Is General Relativity a restricted theory?

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Things should be made simple, but not simpler

I apologize if the title suggests that GR is a wrong theory. This talk should be interpreted as a diagnostic on the *state of art* of GR.

Next year we commemorate the 100-th aniversary of the publication of the General Relativity theory.

Is it time for a revision of its fundamental principles?

I think it is a restricted theory if we admit the Lagrangian description of classical mechanics.

- According to GR, Spacetime is a (pseudo)Riemannian metric space.

- The motion of a test particle (point particle) follows a geodesic on spacetime.

- There are no spinless elementary particles in nature. The concept of **Spin** appeared in the next decade when G.R. was already established.

- The above statement is about the motion of an unexistant object.

- There has been an abusive use of the point particle model for describing experiments which always involve spinning particles.

- Difraction, tunneling, formation of bound pairs of electrons, are features not described by the point particle model.

- These features are contemplated in a classical description of spinning particles.

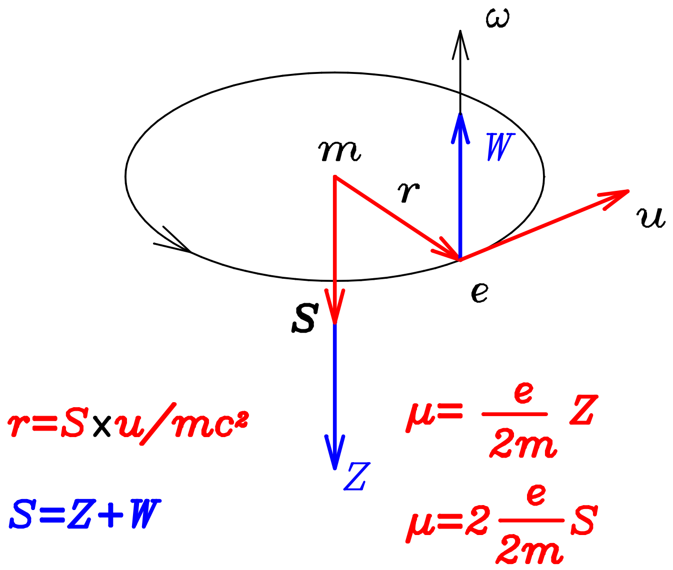
Kinematical theory of spinning particles.
Three fundamental principles:

- Restricted Relativity Principle. Gravity is discarded as a possible interaction.

- Variational formalism. Lagrangian description.

- Atomic Principle. Definition of elementary particle.

It is a complete formalism for describing elementary spinning particles. All spinning models can be quantized. There is a unique classical model which satisfies Dirac's equation when quantized. Published by Kluwer in 2001.



Classical electron structure in the center of mass frame.

The next step was to analyze the behavior of the spinning electron under a gravitational interaction.

The boundary variables manifold in the variational formalism for a spinning particle is larger than spacetime. (It is a homogeneous space of the Poincaré group).

The boundary variables manifold for a spinless point particle is spacetime. It is the metric of this manifold which is modified by gravity.

This manifold for any mechanical system is always a metric space. It is a Finsler metric space.

General Relativity is a restricted theory in two aspects:

- It assumes that spacetime is a (pseudo)Riemannian metric space when it is a Finsler metric space under EM interactions, for the point particle.

- Since there are no spinless particles in nature, the manifold whose geometry is modified by gravity must be larger than spacetime.

Classical Point Particle physics is also a restricted theory:

- It assumes that the center of charge and center of mass of an elementary particle are the same point. This assumption suppress part of the spin content.

- The conclusions we can derive from restricted formalisms cannot be used for the analysis of all forms of matter.

- We cannot use the point particle model for describing experiments performed with spinning particles.

Variational formalism or geodesic statement

The variational formalism of any Lagrangian system can always be interpreted as a Geodesic statement on the manifold of its boundary variables (Kinematical space X).

But this manifold X is not a Riemannian space but rather a metric Finsler space.

Some References:

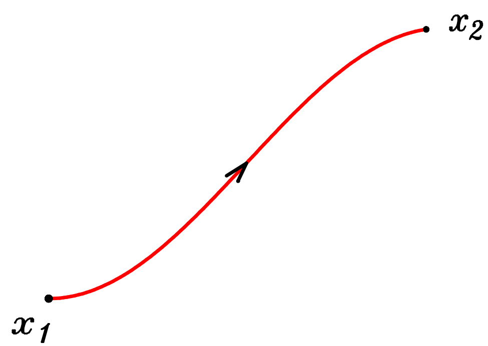
H. Rund, *The differential geometry of Finsler spaces*,
Springer-Verlag, Berlin 1959.

H. Rund, *The Hamilton-Jacobi theory in the calculus of variations*,
Robert Krieger Pub. Co. , Huntington N.Y. 1973.

G.S. Asanov, *Finsler geometry, Relativity and Gauge theories*,
Reidel Pub. Co., Dordrecht 1985.

Any Lagrangian mechanical system of n degrees of freedom q_i,

$$\mathcal{A}[q(t)] = \int_{t_1}^{t_2} L(t, q_i, q_i^{(1)})dt, \quad q_i^{(1)} = dq_i/dt.$$



between the initial state $x_1 \equiv (t_1, q_i(t_1))$ and final state $x_2 \equiv (t_2, q_i(t_2))$ on the X manifold.

X is the $(n+1)$-th dimensional manifold spanned by the time t and the n degrees of freedom q_i. If instead of describing the evolution in terms of time we express the evolution in parametric form $t(\tau)$, $q_i(\tau)$ in terms of some arbitrary evolution parameter τ, then $dq_i/dt = dq_i/d\tau \; d\tau/dt = \dot{q}_i/\dot{t}$, where now $\dot{} \equiv d/d\tau$.

The variational approach will be written as

$$\int_{t_1}^{t_2} L(t, q_i, {q_i}^{(1)})dt = \int_{\tau_1}^{\tau_2} L(t, q_i, \dot{q}_i/\dot{t})\dot{t}d\tau = \int_{\tau_1}^{\tau_2} \widetilde{L}(x, \dot{x})d\tau, \qquad \widetilde{L} = L\dot{t}.$$

But now the Lagrangian $\widetilde{L}$ is independent of the evolution parameter τ and it is a homogeneous function of first degree of the derivatives $\dot{x}$,

$$\widetilde{L} = \frac{\partial \widetilde{L}}{\partial \dot{x}^i}\dot{x}^i = F_i(x, \dot{x})\dot{x}^i,$$

$\widetilde{L}^2$ is a positive definite homogeneous function of second degree of the derivatives $\dot{x}$,

$$\widetilde{L}^2 = \frac{1}{2}\frac{\partial^2 \widetilde{L}^2}{\partial \dot{x}^i \partial \dot{x}^j}\dot{x}^i \dot{x}^j = g_{ij}(x, \dot{x})\dot{x}^i \dot{x}^j,$$

the F_i and g_{ij} are computed from $\widetilde{L}$ and $\widetilde{L}^2$ by

$$F_i(x, \dot{x}) = \frac{\partial \widetilde{L}}{\partial \dot{x}^i}, \qquad g_{ij}(x, \dot{x}) = \frac{1}{2}\frac{\partial^2 \widetilde{L}^2}{\partial \dot{x}^i \partial \dot{x}^j} = g_{ji}.$$

The $F_i(x, \dot{x})$ and $g_{ij}(x, \dot{x})$ are homogeneous functions of zeroth degree of the derivatives $\dot{x}$, and therefore functions of the time derivatives of the degrees of freedom q_i.

Between the allowed boundary states x_1 and x_2, since $\widetilde{L}^2 > 0$, the $g_{ij}(x, \dot{x})$ represent the coefficients of a positive definite metric. The variational problem can be written as

$$\int_{\tau_1}^{\tau_2} \widetilde{L}(x, \dot{x}) d\tau = \int_{\tau_1}^{\tau_2} \sqrt{\widetilde{L}^2(x, \dot{x})} d\tau = \int_{\tau_1}^{\tau_2} \sqrt{g_{ij}(x, \dot{x}) \dot{x}^i \dot{x}^j} d\tau =$$

$$= \int_{x_1}^{x_2} \sqrt{g_{ij}(x, \dot{x}) dx^i dx^j} = \int_{x_1}^{x_2} ds,$$

where ds is the arc length on the X manifold with respect to the metric g_{ij}.

This metric is velocity dependent, and therefore the metric space X is a Finsler space.

The variational statement of the dynamics of any mechanical system is always equivalent to a geodesic problem on the X manifold with a Finsler metric, which can be explicitely constructed by taking the second order partial derivatives of the $\tilde{L}^2$.

Examples of homogeneous Lagrangians (Point particle)

Nonrelativistic

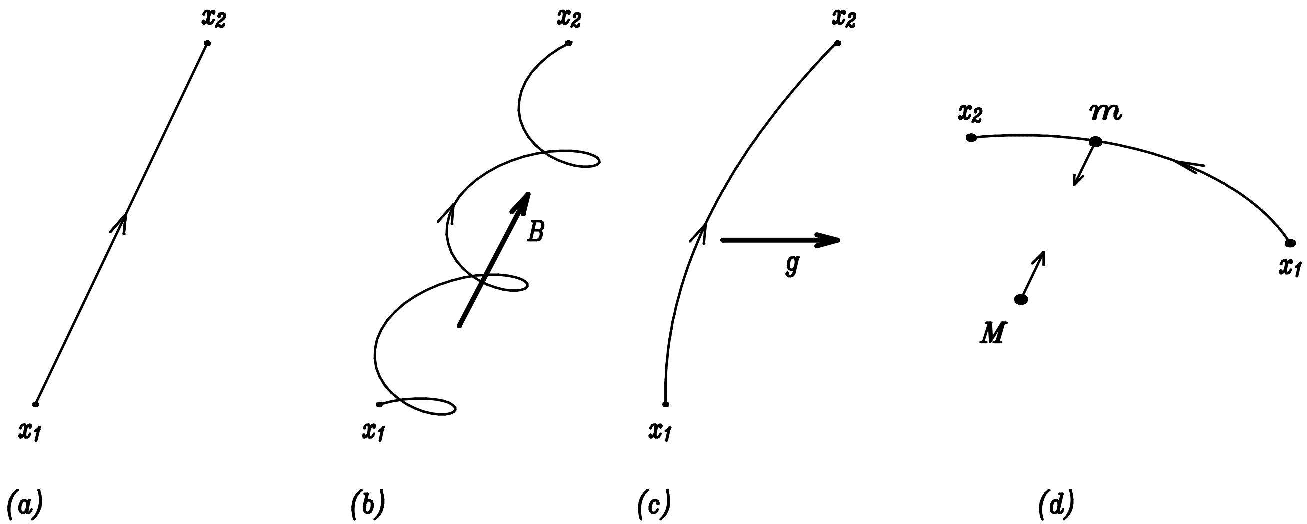
$$L = \frac{1}{2} m \left(\frac{d\boldsymbol{r}}{dt} \right)^2, \quad \widetilde{L} = L\dot{t} = \frac{1}{2} m \left(\frac{\dot{\boldsymbol{r}}}{\dot{t}} \right)^2 \dot{t} = \frac{1}{2} m \frac{\dot{\boldsymbol{r}}^2}{\dot{t}}.$$

Relativistic

$$L = -mc\sqrt{c^2 - (d\boldsymbol{r}/dt)^2}, \quad \widetilde{L} = L\dot{t} = -mc\sqrt{c^2\dot{t}^2 - \dot{\boldsymbol{r}}^2}$$

Both are homogeneous functions of first degree of ALL DERIVATIVES of the boundary variables $x_0 = ct$ and $\boldsymbol{r}$. $\widetilde{L}^2$ is a homogeneous function of second degree.

Finsler metrics (Point particle on spacetime X)



(a) Free motion $\widetilde{L}_0 = -mc\sqrt{\dot{x}_0^2 - \dot{\boldsymbol{r}}^2}$, $\widetilde{L}_0^2 = g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$. The metric is

$$g_{\mu\nu} = \frac{1}{2}\frac{\partial^2 \widetilde{L}_0^2}{\partial \dot{x}^\mu \partial \dot{x}^\nu} = m^2 c^2 \eta_{\mu\nu}, \quad \eta_{\mu\nu} = \quad \mathrm{diag}(1, -1, -1, -1)$$

(b)Motion under a constant magnetic field. The Lagrangian

$$\widetilde{L}_B = -mc\sqrt{\dot{x}_0^2 - \dot{\boldsymbol{r}}^2} + eBx\dot{y}.$$

we get the Lorentz force dynamical equation

$$\frac{d\boldsymbol{p}}{dt} = e\,\boldsymbol{u} \times \boldsymbol{B}.$$

The metric of spacetime becomes, with $k = eB/mc$,

$$g_{00} = 1 + \frac{kxu^2u_y}{(c^2-u^2)^{3/2}}, \quad g_{11} = -1 + \frac{kxu_y}{(c^2-u^2)^{3/2}}\left(c^2 - u_y^2 - u_z^2\right),$$

$$g_{22} = -1 + k^2x^2 + \frac{kxu_y}{(c^2-u^2)^{3/2}}\left(3c^2 - 3u_x^2 - 2u_y^2 - 3u_z^2\right),$$

$$g_{33} = -1 + \frac{kxu_y}{(c^2-u^2)^{3/2}}\left(c^2 - u_x^2 - u_y^2\right),$$

$$g_{01} = -\frac{kxcu_x u_y}{(c^2-u^2)^{3/2}}, \quad g_{02} = -\frac{kxc}{(c^2-u^2)^{3/2}}(c^2 - u_x^2 - u_z^2),$$

$$g_{03} = -\frac{kxcu_y u_z}{(c^2-u^2)^{3/2}}, \quad g_{12} = \frac{kxu_x}{(c^2-u^2)^{3/2}}\left(c^2 - u_x^2 - u_z^2\right),$$

$$g_{13} = \frac{kx}{(c^2-u^2)^{3/2}}u_x u_y u_z, \quad g_{23} = \frac{kxu_z}{(c^2-u^2)^{3/2}}\left(c^2 - u_x^2 - u_z^2\right),$$

metric coefficients are functions of the variable x, and of the velocity of the particle. If the velocity is negligible with respect to c, they become

$$g_{00} = 1, \quad g_{11} = -1, \quad g_{22} = -1 + k^2x^2, \quad g_{33} = -1,$$

vanishing the remaining ones, and since the dependence on the velocity has dissapeared the metric has been transformed into a Riemannian metric.

These metric coefficients represent a Riemannian spacetime, and therefore the dynamics is described by a restricted Lagrangian $\widetilde{L}_R$, quadratic in the derivatives,

$$\widetilde{L}_R^2 = m^2c^2(c^2\dot{t}^2 - \dot{\boldsymbol{r}}^2) + e^2B^2x^2\dot{y}^2, \tag{1}$$

such that when compared with $\widetilde{L}_B$ we have an additional term

$$\widetilde{L}_B^2 = \widetilde{L}_R^2 - 2emcBx\dot{y}\sqrt{c^2\dot{t}^2 - \dot{\boldsymbol{r}}^2}.$$

From the restricted Lagrangian $\widetilde{L}_R$ in a Riemannian spacetime, the force acting on the point particle is not longer the Lorentz force.

From the complete Lagrangian, the Lorentz force dynamical equations are

$$\frac{du_x}{dt} = \frac{1}{\gamma(u)}kcu_y, \quad \frac{du_y}{dt} = -\frac{1}{\gamma(u)}kcu_x, \quad \frac{du_z}{dt} = 0.$$

From the restricted Lagrangian $\widetilde{L}_R$, the geodesic equations are

$$\frac{du_x}{dt} = kcu_y\left(1 - kxu_y/c\right), \quad \frac{du_y}{dt} = -kcu_x\left(1 - kxu_y/c\right), \quad \frac{du_z}{dt} = 0,$$

which reduce to the previous ones in the $u/c \to 0$ limit.

If we accept that the electromagnetic force is the Lorentz force, then spacetime is a Finsler metric space.

Spacetime is Riemannian only in the low velocity approach.

(c)**Motion under a constant gravitational field**

$\widetilde{L}_g = \widetilde{L}_0 + m\boldsymbol{g}\cdot\boldsymbol{r}\dot{t}$, The dynamical equations are

$$\frac{d\boldsymbol{p}}{dt} = m\boldsymbol{g}$$

The Finsler metric from $\widetilde{L}_g^2$ is

$$g_{00} = 1 + \left(\frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right)^2 - \frac{c(2c^2 - 3u^2)}{(c^2-u^2)^{3/2}}\left(\frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right),$$

$$g_{ii} = -1 + \frac{c(c^2 - u^2 + u_i^2)}{(c^2-u^2)^{3/2}}\left(\frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right), \quad i = 1,2,3$$

$$g_{0i} = -\frac{u^2 u_i}{(c^2-u^2)^{3/2}}\left(\frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right), \quad i = 1,2,3,$$

$$g_{ij} = \frac{c u_i u_j}{(c^2-u^2)^{3/2}}\left(\frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right), \quad i \neq j = 1,2,3.$$

The low velocity limit is

$$g_{00} = \left(1 - \frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right)^2, \quad g_{ii} = -\left(1 - \frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right), \quad i = 1, 2, 3,$$

where the g_{00} component is the same as the corresponding component of the Rindler metric corresponding to a noninertial accelerated observer or to the presence of a global uniform gravitational field, in General Relativity.

They satisfy

$$g_{00} = (g_{ii})^2 .$$

(d) **Relativistic point particle under a Newtonian potential.**

$$\widetilde{L}_N = \widetilde{L}_0 + \frac{GmM}{cr}c\dot{t}.$$

Dynamical equations are

$$\frac{d\boldsymbol{p}}{dt} = -\frac{GmM}{r^3}\boldsymbol{r},$$

independent of the mass of the particle.

The Finsler metric from $\widetilde{L}_N^2$ is

$$g_{00} = 1 + \left(\frac{GM}{c^2 r}\right)^2 - \frac{c(2c^2 - 3u^2)}{(c^2 - u^2)^{3/2}}\left(\frac{GM}{c^2 r}\right),$$

$$g_{ii} = -1 + \frac{c(c^2 - u^2 + u_i^2)}{(c^2 - u^2)^{3/2}}\left(\frac{GM}{c^2 r}\right), \quad i = 1, 2, 3,$$

$$g_{0i} = -\frac{u^2 u_i}{(c^2-u^2)^{3/2}}\left(\frac{GM}{c^2 r}\right), \quad i=1,2,3,$$

$$g_{ij} = \frac{c u_i u_j}{(c^2-u^2)^{3/2}}\left(\frac{GM}{c^2 r}\right), \quad i \neq j = 1,2,3$$

It is a Finsler metric, which in the case of low velocities only the diagonal components survive

$$g_{00} = \left(1 - \frac{2GM}{c^2 r} + \frac{G^2 M^2}{c^4 r^2}\right) = \left(1 - \frac{GM}{c^2 r}\right)^2 .$$

If the last term G^2M^2/r^2c^4 is considered negligible, it is the g_{00} coefficient of Schwarzschild's metric. The

$$g_{ii} = -\left(1 - \frac{GM}{c^2 r}\right), \quad i = 1,2,3,$$

are different than in the Schwarzschild case. In any case we see that the nonvanishing coefficients, in the low velocity limit, they satisfy

$$g_{00} = (g_{ii})^2 \, ,$$

as in the previous example.

If we call $R_s = 2GM/c^2$ Schwarzschild's radius, the curvature scalar R and Einstein's tensor $G_{\mu\nu}$ are:

$$R = \frac{R_s^2}{r(2r - R_s)^3}, \quad G_{tt} = \frac{3R_s^2}{8r^3(2r - R_s)}, \quad G_{rr} = \frac{(24r - tR_s)R_s}{4r^2(2r - R_s)^2},$$

$$G_{\theta\theta} = \frac{(R_s - 3r)R_s}{(2r - R_s)^2}, \quad G_{\phi\phi} = \frac{(R_s - 3r)R_s \sin^2\theta}{(2r - R_s)^2}$$

it is not a vacuum solution of E.E., and Einstein tensor is related to the gravitational potential of the mass M.

Z. Chang and X. Li,
Modified Newton's gravity in Finsler spaces as a possible alternative to dark matter hypothesis.
gr-qc/arXiv:0806.2184

A modified Newton's gravity is obtained as the weak field approximation of the Einsteins equation in Finsler space. It is found that a specified Finsler structure makes the modified Newtons gravity equivalent to the modified Newtonian dynamics (MOND). In the framework of Finsler geometry, the flat rotation curves of spiral galaxies can be deduced naturally without invoking dark matter.

For a point particle, and in a special relativity framework, electromagnetism transforms the flat metric of spacetime into a Finsler metric of the form

$$\widetilde{L}_0^2 = \eta_{\mu\nu}\,\dot{x}^\mu \dot{x}^\nu \quad \Rightarrow \quad \widetilde{L}_{EM}^2 = g_{\mu\nu}(x,\dot{x})\dot{x}^\mu \dot{x}^\nu,$$

The proposal of General Relativity is that the motion of a point particle in a gravitational field is a geodesic of

$$\widetilde{L}_0^2 = \eta_{\mu\nu}\,\dot{x}^\mu \dot{x}^\nu \quad \Rightarrow \quad \widetilde{L}_g^2 = g_{\mu\nu}(x)\dot{x}^\mu \dot{x}^\nu,$$

the metric independent of the derivatives $\dot{x}^i$.[First Restriction]

In the spirit of unification of all interactions we should also assume that gravity would produce a Finslerian metric rather than a Riemannian one.

There are no spinless elementary particles

This means that an elementary particle has a boundary space X larger than spacetime because it has more degrees of freedom.

Electromagnetism (and gravity?) modifies the Finsler metric of this manifold X.

Gravitation must be described as a Finsler metric theory on a boundary space X larger than spacetime.[Second Restriction]

Without any statement about general covariance or about the source of the gravitational field, (if we assume that classical mechanics is described in a Variational formalism), it could be interpreted that General Relativity when considered as a gravitational theory of spacetime with a (pseudo)Riemannian metric, is a low velocity limit of a theory of gravitation of spinless matter.

Conclusions

General Relativity could be considered to be a restricted low velocity theory of gravitation of spinless matter.

arXiv: 1203.4076

Thank you for your attention

The Lagrangian of a point particle in an EM field is

$$\widetilde{L}_{EM} = \widetilde{L}_0 - eA_\mu(x)\dot{x}^\mu$$

With

$$p_\mu = \frac{m\dot{x}_\mu}{\sqrt{\dot{x}_0^2 - \dot{\boldsymbol{r}}^2}}, \quad p_0 = \frac{H}{c} = \frac{mc}{\sqrt{1 - u^2/c^2}}, \quad p_i = -\frac{mu_i}{\sqrt{1 - u^2/c^2}},$$

The Finslerian metric associated to the EM field is

$$g_{\mu\nu}(x, \dot{x}) = m^2c^2\eta_{\mu\nu} + e^2A_\mu(x)A_\nu(x) + e(p_\mu A_\nu + p_\nu A_\mu) + eA_\sigma\dot{x}^\sigma\frac{\partial p_\mu}{\partial\dot{x}^\nu}$$

Classical theory of elementary particles

Matter has inertia and therefore it has a Center of Mass (CM).

Matter interacts and therefore it has a Center of Charge (CC).

Classical Physics considers that both points are exactly the same.!!!

Classical spinning particles have a CC and CM which are different points.

If both points are different, physics leads to a unique solution:

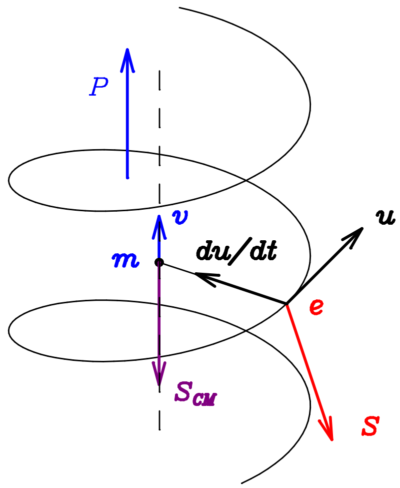
Only a relativistic treatment is allowed.

It suffices to describe the motion of the CC.

The CC has to be moving at the speed of light.

This matches with Dirac's formalism in the quantum case.

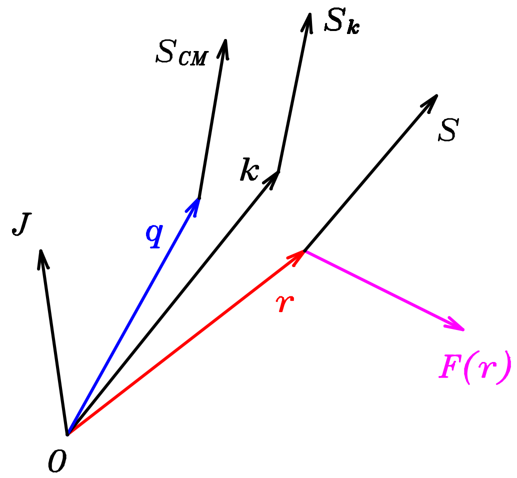
De Broglie was right, elementary matter has a natural frequency, the frequency of the motion of the CC around the CM.



If we consider that they are different, several physical effects can be described which are hidden in the restricted approach:

- Tunneling
- Formation of bound pairs of electrons

Different spin observables



$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p} + \boldsymbol{S}$, o bien $\boldsymbol{J} = \boldsymbol{q} \times \boldsymbol{p} + \boldsymbol{S}_{CM}$, o bien $\boldsymbol{J} = \boldsymbol{k} \times \boldsymbol{p} + \boldsymbol{S}_k$.

The dynamics under some external force $\boldsymbol{F}$, applied at the CC $\boldsymbol{r}$

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{F}, \qquad \frac{d\boldsymbol{J}}{dt} = \boldsymbol{r} \times \boldsymbol{F}$$

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{p} \times \boldsymbol{u}, \quad \frac{d\boldsymbol{S}_{CM}}{dt} = (\boldsymbol{r} - \boldsymbol{q}) \times \boldsymbol{F}, \quad \frac{d\boldsymbol{S}_k}{dt} = (\boldsymbol{r} - \boldsymbol{k}) \times \boldsymbol{F} + \boldsymbol{p} \times \frac{d\boldsymbol{k}}{dt}$$

$\boldsymbol{S}_{CM}$ is conserved if $\boldsymbol{r} = \boldsymbol{q}$. If it is not conserved it means that $\boldsymbol{r} \neq \boldsymbol{q}$

$\boldsymbol{S}$ is the equivalent to Dirac's spin observable.

$\boldsymbol{S}_{CM}$ is equivalent to BMT spin observable.

Conclusions

Classical Point Particle Physics is a restricted theory because all known matter is made from spinning particles.