Homotopically stable particles in the theory of superstrings

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The development of the theory of superstrings opens the possibility of constructing a finite theory of elementary particles and their interactions based on a unified foundation. This theory, in principle, allows one to construct both the gauge symmetry of grand unification and the number of fermion generations. Both of these important consequences of the theory result from the presence of topologically nontrivial string configurations in the compactified dimensions which are general characteristic features of all realistic models of the theory of superstrings. In this article it is shown that the generally accepted scheme of compactification implies that a new type of stable objects appears in the theory. The existence of these objects appears to be an unavoidable consequence of the models of superstring theory, just as models of grand unification inevitably lead to the existence of magnetic monopoles.

In realistic models of superstrings the symmetry of grand unification is broken by compactification into the nonsimply connected manifolds of Calabi-Yau[1] or orbifolds[2] as the result of the formation of nontrivial vacuum expectation values of the Wilson operators $W(C) = \mathrm{Tr}P\exp i\oint A^m dx_m$ on uncontractable contours C.[3] Such manifolds can be constructed by taking the quotient of a manifold M, which has the trivial homotopy group $\pi_1(M) = 0$, by a discrete group G which acts freely. At the same time this leads to a division of the Euler characteristic by the dimension of the group which results in a phenomenologically acceptable number of generations.[1]

The presence of the nontrivial homotopy group $\pi_1(M/G) \neq 0$ causes the appearance of new states of the closed string, which is related to the nontrivial topological sector

$$x(\tau, 2\pi) = gx(\tau, 0), \quad g \in G, \quad x \in M. \tag{1}$$

We shall show how such states appear in the simplest example of a nonsimply connected manifold, the circle S^1, which can be considered as the quotient of R^1 by the discrete group of shifts Z with generator $2\pi r$ where r is the radius of the circle. In this case there are nontrivial homotopic configurations of closed strings

$$x(\tau, 2\pi) = g^n x(\tau, 0) = x(\tau, 0) + 2\pi nr, \quad n \in Z. \tag{2}$$

These "topological string solitons" (see Ref. 4) have a conserved homotopic quantum number n, which corresponds to the winding number on S^1 (see Fig. 1). In interaction processes of strings along the fixed compact manifold the total homotopy number is conserved (see Fig. 2). It is easy to see (see Ref. 4) that such solitons have the following oscillator expansion:

$$x(\tau, \sigma) = x(\tau) + 2nr\sigma + 2\alpha' p\tau + \frac{1}{2}\sum_{m\neq 0}\Big(\frac{a_m}{m}\exp[-2in(\tau+\sigma)] + \frac{\tilde{a}_m}{m}\exp[-2in(\tau-\sigma)]\Big), \tag{3}$$

where $\alpha' \sim (g^2 m_{Pl}{}^2)^{-1}$ is the string tension, g is the gauge constant, and the additional term $2nr\sigma$ is related to the condition (2). The presence of this term leads to a modification of the Virasoro operators L_0 and $\tilde{L}_0$ which determine the mass spectrum. Calculating them by standard rules, we obtain the mass condition

$$-(2\alpha' p)^2 = 4n^2r^2 + \frac{1}{2}\sum_m (:a_m a_{-m} + \tilde{a}_m \tilde{a}_{-m}:),$$

i.e.,

$$M_n{}^2 = M_0{}^2 + n^2r^2(\alpha')^{-2}, \tag{4}$$

where M_0 is the mass of the corresponding state in the topologically trivial sector with $n = 0$.

In the case of the torus $S^1_1 \times \ldots \times S^1_k$ the relation (4) generalizes to

$$M^2_{n_1,\ldots,n_k} = M_0{}^2 + \sum_{i=1}^{k} n_i{}^2 (r_i/\alpha')^2, \tag{4a}$$

when n_i is the winding number on the circles S^1_i and r_i is the corresponding radius. Let us note now that in superstring theory[4] $M_0^2 \geqslant 0$ for all states, so tachyons are absent in the theory, from which it immediately follows that $M_n \sim nr/\alpha' \sim nm_{Pl}(r/r_{Pl})$, where r is the radius of the compactification and m_{Pl} and r_{Pl} are the Planck mass and radius.

In the case of Calabi-Yau manifolds, which were investigated in the construction of realistic models,[1] the discrete group G, in contrast to the group Z, has a finite number of elements. In fact, since the Euler characteristic is $\chi(M/G) = \chi(M)/\mathrm{dim}G$, the groups of interest must be finite (of the type Z_m or $Z_m \times Z_m$). This implies that the charges of the solitons which belong to this group G form a finite set. For example, in the case of the group Z_m the charges of the soliton-antisoliton pair have the form $(k, m-k)$. In the general case of a Calabi-Yau manifold it is impossible in analogy with the case of the torus to obtain explicit mass formulas of the type (4), although for orbifolds[2] this problem can be solved exactly. Thus for orbifolds with the group Z_N the mass of a soliton with topological charge n is equal to [compare with Eq. (4)]

$$M_n{}^2 = M_0{}^2 + \left(\frac{r}{\alpha'}\right)^2 \frac{n^2(N-n)^2}{N^2}, \qquad n = 1, \ldots, N-1.$$

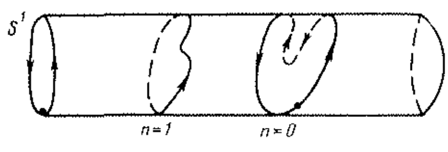

FIG. 1.

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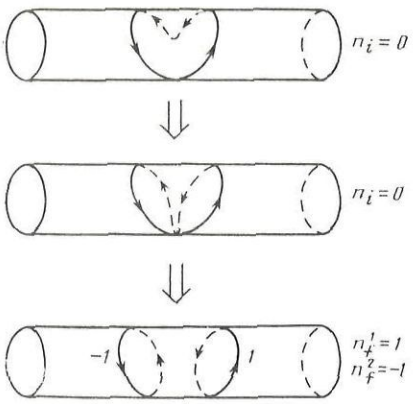

FIG. 2. Creation process for a soliton-antisoliton with $n = \pm 1$.

From this it is obvious that $M_{N-n} = M_n$ as it might be because the soliton with charge $N - n$ is the antisoliton for the soliton with charge n.

If the radius of compactification is $r > r_{P1}/g$, then the mass of the soliton exceeds m_{P1} and the soliton is unstable relative to transitions into a black hole. The probability of such a transition depends on the initial conditions of the formation of the state. The probability works out to be of the order of magnitude of $W \sim (1/R)(R_g/R)^3$ where $R_g = 2M/m_{P1}^2$ is the gravitational radius of a state of mass M, and R is the characteristic dimension of the wave packet of the soliton. If a black hole with a mass m_{P1} is, as was assumed in Ref. (5), a (meta)stable particle—maximon, then the evaporation of the black hole of mass M ends with the formation of a maximon with mass m_{P1}. If maximons do not exist, then the evaporation of the black hole leads to a complete transition of the mass M into relativistic decay products.[6]

A radius of compactification $r > r_{P1}/g$ corresponds to soliton masses which do not exceed m_{P1}. Within the framework of the theory of superstrings the masses of black holes must be larger than m_{P1} (see Ref. 7); phenomenologically, the possibility of existence of black holes with mass smaller than m_{P1} was discussed in Ref. 8. For this reason, solitons are stable relative to a transition to black holes. In this case instability of solitons could be induced only by quantum fluctuations of the topology of the compact space, ideas regarding which do not exist at the present time.

The homotopically stable particles are "sterile" with respect to the charges of strong, weak, and electromagnetic interactions and participate only in the gravitational interaction. Therefore the possibilities of verifying the existence of these particles are related exclusively to their cosmological manifestations. The analysis of these manifestations which will form the subject of a separate article, will permit establishment of acceptable properties of solitons and thereby accomplishment of an indirect verification of the fundamental ideas of superstring theory. Such an analysis will be based on general methods of verifying predictions of elementary-particle theory in astrophysical observations (see the survey in Ref. 9).

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