The Einstein equivalence principle and Doppler tracking in cosmology

Sergei Kopeikin

University of Missouri, Columbia, USA

I THANK
DR. MAXIM YU. KHLOPOV
AND VIRTUAL INSTITUTE OF ASTROPARTICLE PHYSICS IN PARIS
FOR THE GREAT OPPORTUNITY TO PRESENT THIS LECTURE!

What can be in common between the global geometry of the universe and a local physics?

- A quick (but presumably wrong) answer is: nothing.

- This is what the majority of physicists may immediately say on the ground of the Einstein principle of equivalence that states that geometry of any spacetime manifold can be reduced locally to the Minkowski geometry in a neighborhood of any point in space or even around any world line.

- The common opinion (a consensus) is that the background manifold in the solar system is flat Minkowski spacetime which is matched with the curved cosmological manifold very-very far away so that we may forget about cosmology in the local measurements.

- This opinion makes sense but it still remains an opinion which has to be put to the scrutiny test in order to check if the Einstein principle of equivalence is valid on an expanding cosmological manifold.

- The caveat is that the cosmological manifold is conformally flat and the transformation to the local coordinates must be conformal as well because the tangent spacetime is flat. It brings about time-dependent re-scaling of units – a delicate issue.

- This talk discusses possible consequences of this re-scaling for clock/ranging/Doppler-tracking experiments in the solar system.

Large portion of this talk is a logical development of my two recent publications:

PHYSICAL REVIEW D **86,** 064004 (2012)

Celestial ephemerides in an expanding universe

Sergei M. Kopeikin*

Department of Physics and Astronomy, University of Missouri, 322 Physics Building, Columbia, Missouri 65211, USA

PHYSICAL REVIEW D **87,** 044029 (2013)

Post-Newtonian celestial dynamics in cosmology: Field equations

Sergei M. Kopeikin*

Department of Physics and Astronomy, University of Missourì-Columbia, 322 Physics Building, Columbia, Missouri 65211, USA

Alexander N. Petrov†

Sternberg Astronomical Institute, Moscow M. V. Lomonosov State University, Universitetskii Prospect 13, Moscow 119992, Russia

Contents:

Part 1. Theory

Part 2. Applications

Part 1. Theory

The Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + R^2(t)\left(1 + \frac{k}{r^2}\right)\delta_{ij} dy^i dy^j$$

(k – curvature of space; we accept $k = 0$)

$$dt = R[t(\eta)]\mathrm{d}\eta = a(\eta) d\eta$$

$$ds^2 = a^2(\eta) f_{\alpha\beta} dy^\alpha dy^\beta$$

$y^\alpha = (c\eta, y^i)$ – the conformal coordinates ; $\qquad a(\eta) \equiv R[t(\eta)]$

t - the cosmic time = the proper time of the Hubble observer
(can be identified with the barycentric time in the solar system)

η – the conformal time (a convenient mathematical device)

$f_{\alpha\beta} = diag\,(-1,1,1,1)$ – the Minkowski metric

We focus on the Einstein equivalence principle.
It states that there exists a diffeomorphism $y^{\alpha} = y^{\alpha}(x^{\beta})$ from global, y^{α}, to local inertial coordinates, $x^{\alpha} = (x^0, x^i)$, such that in the neighborhood of an observer moving along a time-like world line $\mathcal{P}$

$$ds^2 = f_{\alpha\beta}(\mathcal{P}) dx^{\alpha} dx^{\beta} \quad \text{and} \quad \Gamma^{\alpha}_{\beta\gamma}(\mathcal{P}) = 0$$

and the local equations of motion for test particles and light are geodesics (straight lines) which can be parameterized with a single parameter x^0 which coincides with the proper time τ of the central observer

$$\frac{d^2 x^{\alpha}}{d\tau^2} = 0 \quad \Longrightarrow \quad x^{\alpha}(\tau) = x_0^{\alpha} + v^{\alpha}\tau$$

We simplify the theory by assuming a linearized Hubble approximation in all equations (only terms of the order of *H* are accounted for)

Local diffeomorphism: $$a^2(\eta) f_{\mu\nu} \frac{\partial y^\mu}{\partial x^\alpha} \frac{\partial y^\nu}{\partial x^\beta} = f_{\alpha\beta} \qquad \text{(A)}$$

(the Einstein principle of equivalence)

Special conformal transformation: $$\Omega^2(x) f_{\mu\nu} \frac{\partial y^\mu}{\partial x^\alpha} \frac{\partial y^\nu}{\partial x^\beta} = f_{\alpha\beta} \qquad \text{(B)}$$

$$\Omega(x^\alpha) = 1 - 2b_\alpha x^\alpha + b^2 x^2 \qquad \boxed{y^\alpha = \frac{x^\alpha - b^\alpha x^2}{\Omega(x^\alpha)}}$$

Matching (A) and (B) yields: $x^2 = f_{\alpha\beta} x^\alpha x^\beta \qquad b^2 = f_{\alpha\beta} b^\alpha b^\beta$

$$b^\alpha = \frac{H}{2} u^\alpha \qquad \Omega(x^\alpha) = a(\eta) = 1 + H\eta \qquad u^\alpha = (1,0,0,0)$$

Local Minkowskian metric: $ds^2 = -(dx^0)^2 + \delta_{ij}dx^i dx^j$

$$x^i = a(\eta)y^i \qquad x^0 + \frac{H}{2}\left[(x^0)^2 - \delta_{ij}x^i x^j\right] = a(\eta)\eta$$

We want to identify the coordinate time x^0 with the proper time τ of the Hubble observer $\left(x^i = 0\right)$ which is by definition coincides with the cosmic time t $\quad (x^0 \equiv \tau)$

$$\tau = t = \int a(\eta)d\eta = \int (1 + H\eta)d\eta = \eta + \frac{1}{2}H\eta^2$$

$$\tau + \frac{1}{2}H\tau^2 = \eta + H\eta^2 \quad \Longrightarrow \quad \tau = \eta + \frac{1}{2}H\eta^2$$

Local inertial metric: $ds^2 = -d\tau^2 + \delta_{ij}dx^i dx^j$

Local inertial frame: τ − proper time of static observers

x^i − normal Gaussian coordinates, $u^\alpha = (1,0,0,0)$

Static observers with fixed x^i move with respect to the Hubble observers which have fixed y^i, and vice versa

$x^i = a(\tau)y^i$ $\qquad$ $\frac{dx^i}{d\tau} = Hy^i$ $\qquad$ Hubble law

$\tau = t$ $\qquad$ the proper time τ of the static observers coincides with the cosmic time *t (after clock's synchronization)*

Do the local static observers exist in an expanding universe?

The answer is affirmative. Mathematical analysis of electromagnetic and gravitational forces reveals that they are not influenced by the Hubble expansion in the linearized Hubble approximation (at least, in classical physics)
(*S. Kopeikin*, PRD, **86**, 064004, 2012)

It means that atomic orbitals and planetary orbits are stable (not evolving adiabatically) and can be used to materialize the reference frame of the static observers locally (in lab and in the solar system) .

It also means that the atomic time, the ephemeris time, and the cosmic time *t* flow with the same rate.

Let's now focus on the motion of photons (light). We want to keep the identification $x^0 = \tau$ for photons. Let's see if we can do it.

Equations of motion of light in global coordinates: $\frac{d^2 y^\alpha}{d\eta^2} = 0$ (C)

We expect in local (Gaussian) coordinates: $\frac{d^2 x^\alpha}{d\tau^2} = 0$ (D)

By inspection one can verify that (C) relates to (D) by the conformal coordinate transformation: $\tau = a(\eta)\eta \quad , \quad x^i = a(\eta) y^i$ (E)

We've already seen that operationally $\tau = t$. Hence, (D) and (E) force us to accept that the cosmic time relates to the conformal time by equation

$$t = a(\eta)\eta = \eta + H\eta^2 + O(H^2) \qquad \text{(E)}$$

However, the RW metric tells us that the cosmic and conformal times are related by $t = \int a(\eta)\, d\eta = \eta + \frac{H}{2}\eta^2 + O(H^2)$ (F)

But (E) contradicts (F) !

***Conclusion:* something is wrong with (D)**

Resolving the contradiction: We cannot postulate that the affine parameter on light geodesic is the proper time of the static observers. It must differ from τ.

Let's denote it with λ and replace (D) with $\frac{d^2x^\alpha}{d\lambda^2} = 0$

Conformal transformation: $\lambda = a(\eta)\eta \qquad x^i = a(\eta)y^i$

$$\lambda = a(\eta)\eta = \eta + H\eta^2 + O(H^2) \quad \Rightarrow \quad d\lambda = a(\tau)d\tau$$

$$\tau = \int a(\eta)d\eta = \eta + \frac{H}{2}\eta^2 + O(H^2)$$

Lesson 1: the local inertial metric of static observers $ds^2 = -d\tau^2 + \delta_{ij}dx^i dx^j$ differs from the optical metric on the light cone

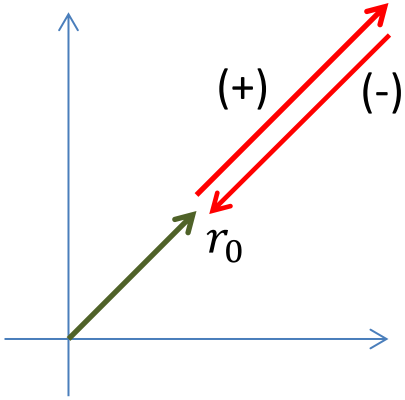
$$\boldsymbol{ds^2 = -a^2(\tau)d\tau^2 + \delta_{ij}dx^i dx^j}$$

Lesson 2: Photons fall differently from massive particles in the local frame built on Robertson-Walker manifold! Correct equations of motion for photons in a local frame parameterized with the proper time

$$\boldsymbol{\frac{d^2x^\alpha}{d\tau^2} = H\left(\frac{dx^\alpha}{d\tau} - u^\alpha\right)}$$

Light propagation:

- Solve $\boldsymbol{ds^2 = -a^2(\tau)d\tau^2 + \delta_{ij}dx^i dx^j = 0}$



- The radial geodesic: $x^i = k^i r(\tau)$
- $dr = \pm \int a(\tau)\, d\tau$

  $(+)$ an outgoing ray $(-)$ an incoming ray

- $r = r_0 + \tau + \frac{H}{2}\tau^2$ $r = r_0 - \tau - \frac{H}{2}\tau^2$

- The coordinate speed of light $\dot{r} = 1 + H\tau >$ c for $\tau > 0$ (should we worry? no – SR is valid)

Part 2.   Applications

Radar and laser ranging (Einstein's procedure):

Radial geodesics, solving $ds = 0$

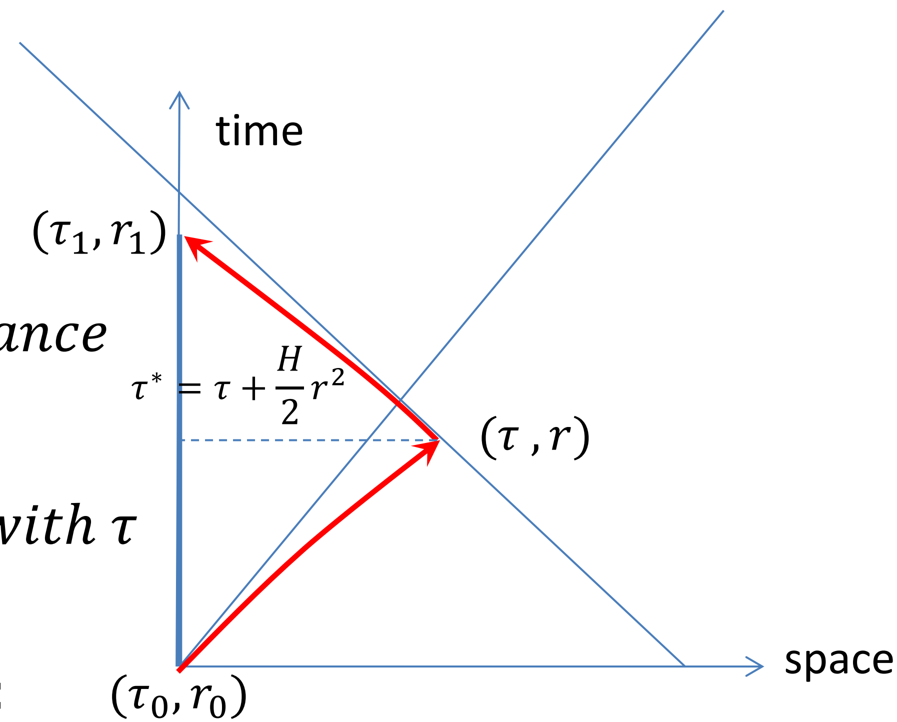
$$r = r_0 + \int_{\tau_0}^{\tau} a(\tau')d\tau'$$

$$a(\tau') = 1 + H(\tau' - \tau_0)$$

$$r_1 = r - \int_{\tau}^{\tau_1} a(\tau')d\tau'$$

$$\ell = \frac{1}{2}(\tau_1 - \tau_0) \quad - the\ proper\ distance$$

$$\tau^* = \frac{1}{2}(\tau_1 + \tau_0) \quad - simultaneous\ with\ \tau$$



For a central observer $(r_0 = r_1 = 0)$:

$$\ell = r - Hr^2$$

$$\tau^* = \tau + \frac{H}{2}r^2$$

Doppler Effect (static observers in the local frame):

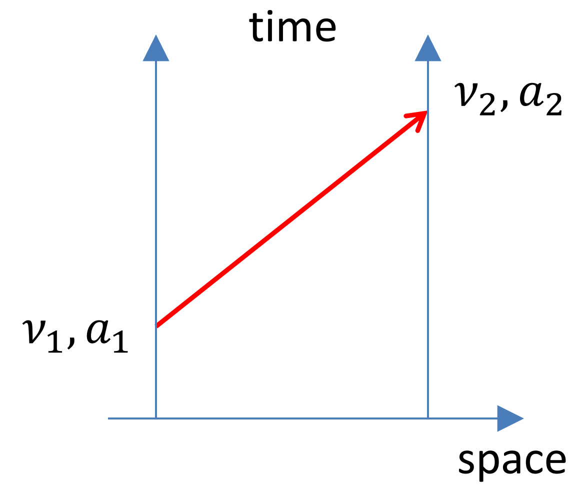
$\nu = -k_\alpha u^\alpha$ the frequency of light

$$\frac{\nu_2}{\nu_1} = \frac{k_0(\tau_2)}{k_0(\tau_1)}$$

$$\frac{dk_\alpha}{d\lambda} = -\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\alpha} k_\mu k_\nu$$ a parallel transport of k_α

$$\frac{k_0(\tau)}{a(\tau)} = \text{const.}$$

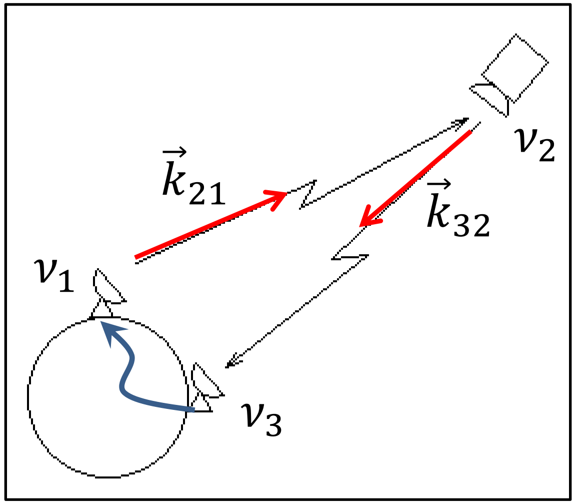
$$\frac{\nu_2}{\nu_1} = \frac{a(\tau_2)}{a(\tau_1)}$$



$$\frac{\nu_2}{\nu_1} = 1 + H(\tau_2 - \tau_1) \quad > 1$$ *a blue shift of frequency!*

Doppler tracking and the Pioneer Anomaly Effect

$$\frac{\nu_2}{\nu_1} = \frac{1 + \mathcal{H}\tau_2 - \vec{k}_{21}\cdot\vec{\beta}_2}{1 + \mathcal{H}\tau_1 - \vec{k}_{21}\cdot\vec{\beta}_1} \qquad \frac{\nu_3}{\nu_2} = \frac{1 + \mathcal{H}\tau_3 - \vec{k}_{32}\cdot\vec{\beta}_3}{1 + \mathcal{H}\tau_2 - \vec{k}_{32}\cdot\vec{\beta}_2}$$



$$\frac{\nu_3}{\nu_1} = \frac{\nu_3}{\nu_2}\frac{\nu_2}{\nu_1} = 1 + 2\mathcal{H}(\tau_2 - \tau_1) - 2\vec{k}_{21}\cdot(\vec{\beta}_2 - \vec{\beta}_1)$$

Assuming that $\boldsymbol{\tau_3 - \tau_2 \simeq \tau_2 - \tau_1}$

Assuming that $\boldsymbol{\vec{k}_{32} \simeq -\vec{k}_{21}}$ and $\boldsymbol{\vec{\beta}_3 \simeq \vec{\beta}_1}$

Integrated Doppler tracking over a long time interval *t* *(decades with modern technology!)*

$$\frac{\nu_{2n+1}}{\nu_1} = \frac{\nu_{2n+1}}{\nu_{2n-1}}\frac{\nu_{2n-1}}{\nu_{2n-3}}\cdots\frac{\nu_3}{\nu_1} \quad \Longrightarrow \quad \frac{\nu_{\mathrm{obs}}}{\nu_{\mathrm{ref}}} = 1 + 2\mathcal{H}\tau - \frac{2}{c}\int_0^\tau a_r(\tau')d\tau'$$

$$\frac{\nu_{\mathrm{obs}}}{\nu_{\mathrm{ref}}} = 1 - \frac{2}{c}\int_0^\tau [a_r(\tau') - \mathcal{H}c]d\tau' \quad \Longrightarrow \quad \boxed{\frac{\nu_{\mathrm{obs}} - \nu_{\mathrm{model}}}{\nu_{\mathrm{ref}}} = 2\mathcal{H}\tau}$$

blue shift

$$\boxed{a_{\mathrm{spacecraft}} = a_{\mathrm{model}} - \mathcal{H}c}$$

The constant "anomalous acceleration"
$\boldsymbol{a_P = -\mathcal{H}c}$ ***is directed toward observer.***

J. Anderson et al 2002 **(a_P has a fundamental physical origin yet unknown)**
C. Lämmerzahl et al 2009 **(a_P can be fully explained by thermal recoil of heat from on-board RTG)**
S. Turyshev et al 2010 **(a_P can be fully explained by thermal recoil of heat from on-board RTG)**

Microwave Cavity, Atomic Clocks and the Hubble Expansion

- Cavity resonator represents a hollow rigid rod. Its size is determined by the chemical (electromagnetic) bonds in the tangent spacetime. The "rigid rod" does not take part in the Hubble expansion.

- Frequencies of atomic transitions in atoms are defined by the energy gap between the energy levels in the atom. They are fully determined by the Dirac equation. The Dirac equation preserves its form after making the conformal transformation to the local coordinates (M. *Ibison, 2007*).
- Atomic frequencies are not affected by the Hubble expansion locally.

Microwave cavity

- Use the RW metric for Maxwell equations

$$-\frac{\partial^2 \vec{E}}{\partial \eta^2} + \frac{\partial^2 \vec{E}}{\partial y^2} = 0 \,,$$

- Look for the standing wave solution in the instantaneous basis

$$\vec{E} = \vec{E}_0 Q_n(\eta) \sin\left[\frac{\pi n y}{l(\eta)}\right]$$

with the boundary condition $\overrightarrow{E_t} = 0$ imposed on the walls of the cavity which are adiabatically contracting in the global coordinates:

$$l(\eta) = L/a(\eta)$$

- Derive equation for the functions Q_n:

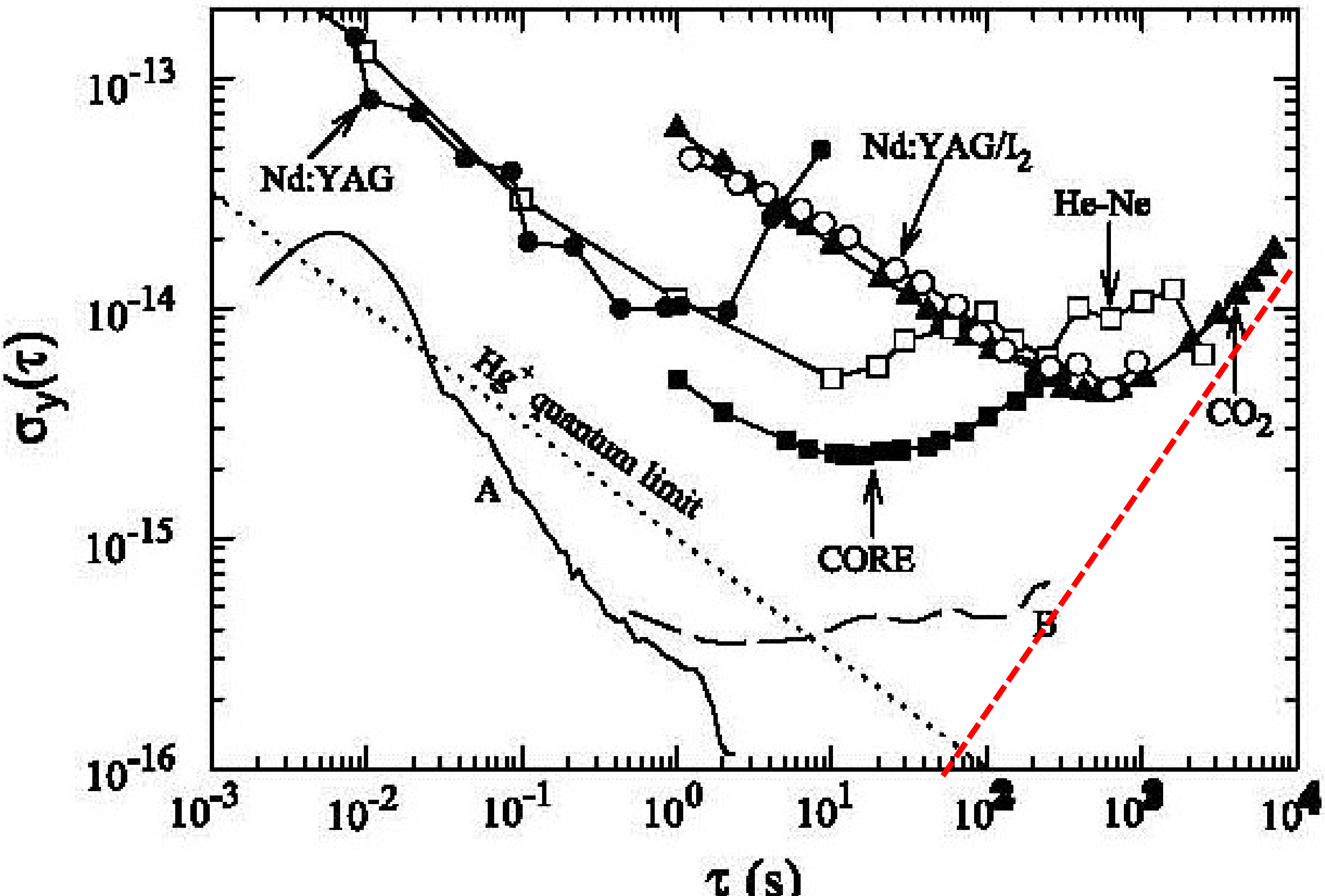
$$Q_n'' + \omega_n(\eta) Q_n = 0 \quad \text{where } \omega_n(\eta) = a(\eta)\Omega_n \text{ and } \Omega_n = \frac{\pi n}{L}.$$

- Solution for $Q_n = \frac{b_n}{\sqrt{a(\eta)}} \cos\left[\int \omega_n(\eta) d\eta + \phi_0\right] = \frac{b_n}{\sqrt{a(\eta)}} \cos[\Omega_n \tau + \phi_0]$

- Solution for electric field

- $\vec{E} = \vec{E}_0 \frac{b_n}{\sqrt{a(\eta)}} \cos[\Omega_n \tau + \phi_0] \sin\left[\frac{\pi n x}{L}\right]$

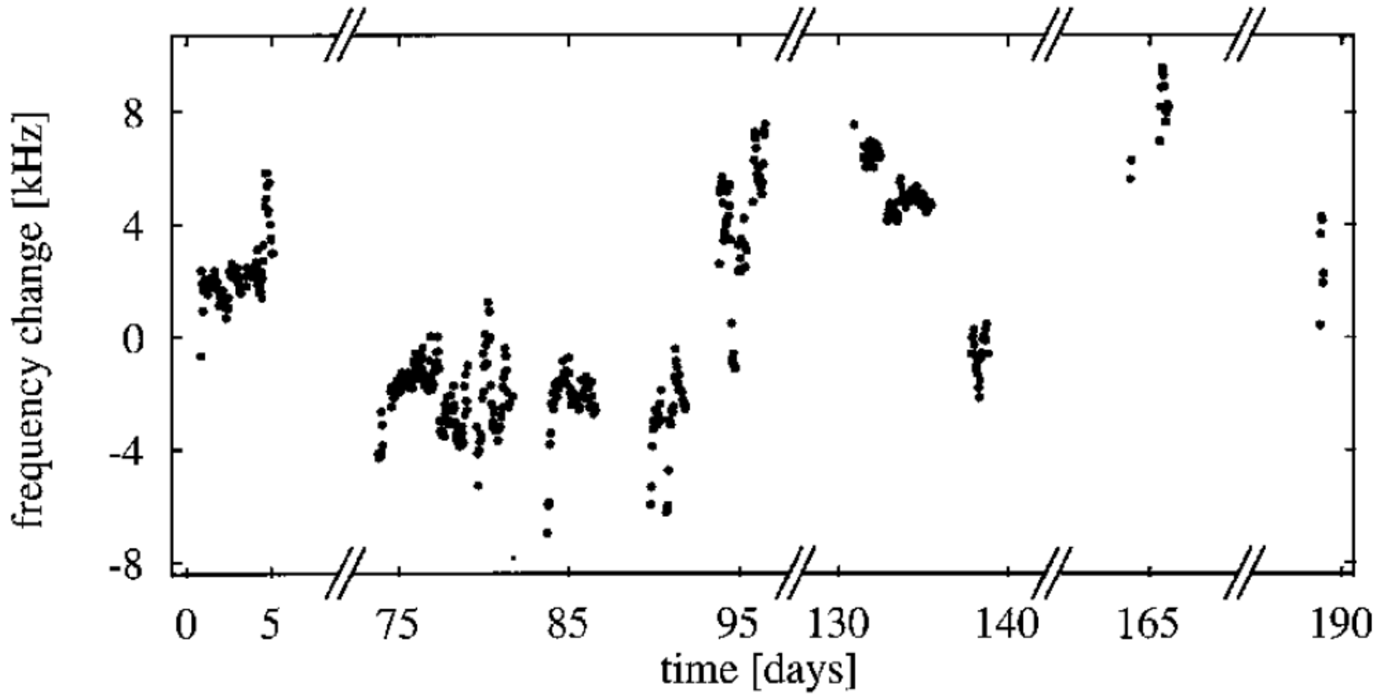
Allan Variance of Cavity-Stabilized Lasers (NIST, Young et al. 1999)



Experimental limits

(we thank Prof. S. Schiller for providing this slide)

Comparsion of a cryogenic optical resonator and an optical transition in molecular iodine I_2



$| \nu^{-1} \Delta\nu/\Delta t | \simeq 1.0 \times 10^{-11}/(190 \text{ days}) = 0.6 \times 10^{-18}/\text{s}$

R. Storz, C. Braxmaier, K. Jäck, O. Pradl, S. Schiller, Opt. Lett. ***23****, 1031 (1998)*

Conclusions:

- We have to be more careful in our theoretical models based on the assumption that spacetime is asymptotically flat.
- EEP may be violated for photons as compared with slowly moving particles because of the Hubble expansion
- New space experiments are required to clarify the origin of the “Pioneer anomaly” effect and to either confirm or disprove its cosmological origin.
- There is no secular drift of microwave cavity frequency with respect to atom/ion frequency caused by the Hubble expansion. Further testing is required.

THANK YOU!