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Cosmological Inflation and High-Scale SUSY as the Origin of Dark Matter

by Sergei V. Ketov

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8+15 July 2024
with Alexei Starobinsky, Maxim Khlopov, Yermek Aldabergenov, Daniel Frolovsky, ...

arXiv:1011.0240, 1203.0895, 1406.0252, 1408.6524, 1607.05366, 2302.06153, 2304.12558, 2401.11651, 2402.02934,.......

$\boxed{\text{COSMOLOGICAL INFLATION}}$

Naively, inflation is a proposal (cosmological paradigm) about the existence of a "short" but "fast" (exponential, or de-Sitter-type) accelerated grow of the scale factor $a(t)$ in the early Universe since $10^{-36}\ s$ until $10^{-32}\ s$, before the radiation-dominated era (Starobinsky, Guth, Linde, around 1981):

$$\boxed{\ddot{a}\ (t) > 0}$$

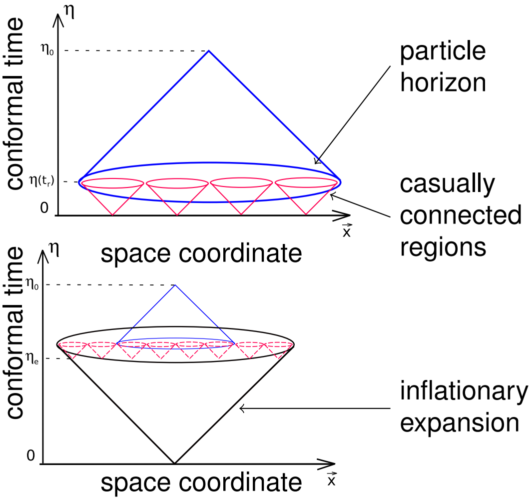
- There is the significant (indirect) evidence for the inflation because of:

(i) the correct prediction of CMB fluctuations and anisotropy (2006 Nobel prize),
(ii) the inflationary origin of the *large scale structure:* inflation can amplify quantum fluctuations that can be the seeds of the formation of structure. It gives us the possibility of testing the inflationary scenario.

- Inflation also solves the problems of the OLD Standard Cosmology.
- However, the detailed mechanism of inflation is still unknown.

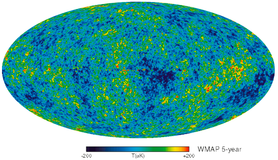
Inflationary solution of Hot Big Bang problems

- no need for infinite density
- all scales grow exponentially, including the radius of the 3-sphere
  the Universe becomes exponentially flat
- any two particles are at exponentially large distances
  no heavy relics
  no traces of previous epochs!
- no particles in post-inflationary Universe
  to solve entropy problem we need post-inflationary reheating

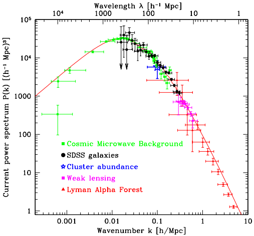


Inflationary solution of Hot Big Bang problems

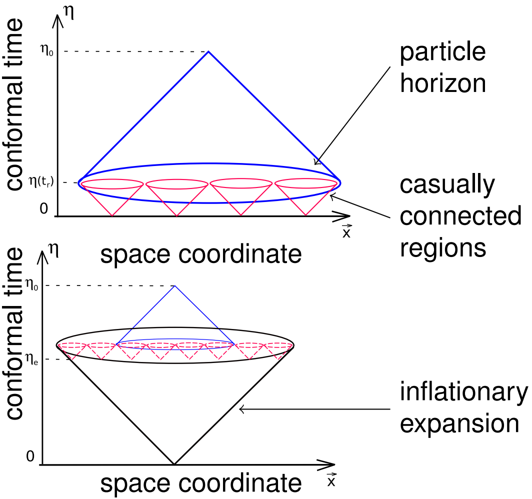
Temperature fluctuations
$\delta T/T \sim 10^{-5}$



Universe is uniform!



$\delta\rho/\rho \sim 10^{-5}$



$\boxed{\text{Models of inflation}}$

- chaotic inflation • natural inflation • hybrid inflation
- Higgs inflation • Starobinsky inflation • hilltop inflation
- modular inflation • Coleman-Weinberg inflation
- supergravity inflation • brane inflation • warm inflation
- tachyon inflation • SUSY breaking inflation • K-inflation
- string moduli inflation • and many more!

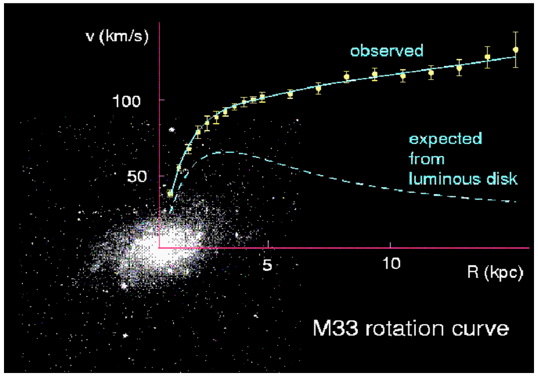
Our approach: is based on the gravitational origin of inflation, by using only gravitational interactions. It distinguishes the Starobinsky (or R^2-inflation) as the only (basic) model of inflation. I will explain why.

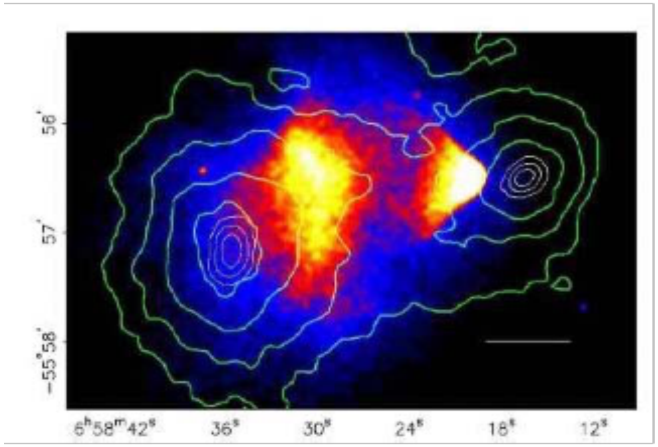
Dark matter postulated in 30's (Zwicky) – 80 years later we know very little about DM

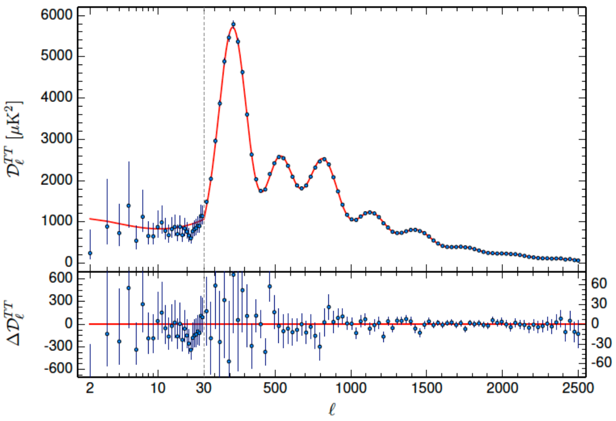
It has gravitational interactions (galaxies – rotation curves- galaxy clusters - Xray, gravitational lensing)

No electromagnetic interactions, must be stable (or meta-stable)

It is cold (or maybe warm) and collisionless (or not)

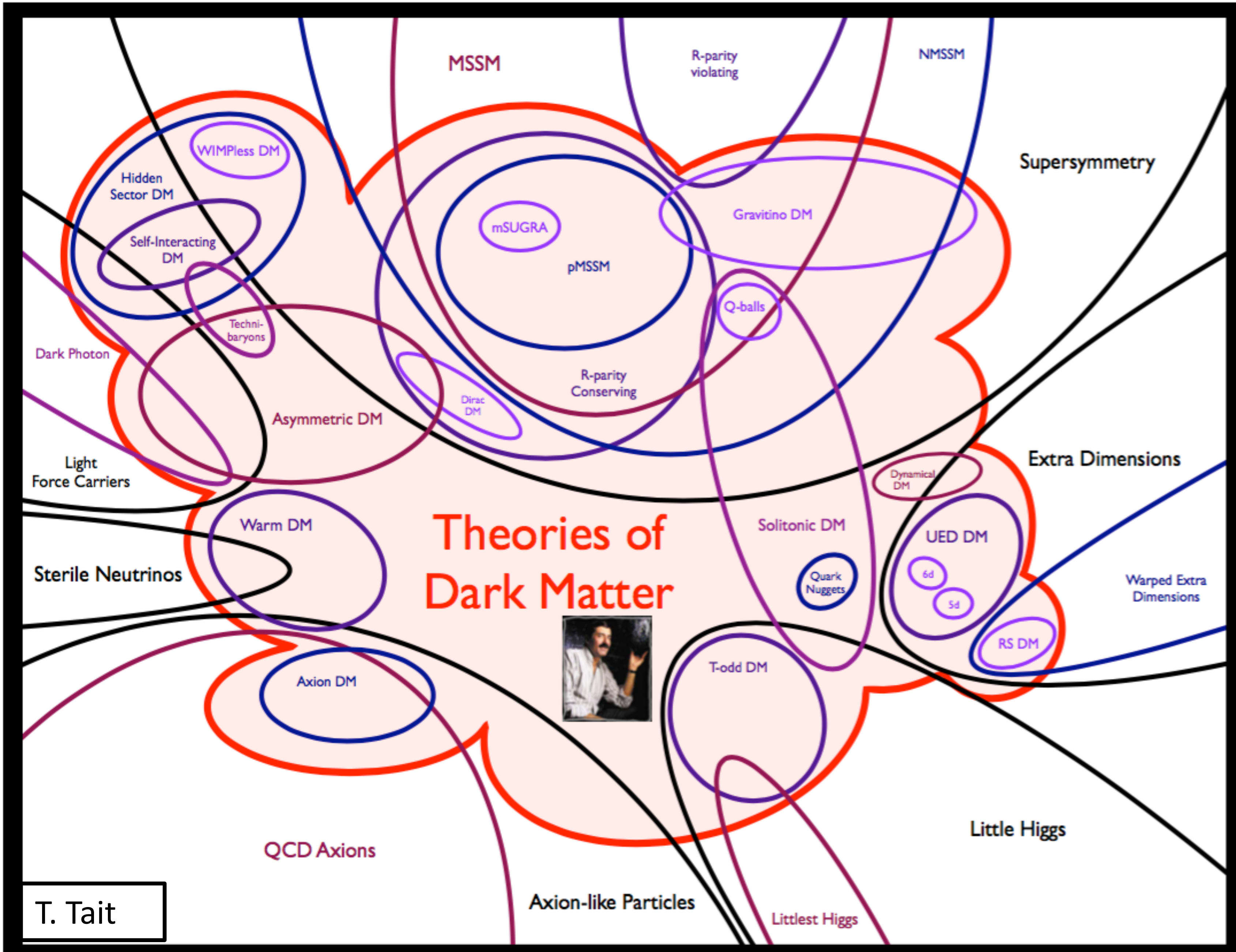






Within ΛCDM model – precisely know DM relic density: 0.3-0.4 GeV/cm^3

$\Omega_{cdm}\, h^2$=0.1193+/- 0.0014 (PLANCK – 1502.01589)


MSSM
R-parity violating
NMSSM
Supersymmetry
WIMPless DM
Hidden Sector DM
Self-Interacting DM
Techni-baryons
mSUGRA
pMSSM
Gravitino DM
Q-balls
Dark Photon
Dirac DM
R-parity Conserving
Asymmetric DM
Light Force Carriers
Extra Dimensions
Dynamical DM
Warm DM
Theories of Dark Matter
Solitonic DM
UED DM
Sterile Neutrinos
Quark Nuggets
6d
5d
Warped Extra Dimensions
RS DM
Axion DM
T-odd DM
Little Higgs
QCD Axions
Axion-like Particles
Littlest Higgs
T. Tait

Leaves us with a lot of possibilities for dark matter

- A new particle? Composite DM? Mass scale? (Meta)stability!

- WIMPs – long time favorite : good theoretical motivation, typical annihilation cross-section leads to correct relic density
- WIMPs : however, no observational evidence despite extensive direct and indirect searches on colliders and in space
- GIMPs (Gravitationally Interacting Massive Particles) and
- Non-particle Dark Matter as Primordial Black Holes!

Plan of talk

- Starobinsky (modified) gravity for inflation and quintessence
- Extension of the Starobinsky model in modified gravity for PBH
- PBH production and induced GW in the extended model
- Inflation and PBH production in Starobinsky supergravity
- Spontaneous high-scale SUSY breaking, dark energy and MSSM
- Higgs mass and dark matter
- Conclusion

Modified gravity

- Modified gravity theories are generally-covariant non-perturbative extensions of Einstein-Hilbert gravity theory by the higher-order terms. These terms are irrelevant in the Solar system but are relevant in the high-curvature regimes (inflation, black holes) or for large cosmological distances (dark energy).
- A modified gravity action has the higher-derivatives and generically suffers from Ostrogradsky instability and ghosts. However, there are exceptions. For example, in the modified gravity Largrangian quadratic in the spacetime curvature, the only ghost-free term is given by R^2 with a positive coefficient. It leads to the Starobinsky model (1980) of modified gravity with the action

$$S_{\text{Star.}} = \alpha \int \mathrm{d}^4x\sqrt{-g}R^2 + \frac{M_{\text{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R\,, \quad \alpha \equiv \frac{M_{\text{Pl}}^2}{12M^2}\ ,$$

with the single dimensionless parameter α, where $M_{\text{Pl}} = 1/\sqrt{8\pi G_{\text{N}}} \approx 2.4 \times 10^{18}$ GeV, and the spacetime signature is $(-,+,+,+,)$.

On a possible origin of EH term

from classically scale-invariant gravity (*Buchbinder 1986, Einhorn, Jones 2015*)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\alpha R^2 + \xi\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \lambda\phi^4\right]$$

that can undergo a phase transition (*dimensional transmutation*) due to quantum (radiative) corrections, known as the Coleman-Weinberg mechanism of spontaneous symmetry breaking (1973). It leads to a massive scalar field ϕ that can be identified with dilaton or Higgs field with a non-vanishing VEV in the effective action UV-fixed point, as can be demonstrated perturbatively (in 1 loop). Therefore, both the Planck mass and the EH term are also generated with

$$\frac{1}{2}M_{\mathrm{Pl}}^2 = \xi\left\langle\phi\right\rangle^2 \ ,$$

though it cannot be served as a full UV-completion of EH or Starobinsky gravity.

Starobinsky attaractor solution

- In a flat FLRW universe $ds^2 = -dt^2 + a^2\left(dx_1^2 + dx_2^2 + dx_3^2\right)$, the EoM read

$$2H\ddot{H} - \left(\dot{H}\right)^2 + H^2\left(6\dot{H} + M^2\right) = 0\,, \quad \text{where} \quad H = \dot{a}/a\ .$$

Using an *ansatz* in the form of left Painlevé series $H(t) = \sum_{k=-\infty}^{k=p} c_k (t_0 - t)^k$, we find

$$H(t) = \frac{M^2}{6}(t_0 - t) + \frac{1}{6(t_0 - t)} - \frac{4}{9M^2(t_0 - t)^3} + \frac{146}{45M^4(t_0 - t)^5} - \frac{11752}{315M^6(t_0 - t)^7} + \mathcal{O}\left((t_0 - t)^{-9}\right)$$

valid for $M(t_0 - t) \gg 1$. This solution is an attractor, it does not have a movable singularity and describes (formally) eternal inflation up to $t \to -\infty$.

Starobinsky model of inflation

- In the high-curvature regime, the EH term can be ignored and the pure R^2-action becomes scale-invariant.
- In the the slow roll-approximation with $\left|\ddot{H}\right| \ll \left|H\dot{H}\right|$ and $\left|\dot{H}\right| \ll H^2$,

$$H(t) \approx \left(\frac{M^2}{6}\right)(t_0 - t) \ .$$

This solution spontaneously breaks the scale invariance of the R^2-gravity and implies an existence of the Nambu-Goldstone boson called scalaron.

- Scalaron is the physical (scalar) excitation of the higher-derivative gravity. It can be revealed by rewriting the Starobinsky action into the quintessence form by the field redefinition (Legendre-Weyl transform) *K.-I. Maeda (1989)*

$$\varphi = \sqrt{\frac{3}{2}} M_{\mathrm{Pl}} \ln F'(\chi) \quad \text{and} \quad g_{\mu\nu} \to \frac{2}{M_{\mathrm{Pl}}^2} F'(\chi) g_{\mu\nu} \ , \quad \chi = R \ ,$$

Scale invariance in Jordan frame = Shift symmetry in Einstein frame !

which leads to

$$S[g_{\mu\nu},\varphi] = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}R - \int {\rm d}^4x\sqrt{-g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right] ,$$

with the potential $V(\varphi) = \frac{3}{4}M_{\rm Pl}^2 M^2\left[1-\exp\left(-\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right)\right]^2 \equiv V_0[1-y]^2$, suitable for describing slow-roll inflation with scalaron φ as the inflaton of mass m due to the infinite plateau of the positive height $\approx V_0$ for $y \ll 1$.

- The UV cutoff is $\Lambda_{\rm UV} = M_{\rm Pl}$ (*Hertzberg, 2010)*. The higher-order curvature terms are suppressed by powers of $H^2/M_{\rm Pl}^2$ relative to Starobinsky. A string theory derivation of the Starobinsky inflation is unknown.
- The inverse transform from quintessence to $F(R)$ gravity reads

$$R = \left(\frac{\sqrt{6}}{M_{\rm Pl}}\frac{dV}{d\varphi} + \frac{4V}{M_{\rm Pl}^2}\right)e^{\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}} , \quad F = \left(\frac{\sqrt{6}}{M_{\rm Pl}}\frac{dV}{d\varphi} + \frac{2V}{M_{\rm Pl}^2}\right)e^{2\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}} .$$

Starobinsky model (1980) and CMB measurements (2020)

The very simple Starobinsky model of inflation is in excellent agreement with the current CMB measurements (Planck, BICEP/Keck).

A duration of inflation is usually measured by the e-foldings number

$$N = \int_{t_*}^{t_{\mathrm{end}}} H(t)dt \approx \frac{1}{M_{\mathrm{Pl}}^2} \int_{\varphi_{\mathrm{end}}}^{\varphi_*} \frac{V}{V'} d\varphi \ .$$

The standard slow roll parameters are defined by

$$\varepsilon_{\mathrm{sr}}(\varphi) = \frac{M_{\mathrm{Pl}}^2}{2} \left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_{\mathrm{sr}}(\varphi) = M_{\mathrm{Pl}}^2 \left(\frac{V''}{V}\right) \ .$$

The amplitude of scalar (curvature) perturbations at the horizon crossing with the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$ is determined by the WMAP normalization,

$$A_s = \frac{V_*^3}{12\pi^2 M_{\mathrm{Pl}}^6 (V_*')^2} = \frac{3M^2}{8\pi^2 M_{\mathrm{Pl}}^2} \sinh^4\left(\frac{\varphi_*}{\sqrt{6}M_{\mathrm{Pl}}}\right) \approx 1.96 \cdot 10^{-9}$$

that implies no free parameters in the Starobinsky model,

$$M \approx 3 \cdot 10^{13}\ \mathrm{GeV} \quad \text{or} \quad \frac{M}{M_{\mathrm{Pl}}} \approx 1.3 \cdot 10^{-5}\,, \quad \text{and} \quad H \approx \mathcal{O}(10^{14})\ \mathrm{GeV}\,.$$

No free parameters but still two predictions!

The CMB measurements give the tilt of scalar perturbations $n_s \approx 1 + 2\eta_{\mathrm{sr}} - 6\varepsilon_{\mathrm{sr}} \approx 0.9649 \pm 0.0042\ (68\%\mathrm{CL})$ and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_{\mathrm{sr}} < 0.032\ (95\%\mathrm{CL})$. The Starobinsky inflation gives $r \approx 12/N^2 \approx 0.003$ and $n_s \approx 1 - 2/N$, with the best fit at $N \approx 55$. *The prediction is* r=3(1-n_s)^2 *and it is the main target for the LiteBIRD and Simons Observatory projects.*

Starobinsky inflation and Swampland Conjectures

about consistency with quantum gravity by pure thoughts aimed to discriminate between EFT (*Vafa, Palti, Valenzuela et al., from 2018*).

- *No exact global symmetries in quantum gravity*: Starobinsky inflation merely implies the approximate scale invariance for limited field values.

- *No de Sitter and no eternal inflation*: the Starobinsky infinite plateau is destabilized by any higher-order terms beyond R^2 in the gravitational EFT, which are certainly present there.

- *Weak gravity Conjecture* means gravity is the weakest force, e.g., against electromagnetic force: during inflation there may be no charged particles yet (they appeared during reheating and after inflaton decay). Also, in the Einstein frame, inflation was due to the inflaton force (from the inflaton potential).

Power spectrum of perturbations

Primordial scalar perturbations (ζ) and tensor perturbations g (primordial GW) are defined by a perturbed FLRW metric,

$$ds^2 = dt^2 - a^2(t)\left(\delta_{ij} + h_{ij}(\vec{r})\right) dx^i dx^j \quad , \qquad i, j = 1, 2, 3 \; ,$$

where

$$h_{ij}(\vec{r}) = 2\zeta(\vec{r})\delta_{ij} + \sum_{b=1,2} g^{(b)}(\vec{r}) e_{ij}^{(b)}(\vec{r}) \quad , \quad H = \frac{da/dt}{a} \; ,$$

in terms of a local basis $e^{(b)}$ with $e_i^{i(b)} = 0$, $g_{,j}^{(b)} e_i^{j(b)} = 0$, $e_{ij}^{(b)} e^{ij(b)} = 1$.
The primordial spectrum $P_\zeta(k)$ of scalar (density) perturbations is defined by the 2-point correlation function of scalar perturbations,

$$\left\langle \frac{\delta\zeta(x)}{\zeta} \frac{\delta\zeta(y)}{\zeta} \right\rangle = \int \frac{d^3k}{k^3} e^{ik\cdot(x-y)} \frac{P_\zeta(k)}{P_0} \quad .$$

For instance, the observed CMB power spectrum is described by the Harrison-Zeldovich fit,

$$P_\zeta^{\mathrm{HZ}}(k) \approx 2.21^{+0.07}_{-0.08} \times 10^{-9} \left(\frac{k}{k_*}\right)^{n_s-1}$$

with the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$. In the slow-roll (SR) approximation, relevant for inflation, one finds

$$P_\zeta = \frac{H^2}{8M_{\mathrm{Pl}}^2\pi^2}\left(\frac{1}{\varepsilon_{\mathrm{sr}}}\right) \quad .$$

Therefore, it is possible to generate a large peak (enhancement) in the power spectrum by engineering $\epsilon_{\mathrm{sr}} \to 0$, called the ultra-slow-roll (USR) regime or the PBH production mechanism based a near-inflection point in the potential. This implies the double inflation scenario (SR $\to$ USR $\to$ SR) with two plateaus in the potential $V(\varphi)$ and in the Hubble function $H(t)$. Warning: USR is not SR !

A large peak means large density perturbations that later gravitationally collapse to PBH !

USR regime

To study the SR and USR regimes, we introduce the Hubble flow functions

$$\epsilon(t) = -\frac{\dot{H}}{H^2}\,, \qquad \eta(t) = \epsilon_H - \frac{\dot{\epsilon}_H}{2H\epsilon_H}\,.$$

During the USR regime, the function $\epsilon(t)$ drops to very low values, whereas the function $\eta(t)$ goes from nearly zero to (-6) and back.
A standard (numerical) procedure for computing the power spectrum $P_\zeta(k)$ of scalar perturbations depending upon scale (co-moving wavenumber) $k = 2\pi/\lambda$ is based on the Mukhanov-Sasaki (MS) equation. The SR approximation in single-field inflation gives the simple formula $P_\zeta = \frac{H^2}{8\pi^2 M_{\mathrm{Pl}}^2 \epsilon_H}$. We found that the difference between our numerical solutions to the MS equation and those derived from the SR formula is very small.

PBH masses

PBH may be formed by gravitational collapse of large density perturbations (Carr, Hawking, 1974). The masses of PBH can be estimated from given peaks (power spectrum enhancement) as follows (*Pi, Sasaki, 2017*):

$$M_{\rm PBH} \simeq \frac{M_{\rm Pl}^2}{H(t_{\rm peak})} \exp\left[2(N_{\rm total} - N_{\rm peak}) + \int_{t_{\rm peak}}^{t_{\rm total}} \varepsilon(t)H(t)dt\right]$$

that is very sensitive to the value of $\Delta N = N_{\rm total} - N_{\rm peak}$, while the integral gives a sub-leading correction. Increasing ΔN leads to decreasing the tilt n_s of CMB, which limits ΔN by 20 from above. On the other hand, ΔN cannot be too small when $M_{\rm PBH}$ have to exceed the Hawking (black hole) evaporation limit of 10^{15} g, which restricts ΔN from below (above 13).

Our model of inflation and PBH production is displayed on the next slide.

After fine-tuning parameters in our model, we obtained the PBH masses in the asteroid-size range between 10^{17} g and 10^{21} g. Compare $M_{\odot} \approx 2 \cdot 10^{33}$ g.

PBH production in the modified Starobinsky model of inflation

We propose the modified Appleby-Battye-Starobinsky (ABS) model (2010) of $F(R)$ gravity for that purpose, defined by the smooth F-function

$$F(R) = (1-g_1)R + gE_{AB}\ln\left[\frac{\cosh\left(\frac{R}{E_{AB}} - b\right)}{\cosh(b)}\right] + \frac{R^2}{6M^2} - \delta\frac{R^4}{48M^6} ,$$

where $g_1 = -g\tanh b$, $g \approx 2.25$ and $b \approx 2.89$, $0 < \delta < 4 \cdot 10^{-6}$, and

$$E_{AB} = \frac{R_0}{2g\ln(1+e^{2b})} \quad \text{with} \quad R_0 \approx 3M^2, \;\; M \sim 10^{-5}M_{\rm Pl} .$$

It is consistent with Starobinsky inflation and CMB measurements, has no ghosts ($F'(R) > 0$, $F''(R) > 0$), and the corresponding inflaton potential has two plateaus, leading to a large peak in the power spectrum. The last term can be interpreted as a quantum correction.

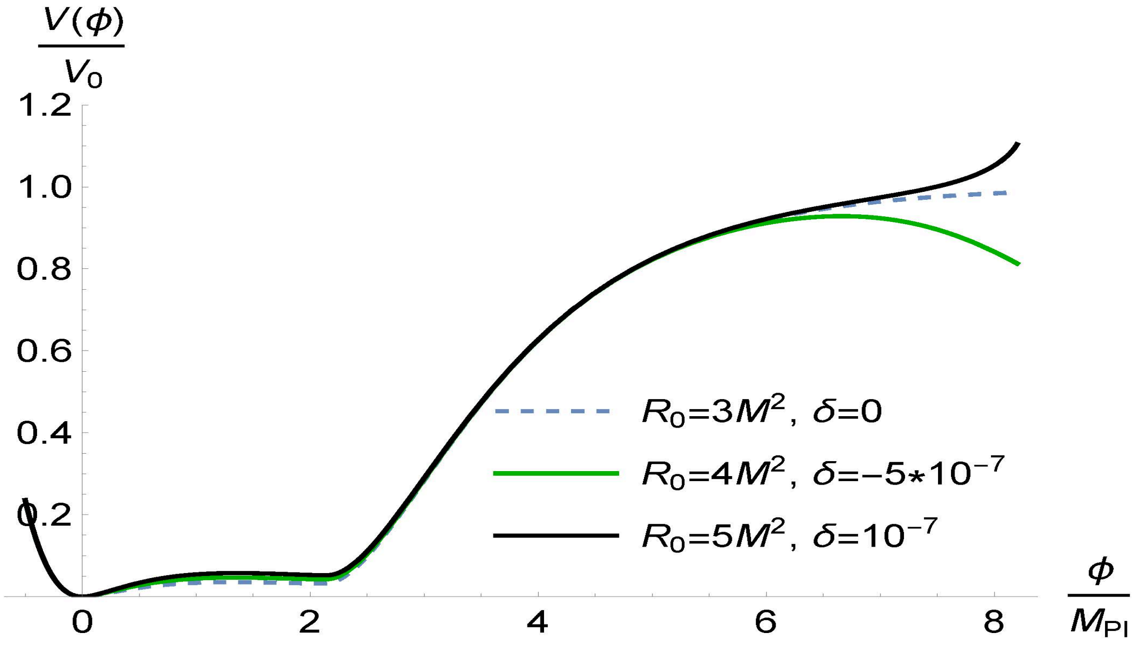
Consistency with CMB, and PBH masses

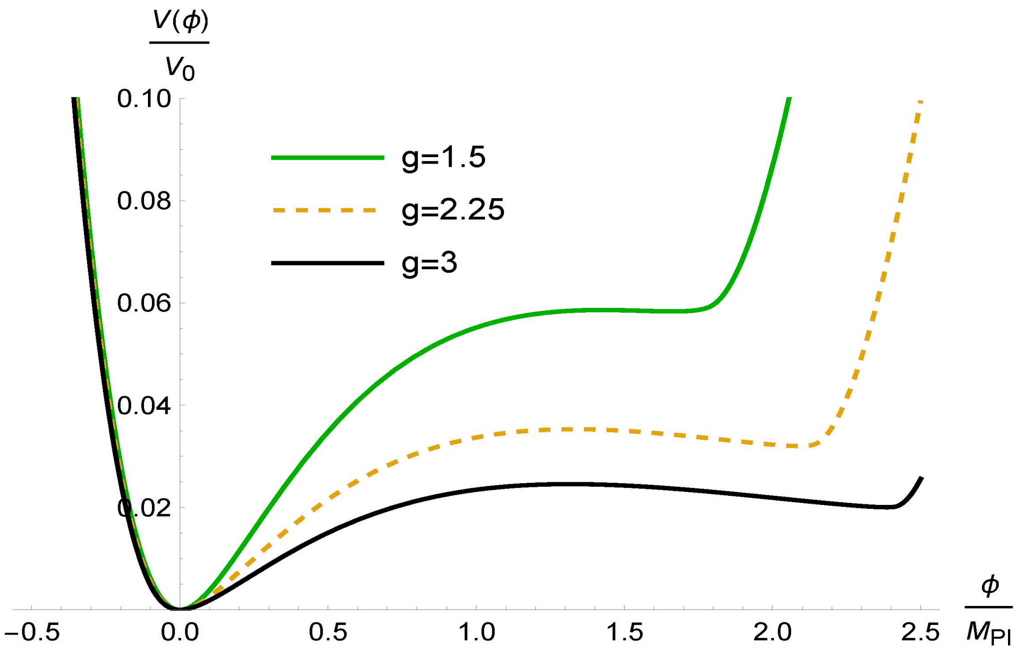
Demanding:

(i) a large enhancement (peak) in the power spectrum by the factor of 10^7 against the CMB level of 10^{-9},
(ii) consistency with the latest CMB measurements,
$n_s = 0.9649 \pm 0.0042$ (within 1σ) and $r < 0.032$, and
(iii) PBH masses beyond 10^{15} g,
we found $\triangle N$ must be restricted between 17 and 22 e-folds, while the total duration of inflation is between 54 and 66 e-folds.

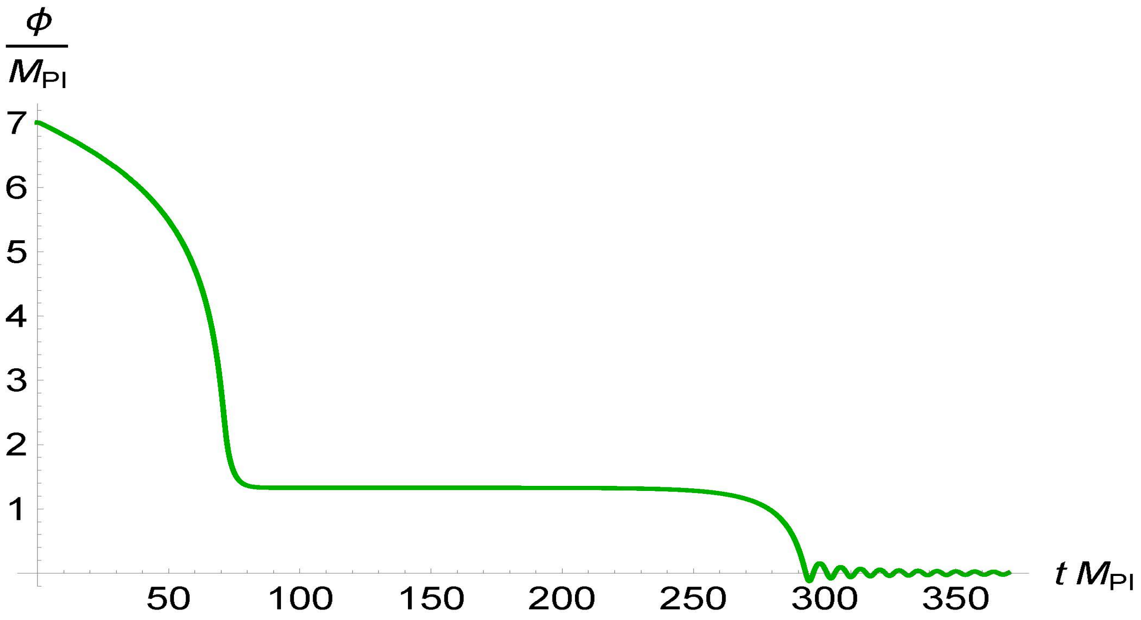
The possible range of the parameter δ is between $1.02 \cdot 10^{-8}$ and $8.74 \cdot 10^{-8}$. The PBH masses found are between 10^{17} g and 10^{21}g, i.e. of the asteroid-size masses.

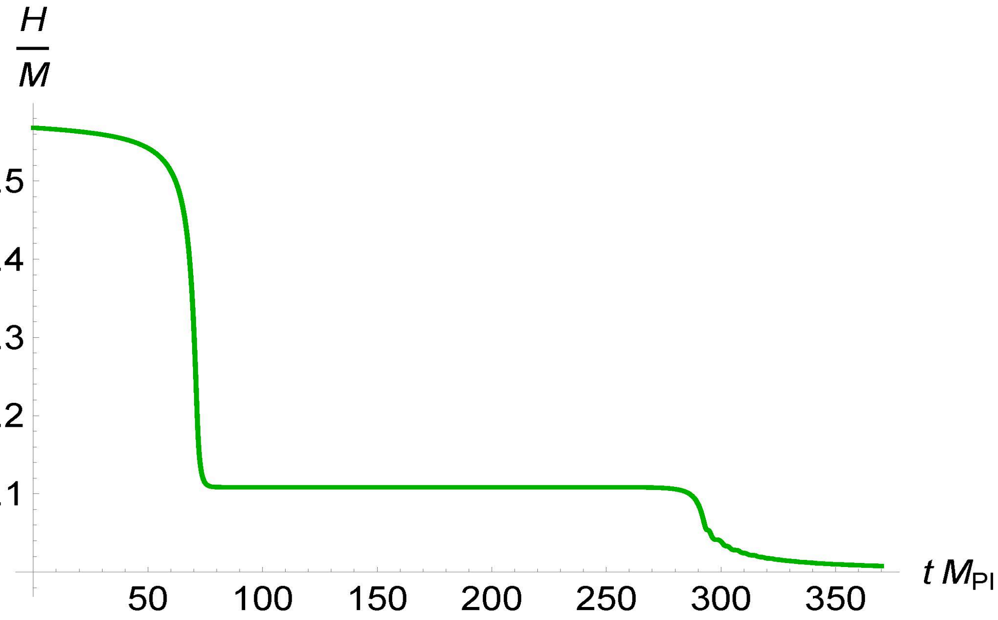
Numerical results



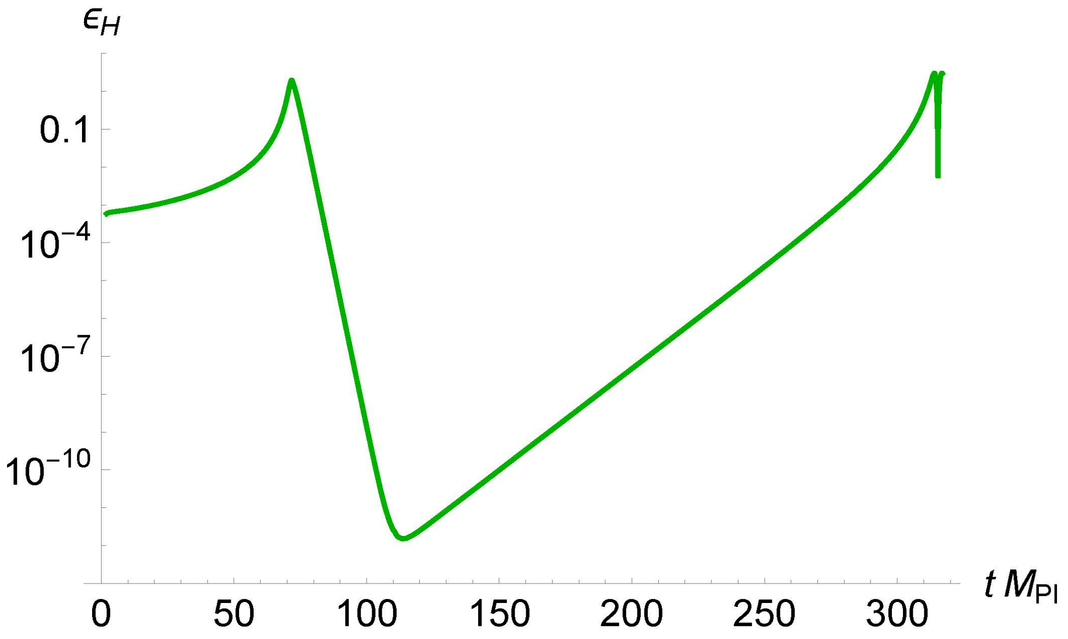


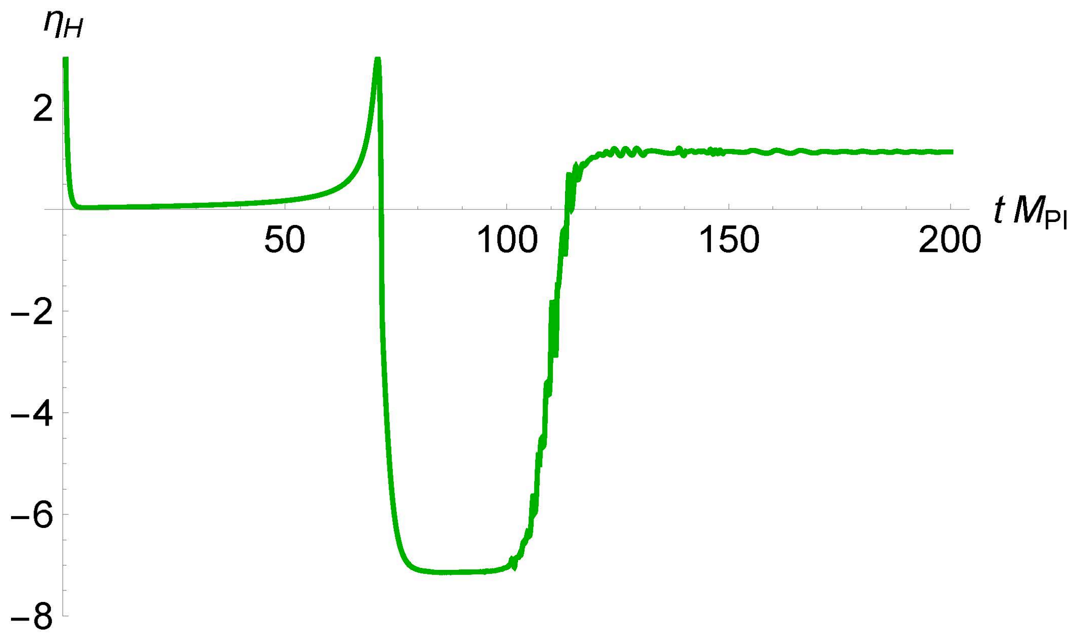
Potential and dynamics



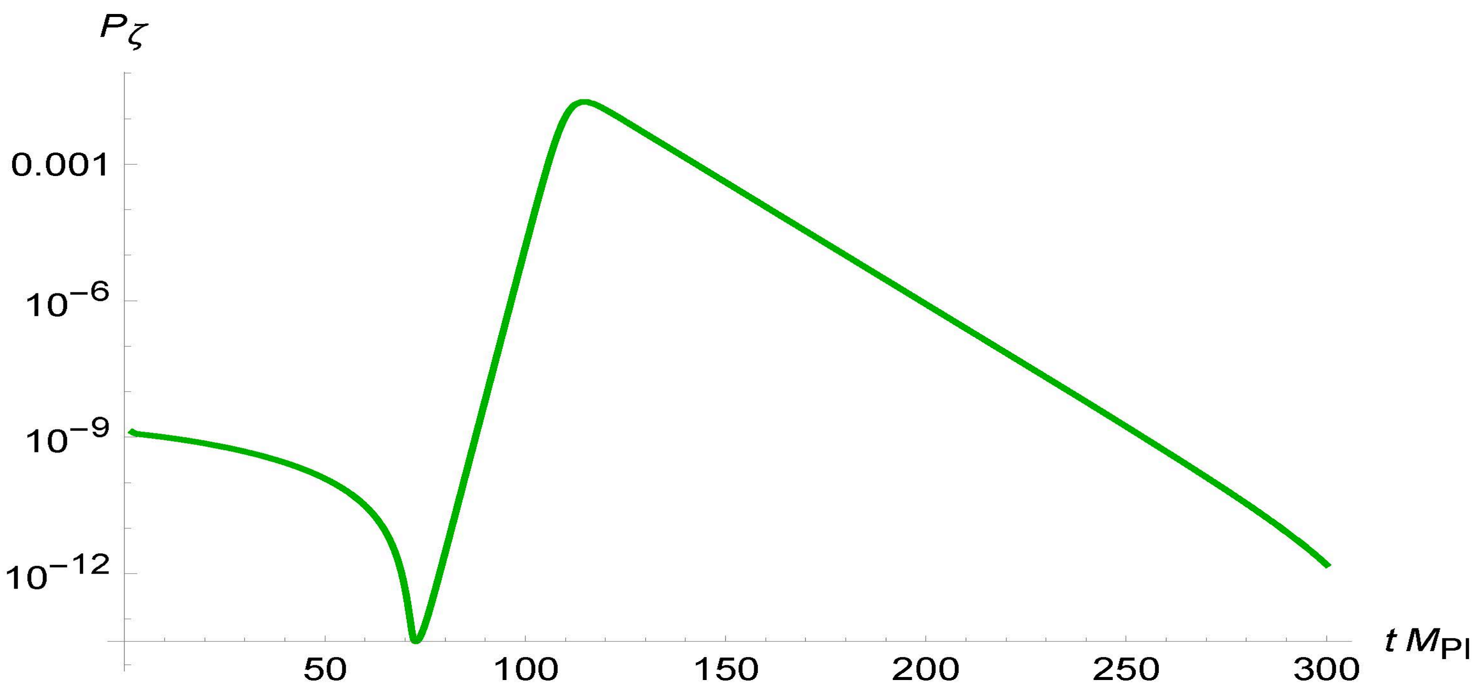


Numerical results

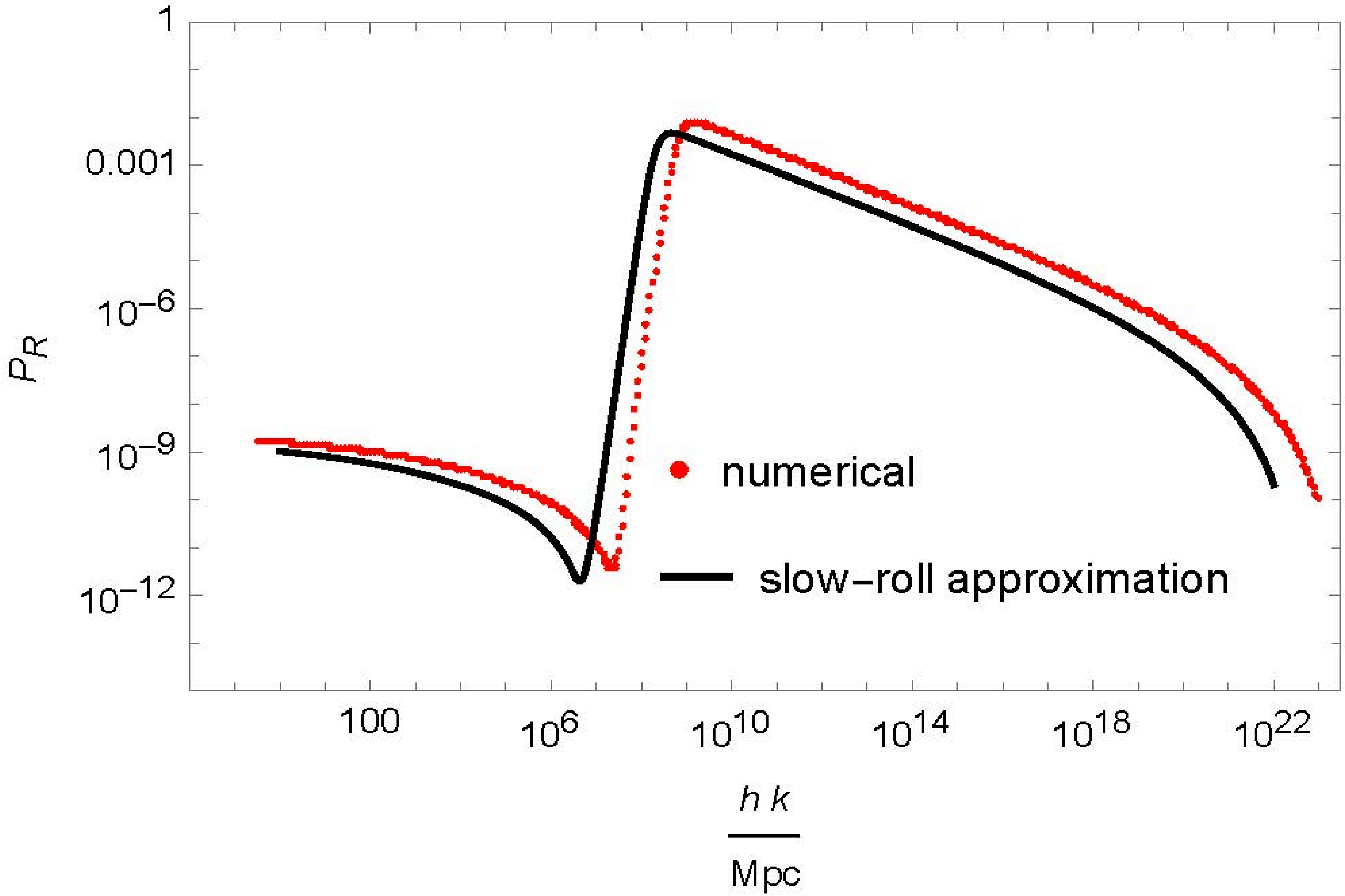




Hubble flow parameters and power spectrum



Comparison of our results from the Mukhanov-Sasaki equation for perturbations and from the slow-roll approximation formula



Energy density of PBH induced GW

The present-day GW density function Ω_{GW} in the 2nd order with respect to perturbations is given by (*Espinosa, Racco, Riotto, 2018*)

$$\frac{\Omega_{\mathrm{GW}}(k)}{\Omega_r} = \frac{c_g}{72}\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(s^2-\frac{1}{3})(d^2-\frac{1}{3})}{s^2+d^2}\right]^2 \times P_\zeta(kx) P_\zeta(ky)\left(I_c^2 + I_s^2\right) ,$$

where the constant $c_g \approx 0.4$ in the SM, and $\Omega_r = 8.6 \cdot 10^{-5}$ according to the present CMB temperature.

The variables (x, y) are related to the integration variables (s, d) as

$$x = \frac{\sqrt{3}}{2}(s+d) , \quad y = \frac{\sqrt{3}}{2}(s-d) .$$

The functions I_c and I_s of $x(s,d)$ and $y(s,d)$ are (Espinosa, Racco, Riotto, 2018)

$$I_c = -36\pi \frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\theta(s-1)\ ,$$
$$I_s = -36 \frac{s^2+d^2-2}{(s^2-d^2)^2}\left[\frac{s^2+d^2-2}{s^2-d^2}\ln\left|\frac{d^2-1}{s^2-1}\right| + 2\right]\ .$$

In our models, $\Omega^{\mathrm{GW}}(k) \sim 10^{-6} P_R^2(k)$. Frequencies of PBH-induced GW are simply related to PBH masses as (*De Luca, Franciolini, Riotto, 2020*)

$$f \approx 5.7 \left(\frac{M_\odot}{M_{\mathrm{PBH}}}\right)^{1/2} 10^{-9}\ \mathrm{Hz}$$

that implies $\sim 10^{-3}$ Hz in our models, cf. NANOGrav GW frequences of 3 to 400 nHz.

Intermediate Summary of the 1st Part

- The Starobinsky model of inflation in 4D is defined by

$$S_{\text{Star.}}[g_{\mu\nu}] = \alpha \int \mathrm{d}^4x\sqrt{-g}R^2 + \frac{M_{\text{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R\,, \quad \alpha \equiv \frac{M_{\text{Pl}}^2}{12M^2}\,,$$

with $M_{\text{inf.}} \sim 10^{-5}M_{\text{Pl}}$, $\alpha \sim 10^9$, $H \sim 10^{14}$ GeV and $R \sim 12H^2$.

- in good agreement with CMB observables (A_s, n_s, r), and $\Lambda_{UV} \sim M_{\text{Pl}}$.
- is equivalent to quintessence (Einstein frame) with the inflaton potential

$$V(\varphi) = \frac{3}{4}M_{\text{Pl}}^2M^2\left[1 - \exp\left(-\sqrt{\tfrac{2}{3}}\varphi/M_{\text{Pl}}\right)\right]^2$$

- V-flatness during inflation is due to scale invariance of the R^2 action,
- EH-term can be generated by radiative corrections *a la* Coleman-Weinberg,
- it is single-large-field inflation model with inflaton field values $\phi \sim M_{\text{Pl}}$,

- the higher-order curvature terms (beyond Starobinsky) are in powers of $H^2/M^2_{\mathsf{Pl}} \sim 10^{-8}$; no control of their coefficients but CMB observations imply those terms to be sub-leading during inflation,
- via modifying $F(R)$-function or $V(\phi)$, it is possible to generate large scalar perturbations at lower scales, leading to PBH production with masses $M_{\mathsf{PBH}} > 10^{15}$ g, while keeping good agreement with CMB,
- the scenario is SR-USR-SR with two plateaus; the mechanism is via a near-inflection point in $V(\phi)$ leading to a large (x10^7) peak in the power spectrum, with the PBH masses in the asteroid-window between 10^{17} g and 10^{21} g,
- fine-tuning of some parameters in modified $F(R)$ or $V(\phi)$ is necessary.

Modified supergravity

Modified supergravity is the (old-minimal) $N = 1$ local SUSY extension of the $(R + \alpha R^2)$ gravity. Manifest SUSY is achieved by using curved superspace. A generic action is given by a sum of D-type and F-type terms,

$$S = \int d^4xd^4\theta E^{-1}N(\mathcal{R},\bar{\mathcal{R}}) + \left[\int d^4xd^2\Theta 2\mathcal{E}F(\mathcal{R}) + h.c\right] \ ,$$

where the covariantly chiral superfield $\mathcal{R}$ has the spacetime scalar curvature R among its field component. *See also Dalianis, Farakos, Kehagias, Riotto, Unge (2015)*.
The Starobinsky inflation scale $H \sim 10^{14}$ GeV (close to the GUT scale) is the scale where SUSY is expected to play a significant role.
The F-term can be included into the D-term (except a constant). We distinguish them by collecting the R-symmetry preserving terms in the N-potential, and the R-symmetry violating terms in the F-potential. *The latter are dangerous for gravitino.*

Field content of modified supergravity

- vierbein e^a_μ, gravitino ψ_μ, complex scalar X, and real vector b_μ,

- form the irreducible (off-shell) supergravity multiplet with linearly realized SUSY and closed SUSY algebra,

- the fields (X, b_μ) are known as the "auxiliary" fields of the old-minimal supergravity (in the textbooks),

- but in modified supergravity (the higher-derivative field theory beyond supergravity textbooks) all these "auxiliary" fields become physical (propagating).

- There are 4 physical scalars in modified supergravity: scalaron φ, complex scalar X and $\widehat{D}_\mu b^\mu/M$ with the nearly equal effective masses of the order M.

Embedding Starobinsky model

Addazi, Ketov, 2017

Expand the functions N and $\mathcal{F}$ in Taylor series and keep only a few *leading* terms, $(M_{\mathsf{Pl}} = 1)$,

$$N = \frac{12}{M^2}\mathcal{R}\overline{\mathcal{R}} - \frac{\xi}{2}(\mathcal{R}\overline{\mathcal{R}})^2 \, , \quad \mathcal{F} = \alpha + 3\beta\mathcal{R} \, ,$$

with real parameters M and ξ, and complex parameters α and β.

- The chiral superfields $\mathcal{R}$ and $\mathcal{E}$ read

$$\begin{aligned} \mathcal{R} &= X + \Theta\left(-\tfrac{1}{6}\sigma^m\bar{\sigma}^n\psi_{mn} - i\sigma^m\overline{\psi}_m X - \tfrac{i}{6}\psi_m b^m\right) + \\ &\quad +\Theta^2\left(-\tfrac{1}{12}R - \tfrac{i}{6}\overline{\psi}^m\bar{\sigma}^n\psi_{mn} - 4X\overline{X} - \tfrac{1}{18}b_m b^m + \tfrac{i}{6}\nabla_m b^m + \right. \\ &\quad \left. + \tfrac{1}{2}\overline{\psi}_m\overline{\psi}^m X + \tfrac{1}{12}\psi_m\sigma^m\overline{\psi}_n b^n - \tfrac{1}{48}\varepsilon^{abcd}(\overline{\psi}_a\bar{\sigma}_b\psi_{cd} + \psi_a\sigma_b\overline{\psi}_{cd})\right) , \\ 2\mathcal{E} &= e\left[1 + i\Theta\sigma^m\overline{\psi}_m + \Theta^2(6\overline{X} - \overline{\psi}_m\bar{\sigma}^{mn}\overline{\psi}_n)\right] \, , \end{aligned}$$

- The standard supergravity is reproduced when $N = 0$ and $\mathcal{F} = -3\mathcal{R}$.
- Starobinsky inflation is realized when $\alpha = 0$, $\beta = -3$, and M equals to the scalaron mass, and dynamics of (X,b) is suppressed ($\xi>0$ is needed).

Effective two-scalar field Lagrangian

Aldabergenov, Addazi, Ketov, 2020

In the notation

$$\frac{M^4\xi}{144} \equiv \zeta \quad \text{and} \quad |X| \equiv \frac{M}{2\sqrt{6}}\sigma \ ,$$

where σ is the radial part of the complex scalar X, after ignoring its angular part together with $b_m = 0$ for simplicity, the bosonic part of the Lagrangian in our model takes the form

$$e^{-1}\mathcal{L} = \frac{1}{2}f(R,\sigma) - \tfrac{1}{2}(1-\zeta\sigma^2)(\partial\sigma)^2 - U \ ,$$

where we have the specific functions dictated by modified supergravity,

$$f(R,\sigma) = \left(1 + \tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4\right)R + \frac{1}{6M^2}(1-\zeta\sigma^2)R^2 \ ,$$

$$U = \frac{1}{2}M^2\sigma^2\left(1 - \tfrac{1}{6}\sigma^2 + \tfrac{3}{8}\zeta\sigma^4\right) \ .$$

(Standard) transfer to Einstein frame in field components

After introducing the auxiliary field χ and rewriting the Lagrangian as

$$e^{-1}\mathcal{L} = \tfrac{1}{2}\left[f_\chi(R-\chi)+f\right] - \tfrac{1}{2}(1-\zeta\sigma^2)(\partial\sigma)^2 - U ,$$

where $f_\chi \equiv \frac{\partial f}{\partial\chi}$ and in $f \equiv f(\chi,\sigma)$, R was replaced by χ, varying w.r.t. χ gives back the initial Lagrangian. On the other hand, after Weyl rescaling,

$$g_{mn} \to f_\chi^{-1} g_{mn} , \quad e \to f_\chi^{-2} e , \quad e f_\chi R \to eR - \tfrac{3}{2} e f_\chi^{-2} (\partial f_\chi)^2 ,$$

with

$$f_\chi = A + B\chi \quad A \equiv 1 + \tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4 , \quad B \equiv \frac{1}{3M^2}(1-\zeta\sigma^2) ,$$

in terms of the canonically normalized scalaron φ defined by

$$f_\chi = \exp\left[\sqrt{\tfrac{2}{3}}\varphi\right] , \quad \chi = \tfrac{1}{B}\left(e^{\sqrt{\frac{2}{3}}\varphi} - A\right) , \quad f = \frac{1}{2B}\left(e^{2\sqrt{\frac{2}{3}}\varphi} - A^2\right) ,$$

the Lagrangian in Einstein frame takes the form

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}(1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi}(\partial\sigma)^2 - V ,$$

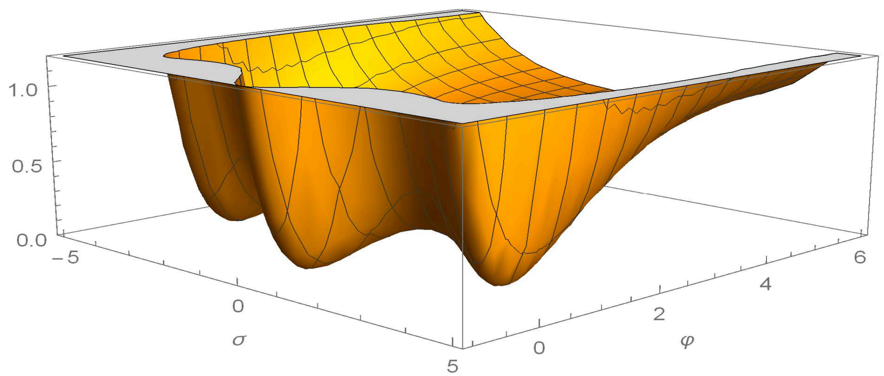
whose two-field scalar potential reads

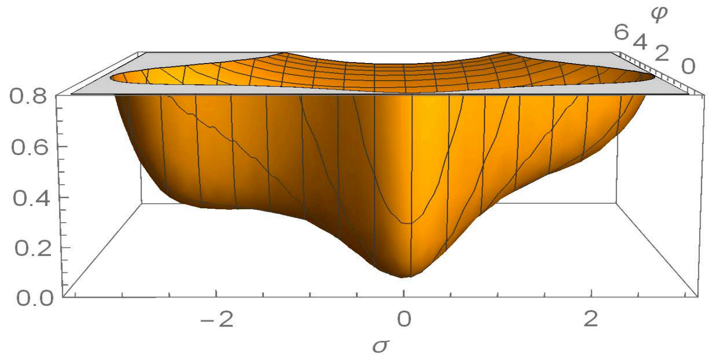
$$
\begin{aligned}
V = \frac{1}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 + e^{-2\sqrt{\frac{2}{3}}\varphi}U = \\
= \frac{3M^2}{4(1-\zeta\sigma^2)}\left[1 - e^{-\sqrt{\frac{2}{3}}\varphi} - \tfrac{\sigma^2}{6}\left(1 - \tfrac{11}{4}\zeta\sigma^2\right)e^{-\sqrt{\frac{2}{3}}\varphi}\right]^2 \\
+ \frac{M^2}{2}e^{-2\sqrt{\frac{2}{3}}\varphi}\sigma^2\left(1 - \tfrac{1}{6}\sigma^2 + \tfrac{3}{8}\zeta\sigma^4\right) \ .
\end{aligned}
$$

When $\sigma^2 > 1/\zeta$, the scalar σ becomes a ghost. However, when approaching $\sigma^2 = 1/\zeta$, the scalar potential becomes singular, so that it would take the infinite amount of energy to turn σ into a ghost (assuming its starting value in the region $\sigma^2 < 1/\zeta$).

Scalar potential in Einstein frame

$$V = \frac{1}{4B}(1 - Ax)^2 + x^2 U\,, \quad e^{-\sqrt{\frac{2}{3}}\varphi} \equiv x\,, \begin{cases} A = 1 + \frac{1}{6}\sigma^2 - \frac{11}{24}\zeta\sigma^4\,, \\ B = \frac{1}{3M^2}(1 - \zeta\sigma^2)\,, \\ U = \frac{M^2}{2}\sigma^2\left(1 - \frac{1}{6}\sigma^2 + \frac{3}{8}\zeta\sigma^4\right)\,. \end{cases}$$





The scalar potential on the left with $\zeta = 1/54 \approx 0.019$ and three Minkowski minima; on the right with $\zeta = 0.027$, a single Minkowski minimum at $\sigma = 0$ and two inflection points. In both cases $M = 1$.

Two-field scalar Lagrangian

takes the form of a non-linear sigma-model (NLSM) minimally coupled to gravity,

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}G_{AB}\partial\phi^A\partial\phi^B - V \ ,$$

where $\phi^A = \{\varphi, \sigma\}$, $A = 1, 2$, and the NLSM target space metric is given by

hyperbolic geometry: $$G_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & (1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi} \end{pmatrix}$$ of negative curvature

With the FLRW spacetime metric $g_{mn} = \mathrm{diag}(-1, a^2, a^2, a^2)$ the EoM read

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{1}{\sqrt{6}}(1-\zeta\sigma^2)e^{-\sqrt{\frac{2}{3}}\varphi}\dot{\sigma}^2 + \partial_\varphi V = 0 \ ,$$

$$\ddot{\sigma} + 3H\dot{\sigma} - \frac{\zeta\sigma\dot{\sigma}^2}{1-\zeta\sigma^2} - \sqrt{\frac{2}{3}}\dot{\varphi}\dot{\sigma} + \frac{e^{\sqrt{\frac{2}{3}}\varphi}}{1-\zeta\sigma^2}\partial_\sigma V = 0 \ ,$$

similar to hybrid inflation

Superfield transfer to Einstein matter-coupled supergravity

After introducing the Lagrange multiplier superfield $\mathbf{T}$ as (*Terada and SVK, 2013*)

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})N(\mathbf{S},\bar{\mathbf{S}}) + \mathcal{F}(\mathbf{S}) + 6\mathbf{T}(\mathbf{S}-\mathcal{R}) \right\} + \text{h.c.} \,,$$

varying the Lagrangian w.r.t. the $\mathbf{T}$ gives back the original Lagrangian. On the other hand, the Lagrangian can be rewritten to the form

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ \tfrac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R}) \left[\mathbf{T} + \bar{\mathbf{T}} - \tfrac{1}{3}N(\mathbf{S},\bar{\mathbf{S}}) \right] + \mathcal{F}(\mathbf{S}) + 6\mathbf{T}\mathbf{S} \right\} + \text{h.c.}$$

that can be put into the standard form in supergravity,

T is inflaton superfield,
S is goldstino superfield

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left[\tfrac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})e^{-K/3} + W \right] + \text{h.c.} \,,$$

where the Kähler potential K takes the no-scale supergravity form

$$K = -3\log(\mathbf{T} + \bar{\mathbf{T}} - \tilde{N}) \,, \quad \tilde{N} \equiv \mathbf{S}\bar{\mathbf{S}} - \tfrac{3}{2}\zeta(\mathbf{S}\bar{\mathbf{S}})^2 \,,$$

but the modified supergravity origin of K and W becomes hidden.

See also Ellis, Nanopoulos and Olive (2013); first observed by Cecotti (1987).

Production of primordial black holes (PBH) in inflation

One needs large curvature fluctuations ($>10^6$ of CMB)!

There are many proposals in the literature:

- gravitational instabilities induced by scalar fields, and collapse of large density fluctuations,
- bubble collisions from first order phase transitions,

and many more !

- critical topological defects, such as cosmic strings and domain walls.

PBH formation due to amplification of the power spectrum (large peak) of scalar perturbations via tachyonic instabilities of the scalar fields present in modified supergravity, during multi-field inflation. This mechanism is different from the standard mechanism of PBH formation in *single-field* models of inflation with a *near-inflection* point in the inflaton scalar potential.

Isocurvature pumping mechanism during inflation

- decompose perturbations into adiabatic Q_a (along inflationary trajectory) and isocurvature Q_s (orthogonal to inflationary trajectory);
- $\ddot{Q}_a + 3H\dot{Q}_a + \Omega Q_a = \hat{f}(d/dt)(\omega Q_s)$, $\ddot{Q}_s + 3H\dot{Q}_s + m_s^2 Q_s = 0$
- When $\ddot{Q}_s \approx 0$, we find the solution $Q_s \approx \exp\left[-\int dt \frac{m_s^2}{3H^2}\right]$
- when the isocurvature mass $m_s^2 < 0$ at the critical point, we get the exp-amplification of Q_s; since Q_a are sourced by Q_s in EoM, we also get an exp-amplification of Q_a when the inflationary trajectory has a sharp turn [*Palma, Sypsas, Zenteno (2020); Fumagalli, Renaux-Petel, Ronayne, Witkwoski (2020)*];
- after the critical point $m_s^2 > 0$ again, the isocurvature modes get suppressed and, hence, no over-amplification (and no PBH overproduction): [*Gundhi, Steinwachs, Ketov (2021)*.

Straightforward generalizations toward PBH

Aldabergenov, Addazi, Ketov, 2020

Adding the next-order terms to the modified supergravity potentials yields

$$N = \frac{12}{M^2}|\mathcal{R}|^2 - \frac{72}{M^4}\zeta|\mathcal{R}|^4 - \frac{768}{M^6}\gamma|\mathcal{R}|^6 \ ,$$
$$F = -3\mathcal{R} + \frac{3\sqrt{6}}{M}\delta\mathcal{R}^2 \ .$$

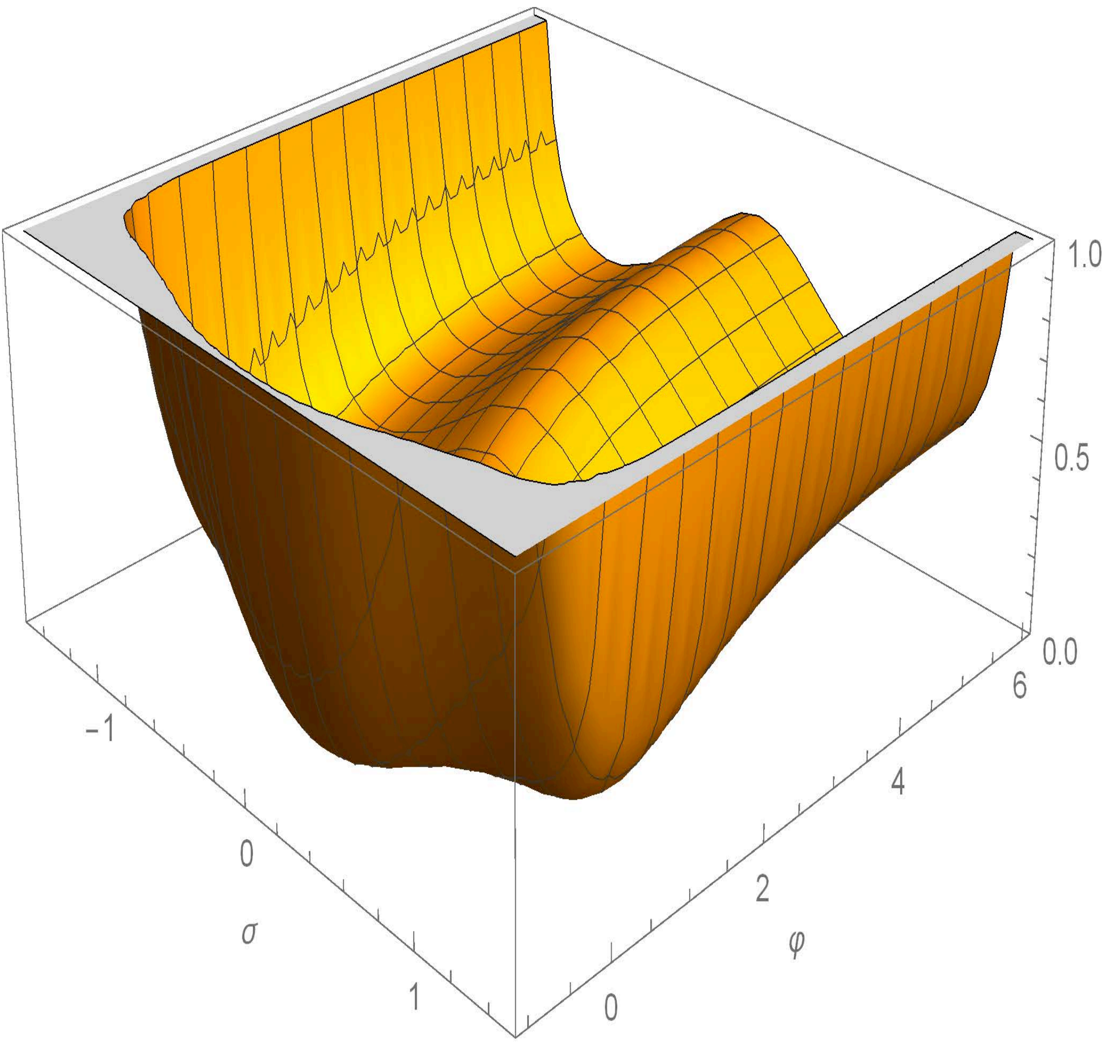
The corresponding Lagrangian in Einstein frame reads

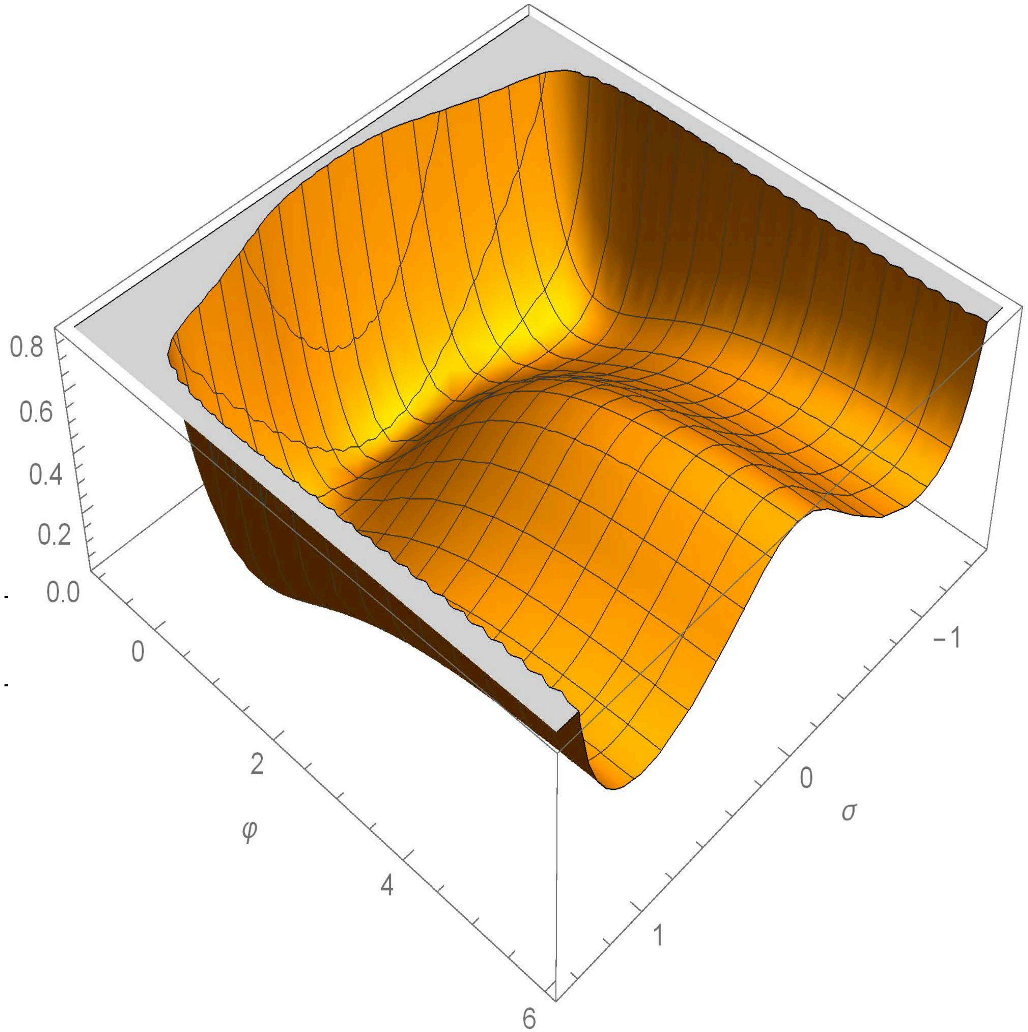
$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{2}(\partial\varphi)^2 - \frac{3M^2}{2}Be^{-\sqrt{\frac{2}{3}}\varphi}(\partial\sigma)^2 - \frac{1}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U \ ,$$

where the functions A, B, U are given by

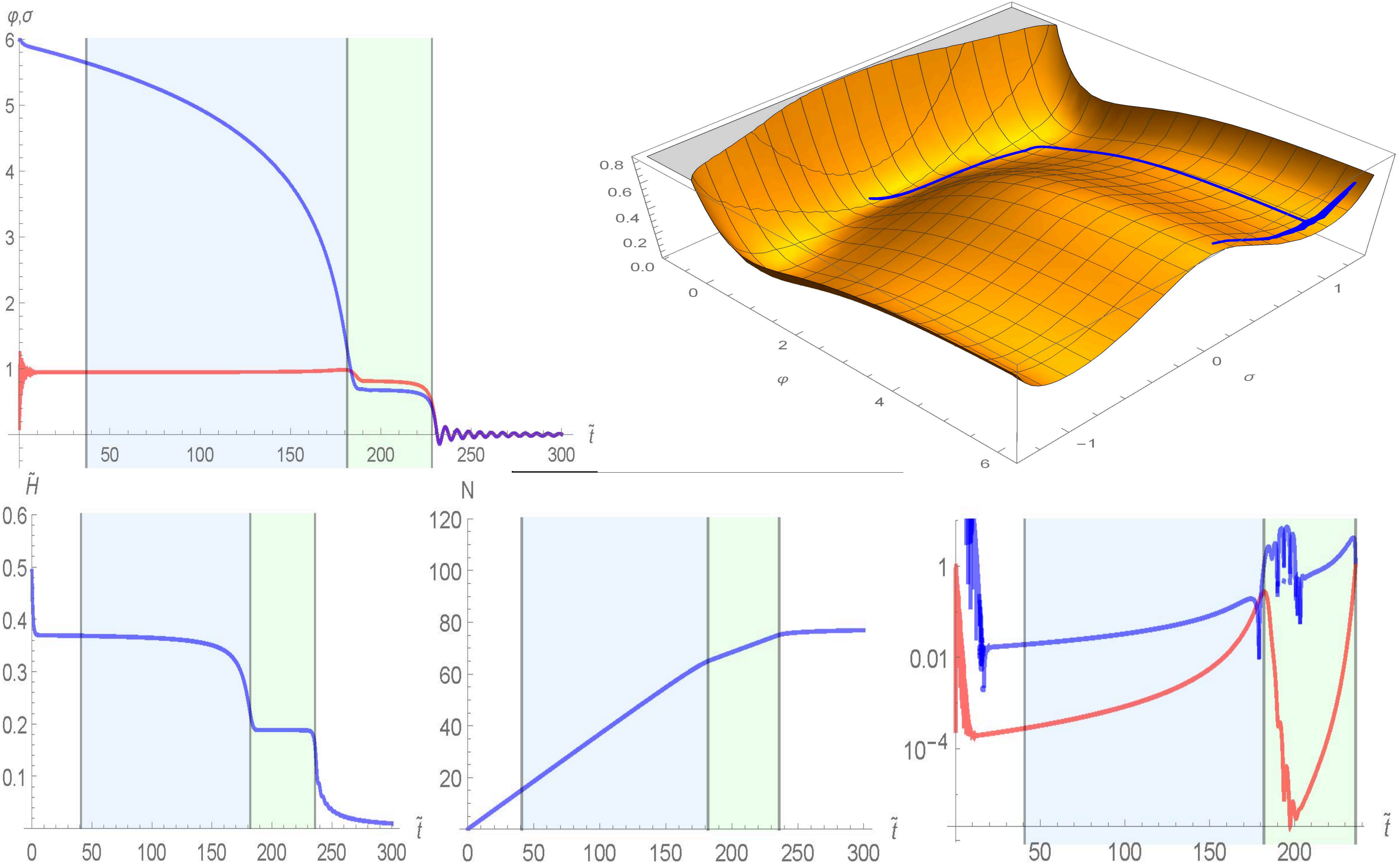
$$A = 1 - \delta\sigma + \tfrac{1}{6}\sigma^2 - \tfrac{11}{24}\zeta\sigma^4 - \tfrac{29}{54}\gamma\sigma^6 \ ,$$
$$B = \frac{1}{3M^2}(1 - \zeta\sigma^2 - \gamma\sigma^4) \ ,$$
$$U = \frac{M^2}{2}\sigma^2\left(1 + \tfrac{1}{2}\delta\sigma - \tfrac{1}{6}\sigma^2 + \tfrac{3}{8}\zeta\sigma^4 + \tfrac{25}{54}\gamma\sigma^6\right) \ .$$

The scalar potential of the gamma-model, delta =0

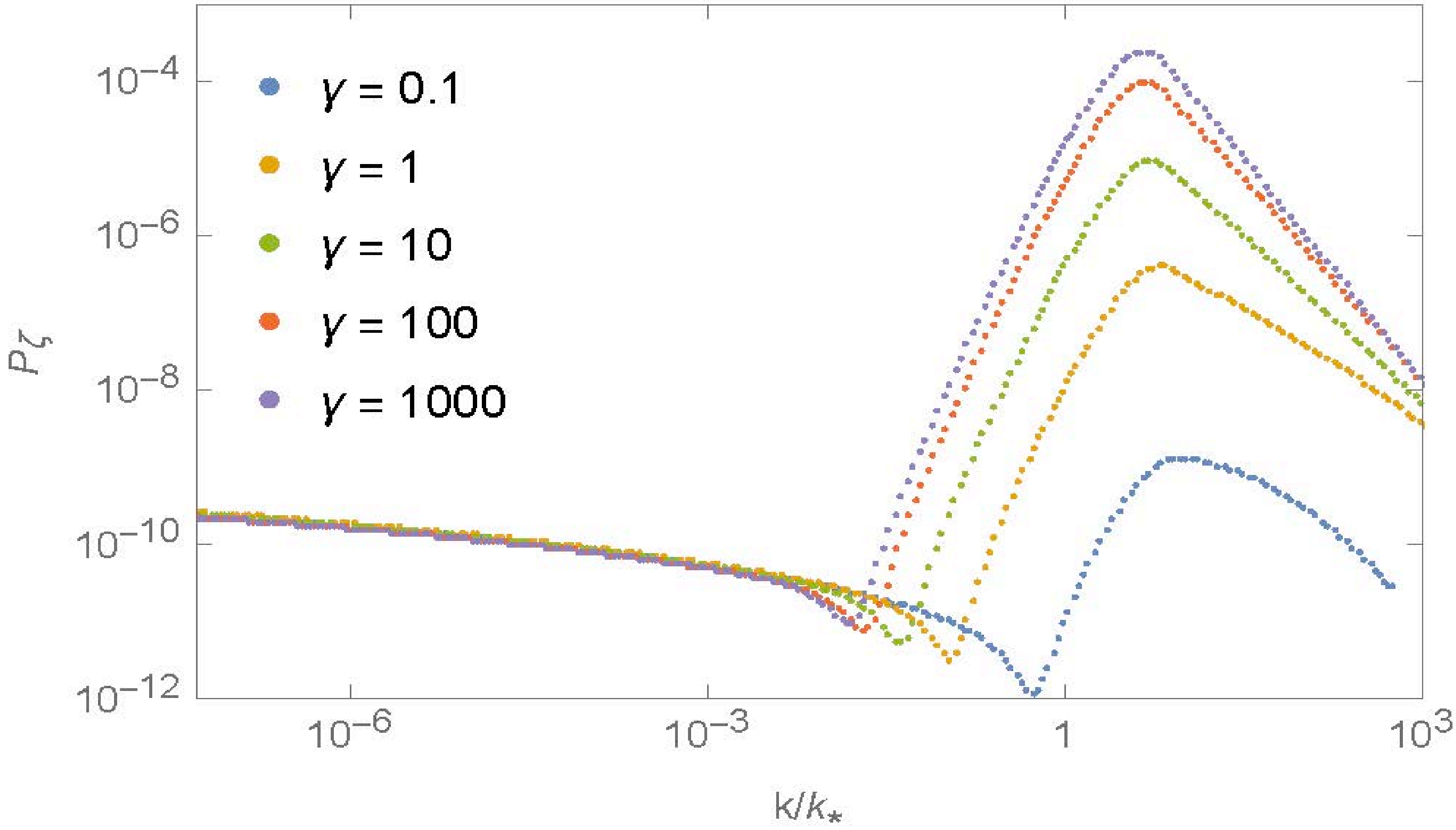




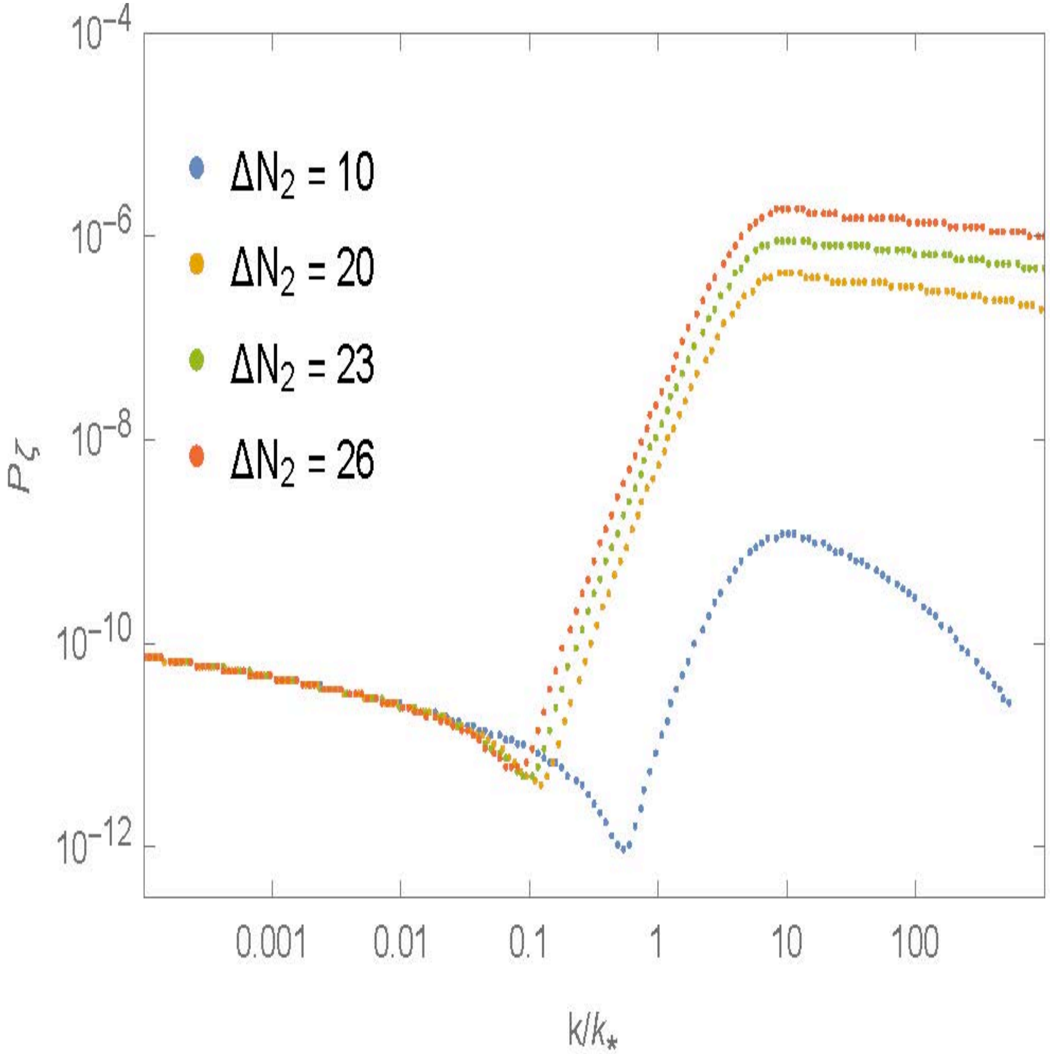
The solution, trajectory, Hubble function, e-foldings, and slow roll parameters

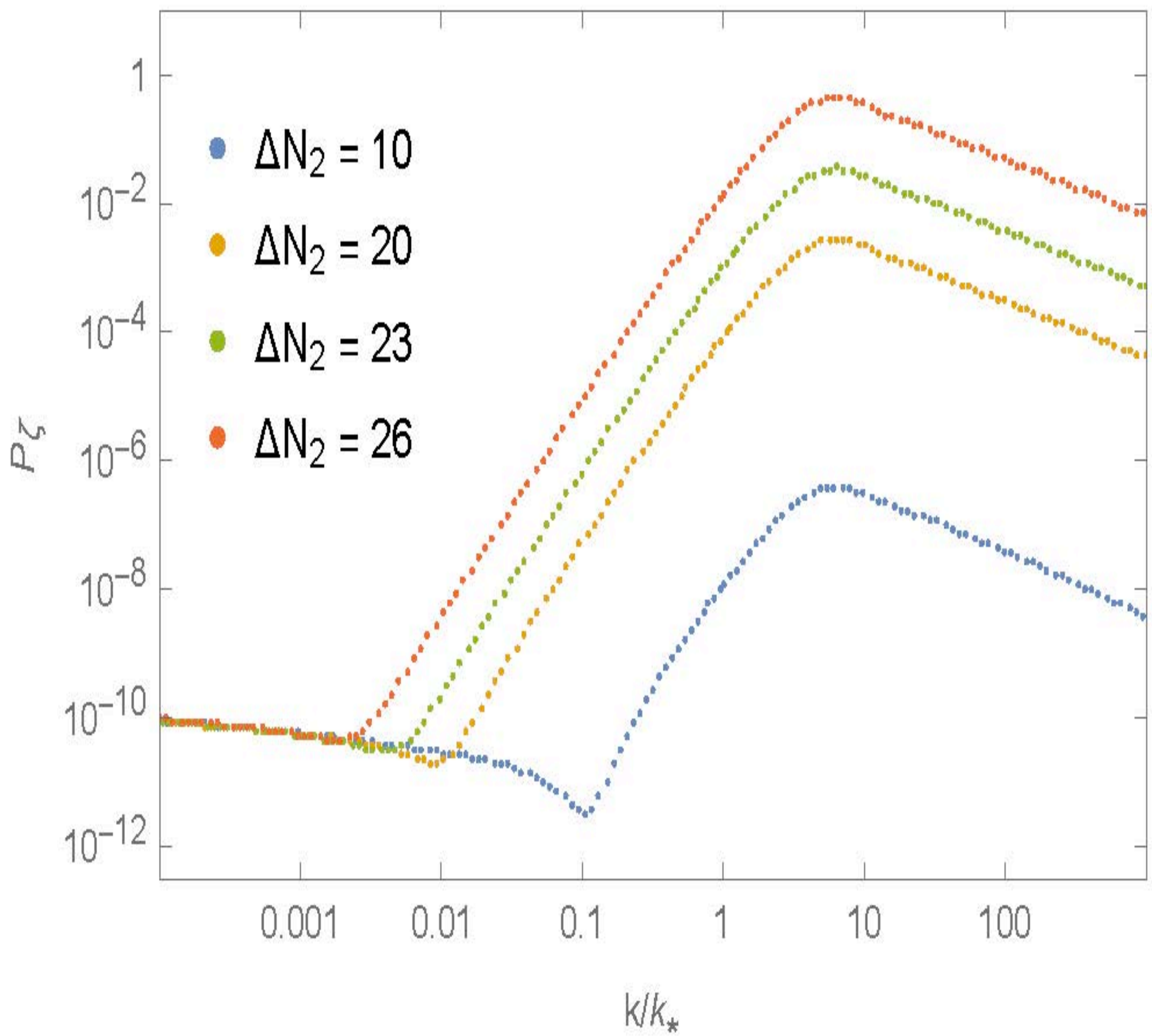


Power spectrum at $\Delta N_2 = 10$ for various values of γ



Power spectrum at $\gamma = 0.1$ (left) and $\gamma = 1$ (right) with changing ΔN_2





PBHs masses in the γ-model

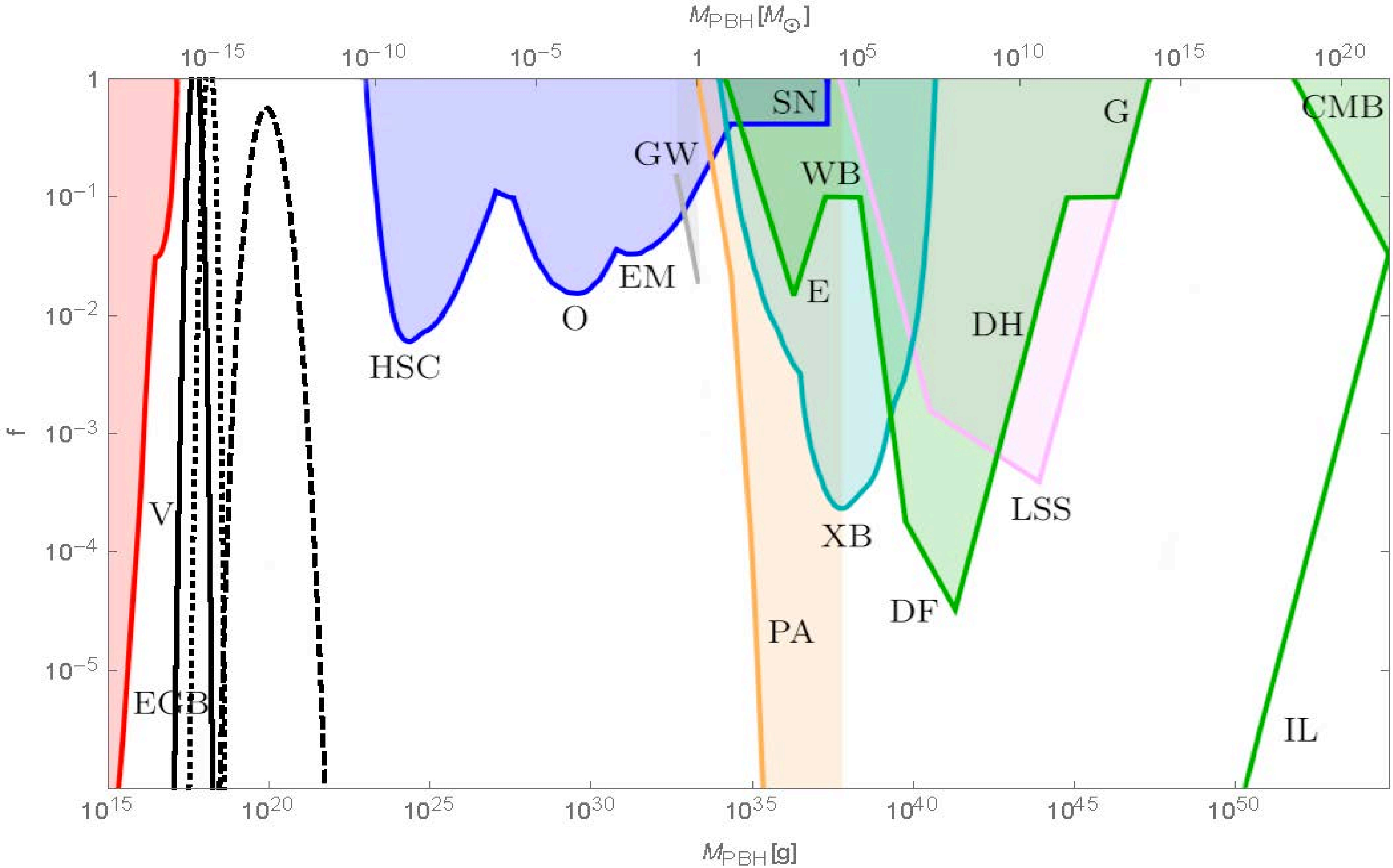
The mass of PBH created by late-inflation overdensities was estimated by Pi, Zhang, Huang and Sasaki in arXiv:1712.09896:

$$M_{\mathrm{PBH}} \simeq \frac{M_{\mathrm{Pl}}^2}{H(t_{\mathrm{peak}})} \exp\left[2(N_{\mathrm{end}} - N_{\mathrm{peak}}) + \int_{t_{\mathrm{peak}}}^{t_{60}} \epsilon(t) H(t) dt\right] ,$$

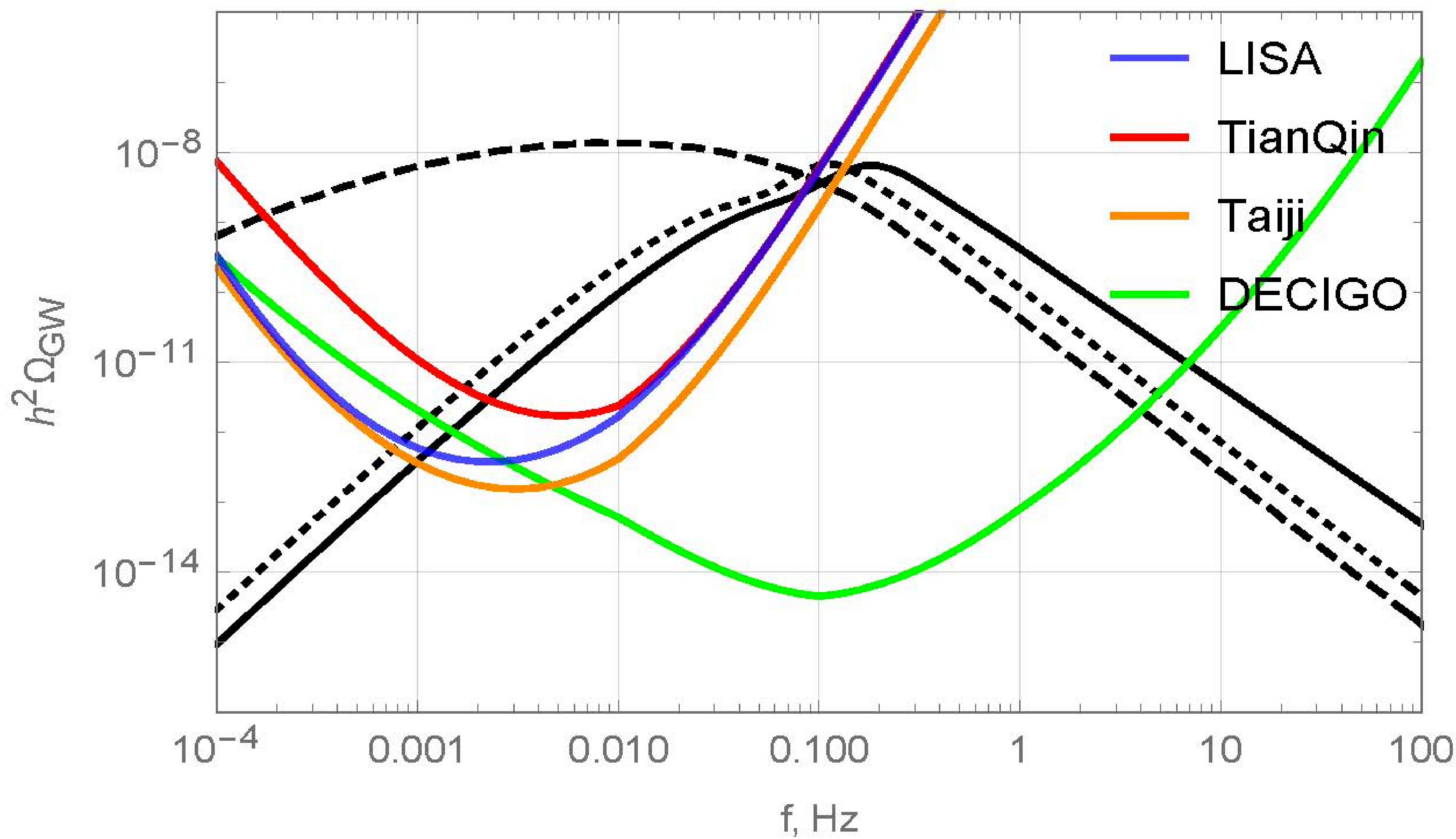
where t_{peak} is the time when the perturbation corresponding to the power spectrum peak (k_{peak}) exits the horizon, whereas t_{60} is the time when k_{60} exits the horizon (the beginning of observable inflation). By using this equation, we estimated the values of M_{PBH} for various values of ΔN_2 in our model:

ΔN_2	10	20	23	26
M_{PBH}, g	10^8	10^{16}	10^{19}	10^{21}
n_s	0.96	0.95	0.945	0.94

The PBH density fraction in the models with γ=1, δ=0, ΔN_2=22.45 (solid line), and δ=0.58 , ΔN_2=23.36 (dotted line). In both cases f_total=1.



The density of stochastic gravitational waves induced by the power spectrum enhancement in the our supergravity models (solid+dashed+dotted black curves) against the expected sensitivity curves of the space-based GW interferometers.



Minimal No-scale Supergravity I

is obtained by identifying the inflaton superfield T with the goldstino superfield S (*Terada, SVK, 2014)*. The scalar potential in supergravity reads ($M_{\rm Pl}=1$)

$$V_{\rm SUGRA} = e^G \left[G_{,T} \left(G_{,T\bar{T}} \right)^{-1} G_{,\bar{T}} - 3 \right]$$

where $G = K + \ln |W|^2$. For example, when $K = -3\ln(T+\bar{T})$ and $W = W_0 T^3$, one gets $V = 0$, while SUSY can be broken along a flat direction. Spontaneous SUSY breaking occurs when $\langle F_T \rangle \neq 0$ with $m_{3/2} = \left\langle e^{G/2} \right\rangle$. Then goldstino is eaten up by gravitino (the well known super-Higgs mechanism). To realize inflation with $V > 0$, one can add a stabilizing term as (*Pallis, 2023*)

$$K = -p \ln \left[T + \bar{T} + \xi^2 (T + \bar{T} - 2v)^4 \right]$$

with the new parameters (p, ξ, v).

Minimal No-scale Supergravity II

The superpotential is fixed by demanding no-scale $(V = 0)$ in the absence of the stabilizing term $(\xi = 0)$. It yields

$$W = W_1 + W_2, \quad W_{1,2} = m_{1,2} T^{q_{1,2}}, \quad q_{1,2} = \frac{1}{2}\left(p \pm \sqrt{3p}\right) \ ,$$

with mass scales $m_{1,2}$. The m_1 is identified with the inflation scale $\sim 10^{13}$ GeV, and m_2 is identified with the dark energy (c.c.) scale $\sim 10^{-3}$ eV.
The stabilizing term breaks no-scale, leading to a positive potential, selects the vacuum with $\langle T \rangle = v$, and stabilizes the inflationary trajectory along $T = \bar{T}$ by giving a mass $\mathcal{O}(m_{\text{inf}})$ to sinflaton (phase of T). Good solutions (without branch cuts) arise for integer powers $q_{1,2}$. For instance, $q_{1,2} = (3, 9)$ for $p = 12$.

Minimal No-scale Supergravity, PBH and MSSM

The remaining parameters can be tuned as $\xi \approx 1.6$ and $v \approx 0.25$ to get a *near-inflection* point in the effective single-field inflaton potential (*Pallis, 2023*).

Spontaneous high-scale SUSY breaking takes place with $m_{3/2} \sim 10^{11}$ GeV. The MSSM can be added to the minimal no-scale supergravity at the high scale by modifying the superpotential and the Kähler potential, $W \to W + W_{\text{MSSM}}$ and $K \to K + K_{\text{MSSM}}$, where

$$W_{\text{MSSM}} = h_{\alpha\beta\gamma}\Phi_\alpha\Phi_\beta\Phi_\gamma + \mu H_u H_d \ , \quad \Phi_\alpha = (Q, L, d^c, u^c, e^c, H_d, H_u) \ ,$$

and $K_{\text{MSSM}} = \sum_\alpha |\Phi_\alpha|^2$. The EFT is then obtained by the RGE renormalizing the parameters $(h_{\alpha\beta\gamma}, \mu)$ by the factor $\langle T\rangle^{-p/2}$ and leading to soft SUSY breaking terms after decoupling of supergravity in the limit $M_{\text{Pl}} \to \infty$, consistently with the observed Higgs mass $M_H = (125.15 \pm 0.25)$ GeV.

High-scale SUSY breaking, MSSM and Higgs mass

$$H_{\rm SM} = H_u \sin\beta + H_d^\dagger \cos\beta \ , \quad \lambda_{\rm SUSY} = \frac{1}{4}\left(g_1^2 + g_2^2\right)\cos^2 2\beta \ ,$$

$$m_{\rm H} = (125.15 \pm 0.25) \ \text{GeV} \ , \quad m_{\rm t} = (173.134 \pm 0.76) \ \text{GeV} \ ,$$

imply via the MSSM 2-loop RGE (*Giudice, Strumia, 2014*)

$$m_{3/2} \leq \mathcal{O}(10^{12}) \ \text{GeV} \quad \text{and} \quad \tan\beta \sim \mathcal{O}(1) \ ,$$

as well as stability of the EW vacuum. However, SUSY cannot be responsible for the hierarchy $m_{\rm H}/M_{\rm Pl} \sim 10^{-16}$, cf. the cosmological constant fine-tuning of 10^{-120}. In the gravity decoupling limit, soft SUSY breaking terms ensure stability.

- Baryogenesis via non-thermal leptogenesis can be activated, cf. *Jeong, Kamada, Starobinsky, Yokoyama (2023)* for Starobinsky inflation+supermassive RH Majorana neutrinos.

Conclusion I

- Our approach is motivated by modified gravity and supergravity, leads to viable inflation, efficient PBH production and induced GW, and can be consistently connected to MSSM and SM.

- The PBH masses are possible in the window from 10^{17} g to 10^{21} g, in all our models. Those PBH may form (the whole or part of) current dark matter.

- The PBH-induced GW may be detectable by the future space-based gravitational interferometers (LISA, DECIGO, TianQin, Taiji) with mHz frequency.

- The near-inflection mechanism of PBH production can be employed in the minimal no-scale supergravity with the stabilizing term and viable single-large-field inflation. *This is different from multi-field inflation and PBH: SUSY2023 in UK.*

Conclusion II

- Unification of inflation and PBH production with high-scale SUSY breaking, MSSM and dark energy (tiny c.c.) is possible in the minimal no-scale supergravity in agreement with the known Higgs mass and the (meta)stable EW scale ($\lambda_H \geq 0$). There is no Polonyi problem and no overproduction problem but no SUSY explanation for the scale hierarchy (fine tuning).

- After inflation, inflaton decays into other particles (gravitinos, etc.), while heavy LSP gravitinos with the mass of $\mathcal{O}(10^{11})$ GeV is also a candidate for dark matter (Addazi, Khlopov, SVK, 2017). In our model, $T_{\rm reh} \sim 10^7$ GeV.

- PBH production during inflation in supergravity leads to the significant constraints on the parameters of high-energy particle physics and strong predictions: (i) high-scale SUSY breaking, (ii) PBH & gravitino DM, (iii) the MSSM mixing angle, $\tan\beta \approx 1$, etc.

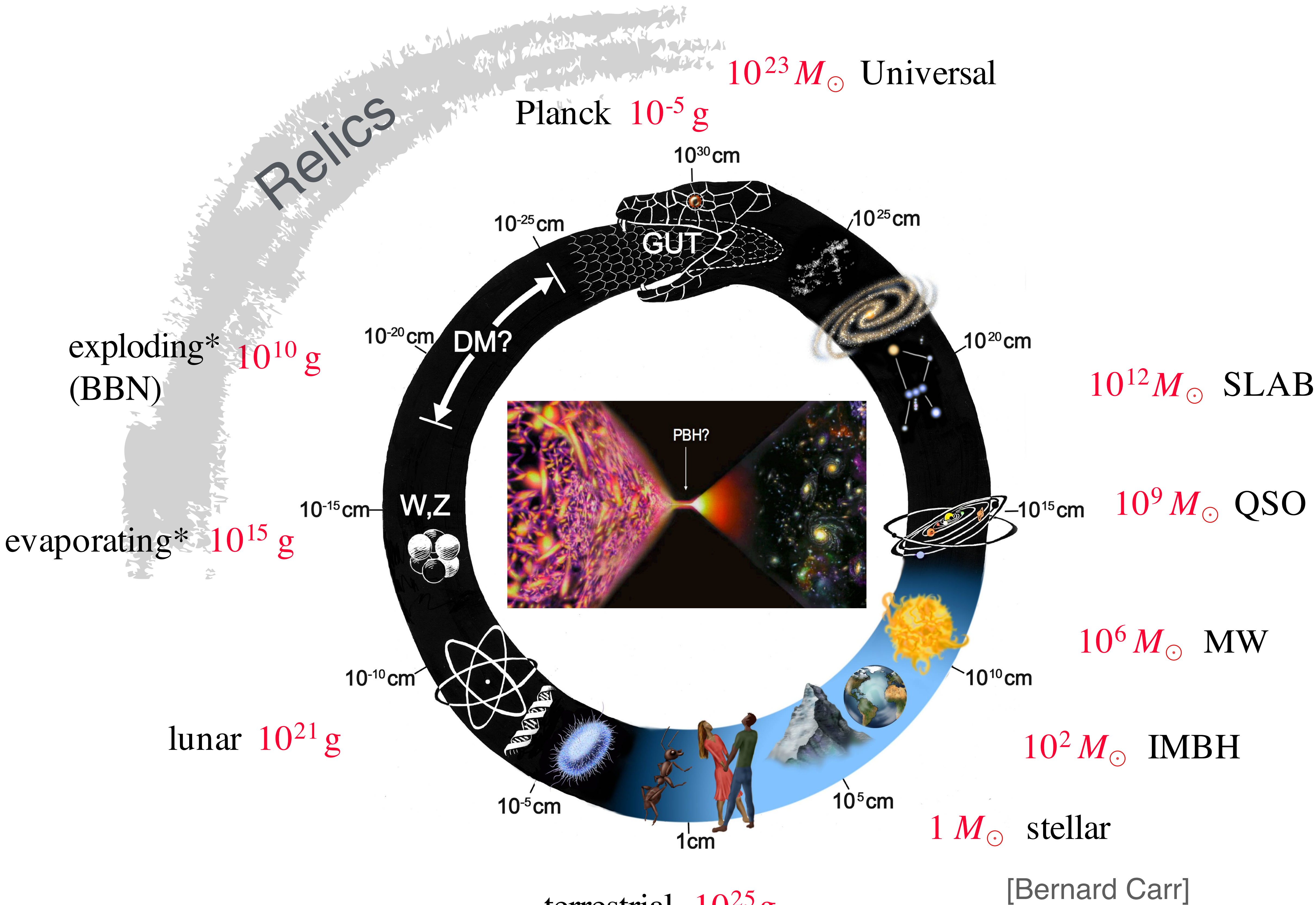
Main message to take home

- CMB (Planck) observations $\Rightarrow$ flat inflaton potential $\Rightarrow$ R^2-inflation
- Starobinsky $(R + \alpha R^2)$ supergravity $\Rightarrow$ Dark Matter as PBH and LSP
- Universal reheating mechanism via transfer to Einstein frame $\Rightarrow$ super-heavy fermions (gravitino, sterile neutrinos)
- Renormalization from inflation scale to electro-weak scale $\Rightarrow$ Minimal SUSY Standard Model $\Rightarrow$ massive active neutrinos via seasaw

High-scale SUSY and SUGRA may solve ALL phenomenological problems!

Thank You for Your Attention!

Black Holes as a Link between Micro and Macro Physics



[Bernard Carr]

*Image of snake from Abrams & Primack 2012

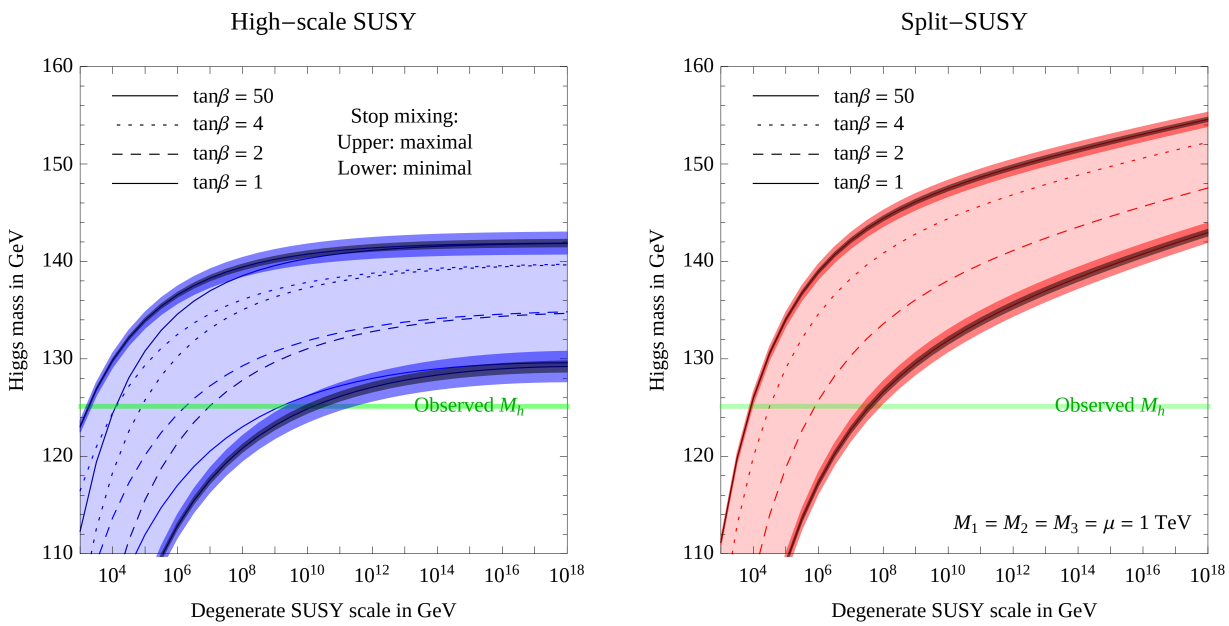


Figure 2: **Left:** *the Higgs mass as a function of the SUSY scale $\tilde{m}$, with a degenerate spectrum of superparticles. We vary the Higgs-stop coupling A_t in such a way as to obtain minimal M_h (lower lines) and maximal M_h (upper lines) at fixed* $\tan\beta = \{1, 2, 4, 50\}$. *The bands around the extremal solid lines are obtained from 1σ variations of $\alpha_3(M_Z)$ (thinner band in gray) and M_t (larger band in color). The green horizontal band indicates the measured Higgs mass.* **Right:** *same as in the left plot, for a split spectrum with gaugino and higgsino masses set to* 1 TeV *and with $A_t = 0$.*

From "Higgs Mass and Unnatural Supersymmetry" arXiv:1407.4081 [hep-ph],
by E. Bagnaschi, G. F. Giudice, P. Slavich and A. Strumia (TH CERN), page 19

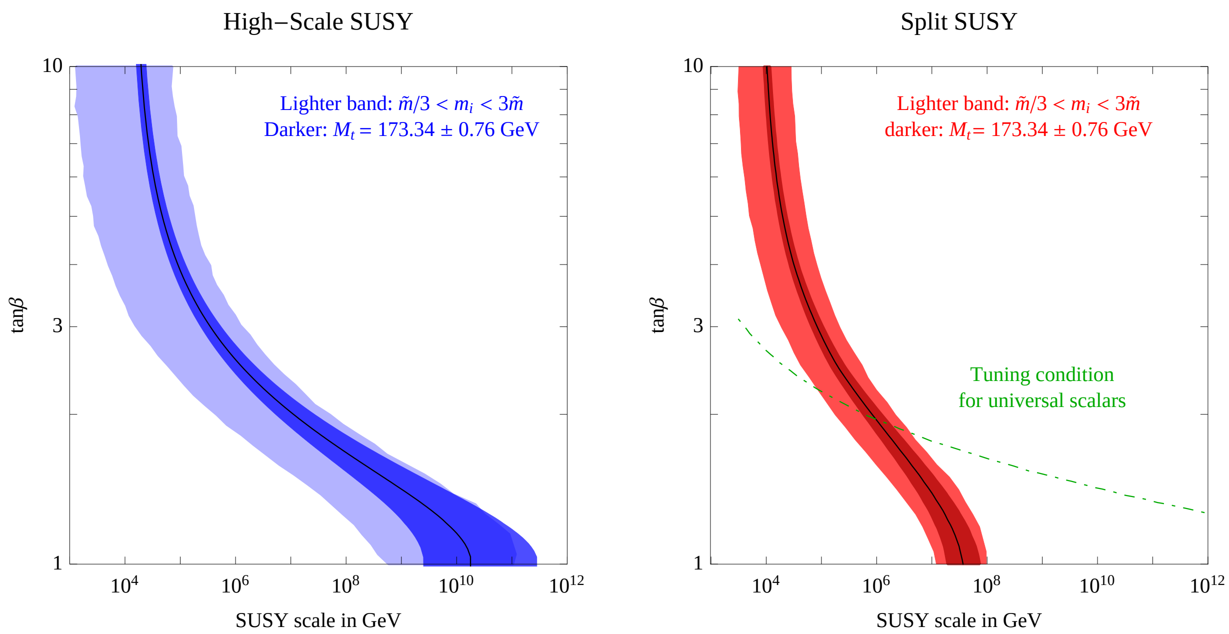


Figure 4: **Left:** *Regions in the* $(\tilde{m}, \tan\beta)$ *plane that reproduce the observed Higgs mass for High-Scale SUSY. The black solid line gives the prediction for* $X_t = 0$*, mass-degenerate superparticles, and central values for the SM parameters. The light-blue band shows the effect of superparticle thresholds by varying the supersymmetric parameters* M_1, M_2, M_3, $m_{Q_i}, m_{U_i}, m_{D_i}, m_{E_i}$, m_{L_i} *and* μ *randomly by up to a factor 3 above or below the scale* $\tilde{m}$*, and* A_t *within the range allowed by vacuum stability. The dark-blue band corresponds to mass-degenerate superparticles, but includes a* 1σ *variation in* M_t. **Right:** *Same as the left plot for the case of Split SUSY. The gaugino and higgsino masses are all set to* $1\,\mathrm{TeV}$*, and* $A_t = 0$*. The dot-dashed curve corresponds to the EW tuning condition in the case of universal scalar masses at the GUT scale.*

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