Solution

For

DARK MATTER

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REFERENCE

P.H. Frampton, *Searching for Dark Matter Constituents with Many Solar Masses.* arXiv:1510.00400[hep-ph]

Has 65 references, none of which is in talk.

Introduction

Astronomical observations have led to a consensus that the energy make-up of the visible universe is approximately 70% dark energy, 25% dark matter and only 5% normal matter. The dark energy remains mainly mysterious; the dark matter problem will be addressed in the present talk; and the normal matter has a successful theory applicable up to at least a few hundred GeV in the form of the standard model*.

General discussions of the history and experiments for dark matter are in books authored or edited by Sciama, Sanders, and Bertone. A recent popular book, The Cosmic Cocktail by Freese, is strong on the panoply of unsuccessful WIMP searches.

*More precise numbers from the Planck Collaboration are 68.4% dark energy, 26.7% dark matter and 4.9% normal matter.

In the present talk we shall make the unjustified assumption that there is only one species of dark matter. Because the luminous matter is far richer than this, with its varied menu of the three families of quarks and leptons, the gauge bosons and the BEH boson, there is no sharp reason why the dark matter should be so different and simpler. One practical reason to make such an assumption is to simplify the research. A better reason is that it is likely, in our opinion, to be correct.

The present ignorance of the dark matter sector is put into perspective by looking at the uncertainty in the values of the constituent mass previously considered. The lightest such candidate is the invisible axion with $M = 1\mu eV$. The heaviest such candidate is the intermediate mass black hole (IMBH) with $M = 100,000 M_{\odot}$ which is a staggering seventy-seven orders of magnitude larger.

Our aim is to reduce this uncertainty.

The result of the present analysis will be that the number of orders of magnitude uncertainty in the dark matter constituent mass can be reduced to three. We shall conclude, after extensive discussion, that the most viable candidate for the constituent which dominates dark matter is the intermediate mass black hole (IMBH) with mass in the range

$$100M_{\odot} < M_{IMBH} < 100,000M_{\odot} \tag{1}$$

Less experimental effort is being invested in searching for IMBHs than for WIMPs. WIMP searches include terrestrial direct detection, astronomical indirect detection and production of WIMPs at the LHC.

An explanation for the neglect of IMBHs may be that the literature is confusing including one study which claimed entirely to rule out Eq.(1). We shall attempt to clarify the situation which actually still permits the whole range in Eq.(1). The present talk is, in part, an attempt to redress the imbalance between the few experimental efforts to search for IMBHs compared to the extensive WIMP searches.

One possible reason for previously overlooking our solution to the dark matter problem is that it had been assumed that all black holes arise only from gravitational collapse of baryonic objects, either normal stars or superheavy early stars.

Axions and WIMPs

One benefit of age in particle phenomenology is to recognize more easily directions which develop an inertia of their own by virtue of many papers and continue past a serious objection, without interruption, with many more papers. This does not necessarily imply that the papers subsequent to the objection were wrong, only that their authors were either unfamiliar with the objection or that they hoped it would go away. It helps to be old enough to remember the time before the research direction was invented.

Axions

It is worth reviewing briefly the history of the axion particle now believed, if it exists, to lie in the mass range

$$10^{-6}eV < M < 10^{-3}eV \tag{2}$$

The lagrangian originally proposed for Quantum Chromodymamics (QCD) was of the simple form, analogous to Quantum Electrodynamics,

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha} - \frac{1}{2}\sum_{i}\bar{q}_{i,a}\gamma^{\mu}D^{ab}_{\mu}q_{i,b} \tag{3}$$

summed over the six quark flavors.

The simplicity of Eq.(3) was only temporary and became more complicated in 1975 by the discovery of instantons which dictated an additional term in the QCD lagrangian must be added

$$\Delta\mathcal{L}_{QCD} = \frac{\Theta}{64\pi^2} G^{\alpha}_{\mu\nu} \tilde{G}^{\mu\nu}_{\alpha} \tag{4}$$

where $\tilde{G}_{\mu\nu}$ is the dual of $G_{\mu\nu}$. Although this extra term is an exact derivative, it cannot be discarded as a surface term because there is now a topologically nontrivial QCD vacuum with an infinite number of different values of the spacetime integral over Eq.(4) all of which correspond to $G^{\alpha}_{\mu\nu} = 0$. Normalized as in Eq.(4), the spacetime integral of this term must be an integer, and an instanton configuation changes this integer, or Pontryagin number, by unity.

When the quark masses are complex, an instanton changes not only Θ but also the phase of the quark mass matrix $\mathcal{M}_{quark}$ and the full phase to be considered is

$$\bar{\Theta} = \Theta + \arg\det||\mathcal{M}_{quark}|| \qquad (5)$$

The additional term, Eq.(4),violates P and CP, and contributes to the neutron electric dipole moment whose upper limit provides a constraint

$$\bar{\Theta} < 10^{-9} \qquad (6)$$

which fine-tuning is the strong CP problem.

The hypothetical axion particle then arises from an ingenious technique to resolve Eq.(6), although as it turns out it may have been too ingenious. The technique is based on the Peccei-Quinn mechanism which introduces a new global $U(1)_{PQ}$ symmetry which allows the vacuum to relax to $\bar{\Theta} = 0$. Because this $U(1)_{PQ}$ symmetry is spontaneously broken, it gives rise to a light pseudoscalar axion with mass in the range $100keV < M < 1MeV$. An axion in this mass range was excluded experimentally but then the theory was modified to one with an invisible axion where the $U(1)_{PQ}$ symmetry is broken at a much higher scale f_a and the coupling of the axion correspondingly suppressed. Nevertheless, clever experiments to detect such so-called invisible axions were proposed by Sikivie.

Over twenty years ago, in 1992, three papers independently pointed out a serious objection to the invisible axion. The point is that the invisible potential is so fine-tuned that adding gravitational couplings for weak gravitational fields at the dimension-five level requires tuning of a dimensionless coupling g to be at least as small as $g < 10^{-40}$, more extreme than the tuning of $\bar{\Theta}$ in Eq.(6).

Although a true statement, it is not a way out of this objection to say that we do not know the correct theory of quantum gravity because for weak gravitational fields, as is the case almost everywhere in the visible universe, one can use an effective field theory as discussed by Donoghue. To our knowledge, this serious objection to the invisible axion which has been generally ignored since 1992 has not gone away and therefore the invisible axion probably does not exist.

There remains the strong CP problem of Eq.(6). One other solution would be a massless up quark but this is disfavored by lattice calculations. For the moment, Eq.(6) must be regarded as fine tuning. We recall that the ratio of any neutrino mass to the top quark mass in the standard model satisfies

$$\left(\frac{M_\nu}{M_t}\right) < 10^{-12}. \tag{7}$$

WIMPs

By Weakly Interacting Massive Particle (WIMP) is generally meant an unidentified elementary particle with mass in the range, say, between 10 GeV and 1000 GeV and with scattering cross section with nucleons (N) satisfying, according to the latest unsuccessful WIMP direct searches,

$$\sigma_{WIMP-N} < 10^{-44} cm^2 \tag{8}$$

which is roughly comparable to the characteristic strength of the known weak interaction.

The WIMP particle must be electrically neutral and be stable or have an extremely long lifetime. In model-building, the stability may be achieved by an *ad hoc* discrete symmetry, for example a Z_2 symmetry under which all the standard model particles are even and others are odd. If the discrete symmetry is unbroken, the lightest odd state must be stable and therefore a candidate for a dark matter. In general, this appears contrived because the discrete symmetry is not otherwise motivated.

By far the most popular WIMP example came from electroweak supersymmetry where a discrete R symmetry has the value R=+1 for the standard model particles and R=-1 for all the sparticles. Such an R parity is less *ad hoc* being essential to prevent too-fast proton decay. The lightest R=-1 particle is stable and, if not a gravitino which has the problem of too-slow decay in the early universe, it was the neutralino, a linear combination of zino, bino and higgsino. The neutralino provided an attractive candidate.

The big problem with the neutralino is that in the LHC Run 1 at 7TeV and 8TeV where electroweak supersymmetry not many years ago confidently predicted sparticles (gluinos, etc.) at the weak scale $\sim$ 250 GeV there is no sign of any additional particle with mass up to at least 1000 GeV so electroweak supersymmmetry probably does not exist.

It is worth briefly recalling the history of electroweak supersymmetry. The standard model was in place by 1971 and its biggest theoretical problem was that, unlike QED with only log divergences, the scalar sector of the standard model generates quadratic divergences which destabilize the mass of the BEH boson.

When supersymmetric field theories were invented in 1974, they provided an elegant solution of the quadratic divergence problem and hence immediately became popular. Even more so in 1983 when the neutralino was identified as a dark matter candidate and more so again in 1991 when it was pointed out that grand unification works better with the supersymmetric partners included.

With the benefit of hindsight, these motivations for supersymmetry can all be otherwise realized.

The quadratic divergence can cancel in non-supersymmetric quiver theories. A dark matter candidate can be invented, in an *ad hoc* fashion, within conformality model building. Historically, the neutralino appeared in particle phenomenology research *before* the WIMP acronym entered the lexicon of cosmology. It is an important point that the WIMP idea came from electroweak supersymmetry.

Precise unification with supersymmetry by adding one parameter, a common sparticle mass, was not miraculous but had at least a 20% probability. Other precise grand unifications are known without supersymmetry in conformality model building.

If we do need[†] a replacement for electroweak supersymmetry, conformal invariance is a contender as discussed in 1998 and a number of subsequent papers as well more recently by 't Hooft and by Mannheim.

Run 2 of the LHC is not necessarily doomed if WIMPs and sparticles do not exist. An important question, independent of naturalness but surely related to anomalies, is the understanding of why there are three families of quarks and leptons. For that reason Run2 may discover additional gauge bosons, siblings of the $W^{\pm}$ and Z^{0}, as occur in the 331-Model.

[†]The unnaturalnesses exhibited in Eq.(6) and Eq.(7) of this talk both may suggest new physics beyond the standard model.

MACHOs

Massive Compact Halo Objects (MACHOs) are commonly defined by the notion of compact objects used in astrophysics as the end products of stellar evolution when most of the nuclear fuel has been expended. They are usually defined to include white dwarfs, neutron stars, black holes, brown dwarfs and unassociated planets, all equally hard to detect because they do not emit any radiation.

This narrow definition implies, however, that MACHOs are composed of normal matter which is too restrictive in the special case of black holes. It is here posited that black holes of arbitrarily high mass up to $100,000 M_{\odot}$ can be produced primordially as calculated and demonstrated in an IPMU paper (2010). Nevertheless the acronym MACHO still nicely applies to dark matter IMBHs which are massive, compact, and in the halo.

Unlike the axion and WIMP elementary particles which would have a definite mass, the black holes will have a range of masses. The lightest PBH which has survived for the age of the universe has a lower mass limit

$$M_{PBH} > 10^{-18} M_{\odot} \sim 10^{36} TeV \qquad (9)$$

already thirty-six orders of magnitude heavier than the heaviest would-be WIMP. This lower limit comes from the lifetime formula derivable from Hawking radiation

$$\tau_{BH}(M_{BH}) \sim \frac{G^2 M_{BH}^3}{\hbar c^4} \sim 10^{64} \left(\frac{M_{BH}}{M_{\odot}} \right)^3 years \qquad (10)$$

Because of observational constraints the dark matter constituents must generally be another twenty orders of magnitude more massive than the lower limit in Eq.(9).

We assert that most dark matter black holes are in the mass range between one hundred and one hundred thousand times the solar mass. The name intermediate mass black holes (IMBHs) is appropriate because they lie in mass above stellar-mass black holes and below the supermassive black holes which reside in galactic cores.

Let us discuss three methods (there may be more) which could be used to search for dark matter IMBHs. While so doing we shall clarify what limits, if any, can be deduced from present observational knowledge.

Before proceeding, it is appropriate first to mention the important Xu-Ostriker upper bound of about a million solar masses from galactic disk stability for any MACHO residing inside the galaxy.

Wide Binaries

There exist in the Milky Way pairs of stars which are gravitationally bound binaries with a separation more than 0.1pc. These wide binaries retain their original orbital parameters unless compelled to change them by gravitational influences, for example, due to nearby IMBHs.

Because of their very low binding energy, wide binaries are particularly sensitive to gravitational perturbations and can be used to place an upper limit on, or to detect, IMBHs. The history of employing this ingenious technique is regretfully checkered. In 2004 a fatally strong constraint was claimed by an Ohio State University group in a paper entitled "End of the MACHO Era" so that, for researchers who have time to read only titles and abstracts, stellar and higher mass constituents of dark matter appeared to be totally excluded.

Five years later in 2009, however, another group this time from Cambridge University reanalyzed the available data on wide binaries and reached a quite different conclusion. They questioned whether *any* rigorous constraint on MACHOs could yet be claimed, especially as one of the important binaries in the earlier sample had been misidentified.

Because of this checkered history, it seems wisest to proceed with caution but to recognize that wide binaries represent a potentially useful source both of constraints on, and the possible discovery of, dark matter IMBHs.

Distortion of the CMB

This approach hinges on the phenomenon of accretion of gas onto the IMBHs. The X-rays emitted by such accretion of gas are downgraded in frequency by cosmic expansion and by Thomson scattering becoming microwaves which distort the CMB, both with regard to its spectrum and to its anisotropy.

One impressive calculation of this effect employs a specific model for the accretion, the Bondi-Hoyle model, and carries through the computation all the way up to a point of comparison with data from FIRAS on CMB spectral distortions, where FIRAS was a sensitive device attached to the COBE satellite. Unfortunately the paper includes the limits from wide binaries discussed *ut supra* (the fatally strong constraint), and preceded the corrective paper so its results might have been influenced.

The results obtained from this approach are interesting if one can be certain that the gas accretion, subsequent X-ray emission and downgrading are well modeled. Like wide binaries, CMB distortion is indirect but could in future lead to useful bounds on, or the possible discovery of, dark matter IMBHs.

Microlensing

Microlensing is the most direct experimental method and has the big advantage that it has successfully found examples of MACHOs. The MACHO Collaboration used a method which had been proposed[‡] by Paczynski where the amplification of a distant source by an intermediate gravitational lens is observed. The MACHO Collaboration discovered several striking microlensing events whose light curves are exhibited in its 2000 paper. The method certainly worked well for $M < 100M_{\odot}$ and so should work equally well for $M > 100M_{\odot}$ provided one can devise a suitable algorithm and computer program to scan enough sources.

[‡] We have read that such gravitational lensing was later found to have been calculated in unpublished 1912 notes by Einstein who did not publish perhaps because at that time he considered its experimental measurement impracticable.

The longevity of a given lensing event is proportional to the square root of the lensing mass and numerically is given by ($\hat{t}$ is longevity)

$$\hat{t} \simeq 0.2yr \left(\frac{M_{lens}}{M_{\odot}} \right)^{1/2} \tag{11}$$

where a transit velocity $200km/s$ is assumed for the lensing object.

The MACHO Collaboration investigated lensing events with longevities ranging between about two hours and two years. From Eq.(11) this corresponds to MACHO masses between approximately $10^{-6} M_{\odot}$ and $100 M_{\odot}$.

The total number and masses of objects discovered by the MACHO Collaboration could not account for all the dark matter known to exist in the Milky Way. At most 10% could be explained. To our knowledge, the experiment ran out of money and was essentially abandoned in about the year 2000. But perhaps the MACHO Collaboration and its funding agency were too easily discouraged.

What is being suggested is that the other 90% of the dark matter in the Milky Way is in the form of MACHOs which are more massive than those detected by the MACHO Collaboration, and which almost certainly could be detected by a straightforward extension of their techniques. In particular, the expected microlensing events have a duration ranging up to two centuries.

Let us consider the entries of Table 1 which merit discussion both with respect to the proposed microlensing experiment and briefly with respect to the entropy of the universe.

We simplify the visible universe without losing anything important by regarding it as containing exactly 10^{11} galaxies, each with mass (dominantly dark matter) of exactly $10^{12} M_{\odot}$. The first three columns of the Table consider one halo of dark matter. To a first approximation, we can temporarily ignore the normal matter. The fourth column gives the additive entropy of the universe for well separated halos and the fifth column gives the corresponding microlensing event longevity in years.

For a black hole with mass $M_{BH} = \eta M_\odot$, the dimensionless entropy is $S_{BH}/k \sim 10^{77}\eta^2$, in other words

$$S_{BH}/k = 10^{77} \left(\frac{M_{BH}}{M_\odot} \right)^2 . \qquad (12)$$

If we study the first five rows of Table 1 we notice that, for a given total halo mass, $M_{Halo} = 10^{12} M_\odot$, a smaller number of heavier black holes gives higher entropy because $S_{BH} \propto M_{BH}^2$. Various arrangements of the allowed black hole mass function have been explored. Arguments using the concept of the entropy of the universe, together with the second law of thermodynamics, are strongly suggestive of many more black holes than the stellar and supermassive types already identified for the simple reason that black holes are, by far, the most efficient concentrators of entropy.

Microlensing Longevity ($\hat{t}$) for the case n IMBHs per halo.
IMBH mass $=\eta M_\odot$.
Halo mass $= 10^{12} M_\odot$.
Universe mass $= 10^{23} M_\odot$.

n/Halo $Log_{10}n$	M $= \eta M_\odot$ $Log_{10}\eta$	Halo Entropy $Log_{10}(S_{Halo}/k)$	Universe Entropy $Log_{10}(S_{Universe}/k)$	Longevity $\hat{t}$ (years)
10	2	91	102	2
9	3	92	103	6
8	4	93	104	20
7	5	94	105	60
6	6	95	106	200
0	*12*	*101*	*112*	n/a

The sixth and last row in Table 1 illustrates how if a halo hypothetically collapsed into one large black hole, its entropy would be $S_{Halo}^{Max}/k \sim 10^{101}$. If the superluminal accelerated expansion prevents coalescence of such collapsed halos the additive entropy of the universe's interior would be $S_{Universe}/k \sim 10^{112}$. If, hypothetically, all the halos would instead combine to one very large black hole with mass $10^{23}M_{\odot}$, the entropy would be $S_{Universe}/k \sim 10^{123}$. The Schwarzschild radius of this very large black hole is $R = 10^{23} \times 3km \sim 30Gly$, not far below the comoving radius ($\sim 45Gly$) of the visible universe.

This discussion implies that the visible universe is, in some sense, close to itself being a black hole inside of which we live. This curious fact seems to have no bearing on dark matter but may be relevant to the more difficult problem of dark energy.

Microlensing experiments involve systematic scans of millions of distant star sources because it requires accurate alignment of the star and the intermediate lensing MACHO. Because the experiments are already highly computer intensive, it makes us more optimistic that the higher longevity events can be successfully analyzed. Study of an event lasting two centuries should not necessitate that long an amount of observation time. It does require suitably ingenious computer programming to track light curves and distinguish them from other variable sources. This experiment is undoubtedly extremely challenging, but there seems no obvious reason it is impracticable.

A fraction of the resources currently being thrown at WIMP searches could be enough to support this desirable pursuit of high-longevity microlensing observations.

Discussion

Axions probably do not exist for theoretical reasons discovered in 1992. Electroweak supersymmetry probably does not exist for the experimental reason of its non-discovery in Run 1 of the LHC. The idea that dark matter experiences weak interactions (WIMPs) came historically from the appearance of an appealing DM constituent, the neutralino, in the theory of electroweak supersymmetry for which there is no experimental evidence.

The only interaction which we know for certain to be experienced by dark matter is gravity and the simplest assumption is that gravity is the only force coupled to dark matter. Why should the dark matter experience the weak interaction when it does not experience the strong and electromagnetic interactions?

All terrestrial experiments searching for dark matter by either direct detection or production may be doomed to failure.

We began with four candidates for dark matter constituent: (1) axion; (2) WIMP; (3) brown dwarf, understood to include all compact baryonic objects; (4) black hole. We eliminated the first two by hopefully persuasive arguments, made within the context of an overview of particle phenomenology including a combination of old and new results. We eliminated the third by the upper limit on baryons imposed by robust Big Bang Nucleosynthesis (BBN) calculations.

We assert that IMBHs can constitute almost all dark matter while maintaining consistency with the BBN calculations. This is an important point because distinguished astronomers have written an opposite assertion *e.g.* Begelman and Rees state that black holes cannot form more than 20 % of dark matter because the remainder is non-baryonic.

These authors are making an implicit assumption which does not apply to the IMBHs which we assert comprise almost all dark matter. That assumption is that black holes can be formed only as the result of the gravitational collapse of baryonic stars. We are claiming, on the contrary, that dark matter black holes can be, and the majority must be, formed primordially in the early universe as calculated and demonstrated in a paper written at IPMU in 2010, and endorsed by Carr *et al.*, unconstrained by the BBN upper limit on baryons.

Our proposal is that the Milky Way contains between ten million and ten billion massive black holes each with between a hundred and a hundred thousand times the solar mass. Assuming the halo is a sphere of radius a hundred thousand light years the typical separation is between one hundred and one thousand light years which is also the most probable distance of the nearest IMBH to the Earth. At first sight, it may be surprising that such a number of massive black holes could have remained undetected in the Milky Way. On second thoughts, it appears reasonable when one bears in mind their large mean separation of a hundred to a thousand light years and their relatively small size, all being physically smaller than our Sun.

Of the detection methods discussed, extended microlensing observations seem the most promising and writing the present paper will have been worthwhile if efforts to detect higher longevity microlensing events are hereby encouraged. It will be exceptionally rewarding if most of the dark matter in our galaxy is confirmed, by microlensing techniques or otherwise, to be in the form of intermediate-mass black holes.

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