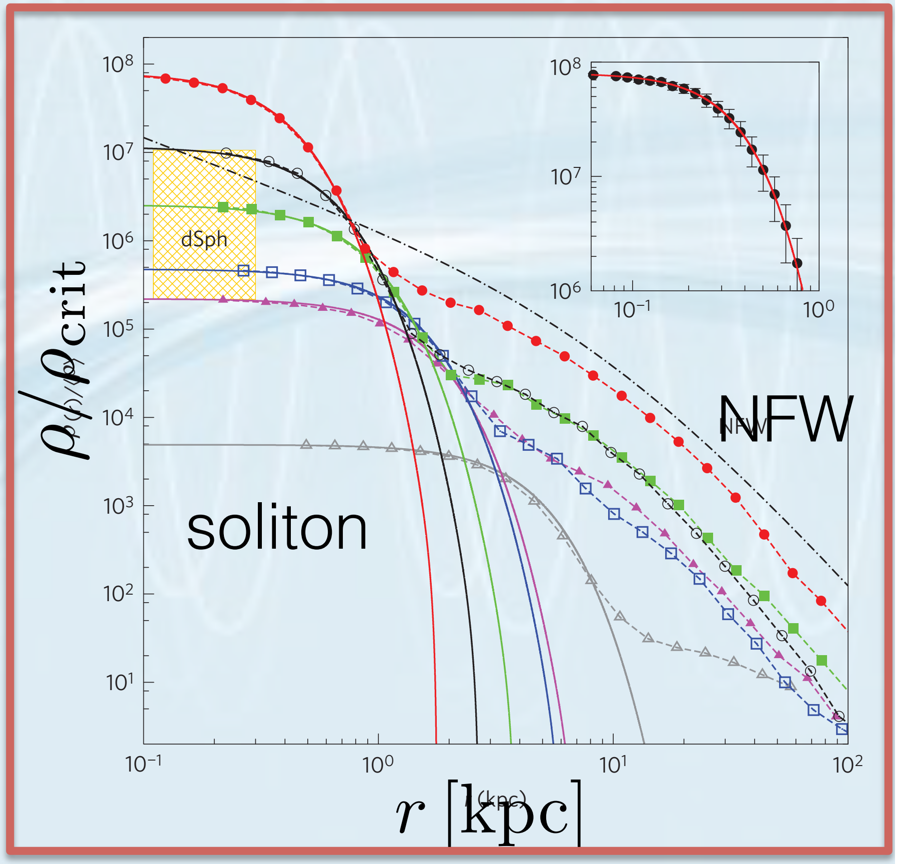


Fig: Schive et al (2014)

A complete model for halos

$$\rho(r) = \Theta(r_\epsilon - r)\rho_{\rm sol}(r) + \Theta(r - r_\epsilon)\rho_{\rm NFW}(r)$$

$$\rho(r_\epsilon) = \epsilon\rho(0)$$

$$\epsilon \sim 10^{-2}$$

Matching radius. Close to “core radius.” Not exactly de Broglie. Fixes conc. from central density.

Defines a three parameter family of halo models:

$$\{\delta_s, \epsilon, r_s\} \qquad (m_a\text{: universal})$$

Explore profiles of this form with MCMC and fit to data…

Bayesian analysis with “emcee”

Easy-peasy! My first ever MCMC all by myself ☺

PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC, **125**:306–312, 2013 March

emcee: The MCMC Hammer

DANIEL FOREMAN-MACKEY,[1] DAVID W. HOGG,[1,2] DUSTIN LANG,[3,4] AND JONATHAN GOODMAN[5]

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ABSTRACT. We introduce a stable, well tested Python implementation of the affine-invariant ensemble sampler for Markov chain Monte Carlo (MCMC) proposed by Goodman & Weare (2010). The code is open source and has already been used in several published projects in the astrophysics literature. The algorithm behind emcee has several advantages over traditional MCMC sampling methods and it has excellent performance as measured by the autocorrelation time (or function calls per independent sample). One major advantage of the algorithm is that it requires hand-tuning of only 1 or 2 parameters compared to $\sim N^2$ for a traditional algorithm in an N-dimensional parameter space. In this document, we describe the algorithm and the details of our implementation. Exploiting the parallelism of the ensemble method, emcee permits *any* user to take advantage of multiple CPU cores without extra effort. The code is available online at http://dan.iel.fm/emcee under the GNU General Public License v2.

Note: If you want to get started immediately with the emcee package, start at Appendix A or visit the online documentation at http://dan.iel.fm/emcee. If you are sampling with emcee and having low-acceptance-rate or other issues, there is some advice in § 4.

Walker and Peñarrubia's method

Velocity dispersion at half-light radius for two distinct stellar sub-components → robust measure of density profile slope.

dSph	$\log_{10}(\sigma^2/\mathrm{km}^2\ \mathrm{s}^{-2})$	Error	$\log_{10}(r/\mathrm{kpc})$	Error
Fornax	2.00	0.05	-0.26	0.04
	2.32	0.04	-0.05	0.04
Sculptor	1.62	0.06	-0.78	0.04
	2.13	0.05	-0.52	0.04

Data from Walker & Peñarrubia (2011)

Use empirical relation instead of full Jeans analysis:

$$\sigma^2(r_h) = \frac{2GM(< r_h)}{5r_h}$$

Full analysis in progress w/ Alma Gonazalez-Morales, following Diez-Tejedor et al (2014)

Likelihood

Treat the data as 2-dim uncorrelated Gaussians.
Central values, errors, and model:

$$(\bar{x}, \bar{y}) \quad (\Sigma_x, \Sigma_y) \quad y = f(x)$$

Reduce to effective 1-dim problem: e.g. Ma et al (2013)

$$\Sigma_{\text{eff}}^2 = \Sigma_y^2 + \left(\frac{df(\bar{x})}{x} \right)^2 \Sigma_x^2$$

$$\mathcal{L} \propto \exp\left[\frac{-(f(\bar{x}) - \bar{y})^2}{2\Sigma_{\text{eff}}^2} \right]$$

Priors

Log-flat uninformative.

Parameter	Prior
$\log_{10}(m_a/\mathrm{eV})$	$U(X,-19)$
$\log_{10}\delta_s^{\mathrm{F,S}}$	$U(0,10)$
$\log_{10}\epsilon^{\mathrm{F,S}}$	$U(-5,\log_{10}0.5)$
$\log_{10}(r_s^{\mathrm{F,S}}/\mathrm{kpc})$	$U(-1,2)$

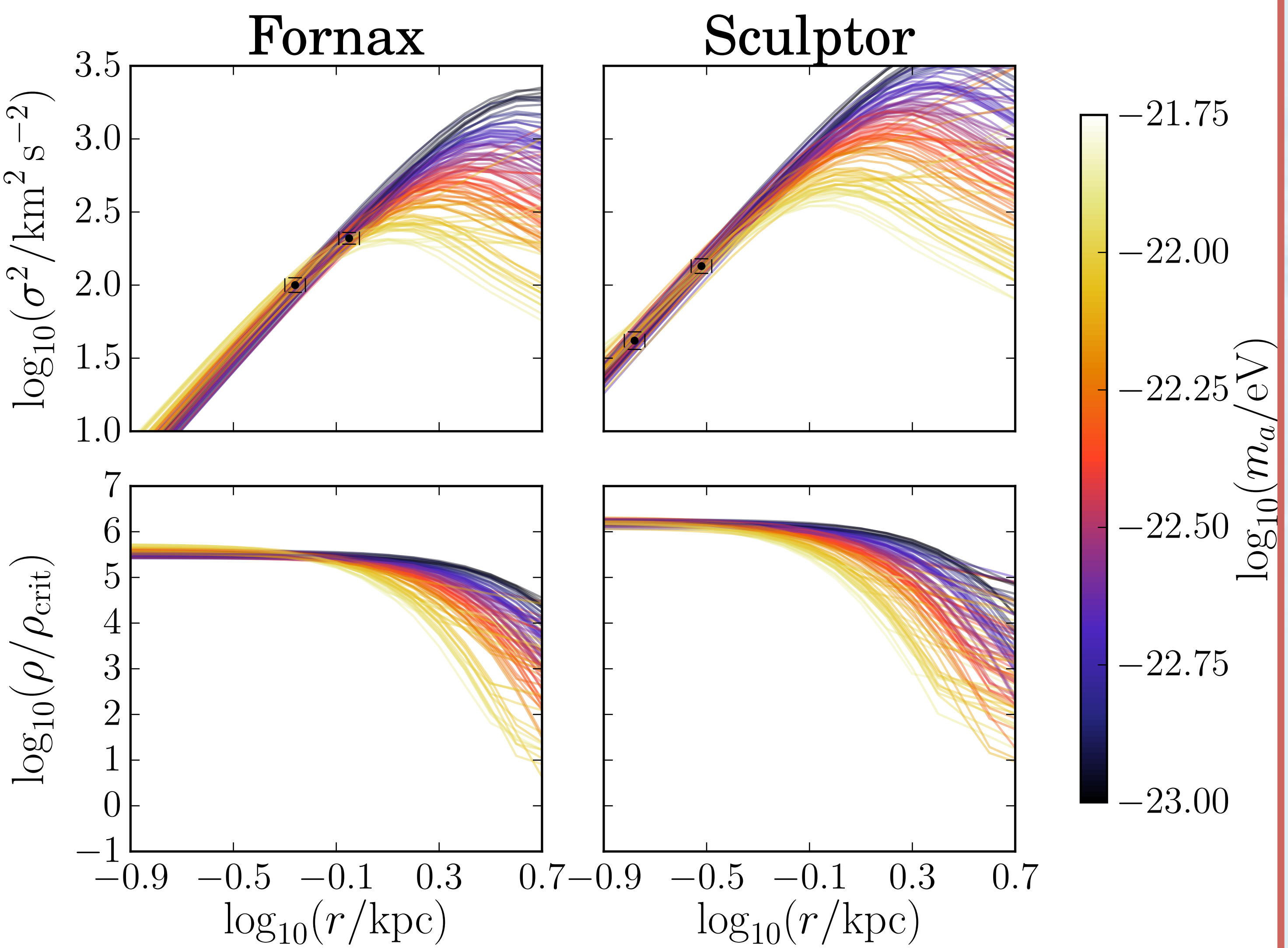
CMB mass prior: Hlozek et al (2014)

$$m_a \gtrsim 10^{-25}\ \mathrm{eV}$$

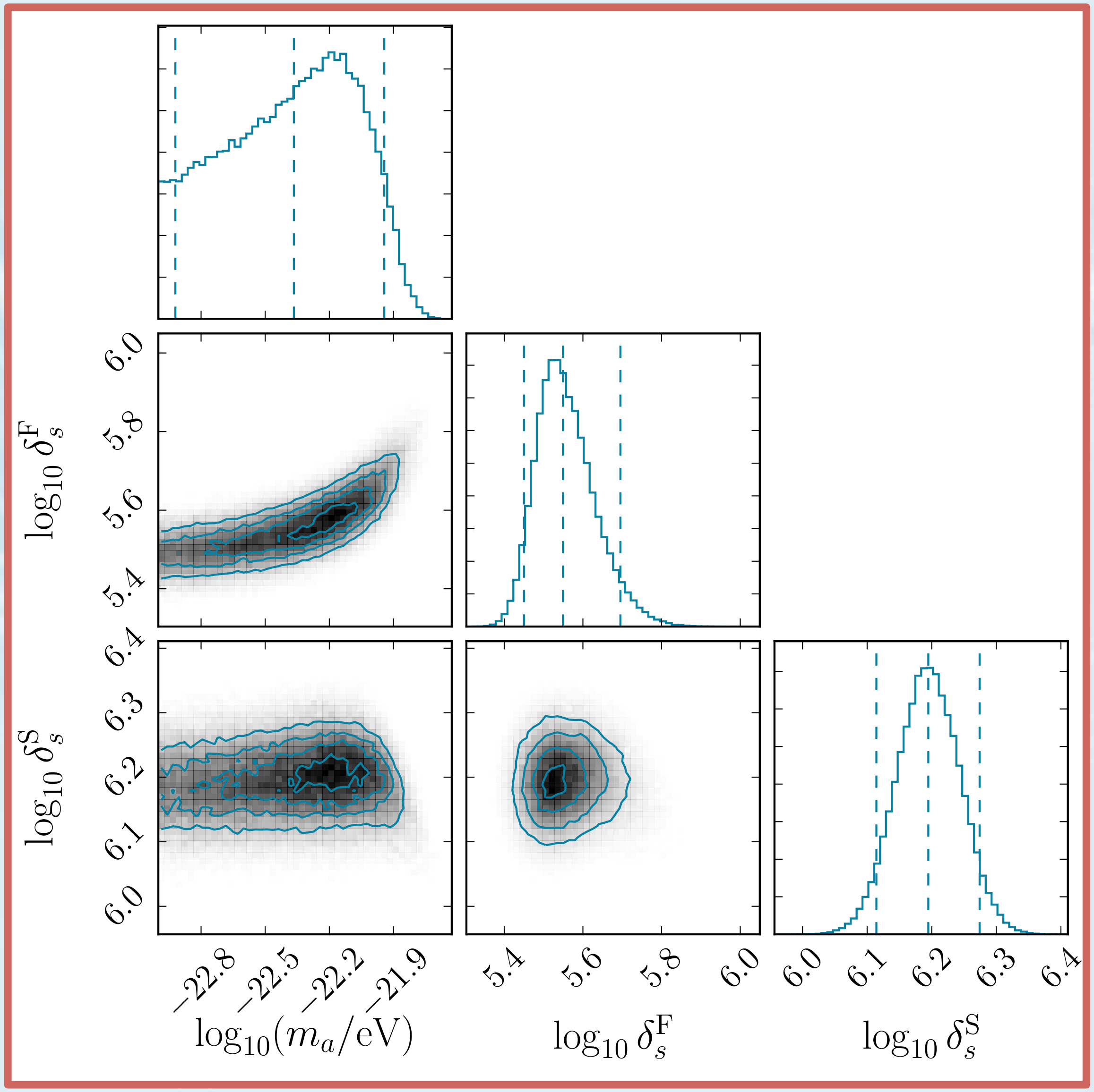
HUDF mass prior: Bozek et al (2014)

$$m_a \gtrsim 10^{-23}\ \mathrm{eV}$$

Results

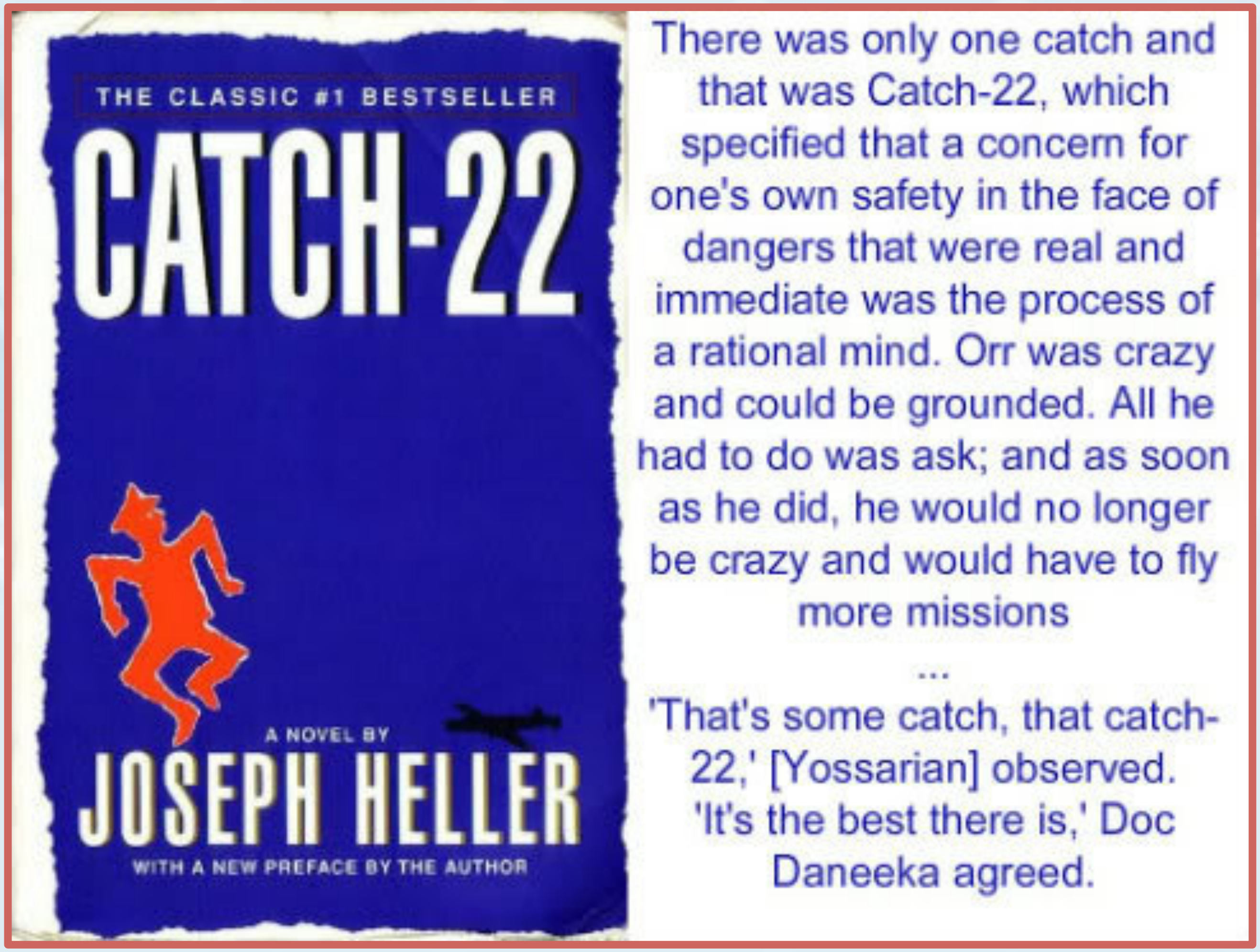

Fornax
Sculptor
$\log_{10}(\sigma^2/\mathrm{km}^2\,\mathrm{s}^{-2})$
3.5
3.0
2.5
2.0
1.5
1.0
$\log_{10}(\rho/\rho_{\mathrm{crit}})$
7
6
5
4
3
2
1
0
−1
−0.9
−0.5
−0.1
0.3
0.7
$\log_{10}(r/\mathrm{kpc})$
$\log_{10}(m_a/\mathrm{eV})$
−21.75
−22.00
−22.25
−22.50
−22.75
−23.00

Results



6.0
5.8
5.6
5.4
$\log_{10} \delta_s^{\mathrm{F}}$
6.4
6.3
6.2
6.1
6.0
$\log_{10} \delta_s^{\mathrm{S}}$
−22.8
−22.5
−22.2
−21.9
$\log_{10}(m_a/\mathrm{eV})$
5.4
5.6
5.8
6.0
$\log_{10} \delta_s^{\mathrm{F}}$
6.0
6.1
6.2
6.3
6.4
$\log_{10} \delta_s^{\mathrm{S}}$

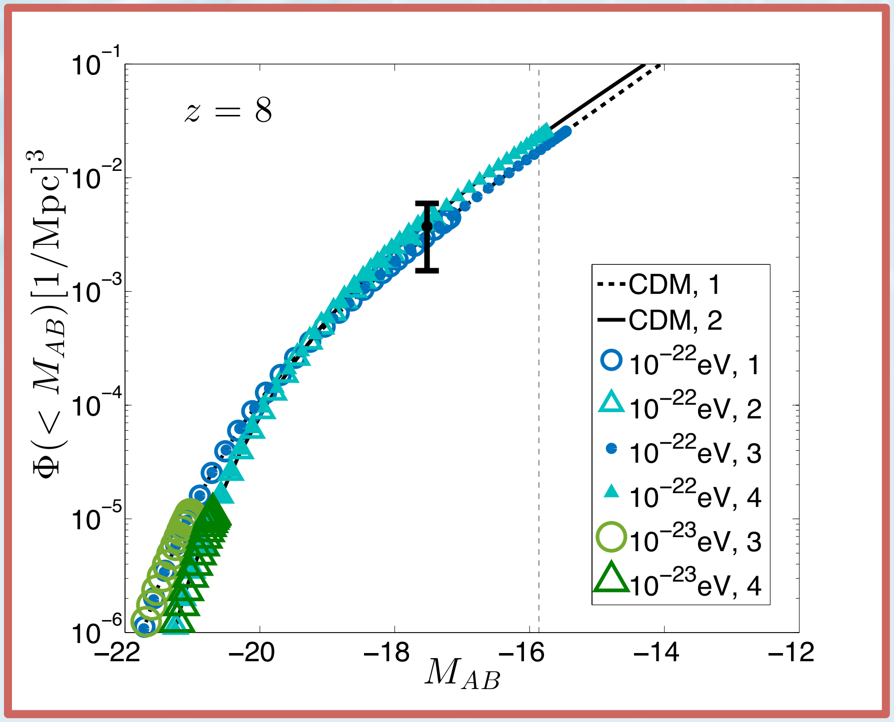
A “Catch 22” for axions?



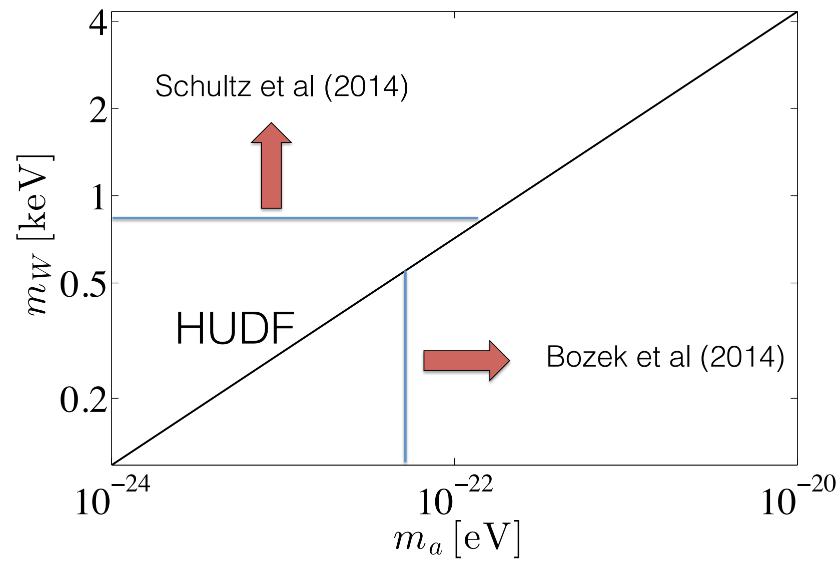
A "Catch 22" for axions?

Parameter	Posterior
$(m_a/\mathrm{eV})_{\mathrm{HUDF}}$	$< 1.1 \times 10^{-22}$ (95% C.L.)
$(m_a/\mathrm{eV})_{\mathrm{CMB}}$	$< 1.0 \times 10^{-22}$ (95% C.L.)
$\log_{10} \delta_s^{\mathrm{F}}$	$5.55^{+0.06}_{-0.08}$
$\log_{10} \delta_s^{\mathrm{S}}$	6.19 ± 0.05

Structure formation only just allows ULAs to be light enough for cores.
→ ULA cusp-core solution can be falsified in near future.



What about Warm DM?



DJEM & Silk (2013)

$$m_W \sim \sqrt{m_a M_{pl}}$$

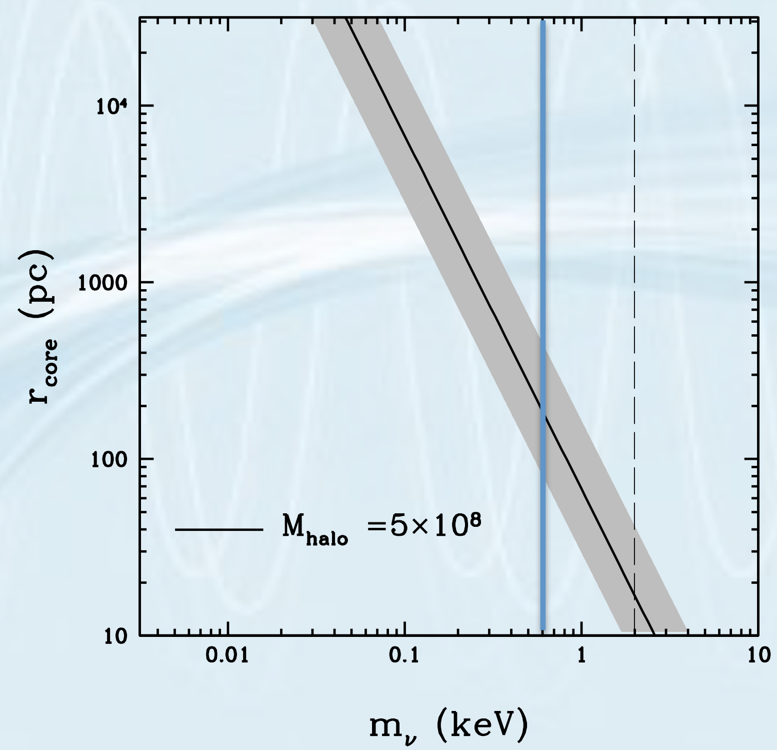


Fig: Maccio et al (2012)

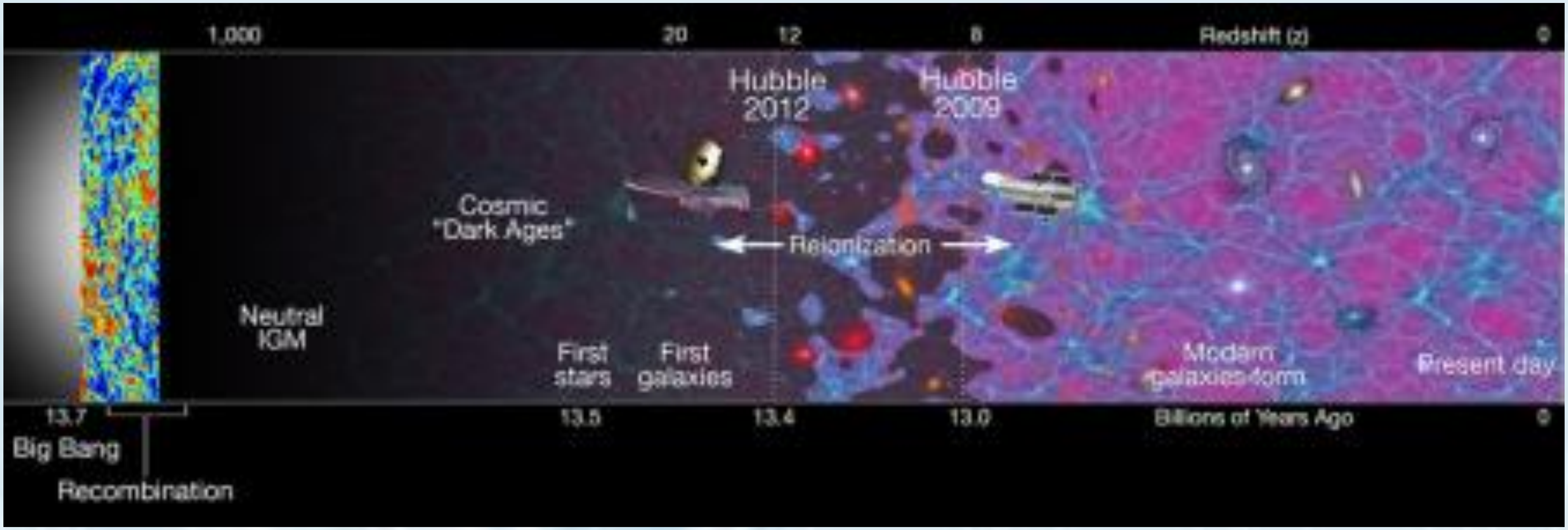
WDM cores are much more compact: really is a Catch 22.
(definitions & methods differ: systematic comparison necessary)

Possible outcomes

- **Pessimistic:** *Structure formation and the requirement of dSph cores put conflicting demands on axion DM. This places the model in a 'Catch 22' analogous to WDM.*[6] *This particle physics model is no longer a catch-all solution to the small-scale crises, and additional mechanisms are required for its consistency.*
- **Optimistic:** *Ultra-light axions are responsible for dSph cores. The cut-off in the power spectrum is just outside current observational reach. Near-future experiments will turn up striking evidence for axions in structure formation and in study of the high-z universe.*

Reionization predictions

Bozek et al (2014)



Credit: NASA/ESA from caltech.edu

Low optical depth → consistent with Planck

$$\tau_{\rm re} \sim 0.05$$

Late and rapid reionization → measure with kSZ or 21cm

$$z_{\rm re} \sim 7 \qquad \delta z_{\rm re} \sim 1.5$$

e.g. Calabrese et al (2014)
Mesinger et al (2014)

Summary

$$i\partial_t\psi = -\frac{1}{2m_a}\nabla^2\psi + m_a V\psi$$

$$\nabla^2 V = 4\pi G|\psi|^2$$