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THESIS:

Primordial black holes as dark matter candidates

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Contents

1 Introduction

A black hole (BH) is an area in space and time possessing a gravitational field so strong that not even photons can escape from it [1]. The boundary beyond which objects become causally disconnected from us (the observer) is called the event horizon of a BH. If any object is within this boundary, it will only move inside the black hole. While an object from the external space can cross the event horizon, it cannot return. In other words, BHs are spherical objects with mass M (in the Schwarzschild model) [2], enclosed within a gravitational radius r_g (or Schwarzschild radius):

$$r_g = \frac{2GM}{c^2}. \tag{1.1}$$

Black holes are classified into two types: astrophysical and primordial. Astrophysical black holes result from the collapse of massive stars with a mass of around 10 solar masses, or from the collapse of large star clusters at the centers of galaxies. However, in 1966, Zel'dovich and Novikov proposed a black hole formation model where formation occurs at the early stages of Universe development, before the formation of large-scale structures [3]. Such BHs are called primordial black holes (PBHs). Historically, this was the first model for the formation of Primordial Black Holes (PBHs). However, the authors did not further develop the idea because their calculations indicated that accretion significantly increases the mass of PBHs by many orders of magnitude. This would lead to an excess of supermassive black holes, which is not observed. Later, B. Carr and S. Hawking showed [4] that significant mass increase due to accretion does not occur, thereby renewing interest in the existence of PBHs. For a long time, PBHs were considered hypothetical objects, the absence of information about which, nonetheless, allowed for constraints on physical processes in the early Universe and on the effects of inhomogeneity. However, in recent years, attention to PBHs has become very wide, as there have been indications that some phenomena can only be explained by PBHs [5]. However, this was not always the case. B. Carr, who is known today for his reviews on PBHs, did not show such keen interest in the PBH topic in the 1990s. At that time, he even believed that only M. Khlopov was truly serious about the PBH topic at that moment.

The possibility of PBH formation is also influenced by the equation of state of matter in the Universe. The influence of the ω parameter in the matter state equation, represented as $p = \omega\rho$, on the probability of PBH formation is investigated in [6]. The probability of PBH formation of mass M:

$$P(M) \propto \exp\left(-\frac{\omega^2}{2\epsilon^2}\right). \tag{1.2}$$

There is an upper limit on the mass of a PBH formed at time t: the mass of the BH cannot exceed the mass of the Hubble horizon at the moment of its formation:

$$M \sim \frac{c^3 t}{G} \sim 10^{15}\left(\frac{t}{10^{-23}\text{s.}}\right)\text{y.} \tag{1.3}$$

Thus, PBHs could have a mass $M_{Pl} \sim 10^{-5}$g if they were formed at the Planck time ($t \sim 10^{-43}$s.), $1M_\odot$ if they were formed during the QCD epoch ($t \sim 10^{-5}$s.) and $10^5 M_\odot$ if they were formed at $t \sim 1$s. Consequently, PBHs can cover a wide range of masses and are the only ones that can have a mass less than the mass of the Sun.

Various mechanisms of formation and subsequent evolution of PBHs are considered in works such as [5; 7–9].

1.1 PBH as dark matter

Primordial black holes (PBHs) hold particular interest for cosmology, especially those with a mass exceeding 10^{15} grams, which are almost unaffected by Hawking radiation. Since dark matter remains unexplained, there is active search for candidates that could constitute this hidden mass. Astrophysical BHs cannot account for all the dark matter, as they form from baryons and are subject to the well-known constraint of primordial nucleosynthesis, according to which baryons contribute no more than 5% of the critical density [10]. The Cosmic Microwave Background (CMB) provides stringent constraints on the abundance and properties of Primordial Black Holes (PBHs), serving as a critical test for models of the early universe.

Based on [11] CMB gives a very strong constraint of $f(M)$ in the range $10^3 < M/M_\odot < 10^{12}$.

Hence, the interest in PBHs stems from their potential role as candidates for dark matter, since they were formed during the radiation-dominated era before primordial nucleosynthesis, circumventing this limitation. Therefore, PBHs are considered a form of non-baryonic dark matter, behaving like any other form of cold dark matter, despite their mass.

Recently, numerous constraints [12] have been examined on the fraction of dark matter $f(M)$ in PBHs of mass M. These constraints suggest that there are only a few mass ranges where f can be significant: a small range ($10^{17} - 10^{23}$ g), a medium range ($10 - 10^5 M_\odot$), and a wide range ($M > 10^{11} M_\odot$).

It should be emphasized that non-evaporating PBHs could be of great cosmological interest, even if they constitute only a small fraction of dark matter. For instance, they could play a certain role in the formation of supermassive BHs in galaxy nuclei. It is also conceivable that dark matter includes a mixture of PBHs and weakly interacting massive particles (WIMPs) [13].

2 Formation of PBHs

The condition for the formation of PBHs in the early Universe is the predominance of gravitational energy over the forces of internal pressure (i.e., dominance over internal energy). Considering a region of the Universe with size R, the gravitational energy can be expressed as $U_g \sim GM^2/R \sim G\rho^2 R^5$, where ρ is the density of the medium. The internal energy of relativistic matter can be written as $U_p = pV$, where $p = c^2\rho/3$. Hence, $U_p \sim c^2\rho R^3$. Thus, the condition for the formation of PBHs can be written as:

$$\frac{G\rho R^2}{c^2} > 1. \tag{2.1}$$

2.1 Primordial Inhomogeneities

This mechanism of PBH formation is based on the collapse of primordial inhomogeneities in the hot plasma and arises within the framework of the standard Big Bang cosmology [14]. Consider a region of the Universe with radius R. The corresponding gravitational energy is on the order of:

$$\Omega \sim -\rho^2 R^5, \tag{2.2}$$

and the kinetic energy of this region's expansion is on the order of:

$$T \sim \rho R^3 \dot{R}^2, \tag{2.3}$$

where ρ is the energy density. In a universe dominated by radiation, pressure and energy density are proportional to R^{-4}, as the state equation parameter $\omega = 1/3$, and the state equation itself is:

$$p = \omega\rho. \tag{2.4}$$

If the matter density is sufficiently high, gravitational forces can overcome the kinetic energy of expansion and pressure forces. As a result, in such a region of the Universe, expansion stops. To overcome pressure forces, the gravitational energy must be greater than the internal energy, which for $p = 1/3\rho$ is on the order of $U \sim \rho R^3$. Thus, a necessary condition for collapse is:

$$\rho R^2 > \sim 1. \tag{2.5}$$

A drawback of this model is that the spectrum of masses of the generated BHs is close to monochromatic, meaning this model cannot explain the existence of BHs of various masses. Also, within this model, it is impossible to generate clusters of BHs. For this reason, this model also cannot explain the rate of BH mergers observed by LIGO/Virgo [15]. Historically, this is the earliest mechanism of generation and does not require additional assumptions beyond the standard Big Bang theory.

2.2 Collapse at the Quantum Chromodynamics Phase Transition

At one time, it was believed that the quantum chromodynamics (QCD) phase transition at the time 10^{-5} s could be first-order. This would mean that quark-gluon plasma and hadrons could coexist. Moreover, cosmic expansion would occur at a constant temperature due to the conversion of quark-gluon plasma into hadrons. During this, the speed of sound decreases, and the effective pressure decreases, significantly lowering the δ_c threshold for collapse. The formation of PBHs during first-order QCD phase transitions was first proposed by Crawford and Schramm [16], and later reconsidered by Jedamzik [17]. Dolgov and Silk also proposed a model of baryonic isocurvature fluctuations as a mechanism for generating PBHs [18]. It is currently considered unlikely that the QCD phase transition is a first-order transition, but some softening in the equation of state can still be expected.

Recently, Byrnes et al. [19] discussed how this softening could lead to a significant jump in the mass function. The mass of PBHs forming during the QCD epoch is:

$$M \approx 0.9 \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{10}\right)^{-1/2} \left(\frac{\xi}{5}\right)^2 M_\odot, \tag{2.6}$$

where g_* is normalized, and $\xi = M_{Pl}/(k_B T) \approx 5$ – the ratio of the proton mass to the temperature of the QCD phase transition. The expression for the mass of PBHs is close to the Chandrasekhar mass. In this case, the QCD phase transition leads to the formation of PBHs with masses close to those of stars. As a result, observed stars and unobserved PBHs have similar masses.

It should be mentioned that a mechanism has been developed that combines cosmological inflation and quark confinement to produce PBHs [20]. In this scenario, PBHs are formed with a mass less than the value obtained by formula (2.6).

2.3 Collapse of Scalar Fields

Supersymmetric extensions of the standard model [21; 22] generally predict that in the early Universe, a scalar condensate can form and fragment into Q-balls before decaying. If Q-balls dominate the energy density for some time, relatively large fluctuations in their concentration can lead to the formation of PBHs. Other scalar fields not associated with supersymmetry can play a similar role. For an arbitrary charged scalar field, this mechanism can lead to the formation of black holes across the entire mass range allowed by observational constraints, with enough abundance to explain all dark matter in certain parameter ranges. In the case of supersymmetry, the mass range is limited to a maximum of 10^{23} g.

The work [23] considered the gravitational instability of a spatially homogeneous relativistic scalar field taking into account self-interaction. It was shown that this instability is similar to the Jeans instability and can lead to the formation of PBHs.

The inflationary stage of the Universe's development is usually explained by the dynamics of a scalar field. After the end of cosmological inflation, a phase of non-relativistic matter dominance may briefly occur, inevitably leading to the formation of PBHs [24].

2.4 Collapse of Domain Walls

A domain wall is a non-trivial field configuration connecting different vacuums of the potential [25]. An example of a domain wall in the form of a hyperbolic tangent for the Higgs field is presented in Figure 1. This mechanism is driven by second-order phase transitions. For its realization, the potential of the corresponding field must have at least two vacuums of equal energy.

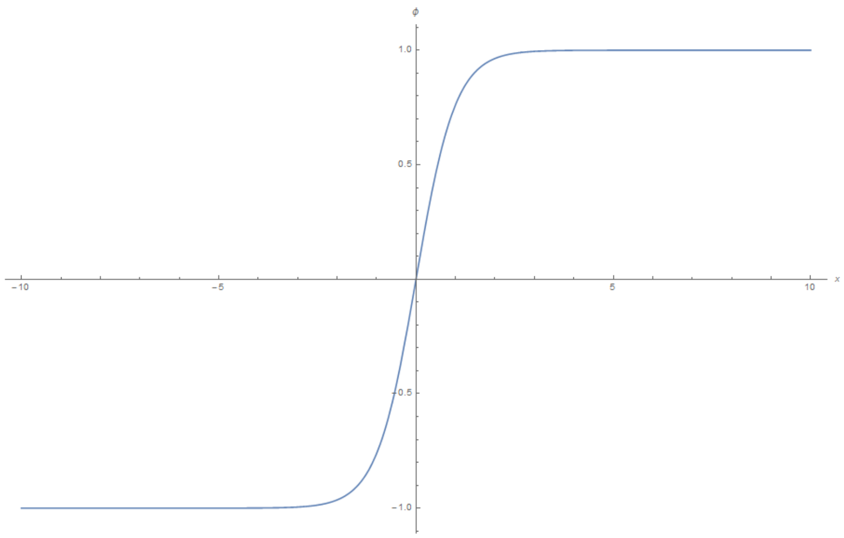

Figure 1 — A domain wall in the form of a hyperbolic tangent for the Higgs field.

Consider first-order phase transitions of a scalar field. The concept of first-order phase transitions as a means of generating PBHs was first proposed in [26]. For this mechanism to be realized, the field's potential must contain at least two minima, one of which must be false. It is assumed that initially, the field is in the false vacuum, and then as a result of field tunneling in one region of space, the field will have a value corresponding to one vacuum, and in another region of space to another vacuum. These regions are called bubbles. In this case, the free energy of a bubble consists of two parts - surface and volume. Let us denote the surface energy density by μ and the difference in potential values at the minima by $\Delta V = E(\phi_0) - E(\phi_1)$, where ϕ_0 corresponds to the true vacuum, and ϕ_1 to the false one. Then the free energy of a bubble with radius R and surface energy density μ can be written as

$$F(R) = 4\pi R^2 \mu - \frac{4\pi}{3} R^3 \Delta V. \tag{2.7}$$

The dependency (2.7) has a maximum at the point $R_{cr} = 2\mu/\Delta V$, after reaching which it becomes energetically favorable for the bubble to expand indefinitely. The expansion of true vacuum bubbles into the false vacuum region leads to the conversion of potential energy of the false vacuum into the kinetic energy of the walls. This transition ensures relativistic speeds of expansion.

When a pair of true vacuum bubbles collide, a new false vacuum bubble can be formed. If the formed bubble is smaller than its gravitational radius, it becomes a black hole to a distant observer. If the bubble's thickness is greater than the gravitational radius, no PBH is formed, and the bubble dissipates.

Also consider second-order phase transitions.

The idea behind this PBH generation mechanism involves producing domain walls that can collapse into PBHs after crossing the Hubble horizon.

There are two ways to create domain walls. The first is based on spontaneous symmetry breaking [27]. The second method is based on the idea of quantum fluctuations of the corresponding field during the stage of cosmological inflation [28].

Let's qualitatively consider the first method of generation. Spontaneous symmetry breaking leads to a change in the potential shape. Once the temperature drops below a certain value, the potential acquires possibly several minima of equal energy. This can lead to the generation of domain walls [29]. In 1993, A. Dolgov proposed a model where, for the first time, cosmological inflation and the Affleck-Dine baryogenesis mechanism were applied to the formation of PBHs, and the resulting mass spectrum in this model is given by the expression:

$$\frac{dN}{dM} = \mu^2 \exp\,[-\gamma \ln^2 \frac{M}{M_0}], \tag{2.8}$$

where γ is a dimensionless constant [30], and $M_0 \sim 10M_\odot$ [31]. The latter value was a theoretical prediction. To date, this is the only mass spectrum that is in good agreement with observed BH masses.

Let's qualitatively consider the second method of generating domain walls, based on quantum fluctuations of the field during the stage of cosmological inflation. In this case, there is no symmetry breaking, yet this mechanism also generates domain walls.

Suppose that at the initial moment, the field is at the top of the potential. During cosmological inflation, due to the large value of the Hubble parameter, the classical motion of the field is "frozen". Thus, the field's equation of motion during inflation, assuming the isotropy of space, is:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) \approx 3H\dot{\phi} + V'(\phi) = 0. \tag{2.9}$$

Classical motion of the field after cosmological inflation will cause the field to "roll down" into one of the potential minima. However, during the stage of cosmological inflation, quantum fluctuations of the field occur, first considered in the work of Andrei Linde [28]. In this work, quantum fluctuations were considered as "random walks"that obey the Fokker-Planck equation. As a result of quantum fluctuations, the field can end up in a region "rolling down" to an alternative potential vacuum. Consequently, in one region of space, the field will "roll down" to one vacuum of the potential, and in another region of space to an alternative vacuum, leading to the formation of a domain wall.

The collapse mechanism of domain walls allows for the generation of PBHs in a wide mass range, however, this mechanism requires fine-tuning of parameters.

Let's briefly consider the process of domain wall formation. The characteristic scale of non-disappearing fluctuations at the stage of inflation is $\mathrm{H}_{\mathrm{inf}}^{-1}$. If a non-disappearing fluctuation forms at time t' during the stage of inflation, after the end of cosmological inflation, the size of this fluctuation will be $e^{\mathrm{N_{inf}} - \mathrm{H_{inf}} t'}$ times larger. Thus, initially, the domain wall exceeds the Hubble horizon in size, hence the domain wall as a whole is initially not causally connected between its "parts". Subsequently, the wall remains at rest relative to Hubble expansion. Over time, the Hubble horizon exceeds the domain wall in size, and it becomes fully enclosed within the cosmological horizon. The wall will then tend to minimize its surface area and contract. Ultimately, if the characteristic thickness of the wall $d < r_g$, where r_g is the Schwarzschild radius corresponding to the domain wall, a black hole is formed. If $d > r_g$, no black hole is formed, and the domain wall will radiate until it fully evaporates.

3 Constraints on Primordial Black Holes

If at least some Primordial Black Holes (PBHs) are stable, they must exist in the Universe; therefore, they should explain at least some of the observational effects of dark matter. The advantage of PBHs over other dark matter candidates lies in the fact that PBHs do not require new physics, except, perhaps, for the new physics involved in the mechanisms of PBH generation. Currently, there are numerous constraints on PBHs as dark matter candidates across a wide range of masses. These constraints arise from the observation of various phenomena, some of which will be discussed in this chapter. A specific limitation on a particular phenomenon, as illustrated in figure 2, is derived from the assumption that PBHs have a monochromatic mass spectrum. However, there are still mass ranges where PBHs are not constrained.

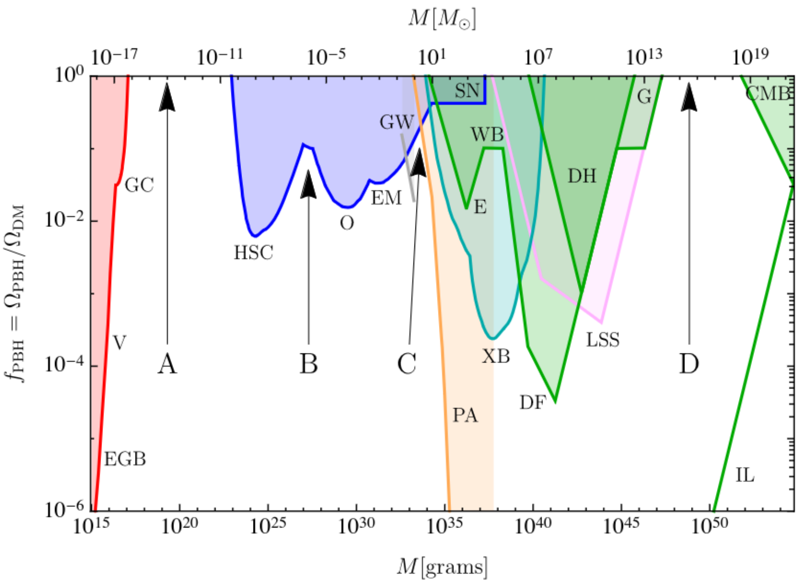

Figure 2 — Contemporary constraints on PBHs as dark matter candidates in the case of a monochromatic mass spectrum, according to [12]. Constraints from evaporation (red), gravitational waves (gray), dynamical effects (green), accretion (blue), cosmic microwave background anisotropies (orange), and large-scale structures (purple).

3.1 Evaporation Constraints

In 1974, Stephen Hawking pointed out the fundamental possibility of black hole evaporation and explosion [32]. A PBH with an initial mass M is assumed to evaporate completely through Hawking radiation over a time scale $\tau \propto M^3$. The rate of black hole evaporation can be expressed as:

$$\dot{M} = -\frac{K_{\text{evp}}}{c^2 M^2}, \tag{3.1}$$

where K_{evp} depends on the mass of the black hole and determines which particles the black hole is capable of emitting.

The age of the Universe imposes a limit on the minimum mass of PBHs that could have survived to the present day. For PBHs with a mass less than

$$M_* \approx 5 \cdot 10^{14}\,\text{g}, \tag{3.2}$$

the evaporation time is less than the age of the Universe [33].

The composition of Hawking radiation from a black hole changes depending on its mass. The mass of the black hole determines the temperature of the radiation, thereby dictating the particles that the black hole can emit. The temperature of the radiation near the event horizon of the black hole is given by the expression:

$$T_{\text{BH}} = \frac{\hbar c^3}{8\pi k G M}. \tag{3.3}$$

In this case, the constraint on the abundance of PBHs follows from the observation of the galactic γ-background. The photon spectrum determines the shape of the constraints. According to [12], the constraint on the abundance of PBHs due to evaporation is as follows:

$$f(M) < 2 \times 10^{-8} \left(\frac{M}{M_*}\right)^{3+\epsilon}, \tag{3.4}$$

where M_* is given by (3.3), and ϵ takes values between 0.1 and 0.4.

There are also other approaches to imposing constraints on PBHs based on their evaporation. For example, in the work [34], the constraint follows from measurements of the annihilation line with an energy of 511 keV.

3.2 Accretion Constraints

It is known that black holes emit radiation as they accrete. PBHs in the early Universe could have emitted very strongly by absorbing nearby gas, and in this case, radiation imposes a strong constraint on the abundance of PBHs. The constraints arise from two considerations: the thermal history of the Universe and another related to the generation of background radiation.

The luminosity corresponding to the accretion rate $\dot{M}$ is expressed as follows:

$$L = \epsilon \dot{M} s^2, \tag{3.5}$$

where ϵ is a dimensionless quantity known as the radiative efficiency, which, generally speaking, is a function of the angular momentum of the black hole [35]. For simplicity, Schwarzschild black holes are considered, for which the radiative efficiency $\epsilon = 0.1$. In the work [36], the authors assumed that the luminosity of a black hole due to accretion cannot exceed the Eddington limit. The Eddington limit is the maximum luminosity that an emitting object can have, assuming it is spherically symmetric. It is given by the expression:

$$L_{edd} = \frac{4\pi c G M m_p}{\sigma_{th}}, \tag{3.6}$$

where m_p is the mass of the proton, σ_{th} is the Thomson cross-section for electron scattering.

The strongest constraints follow from the observation of radiation from black hole accretion in the present day. For example, in the work [37], the accretion of gas by a population of PBHs in the Milky Way is modeled, and the observed radiation is compared to the predictions of the model.

Also, the interaction of PBHs with interstellar matter should lead to a strong flux of X-ray radiation in galaxies. In the work [38], data on observed X-ray radiation are used to establish constraints on the abundance of PBHs.

3.3 Dynamical Constraints

Observation of certain objects in the Universe can impose constraints on PBHs in the Universe. For example, observing a specific set of neutron stars or white dwarfs places constraints on the PBH population [39; 40].

If PBHs have a density ρ and velocity dispersion v, while the surrounding objects have a mass M_c characteristic size R_c, velocity dispersion v_c, and lifetime t_L, then the constraint on the abundance of PBHs is as follows:

$$f(M) < \begin{cases} M_c v/(G M \rho t_L R_c) & [M < M_c(v/v_c)] \\ M_c/(\rho v_c t_L R_c^2) & [M_c(v/v_c) < M < M_c(v/v_c)^3]] \\ M v_c^2/(\rho R_c^2 v^3 t_L) \exp\left[(M/M_c)(v_c/V)^3\right] & [M_c(v/v_c)^3 < M] \end{cases} \tag{3.7}$$

Wide binary systems are considered to be the most sensitive to the presence of PBHs nearby [41; 42]. When compact massive objects pass through a wide binary system, it is highly likely that one of the bodies in the binary system can be imparted with the energy needed to leave its orbit in the binary system, leading to the disruption of the binary system.

One of the known and stringent constraints on the abundance of PBHs comes from the observation of a star cluster within the dwarf galaxy Eridanus II [43] (the area marked with the letter "E"in figure 2). It is worth noting that the author in his work does not specifically consider PBHs, but looks at Massive Astrophysical Compact Halo Objects (MACHOs). MACHOs, in turn, are not necessarily PBHs, but nonetheless, the constraint obtained for MACHOs can be applied to PBHs as well, due to their compact nature.

The observation of the star cluster in the dwarf galaxy Eridanus II imposes a constraint on PBHs. Physically, this constraint is based on gravitational dynamical friction. There is an approach to considering dynamical friction using the Fokker-Planck equation. In this case, its solution is called diffusion coefficients, which represent the time-averaged rate of energy exchange. In the work [43], the author uses diffusion coefficients obtained using the Fokker-Planck equation. The derivation of the diffusion coefficients can be found, for example, in [44].

The change in the total energy of the star cluster per unit mass per unit time is given by the expression:

$$\frac{\dot{E}_{\rm tot}}{M_{\rm star\ cluster}} = \frac{D[\Delta E]}{M_{\rm star\ cluster}} = \frac{1}{2}\,D[(\Delta v^2)] + vD[\Delta v_{||}] = \tag{3.8}$$
$$= \frac{2\sqrt{2}\pi G^2 \rho M \ln\Lambda}{\sigma}\,\frac{\mathrm{erf}(X)}{X} - \frac{4\pi G^2 \rho v (m+M)\ln\Lambda}{\sigma^2}\,\mathrm{G}(X).$$

In the expression (3.8) $X = v/\sqrt{2}\sigma$, $\mathrm{G}(X) = \frac{1}{2X^2}\left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}}\,e^{-X^2}\right]$, σ is the velocity dispersion of PBHs, ρ is the density of PBHs in the galaxy, M is the mass of PBHs, m is the characteristic mass of a star, v is the velocity of PBHs relative to stars, $\ln\Lambda \approx 10$ is the Coulomb logarithm.

The potential energy per unit mass for a star cluster in the Eridanus II galaxy is given by the expression

$$\frac{U}{M_{\rm star\ cluster}} = \mathrm{const} + \beta G \rho r_{\rm h}^2 - \alpha \frac{GM_*}{r_{\rm h}}, \tag{3.9}$$

where $\alpha \approx 0.36$ and $\beta \approx 7.2$ as shown in [45; 46], and $M_* \sim 10^5\,M_\odot$ is the stellar mass of the cluster inside Eridanus II. Further, replace ρ with the product $\rho_{\rm DM} \cdot f_{\rm DM}$. Next, using the virial theorem

$$E_{\rm tot} = \frac{1}{2}\,U, \tag{3.10}$$

and equation (3.9), it is possible to derive the evolution equation for $r_{\rm h}$.

Omitting transformations:

$$\dot{r}_{\rm h}\left(\alpha\frac{M_*}{\rho_{\rm DM} r_{\rm h}^2} + 2\beta r_{\rm h}\right) = \frac{4\sqrt{2}\pi G f_{\rm DM}\ln\Lambda}{\sigma}\,M. \tag{3.11}$$

Integrating equation (3.11), using the observed parameters of the star cluster, provides a constraint on the abundance of PBHs. There are three observed parameters: $r_{\rm h}$, M_* and σ. $r_{\rm h}$ is the radius at which half of the luminosity is emitted, M_* is the stellar mass of the cluster, and σ is the velocity dispersion of objects in the galaxy.

The star cluster in the Eridanus II galaxy excludes PBHs with a mass of $M \sim 10^3\, M_\odot$ as the main component of dark matter.

The overview presented above on deriving constraints on the abundance of PBHs by observing the star cluster in the dwarf galaxy Eridanus II is based on the assumption that PBHs have a monochromatic mass spectrum. However, PBHs may be born clustered [47]. In the case of replacing PBHs with a monochromatic spectrum with clusters, this constraint needs to be revisited.

3.4 Gravitational-Wave Constraints

The PBH population should emit gravitational waves, and particularly interesting sources would be binary PBH systems. Therefore, the frequency of gravitational wave detections indicates a certain population of black holes. In turn, the non-observation of gravitational waves places a constraint on the fraction of PBHs in dark matter.

As shown in the work [48], early results from LIGO/Virgo placed strong constraints on the abundance of PBHs in the mass range of $0.5 - 30 M_\odot$. Currently, this constraint has been revisited using both LIGO/Virgo data [49] and pulsar observations [50].

A more direct constraint is the rate of gravitational-wave events observed by LIGO/Virgo. An important question is whether binary black hole systems formed in the early Universe, as suggested by Sasaki et al., or after the formation of galaxies, as suggested by Bird et al.

Recently, Bœhm et al. [51] stated that binary PBH systems formed early on merge long before LIGO/Virgo observations, which weakens the constraints and may even remove them altogether. Also, in the work [52], it is claimed that the constraints can be weakened or even lifted if PBHs are in clusters.

Another type of gravitational-wave constraint on the abundance of PBHs arises from large second-order tensor perturbations generated by scalar perturbations that produce PBHs [53].

The reliability of LIGO/Virgo constraints on PBHs, with masses on the order of $10M_{\odot}$, depends on the accuracy with which the formation of binary PBH systems in the early Universe can be described.

4 Evidence of Observing PBHs

4.1 Lensing Evidence

Detecting a cosmological distribution of compact bodies through quasar microlensing is a challenging task. The complexity arose from the fact that if compact bodies constitute a significant portion of dark matter, it would create a caustic network, leading to unstable changes in quasar light as it passes through a complex amplification scheme. The resulting light curve would not have an easily identifiable shape, making it difficult to distinguish from internal variations of the quasar's accretion disk. There are precise characteristics associated with caustic crossings that have been observed and cannot be linked to internal variations of quasar accretion disks [54].

In 1993, Hawkins [55] presented the first evidence of a cosmological distribution of PBHs based on microlensing of quasar light curves.

Consider one of the areas where compact objects are searched for through microlensing. The work [56] suggested searching for compact objects in the halo of the Milky Way by microlensing stars in the Magellanic Clouds. The proposal was based on the fact that the half-crossing time of the brightness curve depends on three quantities: the mass of the lensing body M_D, the inverse distance $1/d = 1/d_{OD} + 1/d_{DS}$, where d_{OD} is the distance from the observer to the lensing body, and d_{DS} is the distance from the lensing body to the source, and the velocity of the lens v. This gives:

$$\Delta t_{1/2} = \frac{\sqrt{4GM_D d}}{v}. \tag{4.1}$$

Microlensing results [57; 58] indicate that compact objects of near-Solar masses cannot dominate the composition of the galaxy's dark matter. It is also claimed that such objects cannot make up more than 40% of the halo mass.

4.2 LIGO/Virgo Evidence

Following the detection of gravitational waves (GWs), Bird [59] and Garcia-Bellido [60] claimed that the expected merger rates of binary PBH systems, formed in late times in compact halos, are compatible with LIGO/Virgo analysis results if they include all of dark matter.

Mergers of binary PBH systems formed before the end of the radiation-dominated stage, when neighboring PBHs are sufficiently close to form pairs, occur much more frequently, meaning that the dark matter fraction in PBHs with a mass of $30M_{\odot}$ could constitute no more than 1%. The evolution and merger of early binary PBH systems have been thoroughly studied using $N - body$ simulations [48]. It was found that the rates of such evolution are significantly suppressed if PBHs contribute significantly to dark matter, due to the disruption of binary systems by nearby PBHs, early formation of PBH clusters, and dark matter inhomogeneities. Other groups have obtained similar results using both analytical and numerical methods [61], but without accounting for all these effects.

4.3 Clusters of PBHs

The main difference between dark matter particles and PBHs lies in the greater mass of the latter. This leads to significant Poisson fluctuations in the spatial distribution of PBHs and ultimately results in the formation of gravitationally-bound PBH clusters. This was first pointed out by Meszaros [62]. The idea of PBH clusters was developed further in joint work by Khlopov, Belotsky, Eroshenko, Rubin, and others [47]. The effects of Poisson fluctuations are crucial for sufficiently massive PBHs and have several important implications:

- Some constraints on PBHs, especially dynamic ones, can be weakened;
- Clustering affects the rate of PBH mergers and thereby provides a connection between dark matter and gravitational wave (GW) observations;
- Clustering suggests the formation of nonlinear structures at higher redshifts than in standard cosmology, which has implications for the cosmic background and observations of high-luminosity galaxies;

Clustering could explain the minimum size and large mass-to-light ratio of ultra-faint dwarf galaxies, and identifying a subset of PBH clusters with them means we can consider clustering as positive evidence for the existence of PBHs. The Coulomb effect of individual PBHs, which can be viewed as a specific case of the Poisson effect, is also important.

The formation of Poisson-induced PBH clusters has been studied numerically using $N-body$ simulations [63] and analytically using the Press-Schechter formalism and the theory of spherical collapse [64].

5 Conclusion

This paper presents a review and analysis of some existing mechanisms of PBH formation, constraints on the abundance of PBHs, and mentions some evidence in favor of the existence of PBHs.

The current constraints on the abundance of PBHs do not rule out PBHs as a principal candidate for dark matter — dark matter could entirely consist of PBHs of various masses. Thus, PBHs remain a viable candidate for dark matter.

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