Can the *spin-charge-family* theory be related to *string theories* if the point fields in the *spin-charge-family* theory are extended to strings? :

A short overview of the internal degrees of freedom of fermions and boson, used in the spin-charge-family theory will be presented

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** Some publications:

- *Phys. Lett.* **B 292**, 25-29 (**1992**), *J. Math. Phys.* **34**, 3731-3745 (**1993**), *Mod. Phys. Lett.* **A 10**, 587-595 (**1995**),
- *Phys. Rev.* **D 62** (04010-14) (**2000**), *Phys. Lett.* **B 633** (**2006**) 771-775, **B 644** (**2007**) 198-202, **B** (**2008**) 110.1016, *JHEP* **04** (**2014**) 165, *Fortschritte Der Physik-Progress in Physics*, **(2017)**1700046, J. of Math. Phys. **43** (2002), (5782-5803), hep-th/0111257, *J. of Math. Phys.* **44** (2003) 4817-4827, hep-th/0303224, *Jour. of High Energy Phys.* **04** (2014)165,doi:10.1007, [http://arxiv.org/abs/1212.2362v3].
- Rev. Artile in **Progress in Particle and Nuclear Physics**, vol.121(2021)103890, **http://doi.org/10.1016.j.ppnp.2021.103890**
- Nucl. Phys. B, j.nuclphysb.2023.116326, Symmetry 2023,15,818-12-V2 94818, https:doi.org/10.3390/sym15040818

o To represent (explain) the internal spaces of fermions and bosons usually the groups are used.
o The internal space of fermions is in this case described by the fundamental representations of the groups,
o the internal space of bosons is correspondingly described by the adjoint representations of the groups.

In theories assuming more than the observed $d = (3+1)$, that is $d > (3+1)$,
with one time and $d-1$ space dimensions, (and the Lorentz symmetry in all dimensions),
fermions carry two kinds of half integer spins in $d = (3+1)$, $(\pm\frac{i}{2}, \pm\frac{1}{2})$, and also half integer spins, $\pm\frac{1}{2}$, in all other dimensions,
bosons carry two kinds of integer spins in $d = (3+1)$, $(\pm i, 0), (\pm 1, 0)$ and also integer spins, $(\pm 1, 0)$ also in all other dimensions.

o Can the internal spaces of fermions and bosons be treated in an equivalent way as the ordinary space?

o Can we replace the group theories in the way so that we do not need to invent groups for each observed properties of fermions and bosons?: In the way as the ordinary space is automatically enlarged with d,

o having in mind that the large enough orthogonal group includes all the other groups?

Something like that string theories do.

o Why do we need to understand internal spaces of fermions and bosons?

- o Do we understand internal spaces of fermions and bosons in an unique way?
- o Do we understand why fermions appear in families while bosons do not?
- o Do we understand the postulates of the second quantized fields; why fermion fields anti-commute while boson fields commute?
- Do we understand why fermions and bosons interact?
- Can we understand our cosmos if we do not understand the appearance of fermions and bosons?

And many other questions

- o In a long series of works the author, together with collaborators, has found the phenomenological success with the model named the *spin-charge-family* theory with the properties: The creation and annihilation operators for fermions and bosons fields are described as tensor products of the Clifford odd (for fermions) and the Clifford even (for bosons) "basis vectors" and basis in ordinary space, explaining the second quantization postulates.
- o The theory offers the explanation for the observed properties of fermion and bosons and for several cosmological observations.
- o The number of creation and annihilation operators for fermions and bosons is the same, manifesting correspondingly a kind of supersymmetry.

- o This workshop should present the properties of the creation and annihilation operators if extending the point fermions and bosons into strings, expecting that this theory offers the low energy limit for the *string theory*.
- o We are making the first steps in this study: We try to reproduce the internal wave functions for the boson fields, represented in the "string theories" with the tensor products of the left and right movers, with the algebraic products of the Clifford odd "basis vectors" and their Hermitian conjugated partners.

- o Let us start with a brief introduction into the description of the internal spaces of fermions and bosons with the Clifford odd and even algebra, respectively, starting with the Grassmann algebra.

o Let us notice properties of the Grassmann algebra first.

- In Grassmann d-dimensional space there are d anti-commuting (operators) θ^a, and d anti-commuting operators which are derivatives with respect to θ^a, $\frac{\partial}{\partial\theta_a}$,

$$\{\theta^a, \theta^b\}_+ = 0\,, \qquad \{\frac{\partial}{\partial\theta_a}, \frac{\partial}{\partial\theta_b}\}_+ = 0\,,$$

$$\{\theta_a, \frac{\partial}{\partial\theta_b}\}_+ = \delta_{ab}\,,\ (a,b) = (0,1,2,3,5,\cdots,d)\,.$$

θ^a's and p^θ_a's, $p^\theta_a = \frac{\partial}{\partial\theta_a}$

have the property

$$(\theta^a)^\dagger = \eta^{aa}\frac{\partial}{\partial\theta_a}\,,$$

with $\eta^{ab} = diag\{1,-1,-1,\cdots,-1\}$

Grassmann algebra is offering together $2\cdot 2^d$ operators.

J. of Math. Phys. **34** (1993) 3731

- o There are **two kinds of the Clifford algebra objects,** γ^a **and** $\tilde{\gamma}^a$**, in any d, expressible with** θ_a **and** $\frac{\partial}{\partial \theta_b}$.

$$\begin{aligned} \gamma^a &= (\theta^a + \frac{\partial}{\partial \theta_a})\,, \quad \tilde{\gamma}^a = i\,(\theta^a - \frac{\partial}{\partial \theta_a})\,, \\ \theta^a &= \frac{1}{2}\,(\gamma^a - i\tilde{\gamma}^a)\,, \quad \frac{\partial}{\partial \theta_a} = \frac{1}{2}\,(\gamma^a + i\tilde{\gamma}^a)\,, \end{aligned}$$

  offering together $2 \cdot 2^d$ operators: 2^d are superposition of products of γ^a and 2^d of $\tilde{\gamma}^a$.

- **The two kinds of the Clifford algebra objects anti-commute in the sense**

$$\begin{aligned} \{\gamma^{\mathbf{a}}, \gamma^{\mathbf{b}}\}_+ &= \mathbf{2}\eta^{\mathbf{ab}} = \{\tilde{\gamma}^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_+, \\ \{\gamma^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_+ &= 0, \end{aligned}$$

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- **o Grassmann algebra is describing the anti-commuting fermion fields with integer spins and commuting boson fields with integer spins.**
- **o There are no anti-commuting fermion fields with integer spins observed so far.**
  And there are one kind of anti-commuting fermion fields with half integer spins and commuting boson fields with integer spins observed so far.
- **o the postulate**

$$
\begin{aligned}
(\tilde{\gamma}^{\mathbf{a}}\mathbf{B} &= \mathbf{i}(-)^{\mathbf{n_B}}\mathbf{B}\gamma^{\mathbf{a}}\,)\,|\psi_0\, >,\\
(\mathbf{B} &= a_0 + a_a\gamma^a + a_{ab}\gamma^a\gamma^b + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\ldots\gamma^{a_d})|\psi_o >
\end{aligned}
$$

with $(-)^{n_B} = +1, -1$, if B has a Clifford even or odd character, respectively, $|\psi_o >$ is a vacuum state on which the operators γ^a **apply**, **reduces the Clifford space for fermions and bosons for the factor of two**, while the operators $\tilde{\gamma}^a\tilde{\gamma}^b = -2i\tilde{S}^{ab}$ **define the family quantum numbers.**

o We have in each even-dimensional space

- $2^{\frac{d}{2}-1}$ **members, m, in each of $2^{\frac{d}{2}-1}$ families, f, the Clifford odd "basis vectors" $\hat{b}_f^{m\dagger}$ and the same number, $2^{\frac{d}{2}-1}\times\, 2^{\frac{d}{2}-1}$, of their Hermitian conjugated partners, $(\hat{b}_f^{m\dagger})^{\dagger}$, offering description of internal space of fermions,.**
- **We have the same number, twice $2^{\frac{d}{2}-1}\,\times 2^{\frac{d}{2}-1}$, of two kinds of the Clifford even "basis vectors", ${}^{I}\hat{\mathcal{A}}_f^{m\dagger}$ and ${}^{II}\hat{\mathcal{A}}_f^{m\dagger}$, having their Hermitian conjugated partners within the same group, offering the description of the internal space of bosons.**

To show this let us first "build" the building blocks: nilpotents and projectors, the eigenvectors of the Cartan subalgebra of the Lorentz algebra , so that the internal spaces of fermions and bosons will be algebraic products of nilpotents and projectors.

- **o** It is convenient to write all the **"basis vectors"** **describing the internal space of either fermion fields or boson fields as products of nilpotents and projectors, which are eigenvectors of the chosen Cartan subalgebra**

$$
\begin{aligned}
&S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\; d}\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\; d}\,,\\
&\mathbf{S}^{ab} = S^{ab} + \tilde{S}^{ab}\,.
\end{aligned}
$$

nilpotents

$$
S^{ab}\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b) = \frac{k}{2}\,\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \quad \overset{\mathbf{ab}}{(\mathbf{k})} := \frac{\mathbf{1}}{\mathbf{2}}(\gamma^{\mathbf{a}} + \frac{\eta^{\mathbf{aa}}}{\mathbf{ik}}\gamma^{\mathbf{b}})\,,
$$

projectors

$$
S^{ab}\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b) = \frac{k}{2}\,\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,, \quad \overset{\mathbf{ab}}{[\mathbf{k}]} := \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{1} + \frac{\mathbf{i}}{\mathbf{k}}\gamma^{\mathbf{a}}\gamma^{\mathbf{b}})\,,
$$

$$
(\overset{\mathbf{ab}}{(\mathbf{k})})^2 = \mathbf{0}\,, \qquad (\overset{\mathbf{ab}}{[\mathbf{k}]})^2 = \overset{\mathbf{ab}}{[\mathbf{k}]}\,,
$$

$$
\overset{\mathbf{ab}}{(\mathbf{k})}{}^{\dagger} = \eta^{\mathbf{aa}}\,\overset{\mathbf{ab}}{(-\mathbf{k})}\,, \quad \overset{\mathbf{ab}}{[\mathbf{k}]}{}^{\dagger} = \overset{\mathbf{ab}}{[\mathbf{k}]}\,.
$$

It is easy to find the relations

$$
\begin{aligned}
\mathbf{S}^{ab}\,\overset{ab}{(\mathbf{k})} &= \frac{k}{2}\overset{ab}{(\mathbf{k})}, \quad \mathbf{S}^{ab}\,\overset{ab}{[\mathbf{k}]} = \frac{k}{2}\overset{ab}{[\mathbf{k}]},\\
\tilde{\mathbf{S}}^{ab}\overset{ab}{(\mathbf{k})} &= \frac{k}{2}\overset{ab}{(\mathbf{k})}, \quad \tilde{\mathbf{S}}^{ab}\overset{ab}{[\mathbf{k}]} = -frack2\overset{ab}{[\mathbf{k}]}\,.
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{a}\,\overset{ab}{(\mathbf{k})} &= \eta^{aa}\overset{ab}{[-\mathbf{k}]}\,,\ \gamma^{b}\,\overset{ab}{(\mathbf{k})} = -ik\overset{ab}{[-\mathbf{k}]}\,,\ \gamma^{a}\overset{ab}{[\mathbf{k}]} = \overset{ab}{(-\mathbf{k})}\,,\ \gamma^{b}\overset{ab}{[\mathbf{k}]} = -ik\eta^{aa}\overset{ab}{(-\mathbf{k})}\,,\\
\tilde{\gamma}^{a}\,\overset{ab}{(\mathbf{k})} &= -i\eta^{aa}\overset{ab}{[\mathbf{k}]}\,,\ \tilde{\gamma}^{b}\,\overset{ab}{(\mathbf{k})} = -k\overset{ab}{[\mathbf{k}]}\,,\ \tilde{\gamma}^{a}\overset{ab}{[\mathbf{k}]} = \ i\overset{ab}{(\mathbf{k})}\,,\ \tilde{\gamma}^{b}\overset{ab}{[\mathbf{k}]} = -k\eta^{aa}\overset{ab}{(\mathbf{k})}\,,\\
\overset{ab}{(\mathbf{k})}\overset{ab}{(-\mathbf{k})} &= \eta^{aa}\overset{ab}{[\mathbf{k}]}\,,\ \overset{ab}{[\mathbf{k}]}\overset{ab}{(\mathbf{k})} = \overset{ab}{(\mathbf{k})}\,,\ \overset{ab}{(\mathbf{k})}\overset{ab}{[-\mathbf{k}]} = \overset{ab}{(\mathbf{k})}\,,\\
\overset{ab}{(\mathbf{k})}\overset{ab}{[\mathbf{k}]} &= \mathbf{0}\,,\ \overset{ab}{[\mathbf{k}]}\overset{ab}{(-\mathbf{k})} = \mathbf{0}\,,\ \overset{ab}{[\mathbf{k}]}\overset{ab}{[-\mathbf{k}]} = \mathbf{0}\,,\\
\overset{ab}{\widetilde{(-\mathbf{k})}}\overset{ab}{(\mathbf{k})} &= -i\eta^{aa}\overset{ab}{[\mathbf{k}]}\,,\ \overset{ab}{\widetilde{[\mathbf{k}]}}\overset{ab}{(\mathbf{k})} = \overset{ab}{(\mathbf{k})}\,,\ \overset{ab}{\widetilde{(\mathbf{k})}}\overset{ab}{[\mathbf{k}]} = i\overset{ab}{(\mathbf{k})}\,,\ \overset{ab}{\widetilde{[-\mathbf{k}]}}\overset{ab}{[\mathbf{k}]} = \overset{ab}{[\mathbf{k}]}\,,\\
\overset{ab}{\widetilde{(\mathbf{k})}}\overset{ab}{(\mathbf{k})} &= \mathbf{0}\,,\ \overset{ab}{\widetilde{[-\mathbf{k}]}}\overset{ab}{(\mathbf{k})} = \mathbf{0}\,,\ \overset{ab}{\widetilde{(\mathbf{k})}}\overset{ab}{[-\mathbf{k}]} = \mathbf{0}\,,\ \overset{ab}{\widetilde{[\mathbf{k}]}}\overset{ab}{[\mathbf{k}]} = \mathbf{0}\,.
\end{aligned}
$$

o

- γ^a **transforms** $\overset{ab}{(k)}$ **into** $\overset{ab}{[-k]}$, **never to** $\overset{ab}{[k]}$.

- $\tilde{\gamma}^a$ **transforms** $\overset{ab}{(k)}$ **into** $\overset{ab}{[k]}$, **never to** $\overset{ab}{[-k]}$.

- **There are the Clifford odd "basis vectors", that is the "basis vectors" with an odd number of nilpotents, at least one, the rest are projectors, such "basis vectors" anti-commute among themselves.**

- **There are the Clifford even "basis vectors", that is the "basis vectors" with an even number of nilpotents, the rest are projectors, such "basis vectors" commute among themselves.**

o A. Let us start with the Clifford odd "basis vectors".

- **Let us see how does one family of the Clifford odd "basis vectors" in $d = (13+1)$ look like, if spins in $d = (13+1)$ are analysed with respect to the** *standard model groups.*

- **One irreducible representation** of one **family contains $2^{\frac{(13+1)}{2}-1} = 64$ members which include all the family members, quarks and leptons with the right handed neutrinos included, as well as all the anti-members, antiquarks and antileptons, reachable by either S^{ab} (or by $\mathbb{C}_{\mathcal{N}}$ $\mathcal{P}_{\mathcal{N}}$ on a family member).**
- S^{ab} generate **all the members of one family**.
  $\tilde{S}^{ab}$ generate **all the families**.

Jour. of High Energy Phys. **04 (2014)** 165
J. of Math. Phys. **34**, 3731 (**1993**),
Int. J. of Modern Phys. **A 9**, 1731 (**1994**),
J. of Math. Phys. **44** 4817 (**2003**), hep-th/030322.

o The eightplet (represent. of $SO(7,1)$) **of quarks of a particular colour charge are presented. Theyare Clifford odd "basis vectors" , the eigenvectors of all the Cartan subalgebra members.** ($\tau^{33}=1/2$, $\tau^{38}=1/(2\sqrt{3})$ and $\tau^{4}=1/6$)

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Octet, $\Gamma^{(7,1)}=1$, $\Gamma^{(6)}=-1$, of quarks							
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform u_R of the 1^{st} row into u_L of the 7^{th} row, and d_R of the 4^{rd} row into d_L of the 6^{th} row, doing what the Higgs scalars and γ^0 do in the *standard model*.

o S^{ab} generate all the members of one family with leptons included. Here is The **eightplet** (represent. of $SO(7,1)$) **of leptons colour chargeless. The $SO(7,1)$ part is identical with the one of quarks.**

i		$\|^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(7,1)}=1,\ \Gamma^{(6)}=-1$, of leptons							
1	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
2	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
3	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
4	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
5	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
6	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
7	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
8	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform ν_R of the 1^{st} line into ν_L of the 7^{th} line, and e_R of the 4^{rd} line into e_L of the 6^{th} line, doing what the Higgs scalars and γ^0 do in the *standard model*.

o S^{ab} generate also all the anti-eightplet (repres. of $SO(7,1)$) of **anti-quarks of the anti-colour charge belonging to the same family of the Clifford odd basis vectors. Also eightplet of anti leptons.**

i		$\vert {}^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Antioctet, $\Gamma^{(7,1)}=-1$, $\Gamma^{(6)}=1$, of antiquarks							
33	$\bar{d}^{\bar{c}1}_{L}$	$\overset{03}{[-i]}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{(+)} \Vert \overset{9\,10}{[-]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
34	$\bar{d}^{\bar{c}1}_{L}$	$\overset{03}{(+i)}\overset{12}{[-]} \mid \overset{56}{(+)}\overset{78}{(+)} \Vert \overset{9\,10}{[-]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
35	$\bar{u}^{\bar{c}1}_{L}$	$\overset{03}{[-i]}\overset{12}{(+)} \mid \overset{56}{[-]}\overset{78}{[-]} \Vert \overset{9\,10}{[-]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
36	$\bar{u}^{\bar{c}1}_{L}$	$\overset{03}{(+i)}\overset{12}{[-]} \mid \overset{56}{[-]}\overset{78}{[-]} \Vert \overset{9\,10}{[-]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	- 1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
37	$\bar{d}^{\bar{c}1}_{R}$	$\overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{[-]} \Vert \overset{9\,10}{[-]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
38	$\bar{d}^{\bar{c}1}_{R}$	$\overset{03}{[-i]}\overset{12}{[-]} \mid \overset{56}{(+)}\overset{78}{[-]} \Vert \overset{9\,10}{[-]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
39	$\bar{u}^{\bar{c}1}_{R}$	$\overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{[-]}\overset{78}{(+)} \Vert \overset{9\,10}{[-]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
40	$\bar{u}^{\bar{c}1}_{R}$	$\overset{03}{[-i]}\overset{12}{[-]} \mid \overset{56}{[-]}\overset{78}{(+)} \Vert \overset{9\,10}{[-]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\bar{d}_L$ of the 1^{st} row into $\bar{d}_R$ of the 5^{th} row, and $\bar{u}_L$ of the 4^{rd} row into $\bar{u}_R$ of the 8^{th} row.

o

- The **Hermitian conjugated partners of the Clifford odd "basis vectors"** $\hat{b}^{m\dagger}_{f}$, **follow if all nilpotents** $\stackrel{ab}{(k)}$ **of** $\hat{b}^{m\dagger}_{f}$ **are transformed into** $\eta^{aa}\,\stackrel{ab}{(-k)}$**. Projectors** $\stackrel{ab}{[k]}$ **are selfadjoint.**
- **All the "basis vectors" within any family, as well as the "basis vectors" among families, are orthogonal; that is, their algebraic product is zero. The same is true within their Hermitian conjugated partners.**

$$\mathbf{\hat{b}^{m\dagger}_{f}} *_{\mathbf{A}} \mathbf{\hat{b}^{m`\dagger}_{f`}} \;=\; 0\,, \quad \mathbf{\hat{b}^{m}_{f}} *_{\mathbf{A}} \mathbf{\hat{b}^{m`}_{f`}} = 0\,, \quad \forall m, m', f, f`\,.$$

$$\begin{aligned}
\hat{b}^{m}_{f}{}_{*A}|\psi_{oc}> &= 0.\,|\psi_{oc}>\,,\\
\hat{b}^{m\dagger}_{f}{}_{*A}|\psi_{oc}> &= |\psi^{m}_{f}>\,,\\
\{\hat{b}^{m}_{f}, \hat{b}^{m'}_{f`}\}_{*A+}|\psi_{oc}> &= 0.\,|\psi_{oc}>\,,\\
\{\hat{b}^{m\dagger}_{f}, \hat{b}^{m'\dagger}_{f`}\}_{*A+}|\psi_{oc}> &= 0.\,|\psi_{oc}>\,,\\
\{\hat{b}^{m}_{f}, \hat{b}^{m'\dagger}_{f`}\}_{*A+}|\psi_{oc}> &= \delta^{mm'}\,\delta_{ff`}|\psi_{oc}>\,,\\
|\psi_{oc}> &= \sum_{f=1}^{2^{\frac{d}{2}-1}} \hat{b}^{m}_{f}{}_{*A}\hat{b}^{m\dagger}_{f}\,|\,1\,>\,,
\end{aligned}$$

o B. Let us discuss the properties of the Clifford even "basis vector".

- **While the Clifford odd "basis vectors" must be products of an odd number of nilpotents, at least one, the rest, up to $\frac{d}{2}$, of projectors, the Clifford even "basis vectors" must be products of an even number of nilpotents and the rest, up to $\frac{d}{2}$, of projectors; Each nilpotent and each projector is chosen to be the "eigenstate" of one of the members of the Cartan subalgebra of the Lorentz algebra,**
  $\mathcal{S}^{ab} = S^{ab} + \tilde{S}^{ab}$.
  Correspondingly the "basis vectors" are the eigenstates of all the members of the Cartan subalgebra of the Lorentz algebra.

o Let us call the Clifford even "basis vectors" ${}^{i}\hat{\mathcal{A}}^{m\dagger}_{f}$, $i = (I, II)$ denotes two groups of Clifford even "basis vectors", while m and f determine membership of "basis vectors" in any of the two groups, I or II.

-

$$
\begin{array}{rclcrcl}
 & & d & = & 2(2n+1) & & \\
{}^{I}\hat{\mathcal{A}}^{1\dagger}_{1} &=& \stackrel{03}{(+i)}\stackrel{12}{(+)}\cdots\stackrel{d-1\,d}{[+]} & & {}^{II}\hat{\mathcal{A}}^{1\dagger}_{1} &=& \stackrel{03}{(-i)}\stackrel{12}{(+)}\cdots\stackrel{d-1\,d}{[+]}\,, \\
{}^{I}\hat{\mathcal{A}}^{2\dagger}_{1} &=& \stackrel{03}{[-i]}\stackrel{12}{[-]}\stackrel{56}{(+)}\cdots\stackrel{d-1\,d}{[+]} & & {}^{II}\hat{\mathcal{A}}^{2\dagger}_{1} &=& \stackrel{03}{[+i]}\stackrel{12}{[-]}\stackrel{56}{(+)}\cdots\stackrel{d-1\,d}{[+]}\,, \\
{}^{I}\hat{\mathcal{A}}^{3\dagger}_{1} &=& \stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{(+)}\cdots\stackrel{d-3\,d-2}{[-]}\;\stackrel{d-1\,d}{(-)} & & {}^{II}\hat{\mathcal{A}}^{3\dagger}_{1} &=& \stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{(+)}\cdots\stackrel{d-3\,d-2}{[-]}\;\stackrel{d-}{(} \\
 & & \cdots & & \cdots & &
\end{array}
$$

- **Similarly for $d = 4n$**
  In both cases the **Clifford even basis vectors can have only even number of nilpotents: (0,2,...).**

o There are $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ the Clifford even "basis vectors" of the kind ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and there are $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ Clifford even "basis vectors" of the kind ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$.

- $$ {}^{\mathbf{i}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{i}}\hat{\mathcal{A}}^{\mathbf{m}'\dagger}_{\mathbf{f}`} \to \left\{ \begin{array}{r} {}^{\mathbf{i}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}`}\,, \\ \text{or}\, \mathbf{0}\,, \mathbf{i} = (\mathbf{I}, \mathbf{II})\,. \end{array} \right. $$

- $$ {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{II}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} = \mathbf{0} = {}^{\mathbf{II}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}}\,. $$

o It remains to evaluate the algebraic application, $*_A$, of the Clifford even "basis vectors" ${}^{I,II}\hat{\mathcal{A}}^{m\dagger}_{f}$ on the Clifford odd "basis vectors" $\hat{b}^{m'\dagger}_{f`}$.

- $$ {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} \hat{\mathbf{b}}^{\mathbf{m}'\dagger}_{\mathbf{f}`} \rightarrow \left\{ \begin{array}{c} \hat{\mathbf{b}}^{\mathbf{m}\dagger}_{\mathbf{f}`} \;, , \\ \text{or zero} \,, \end{array} \right. $$

- $$ \hat{\mathbf{b}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_A {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}'\dagger}_{\mathbf{f}`} = 0 \,, \quad \forall (m, m`, f, f`) \,. $$

- $$ {}^{\mathbf{II}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_A \hat{\mathbf{b}}^{\mathbf{m}'\dagger}_{\mathbf{f}`} = 0 \,, \;\; \forall (m, m', f, f`) \,, $$

- $$ \hat{\mathbf{b}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_A {}^{\mathbf{II}}\hat{\mathcal{A}}^{\mathbf{m}'\dagger}_{\mathbf{f}`} \rightarrow \left\{ \begin{array}{c} \hat{\mathbf{b}}^{\mathbf{m}\dagger}_{\mathbf{f}``} \;, \\ \text{or zero} \,, \end{array} \right. $$

o Let be pointed out again that although there is the same number of the Clifford odd and the Clifford even "basis vectors" and their Hermitian conjugated partners, each have $2\times 2^{\frac{d}{2}-1}\times 2^{\frac{d}{2}-1}$, yet they have completely different properties:

- $\hat{b}^{m\dagger}_{f}$ appear in families and have their Hermitian conjugated partners $\hat{b}^{m}_{f}$ in a separate group, they anticommute, explaining the second quantization postulates for fermions,
- ${}^{i}\hat{\mathcal{A}}^{m\dagger}_{f}$ have no families, appear in two groups, have their Hermitian conjugated partners within the same group, they commute, explaining the second quantization postulates for bosons.

Yet it is a small step from the Clifford even to the Clifford odd objects: the algebraic multiplication of $\hat{b}^{m\dagger}_{f}$ by γ^{a} or $\tilde{\gamma}^{a}$ transform $\hat{b}^{m\dagger}_{f}$ or $\hat{b}^{m}_{f}$ to ${}^{i}\hat{\mathcal{A}}^{m'\dagger}_{f'}$ and vice versa.

Let us see that γ^a, applying on **the Clifford odd** $\hat{b}^{m\dagger}_{f}$, changes it to ${}^{i}\hat{\mathcal{A}}^{m'\dagger}_{f'}$, and γ^a, applying on **the Clifford even** ${}^{i}\hat{\mathcal{A}}^{m'\dagger}_{f'}$ changes it to $\hat{b}^{m\dagger}_{f}$, changing the number of **nilpotents** for one, and similarly for $\tilde{\gamma}^a$:

- γ^a **transforms** $\overset{ab}{(k)}$ **into** $\overset{ab}{[-k]}$,
  γ^a **transforms** $\overset{ab}{[k]}$ **into** $\overset{ab}{(-k)}$,
- $\tilde{\gamma}^a$ **transforms** $\overset{ab}{(k)}$ **into** $\overset{ab}{[k]}$,
  $\tilde{\gamma}^a$ **transforms** $\overset{ab}{[k]}$ **into** $\overset{ab}{(k)}$.

o Let us demonstrate the difference in the **Clifford odd** and the **Clifford even** "basis vectors" in $d = (5+1)$ case.

- In $d = (5+1)$ there are $2^{\frac{6}{2}-1}$ members of **Clifford odd "basis vectors"** appearing in $2^{\frac{6}{2}-1}$ **Clifford odd families**.
- **Clifford odd "basis vectors"**, $\hat{b}^{m\dagger}_{f}$, have their **Hermitian conjugated partners**, $\hat{b}^{m}_{f}$, in the separate group not reachable either by S^{ab} or by $\tilde{S}^{ab}$. Due to

$$\overset{ab}{(\mathbf{k})}^{\dagger} \;=\; \eta^{aa}\,\overset{ab}{(-\mathbf{k})}\,,\;\overset{ab}{[\mathbf{k}]}^{\dagger} = \overset{ab}{[\mathbf{k}]}\,.$$

- **Clifford even "basis vectors"**, ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$, have their **Hermitian conjugated partners**, ${}^{I}\hat{\mathcal{A}}^{m}_{f}$, within the same group reachable by S^{ab} or by $\tilde{S}^{ab}$.

basis vect. $\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}$	m $\rightarrow$	$f=1$ $\frac{i}{2}, -\frac{1}{2}, -\frac{1}{2}$	$f=2$ $-\frac{i}{2}, -\frac{1}{2}, \frac{1}{2}$	$f=3$ $-\frac{i}{2}, \frac{1}{2}, -\frac{1}{2}$	$f=4$ $\frac{i}{2}, \frac{1}{2}, \frac{1}{2}$	S^{03}	S^{12}	S^{56}
odd I $\hat{b}_f^{m\dagger}$	1 2 3 4	$\overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}$ $[-i](-)[+]$ $[-i][+](-)$ $(+i)(-)(-)$	$\overset{03}{[+i]}\overset{12}{[+]}\overset{56}{(+)}$ $(-i)(-)(+)$ $(-i)[+][-]$ $[+i](-)[-]$	$\overset{03}{[+i]}\overset{12}{(+)}\overset{56}{[+]}$ $(-i)[-][+]$ $(-i)(+)(-)$ $[+i][-](-)$	$\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)}$ $[-i][-](+)$ $[-i](+)[-]$ $(+i)[-][-]$	$\frac{i}{2}$ $-\frac{i}{2}$ $-\frac{i}{2}$ $\frac{i}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$
S^{03}, S^{12}, S^{56}	$\rightarrow$	$-\frac{i}{2}, \frac{1}{2}, \frac{1}{2}$ 03 12 56	$\frac{i}{2}, \frac{1}{2}, -\frac{1}{2}$ 03 12 56	$\frac{i}{2}, -\frac{1}{2}, \frac{1}{2}$ 03 12 56	$-\frac{i}{2}, -\frac{1}{2}, -\frac{1}{2}$ 03 12 56	$\tilde{S}^{03}$	$\tilde{S}^{12}$	$\tilde{S}^{56}$
odd II $\hat{b}_f^{m}$	1 2 3 4	$(-i)[+][+]$ $[-i](+)[+]$ $[-i][+](+)$ $(-i)(+)(+)$	$[+i][+](-)$ $(+i)(+)(-)$ $(+i)[+][-]$ $[+i](+)[-]$	$[+i](-)[+]$ $(+i)[-][+]$ $(+i)(-)(+)$ $[+i][-](+)$	$(-i)(-)(-)$ $[-i][-](-)$ $[-i](-)[-]$ $(-i)[-][-]$	$-\frac{i}{2}$ $\frac{i}{2}$ $\frac{i}{2}$ $-\frac{i}{2}$	$-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$	$-\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}$	$\rightarrow$	$-\frac{i}{2}, \frac{1}{2}, \frac{1}{2}$ 03 12 56	$\frac{i}{2}, -\frac{1}{2}, \frac{1}{2}$ 03 12 56	$-\frac{i}{2}, -\frac{1}{2}, -\frac{1}{2}$ 03 12 56	$\frac{i}{2}, \frac{1}{2}, -\frac{1}{2}$ 03 12 56	S^{03}	S^{12}	S^{56}
even I ${}^{I}\mathcal{A}_f^{m}$	1 2 3 4	$[+i](+)(+)$ $(-i)[-](+)$ $(-i)(+)[-]$ $[+i][-][-]$	$(+i)[+](+)$ $[-i](-)(+)$ $[-i][+][-]$ $(+i)(-)[-]$	$[+i][+][+]$ $(-i)(-)[+]$ $(-i)[+](-)$ $[+i](-)(-)$	$(+i)(+)[+]$ $[-i][-][+]$ $[-i](+)(-)$ $(+i)[-](-)$	$\frac{i}{2}$ $-\frac{i}{2}$ $-\frac{i}{2}$ $\frac{i}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$
$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}$	$\rightarrow$	$\frac{i}{2}, \frac{1}{2}, \frac{1}{2}$ 03 12 56	$-\frac{i}{2}, -\frac{1}{2}, \frac{1}{2}$ 03 12 56	$\frac{i}{2}, -\frac{1}{2}, -\frac{1}{2}$ 03 12 56	$-\frac{i}{2}, \frac{1}{2}, -\frac{1}{2}$ 03 12 56	S^{03}	S^{12}	S^{56}
even II ${}^{II}\mathcal{A}_f^{m}$	1 2 3 4	$[-i](+)(+)$ $(+i)[-](+)$ $(+i)(+)[-]$ $[-i][-][-]$	$(-i)[+](+)$ $[+i](-)(+)$ $[+i][+][-]$ $(-i)(-)[-]$	$[-i][+][+]$ $(+i)(-)[+]$ $(+i)[+](-)$ $[-i](-)(-)$	$(-i)(+)[+]$ $[+i][-][+]$ $[+i](+)(-)$ $(-i)[-](-)$	$-\frac{i}{2}$ $\frac{i}{2}$ $\frac{i}{2}$ $-\frac{i}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$

- **o Clifford odd "basis vectors"** describing the internal space of **fermions in the case of** $d = (5+1)$ are presented in the table as *odd I* $\hat{b}_f^{m\dagger}$, **having odd numbers of nilpotents,**
- **their Hermitian conjugated partners** $\hat{b}_f^{m}$ **appear in a separate group presented in the same table as** *odd II* $\hat{b}_f^{m}$**.** The two groups are not reachable by either S^{ab} or by $\tilde{S}^{ab}$.
- **Clifford even "basis vectors"** describing the internal space of **bosons in the case of** $d = (5+1)$ are presented in the table as *even I, II* ${}^{I,II}\hat{\mathcal{A}}_f^{m\dagger}$, **having an even numbers of nilpotents.**
- **Their Hermitian conjugated partner appear within the same group of "basis vectors", either I or II, demonstrating correspondingly the properties of the internal space of the gauge fields with respect to the fermion "basis vectors".**

*

- **Clifford odd "basis vector"** describing the internal space of **quark** $u^{c1\dagger}_{\uparrow R}$, $\Leftrightarrow b^{1\dagger}_{1} := \overset{03}{(+i)}\overset{12}{[+]} \mid \overset{56}{[+]}\overset{78}{(+)} \mid\mid \overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$,
  has the Hermitian conjugated partner equal to
  $u^{c1}_{\uparrow R} \Leftrightarrow (\mathbf{b}^{1\dagger}_{1})^{\dagger} = \overset{13\,14}{[-]}\;\overset{11\,12}{[-]}\;\overset{9\,10}{(-)} \mid\mid \overset{78}{(-)}\overset{56}{[+]} \mid \overset{12}{[+]}\overset{03}{(-\mathbf{i})}$, both with an odd number of nilpotents,
  both are the Clifford odd objects, belonging to two group.
- **Quarks "basis vectors"** contain $b^{1\dagger}_{1} = \overset{03}{(+i)}\overset{12}{[+]} \mid \overset{56}{[+]}$ from d=(5+1).
- **Clifford even "basis vectors"**, having an even number of nilpotents, describe the internal space of the corresponding boson field
  ${}^{I}\mathcal{A}^{m}_{f} = \overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{[+]}\overset{78}{(+)} \mid\mid \overset{9\,10}{(+)}\;\overset{11\,12}{[-]}\;\overset{13\,14}{[-]}$,
- it contains ${}^{I}\mathcal{A}^{m}_{f} = \overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{[+]}$ from d=(5+1).

*

Repeating the anti-commutation relations for Clifford odd "basis vectors",
representing the internal space of fermion fields of quarks and leptons ($i = (u^{c,f,\uparrow,\downarrow}_{R,L}, d^{c,f,\uparrow,\downarrow}_{R,L}, \nu^{f,\uparrow,\downarrow}_{R,L}, e^{f,\uparrow,\downarrow}_{R,L})$),
and anti-quarks and anti-leptons, with the family quantum number f.

- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*_{\mathbf{A}}+}|\psi_{\mathbf{o}}> = \delta_{\mathbf{f}\,\mathbf{f}'}\,\delta^{\mathbf{mk}}\,|\psi_{\mathbf{o}}>$,
- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}}_{\mathbf{f}'}\}_{*_{\mathbf{A}}+}|\psi_{\mathbf{o}}> = 0\cdot|\psi_{\mathbf{o}}>$,
- $\{\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*_{\mathbf{A}}+}|\psi_{\mathbf{o}}> = 0\cdot|\psi_{\mathbf{o}}>$,
- $\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}\,|\psi_{\mathbf{o}}> = 0\cdot|\psi_{\mathbf{o}}>$,
- $\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}\,|\psi_{\mathbf{o}}> = |\psi^{\mathbf{m}}_{\mathbf{f}}>$,

$$|\psi_{\mathbf{o}}> = \overset{03}{[-\mathbf{i}]}\overset{12}{[-]}\overset{56}{[-]}\cdots\overset{13\,14}{[-]}\;|\mathbf{1}>$$

define the vacuum state for quarks and leptons and antiquarks and antileptons of the family f .

[arXiv:1802.05554v1], [arXiv:1802.05554v4], [arXiv:1902.10628]

*** Let us come back to d=(5+1) case and to the properties of the Clifford odd and the Clifford even "basiss vectors"**

Let us first treat the properties **of the "basis vectors" for fermion fields in** $d = (5+1)$, then we shall treat properties **of the "basis vectors" for boson fields in** $d = (5+1)$, as well as their mutual interaction.

The "basis vectors" for fermion fields in $d = (5+1)$, **appear in four families, each family is identical with respect to** $S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)$, **distinguishing only in** $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$.

The nilpotents and projectors are chosen to be eigenstates of the Cartan subalgebra of the Lorentz algebra

$$\mathbf{S}^{ab} \stackrel{ab}{(\mathbf{k})} = \frac{k}{2}\stackrel{ab}{(\mathbf{k})}, \quad \mathbf{S}^{ab} \stackrel{ab}{[\mathbf{k}]} = \frac{k}{2}\stackrel{ab}{[\mathbf{k}]},$$

$$\tilde{\mathbf{S}}^{ab}\stackrel{ab}{(\mathbf{k})} = \frac{k}{2}\stackrel{ab}{(\mathbf{k})}, \quad \tilde{\mathbf{S}}^{ab}\stackrel{ab}{[\mathbf{k}]} = -\frac{k}{2}\stackrel{ab}{[\mathbf{k}]}.$$

$\tilde{S}^{01} \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]} = -\frac{i}{2}\stackrel{03}{[+i]}\stackrel{12}{(+)}\stackrel{56}{[+]}$,

and the $\hat{b}^{m\dagger}_{f}$ are eigenvectors of all the Cartan subalgebra members.

"Basis vectors" for fermions

f	m	$\hat{b}^{m\dagger}_f$	S^{03}	S^{12}	S^{56}	Γ^{3+1}	N^3_L	N^3_R	τ^3	τ^8	τ	$\tilde{S}^{03}$	$\tilde{S}$
I	1	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{[+]}$	$\frac{i}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{i}{2}$	$-$
	2	$\stackrel{03}{[-i]}\stackrel{12}{(-)} \mid \stackrel{56}{[+]}$	$-\frac{i}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	$-$
	3	$\stackrel{03}{[-i]}\stackrel{12}{[+]} \mid \stackrel{56}{(-)}$	$-\frac{i}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	$-$
	4	$\stackrel{03}{(+i)}\stackrel{12}{(-)} \mid \stackrel{56}{(-)}$	$\frac{i}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	$-$
II	1	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{[+]}$	$\frac{i}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{i}{2}$	
	2	$\stackrel{03}{(-i)}\stackrel{12}{[-]} \mid \stackrel{56}{[+]}$	$-\frac{i}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	
	3	$\stackrel{03}{(-i)}\stackrel{12}{(+)} \mid \stackrel{56}{(-)}$	$-\frac{i}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	
	4	$\stackrel{03}{[+i]}\stackrel{12}{[-]} \mid \stackrel{56}{(-)}$	$\frac{i}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	
III	1	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}$	$\frac{i}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{i}{2}$	$-$
	2	$\stackrel{03}{(-i)}\stackrel{12}{(-)} \mid \stackrel{56}{(+)}$	$-\frac{i}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	$-$
	3	$\stackrel{03}{(-i)}\stackrel{12}{[+]} \mid \stackrel{56}{[-]}$	$-\frac{i}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	$-$
	4	$\stackrel{03}{[+i]}\stackrel{12}{(-)} \mid \stackrel{56}{[-]}$	$\frac{i}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	$-$
IV	1	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}$	$\frac{i}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{i}{2}$	
	2	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \mid \stackrel{56}{(+)}$	$-\frac{i}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	
	3	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \mid \stackrel{56}{[-]}$	$-\frac{i}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	
	4	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \mid \stackrel{56}{[-]}$	$\frac{i}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	

Let us demonstrate properties of the internal space of fermions using the odd Clifford subalgebra in two ways:
a. Let us use the superposition of members of Cartan subalgebra for the subgroup $SO(3,1)\times U(1)$: $(N^3_{\pm}, \tau)$

$$N^3_{\pm}(=N^3_{(L,R)}) := \frac{1}{2}(S^{12}\pm iS^{03})\,,\quad \tau=S^{56}\,,$$

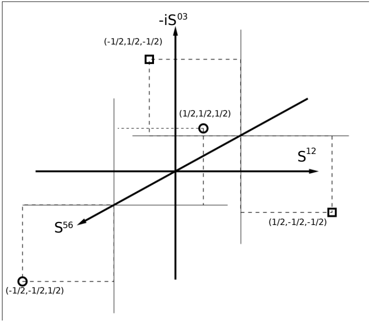
what is meaningful if we understand S^{03} and S^{12} as spins of fermions , S^{56} as their charge,

o b. for the subgroup $SU(3)\times U(1)$: (τ', τ^3, τ^8)

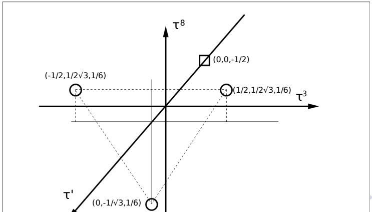
$$\tau^3 := \quad \frac{1}{2}(-S^{1\,2}-iS^{0\,3})\,,\qquad \tau^8=\frac{1}{2\sqrt{3}}(-iS^{0\,3}+S^{1\,2}-2S^{5\,6})\,,$$
$$\tau' = \quad -\frac{1}{3}(-iS^{0\,3}+S^{1\,2}+S^{5\,6})\,,$$

if we treat the colour properties for fermions to learn from this toy model as much as we can. The number of commuting operators is three in both cases.

* **a.** **We recognize twice 2 "basis vectors" with charge $\pm\frac{1}{2}$, and with spins up and down.**



o **b.** **We recognize one colour triplet of "basis vectors" with $\tau' = \frac{1}{6}$ and one colour singlet with $\tau' = -\frac{1}{2}$.**



- o To see that the Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ are "the gauge" fields of the Clifford odd "basis vectors", let us algebraically, $*_A$, apply the Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f=3}, m=(1,2,3,4)$ on the Clifford odd "basis vectors" .

  * Let the Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f=3}, m=(1,2,3,4)$ be taken from the third column of *even I*, and $\hat{b}^{m=1\dagger}_{f=1}$, is present as the first Clifford *odd I* "basis vector" on the first and the second table.

- The algebraic application, $*_A$, can easily be evaluated by taking into account

$$
\begin{aligned}
*\overset{ab}{(\mathbf{k})}\overset{ab}{(-\mathbf{k})} &= \eta^{aa}\overset{ab}{[\mathbf{k}]}\,,\quad \overset{ab}{[\mathbf{k}]}\overset{ab}{(\mathbf{k})}=\overset{ab}{(\mathbf{k})}\,,\quad \overset{ab}{(\mathbf{k})}\overset{ab}{[-\mathbf{k}]}=\overset{ab}{(\mathbf{k})}\,,\\
\overset{ab}{(\mathbf{k})}\overset{ab}{[\mathbf{k}]} &= \mathbf{0}\,,\overset{ab}{[\mathbf{k}]}\overset{ab}{(-\mathbf{k})}=\mathbf{0}\,,\quad \overset{ab}{[\mathbf{k}]}\overset{ab}{[-\mathbf{k}]}=\mathbf{0}\,,
\end{aligned}
$$

for any m and f.

We obtain:

$$
\begin{aligned}
&{}^{I}\hat{\mathcal{A}}^{1\dagger}_{3}(\equiv \overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]})*_{A}\hat{\mathbf{b}}^{1\dagger}_{1}(\equiv \overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}) \rightarrow \hat{\mathbf{b}}^{1\dagger}_{1}\,, \text{selfadjoint}\\
&{}^{I}\hat{\mathcal{A}}^{2\dagger}_{3}(\equiv \overset{03}{(-i)}\overset{12}{(-)}\overset{56}{[+]})*_{A}\hat{\mathbf{b}}^{1\dagger}_{1} \rightarrow \hat{\mathbf{b}}^{2\dagger}_{1}(\equiv \overset{03}{[-i]}\overset{12}{(-)}\overset{56}{[+]})\,,\\
&{}^{I}\hat{\mathcal{A}}^{3\dagger}_{3}(\equiv \overset{03}{(-i)}\overset{12}{[+]}\overset{56}{(-)})*_{A}\hat{\mathbf{b}}^{1\dagger}_{1} \rightarrow \hat{\mathbf{b}}^{3\dagger}_{1}(\equiv \overset{03}{[-i]}\overset{12}{[+]}\overset{56}{(-)})\,,\\
&{}^{I}\hat{\mathcal{A}}^{4\dagger}_{3}(\equiv \overset{03}{[+i]}\overset{12}{(-)}\overset{56}{(-)})*_{A}\hat{\mathbf{b}}^{1\dagger}_{1} \rightarrow \hat{\mathbf{b}}^{4\dagger}_{1}(\equiv \overset{03}{(+i)}\overset{12}{(-)}\overset{56}{(-)})\,.
\end{aligned}
$$

Looking at the eigenvalues of the $\hat{b}^{m\dagger}_{1}$ we see that ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{3}$ obviously carry the integer eigenvalues of $\mathcal{S}^{03}, \mathcal{S}^{12}, \mathcal{S}^{56}$.

Let us look at the eigenvalues of (τ^3, τ^8, τ') of $\hat{b}_1^{m\dagger}$.

$\hat{b}_1^{1\dagger}$ has $(\tau^3, \tau^8, \tau') = (0, 0, -\frac{1}{2})$,
$\hat{b}_1^{2\dagger}$ has $(\tau^3, \tau^8, \tau') = (0, -\frac{1}{\sqrt{3}}, \frac{1}{6})$,
$\hat{b}_1^{3\dagger}$ has $(\tau^3, \tau^8, \tau') = (-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{6})$,
$\hat{b}_1^{4\dagger}$ has $(\tau^3, \tau^8, \tau') = (\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{6})$.

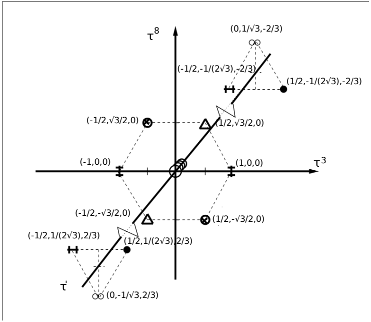
The eigenvalues of (τ^3, τ^8, τ') of ${}^{I}\hat{\mathcal{A}}_3^{1\dagger}$ are obviously
${}^{I}\hat{\mathcal{A}}_3^{1\dagger}$ has $(\tau^3, \tau^8, \tau') = (0, 0, 0)$,
${}^{I}\hat{\mathcal{A}}_3^{2\dagger}$ has $(\tau^3, \tau^8, \tau') = (0, -\frac{1}{\sqrt{3}}, \frac{2}{3})$,
${}^{I}\hat{\mathcal{A}}_3^{3\dagger}$ has $(\tau^3, \tau^8, \tau') = (-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{2}{3})$,
${}^{I}\hat{\mathcal{A}}_3^{4\dagger}$ has $(\tau^3, \tau^8, \tau') = (\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{2}{3})$,

It can be concluded: $\mathcal{S}^{ab} = S^{ab} + \tilde{S}^{ab}$. Using this recognition we find the properties of the Clifford even "basis vectors":

f	m	$*$	$^I\hat{\mathcal{A}}^{m\dagger}_f$	$\mathcal{S}^{03}$	$\mathcal{S}^{12}$	$\mathcal{S}^{56}$	$\mathcal{N}^3_L$	$\mathcal{N}^3_R$	τ^3	τ^8	τ'
I	1	$\star\star$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\stackrel{56}{(+)}$	0	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$
	2	$\triangle$	$\stackrel{03}{(-i)}\stackrel{12}{[-]}\stackrel{56}{(+)}$	$-i$	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2\sqrt{3}}$	0
	3	$\ddagger$	$\stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{[-]}$	$-i$	1	0	1	0	-1	0	0
	4	$\bigcirc$	$\stackrel{03}{[+i]}\stackrel{12}{[-]}\stackrel{56}{[-]}$	0	0	0	0	0	0	0	0
II	1	$\bullet$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{(+)}$	i	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$
	2	$\otimes$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{(+)}$	0	-1	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2\sqrt{3}}$	0
	3	$\bigcirc$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{[-]}$	0	0	0	0	0	0	0	0
	4	$\ddagger$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\stackrel{56}{[-]}$	i	-1	0	-1	0	1	0	0
III	1	$\bigcirc$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}$	0	0	0	0	0	0	0	0
	2	$\odot\odot$	$\stackrel{03}{(-i)}\stackrel{12}{(-)}\stackrel{56}{[+]}$	$-i$	-1	0	0	-1	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$
	3	$\bullet$	$\stackrel{03}{(-i)}\stackrel{12}{[+]}\stackrel{56}{(-)}$	$-i$	0	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$
	4	$\star\star$	$\stackrel{03}{[+i]}\stackrel{12}{(-)}\stackrel{56}{(-)}$	0	-1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$
IV	1	$\odot\odot$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{[+]}$	i	1	0	0	1	0	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$
	2	$\bigcirc$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\stackrel{56}{[+]}$	0	0	0	0	0	0	0	0
	3	$\otimes$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{(-)}$	0	1	-1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2\sqrt{3}}$	0
	4	$\triangle$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\stackrel{56}{(-)}$	i	0	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2\sqrt{3}}$	0

Selfadjoint members are denoted by $\bigcirc$, Hermitian conjugated partners are denoted by the same symbol.

o Fig. analyses ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ with respect to Cartan subalgebra members (τ^3, τ^8, τ'). There are
one sextet with $\tau' = 0$,
four singlets with $(\tau^3 = 0, \tau^8 = 0, \tau' = 0)$,
one “anti-triplet” with $\tau' = \frac{2}{3}$ and one “triplet” with $\tau' = -\frac{2}{3}$.
NO FAMILIES!



* We learned that the description of the internal spaces of fermions and bosons with the Clifford algebra odd, for fermions, and even, for bosons behave so that they offer:

a. families and all the observed charges of quarks and leptons and anti-quarks and anti-leptons,

b. two kinds of the boson fields, the gauge fields of the corresponding fermion fields, what looks very promising.

Can the Clifford algebra and the *spin-charge-family* theory offer more if we extend the point fields in the ordinary space to strings?

- **In odd dimensional spaces fermion fields and boson fields have completely different properties.**

**

- In In odd dimensional spaces, $d = 2n + 1$, only half of "basis vectors" demonstrate properties which they demonstrate in even dimensional spaces,
- the properties which empower the Clifford odd "basis vectors" to describe the internal space of fermions and
- the Clifford even "basis vectors" to describe the internal space of bosons:
- This half belongs to $d' = 2n$ and does demonstrate these properties.
- The other half, obtained from the first half by the application of $S^{0\,2n+1}$
- This second half of the Clifford odd "basis vectors", although anticommuting, demonstrate properties of the Clifford even "basis vectors",
  and the second half of the Clifford even "basis vectors", although commuting, demonstrate properties of the Clifford odd "basis vectors" in even dimensional spaces.

* **Still anticommuting Clifford odd "basis vectors"** (the Clifford even operators $S^{0\,2n+1}$ do not change either oddness or evenness of the "basis vectors")

appear in two separate groups with $2^{\frac{2n}{2}-1} \times 2^{\frac{2n}{2}-1}$ members, each with their Hermitian conjugated partners within the same group having no families;

Still commuting Clifford even "basis vectors" appear in $2^{\frac{2n}{2}-1}$ families, each with $2^{\frac{2n}{2}-1}$ members, having their Hermitian conjugated partners $2^{\frac{2n}{2}-1} \times 2^{\frac{2n}{2}-1}$ in a separate group.

For illustration let me treat the special case for $d = (4+1)$.

**

$$
\begin{array}{rl}
d = & 4+1 \\
 & \text{Clifford odd} \\
\hat{\mathbf{b}}^{1\dagger}_{\mathbf{1}} = \stackrel{03}{(+\mathbf{i})}\stackrel{12}{[+]}\,,\quad \hat{\mathbf{b}}^{1\dagger}_{\mathbf{2}} = \stackrel{03}{[+\mathbf{i}]}\stackrel{12}{(+)}\,, & \hat{b}^{1\dagger}_3 = \stackrel{03}{[-\mathbf{i}]}\stackrel{12}{[+]}\gamma^5\,,\quad \hat{b}^{1\dagger}_4 = \stackrel{03}{(-\mathbf{i})}\stackrel{12}{(+)}\gamma^5\,, \\
\hat{b}^{2\dagger}_1 = \stackrel{03}{[-i]}\stackrel{12}{(-)}\,,\quad \hat{b}^{2\dagger}_2 = \stackrel{03}{(-i)}\stackrel{12}{[-]}\,, & \hat{b}^{2\dagger}_3 = \stackrel{03}{(+\mathbf{i})}\stackrel{12}{(-)}\gamma^5\,,\quad \hat{b}^{2\dagger}_4 = \stackrel{03}{[+i]}\stackrel{12}{[-]}\,\gamma^5\,, \\
\hat{\mathbf{b}}^{1}_{\mathbf{1}} = \stackrel{03}{(-\mathbf{i})}\stackrel{12}{[+]}\,,\quad \hat{b}^{1}_2 = \stackrel{03}{[+i]}\stackrel{12}{(-)}\,, & \hat{b}^{1}_3 = \stackrel{03}{[+i]}\stackrel{12}{[+]}\,\gamma^5\,,\quad \hat{b}^{1}_4 = \stackrel{03}{(-i)}\stackrel{12}{(-)}\,\gamma^5\,, \\
\hat{b}^{2}_1 = \stackrel{03}{[-i]}\stackrel{12}{(+)}\,,\quad \hat{b}^{2}_2 = \stackrel{03}{(+i)}\stackrel{12}{[-]}\,, & \hat{b}^{2}_3 = \stackrel{03}{(+i)}\stackrel{12}{(+)}\,\gamma^5\,,\quad \hat{b}^{2}_4 = \stackrel{03}{[-i]}\stackrel{12}{[-]}\,\gamma^5\,, \\
 & \text{Clifford even} \\
{}^{I}\mathcal{A}^{1\dagger}_1 = \stackrel{03}{[+i]}\stackrel{12}{[+]}\,,\quad {}^{I}\mathcal{A}^{1\dagger}_2 = \stackrel{03}{(+i)}\stackrel{12}{(+)}\,, & {}^{I}\mathcal{A}^{1}_3 = \stackrel{03}{(-i)}\stackrel{12}{[+]}\,\gamma^5\,,\quad {}^{I}\mathcal{A}^{1}_4 = \stackrel{03}{[-i]}\stackrel{12}{(+)}\,\gamma^5\,, \\
{}^{I}\mathcal{A}^{2\dagger}_1 = \stackrel{03}{(-i)}\stackrel{12}{(-i)}\,,\quad {}^{I}\mathcal{A}^{2\dagger}_2 = \stackrel{03}{[-i]}\stackrel{12}{[-]}\,, & {}^{I}\mathcal{A}^{2}_3 = \stackrel{03}{[+i]}\stackrel{12}{(-)}\,\gamma^5\,,\quad {}^{I}\mathcal{A}^{2}_4 = \stackrel{03}{(+i)}\stackrel{12}{[-]}\,\gamma^5\,, \\
{}^{II}\mathcal{A}^{1\dagger}_{\mathbf{1}} = \stackrel{03}{[-\mathbf{i}]}\stackrel{12}{[+]}\,,\quad {}^{II}\mathcal{A}^{1\dagger}_{\mathbf{2}} = \stackrel{03}{(-\mathbf{i})}\stackrel{12}{(+)}\,, & {}^{II}\mathcal{A}^{1\dagger}_3 = \stackrel{03}{(+\mathbf{i})}\stackrel{12}{[+]}\gamma^5\,,\quad {}^{II}\mathcal{A}^{1\dagger}_4 = \stackrel{03}{[+\mathbf{i}]}\stackrel{12}{(+)}\gamma^5\,, \\
{}^{II}\mathcal{A}^{2\dagger}_1 = \stackrel{03}{(+i)}\stackrel{12}{(-)}\,,\quad {}^{II}\mathcal{A}^{2\dagger}_2 = \stackrel{03}{[+i]}\stackrel{12}{[-]}\,, & {}^{II}\mathcal{A}^{2\dagger}_3 = \stackrel{03}{[-i]}\stackrel{12}{(-)}\,\gamma^5\,,\quad {}^{II}\mathcal{A}^{2\dagger}_4 = \stackrel{03}{(-i)}\stackrel{12}{[-]}\,\gamma^5\,.
\end{array}
\qquad (1)
$$

It can clearly be seen that the left-hand side of the Clifford odd "basis vectors" and the right-hand side of the Clifford even "basis vectors", although the former are the Clifford odd objects and the latter are Clifford even objects, have similar properties.

o This is the first step to compare the properties of the Clifford odd and the Clifford even "basis vectors" with the properties of the *string theories* in the case of $d=(9+1)$, for which *strings theory* experts declare that it is favourable.

o We shall demonstrate how do the Clifford odd and Clifford even "basis vectors" reproduce left and right movers of the *string theory* IIA and IIB. Let us repeat:

$$\hat{\mathbf{b}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} \hat{\mathbf{b}}^{\mathbf{m}`\dagger}_{\mathbf{f}`} \;=\; 0\,, \quad \hat{\mathbf{b}}^{\mathbf{m}}_{\mathbf{f}} *_{\mathbf{A}} \hat{\mathbf{b}}^{\mathbf{m}`}_{\mathbf{f}`} = 0\,, \quad \forall m, m', f, f`\,.$$

o *One can obtain the* Clifford even "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, as algebraic products of the Clifford odd "basis vectors" and their Hermitian conjugated partners,

- ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} = \hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`})^{\dagger}$,
- ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f} = (\hat{b}^{m'\dagger}_{f`})^{\dagger} *_A \hat{b}^{m'\dagger}_{f``}$.

- o One can check that all $2^{\frac{d}{2}-1}\times 2^{\frac{d}{2}-1}$ of ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ are generated by any of $2^{\frac{d}{2}-1}$ $f`$ by the relation
  ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} = \hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f''=f`})^{\dagger}$,
  when m' **and** m'' **run** $(1, 2, ..., 2^{\frac{d}{2}-1})$.
- o One can check that all $2^{\frac{d}{2}-1}\times 2^{\frac{d}{2}-1}$ of ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ are generated by any of $2^{\frac{d}{2}-1}$ $m`$ by the relation
  ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f} = (\hat{b}^{m'\dagger}_{f`})^{\dagger} *_A \hat{b}^{m'\dagger}_{f''}$,
  when f' **and** f'' **run** $(1, 2, ..., 2^{\frac{d}{2}-1})$.
- One finds that $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`})^{\dagger}$ applying on $\hat{b}^{m'''\dagger}_{f`''}$ obey

$$
{}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} \hat{\mathbf{b}}^{\mathbf{m}'\dagger}_{\mathbf{f}`} \rightarrow \left\{ \begin{array}{c} \hat{\mathbf{b}}^{\mathbf{m}\dagger}_{\mathbf{f}`}\,,, \\ \text{or zero}\,, \end{array} \right.
$$

and that $\hat{b}^{m'''\dagger}_{f``'}$ applying on $(\hat{b}^{m'\dagger}_{f`})^{\dagger} *_A \hat{b}^{m'\dagger}_{f``}$ obey

$$
\hat{\mathbf{b}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_A {}^{\mathbf{II}}\hat{\mathcal{A}}^{\mathbf{m}'\dagger}_{\mathbf{f}`} \rightarrow \left\{ \begin{array}{c} \hat{\mathbf{b}}^{\mathbf{m}\dagger}_{\mathbf{f}``}\,, \\ \text{or zero}\,, \end{array} \right.
$$

o If the handedness of the Clifford odd "basis vectors" is chosen to be the right handedness,

$$\Gamma^{(d)} = \prod_a (\sqrt{\eta^{aa}}\gamma^a) \cdot \begin{cases} (i)^{\frac{d}{2}}\,, & \text{for d even}\,,\\ (i)^{\frac{d-1}{2}}\,, & \text{for d odd}\,, \end{cases}$$

then their Hermitian conjugated parters have left handedness (for either $\mathcal{S}^{12} = +1$ and $\mathcal{S}^{12} = -1$), resembling left and right movers contributing to boson strings in *string theories* AII and BII.

o The Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$, with $\mathcal{S}^{12} = 1$ and -1, for $d = (5+1)$ is presented below as, $\hat{b}^{m'\dagger}_1 *_A (\hat{b}^{m''\dagger}_1)^\dagger$.

(**There are equivalently the same number of Clifford even "basis vectors"** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$, for $\mathcal{S}^{12} = 0$.)

$\mathcal{S}^{12}$	symbol	${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} =$	$\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`})^{\dagger}$
1	$\star\star$	${}^{I}\hat{\mathcal{A}}^{1\dagger}_{1} =$	$\hat{b}^{1\dagger}_{1} *_A (\hat{b}^{4\dagger}_{1})^{\dagger}$
		$\stackrel{03}{[+i]}\stackrel{12}{(+)}\stackrel{56}{(+)}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]} *_A \stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{(+)}$
1	$\ddagger$	${}^{I}\hat{\mathcal{A}}^{3\dagger}_{1} =$	$\hat{b}^{3\dagger}_{1} *_A (\hat{b}^{4\dagger}_{1})^{\dagger}$
		$\stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{[-]}$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)} *_A \stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{(+)}$
1	$\odot\odot$	${}^{I}\hat{\mathcal{A}}^{1\dagger}_{4} =$	$\hat{b}^{1\dagger}_{1} *_A (\hat{b}^{2\dagger}_{1})^{\dagger}$
		$\stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{[+]}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]} *_A \stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{[+]}$
1	$\otimes$	${}^{I}\hat{\mathcal{A}}^{3\dagger}_{4} =$	$\hat{b}^{3\dagger}_{1} *_A (\hat{b}^{2\dagger}_{1})^{\dagger}$
		$\stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{(-)}$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)} *_A \stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{[+]}$
-1	$\otimes$	${}^{I}\hat{\mathcal{A}}^{2\dagger}_{2} =$	$\hat{b}^{2\dagger}_{1} *_A (\hat{b}^{3\dagger}_{1})^{\dagger}$
		$\stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{(+)}$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{[+]} *_A \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(+)}$
-1	$\ddagger$	${}^{I}\hat{\mathcal{A}}^{4\dagger}_{2} =$	$\hat{b}^{4\dagger}_{1} *_A (\hat{b}^{3\dagger}_{1})^{\dagger}$
		$\stackrel{03}{(+i)}\stackrel{12}{(-)}\stackrel{56}{[-]}$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\stackrel{56}{(-)} *_A \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(+)}$
-1	$\odot\odot$	${}^{I}\hat{\mathcal{A}}^{2\dagger}_{3} =$	$\hat{b}^{2\dagger}_{1} *_A (\hat{b}^{1\dagger}_{1})^{\dagger}$
		$\stackrel{03}{(-i)}\stackrel{12}{(-)}\stackrel{56}{[+]}$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{[+]} *_A \stackrel{03}{(-i)}\stackrel{12}{[+]}\stackrel{56}{[+]}$
-1	$\star\star$	${}^{I}\hat{\mathcal{A}}^{4\dagger}_{3} =$	$\hat{b}^{4\dagger}_{1} *_A (\hat{b}^{1\dagger}_{1})^{\dagger}$
		$\stackrel{03}{[+i]}\stackrel{12}{(-)}\stackrel{56}{(-)}$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\stackrel{56}{(-)} *_A \stackrel{03}{(-i)}\stackrel{12}{[+]}\stackrel{56}{[+]}$

o To keep in mind:

The Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ are products of one projector and two nilpotents, the Clifford odd "basis vectors" and their **Hermitian conjugated partners are products of one nilpotent and two projectors or of three nilpotents.**

The Clifford even and Clifford odd objects are eigenvectors of all the corresponding Cartan subalgebra members. There are $2^{\frac{6}{2}-1} \times 2^{\frac{6}{2}-1}$ **algebraic products of** $\hat{b}^{m'\dagger}_{1} *_{A} \; (\hat{b}^{m''\dagger}_{1})^{\dagger}$.
The rest 8 **of** 16 **members present** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ **with** $\mathcal{S}^{12} = 0$.

o The members $\hat{b}^{m'\dagger}_{f}$ **together with their Hermitian conjugated partners of each of the four families**,
$f = (1, 2, 3, 4)$**, offers the same** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ **with** $\mathcal{S}^{12} = \pm 1$ **as the ones presented in this table**.

o The **Clifford even "basis vectors"** $^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, **belonging to transverse momentum in internal space,** $\mathcal{S}^{12}$ **equal to** 1, **the first half** $^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, **and** -1, **the second half** $^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, **for** $d=(5+1)$, **are presented as algebraic products of the first,** $m'=1$, **member of the "basis vectors"** $\hat{b}^{m'=1\dagger}_{f'}$ **and the Hermitian conjugated partners** $(\hat{b}^{m'=1\dagger}_{f''})^{\dagger}$.
The **Hermitian conjugated partners of two** $^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ **are marked with the same symbol**.

The **Clifford even "basis vectors"** $^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ **are products of one projector and two nilpotents**, the **Clifford odd "basis vectors" and the Hermitian conjugated partners are products of one nilpotent and two projectors or of three nilpotents.**

There are again $2^{\frac{6}{2}-1}\times 2^{\frac{6}{2}-1}$ **algebraic products of** $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m'\dagger}_{f``})^{\dagger}$, $f`$ **and** $f``$ **run over all four families. The rest** 8 **of** 16 **members presents** $^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ **with** $\mathcal{S}^{12}=0$.

The **members** $\hat{b}^{m'\dagger}_{f`}$ **together with** $(\hat{b}^{m'\dagger}_{f`'}$ $m'=(1,2,3,4)$, **offers the same** $^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ with $\mathcal{S}^{12}=\pm 1$ **as the ones presented in this table.**

(And equivalently for $\mathcal{S}^{12}=0$.)

$\mathcal{S}^{12}$	*symbol*	${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f} =$	$(\hat{b}^{1\dagger}_{f`})^{\dagger} *_{A} \hat{b}^{1\dagger}_{f``}$
1	$\star\star$	${}^{II}\hat{\mathcal{A}}^{1\dagger}_{1} =$	$(\hat{b}^{1\dagger}_{1})^{\dagger} *_{A} \hat{b}^{1\dagger}_{4}$
		$\overset{03}{[-i]}\,\overset{12}{(+)}\overset{56}{(+)}$	$\overset{03}{(-i)}\,\overset{12}{[+]}\overset{56}{[+]} *_{A} \overset{03}{(+i)}\,\overset{12}{(+)}\overset{56}{(+)}$
1	$\odot\odot$	${}^{II}\hat{\mathcal{A}}^{3\dagger}_{1} =$	$(\hat{b}^{1\dagger}_{2})^{\dagger} *_{A} \hat{b}^{1\dagger}_{4}$
		$\overset{03}{(+i)}\,\overset{12}{(+)}\overset{56}{[-]}$	$\overset{03}{[+i]}\,\overset{12}{[+]}\overset{56}{(-)} *_{A} \overset{03}{(+i)}\,\overset{12}{(+)}\overset{56}{(+)}$
1	$\ddagger$	${}^{II}\hat{\mathcal{A}}^{1\dagger}_{4} =$	$(\hat{b}^{1\dagger}_{1})^{\dagger} *_{A} \hat{b}^{1\dagger}_{3}$
		$\overset{03}{(-i)}\,\overset{12}{(+)}\overset{56}{[+]}$	$\overset{03}{(-i)}\,\overset{12}{[+]}\overset{56}{[+]} *_{A} \overset{03}{[+i]}\,\overset{12}{(+)}\overset{56}{[+]}$
1	$\otimes$	${}^{II}\hat{\mathcal{A}}^{3\dagger}_{4} =$	$(\hat{b}^{1\dagger}_{2})^{\dagger} *_{A} \hat{b}^{1\dagger}_{3}$
		$\overset{03}{[+i]}\,\overset{12}{(+)}\overset{56}{(-)}$	$\overset{03}{[+i]}\,\overset{12}{[+]}\overset{56}{(-)} *_{A} \overset{03}{[+i]}\,\overset{12}{(+)}\overset{56}{[+]}$
-1	$\otimes$	${}^{II}\hat{\mathcal{A}}^{2\dagger}_{2} =$	$(\hat{b}^{1\dagger}_{3})^{\dagger} *_{A} \hat{b}^{1\dagger}_{2}$
		$\overset{03}{[+i]}\,\overset{12}{(-)}\overset{56}{(+)}$	$\overset{03}{[+i]}\,\overset{12}{(-)}\overset{56}{[+]} *_{A} \overset{03}{[+i]}\,\overset{12}{[+]}\overset{56}{(+)}$
-1	$\otimes\otimes$	${}^{II}\hat{\mathcal{A}}^{4\dagger}_{2} =$	$(\hat{b}^{1\dagger}_{4})^{\dagger} *_{A} \hat{b}^{1\dagger}_{2}$
		$\overset{03}{(-i)}\,\overset{12}{(-)}\overset{56}{[-]}$	$\overset{03}{(-i)}\,\overset{12}{(-)}\overset{56}{(-)} *_{A} \overset{03}{[+i]}\,\overset{12}{[+]}\overset{56}{(+)}$
-1	$\ddagger$	${}^{II}\hat{\mathcal{A}}^{2\dagger}_{3} =$	$(\hat{b}^{1\dagger}_{3})^{\dagger} *_{A} \hat{b}^{1\dagger}_{1}$
		$\overset{03}{(+i)}\,\overset{12}{(-)}\overset{56}{[+]}$	$\overset{03}{[+i]}\,\overset{12}{(-)}\overset{56}{[+]} *_{A} \overset{03}{(+i)}\,\overset{12}{[+]}\overset{56}{[+]}$
-1	$\star\star$	${}^{II}\hat{\mathcal{A}}^{4\dagger}_{3} =$	$(\hat{b}^{1\dagger}_{4})^{\dagger} *_{A} \hat{b}^{1\dagger}_{1}$
		$\overset{03}{[-i]}\,\overset{12}{(-)}\overset{56}{(-)}$	$\overset{03}{(-i)}\,\overset{12}{(-)}\overset{56}{(-)} *_{A} \overset{03}{(+i)}\,\overset{12}{[+]}\overset{56}{[+]}$

Let us repeat what we have learned about the Clifford even and the **Clifford odd "basis vectors" in even dimensional spaces**.

There are in even dimensional spaces $2^{\frac{d}{2}-1}$ **Clifford odd families**, **each family having** $2^{\frac{d}{2}-1}$ **members**. The **Clifford odd "basis vectors"** have their **Hermitian conjugated partners in a separate group of** $2^{\frac{d}{2}-1}$ **families with** $2^{\frac{d}{2}-1}$ **members**.

There are in even dimensional spaces two times $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ **Clifford even basis vectors, with their Hermitian conjugated partners within the same group.**

In a tensor product with the basis in ordinary space the **Clifford odd "basis vectors", together with their Hermitian conjugated partners, and** the **Clifford even "basis vectors", form creations and annihilation operators, which fulfil on the vacuum state the postulates of the second quantized fermion and boson fields**.

o Both are represented by the points in the ordinary space.

o **Looking at the properties of both kinds of** the **Clifford even "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$** and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, **manifesting momentum in only transverse dimensions (with $\mathcal{S}^{ab}$ not equal $\mathcal{S}^{03}$), we found in both Tables**, **that to both groups of the Clifford even "basis vectors"** all the **family members** m and all the **families** f **contribute**:

o **a.** To ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$, **all** the **family members** m **for a particular family** f **and their Hermitian conjugated partners contribute in** $\hat{b}^{m'\dagger}_{f'} *_A (\hat{b}^{m''\dagger}_{f'})^{\dagger}$, **using only half of possibilities ($\frac{1}{2} \times 2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$), the other half possibilities contribute to** $\mathcal{S}^{12} = 0$. **Each family** f' of $\hat{b}^{m'\dagger}_{f'} *_A (\hat{b}^{m''\dagger}_{f'})^{\dagger}$ **generates the same eight Clifford even** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ as are the **ones presented in the first of the above Tables for** $f' = 1$.

o **b.** To ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, all the **families** f' **of a particular member** m' **and their Hermitian conjugated partners contribute in** $(\hat{b}^{m'\dagger}_{f'})^{\dagger} *_A \hat{b}^{m'\dagger}_{f''}$, **using only half of possibilities, the other half contribute to** $\mathcal{S}^{12} = 0$. Each **family member** m' **generates in** $(\hat{b}^{m'\dagger}_{f'})^{\dagger} *_A \hat{b}^{m'\dagger}_{f''}$ **the same eight Clifford even** ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ **as are the ones presented in the second one of above Tables above for** $m' = 1$.

o **We find in** $d=(9+1)$ **which is the boson** *string* **AII and BII case, according to what it is discussed so far on the case of** $d=(5+1)$**, in the case that we are interested only on those internal degrees of freedom of** the **Clifford even "basis vectors" of each of the two kinds**,
${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$**, which manifest momentum in only transverse dimensions (with** $\mathcal{S}^{ab}$ **not equal** $\mathcal{S}^{03}$**),**
$\frac{1}{2}\times 2^{\frac{d=10}{2}-1}\times 2^{\frac{d=10}{2}-1}=8\times 16=128$ **of** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$**, and** 128 **of** ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$**, together** 256 **of both kinds of** the **Clifford even "basis vectors", representing** the **boson fields**.

These are also possibilities suggested in reference of **Kevin Wray ("An Introduction to String Theory", Preprint typeset in JHEP style - paper version)**, for **closed strings in** $d=(9+1)$**; for** the **left-right movers** or **right-left movers forming the closed** *boson strings* **of AII and BII kinds, manifesting the momentum in only transverse dimensions they found** 256 **possibilities**.

Our way of presenting the Clifford even "basis vectors" of two kinds ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, **which manifest momentum in only transverse dimensions agrees** with the **properties of the** *closed strings* **in** $d=(9+1)$.

o The *strings theories* **seems to offer the way for explaining the so far observed fermion and boson second quantized fields, with gravity included, by offering the renormalizability of the theory by extending the point fermions and bosons into strings and by offering the supersymmetry among fermions and bosons.**

o **We expect that in the low energy regime the** *string theories* **coincide with our predictions presented in this workshop provided that we extend points in the ordinary space to strings, hoping that this would help to solve the problem of renormalisability** of the *spin-charge-family* **theory.**

o Still a hard work is needed to make the next step towards relating the *string theories* and the *spin-charge-family* theory.

o However, the description of the internal spaces of **fermion** and **boson fields** with the **Clifford odd** and **Clifford even "basis vectors", respectively, is simple and well defined, it might bring a new understanding of the theory of our world**.

o The first to be discovered is why the string theories find as the only acceptable dimensions $d = (9+1)$ **and** $d = (25+1)$.

o Our way of presenting internal spaces of **fermions** and **bosons seems to treat all** $d = 2(2n+1)$ in an **equivalent way**.

o The *spin-charge-family* **theory sees** $d = (13 + 1)$ **as an elegant possibility which allows the explanation of all the assumptions of the** *standard model* **before the electroweak break, with the higgs and Yukawa couplings included**,

o **offering the explanation of the second quantization** of **fermion** and **boson fields**, **explaining also the appearance** of the **dark matter, matter-antimetter asymmetry**, and **other observations included, with the choice of the simple and elegant action.**

o **The extension of the point fields in ordinary space to strings brings the hope for assuring renormalizability of the** *spin-charge-family* **theory**.