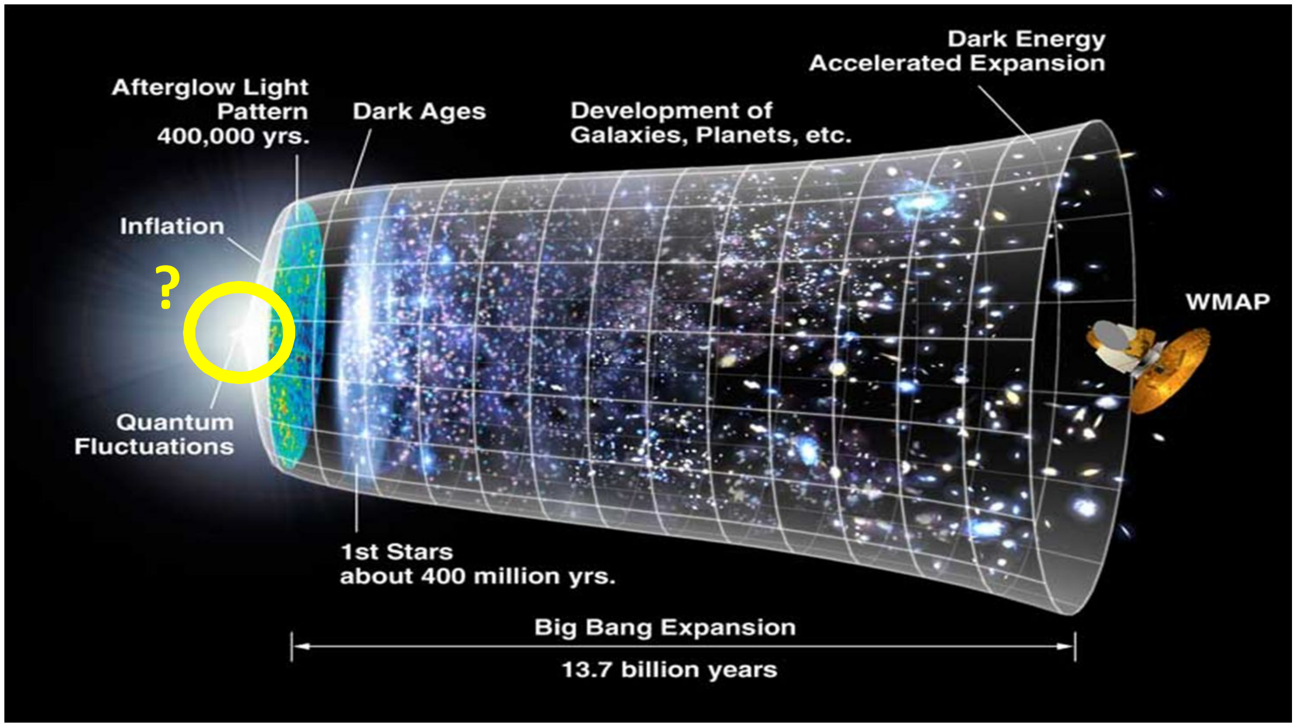
Contents

Introduction

? Physics in the early Universe ?



?

Inflation

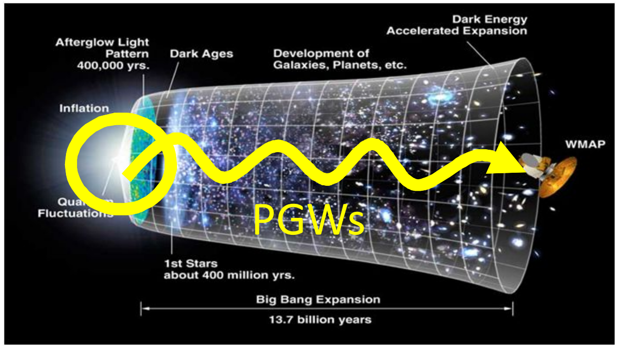
(inflaton＋graviton)

$\xrightarrow{\delta\varphi}$ Fluctuations of CMB anisotropy, large scale structures

$\xrightarrow{\delta g_{ij}}$ **Primordial gravitational waves (PGWs)**

(today's talk)

PGWs observations



- Observational method
  - B-mode polarization of CMB
  - （Space）gravitational wave interferometer
  - Pulsar timing array etc.

- Observational quantity
  - Tensor-to-scalar ratio $$r = \frac{\Delta_t}{\Delta_s} < 0.07$$ BICEP2/Keck Array (2016)
  - Energy intensity of GWs $$\Omega_{\rm gw}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\ln f} \qquad (\rho_c = 3M_p^2 H_0^2)$$

How are primordial GWs generated?

Metric of spacetime

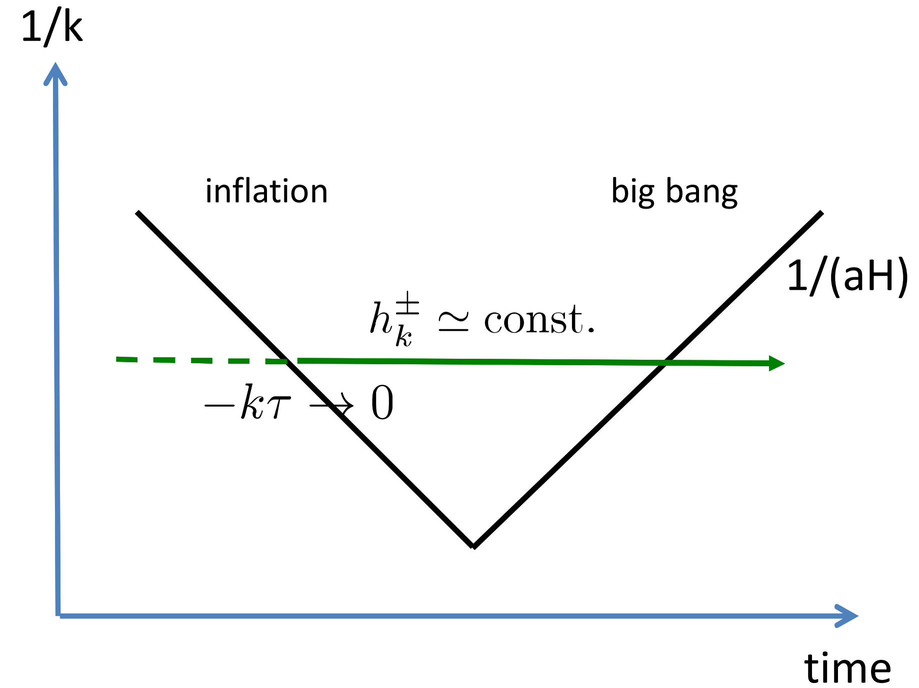
$$g_{\mu\nu}dx^\mu dx^\nu = a(\tau)^2[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

Substituting in Einstein eq.

$$(ah_{ij})'' + \left(k^2 - \frac{a''}{a}\right)(ah_{ij}) = 0$$

Solving in Fourier mode...

$$\boxed{h_k^\pm = \frac{H}{\sqrt{2k^3}}(i - k\tau)e^{-ik\tau}}$$



How are primordial GWs generated?

Metric of spacetime

$$g_{\mu\nu}dx^\mu dx^\nu = a(\tau)^2[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

GWs powerspectrum

$$\boxed{\Delta_h^{\pm}(k) = \left.\frac{H^2}{\pi^2 M_p^2}\right|_{k=aH}}$$

(nearly) scale-invariant

parity-symmetric

CMB observations (COBE, WMAP,…)

$$\Rightarrow\ E_{\rm inf} \simeq 10^{16}\,{\rm GeV}\left(\frac{r}{0.01}\right)^{1/4} \longleftarrow$$

High energy physics?

(GUT, string theory, supergravity…)

How are primordial GWs generated?

GWs spectrum is related to the energy density of GWs

$$h_0^2\Omega_{\rm gw}(f) = \frac{h_0^2}{\rho_c}\frac{d\rho_{\rm gw}}{d\ln f} \simeq 10^{-6}\Delta_h^{\pm}(k) \qquad (h_0 : \text{dimensionless Hubble parameter})$$

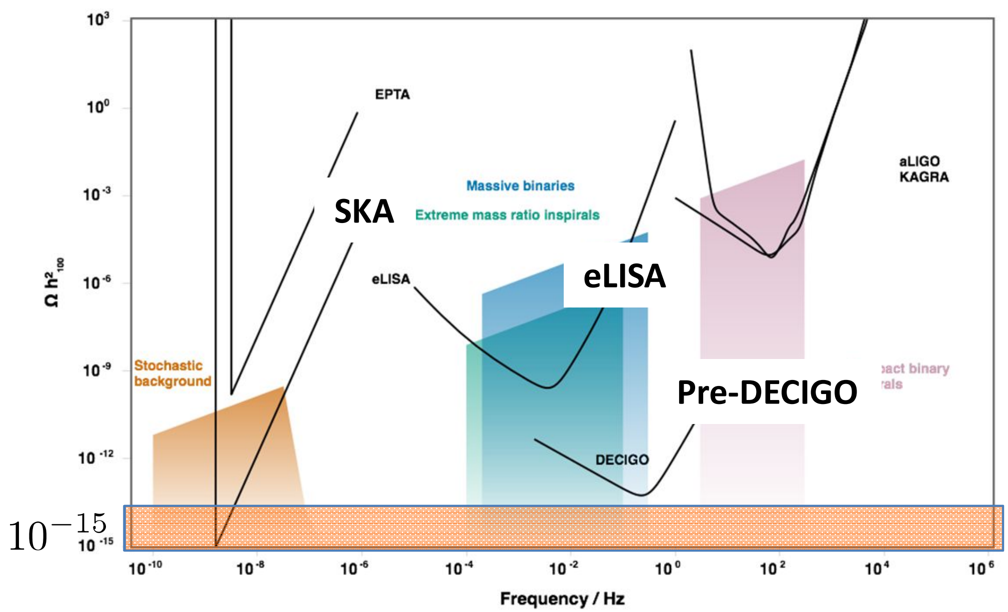
$(f > 10^{-16}\ \mathrm{Hz})$

(simple inflation models…)

$$\boxed{h_0^2\Omega_{\rm gw}(f) \simeq 10^{-15}\left(\frac{E_{\rm inf}}{2\times 10^{16}\mathrm{GeV}}\right)^4}$$

$(f > 10^{-16}\ \mathrm{Hz})$

Sensitivity curves of GWs



http://rhcole.com/apps/GWplotter/

Today's talk...

Reconsidering PGWs production...

Ordinal EOM

$$(ah_{ij})'' + \left(k^2 - \frac{a''}{a}\right)(ah_{ij}) = 0$$

(vacuum fluctuations)

In fact...

$$(ah_{ij})'' + \left(k^2 - \frac{a''}{a}\right)(ah_{ij}) = \underline{S_{ij}}$$

matter

$$\longrightarrow \quad h_{ij} = (h_{ij})_{\text{vacuum}} + \underline{(h_{ij})_{\text{source}}}$$

Scale-dependent!

Parity-violated!

Etc.

Particle production of GWs predicted by high energy physics!

Today's talk…

Axion phyrsics

（string theory, supergravity）

inflation

Particle production of gauge fields

GWs provided by gauge fields

- ✧ **Detectable amplitude**
- ✧ **Parity-violated spectrum**

Lagrangian density for Axion system

Fourier mode

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) - \frac{1}{4}FF - \boxed{\frac{\lambda}{4}\frac{\varphi}{f}F\tilde{F}} \supset \pm \lambda k \frac{\bar{\varphi}'}{f}\delta A_k^{\pm}\delta A_k^{\pm}$$

$$|\delta A_k^{+}| \neq |\delta A_k^{-}| \longrightarrow |\delta g_k^{+}| \neq |\delta g_k^{-}|$$

Consistent with CMB data

goal：researching an inflationary model to generate chiral gravitational waves

Chiral GWs from an axionic inflation

Axionic inflation with SU(2) gauge field

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

Chromo-natural inflation

P.Adshead & M. Wyman 2012

$$\begin{aligned} S &= S_{\rm EH} + S_{\rm axion} + S_{\rm gauge} + S_{\rm CS} \\ &= \int d^4x \left[\frac{1}{2}R - \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\lambda\frac{\varphi}{f}\tilde{F}^{a\mu\nu}F^a_{\mu\nu} \right] \end{aligned}$$

$$V(\varphi) = \Lambda^4 \left[1 - \cos\left(\frac{\varphi}{f}\right) \right]$$

$$\tilde{F}^{a\mu\nu} \equiv \frac{1}{2}\sqrt{-g}\epsilon^{\mu\nu\lambda\sigma}F^a_{\lambda\sigma}$$

Background configurations

space-time (flat FLRW) : $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$

inflaton + SU(2) gauge field : $\varphi = \varphi(t)$

$$\boxed{A^a_i = a(t)Q(t)\delta^a_i}$$

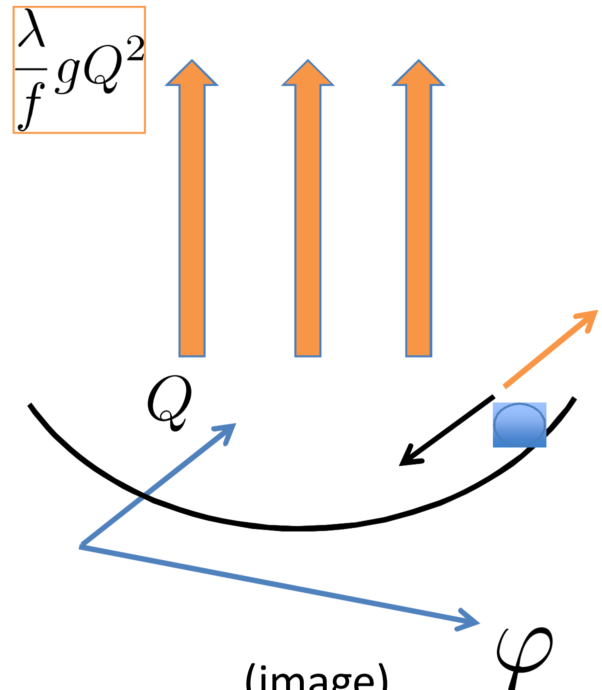
Solving background equations for not only ϕ(t) but Q(t) !

Axionic inflation with SU(2) gauge field

Equations of motions for ϕ(t) and Q(t)

$$\cancel{\ddot{\varphi}} + 3H\dot{\varphi} + V_{\varphi} = \underline{-3\frac{\lambda}{f}gQ^2(\dot{Q} + HQ)}$$

$$\cancel{\ddot{Q}} + 3H\dot{Q} + (2H^2 + \cancel{\dot{H}})Q + 2g^2Q^3 = \underline{\frac{\lambda}{f}gQ^2\dot{\varphi}}$$



(image)

$\boxed{\text{Assuming } \lambda \gg 1}$

``Magnetic drift force” dominates over Hubble friction!

(gauge field assists slow-roll motion of inflaton !)

Slow-roll solutions

$$Q(t) \simeq Q_{\min} \equiv -\left(\frac{V_{\tilde{\varphi}}}{3\lambda g H}\right)^{1/3} \qquad \frac{\lambda\dot{\varphi}}{2fH} \simeq m_Q + \frac{1}{m_Q}$$

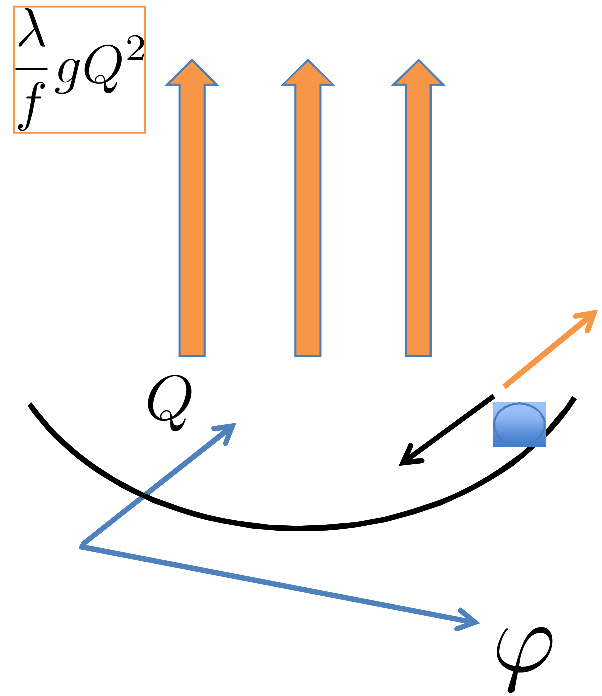
$$\boxed{m_Q \equiv \frac{gQ}{H}} \qquad \boxed{\tilde{\varphi} \equiv \frac{\varphi}{f}}$$

Axionic inflation with SU(2) gauge field

Equations of motions for ϕ(t) and Q(t)

$$\cancel{\ddot{\varphi}} + 3H\dot{\varphi} + V_\varphi = -3\frac{\lambda}{f}gQ^2(\dot{Q}+HQ)$$

$$\cancel{\ddot{Q}} + 3H\dot{Q} + (2H^2 + \cancel{\dot{H}})Q + 2g^2Q^3 = \frac{\lambda}{f}gQ^2\dot{\varphi}$$



$\boxed{\text{Assuming } \lambda \gg 1}$

``Magnetic drift force” dominates over Hubble friction!

Inflationary dynamics (slow-roll parameters + energy density of elemag fields)

$$\epsilon_H = -\frac{\dot{H}}{H^2} \simeq \frac{1}{\lambda}\frac{1+m_Q^2}{m_Q}\frac{V_{\tilde{\varphi}}}{V} \qquad \boxed{m_Q \equiv \frac{gQ}{H}}$$

$$\eta_H = \frac{\dot{\epsilon}_H}{\epsilon_H H} \simeq \frac{1}{\lambda}\frac{1+m_Q^2}{m_Q}\left(\frac{2V_{\tilde{\varphi}}}{V} - \frac{V_{\tilde{\varphi}\tilde{\varphi}}}{V_{\tilde{\varphi}}}\right)$$

$$\rho_E \equiv \frac{3}{2}E^2 \simeq \frac{3}{2}H^2Q_{\rm min}^2$$

$$\rho_B \equiv \frac{3}{2}B^2 \simeq \frac{3}{2}g^2Q_{\rm min}^4$$

Axionic inflation with SU(2) gauge field

$$S = S_{\rm EH} + S_{\rm axion} + S_{\rm gauge} + S_{\rm CS}$$
$$= \int d^4x \left[\frac{1}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi) - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\lambda\frac{\varphi}{f}\tilde{F}^{a\mu\nu}F^a_{\mu\nu}\right]$$

Considering perturbation…

$$\varphi = \bar{\varphi}(t) + \delta\varphi$$
$$A^a_i = a(t)Q(t)\delta^a_i + \boxed{\delta A^a_i}$$

$$\delta A^a_i \supset t^a_i \leftrightarrow \delta g_{\mu\nu}$$

Couples to metric tensor mode!

Parity-violating interaction

Fourier mode

$$\lambda\frac{\varphi}{f}{\rm Tr}F\tilde{F} \supset \underline{\pm}\lambda k\frac{\bar{\varphi}'}{f}\delta A^\pm_k\delta A^\pm_k$$

$$|\delta A^+_k| \neq |\delta A^-_k|$$

Axionic inflation with SU(2) gauge field

quantization

$$t_{ij}(\boldsymbol{x},\tau) = \sum_{A=\pm} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} e^A_{ij}(\hat{\boldsymbol{k}}) t^A_{\boldsymbol{k}}(\tau) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$= \sum_{A=\pm} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \left[e^A_{ij}(\hat{\boldsymbol{k}}) t^A_k(\tau) b^A_{\boldsymbol{k}} + e^{A*}_{ij}(-\hat{\boldsymbol{k}}) t^{A*}_k(\tau) b^{A\dagger}_{-\boldsymbol{k}} \right] e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$[b^A_{\boldsymbol{k}}, b^{B\dagger}_{-\boldsymbol{k}'}] = (2\pi)^3 \delta_{AB} \delta^3(\boldsymbol{k}+\boldsymbol{k}')$$

EOM for free gauge particle $(x \equiv -k\tau$: dimensionless time variable$)$

$$\frac{d^2 t^\pm_k}{dx^2} + \left(1 + \frac{A}{x^2} \mp \frac{2B}{x}\right) t^\pm_k \simeq 0$$

$$A = 2(m_Q^2 + 1) > 0$$
$$B = 2m_Q + m_Q^{-1} > 0$$

$$t^+_k : \quad m^2 < 0 \quad \text{for} \quad \frac{1}{2}(B - \sqrt{B^2 - A}) < x < \frac{1}{2}(B + \sqrt{B^2 - A})$$

One helicity mode is enhanced due to a tachyonic instability!

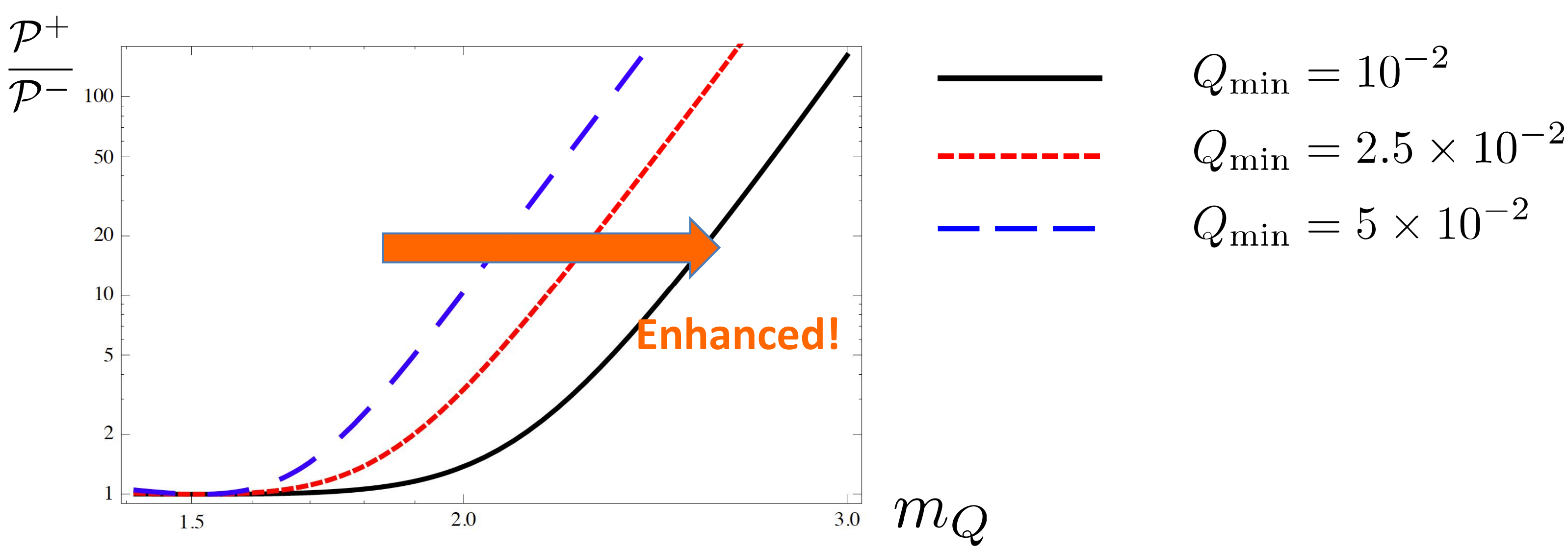
$\longrightarrow$ producing parity-violated gravitational tensor modes!

Axionic inflation with SU(2) gauge field

Gravitational waves power spectrum in this model

$$\mathcal{P}_h^-(k) = \langle in|\, h_k^-(\tau)^2 \,|in\rangle \simeq \frac{2H^2}{k^3}$$

$$\mathcal{P}_h^+(k) = \langle in|\, h_k^+(\tau)^2 \,|in\rangle \simeq \frac{2H^2}{k^3}\left[1 + 8|C_2|^2 Q_{\rm min}^2 \left|\mathcal{I}_0(m_Q) - m_Q \mathcal{I}_1(m_Q) + m_Q^2 \mathcal{I}_2(m_Q)\right|^2\right]$$



P.Adshead, E. Martinec & M. Wyman 2013

Axionic inflation with SU(2) gauge field

Observational constraint on CMB (scalar modes)

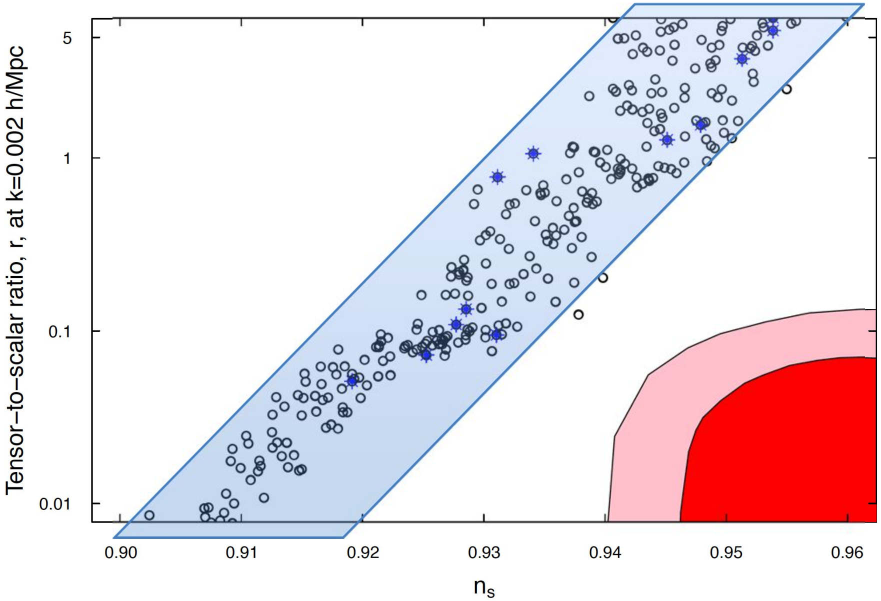
$$\mathcal{R} \simeq \frac{1}{\sqrt{2k^3}} \frac{H}{\sqrt{2\epsilon_H}} \frac{m_Q}{(1+m_Q^2)^{1/2}(m_Q-\sqrt{2})\sqrt{3/2}} \quad (m_Q \lesssim 2)$$

Gauge field also produces scalar part.

$$\delta A + \bar{A} \longrightarrow \delta\varphi \ (\mathcal{R})$$

affect

n_s , r (CMB observation)



P.Adshead, E. Martinec & M. Wyman 2013

In conflict with CMB observation…

Our study

Reconsidering conventional model...

Chromo-natural inflation

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) - \frac{1}{4}FF - \frac{\lambda}{4}\frac{\varphi}{f}F\tilde{F}$$

✕ **CMB observation**

(large amount of chiral GWs)

Considering from the point of high energy physics...

$$FF \to I(t)^2 FF \quad \left(\lambda \to \frac{\lambda}{I(t)^2} \equiv \lambda_{\mathrm{eff}}(t) : \mathrm{dynamical}\right)$$

A gauge-kinetic function of scalar field "dilaton"

Chiral GWs generation should depend on time!

Dilaton and Axion with SU(2) fields

Model building

IO and J. Soda 2016

Action

$$S = S_{\mathrm{EH}} + S_{\mathrm{dilaton}} + S_{\mathrm{axion}} + S_{\mathrm{gauge}} + S_{\mathrm{CS}}$$
$$= \int dx^4 \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi) - \frac{1}{2}(\partial_\mu\sigma)^2 - W(\sigma) - \frac{1}{4}I(\varphi)^2 F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\lambda\frac{\sigma}{f}\tilde{F}^{a\mu\nu}F^a_{\mu\nu}\right]$$

The scalar fields and SU(2) gauge field

$\varphi(t)$: dilaton

$Q(t)$: VEV of the gauge fields

$$A_i^a = A(t)\delta_i^a = a(t)Q(t)\delta_i^a$$

$\sigma(t)$: axion

Dilaton and Axion with SU(2) fields

Friedmann equation and EOM for a(t)

$$3H^2 = \frac{1}{2}\dot{\varphi}^2 + V + \frac{1}{2}\dot{\sigma}^2 + W + \rho_E + \rho_B$$

$$\dot{H} = -\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\sigma}^2 + \frac{2}{3}(\rho_E + \rho_B)\right)$$

$$\rho_E \equiv \frac{3}{2}I^2E^2 = \frac{3}{2}I^2\frac{\dot{A}^2}{a^2}$$

$$\rho_B \equiv \frac{3}{2}I^2B^2 = \frac{3}{2}I^2\frac{g^2A^4}{a^4}$$

EOMs for ϕ(t), σ(t) and A(t)

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 2\frac{I_\varphi}{I}(\rho_E - \rho_B)$$

$$\ddot{\sigma} + 3H\dot{\sigma} + W_\sigma = -3\frac{\lambda}{f}EB$$

$$\ddot{A} + \left(H + 2\frac{\dot{I}}{I}\right)\dot{A} + 2g^2\frac{A^3}{a^2} = \frac{\lambda}{f}\dot{\sigma}g\frac{A^2}{aI^2}$$

$$I(\varphi) = I_0 \exp[-n\int_0 \frac{V}{V_\varphi}d\varphi]$$

Dilaton and Axion with SU(2) fields

Initial conditions of the background motion

- dilaton is energy dominant (playing a role of an inflaton) :

$$3H^2 \simeq V \quad (\varphi(t) : \text{inflaton})$$

→ Producing CMB fluctuations!

- weak gauge kinetic coupling function :

$$I(\varphi) \propto a(t)^n \ , \ n \leq -2 \quad \left(\frac{\lambda}{I(\varphi_i)^2} \ll 1 \right)$$

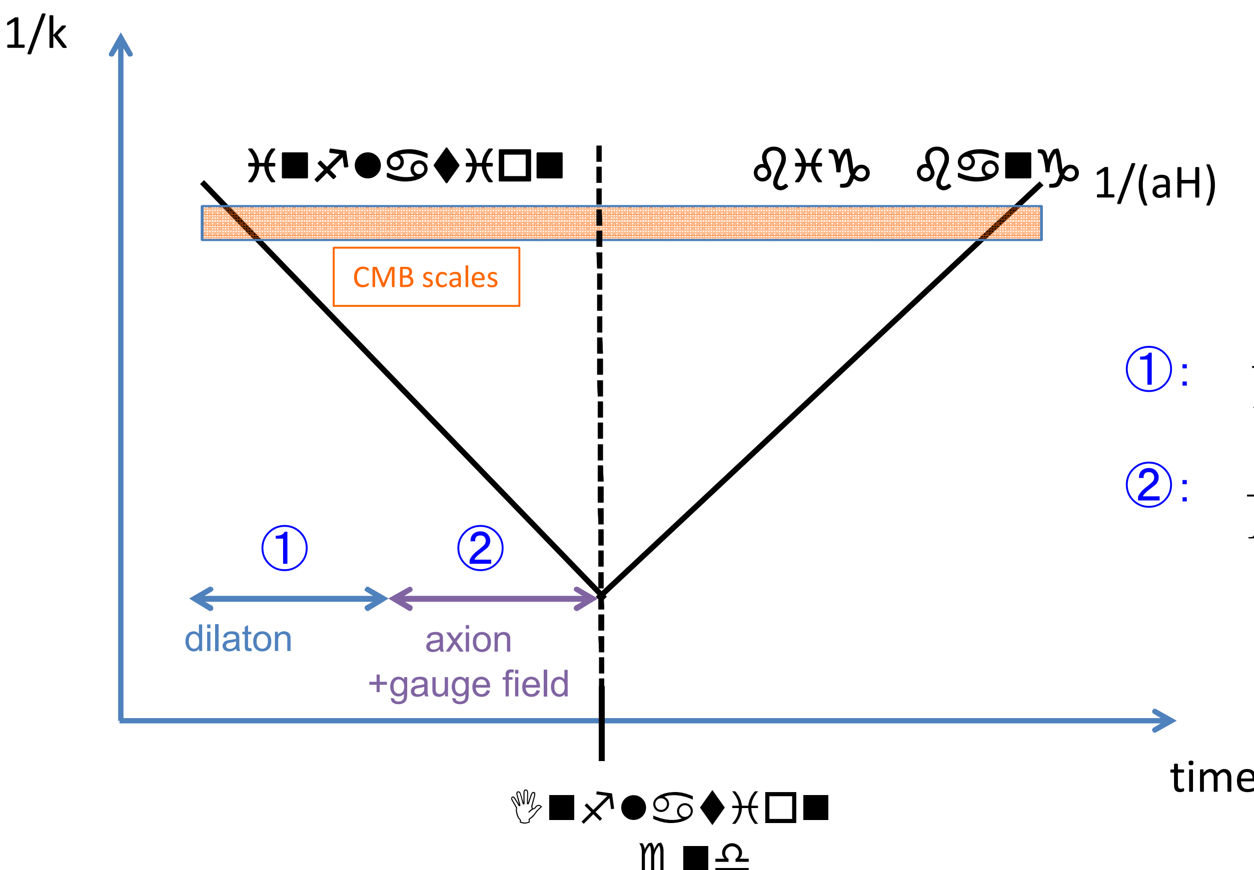
(Suppressing axion-gauge interactions)

- The vev of gauge field is near the origin of its effective potential:

$$U_{\text{eff}}(Q) = \left(1 + \frac{\dot{I}}{HI}\right) H^2 Q^2 - \frac{1}{3}\frac{\lambda}{f}\dot{\sigma}\frac{gQ^3}{I^2} + \frac{1}{2}g^2 Q^4$$

Dilaton and Axion with SU(2) fields

A background inflationary dynamics (two stages)

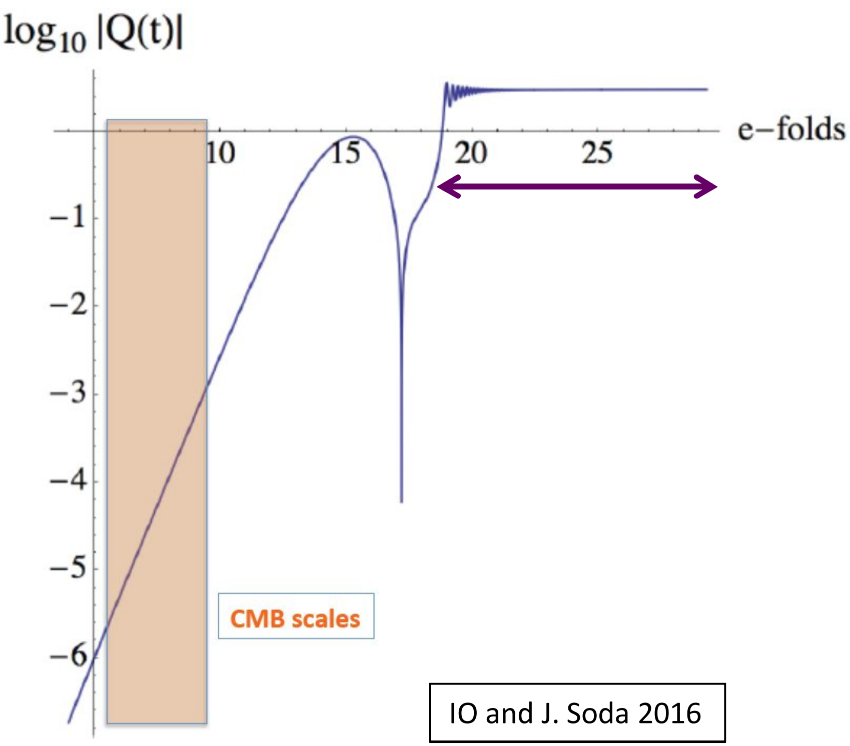


①: $\dfrac{\lambda}{I(\varphi)^2} \ll 1$ (× Chiral GWs)

②: $\dfrac{\lambda}{I(\varphi)^2} \gtrsim 1$ (○ Chiral GWs)

Dilaton and Axion with SU(2) fields

The dynamics of the vev of gauge field



$(\Lambda_\varphi, \Lambda_\sigma, r, n, g, \lambda, f) = (10^{-2}, 2\times10^{-3}, 1, -2.01, 10^{-6}, 10^{-1}, 10)$

$V(\varphi) = \Lambda_1^4 \exp[r\varphi] \quad W(\sigma) = \Lambda_2^4\left(1 - \cos\left(\frac{\sigma}{f}\right)\right)$

Slow-roll solutions of ⟷

$$3H^2 \simeq W$$

$$Q \simeq Q_{\min} = -\left(\frac{fW_\sigma}{3\lambda g H}\right)^{1/3} \quad \text{(const.)}$$

$$\frac{1}{2}\frac{\lambda}{I^2}\frac{\dot\sigma}{fH} \simeq m_Q + \frac{1}{m_Q}\left(1 + \frac{\dot I}{HI}\right)$$

Chromo-natural inflation

(Providing chiral GWs!) (>nHz)

Dilaton and Axion with SU(2) fields

Tensor mode dynamics

Metric: $\psi_{ij} = a h_{ij}$

Gauge field: $I(\delta A_i^a)_{TT} = t_i^a$

$$\psi_{ij}(\boldsymbol{x},\tau) = 2\sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} e_{ij}^A(\hat{\boldsymbol{k}})\psi_{\boldsymbol{k}}^A(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$
$$= 2\sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[e_{ij}^A(\hat{\boldsymbol{k}})\psi_k^A(\tau)a_{\boldsymbol{k}}^A + e_{ij}^{A*}(-\hat{\boldsymbol{k}})\psi_k^{A*}(\tau)a_{-\boldsymbol{k}}^{A\dagger}\right]e^{i\boldsymbol{k}\cdot\boldsymbol{x}} ,$$
$$t_{ij}(\boldsymbol{x},\tau) = \sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} e_{ij}^A(\hat{\boldsymbol{k}})t_{\boldsymbol{k}}^A(\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$
$$= \sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[e_{ij}^A(\hat{\boldsymbol{k}})t_k^A(\tau)b_{\boldsymbol{k}}^A + e_{ij}^{A*}(-\hat{\boldsymbol{k}})t_k^{A*}(\tau)b_{-\boldsymbol{k}}^{A\dagger}\right]e^{i\boldsymbol{k}\cdot\boldsymbol{x}} ,$$

$$[a_{\boldsymbol{k}}^A, a_{-\boldsymbol{k}'}^{B\dagger}] = [b_{\boldsymbol{k}}^A, b_{-\boldsymbol{k}'}^{B\dagger}] = (2\pi)^3\delta_{AB}\delta^3(\boldsymbol{k}+\boldsymbol{k}')$$

EOMs for free tensor modes in dS space-time $(x = -k\tau)$

$$\frac{d^2\psi_k^\pm}{dx^2} + \left(1 - \frac{2}{x^2}\right)\psi_k^\pm = 0 ,$$
$$\frac{d^2t_k^\pm}{dx^2} + \left(1 - \frac{d^2I/dx^2}{I} + \frac{2m_Q\xi}{x^2} \mp \frac{2(m_Q+\xi)}{x}\right)t_k^\pm = 0$$

$$\xi \equiv \frac{\lambda}{2I^2}\frac{\dot{\sigma}}{fH}$$

$$m_Q \equiv \frac{gQ}{H}$$

Dilaton and Axion with SU(2) fields

Tensor mode dynamics

Weinberg 2005

Calculating tensor spectrum…(using in-in formalism)

$$
\begin{aligned}
&(2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}')\,\langle in|\,h_k^A(\tau)^2\,|in\rangle = \sum_{N=0}^{\infty}(-i)^N\int^{\tau}d\tau_1\int^{\tau_1}d\tau_2\,...\int^{\tau_{N-1}}d\tau_N \\
&\times\langle 0|\left[\left[\left[h_{\boldsymbol{k}}^A(\tau)h_{\boldsymbol{k}'}^A(\tau)\,,\;H_I(\tau_1)\right]\,,H_I(\tau_2)\right]\,...\,,H_I(\tau_N)\right]|0\rangle\;.
\end{aligned}
$$

$$
|in\rangle \equiv T\exp\left(-i\int_{-\infty(1+\epsilon)}^{\tau}d\tilde{\tau}H_I(\tilde{\tau})\right)|0\rangle
$$

$$
\begin{aligned}
H_I(\tau) &= -\int d^3\boldsymbol{x}\left[\frac{\sqrt{\epsilon_E}}{\tau}\psi^{ij}v_{ij}-\frac{\sqrt{\epsilon_B}}{\tau}\psi^{jm}\epsilon_{ij}^a t_{m,i}^a+\frac{\sqrt{\epsilon_B}m_Q}{\tau^2}\psi^{ij}t_{ij}+\frac{\epsilon_E-\epsilon_B}{4\tau^2}\psi^{ij}\psi_{ij}\right] \\
&= -2\sum_{A=\pm}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[J_{\boldsymbol{k}}^A\,\psi_{-\boldsymbol{k}}^A+\frac{\epsilon_E-\epsilon_B}{2\tau^2}\psi_{\boldsymbol{k}}^A\,\psi_{-\boldsymbol{k}}^A\right]\,,
\end{aligned}
$$

$$
v_{ij}\equiv t_{ij}'-\frac{I'}{I}t_{ij}\,,\qquad v_{\boldsymbol{k}}^A=t_{\boldsymbol{k}}^{A\prime}-\frac{I'}{I}t_{\boldsymbol{k}}^A\,,
$$

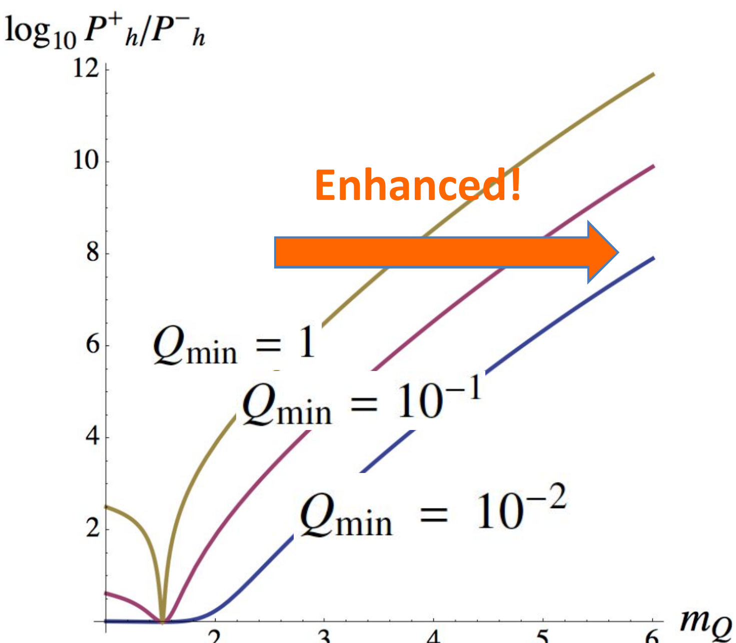
$$
J_{\boldsymbol{k}}^{\pm}(\tau)\equiv\frac{\sqrt{\epsilon_E}}{\tau}v_{\boldsymbol{k}}^{\pm}+\left(\frac{\sqrt{\epsilon_B}m_Q}{\tau^2}\pm\frac{k\sqrt{\epsilon_B}}{\tau}\right)t_{\boldsymbol{k}}^{\pm}
$$

Dilaton and Axion with SU(2) fields

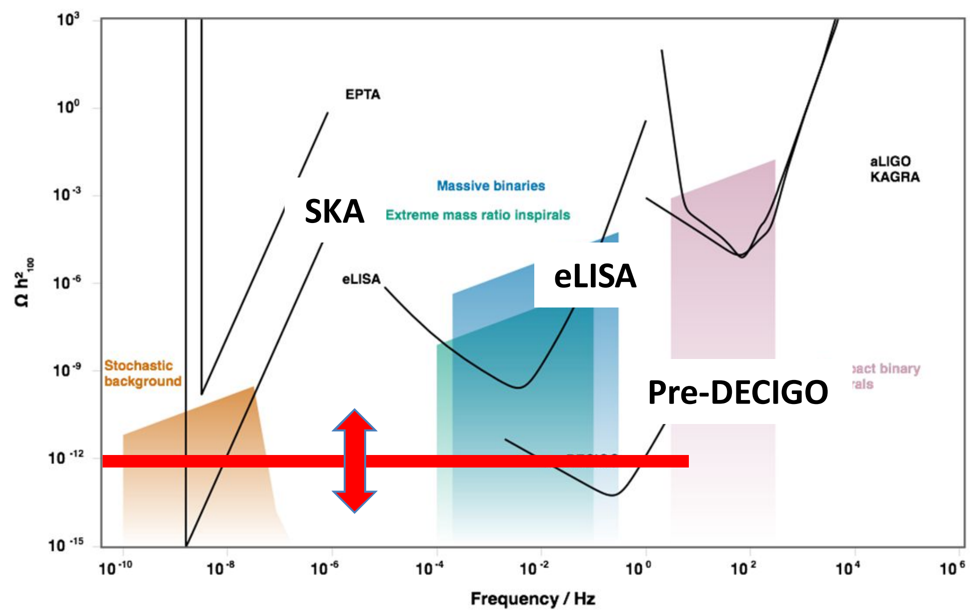
Power spectrum of chiral GWs in late time periods :

$$\mathcal{P}_h^-(k) \simeq \frac{2H^2}{k^3}$$

$$\mathcal{P}_h^+(k) \simeq \frac{2H^2}{k^3}\left[1 + 8Q_{\rm min}^2 \left|\mathcal{I}_0(m_{\bar{Q}}) - m_{\bar{Q}}\mathcal{I}_1(m_{\bar{Q}}) + (-2 + m_{\bar{Q}}^2)\mathcal{I}_2(m_{\bar{Q}})\right|^2\right]$$



IO and J. Soda 2016



Summary & outlook

Summary & Outlook

- We study the mechanism of generating chiral primordial gravitational waves from particle production, which are consistent with CMB observations.
- We introduce a dilatonic field in the conventional model , chromo-natural inflation, and generalize an axion-gauge interaction dynamically based on the fundamental theory.
- We might discover the parameter region where chiral GWs consistent with CMB data are produced, which might be detectable in future GW experiments (DECIGO, eLISA, SKA…).
- We must check the reheating age in this model and consider the dynamics of anisotropic background.

Appendix

Dilaton and Axion with SU(2) fields

EOMs for A(t) in an early period

$$\ddot{A} + \left(H + 2\frac{\dot{I}}{I}\right)\dot{A} \simeq 0 \quad \longrightarrow \quad aI^2\dot{A} = \text{const.} \equiv C$$

$$\dashrightarrow \quad \rho_E = \frac{3}{2}I^2\frac{\dot{A}^2}{a^2} = \frac{3C^2}{2a^4I^2}$$

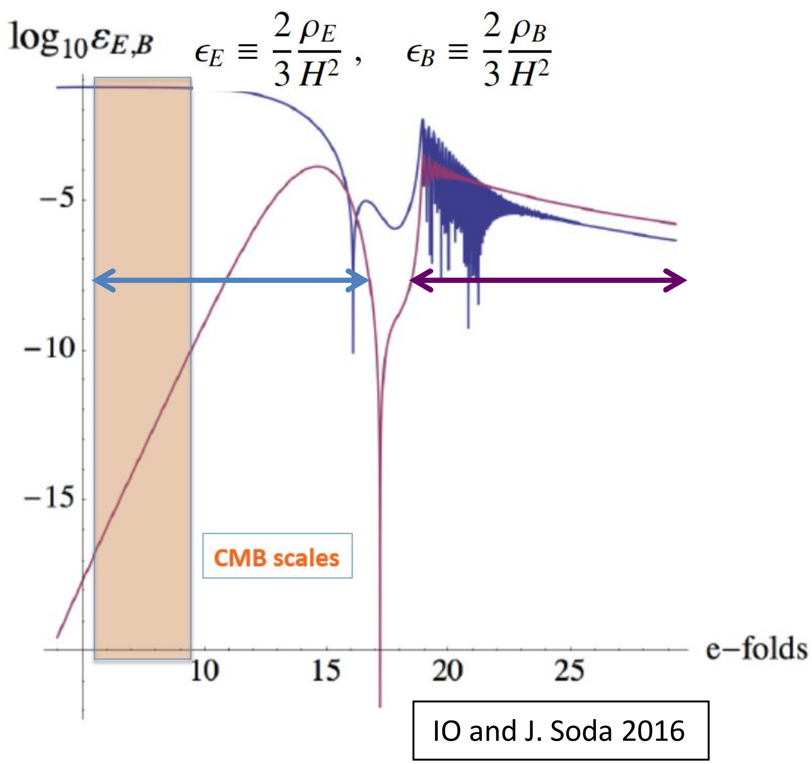
Slow-roll equation for ϕ(t) in an early period

$$3H\dot{\varphi} + V_\varphi \simeq 2\frac{I_\varphi}{I}\rho_E \quad \longrightarrow \quad a^4I^2 \simeq -\frac{n^2}{n+2}\frac{3C^2}{2\epsilon_V V}(1 + D_1 a^{2(n+2)})$$

$$\rho_E \simeq -\frac{n+2}{n^2}\epsilon_V V$$

Dilaton and Axion with SU(2) fields

The dynamics of the energy density of gauge field



$(\Lambda_\varphi,\ \Lambda_\sigma,\ r,\ n,\ g,\ \lambda,\ f\ ) = (\ 10^{-2},\ 2\times10^{-3}, 1,\ -2.01,\ 10^{-6},\ 10^{-1},\ 10\ )$

$V(\varphi) = \Lambda_1^4 \exp[r\varphi] \quad W(\sigma) = \Lambda_2^4\left(1-\cos\left(\frac{\sigma}{f}\right)\right)$

Slow-roll solutions of ⟷

$$3H^2 \simeq V \quad 3H\dot{\varphi} + V_\varphi \simeq 2\frac{I_\varphi}{I}\rho_E$$

n<-2

$$a^4 I^2 \simeq -\frac{n^2}{n+2}\frac{3C^2}{2\epsilon_V V} \simeq \text{const.}$$

$$\longrightarrow \quad \rho_E = \frac{3C^2}{2a^4 I^2} \simeq \text{const.}$$

Dilaton and Axion with SU(2) fields

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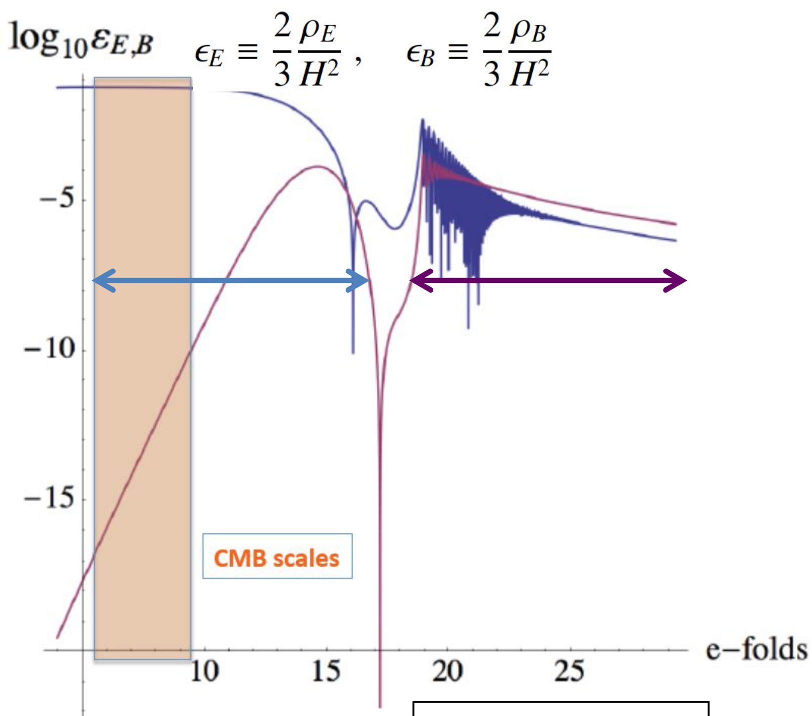

IO and J. Soda 2016

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Slow-roll solutions of ⟷

$$3H^2 \simeq W$$

$$Q \simeq Q_{\min} = -\left(\frac{fW_\sigma}{3\lambda g H}\right)^{1/3} \quad \text{(const.)}$$

$$\frac{1}{2}\frac{\lambda}{I^2}\frac{\dot{\sigma}}{fH} \simeq m_Q + \frac{1}{m_Q}\left(1 + \frac{\dot{I}}{HI}\right)$$

$$\rho_E \simeq \frac{3}{2}I^2H^2Q_{\min}^2 \qquad \rho_B \simeq \frac{3}{2}I^2g^2Q_{\min}^4$$

Dilaton and Axion with SU(2) fields

Phenomenology in CMB scales

$$\mathcal{P}_h^{\pm}(k) \simeq \frac{2H^2}{k^3}\left[1 + 4\epsilon_E (\ln x_f)^2\right] \qquad \epsilon_E \equiv \frac{2}{3}\frac{\rho_E}{H^2} \ll 1$$

$$n_t = \frac{d\ln k^3 \mathcal{P}_h^{\pm}}{d\ln k} \simeq \frac{8\epsilon_E\,(\ln x_f)}{1 + 4\epsilon_E\,(\ln x_f)^2}$$

$$\boxed{-60 \le \ln x_f \le -50}$$

(scalar modes spectrum)

$$\boxed{\frac{k^3}{2\pi^2}\mathcal{P}_\zeta(k) \simeq \frac{H^2}{8\pi^2\epsilon_\varphi}\left(1 + \mathcal{A}\,\mathcal{E}_E^2\,(\ln x_f)^2\right) \quad n_s - 1 = \frac{d\ln k^3\mathcal{P}_\zeta}{d\ln k} \simeq \frac{2\mathcal{A}\,\mathcal{E}_E^2\,(\ln x_f)}{1 + \mathcal{A}\,\mathcal{E}_E^2\,(\ln x_f)^2}}$$

$$\boxed{\mathcal{E}_E \equiv \sqrt{\frac{\epsilon_E}{\epsilon_\varphi}}}$$

Tensor-to-scalar ratio: $$r = \frac{\mathcal{P}_h^+ + \mathcal{P}_h^-}{\mathcal{P}_\zeta} \simeq 16\epsilon_\varphi^2 \frac{\epsilon_\varphi^{-1} + 4\mathcal{E}_E^2(\ln x_f)^2}{1 + \mathcal{A}\,\mathcal{E}_E^2\,(\ln x_f)^2}$$

$$\boxed{\mathcal{A} = \mathcal{O}(10)}$$