PBHs and Primordial non-gaussianity: from their abundance to gravitational waves

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Sapienza Università di Roma

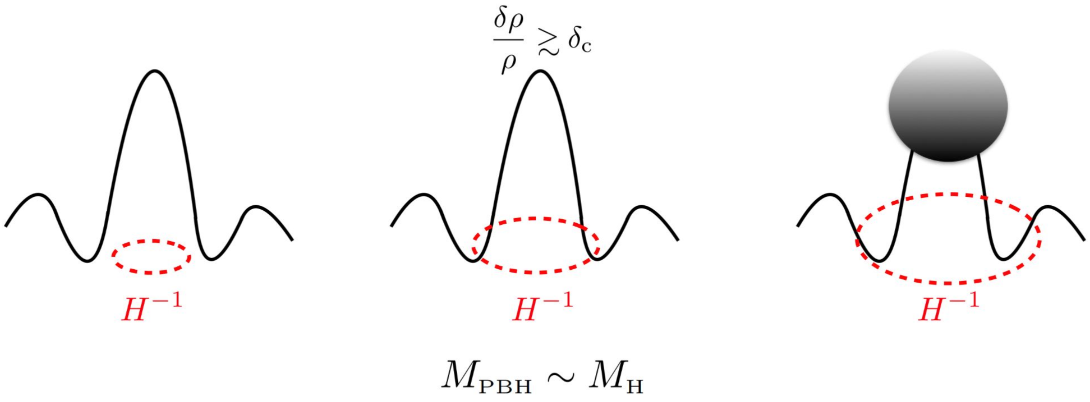
ZOOMing in on PBH,
December 4th, 2023

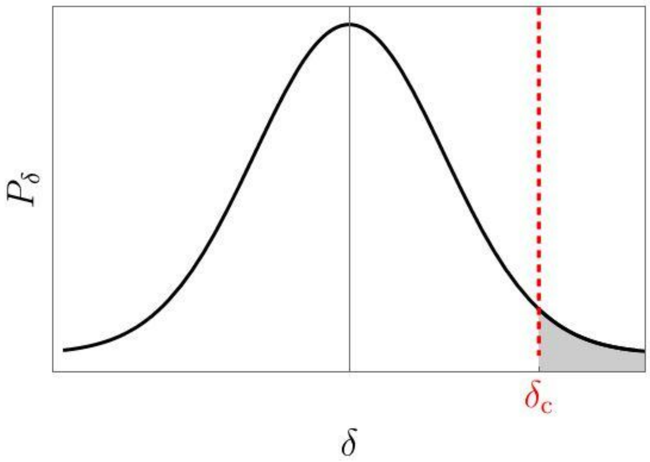
Primordial Black Holes: outline presentation

REVIEW: A. M. Green and B. J. Kavanagh.– arXiv:2007.10722

PART 1) ABUNDANCE of PBH:
The role of NGs
arXiv:2211.01728 (Published on PRD)
G.Ferrante, G.Franciolini, A.J.I., A.Urbano
PART 2) GRAVITATIONAL WAVES:
PBH as possible explanation of PTA
arXiv:2306.17149 (Published on PRL)
G.Franciolini, A.J.I., H. Veermae, V. Vaskonen

Abundance of PBHs

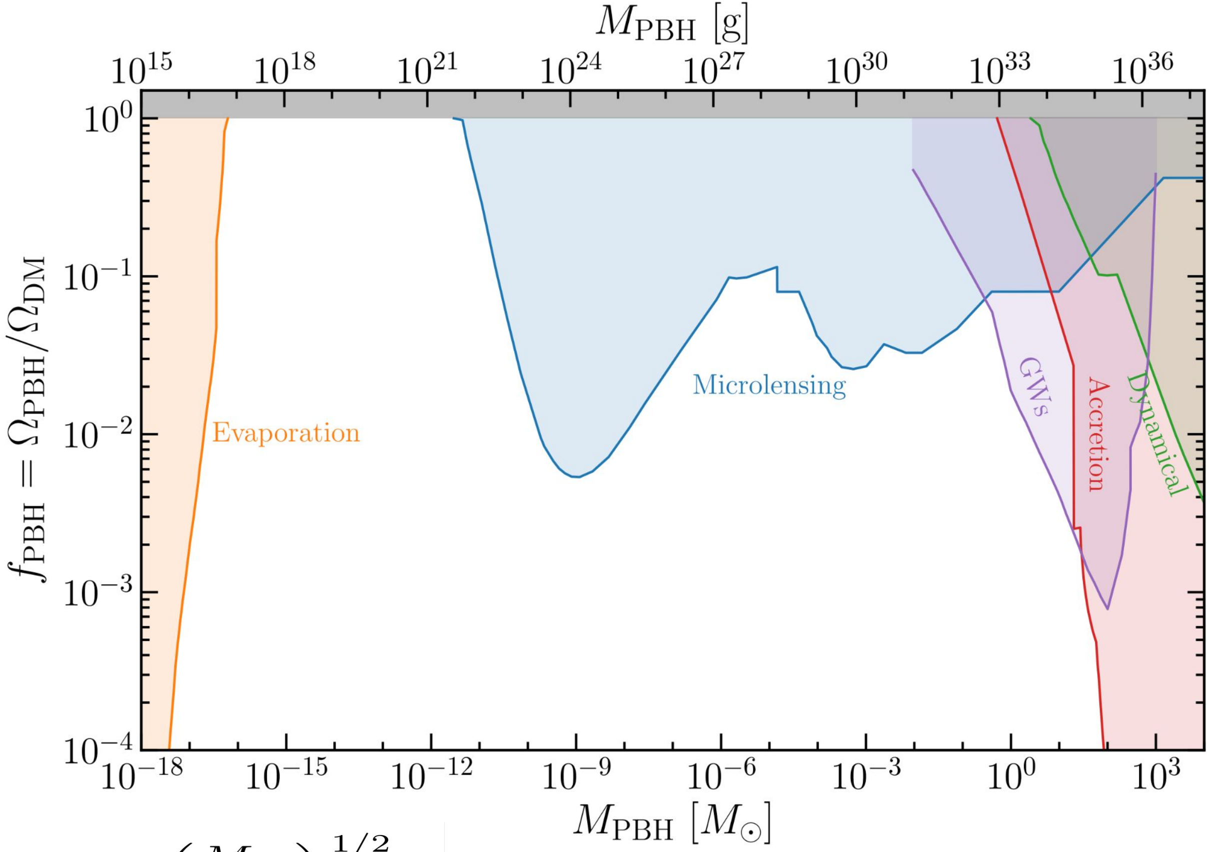




$$\beta = \int_{\delta_c}^{\infty} \mathcal{K}(\delta - \delta_c)^{\gamma}\, \mathrm{P}_\delta(\delta) d\delta$$

Primordial Black Holes as DM candidates

Mass Distribution



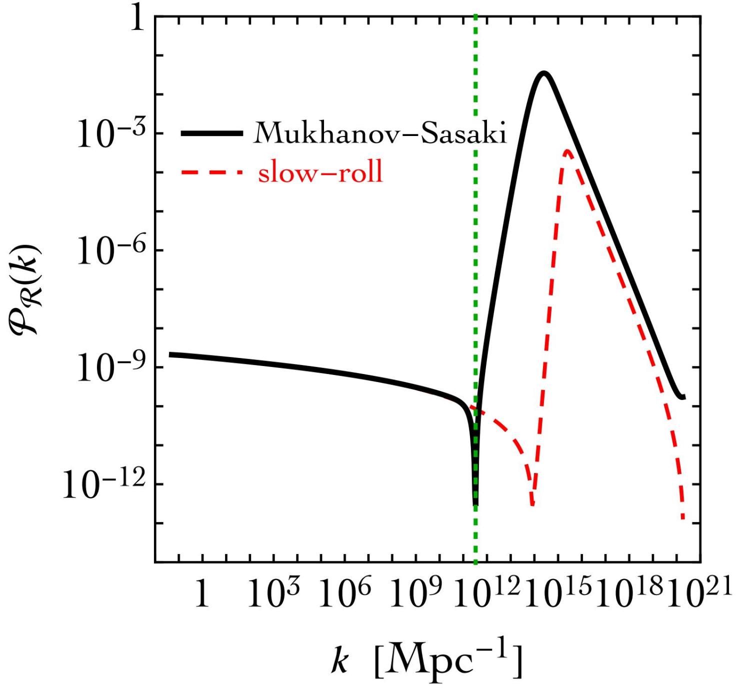
PBH as Dark Matter:

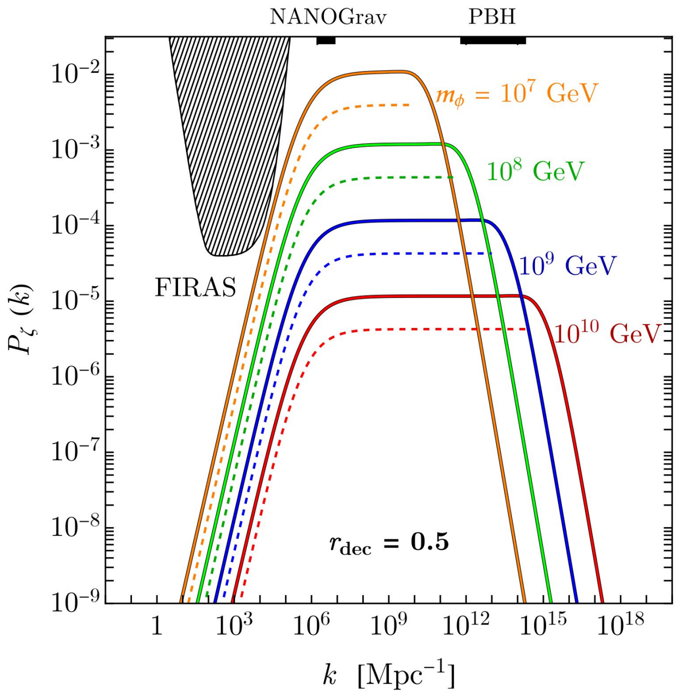
- USR models
- Hybrid inflation
- Curvaton field
- exotic formation mechanisms (bubble collisions and so on)
- And etc etc…

$$\Omega_{\rm PBH} = \int d\log M_H \left(\frac{M_{\rm eq}}{M_H}\right)^{1/2} \beta$$

Primordial Black Holes as DM candidates

What we can compute during inflation is the curvature perturbation field ζ (or *R*).





PBH as Dark Matter:

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$$M_H \simeq 17 M_\odot \left(\frac{g_\star}{10.75}\right)^{-1/6} \left(\frac{k/\kappa}{10^6 \mathrm{Mpc}^{-1}}\right)^{-2}$$

Abundance of PBHs: The role of Non-Gaussianities (NG).

An exact formalism for the computation of PBHs mass fraction abundance NGs in the curvature perturbation field ζ:

NON-LINEARITIES (NL)

$$\delta(\vec{x},t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(\vec{x})}\left[\nabla^2\zeta(\vec{x}) + \frac{1}{2}\partial_i\zeta(\vec{x})\partial_i\zeta(\vec{x})\right]$$

T. Harada, C. M. Yoo, T. Nakama and Y. Koga,.– arXiv:1503.03934

PRIMORDIAL NG IN $\zeta = F(\zeta_G)$

$$\zeta = F(\zeta_{\rm G})$$

power-series expansion [28–34, 36–39]

$$\zeta = \zeta_{\rm G} + \tfrac{3}{5} f_{\rm NL}\zeta_{\rm G}^2 + \tfrac{9}{25} g_{\rm NL}\zeta_{\rm G}^3 + \dots$$

ultra slow-roll with an upward step [50]

$$\zeta = -\tfrac{2}{|h|}\left[\sqrt{1-|h|\zeta_{\rm G}} - 1\right]$$

curvaton [48, 49]

$$\zeta = \log\left[X(r_{\rm dec}, \zeta_{\rm G})\right]$$

ultra slow-roll [42]

$$\zeta = -\left(\tfrac{6}{5} f_{\rm NL}\right)^{-1}\log\left(1 - \tfrac{6}{5} f_{\rm NL}\zeta_{\rm G}\right)$$

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$$\zeta = \log\left[X(r_{dec},\zeta_G)\right]$$

$$r_{dec} \equiv \left.\frac{3\rho_\phi}{3\rho_\phi + 4\rho_\gamma}\right|_{\text{curvaton decay}}$$

ultra slow-roll [42]

$$\zeta = -\left(\tfrac{6}{5} f_{NL}\right)^{-1}\log\left(1 - \tfrac{6}{5} f_{NL}\zeta_G\right)$$

Mathematical formulation

S.Young
arXiv:2201.13345

By integrating δ over the radial coordinate r we get the compaction function C

$$\mathcal{C}(r) = -2\Phi\, r\, \zeta'(r)\left[1+\frac{r}{2}\zeta'(r)\right] = \mathcal{C}_1(r) - \frac{1}{4\Phi}\mathcal{C}_1(r)^2\,, \qquad \mathcal{C}_1(r) := -2\Phi\, r\, \zeta'(r)$$

Later on confirmed also by A.Gow et al arXiv:2211.08348

In the presence of NG C_1 takes the form

$$\mathcal{C}_1(r) = -2\Phi\, r\, \zeta'_{\mathrm{G}}(r)\frac{dF}{d\zeta_{\mathrm{G}}} = \mathcal{C}_{\mathrm{G}}(r)\frac{dF}{d\zeta_{\mathrm{G}}}\,, \qquad \text{with} \quad \mathcal{C}_{\mathrm{G}}(r) := -2\Phi\, r\, \zeta'_{\mathrm{G}}(r)$$

From the two-dimensional joint PDF of ζ_G and C_G, called P_G

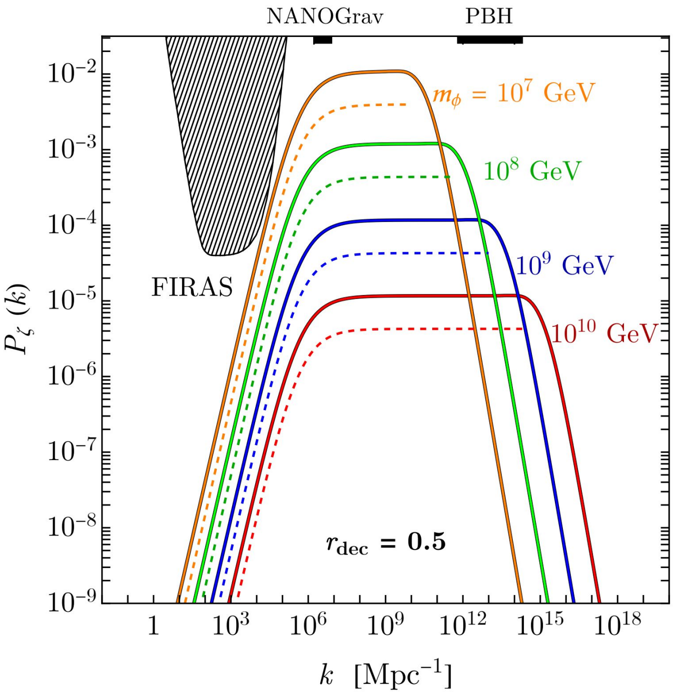
NG PBH mass fraction adopting threshold statistics on the compaction function

$$\beta_{\mathrm{NG}} = \int_{\mathcal{D}} \mathcal{K}(\mathcal{C}-\mathcal{C}_{\mathrm{th}})^{\gamma}\,\mathrm{P_G}(\mathcal{C}_{\mathrm{G}},\zeta_{\mathrm{G}})\,d\mathcal{C}_{\mathrm{G}}\,d\zeta_{\mathrm{G}}\,, \tag{56}$$

$$\mathrm{P_G}(\mathcal{C}_{\mathrm{G}},\zeta_{\mathrm{G}}) = \frac{1}{(2\pi)\sigma_c\sigma_r\sqrt{1-\gamma_{cr}^2}}\exp\left(-\frac{\zeta_{\mathrm{G}}^2}{2\sigma_r^2}\right)\exp\left[-\frac{1}{2(1-\gamma_{cr}^2)}\left(\frac{\mathcal{C}_{\mathrm{G}}}{\sigma_c}-\frac{\gamma_{cr}\zeta_{\mathrm{G}}}{\sigma_r}\right)^2\right], \tag{57}$$

$$\mathcal{D} = \{\mathcal{C}_{\mathrm{G}},\, \zeta_{\mathrm{G}} \in \mathbb{R} \;:\; \mathcal{C}(\mathcal{C}_{\mathrm{G}},\zeta_{\mathrm{G}}) > \mathcal{C}_{\mathrm{th}} \;\wedge\; \mathcal{C}_1(\mathcal{C}_{\mathrm{G}},\zeta_{\mathrm{G}}) < 2\Phi\}\,, \tag{58}$$

Mathematical formulation



$$\langle \mathcal{C}_{\rm G}\mathcal{C}_{\rm G}\rangle = \sigma_c^2 = \frac{4\Phi^2}{9}\int_0^\infty \frac{dk}{k}(kr_m)^4 W^2(k,r_m)T^2(k,r_m)P_\zeta(k)\,,$$

$$\langle \mathcal{C}_{\rm G}\zeta_{\rm G}\rangle = \sigma_{cr}^2 = \frac{2\Phi}{3}\int_0^\infty \frac{dk}{k}(kr_m)^2 W(k,r_m)W_s(k,r_m)T^2(k,r_m)P_\zeta(k)\,,$$

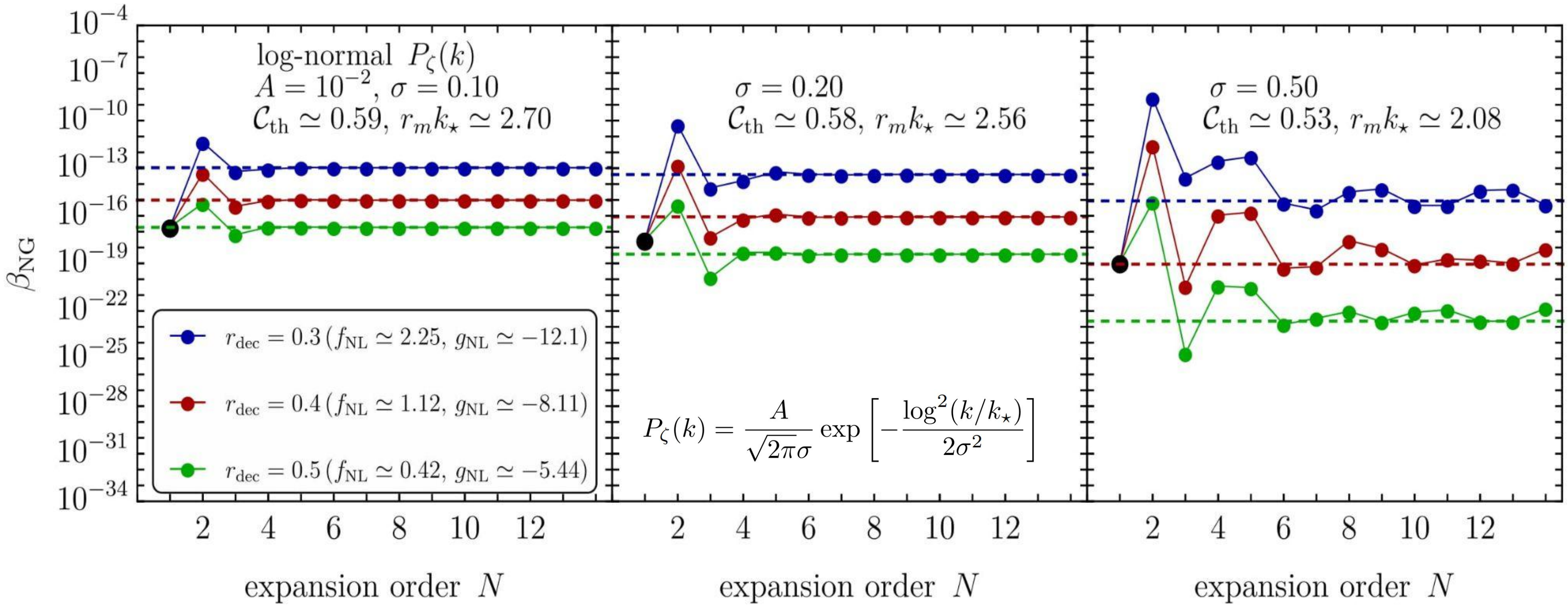
$$\langle \zeta_{\rm G}\zeta_{\rm G}\rangle = \sigma_r^2 = \int_0^\infty \frac{dk}{k}W_s^2(k,r_m)T^2(k,r_m)P_\zeta(k)\,,$$

Application to the curvaton model

Failure of the perturbative approach (Narrow)

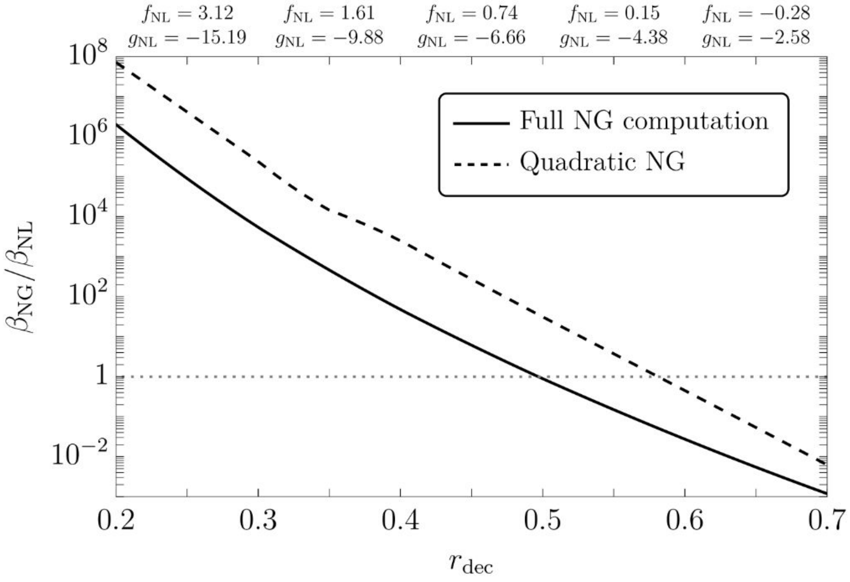
- - - - $\zeta = \log\left[X(r_{\rm dec}, \zeta_{\rm G})\right]$

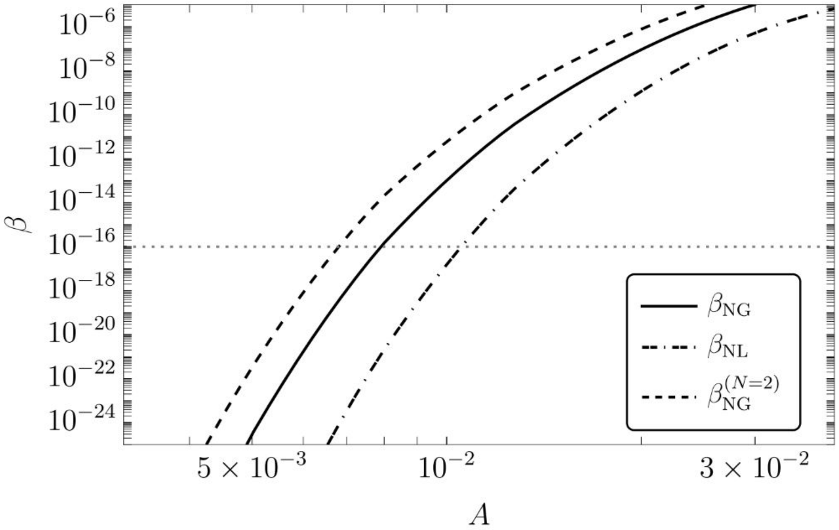
● $\zeta_N = \sum_{n=1}^{N} c_n(r_{\rm dec})\zeta_{\rm G}^n$



Application to the curvaton model

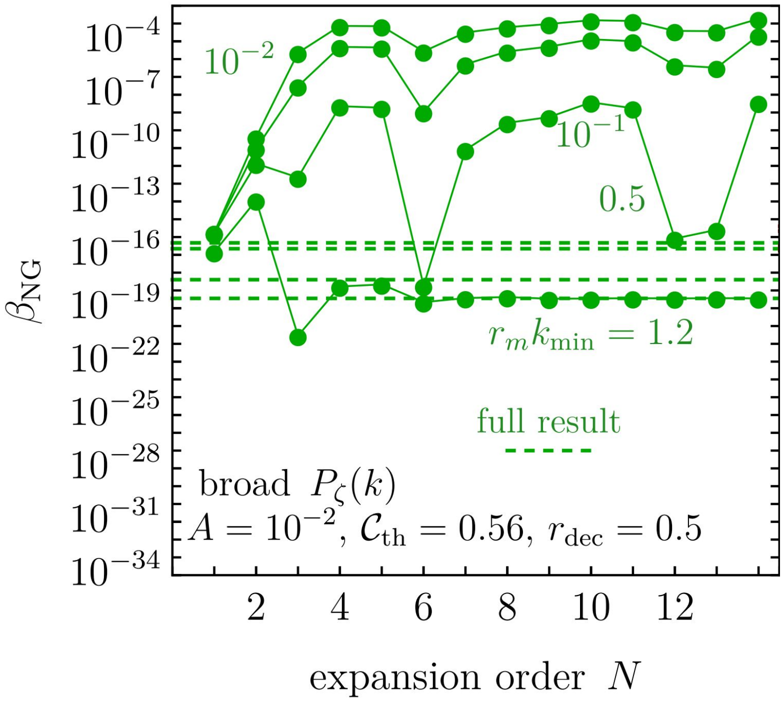
Quadratic app. overestimates the abundance





Application to the curvaton model

Failure of the perturbative approach (Broad)

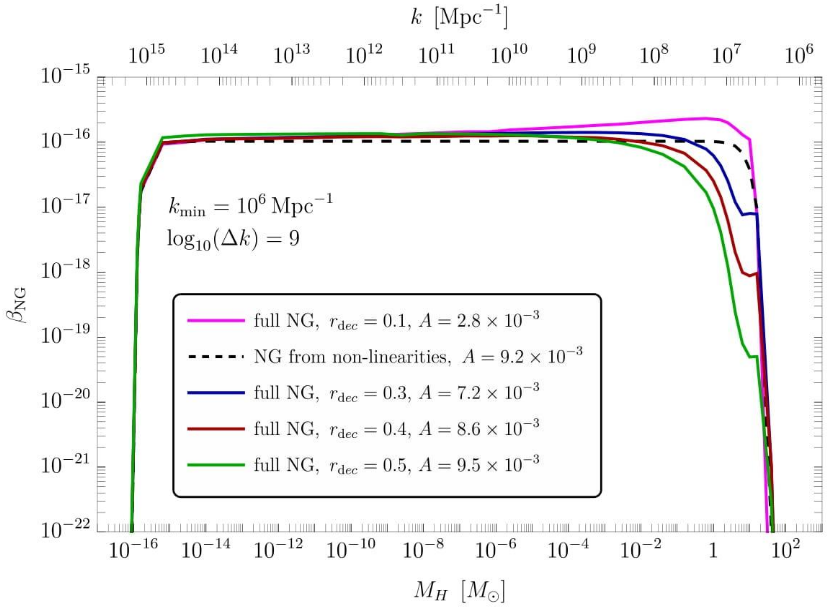


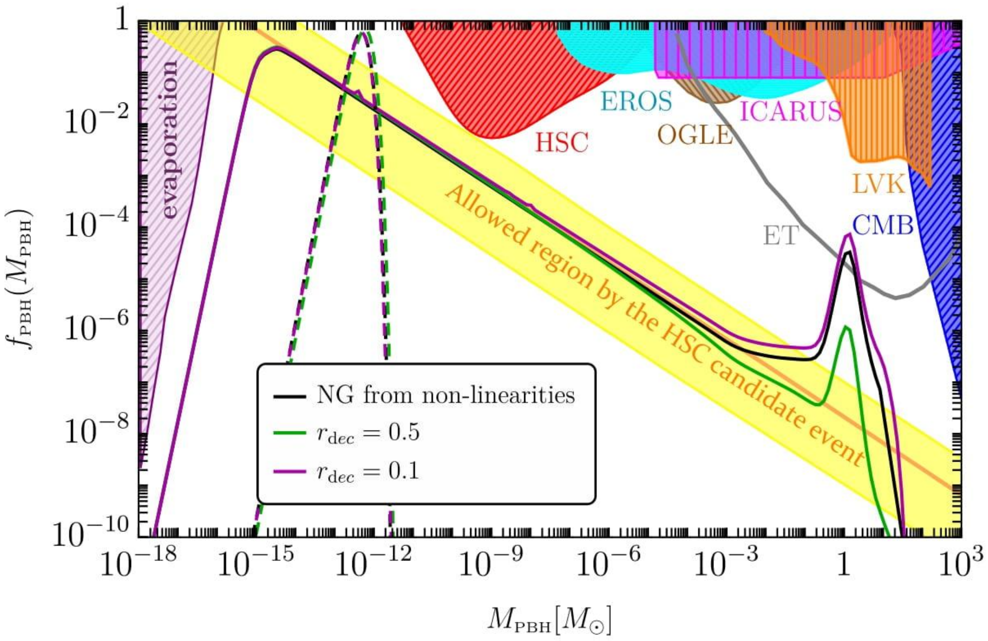
$$P_\zeta(k) = A\,\Theta(k - k_{\min})\,\Theta(k_{\max} - k)$$

For a broad Power spectrum the power-series expansion is simply wrong and one is forced to use the full result NG.

Application to the curvaton model (2)

Breaking of M_H-Independence





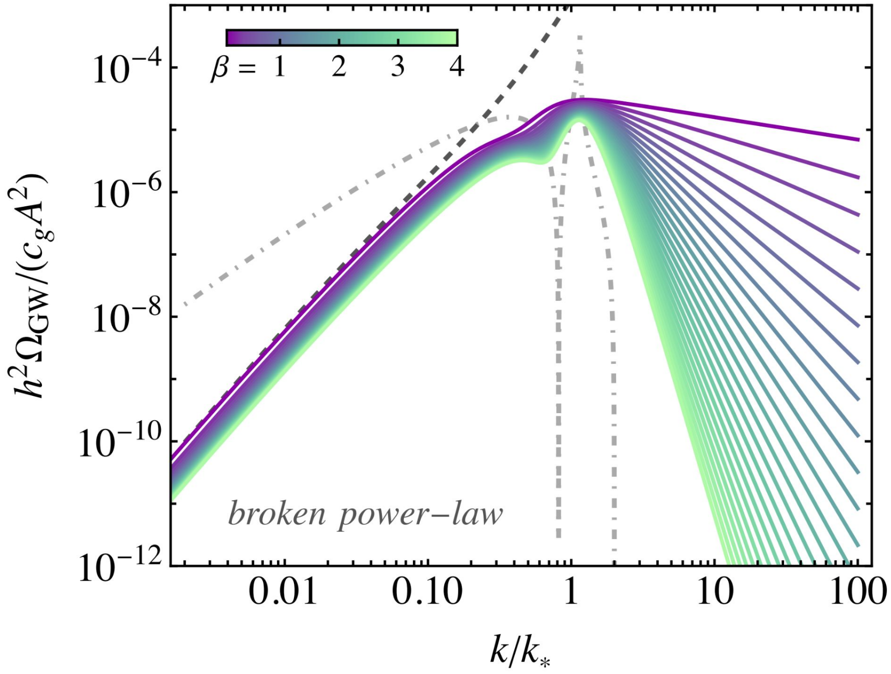
We need another observable: The induced Gravitational waves

PBH and SGWB

SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

$$h^2\Omega_{\rm GW}(k) = \frac{h^2\Omega_r}{24}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*s}}{g_{*s}^0}\right)^{-\frac{4}{3}}\mathcal{P}_h(k)$$

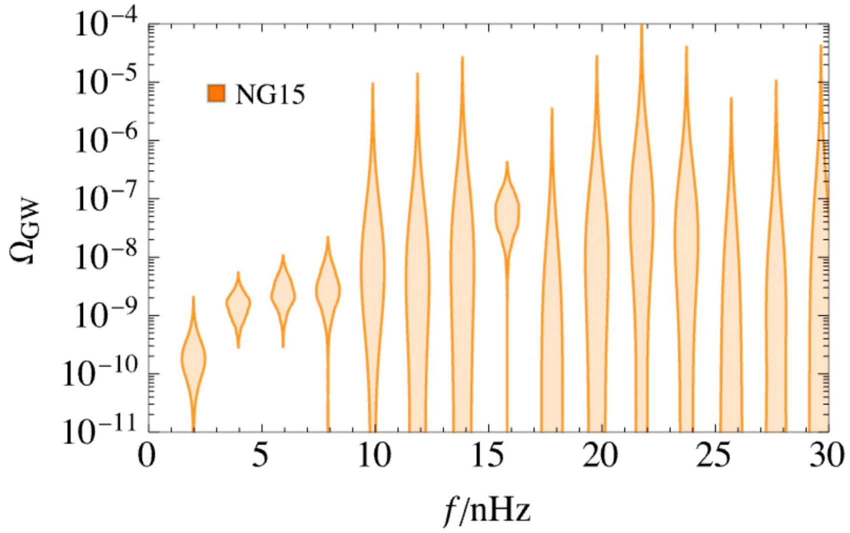
$$\mathcal{P}_h(k) \propto \mathcal{P}_\zeta^2(k)$$

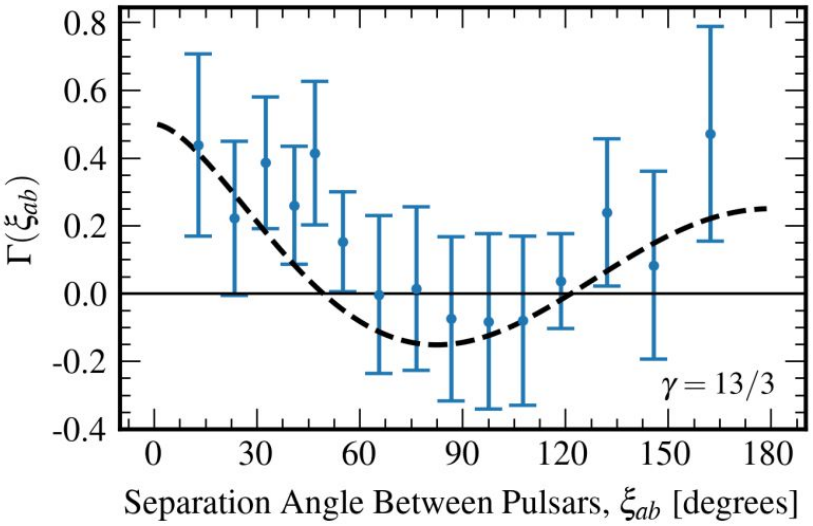


PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

NANOGrav – arXiv:2306.16213
arXiv:2306.16219





PBH and SGWB

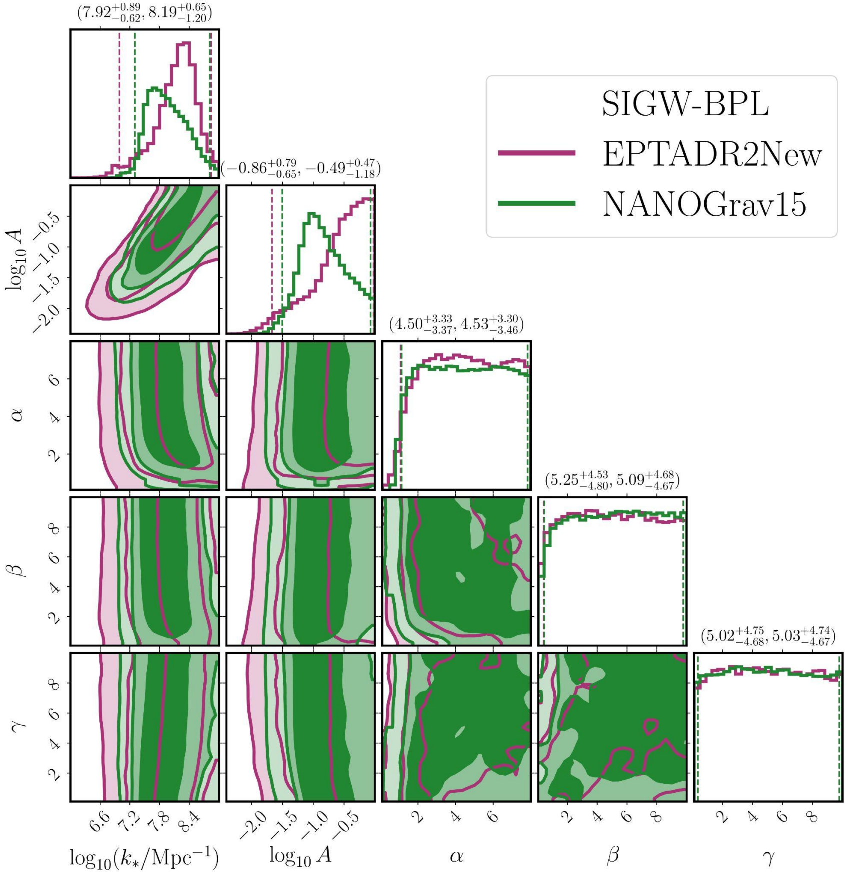
SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

Log-likelihood analysis

Fitting the posterior distributions

$$\mathcal{P}_\zeta^{\mathrm{BPL}}(k) = A\frac{(\alpha+\beta)^\gamma}{\left(\beta\,(k/k_*)^{-\alpha/\gamma}+\alpha\,(k/k_*)^{\beta/\gamma}\right)^\gamma}$$

$$\mathcal{P}_\zeta^{\mathrm{LN}}(k) = \frac{A}{\sqrt{2\pi}\Delta}\,\exp\left(-\frac{1}{2\Delta^2}\,\ln^2(k/k_*)\right)$$



Improvement respect to NANOGrav analysis.

NANOGrav collaboration
arXiv:2306.16219

Power spectrum <> Abundance <> GWs

- Non-Gaussianities in the abundance.
- Dependency of the PBH formation parameters on the PS shape.
- QCD impact on threshold.

NGs in the abundance: Cases under consideration

NON-LINEARITIES (NL)

$$\delta(\vec{x},t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(\vec{x})}\left[\nabla^2\zeta(\vec{x}) + \frac{1}{2}\partial_i\zeta(\vec{x})\partial_i\zeta(\vec{x})\right]$$

$$\delta(\vec{x},t) = -\frac{4}{9a^2H^2}\nabla^2\zeta(\vec{x})$$

PRIMORDIAL NG IN $\zeta = F(\zeta_G)$

$$\zeta = \log\left[X(r_{\rm dec}, \zeta_{\rm G})\right]$$

curvaton case

$$\zeta = -\frac{2}{\beta}\log\left(1 - \frac{\beta}{2}\zeta_{\rm G}\right)$$

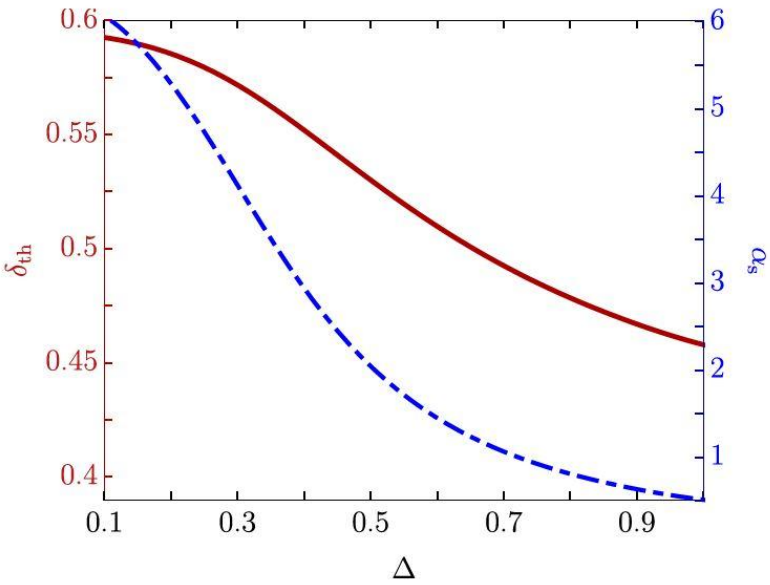
Inflection-point (USR) case

$$\zeta = \zeta_{\rm G} + \frac{3}{5}f_{\rm NL}\zeta_{\rm G}^2$$

Quadratic approx.

Abundance of PBHs: Shape dependencies

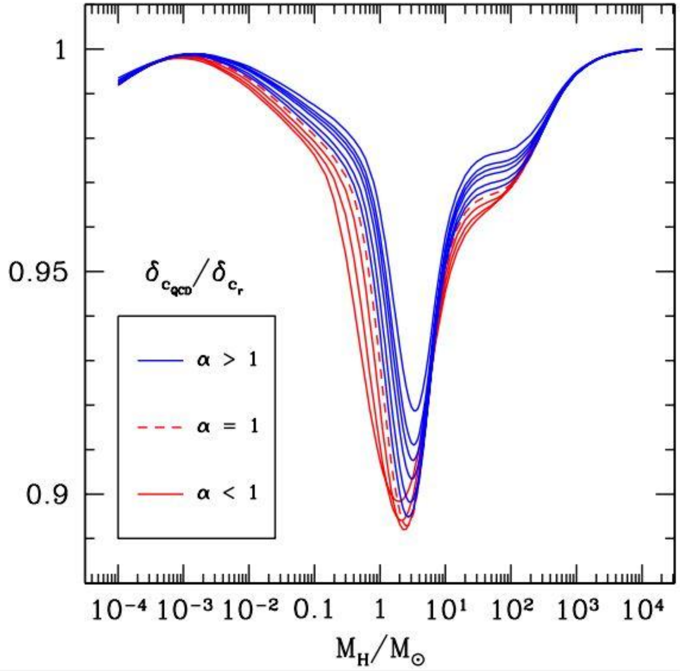
I. Musco, V. De Luca, G. Franciolini, A. Riotto.– arXiv:2011.03014



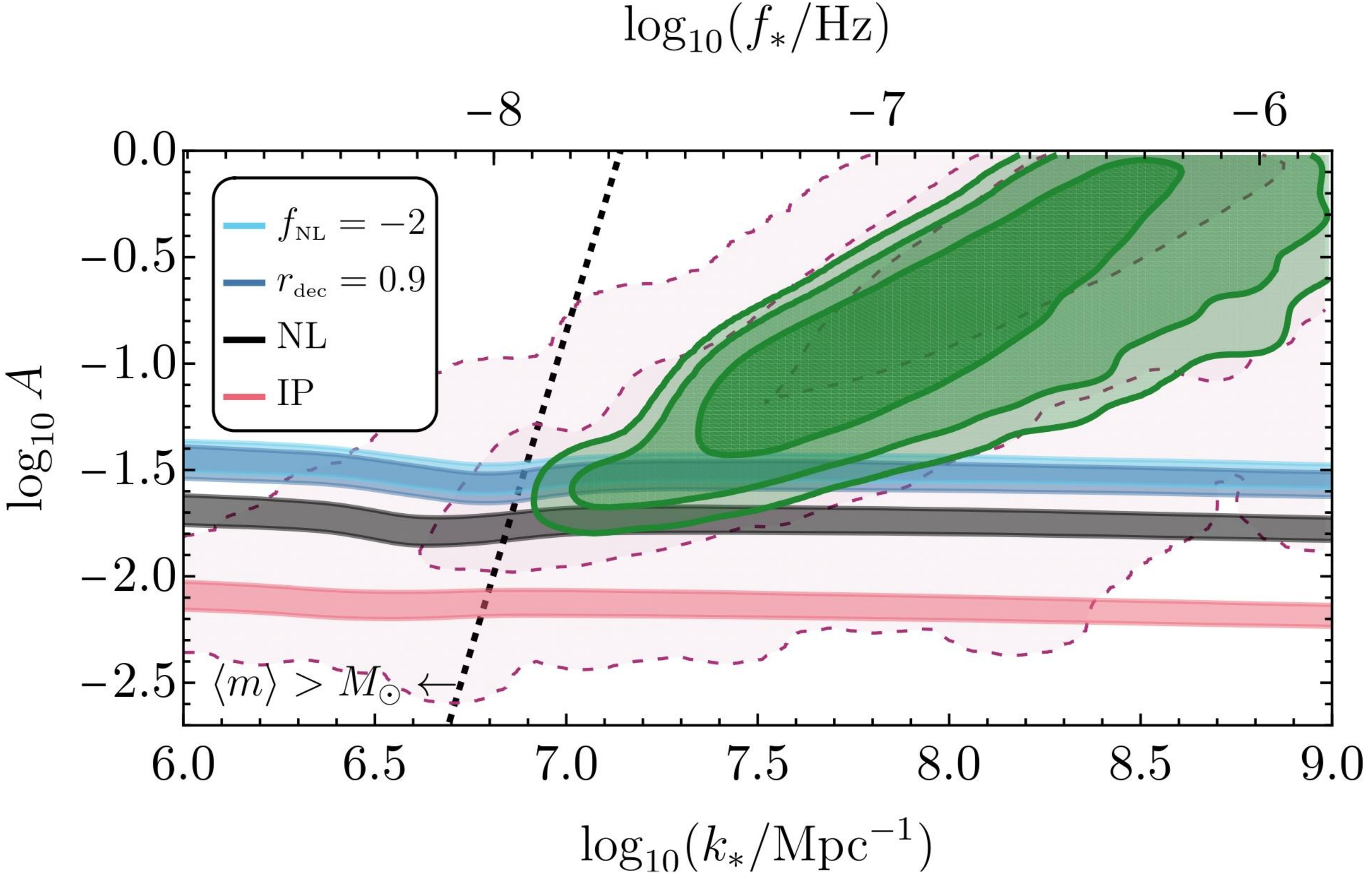
$$\mathcal{P}_\zeta^{\rm LN}(k) = \frac{A}{\sqrt{2\pi}\Delta}\, \exp\left(-\frac{1}{2\Delta^2}\ln^2(k/k_*)\right)$$

QCD phase transitions

I. Musco, K. Jedamzik, S. Young.– arXiv:2303.07980



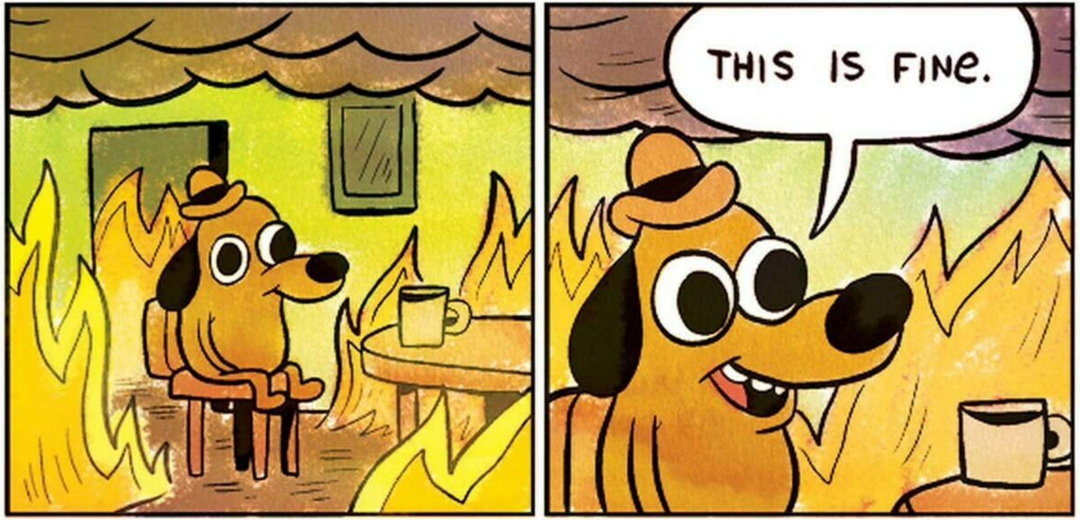
Tension between NANOGrav and PBHs



Conclusions

- *Fundamental to take into account both kind of NGs in the computation for the abundance.*

- *Negative NGs to alleviate the tension between PTA and PBH overproduction.*



A potential issue

Threshold values maybe are not correct? Different super-horizon threshold conditions may lead to an overestimation of the abundance, due to non-linear effects not included in the linear transfer function.

$$\mathcal{C}(r) = -2\Phi\, r\, \zeta'(r)\left[1 + \frac{r}{2}\zeta'(r)\right] = \mathcal{C}_1(r) - \frac{1}{4\Phi}\mathcal{C}_1(r)^2\,, \qquad \mathcal{C}_1(r) := -2\Phi\, r\, \zeta'(r)$$

V. De Luca, A. Kehagias, A. Riotto.– arXiv:2307.13633

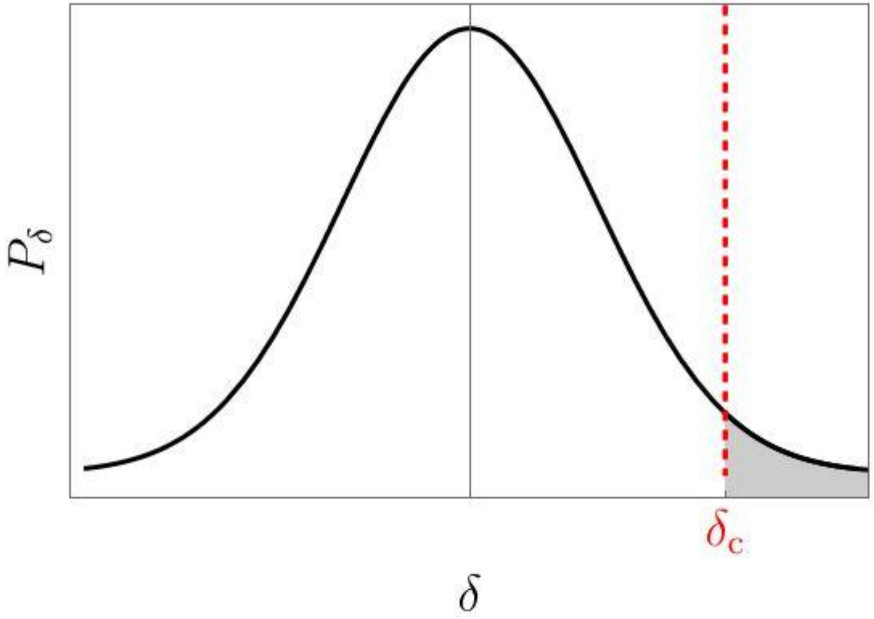
Next step: finding a new prescription.

Backup Slides

Abundance of PBHs

Mass Fraction

$$\beta = \int_{\delta_c}^{\infty} \mathcal{K}(\delta - \delta_c)^{\gamma}\, \mathrm{P}_{\delta}(\delta) d\delta$$



Mass distribution

$$f_{\rm PBH}(M_{\rm PBH}) \equiv \frac{1}{\Omega_{\rm DM}} \frac{d\Omega_{\rm PBH}}{d\log M_{\rm PBH}}\,, \qquad \Omega_{\rm PBH} = \int d\log M_H \left(\frac{M_{\rm eq}}{M_H}\right)^{1/2} \beta_{\rm NG}(M_H)\,,$$

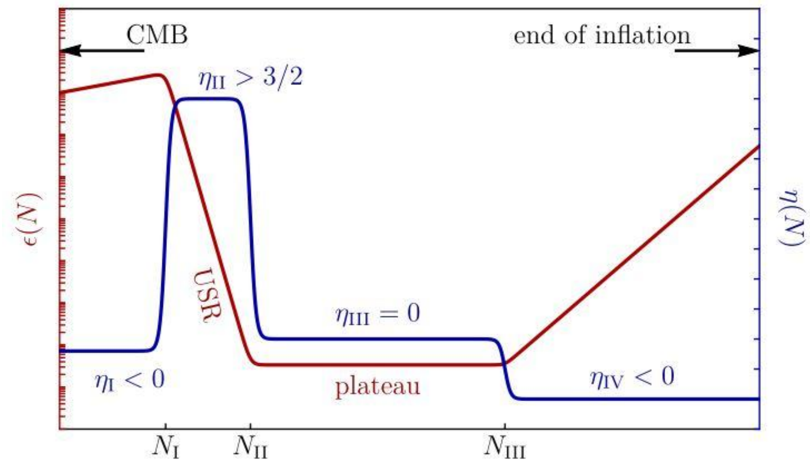
Abundance of PBHs: The role of curvature perturbation ζ (or R).

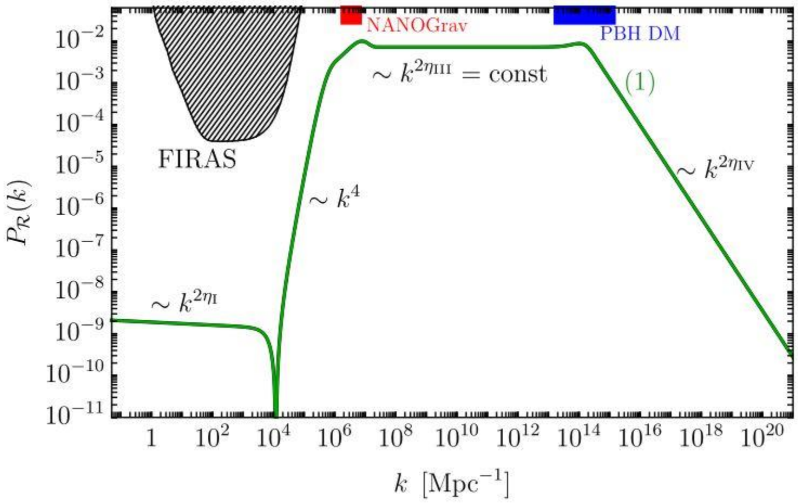
For the moment:
- ζ is gaussian
- $\delta(\vec{x},t) = -\frac{4}{9a^2H^2}\nabla^2\zeta(\vec{x})$ $\qquad P_\delta(k,t) = \frac{16}{81}\frac{k^4}{a^4H^4}P_\zeta(k)$

In order to get 100 % of DM $P_\zeta \simeq$10^-2

USR $\quad \epsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \eta \equiv -\frac{\ddot{H}}{2H\dot{H}} = \epsilon - \frac{1}{2}\frac{d\log\epsilon}{dN}$

G.Franciolini A.Urbano– arXiv:2207.10056

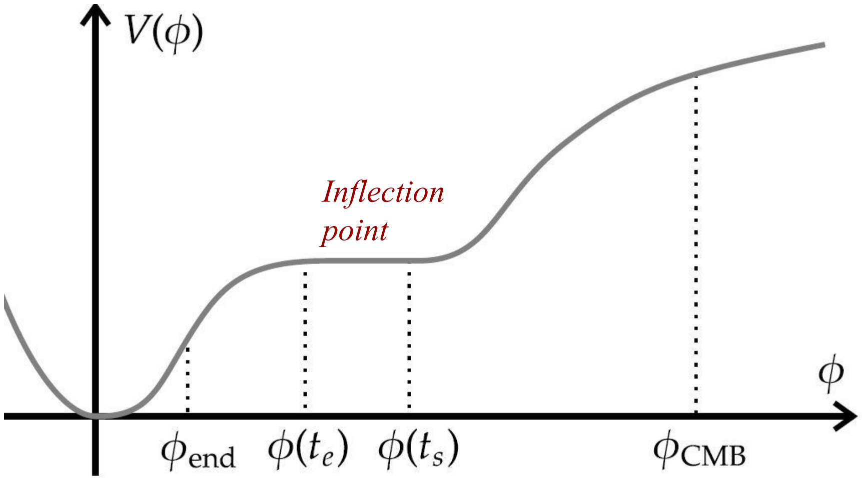




USR models

$$S=\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_{\rm pl}^2R-(\partial_\mu\phi)^2-2V(\phi)\right]$$

$$\phi(\mathbf{x},t)=\phi(t)+\delta\phi(\mathbf{x},t),$$
$$\mathrm{d}s^2=-N^2\mathrm{d}t^2+\gamma_{ij}(\mathrm{d}x^i+N^i\mathrm{d}t)(\mathrm{d}x^j+N^j\mathrm{d}t),$$



We choose comoving gauge condition

$$\delta\phi(\mathbf{x},t)=0,\ \ \gamma_{ij}(\mathbf{x},t)=a^2(t)[1+2\zeta(\mathbf{x},t)]\delta_{ij},$$

$$S^{(2)}=M_{\rm pl}^2\int \mathrm{d}t\ \mathrm{d}^3x\ a^3\epsilon\left[\dot{\zeta}^2-\frac{1}{a^2}(\partial_i\zeta)^2\right]$$

Famous Mukhanov-Sasaki equation

USR models

SR

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{e^{-ik\tau}}{k^{3/2}}(1+ik\tau)$$

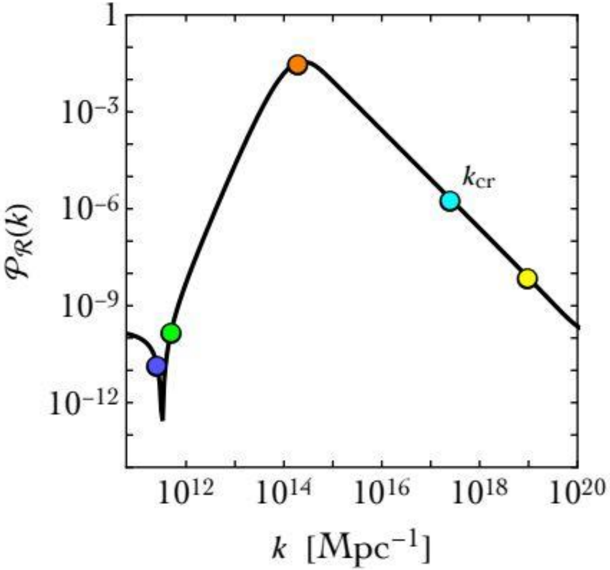
$$P_{\zeta(\rm SR)}(k) = \left(\frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_{\rm SR}}\right)_\star$$

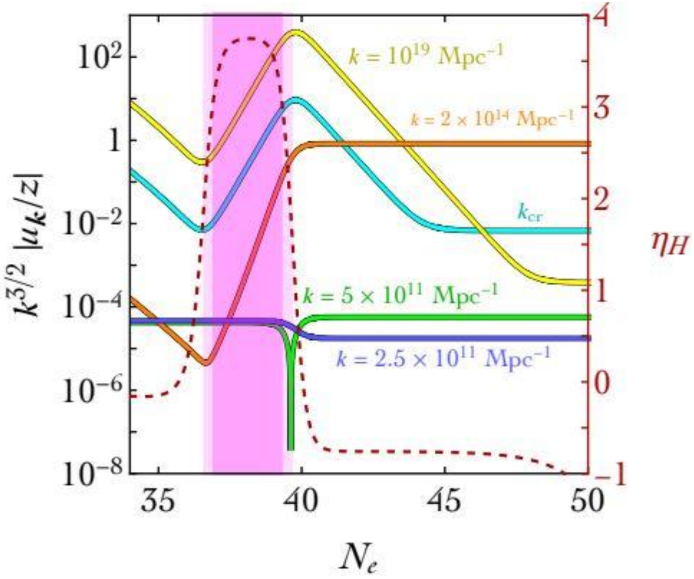
USR

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{1}{k^{3/2}}\mathcal{F}_k(\tau)$$

$$P_{\zeta(\rm PBH)} \approx P_{\zeta(\rm SR)}(k_s)\left(\frac{k_e}{k_s}\right)^6$$

G.Ballesteros,
J. Rey,
M.Taoso,
A.Urbano.
ArXiv:
2001.08220





USR models

SR

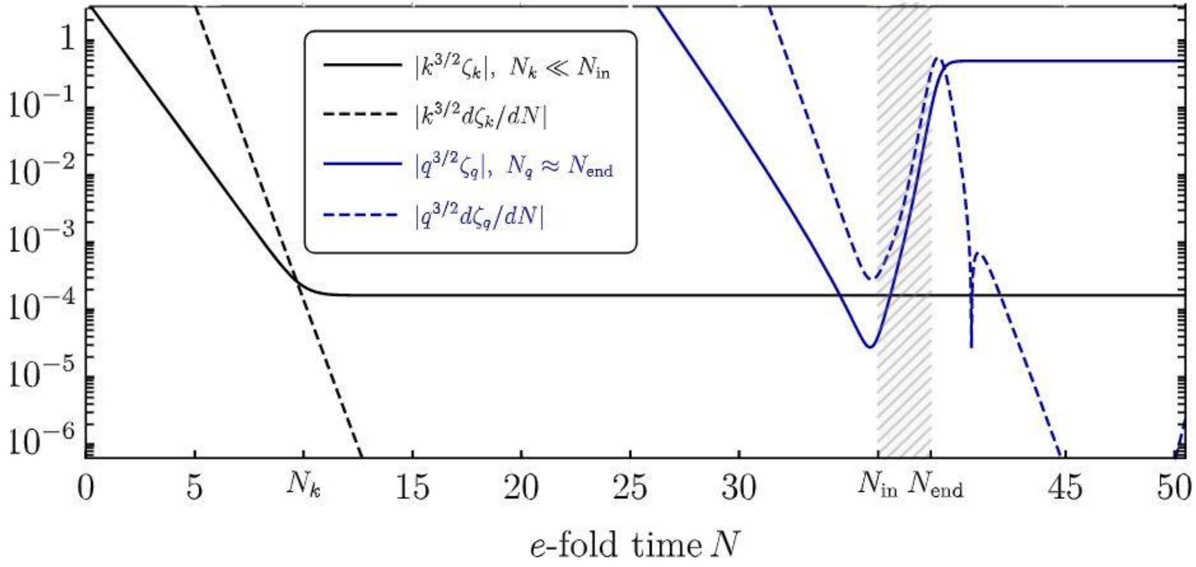
$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{e^{-ik\tau}}{k^{3/2}}(1+ik\tau)$$

$$P_{\zeta({\rm SR})}(k) = \left(\frac{H^2}{8\pi^2 M_{\rm pl}^2\epsilon_{\rm SR}}\right)_\star$$

USR

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{1}{k^{3/2}}\mathcal{F}_k(\tau)$$

$$P_{\zeta({\rm PBH})} \approx P_{\zeta({\rm SR})}(k_s)\left(\frac{k_e}{k_s}\right)^6$$

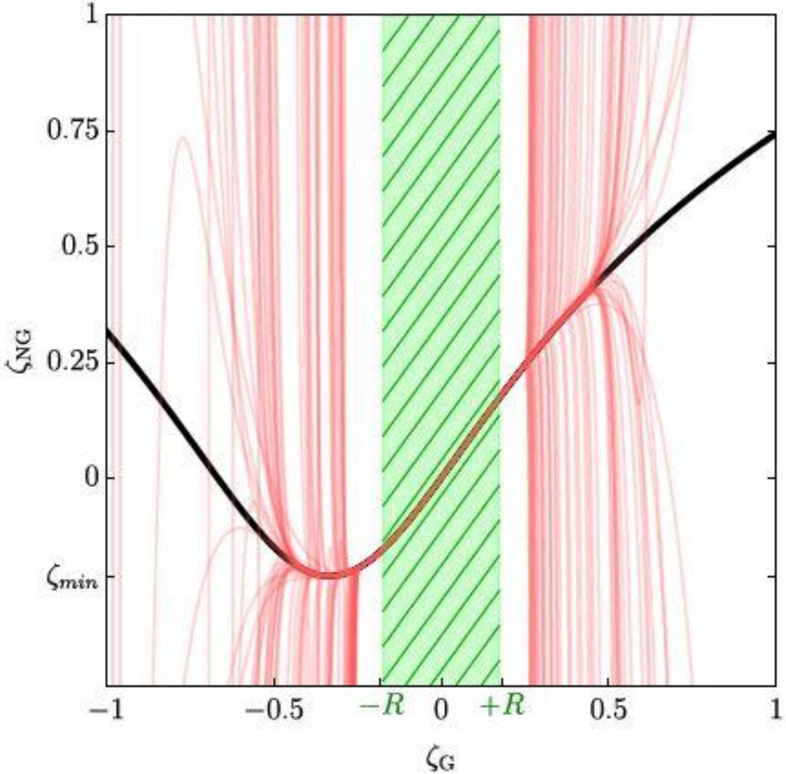


Failure of perturbative approach

$$\sum_{n=1}^{\infty} c_n(r_{\rm dec})\zeta_{\rm G}^n = \log\left[X(r_{\rm dec},\zeta_{\rm G})\right]$$

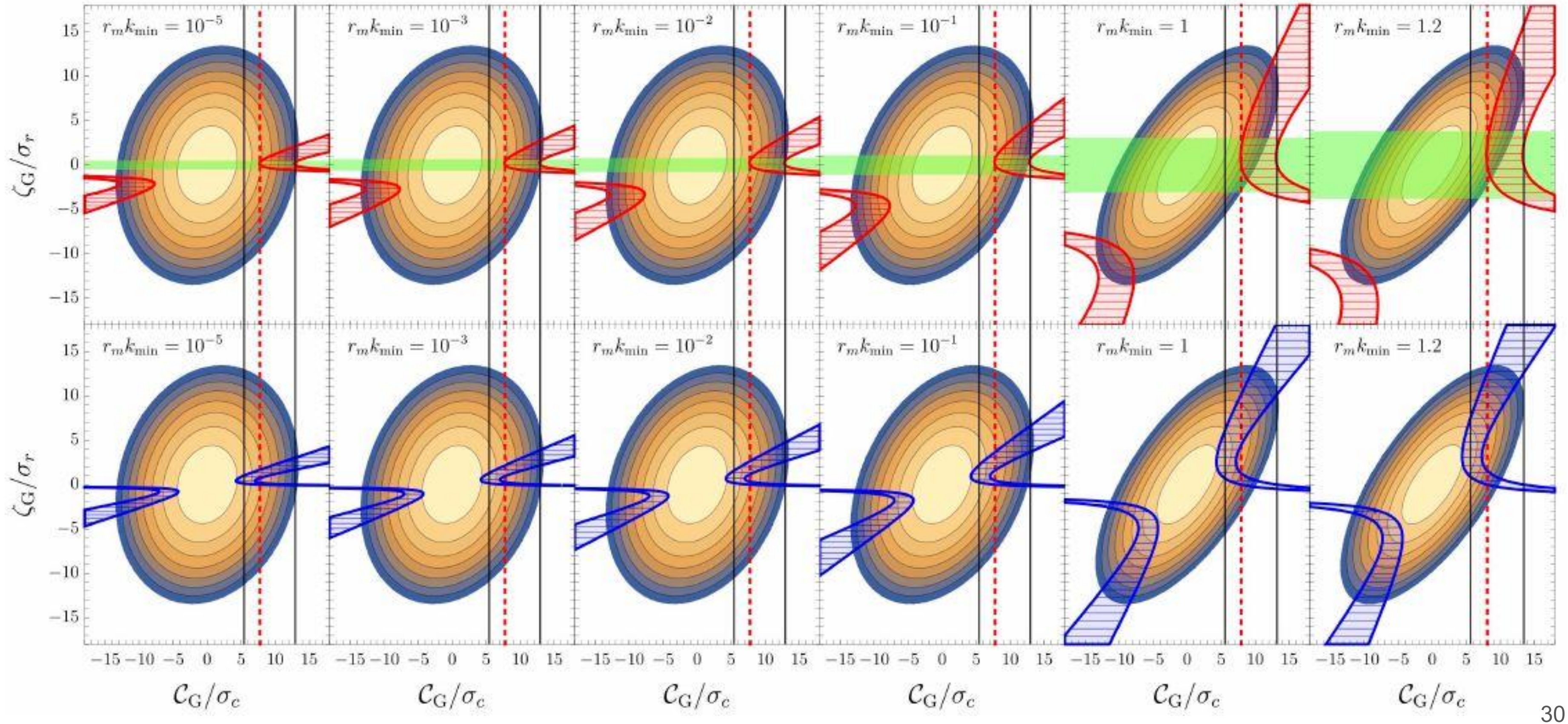
$$c_n(1) = \frac{(-1)^{n+1}}{n(2/3)^{n-1}}\,,$$

$$R = \lim_{n\to\infty}\left|\frac{c_n(1)}{c_{n+1}(1)}\right| = \frac{2}{3}\,,$$



$$\frac{4(1-\sqrt{1-3\mathcal{C}_{\rm th}/2})}{3} < \mathcal{C}_{\rm G}\frac{dF}{d\zeta_{\rm G}} < \frac{4}{3}$$

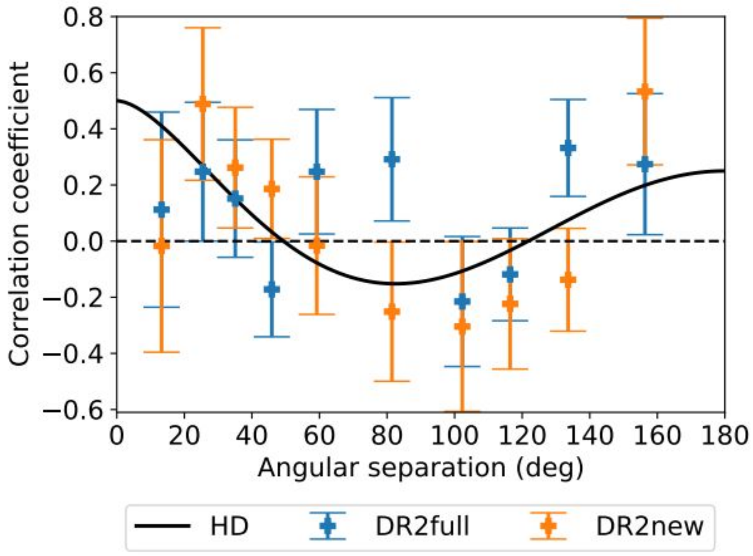
Breaking of scale invariance



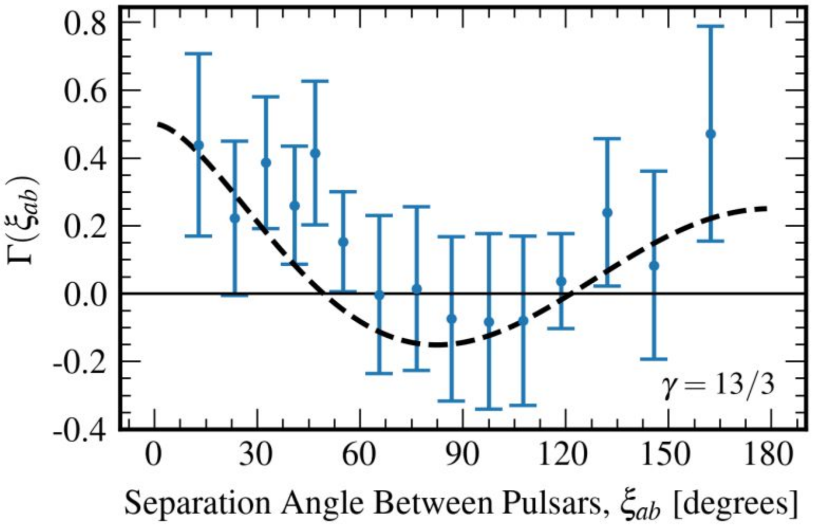
PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

EPTA – arXiv:2306.16214



NANOGrav – arXiv:2306.16213
arXiv:2306.16219

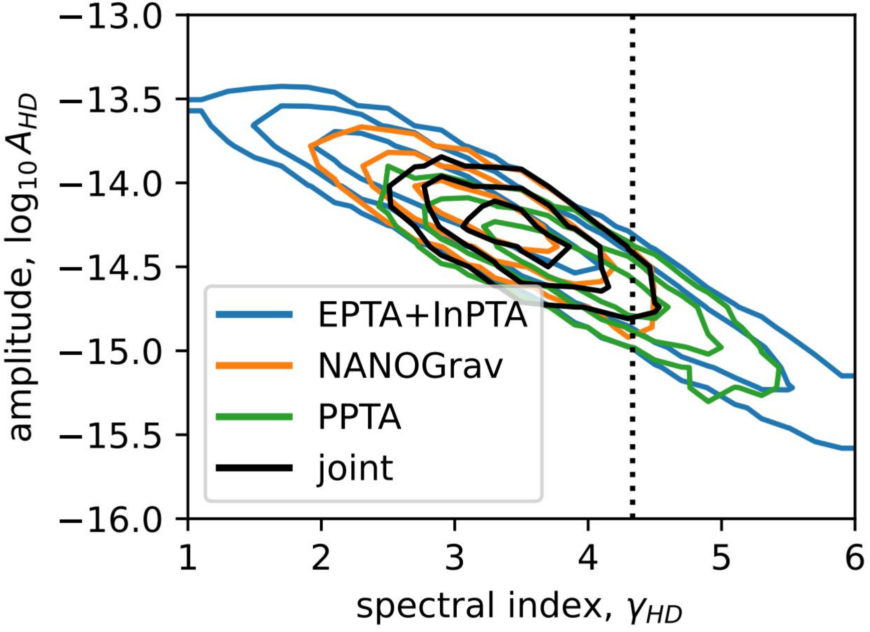


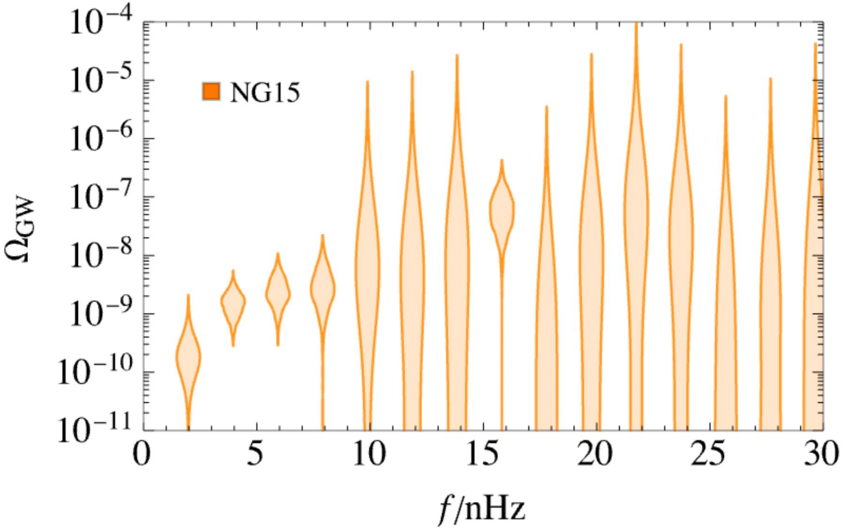
PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

IPTA – arXiv:2309.00693

NANOGrav – arXiv:2306.16213
arXiv:2306.16219





PBH and SGWB

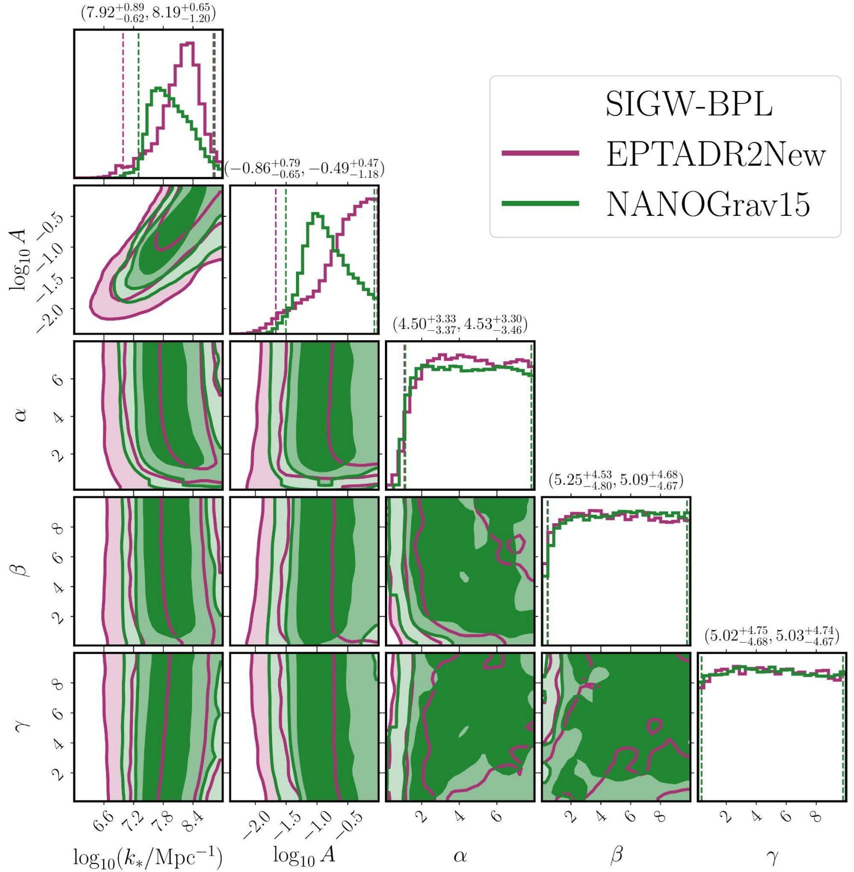
SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

Log-likelihood analysis

Fitting the posterior distributions

$$\mathcal{P}_\zeta^{\mathrm{BPL}}(k) = A\frac{(\alpha+\beta)^\gamma}{\left(\beta\,(k/k_*)^{-\alpha/\gamma} + \alpha\,(k/k_*)^{\beta/\gamma}\right)^\gamma}$$

$$\mathcal{P}_\zeta^{\mathrm{LN}}(k) = \frac{A}{\sqrt{2\pi}\Delta}\exp\left(-\frac{1}{2\Delta^2}\ln^2(k/k_*)\right)$$



Results:

The casuality tail is not good:

$$\Omega_{\mathrm{GW}}(k \ll k_*) \propto k^3(1+\tilde{A}\ln^2(k/\tilde{k}))$$

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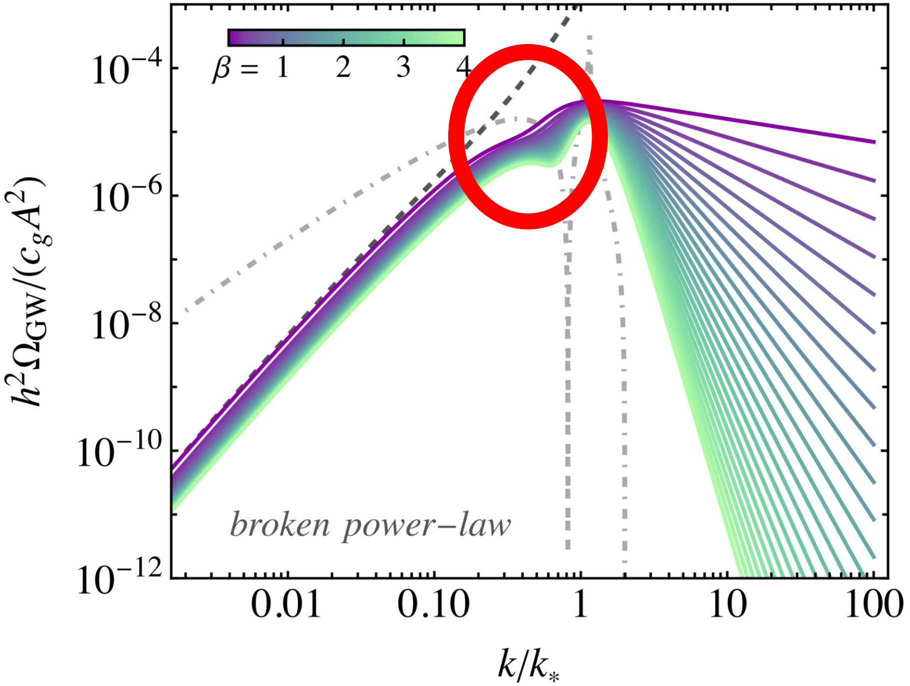

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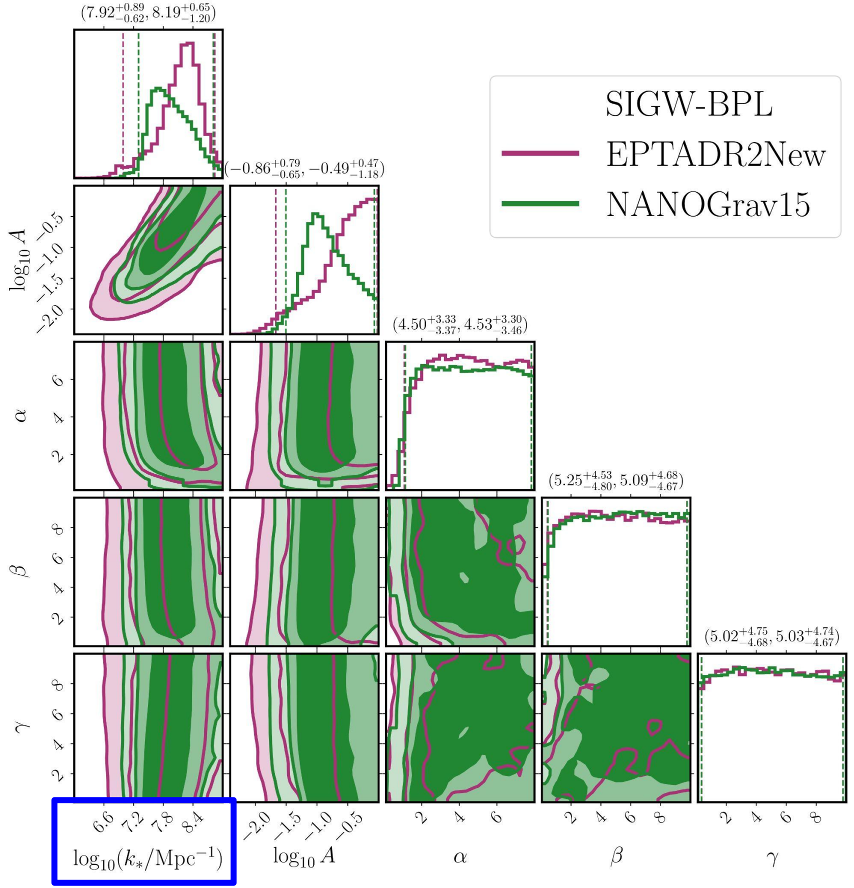
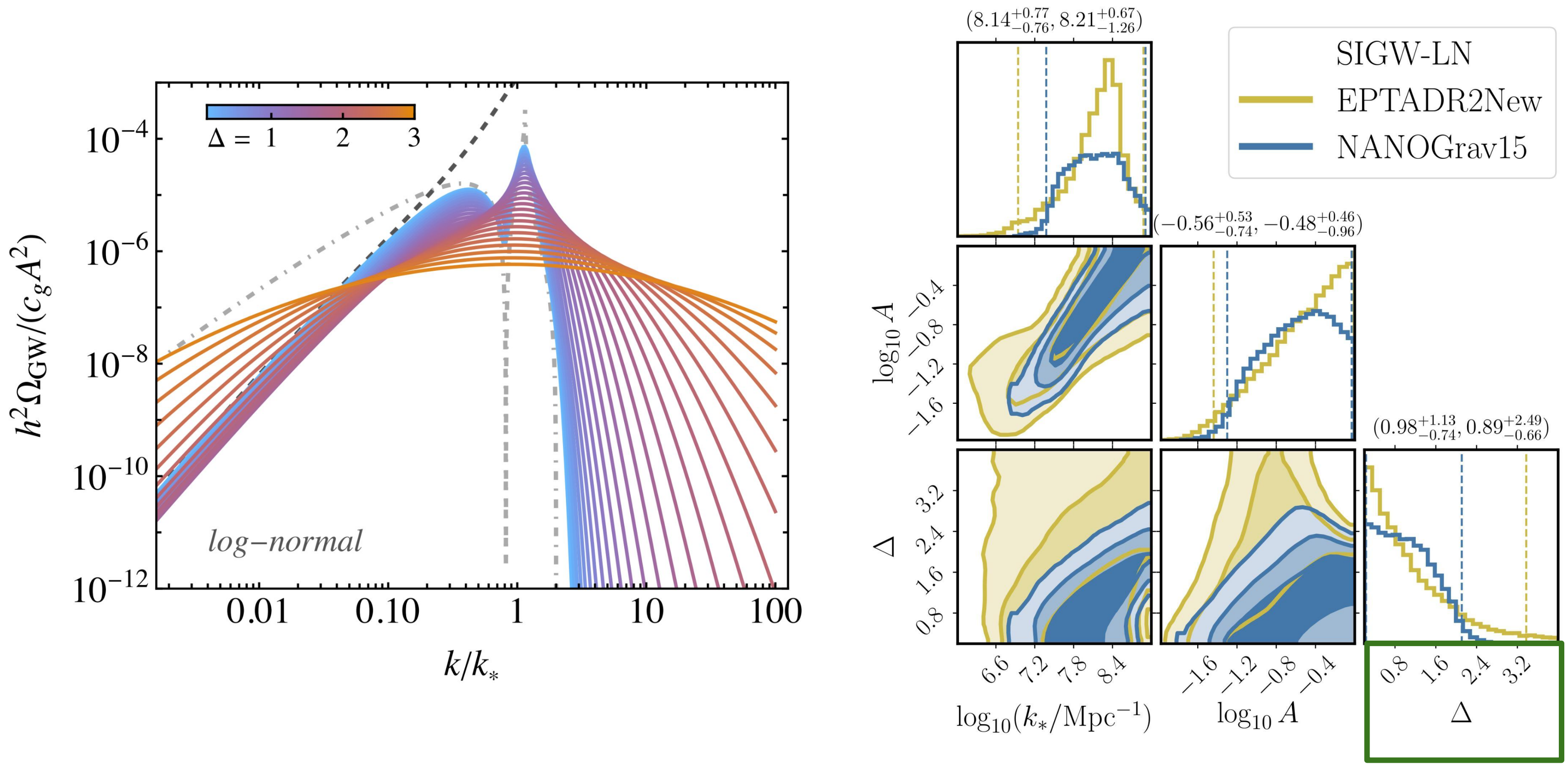
Results:

Position of the peak at higher frequencies.

Broad spectrum does not fit so well.





$$\mathcal{P}_\zeta^{\rm LN}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{1}{2\Delta^2} \ln^2(k/k_*)\right)$$

Improvement respect to NANOGrav analysis.

NANOGrav collaboration
arXiv:2306.16219

Power spectrum <> Abundance <> GWs

- Non-Gaussianities in the abundance.
- Dependency of the PBH formation parameters on the PS shape.
- QCD impact on threshold.

NGs in the abundance: Cases under consideration

NON-LINEARITIES (NL)

$$\delta(\vec{x},t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(\vec{x})}\left[\nabla^2\zeta(\vec{x}) + \frac{1}{2}\partial_i\zeta(\vec{x})\partial_i\zeta(\vec{x})\right]$$

$$\delta(\vec{x},t) = -\frac{4}{9a^2H^2}\nabla^2\zeta(\vec{x})$$

PRIMORDIAL NG IN $\zeta = F(\zeta_G)$

$$\zeta = \log\left[X(r_{\rm dec},\zeta_{\rm G})\right]$$

curvaton case

$$\zeta = -\frac{2}{\beta}\log\left(1 - \frac{\beta}{2}\zeta_{\rm G}\right)$$

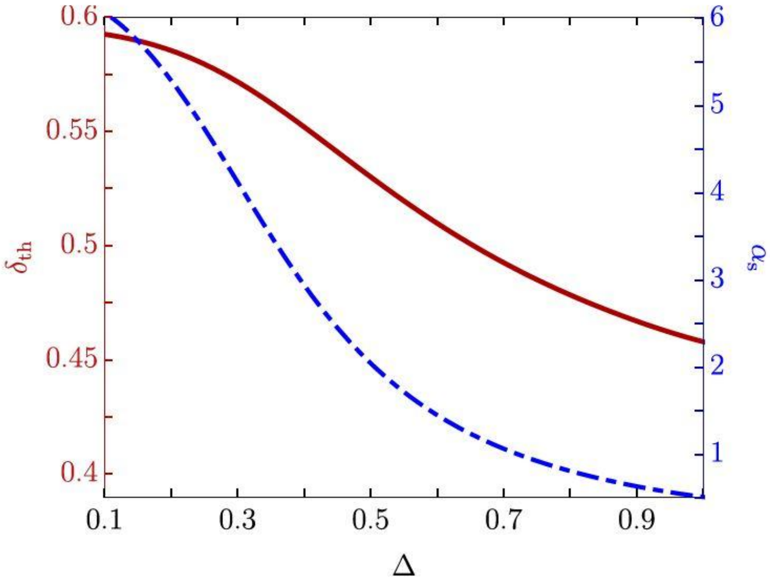
Inflection-point (USR) case

$$\zeta = \zeta_{\rm G} + \frac{3}{5}f_{\rm NL}\zeta_{\rm G}^2$$

Quadratic approx.

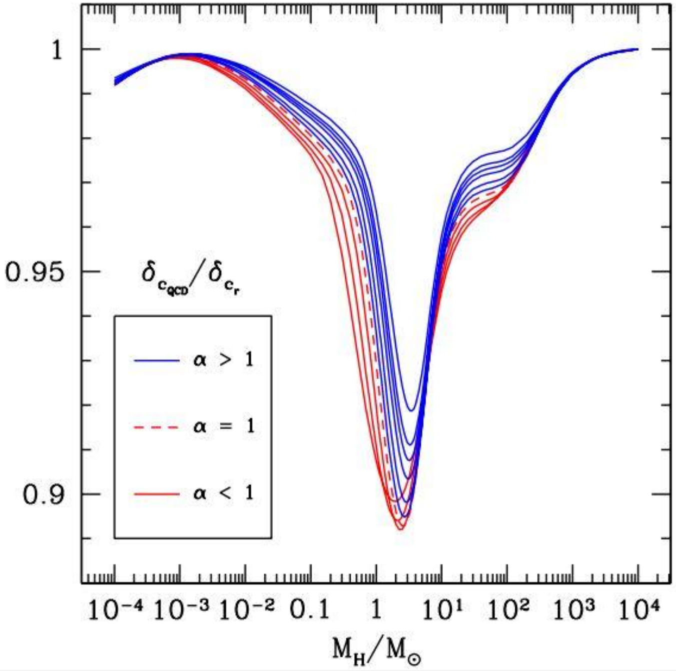
Abundance of PBHs:
Shape dependencies

I. Musco,V. De Luca, G. Franciolini, A.Riotto.– arXiv:2011.03014

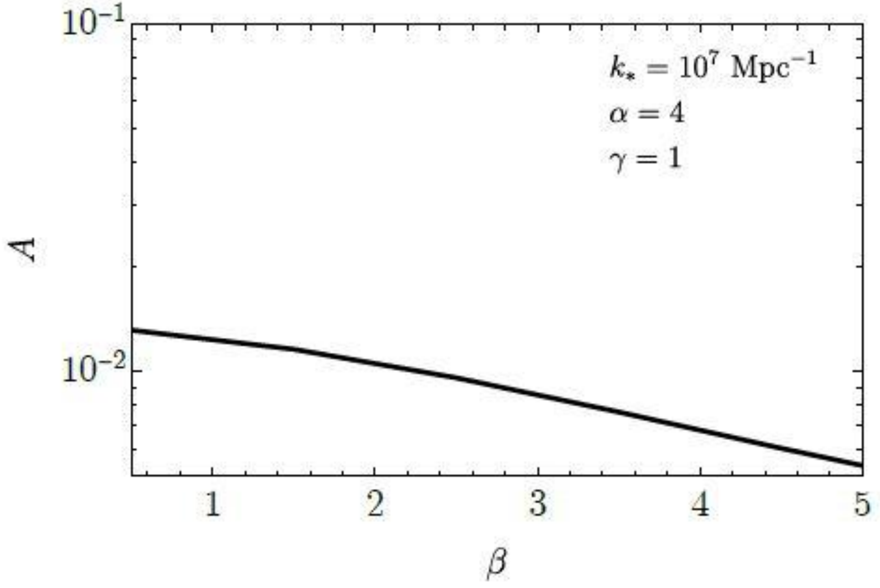


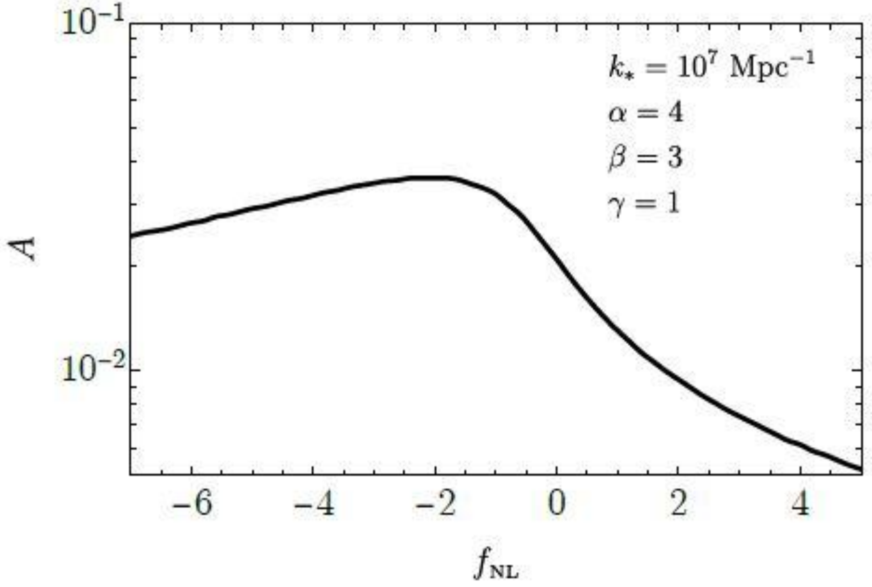
QCD phase transitions

I. Musco, K.Jedamzik, S.Young.– arXiv:2303.07980



NG generic features





NG generic features

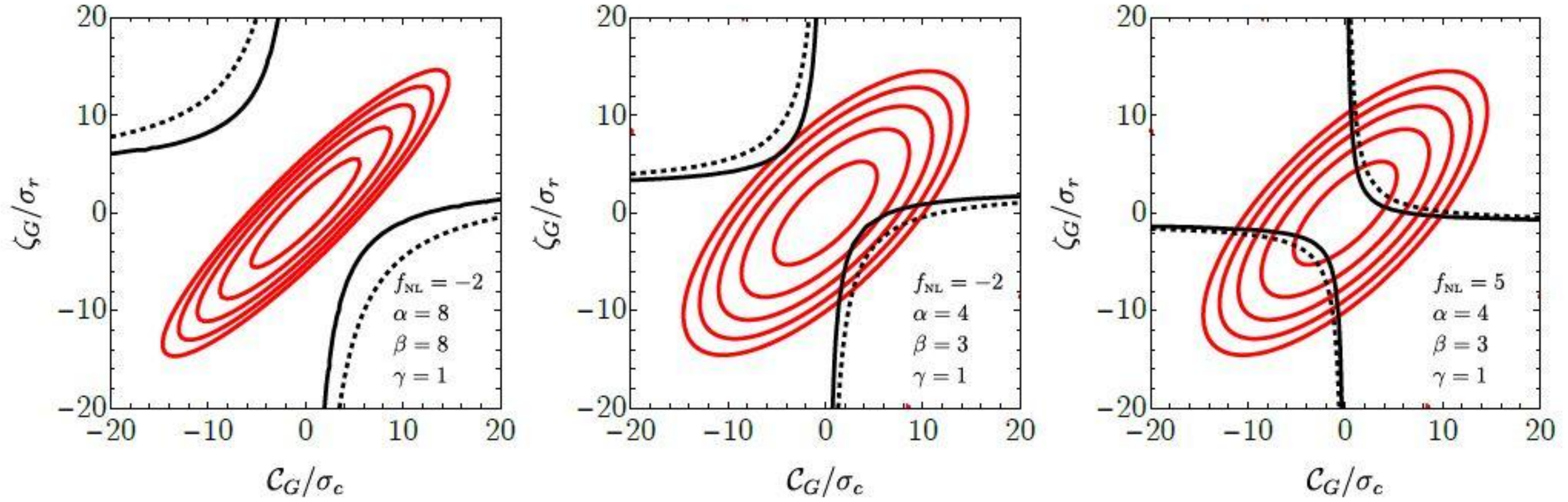


FIG. S4. Two dimensional PDF as a function of $(\mathcal{C}_G, \zeta_G)$ compared to the over-threshold condition $\mathcal{C} > \mathcal{C}_{th}$. In all panels, we considered the BPL power spectrum with an amplitude $A = 0.05$. The red lines indicates the contour lines corresponding to $\log_{10}(P_G) = -45, -35, -25, -15, -5$. The collapse of type-I PBHs take place between the black solid and dashed lines (see more details in Ref. [195]). *Left panel:* Example of a very narrow power spectrum with $\alpha = \beta = 8$. The abundance is suppressed in the presence of negative f_{NL} by the strong correlation between $\mathcal{C}_G$ and ζ_G obtained for narrow spectra. *Center panel:* Example of negative non-Gaussianity and representative BPL spectrum. The PBH formation is sourced by regions of small ζ_G and positive $\mathcal{C}_G$ or both negative $\mathcal{C}_G$ and ζ_G. *Right panel:* Example with positive f_{NL}, showing the region producing PBHs populates the correlated quadrants of the plot, at odds with that is found in the other panels.

Small improvements left

- Compute the threshold including NGs

  A. Escriva, Y. Tada, S. Yokoyama, and C.Yoo.– arXiv:2202.01028

- Variation of speed of sound due to QCD

  K.T.Abe, Y. Tada, I.Ueda.– arXiv:2010.06193

- NGs directly in GWs $\Omega_{\rm GW}^{\rm NLO}/\Omega_{\rm GW} \propto A(3f_{\rm NL}/5)^2$

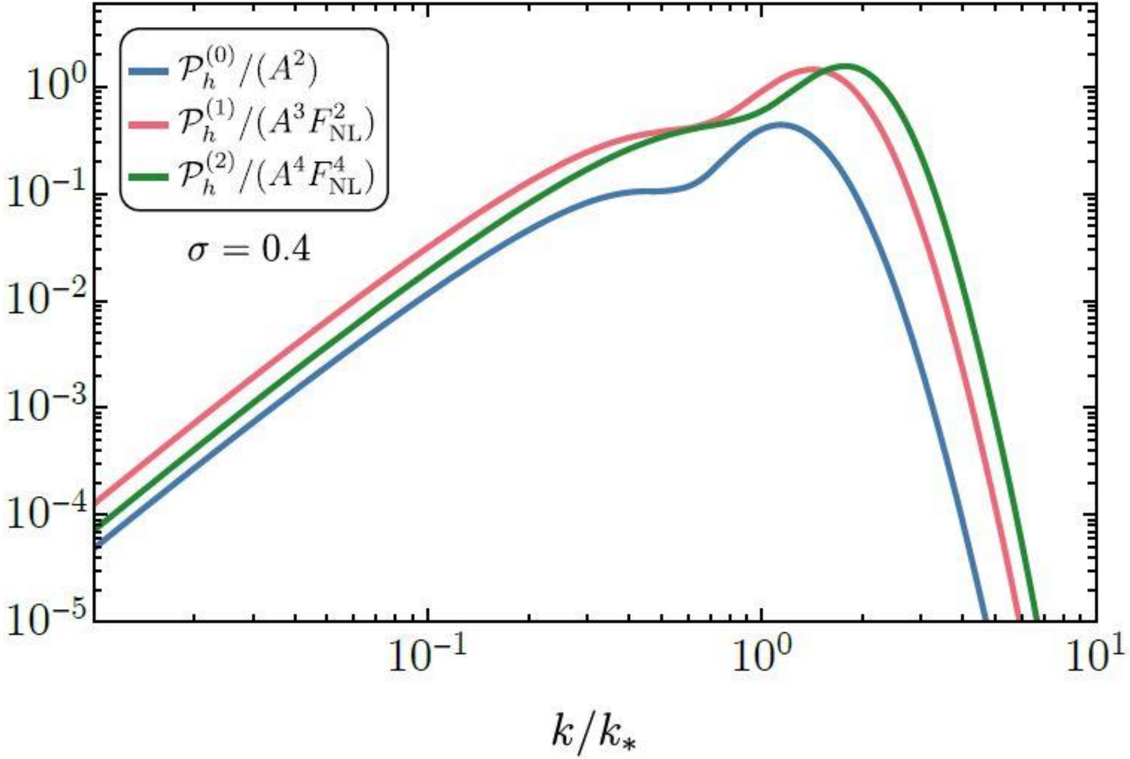
  R. Cai, S. Pi, and M. Sasaki– arXiv:1810.11000
  K. T. Abe, R. Inui, Y. Tada, and S. Yokoyama-arXiv:2209.13891

NGs directly in GWs

arXiv:2308.08546
J. Ellis, M. Fairbarn, A.J.I. et al

PTA prefers "Intermediate region", so HO corrections do not affect significantly the results showed before.

$$\Omega_{\rm GW}^{\rm NLO}/\Omega_{\rm GW} \propto A(3f_{\rm NL}/5)^2$$

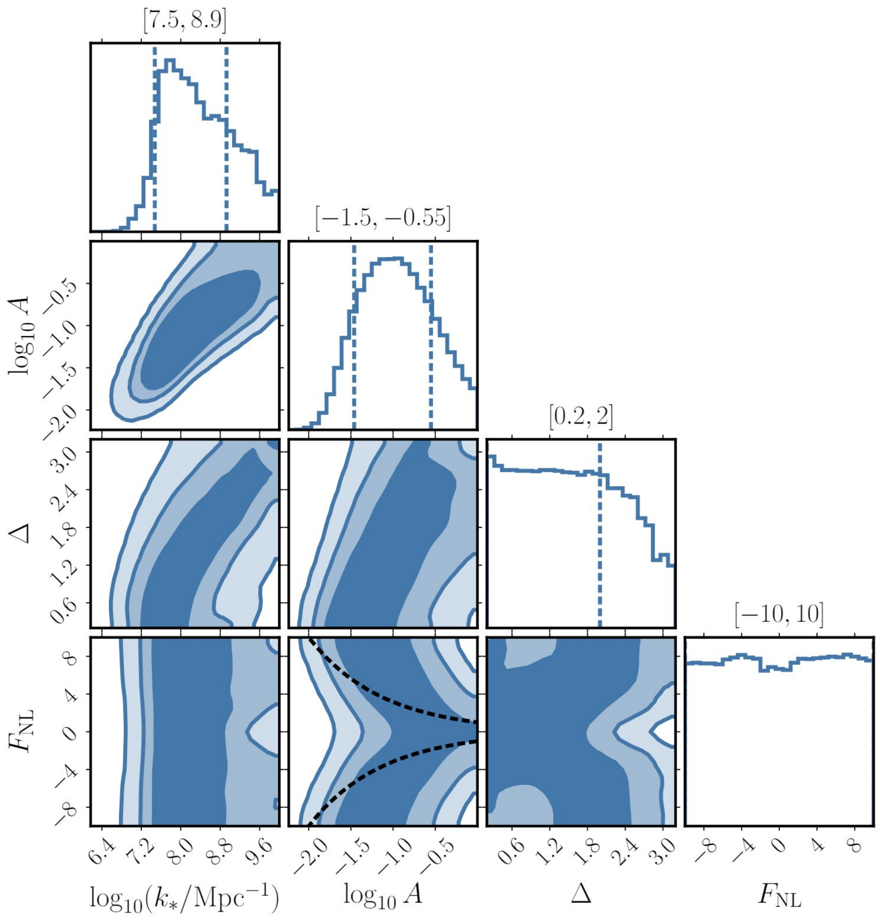


NGs directly in GWs

We cannot constrain the presence of NGs at PTA scales,because large values of FNL remain possible, provided the curvature power spectral amplitude is sufficiently small.

arXiv:2308.08546

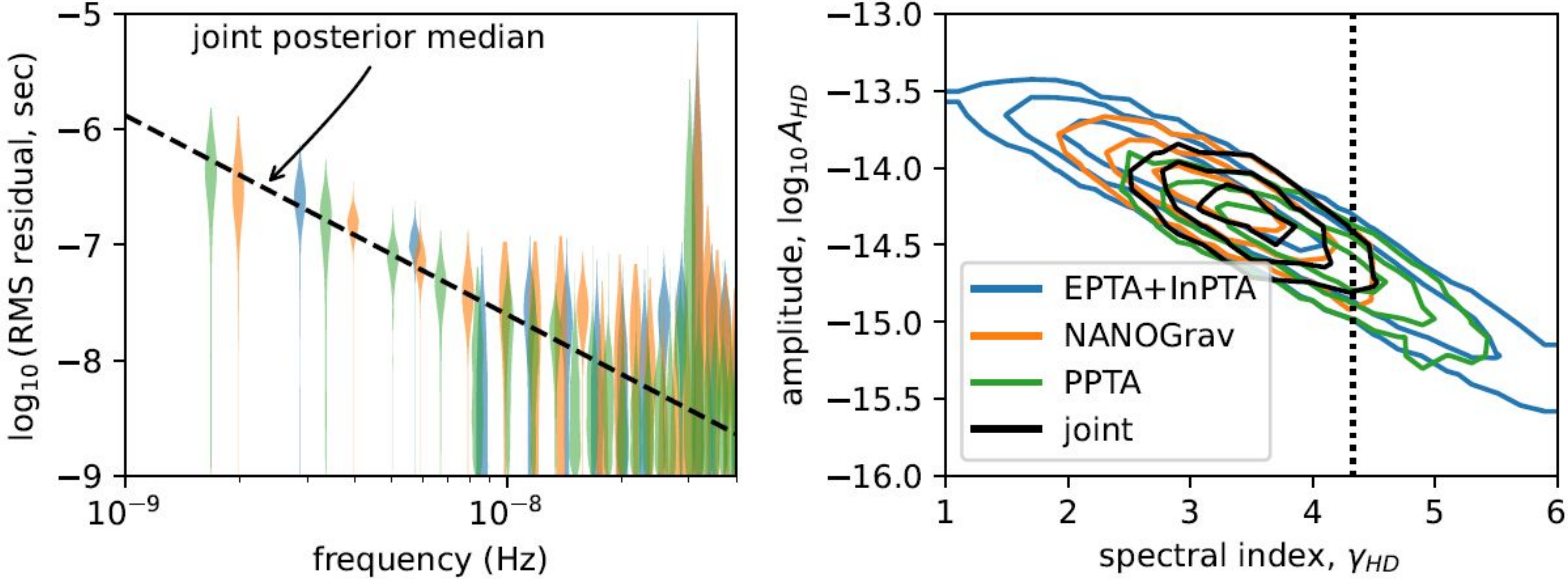
J. Ellis, M. Fairbarn, A.J.I. et al



PBH and SGWB

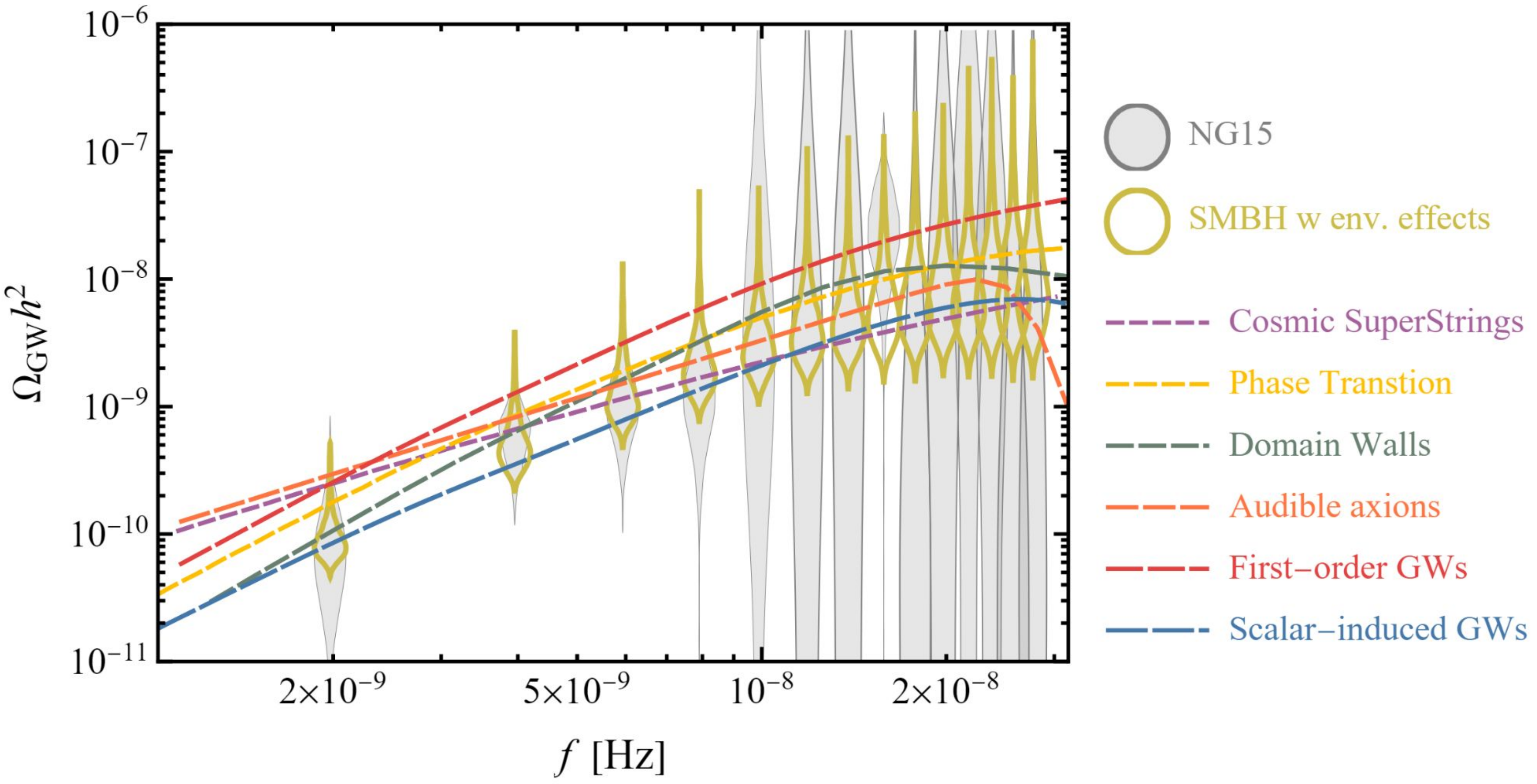
Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

IPTA – arXiv:2309.00693



Are PBHs the end of the story?

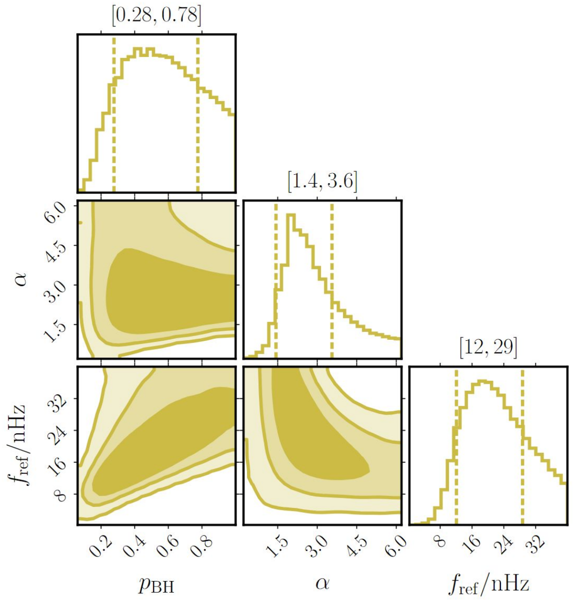
All the PTA possible sources for NANOGrav: Astro vs Cosmo

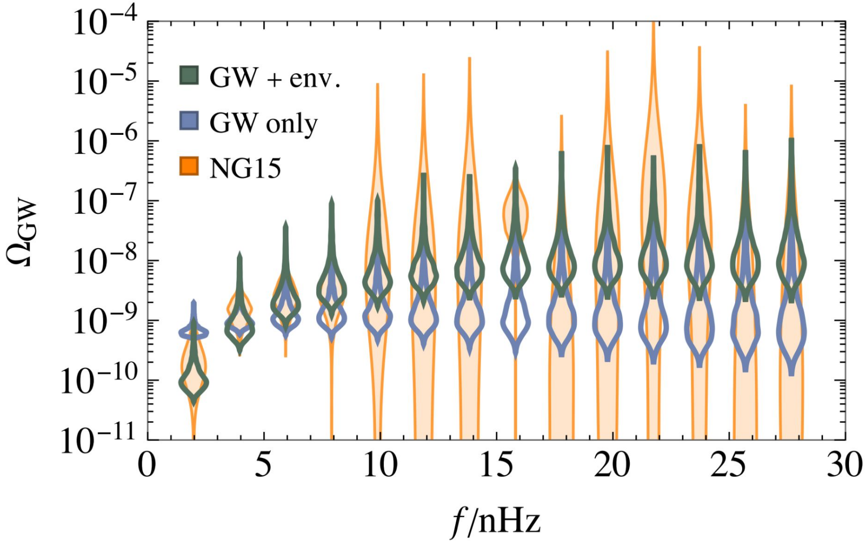


SMBH: Extra mechanism to lose energy with a different time scale

$$t_{\rm GW} \equiv |E|/\dot{E}_{\rm GW} = 4\tau\,, \qquad t_{\rm env} \equiv |E|/\dot{E}_{\rm env}$$

$$\frac{t_{\rm env}}{t_{\rm GW}} = \left(\frac{f_r}{f_{\rm GW}}\right)^{\alpha}, \quad f_{\rm GW} = f_{\rm ref}\left(\frac{\mathcal{M}}{10^9 M_\odot}\right)^{-\beta}$$





The interactions with the environments reduce the period over which the binaries emit GWs.

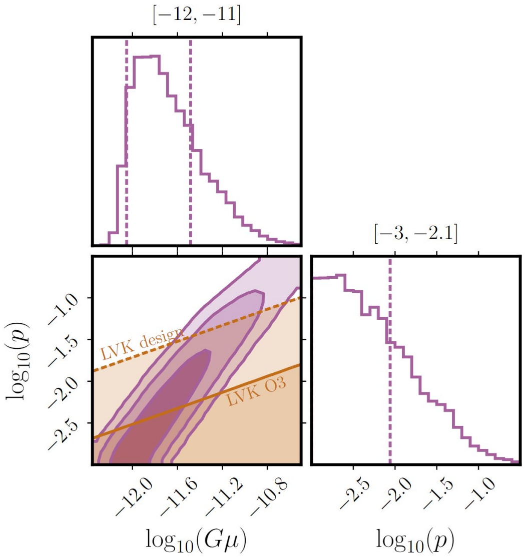
See also arXiv:2306.17021 J.Ellis, J.Urrutia et al

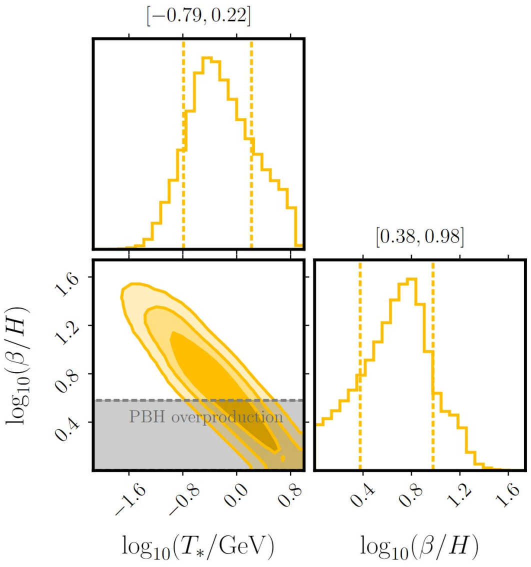
Cosmic Superstring: Evolution of a network of cosmic strings generate a spectrum of GWs whose magnitude and shape are determined by the string tension $G\mu$ and intercommunication probability p.

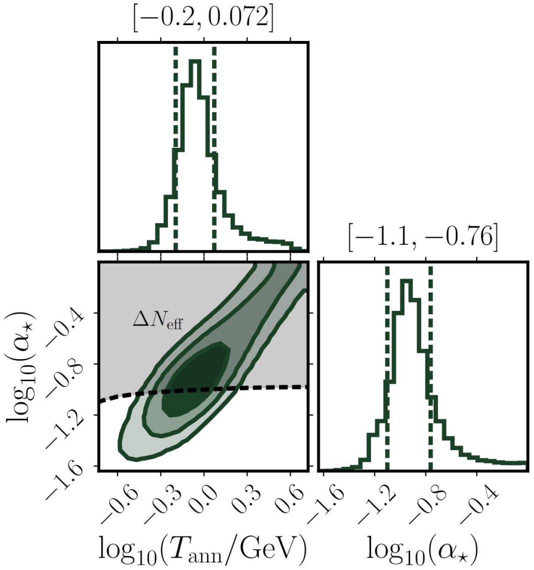
See also arXiv:2306.17147 J.Ellis, M.Lewicki, C.Lin and V.Vaskonen

Phase Transition: GWs from bubbles collisions and motion by inhomogeneities in the fluid. Temperature at the end of the transition. If it is slow (small β) PBH overproduction.

Domain Walls: emission of GWs due to DWs annihilation. Temperature of the annihilation and energy density of the domain walls. See also arXiv:2306.17841 Y.Gouttenoire and E. Vitagliano





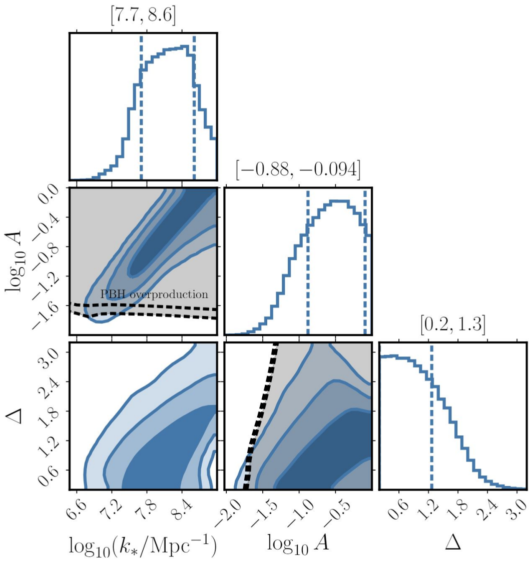


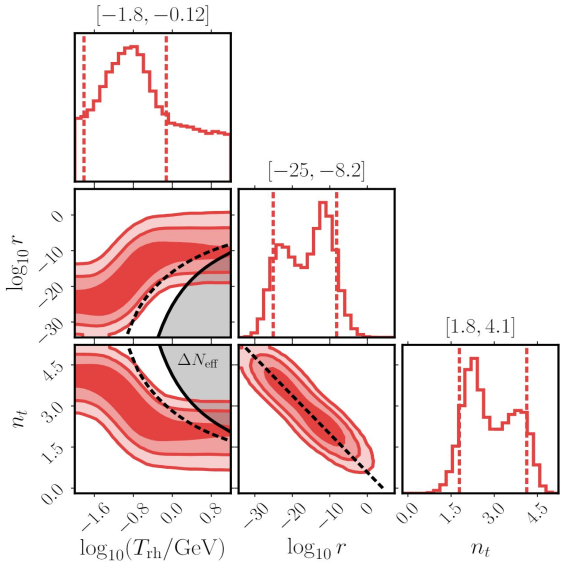
SIGWs: Large scalar cosmological perturbations. See this talk

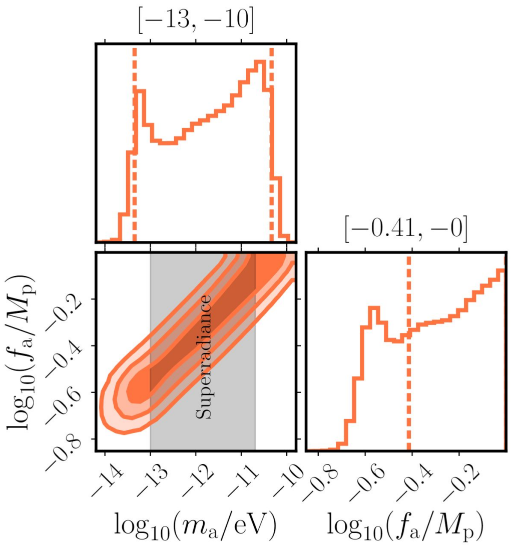
FOGWs: Tensorial perturbations during Inflation. Reheating temperature and tensorial spectral index

See also arXiv:2306.16912 S. Vagnozzi

Audible axions: Coupling to dark photon (DP). While the axion rolls down, tachionic instability for one of the DP helicities, causing vacuum fluctuations to grow. Anisotropic stress in energy-momentum tensor and then GWs.







Results from Multi-Model Analysis (MMA)

Scenario	Best-fit parameters	ΔBIC	Signatures
GW-driven SMBH binaries	$p_{\mathrm{BH}} = 0.07$	6.0	FAPS, LISA, mid-f, ~~LVK, ET~~
GW + environment-driven SMBH binaries	$p_{\mathrm{BH}} = 0.84$ $\alpha = 2.0$ $f_{\mathrm{ref}} = 34$ nHz	Baseline (BIC = 53.9)	FAPS, LISA, mid-f, ~~LVK, ET~~
Cosmic (super)strings (CS)	$G\mu = 2 \times 10^{-12}$ $p = 6.3 \times 10^{-3}$	-1.2 (4.6)	~~FAPS~~, LISA, mid-f, LVK, ET
Phase transition (PT)	$T_* = 0.34$ GeV $\beta/H = 6.0$	-4.9 (2.9)	~~FAPS, LISA, mid-f, LVK, ET~~
Domain walls (DWs)	$T_{\mathrm{ann}} = 0.85$ GeV $\alpha_* = 0.11$	-5.7 (2.2)	~~FAPS~~, LISA?, ~~mid-f, LVK, ET~~
Scalar-induced GWs (SIGWs)	$k_* = 10^{7.7}/\mathrm{Mpc}$ $A = 0.06$ $\Delta = 0.21$	-2.1 (5.8)	~~FAPS, LISA, mid-f, LVK, ET~~
First-order GWs (FOGWs)	$\log_{10} r = -14$ $n_{\mathrm{t}} = 2.6$ $\log_{10}(T_{\mathrm{rh}}/\mathrm{GeV}) = -0.67$	-2.0 (6.0)	~~FAPS, LISA, mid-f, LVK, ET~~
"Audible" axions	$m_a = 3.1 \times 10^{-11}$ eV $f_a = 0.87\, M_{\mathrm{P}}$	-4.2 (3.7)	~~FAPS, LISA, mid-f, LVK, ET~~

FAPS ≡ fluctuations, anisotropies, polarization, sources, mid-f ≡ mid-frequency experiment, e.g., AION [308], AEDGE [310], LVK ≡ LIGO/Virgo/KAGRA [161–163], ET ≡ Einstein Telescope [312] (or Cosmic Explorer [313]), ~~signature~~ ≡ not detectable

TABLE I. *The parameters of the different models are defined in the text. For each model, we tabulate their best-fit values, and the Bayesian information criterion* $BIC \equiv -2\ell + k \ln 14$*, where* k *denotes the number of parameters, relative to that for the purely SMBH model with environmental effects that we take as the baseline. The quantity in the parentheses in the third column shows the* ΔBIC *for the best-fit combined SMBH+cosmological scenario. The last column summarizes the prospective signatures.*

Other experiments?