Special Relativity and Its $SO(3)$ in Euclidean Space — Time-Independent Special Theory of Relativity

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In memory of the late

Prof. Ichiro Yokota

of Shinshu University

and

known for his research of cellular decompositions of classical Lie groups and realizations of exceptional Lie groups.

Abstract

In the process of Albert Einstein establishing the theory of special relativity, the principle of relativity is completely based on geometrical description. On the other hand, the electromagnetic theory is purely algebraic and complicated. Minkowski's work extended it for the 4-dimensional space-time which is purely algebraic as well.

However, we are able to understand Einstein's idea more simply and phenomenally. Such a description of special relativity will facilitate researches in spintropics to consider the relativistic effect. Besides, it leads to an unknown special orthogonal group in real space, unlike the Lorentz group $SL(2, C)$. In this talk, we discuss so-to-speak the 'complete' geometric special relativity and its new Lie group in real space.

Chapter 1. Another derivation of special relativity different from Einstein

First of all, let us think of an observer when in the static system κ as shown in figure 1 and inertia system κ' in figure 2. A ray of light to the observer, declines to the x' axis by the aberration based on the *principle of the constancy of the speed of light,* as the velocity of the observer v approaches to the velocity of light.

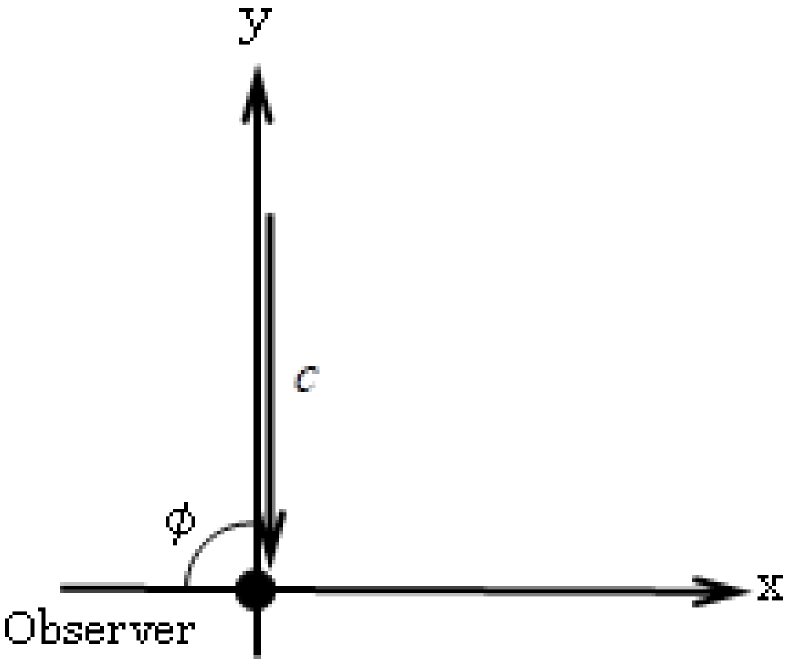

Figure 1

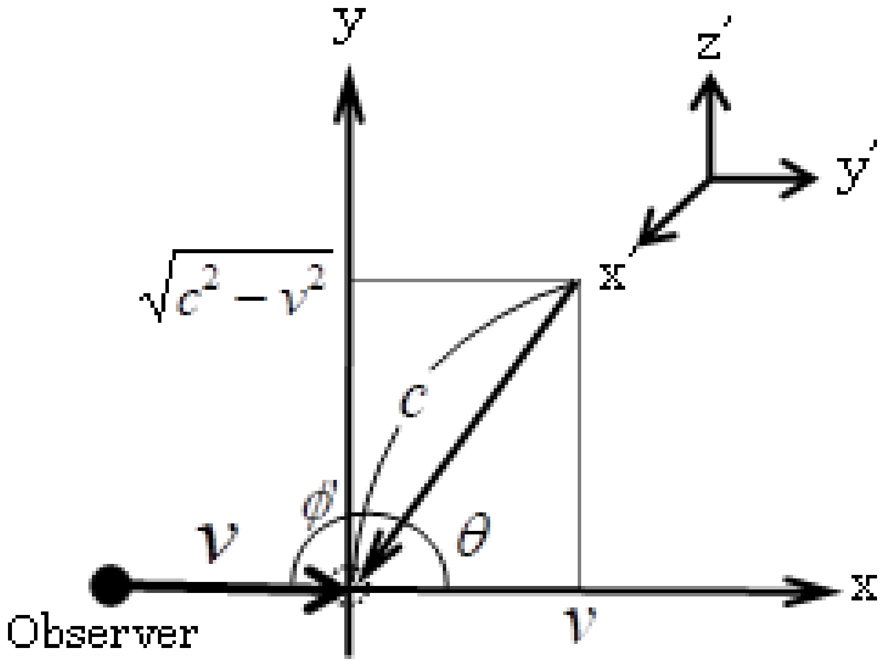

Figure 2

The Formula of Relativistic Aberration

$$cos\phi' = \frac{cos\phi - v/c}{1-(v/c)cos\phi}\ , \cdots\ (1.1)$$

where ϕ and ϕ' are angles between the ray of light and the x axis in κ and κ', and c is the velocity of light. Since $\phi = \pi/2$, in this case as shown in figure 1, it results in:

$$cos\phi' = -v/c, \cdots\ (1.2)$$

$$\cos(\pi - \phi') = cos\theta = v/c.\ \ \cdots(1.3)$$

It corresponds to what figure 2 shows. Discussing special relativity from the geometrical viewpoint, relativistic aberration plays a key role.

Relativistic Coulomb Force

We know that the Coulomb force between an electron and a positron moving with relativistic speed is considerably reduced. It will be explained by the aberration we discussed above. As shown in figure 3, when the pair of electron and positron travel horizontally at relativistic speed, an observer detects the electric field E (or $E'_{\perp}$ if expressing it by the co-ordinates of the inertia system as viewed by the observer in the static system), which is declined by the relativistic aberration.

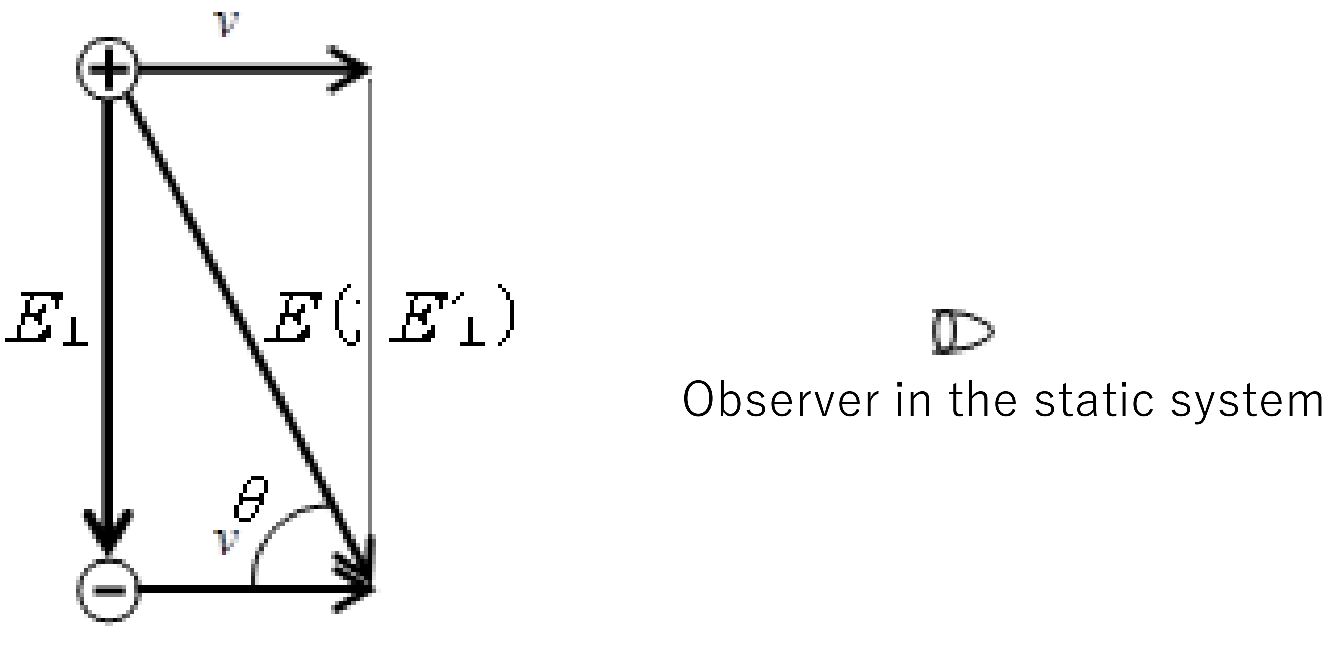

Figure 3

Since only the vertical component of the electric field is active for the electron, relativistic Coulomb force F' from the observer's viewpoint is

$$F' = qE_{\perp}\,, \quad \cdots\ (1.4)$$

where q is quantity of electric charge, $E_{\perp}$ is vertical component of E expressed by the static system. Since $E_{\perp} = Esin\theta$ from Fig. 2, eq. 4 is

$$F' = qEsin\theta. \quad \cdots\ (1.5)$$

From Eq. 1.3, Eq. 1.5 is

$$F' = qE\sqrt{1 - cos^2\theta} = qE\sqrt{1 - (v/c)^2}. \quad \cdots\ (1.6)$$

It corresponds to the general expression.

Relativistic Electromagnetic Field

Relativistic magnetic flux density B'_x, B'_y, and B'_z are expressed by

$$B'_x = B_x, \cdots (1.7)$$

$$B'_y = \beta\left(B_y + \frac{v}{c^2}E_z\right), \cdots (1.8)$$

$$B'_z = \beta\left(B_z - \frac{v}{c^2}E_y\right), \cdots (1.9)$$

and the electric field E'_x, E'_y, and E'_z are expressed by

$$E'_x = E_x, \cdots (1.10)$$

$$E'_y = \beta(E_y - vB_z), \cdots (1.11)$$

$$E'_z = \beta(E_z + vB_y), \cdots (1.12)$$

where $\beta = 1/\sqrt{1-(v/c)^2}$. Let us discuss those equations from a geometric viewpoint.

First of all, let us think of the magnetic flux:

$$B'_x = B_x,$$

$$B'_y = \beta\left(B_y + \frac{v}{c^2}E_z\right),$$

$$B'_z = \beta\left(B_z - \frac{v}{c^2}E_y\right).$$

In the original paper by Albert Einstein [1], those equations of 1.7 to 1.12 are derived by very complicated algebraic expansions. However, contrarily, let us derive them by simple geometry.

As shown in figure 4, an observer moves at a speed v along the $+y$-axis and detects the declined magnetic flux of the static system.

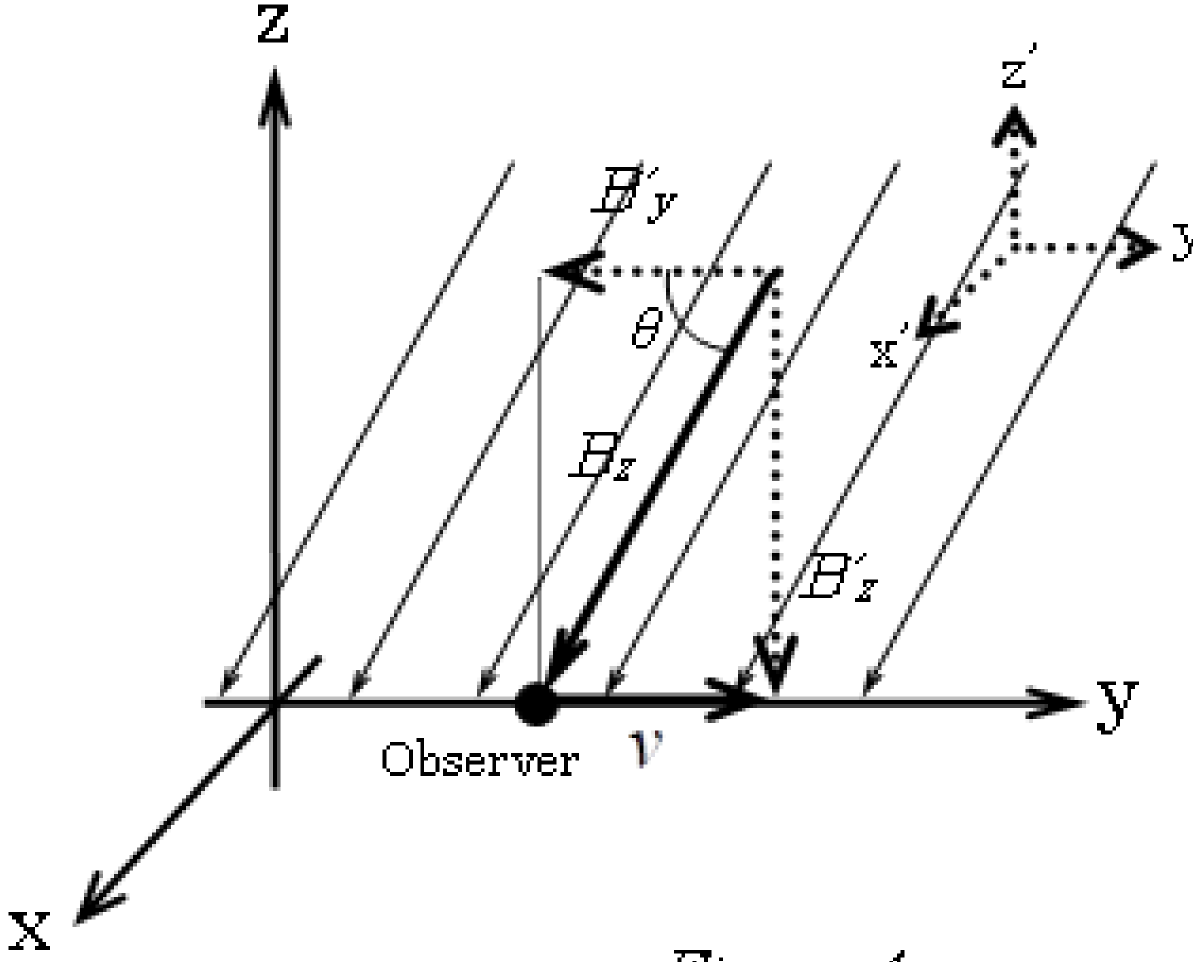

Figure 4

Expressing the magnetic flux by the co-ordinates of the inertia system,

$$B_z = B'_y sin\theta + B'_z cos\theta. \quad \cdots (1.13)$$

Since $sin\theta = \sqrt{1-(v/c)^2}$ and $\cos\theta = v/c$ from figure 2 and Eq. 1.3, Eq. 1.13 is expanded as

$$B_z = \sqrt{1-(v/c)^2} B'_Z + \frac{v}{c} B'_y. \quad \cdots (1.14)$$

Since $\beta = 1/\sqrt{1-(v/c)^2}$, Eq. 1.14 is expanded as

$$B_z = \frac{1}{\beta} B'_Z + \frac{v}{c} B'_y. \quad \cdots (1.14)'$$

B_z denotes a vector of the declined magnetic flux density of the static system, B'_z the vertical component and B'_z the horizontal by the inertia system.

Since B'_y is the horizontal to the direction of the observer moving at the speed of v, $B_y = B'_y$ by the ‘constancy of light’ speed claimed by Einstein in [1]. So that, Eq.1.13 is

$$B_z = \frac{1}{\beta} B'_Z + \frac{v}{c} B_y. \quad \cdots (1.15)$$

By the way, since B'_y (or B_y) is perpendicular to B'_z, it makes the observer look electric field. Considering it as if another field, then substituting Maxwell's equation $B = E/c$ to correct the magnitude gap between B and E, Eq. 15 is

$$B_z = \frac{1}{\beta} B'_Z + \frac{v}{c^2} E_y. \quad \cdots (1.16)$$

Solving it for B'_z,

$$B'_Z = \beta \left(B_z - \frac{v}{c^2} E_y \right). \quad \cdots (1.17)$$

It corresponds to the original equation (1.9).

Let us think of B'_y in the same way.

Remark. We discuss them in the left-hand system.

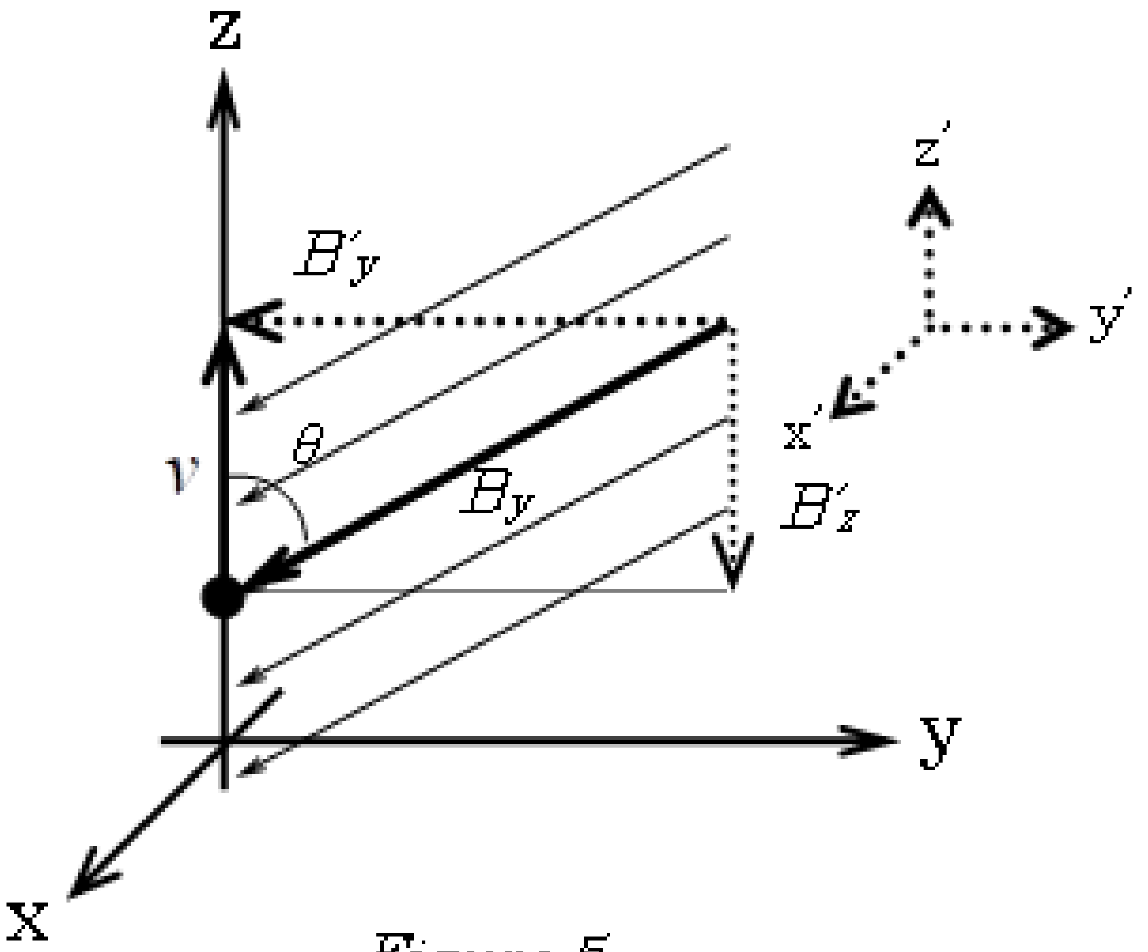

Figure 5

As shown in Figure 5, B_y is

$$B_y = B_y' sin\theta + B_z' cos\theta. \quad \cdots (1.18)$$

From Fig. 2 and Eq. 1.3 as well,

$$B_y = \sqrt{1-(v/c)^2} B_y' + \frac{v}{c} B_z' = \frac{1}{\beta} B_y' + \frac{v}{c} B_z'. \quad \cdots (1.19)$$

Since B'_z is parallel to direction of the moving observer, $B_z' = B_z$ by constancy of light speed. So that, Eq. 1.19 is

$$B_y = \frac{1}{\beta} B_y' + \frac{v}{c} B_z. \quad \cdots (1.20)$$

Since B'_z (or B_z) is perpendicular to B'_y, it looks electric field from the observer. Substituting Maxwell's equation $B = E/c$ to correct the magnitude gap between B and E, Eq. 1.21 is

$$B_y = \frac{1}{\beta} B_y' + \frac{v}{c^2} E_z. \quad \cdots (1.21)$$

Solving it for B'_y,

$$B'_y = \beta\left(B_y - \frac{v}{c^2}E_z\right). \quad \cdots (1.22)$$

However, it does not correspond to the original equitation (1.8).

To solve the problem, let us introduce Fleming's left hand rule or the left-handed system applied to Lorentz force

$F = q(E + v \times B)$ by assuming that the moving observer were electron or positron and then pay attention to each direction of vectors B_y, B'_y, B'_z for the coordinates.

First of all, let us set the rule in the coordinates as shown in Figure 6. Then, let us see the coordinates from different viewpoint as shown in Figure 7. Although the Lorenz force is negative(; $-F'_x$) in the coordinates of Figure 6, it is positive (; $+F'_x$) from different viewpoint in the coordinates of Figure 7. However, they are equivalent with each other. So that, let us unify all the settings by positive direction(; $+F'_x$) from now on.

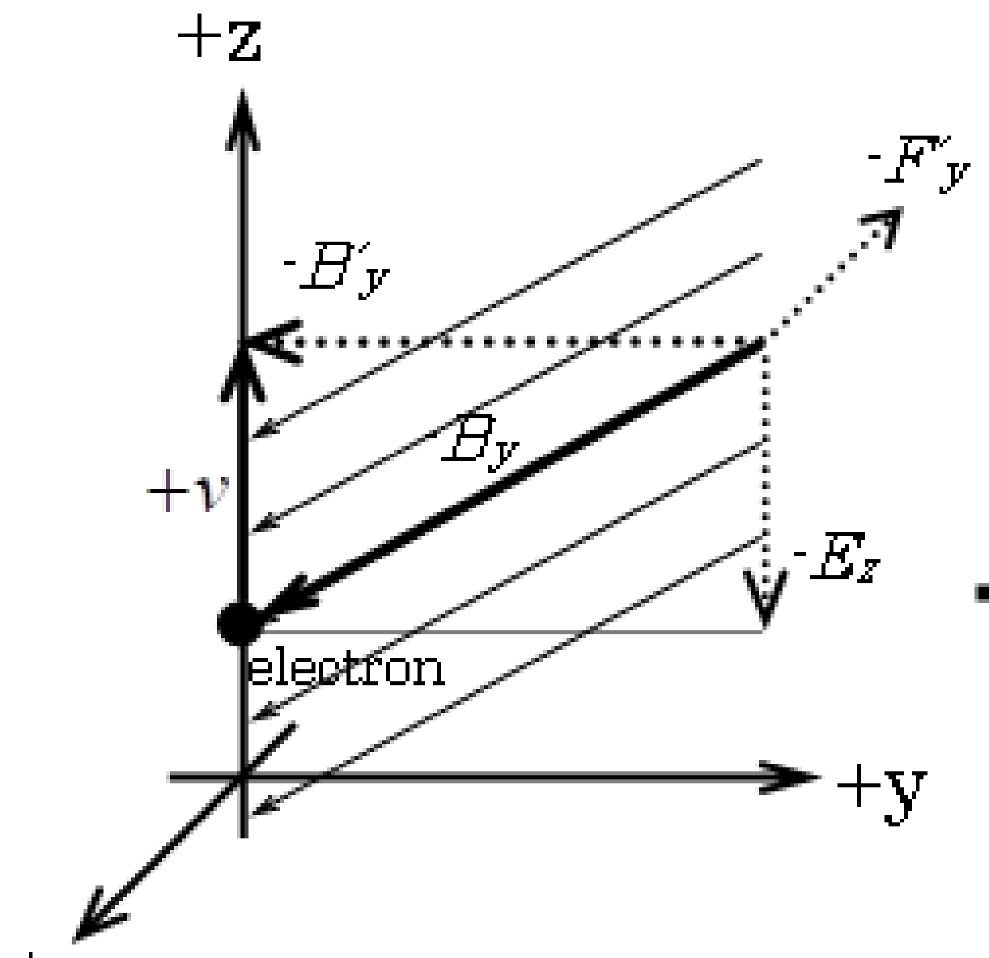

Figure 6

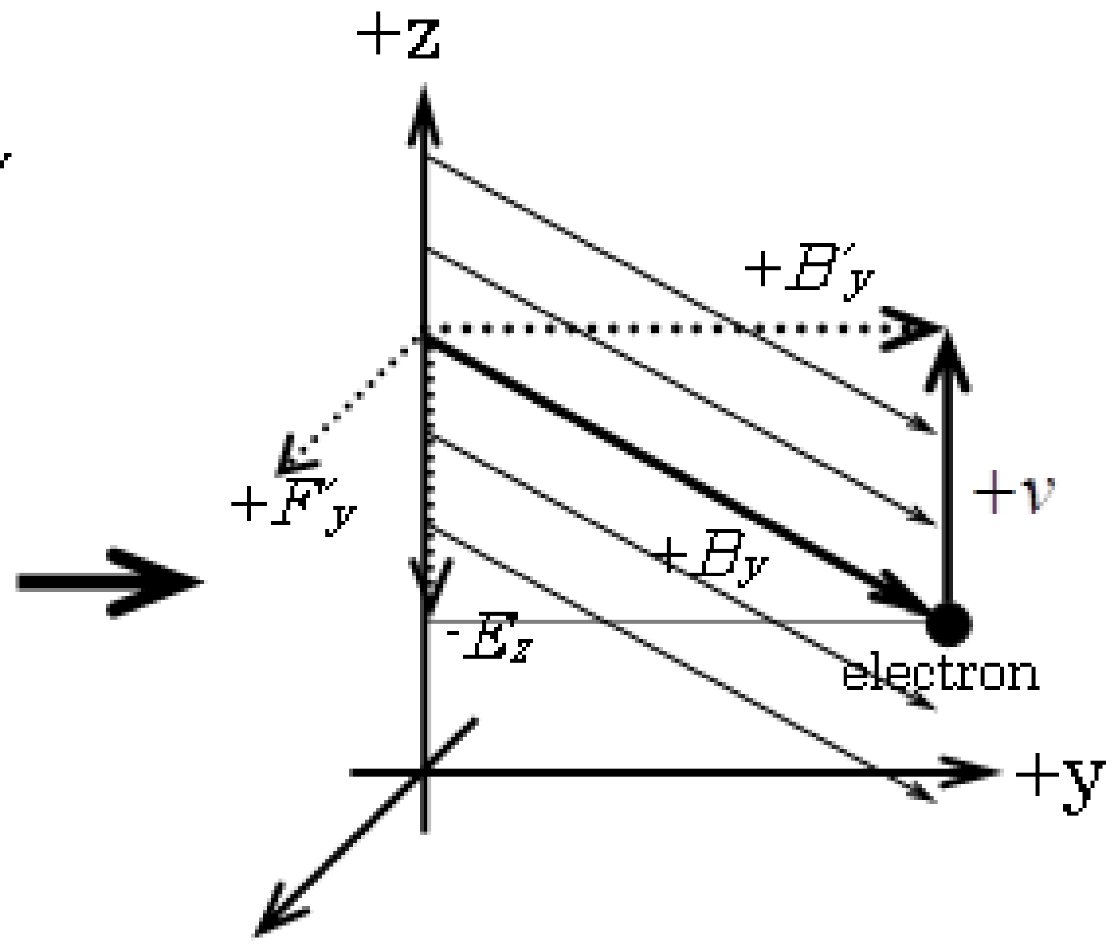

Figure 7

Again, let us reconsider Eq. (1.21) and (1.22) from the latter viewpoint. The revised equation is

$$B_y = \frac{1}{\beta} B'_y + \frac{v}{c^2}(-E_z). \quad \cdots (1.23)$$

Solving it by B'_y,

$$B'_y = \beta \left(B_y + \frac{v}{c^2} E_z \right). \quad \cdots (1.24)$$

It eventually corresponds to the original equation (1.8).

Now let us reconsider a series of Eq. 1.13 to 1.17 similarly.

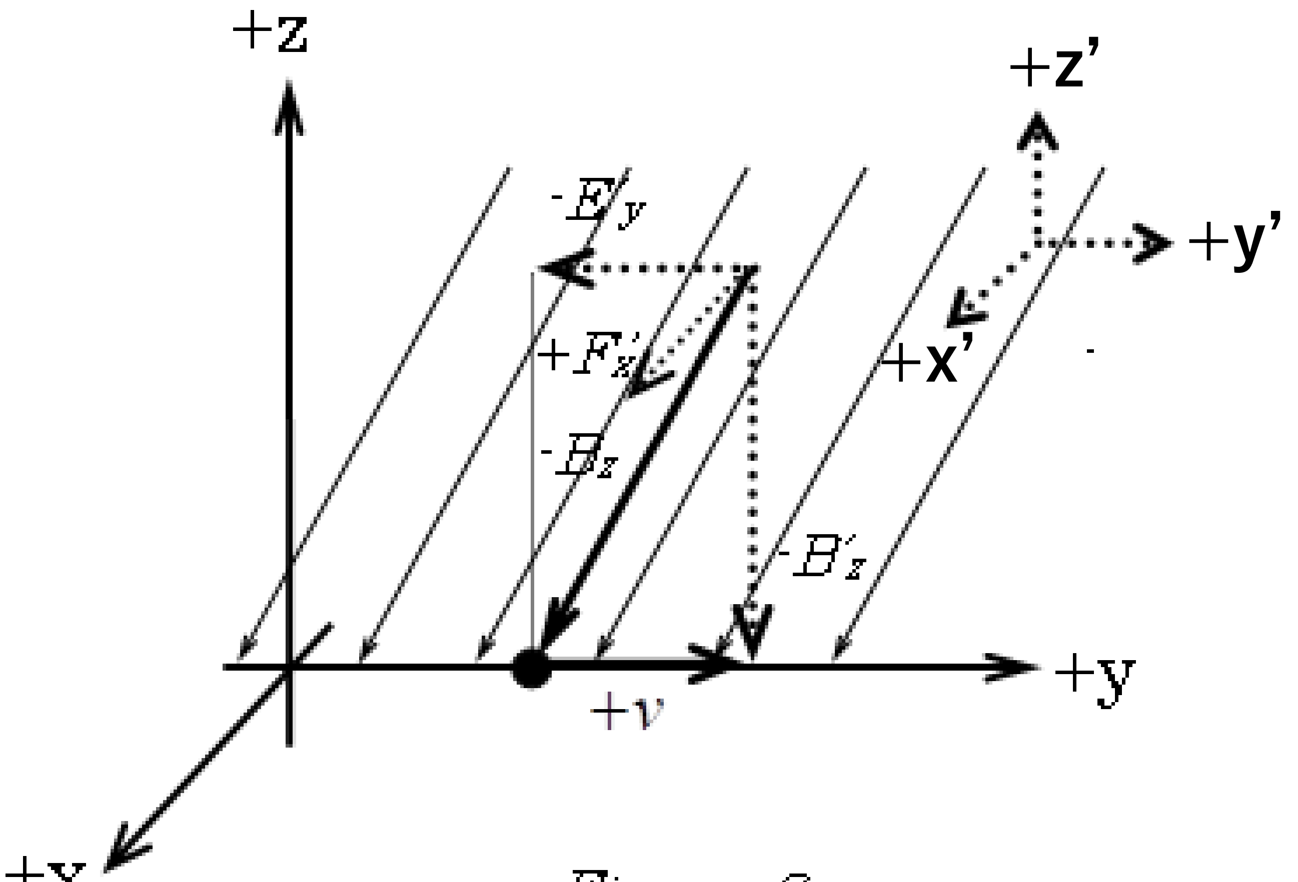

Figure 8

Since Figure 4 is redrawn as shown in Figure 8, those therefore need to be corrected as follows: Eq. (1.13) is revised as

$$-B_z = -B_z' sin\theta + (-B_y') cos\theta. \quad \cdots (1.25)$$

Multiplying both sides of the equation by -1, it corresponds to Eq. (1.13).

Secondarily, let us think of relativistic electric field.

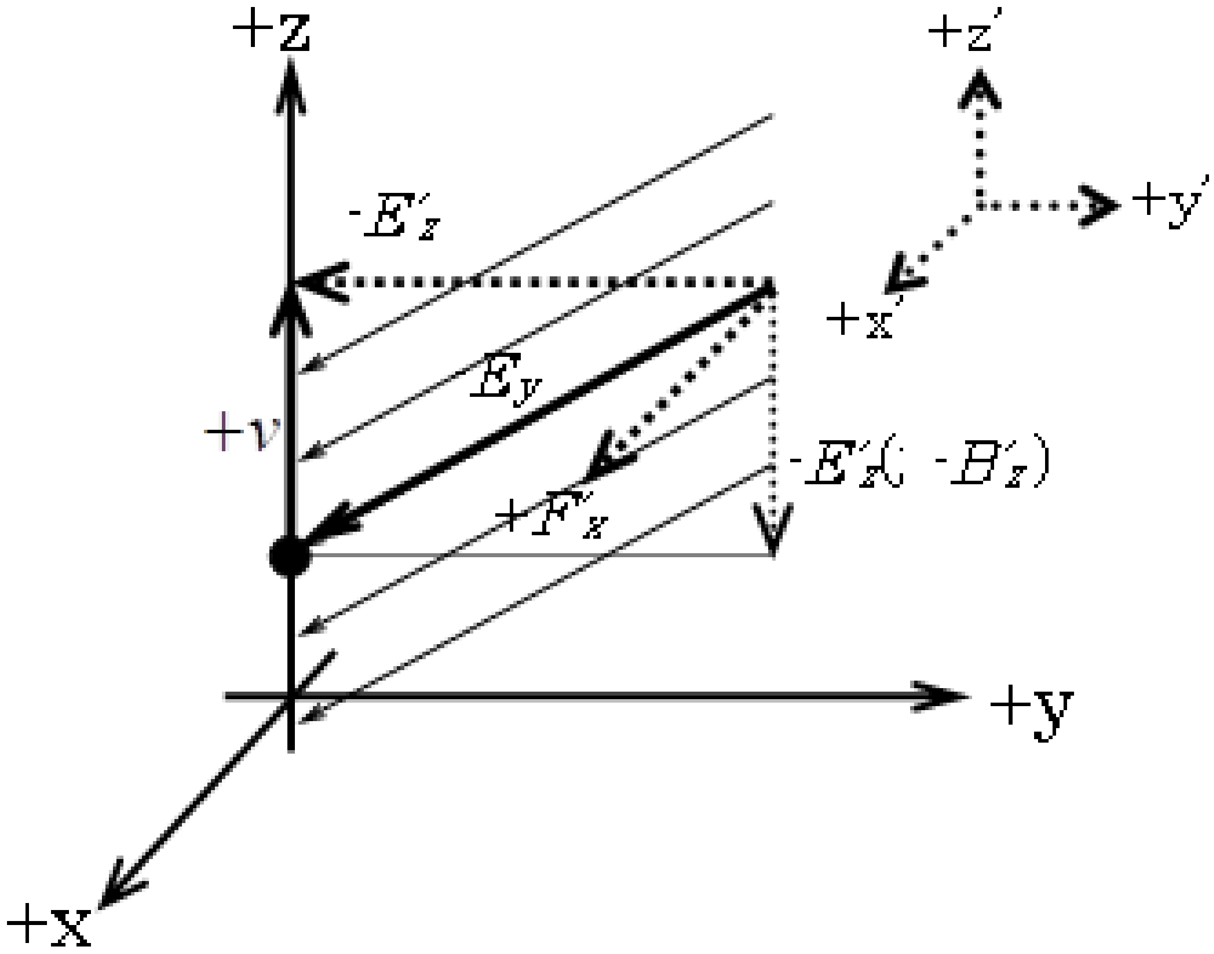

Figure 9

As shown in Figure 9, the equation of electric field of the static system E_y is

$$-E_y = -E'_y sin\theta + (-E'_z) cos\theta. \quad \cdots (1.26)$$

Multiplying both sides of each equation by -1,

$$E_y = E'_y sin\theta + E'_z cos\theta. \quad \cdots (1.27)$$

From Eq. 1.3,

$$E_y = \frac{1}{\beta} E'_y + \frac{v}{c} E'_z. \quad \cdots (1.28)$$

E'_y denotes horizontal component and E'_z vertical one by the inertia system. Since E'_z is parallel to direction of moving observer, $E'_z = E_z$ by the constancy of light speed. So that, Eq. 1.28 is

$$E_y = \frac{1}{\beta} E'_y + \frac{v}{c} E_z. \quad \cdots (1.29)$$

Since E’z (or Ez) is perpendicular to E’y , it makes observer look magnetic flux. Substituting Maxwell’s equation $E = cB$ to correct the magnitude gap between B and E , Eq. (1.29) is

$$E_y = \frac{1}{\beta} E'_y + vB_z. \quad \cdots (1.30)$$

Solving it for $E'y$,

$$E'_y = \beta\left(E_y - vB_z\right). \quad \cdots (1.31)$$

It corresponds to the original equation (1.11).

As shown in Figure 10, let us think of *E'z* in the same way.

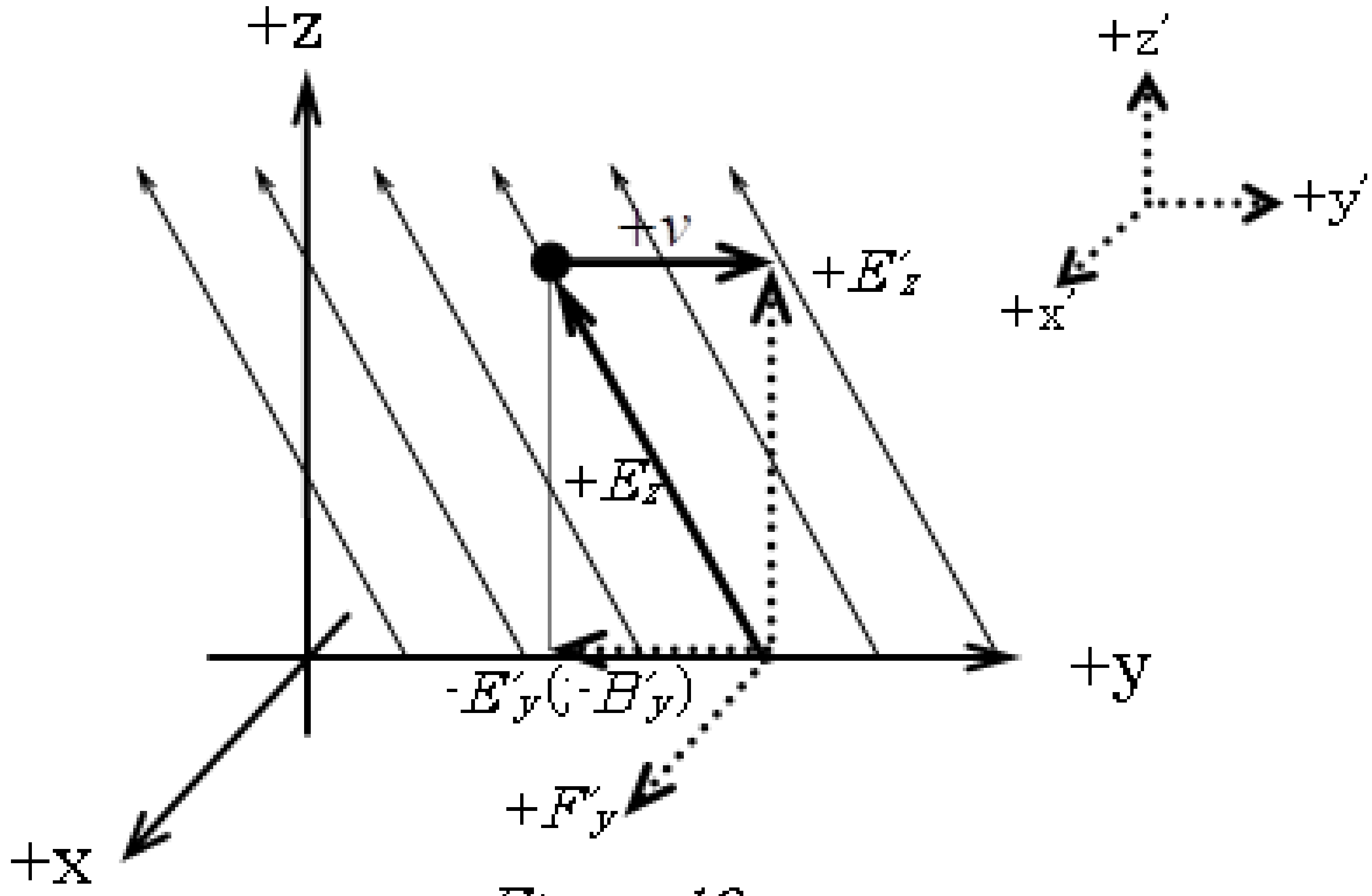

Figure 10

The equation of E_z is

$$E_z = E'_z sin\theta + (-E'_y)cos\theta. \quad \cdots (1.32)$$

From Eq. 1.3,

$$E_z = \frac{1}{\beta}E'_z - \frac{v}{c}E'_y. \quad \cdots (1.33)$$

Since E'_y is horizontal to direction of moving observer, $E'_y = E_y$, by constancy of light speed. Therefore, Eq. 1.33 is

$$E_z = \frac{1}{\beta}E'_z - \frac{v}{c}E_y. \quad \cdots (1.34)$$

Since E'y , (or Ey) is perpendicular to E'z , it makes observer magnetic flux. Substituting Maxwell's equation $E = cB$ to correct the magnitude gap between B and E, Eq. 1.34 is

$$E_z = \frac{1}{\beta}E'_z - vB_y. \quad \cdots (1.35)$$

Solving it for *E'z* ,

$$E'_z = \beta(E_z + vB_y). \quad \cdots (1.36)$$

It corresponds to the original equation (1.12).

At last, original equations (1.7) and (1.10) are easily verified by constancy of light velocity as shown in Fig.11.

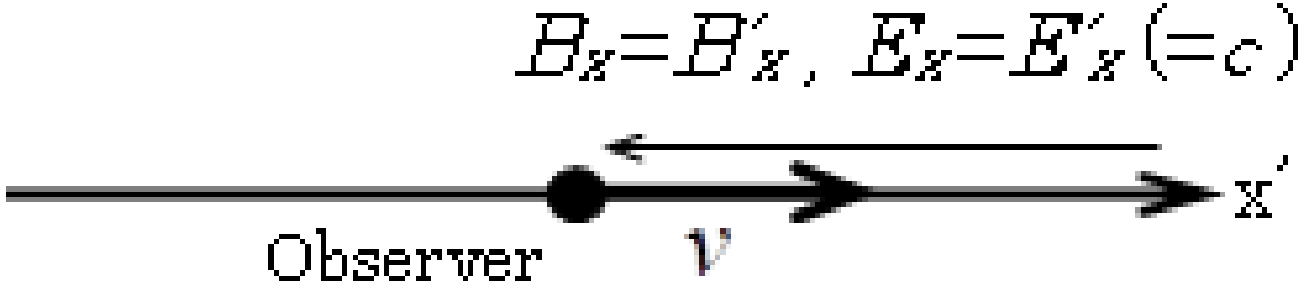

Figure 11

By the way, if $v = c$ (then $sin\theta = 0$ and $\cos\theta = 1$), how does it go and what does it mean from our viewpoint?

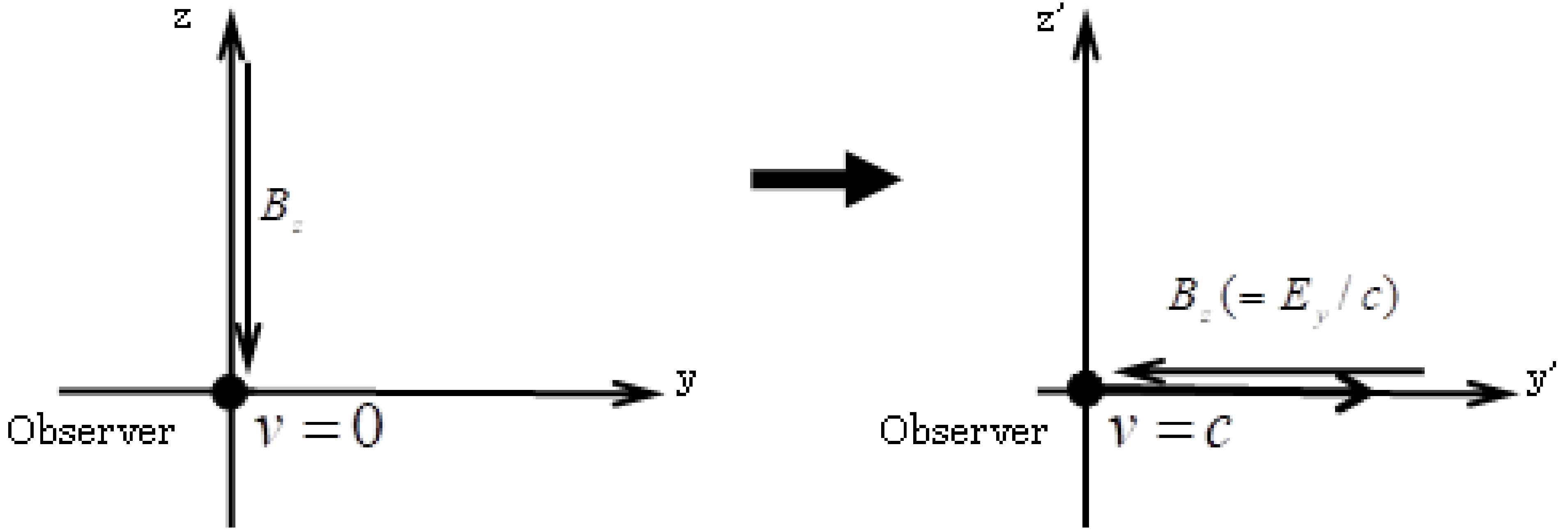

Figure 12

It phenomenally suggests that the magnetic flux totally decline by relativistic aberration as shown in Fig. 12. It totally corresponds to Maxwell's equations then.

Chapter 2. A derivation of a new Lorentz group SO(3) in real space

- From our discussion, it obviously suggests rotations in the static system of Euclidean space.
- It naturally looks $SO(3, R)$ in Euclidean space, not $SL(2, C)$ in the Minkowski space.

So let us derive the group in the same way we have discussed.

Relativistic Aberration, Again

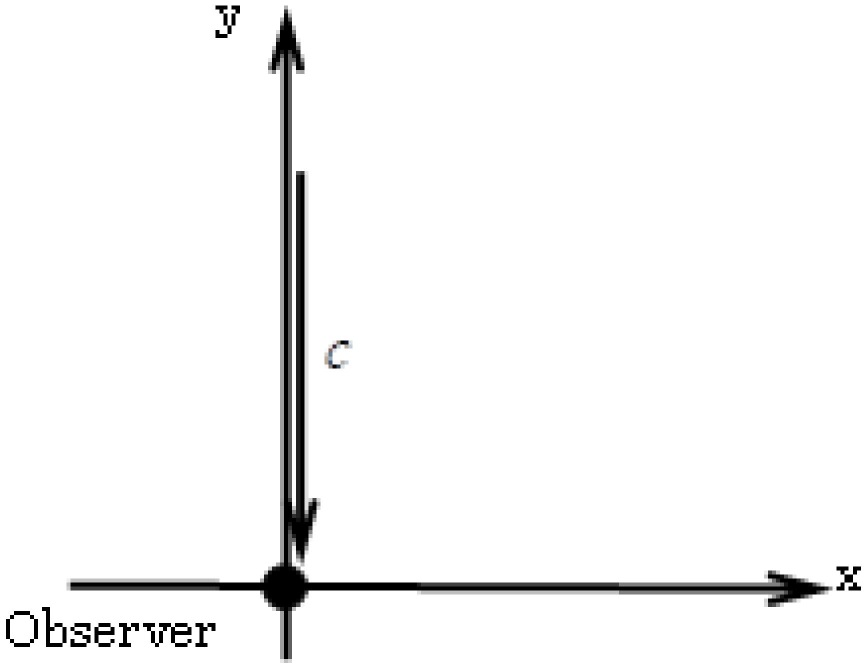

Figure 13

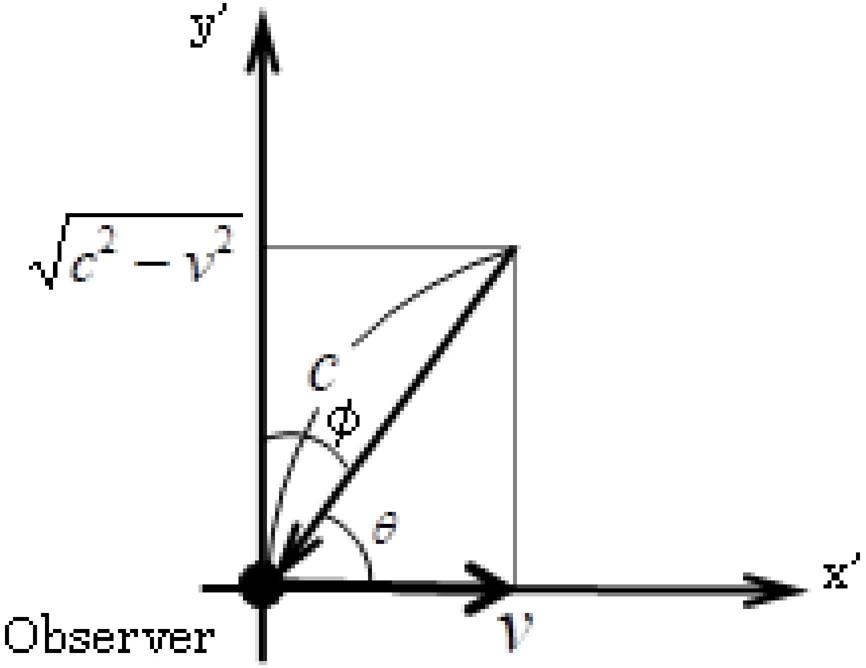

Figure 14

Based on Figure 2, we can denote as follows.

$$sin\phi(= cos\theta) = v/c, \cdots (2.1)$$

$$cos\phi(= sin\theta) = \sqrt{1-(v/c)^2} = \frac{1}{\beta}, \quad \cdots (2.2)$$

where ϕ is angle between the ray of light and y axis in κ' as shown in Figure 14. Since ϕ is clockwise, it is inverse rotation for θ. In this chapter, we mainly use ϕ for descriptive purpose.

First of all, let us think of a specific case as Fig. 12. Multiplying both sides of each equation (1.9) by $\sqrt{1-(v/c)^2}$, then $\sqrt{1-(v/c)^2}B_z' = B_z - \frac{v}{c^2}E_y$.

When $v = c$, $LHS = \sqrt{1-(v/c)^2}B_z' = B_z' \cos\left(\frac{\pi}{2}\right) = 0$ and $RHS = B_y + \frac{c}{c}\left(\frac{1}{c}E_z\right) = B_y + \frac{1}{c}E_z$.

$\therefore B_y = -\frac{1}{c}E_z$.

,

Eq. 1.7 is $E_y = vB_z$ and Eq. 1.18 is $E_z = -vB_y$ when v = c in the same manner. From the results,

$$\begin{pmatrix} B_y \\ B_z \end{pmatrix} = \frac{1}{c}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} E_y \\ -E_z \end{pmatrix} \text{ and } \begin{pmatrix} E_y \\ E_z \end{pmatrix} = v\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -B_y \\ B_z \end{pmatrix}.$$

They are equivalent to

$$\begin{pmatrix} B_y \\ B_z \end{pmatrix} = \frac{1}{c}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} E_y \\ E_z \end{pmatrix} \text{ and } \begin{pmatrix} E_y \\ E_z \end{pmatrix} = v\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} B_y \\ B_z \end{pmatrix}$$

Then, they are naturally related to $SO(2, R)$:

$$\begin{pmatrix} B_y \\ B_z \end{pmatrix} = \frac{1}{c}\begin{pmatrix} \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{pmatrix}\begin{pmatrix} E_y \\ E_z \end{pmatrix} \text{ and } \begin{pmatrix} E_y \\ E_z \end{pmatrix} = v\begin{pmatrix} \cos(\frac{\pi}{2}) & \sin(\frac{\pi}{2}) \\ -\sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{pmatrix}\begin{pmatrix} B_y \\ B_z \end{pmatrix}$$

Based on the fact above, let us discuss Eqs. 1.7 to 1.12 in the same way. Firstly, let us think of the magnetic flux density. Solving Eq. 1.9 for Bz, the expansion is denoted by Eqs. 2.1 and 2.2 as follows.

$$B_z = \frac{1}{\beta}B_z' + \frac{v}{c^2}E_y = B_z' cos\phi + \frac{1}{c}E_y sin\phi.$$

Since $E/c = B = B'$ by constancy of light speed,

$$B_z = \frac{1}{\beta} B_z' + \frac{v}{c^2} E_y = B_z' cos\phi + B_y' sin\phi. \quad \cdots (2.3)$$

Likewise, solving Eq. 1.5 for *By* , the expansion is

$$B_y = \frac{1}{\beta} B_y' - \frac{v}{c^2} E_z = B_y' cos\phi - \frac{1}{c} E_z sin\phi.$$

Since $E/c = B = B'$ by constancy of light speed,

$$B_y = B_y' cos\phi - B_z' sin\phi. \quad \cdots (2.4)$$

Denoting Eqs. 2.3 and 2.4 by 2-by-2 matrix,

$$\begin{pmatrix} B_z \\ B_y \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} B'_z \\ B'_y \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} B'_y \\ B'_z \end{pmatrix}$$

$$= \begin{pmatrix} \sin\phi & \cos\phi \\ \cos\phi & -\sin\phi \end{pmatrix} \begin{pmatrix} B'_y \\ B'_z \end{pmatrix}. \quad \ldots(2.5)$$

Multiplying both sides of the equation (2.5) by $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$,

$LHS = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} B_z \\ B_y \end{pmatrix} = \begin{pmatrix} B_y \\ B_z \end{pmatrix}$, and

$RHS = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \sin\phi & \cos\phi \\ \cos\phi & -\sin\phi \end{pmatrix} \begin{pmatrix} B'_y \\ B'_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} B'_y \\ B'_z \end{pmatrix}.$

$$\therefore \begin{pmatrix} B_y \\ B_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} B'_y \\ B'_z \end{pmatrix}. \quad \ldots(2.6)$$

Then, multi plying both sides of each equation (2.6) by $\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}^{-1}$,

$$\begin{pmatrix} B'_y \\ B'_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}^{-1} \begin{pmatrix} B_y \\ B_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} B_y \\ B_z \end{pmatrix}. \quad \ldots(2.7)$$

Secondarily, let us think of the electric field *Ey* , Solving Eq. (1.11) for E_y , the expansion is

$$E_y = \frac{1}{\beta} E'_y + vB_z = E'_y \cos\phi + vB_z.$$

Since $B = B' = E'/c$,

$$E'_y \cos\phi + vB_z = E'_y \cos\phi + \frac{v}{c} E'_z = E'_y \cos\phi + E'_z \sin\phi. \quad \cdots (2.8)$$

Likewise, solving Eq. (1.12) for E_y, the expansion is

$$E_z = \frac{1}{\beta} E'_z - vB_y = E'_z \cos\phi - vB_y$$

Since $B = B' = E'/c$,

$$E_z = E'_z \cos\phi - \frac{v}{c} E'_z = E'_z \cos\phi - E'_z \sin\phi. \quad \cdots (2.9)$$

Denoting equations (2.8) and (2.9) by 2-by-2 matrix,

$$\begin{pmatrix} E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} E'_y \\ E'_z \end{pmatrix}. \quad \ldots(2.10)$$

Multiplying both sides of each equation (2.10) by $\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}^{-1}$,

$$\begin{pmatrix} E'_y \\ E'_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}^{-1} \begin{pmatrix} E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} E_y \\ E_z \end{pmatrix}. \quad \ldots(2.11)$$

The rotation matrices of Eq. (2.7) and (2.11) can naturally make a special orthogonal group $SO(2, R)$ in a series of Lorentz transformations.

Furthermore, including equations (1.7) and (1.10), the matrices of (2.7) and (2.11) are also denoted as

$SO(3, R)$ as follows:

$$\begin{pmatrix} B'_y \\ B'_y \\ B'_y \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}^{-1} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}. \quad \ldots (2.12)$$

Similarly, 3-D to 2-D is operated as:

$$\begin{pmatrix} E'_y \\ E'_y \\ E'_y \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}. \quad \ldots (2.13)$$

Application for Spintronics

Spin of electron in the static and inertia system

As shown in Figure 15, the spin in the magnetic field leans as it speeds. The angle between the spin axis and horizon is naturally denoted by $sin\phi = cos\theta = v/c$ as we have discussed.

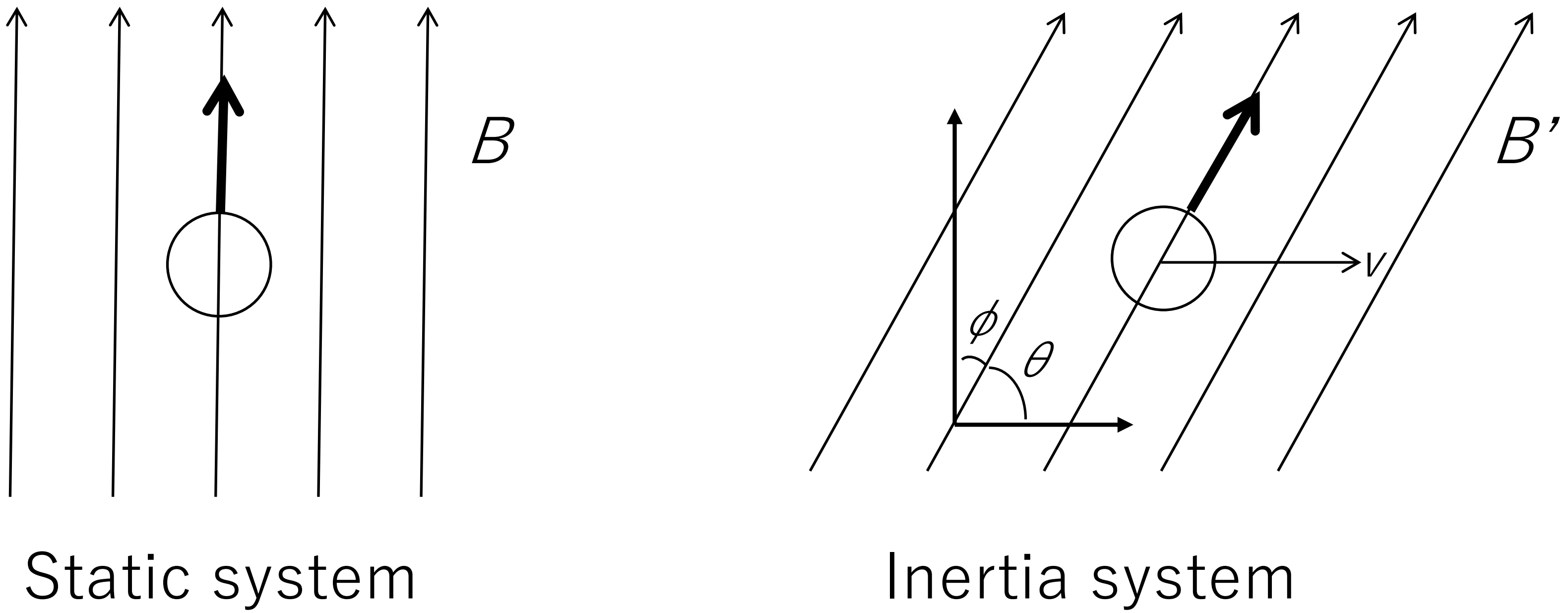

Figure 15

Chapter 3. Discussion of Ampère-Maxwell equation in the inertia system (derivation of displacement current from relativistic helix)

Magnetic Helix — Relativistic Ampère-Maxwell Law from Geometric Viewpoint

As shown in Figure 16, a magnetic field in the static system κ is around the observer on the x-axis. However, as shown in Figure 17, it will converge onto line in the x-axis drawing helix in the inertia system κ'. The spin in the internal space aside, it suggests that any photon has no spin in *real space*.

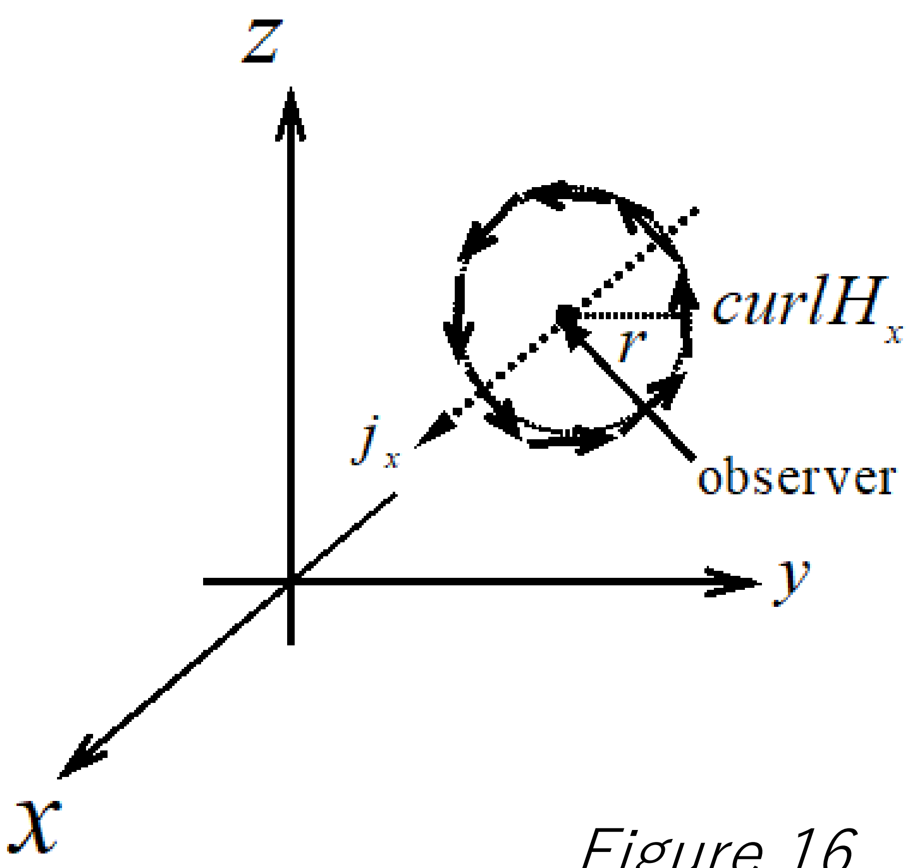

Figure 16

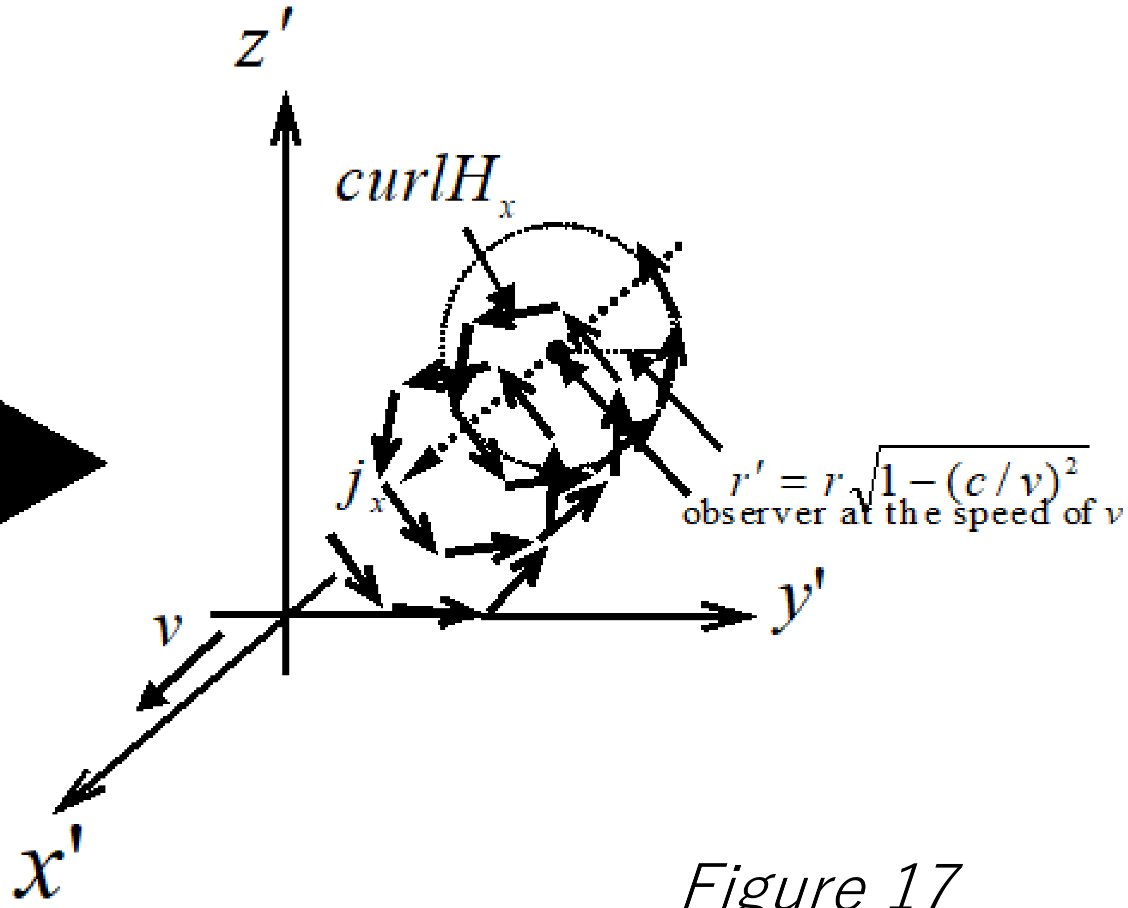

Figure 17

Let us think of the magnetic flux H around the observer in κ' as shown in Figure 17. The original magnetic flux H in y-z plane converges onto x-axis as the observer's speed is close to the light one. Since the magnetic flux in κ' consists of two components of vector in the y-z plane and parallel to x-axis as shown in Figure 18, $curlH_x$ in κ' is

$$curlH_x = \sqrt{1-(v/c)^2}H'_x - (v/c)curlH'_x. \cdots(3.1)$$

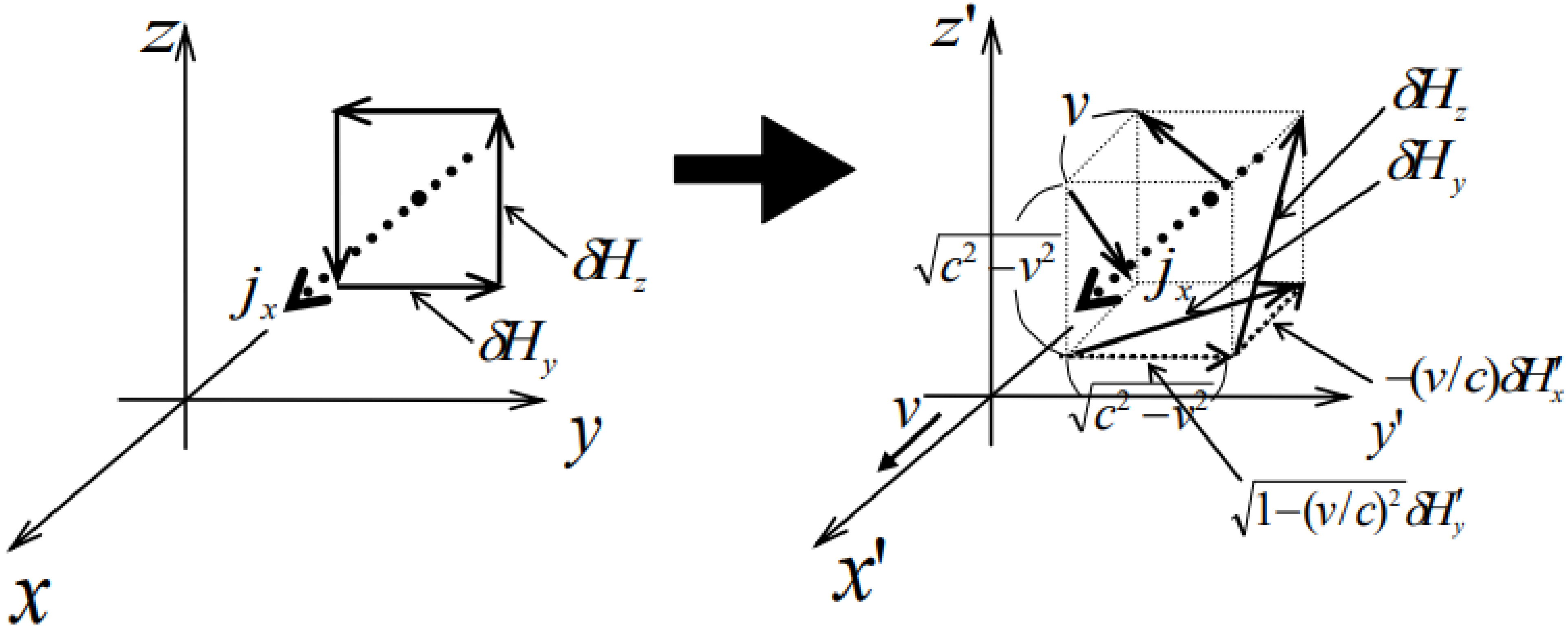

Figure 18 (intuitive images for Figures 16 and 17)

Since $(v/c)curlH_x'$ is vertical to $\sqrt{1-(v/c)^2}curlH_x'$, it could be considered as D' in the manner of our discussion in the former chapters. Then, from $H = cD$ (from $B = \frac{1}{c}E, B = \mu_0 H, D = \epsilon_0 E$ and $c = 1/\sqrt{\epsilon_0\mu_0}$),

$$curlH_x = \sqrt{1-\left(\frac{v}{c}\right)^2}\,curlH_x' - v\frac{\partial D_x'}{\partial x} = \sqrt{1-\left(\frac{v}{c}\right)^2}\,curlH_x' - \frac{\partial x}{\partial t}\frac{\partial D_x'}{\partial x} = \sqrt{1-\left(\frac{v}{c}\right)^2}\,curlH_x' - \frac{\partial D_x'}{\partial t}. \cdots(3.1)$$

Since D’ is equivalent to by the consistency of light speed,

$$curlH_x = \sqrt{1-\left(\frac{v}{c}\right)^2}\,curlH_x' - \frac{\partial D_x}{\partial t}, \ldots(3.2)$$

Therefore, the relativistic Ampère-Maxwell law is,

$$j_x = curlH_x = \sqrt{1-\left(\frac{v}{c}\right)^2}\,curlH_x' - \frac{\partial D_x}{\partial t},$$

$$\therefore j_x + \frac{\partial D_x}{\partial t} = \sqrt{1-\left(\frac{v}{c}\right)^2}\,curlH_x' \ldots(3.3)$$

Likewise,

$$j_y + \frac{\partial D_y}{\partial t} = \sqrt{1 - \left(\frac{v}{c}\right)^2}\, curlH'_y,$$

$$j_z + \frac{\partial D_z}{\partial t} = \sqrt{1 - \left(\frac{v}{c}\right)^2}\, curlH'_z$$ hold. Therefore,

$$j + \frac{\partial D}{\partial t} = \sqrt{1 - \left(\frac{v}{c}\right)^2}\, curlH'. \ldots(3.4)$$

As we shall see the results with Figures 17 and 18, displacement current cannot generate magnetic field.

Furthermore, since the propagation rate of electro-magnetic wave is always c, the equation of helix in κ' is

$x' = -vt$, ..(3.5) $y' = r\sqrt{1-(v/c)^2}\cos(c/r)t$ ... (3.6)

and $z' = \sqrt{1-(v/c)^2}\,\sin(c/r)t$. ...(3.7)

Remark

Any spin in real space disappears at relativistic speed, but another spin in the internal space is ‘not’ influenced by the relativistic speed at all. Therefore, photon has spin 1 given by the internal space as we know. It strongly suggests that the internal space is neither a subspace nor compactified in our universe.

Not only from the results, we know the fact that non-relativistic Pauli equation is for spin 1/2 particles as well as relativistic Dirac equation for 1/2 ones. So to speak, spin in the internal space is always the same. In other words, it also suggests that the internal space is not a subspace of our space. If not, spin in the internal space must be influenced by relativistic effect.

However, AdS/CFT, braneworld and M-theory highly depend on a conventional idea that such a low dimensional space is a subspace of higher dimensional space, against what those phenomena in nature strongly suggests.

Conclusions

As we have discussed, electric and magnetic fields oriented perpendicular to an electron traveling at relativistic speed will be leaned by the relativistic aberration. Similarly, the spin angle of the electron will be affected. In spintronics in such the inertia system, the rotation group on Euclidean space $SO(3, R)$ would be useful.

Acknowledgements

I appreciate Prof. Maxim Khlopov, who gave me some good advice. I appreciate Prof. Susan Hansen, who voluntarily and patiently listened to explanations of my research; proofread all the slides to correct grammatical errors; and offered me significant suggestions and opinions for English expression.

References

[1] Albert Einstein (1905) “Zur Elektrodynamik bewegter Körper”, *Annalen der Physik* 17: 891; English translation “On the Electrodynamics of Moving Bodies”. See also http://www.fourmilab.ch/etexts/einstein/specrel/specrel.pdf

[2] Euich Muztani, Special Relativity from Geometric Viewpoint, *Communications in Applied Geometry*, ISSN 2249-4986, 2011, Vol. 1, No. 1, pp. 17-25.

[3] Euich Miztani, Lorentz Group *SO*(3, *R*) in non-Minkowski Space, *Communications in Applied Geometry*, ISSN 2249-4986, 2016, Vol. 3, No. 1, pp. 11-16.

[4] Euich Miztani, Magnetic Helix — Relativistic Ampère-Maxwell Law from Geometric Viewpoint, *Communications in Applied Geometry*, ISSN 2249-4986, 2016, Vol. 3, No. 1, pp. 17-19.