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THESIS:

Primordial black holes as dark matter candidates

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Contents

1 Introduction

A black hole (BH) is an area in space and time possessing a gravitational field so strong that not even photons can escape from it [1]. The boundary beyond which objects become causally disconnected from us (the observer) is called the event horizon of a BH. If any object is within this boundary, it will only move inside the black hole. While an object from the external space can cross the event horizon, it cannot return. In other words, BHs are spherical objects with mass M (in the Schwarzschild model) [2], enclosed within a gravitational radius r_g (or Schwarzschild radius):

$$r_g = \frac{2GM}{c^2}. \tag{1.1}$$

Black holes are classified into two types: astrophysical and primordial. Astrophysical black holes result from the collapse of massive stars with a mass of around 10 solar masses, or from the collapse of large star clusters at the centers of galaxies. However, in 1966, Zel'dovich and Novikov proposed a black hole formation model where formation occurs at the early stages of Universe development, before the formation of large-scale structures [3]. Such BHs are called primordial black holes (PBHs). Historically, this was the first model for the formation of PBHs, but the authors did not further develop the idea because their calculations indicated that accretion significantly increases the mass of PBHs by many orders of magnitude, leading to an excess of supermassive black holes, which is not observed. Later, B. Carr and S. Hawking showed [4] that significant mass increase due to accretion does not occur, thereby renewing interest in the existence of PBHs. For a long time, PBHs were considered hypothetical objects, the absence of information about which, nonetheless, allowed for constraints on physical processes in the early Universe and on the effects of inhomogeneity. However, in recent years, attention to PBHs has become very wide, as there have been indications that some phenomena can only be explained by PBHs [5]. However, this was not always the case. B. Carr, who is known today for his reviews on PBHs, did not show such keen interest in the PBH topic in the 1990s. At that time, he even believed that only M. Khlopov was truly serious about the PBH topic at that moment.

The possibility of PBH formation is also influenced by the equation of state of matter in the Universe. The influence of the ω parameter of the equation of state of matter in the form $p = \omega\rho$ on the probability of PBH formation is investigated in [6]. The probability of PBH formation of mass M:

$$P(M) \propto \exp\left(-\frac{\omega^2}{2\epsilon^2}\right). \tag{1.2}$$

There is an upper limit on the mass of a PBH formed at time t: the mass of the BH cannot exceed the mass of the Hubble horizon at the moment of its formation:

$$M \sim \frac{c^3 t}{G} \sim 10^{15}\left(\frac{t}{10^{-23}\text{s.}}\right)\text{y.} \tag{1.3}$$

Thus, PBHs could have a mass $M_{Pl} \sim 10^{-5}$g if they were formed at the Planck time ($t \sim 10^{-43}$s.), $1M_\odot$ if they were formed during the QCD epoch ($t \sim 10^{-5}$s.) and $10^5 M_\odot$ if they were formed at $t \sim 1$s. Consequently, PBHs can cover a wide range of masses and are the only ones that can have a mass less than the mass of the Sun.

Various mechanisms of formation and subsequent evolution of PBHs are considered in works such as [5; 7–9].

1.1 PBH as dark matter

Primordial black holes (PBHs) hold particular interest for cosmology, especially those with a mass exceeding 10^{15} grams, which are almost unaffected by Hawking radiation. Since dark matter remains unexplained, there is active search for candidates that could constitute this hidden mass. Astrophysical BHs cannot account for all the dark matter, as they form from baryons and are subject to the well-known constraint of primordial nucleosynthesis, according to which baryons contribute no more than 5% of the critical density [10].

Hence, the interest in PBHs stems from their potential role as candidates for dark matter, since they were formed during the radiation-dominated era before primordial nucleosynthesis, circumventing this limitation. Therefore, PBHs are considered a form of non-baryonic dark matter, behaving like any other form of cold dark matter, despite their mass.

Recently, numerous constraints [11] have been examined on the fraction of dark matter $f(M)$ in PBHs of mass M. These constraints suggest that there are only a few mass ranges where f can be significant: a small range ($10^{17} - 10^{23}$ g), a medium range ($10 - 10^{5} M_{\odot}$), and a wide range ($M > 10^{11} M_{\odot}$).

It should be emphasized that non-evaporating PBHs could be of great cosmological interest, even if they constitute only a small fraction of dark matter. For instance, they could play a certain role in the formation of supermassive BHs in galaxy nuclei. It is also conceivable that dark matter includes a mixture of PBHs and weakly interacting massive particles (WIMPs) [12].

2 Formation of PBHs

The condition for the formation of PBHs in the early Universe is the predominance of gravitational energy over the forces of internal pressure (i.e., dominance over internal energy). Considering a region of the Universe with size R, the gravitational energy can be expressed as $U_g \sim GM^2/R \sim G\rho^2 R^5$, where ρ is the density of the medium. The internal energy of relativistic matter can be written as $U_p = pV$, where $p = c^2\rho/3$. Hence, $U_p \sim c^2\rho R^3$. Thus, the condition for the formation of PBHs can be written as:

$$\frac{G\rho R^2}{c^2} > 1. \tag{2.1}$$

2.1 Primordial Inhomogeneities

This mechanism of PBH formation is based on the collapse of primordial inhomogeneities in the hot plasma and arises within the framework of the standard Big Bang cosmology [13]. Consider a region of the Universe with radius R. The corresponding gravitational energy is on the order of:

$$\Omega \sim -\rho^2 R^5, \tag{2.2}$$

and the kinetic energy of this region's expansion is on the order of:

$$T \sim \rho R^3 \dot{R}^2, \tag{2.3}$$

where ρ is the energy density. In a universe dominated by radiation, pressure and energy density are proportional to R^{-4}, as the state equation parameter $\omega = 1/3$, and the state equation itself is:

$$p = \omega\rho. \tag{2.4}$$

If the matter density is sufficiently high, gravitational forces can overcome the kinetic energy of expansion and pressure forces. As a result, in such a region of the Universe, expansion stops. To overcome pressure forces, the gravitational energy must be greater than the internal energy, which for $p = 1/3\rho$ is on the order of $U \sim \rho R^3$. Thus, a necessary condition for collapse is:

$$\rho R^2 >\sim 1. \tag{2.5}$$

A drawback of this model is that the spectrum of masses of the generated BHs is close to monochromatic, meaning this model cannot explain the existence of BHs of various masses. Also, within this model, it is impossible to generate clusters of BHs. For this reason, this model also cannot explain the rate of BH mergers observed by LIGO/Virgo [14]. Historically, this is the earliest mechanism of generation and does not require additional assumptions beyond the standard Big Bang theory.

2.2 Collapse at the Quantum Chromodynamics Phase Transition

At one time, it was believed that the quantum chromodynamics (QCD) phase transition at the time 10^{-5} s could be first-order. This would mean that quark-gluon plasma and hadrons could coexist. Moreover, cosmic expansion would occur at a constant temperature due to the conversion of quark-gluon plasma into hadrons. During this, the speed of sound decreases, and the effective pressure decreases, significantly lowering the δ_c threshold for collapse. The formation of PBHs during first-order QCD phase transitions was first proposed by Crawford and Schramm [15], and later reconsidered by Jedamzik [16]. Dolgov and Silk also proposed a model of baryonic isocurvature fluctuations as a mechanism for generating PBHs [17]. It is currently considered unlikely that the QCD phase transition is a first-order transition, but some softening in the equation of state can still be expected.

Recently, Byrnes et al. [18] discussed how this softening could lead to a significant jump in the mass function. The mass of PBHs forming during the QCD epoch is:

$$M \approx 0.9 \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{10}\right)^{-1/2} \left(\frac{\xi}{5}\right)^2 M_\odot, \tag{2.6}$$

where g_* is normalized, and $\xi = M_{Pl}/(k_B T) \approx 5$ – the ratio of the proton mass to the temperature of the QCD phase transition. The expression for the mass of PBHs is close to the Chandrasekhar mass. In this case, the QCD phase transition leads to the formation of PBHs with masses close to those of stars. As a result, observed stars and unobserved PBHs have similar masses.

It should be mentioned that a mechanism has been developed that combines cosmological inflation and quark confinement to produce PBHs [19]. In this scenario, PBHs are formed with a mass less than the value obtained by formula (2.6).

2.3 Collapse of Scalar Fields

Supersymmetric extensions of the standard model [20; 21] generally predict that in the early Universe, a scalar condensate can form and fragment into Q-balls before decaying. If Q-balls dominate the energy density for some time, relatively large fluctuations in their concentration can lead to the formation of PBHs. Other scalar fields not associated with supersymmetry can play a similar role. For an arbitrary charged scalar field, this mechanism can lead to the formation of black holes across the entire mass range allowed by observational constraints, with enough abundance to explain all dark matter in certain parameter ranges. In the case of supersymmetry, the mass range is limited to a maximum of 10^{23} g.

The work [22] considered the gravitational instability of a spatially homogeneous relativistic scalar field taking into account self-interaction. It was shown that this instability is similar to the Jeans instability and can lead to the formation of PBHs.

The inflationary stage of the Universe's development is usually explained by the dynamics of a scalar field. After the end of cosmological inflation, a phase of non-relativistic matter dominance may briefly occur, inevitably leading to the formation of PBHs [23].

2.4 Collapse of Domain Walls

A domain wall is a non-trivial field configuration connecting different vacuums of the potential [24]. An example of a domain wall in the form of a hyperbolic tangent for the Higgs field is presented in Figure 1. This mechanism is driven by second-order phase transitions. For its realization, the potential of the corresponding field must have at least two vacuums of equal energy.

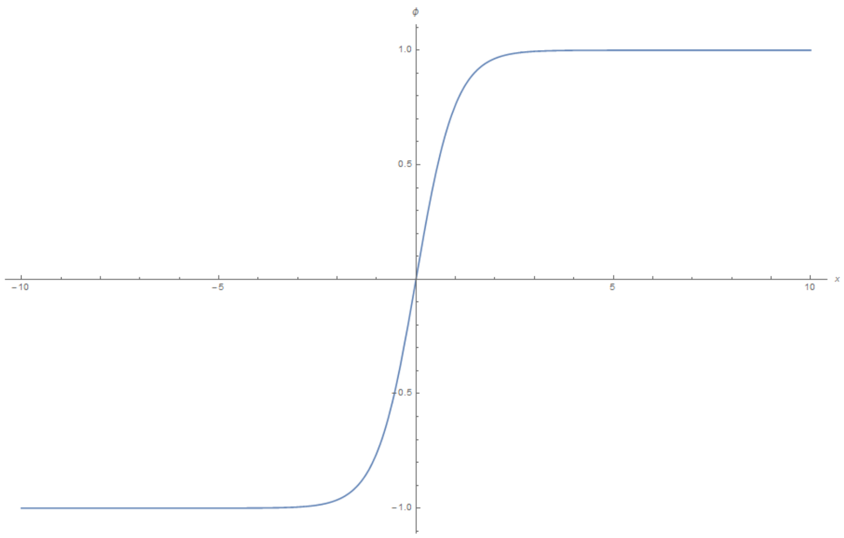

Figure 1 — A domain wall in the form of a hyperbolic tangent for the Higgs field.

Consider first-order phase transitions of a scalar field. The concept of first-order phase transitions as a means of generating PBHs was first proposed in [25]. For this mechanism to be realized, the field's potential must contain at least two minima, one of which must be false. It is assumed that initially, the field is in the false vacuum, and then as a result of field tunneling in one region of space, the field will have a value corresponding to one vacuum, and in another region of space to another vacuum. These regions are called bubbles. In this case, the free energy of a bubble consists of two parts - surface and volume. Let us denote the surface energy density by μ and the difference in potential values at the minima by $\Delta V = E(\phi_0) - E(\phi_1)$, where ϕ_0 corresponds to the true vacuum, and ϕ_1 to the false one. Then the free energy of a bubble with radius R and surface energy density μ can be written as

$$F(R) = 4\pi R^2 \mu - \frac{4\pi}{3} R^3 \Delta V. \tag{2.7}$$

The dependency (2.7) has a maximum at the point $R_{cr} = 2\mu/\Delta V$, after reaching which it becomes energetically favorable for the bubble to expand indefinitely. The expansion of true vacuum bubbles into the false vacuum region leads to the conversion of potential energy of the false vacuum into the kinetic energy of the walls. This transition ensures relativistic speeds of expansion.

When a pair of true vacuum bubbles collide, a new false vacuum bubble can be formed. If the formed bubble is smaller than its gravitational radius, it becomes a black hole to a distant observer. If the bubble's thickness is greater than the gravitational radius, no PBH is formed, and the bubble dissipates.

Also consider second-order phase transitions.

The idea behind this PBH generation mechanism involves producing domain walls that can collapse into PBHs after crossing the Hubble horizon.

There are two ways to create domain walls. The first is based on spontaneous symmetry breaking [26]. The second method is based on the idea of quantum fluctuations of the corresponding field during the stage of cosmological inflation [27].

Let's qualitatively consider the first method of generation. Spontaneous symmetry breaking leads to a change in the potential shape. Once the temperature drops below a certain value, the potential acquires possibly several minima of equal energy. This can lead to the generation of domain walls [28]. In 1993, A. Dolgov proposed a model where, for the first time, cosmological inflation and the Affleck-Dine baryogenesis mechanism were applied to the formation of PBHs, and the resulting mass spectrum in this model is given by the expression:

$$\frac{dN}{dM} = \mu^2 \exp{[-\gamma \ln^2 \frac{M}{M_0}]}, \tag{2.8}$$

where γ is a dimensionless constant [29], and $M_0 \sim 10 M_\odot$ [30]. The latter value was a theoretical prediction. To date, this is the only mass spectrum that is in good agreement with observed BH masses.

Let's qualitatively consider the second method of generating domain walls, based on quantum fluctuations of the field during the stage of cosmological inflation. In this case, there is no symmetry breaking, yet this mechanism also generates domain walls.

Suppose that at the initial moment, the field is at the top of the potential. During cosmological inflation, due to the large value of the Hubble parameter, the classical motion of the field is "frozen". Thus, the field's equation of motion during inflation, assuming the isotropy of space, is:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) \approx 3H\dot{\phi} + V'(\phi) = 0. \tag{2.9}$$

Classical motion of the field after cosmological inflation will cause the field to "roll down" into one of the potential minima. However, during the stage of cosmological inflation, quantum fluctuations of the field occur, first considered in the work of Andrei Linde [27]. In this work, quantum fluctuations were considered as "random walks"that obey the Fokker-Planck equation. As a result of quantum fluctuations, the field can end up in a region "rolling down" to an alternative potential vacuum. Consequently, in one region of space, the field will "roll down" to one vacuum of the potential, and in another region of space to an alternative vacuum, leading to the formation of a domain wall.

The collapse mechanism of domain walls allows for the generation of PBHs in a wide mass range, however, this mechanism requires fine-tuning of parameters.

Let's briefly consider the process of domain wall formation. The characteristic scale of non-disappearing fluctuations at the stage of inflation is $\mathrm{H}_{\mathrm{inf}}^{-1}$. If a non-disappearing fluctuation forms at time t' during the stage of inflation, after the end of cosmological inflation, the size of this fluctuation will be $e^{\mathrm{N}_{\mathrm{inf}}-\mathrm{H}_{\mathrm{inf}}t'}$ times larger. Thus, initially, the domain wall exceeds the Hubble horizon in size, hence the domain wall as a whole is initially not causally connected between its "parts". Subsequently, the wall remains at rest relative to Hubble expansion. Over time, the Hubble horizon exceeds the domain wall in size, and it becomes fully enclosed within the cosmological horizon. The wall will then tend to minimize its surface area and contract. Ultimately, if the characteristic thickness of the wall $d < r_g$, where r_g is the Schwarzschild radius corresponding to the domain wall, a black hole is formed. If $d > r_g$, no black hole is formed, and the domain wall will radiate until it fully evaporates.

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