transition in axion models with $N = 1$ assuming the axion field is not homogenized by inflation. In the simulations, the walls decay immediately, i.e. in a time scale of order the light travel time. The simulations provide an estimate of the average energy of the axions emitted in the decay of the walls: $\langle\omega_a\rangle \simeq 7m_a$ when $\sqrt{\lambda}v_a/m_a \simeq 20$.

Because of restrictions on the available lattice sizes, the simulations are for values of v_a/m_a of order 100, whereas in axion models of interest v_a/m_a is of order 10^{26}. To address this problem, we have investigated the dependence of $\langle\omega_a\rangle/m_a$ upon $\sqrt{\lambda}v_a/m_a$ and found that it increases approximately as the logarithm of this quantity. This is because the decay process occurs in two steps: 1) wall energy converts into moving string energy because the wall accelerates the string, and the energy spectrum hardens accordingly; 2) the strings annihilate into axions without qualitative change in the energy spectrum. If this behaviour persists all the way to $\sqrt{\lambda}v_a/m_a \sim 10^{26}$, then $\langle\omega_a\rangle/m_a \sim 60$ for axion models of interest.

This has interesting consequences for the axion cosmology and the cavity detector experiments of dark matter axion as discussed in Ref. [8].

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REFERENCES

1. R. D. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. **D16**, 1791 (1977); S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
2. C. Hagmann and P. Sikivie, Nucl. Phys. **B363**, 247 (1991).
3. D. H. Lyth, Phys. Lett. **B275**, 279 (1992).
4. M. Nagasawa and M. Kawasaki, Phys. Rev. **D50**, 4821 (1994).
5. M.C. Huang and P. Sikivie, Phys. Rev. **32**, 1560 (1985).
6. D. Harari and P. Sikivie, Phys. Lett. **B195**, 361 (1987).
7. P. Sikivie in *Where are the elementary particles*, Proc. of the 14th Summer School on Particle Physics, Gif-sur-Yvette, 1982, edited by P. Fayet et al. (Inst. Nat. Phys. Nucl. Phys. Particules, Paris, 1983).
8. S. Chang, C. Hagmann and P. Sikivie, hep-ph/9807374.

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SUPPLEMENTS

The nonlinear modulation of the density distribution in standard axionic CDM and its cosmological impact

M. Yu. Khlopov[a,b], A. S. Sakharov[a,b] and D. D. Sokoloff[c]

[a]Center for CosmoParticle Physics "Cosmion", Miusskaya Pl.4, 125047 Moscow, Russia

[b]Moscow Engineering Physics Institute (Technical University), Kashirskoe Sh.31, 115409 Moscow, Russia

[c]Department of Physics, Moscow State University, 119899 Moscow, Russia

It is shown, that the energy density of coherent axion field oscillations in the cosmology of standard invisible axion should be distributed in the Universe in the form of archioles, being nonlinear inhomogeneous structure, reflecting the large scale distribution of Brownian structure of axion strings in the very early Universe. Spectrum of inhomogeneities, generated by archioles, is obtained and their effects in the spectrum and quadrupole anisotropy of relic radiation are considered. The axionic-string-decay-model-independent restriction on the scale of axion interaction is obtained.

1. INTRODUCTION

In this paper we analyze the problem, inherent to practically all the cosmological cold dark matter models of invisible axion, that concerns primordial inhomogeneity in the distribution of the energy of coherent oscillations of the axion field. This problem, referred to as the problem of *archioles*, invokes non-Gaussian component in the initial perturbations for axionic cold dark matter. Archioles are the formation that represents a replica of the percolation Brownian vacuum structure of axionic walls bounded by strings, which is fixed in the strongly inhomogeneous primeval distribution of cold dark matter. They can give rise to interesting alternative scenarios of axion mini-clusters formation that relate the mechanism responsible for structure formation to inhomogeneities of the type of topological defects. The analysis of observable effects associated with archioles leads to a new model-independent constraint on the mass of invisible axion.

2. FORMATION OF THE ARCHIOLES

In the standard invisible axion scenario [1] the breaking of the Peccei-Quinn symmetry is induced by the complex $SU(3)\otimes SU(2)\otimes U(1)$- singlet Higgs field φ with a "Mexican hat" potential

$$V(\varphi) = \frac{\lambda}{2}\left(\varphi^{+}\varphi - F_a^{\,2}\right)^2. \qquad (1)$$

Such field can be represented as $\varphi = F_a \exp(i\theta)$, where $\theta = a/F_a$ and a is the angular Goldstone mode-axion. QCD instanton effects remove the vacuum degeneracy and induce effective potential for θ

$$V(\theta) = \Lambda_1^{\,4}\left(1 - \cos(\theta \cdot N)\right). \qquad (2)$$

Below we will simply assume for standard axion that $N = 1$. In the context of standard big bang sce-

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nario it is usually assumed that the phase transition with U(1)- symmetry breaking occurs when the Universe cools below the temperature $T \cong F_a$. Thus, in the standard case the crucial assumption is that from the moment of the PQ phase transition and all the way down to the temperatures $T \cong \Lambda_{QCD}$, the bottom of the potential (1) is exactly flat and there is no preferred value of a during this period ((2) vanishes). Consequently, at the moment of the QCD phase transition, when the instanton effects remove vacuum degeneracy, a rolls to the minimum and starts coherent oscillations (CO) about it with energy density [1]

$$\rho_a(T,\theta) = 19{,}57\left(\frac{T_1^2 m_a}{M_P}\right)\left(\frac{T}{T_1}\right)^3 \theta^2 F_a^{\ 2}. \qquad (3)$$

The coherent axion field oscillations turn on at the moment $\tilde{t} \approx 8{,}8 \cdot 10^{-7} s$.

It is generally assumed, that PQ transition takes place after inflation and the axion field starts oscillations with different phase in each region causally connected at, $T \cong F_a$ and, so one has an average over all the possibilities to obtain the present-day axion density. Thus in the standard cosmology of invisible axion, it is usually assumed that the energy density of coherent oscillations is distributed uniformly and that it corresponds to the averaged phase value of $\bar{\theta} = \pi/\sqrt{3}$ ($\bar{\rho}_a = \rho(\bar{\theta})$). However, the local value of the energy density of coherent oscillations depends on the local phase θ that determines the local amplitude of these coherent oscillations. It was first found in [2], that the initial large-scale (LS) inhomogeneity of the distribution of θ must be reflected in the distribution of the energy density of coherent oscillations of the axion field. Such LS modulation of the distribution of the phase θ and consequently of the energy density of CO appears when we take into account the vacuum structures leading to the system of axion topological defects.

As soon as the temperature of Universe becomes less then F_a, the field φ acquires the vacuum expectation value (VEV) $\langle\varphi\rangle = F_a e^{i\theta}$, where θ varies smoothly at the scale F_a^{-1}. The existence of noncontractable closed loops that change the phase by $2k\pi$ leads to emergence of axion strings. These strings can be infinite or closed. The numerical simulation of global string formation [3] revealed that about 66%-80% of the length of strings correspond to infinite Brownian lines. The remaining 34%-20% of this length is contributed by closed loops. Infinite strings form a random Brownian network with the step $L(t) \approx t$. After string formation when the temperature becomes as low as $T \cong \Lambda_{QCD}$, the expression (2) makes a significant contribution to the total potential so that the minimum of energy corresponds to a vacuum with $\theta = 2k\pi$, where k is an integer- for example, $k = 0$. However, the vacuum value of the θ cannot be zero everywhere, since the phase must change by $\Delta\theta = 2\pi$ upon a loop around a string. Hence, we come from the vacuum with $\theta = 0$ to the vacuum with $\theta = 2\pi$ as the result of such circumvention. The vacuum value of θ is fixed at all points with the exception of the point $\theta = \pi$. At this point, a transition from one vacuum to another occurs, and the vacuum axion wall is formed simultaneously with CO turning on. The width of such wall bounded by string is $\delta \cong m_a^{-1}$. Thus, the initial value of θ must be close to π near the wall, and the amplitude of CO in (3) is determined by the difference of the initial local phase $\theta(x)$ and the vacuum value, which is different from zero only in a narrow region within the wall of thickness $\delta \cong m_a^{-1}$. Therefore in this region we can write $\theta(x) = \pi - \varepsilon(x)$, where [2] $\varepsilon(x) = 2\tan^{-1}(\exp(m_a x))$ and $x \cong m_a^{-1}$. Thereby the energy density of CO in such regions is given by

$$\rho^A \approx \pi^2 \bar{\rho}_a. \qquad (4)$$

And so we obtain, that the distribution of CO of axion field is modulated by nonlinear inhomogeneities in which relative density contrasts are $\delta\rho/\rho = (\rho^A - \bar{\rho}_a)/\bar{\rho}_a > 1$. Such inhomogeneities were called *archioles*. In the other words *archioles* are a formation that represents a replica of the percolation Brownian vacuum structure of axionic walls bounded by strings and which is fixed in the strongly inhomogeneous initial distribution of axionic CDM. The scale of this modulation of density distribution exceeds the cosmological horizon because of the presence of infinite component in the structure of axionic wall bounded by strings system. The superweakness of the axion field self-interaction results in the separation of archioles and of the vacuum structure of axionic walls-bounded-by-strings. So these two structures evolve independently. The structure of walls bounded by strings disappears rapidly due to axion emission. The structure of archioles remains frozen at the RD stage. On the large scales, the structure of archioles is an initially nonlinear formation, a Brownian network of quasi-one-dimensional filaments of dustlike matter with the step

$$L^A(t) = \lambda'\tilde{t} \qquad (5)$$

(where $\lambda' \cong 1$). At the moment of creation $\tilde{t}$, the linear density of this quasi-linear filamentary formations given by

$$\mu_A = \pi^2\bar{\rho}_a\tilde{t}\delta. \qquad (6)$$

The nonlinear character of dustlike infinite filaments along to crosscut direction allow us to adopt following consideration, namely the cosmological evolution of archioles in the expanding Universe is reduced to the extension of lines along only one direction (see [4] more detail).

We have studied in [4] the spectrum of inhomogeneities that the density develops in response to the large-scale Brownian modulation of the distribution of CO of axion field. Density perturbations, associated with Brownian network of archioles, may be described in the terms of a two- point auto-correlation function [4]. To obtain such auto-correlation function, it is necessary to perform averaging of energy density of infinite Brownian lines over all lines and over the Winner measure, which corresponds to the position along of Brownian line (see [4]).

The two-point auto-correlation function in the Fourier representation has the form

$$\langle \frac{\delta\rho}{\rho_0}(\mathbf{k})\frac{\delta\rho}{\rho_0}(\mathbf{k}')\rangle = 12\rho_A\mu_A k^{-2}\delta^3(\mathbf{k}+\mathbf{k}')\times \qquad (7)$$
$$\times\tilde{t}^{-1}f^{-2}t^4G^2,$$

where ρ_0 is background density, $f_{MD} = 3/(32\pi)$ for dust-like stage, $f_{RD} = (6\pi)^{-1}$ for RD stage, G is the gravitational constant, ρ_A is the total energy density of the Brownian lines. The mean- square fluctuation of the mass is given by

$$\left(\frac{\delta M}{M}\right)^2(k,t) = 12\rho_A\mu_A\tilde{t}^{-1}f^{-2}G^2kt^4. \qquad (8)$$

3. COSMOLOGICAL IMPACT OF ARCHIOLES

3.1 CMBR anisotropy

Let us estimate the perturbation amplitude that is induced by archioles in relic radiation at the scale of the modern horizon. One can see from (8) that this amplitude is expressed by following formula

$$\left(\frac{\delta M}{M}\right)^2 = 7\cdot10^{-26}\left(\frac{F_a}{10^{10}\,\mathrm{GeV}}\right)^4\left(\frac{t_{RD}}{1\mathrm{s.}}\right)^{2/3}\times \qquad (9)$$
$$\times\left(\frac{t_{pres}}{1\mathrm{s.}}\right)^{1/3}\left(k_{hor}t_{pres}\right)$$

It implies that

$$\delta_H^A \cong 1.3\cdot10^{-6}\left(\frac{F_A}{10^{10}\,\mathrm{GeV}}\right)^2 \qquad (10)$$

there δ_H is the perturbation amplitude at the present Hubble radius.

The COBE data determine the perturbation amplitude with the accuracy of about 10%. Its most useful application is to normalized the density spectrum [5]

$$\delta_H = 1.91\cdot10^{-5}\frac{\exp(1.01(1-n))}{\sqrt{1+0.75r}} \approx 2\cdot10^{-5}. \qquad (11)$$

Here r is approximately the measure of the relative contribution of gravitational waves and density perturbations in the anisotropy and n is the spectral index. From the condition that the effect of archioles does not exceed the measured amplitude (11) one puts the restriction on the model of an invisible axion.

If we take into account the uncertainties of our consideration such as the uncertainties in correlation length scale of Brownian network ($\lambda' \approx 1 \div 13$) and in temperature dependence of axion mass we can obtain a constraint on the scale of PQ symmetry breaking and on the mass of axion given by

$$F_a \le 1.7 \cdot 10^{10}\,\mathrm{GeV} \div 4.2 \cdot 10^{9}\,\mathrm{GeV}, \quad m_a \ge 370\,\mu eV \div 1400\,\mu eV. \tag{12}$$

This upper limit for F_a is close to the strongest upper limits in [6-8], obtained by comparing the density of axions from decays of axionic strings with the critical density, but has an essentially different character.

The point is that the density of axions formed in decays of axionic strings depends critically on the assumption about the spectrum of such axions (see [6-7]) and on the model of axion radiation from the strings (see [8]). For example, Davis [6] assumed that radiated axions have a maximum wavelength of $\omega(t) \cong t^{-1}$ while Harari and Sikivie [7] have argued that the motion of global strings was suppressed, leading to an axion spectrum emitted from infinite strings or loops with a flat frequency spectrum $\propto k^{-1}$. This leads to an uncertainty factor of 100 in the estimate of the density of axions from strings and to the corresponding uncertainty in the estimated upper limit on F_a

$$F_a \le 2 \cdot 10^{10}\varsigma\,\mathrm{GeV}, \quad m_a \ge 300/\varsigma\,\mu\mathrm{eV}. \tag{13}$$

Here, $\varsigma = 1$ for the spectrum from Davis [6], and $\varsigma = 70$ for the spectrum from Harari and Sikivie [7].

In their treatment of axion radiation from global strings, Battye and Shellard [8] found that the dominant source of axion radiation are string loops rather than long strings, contrary to what was assumed by Davis [6]. This leads to the estimations

$$F_a \le 2.5 \cdot 10^{11}\,\mathrm{GeV} \div 4.7 \cdot 10^{10}\,\mathrm{GeV}, \quad m_a \ge 24\,\mu\mathrm{eV} \div 130\,\mu\mathrm{eV}. \tag{14}$$

Arguments that lead to the constraint (12) are free from these uncertainties, since they have a global string decay model- independent character.

3.2 Axionic mini-clusters

The creation of axionic mini-clusters [9,10] can find interpretation in framework of archioles. Actually the Brownian step $L^A(t) = \lambda'\tilde{t}$ extracts the scale at which the evolution of archioles just in the beginning of axionic CDM dominance in the Universe (at the moment $t_{eq} \approx 3 \cdot 10^{10} h^{-4}\mathrm{s}$.) should lead to formation of the smallest gravitational bound axionic mini-clusters with the minimal mass

$$M \cong \rho_a(T_1)L^3 = \lambda^3 \frac{\pi^2}{30}\sqrt{g(T_1)}T_{eq}\left(\frac{m_{Pl}}{T_1}\right)^3 \cong \cong \lambda^3 \cdot 10^{-8} M_{Solar} \tag{15}$$

and of typical size 10^{13}sm. (here $g(T_1) \approx 62$ is the number of an effective massless degrees of freedom). To sustain this consideration it is useful to analyze the process of accretion of surrounding axionic CDM onto archioles filaments. For reasonable estimations we have used Zeldovich approximation and revealed that the scales which has gone non-linear just after beginning matter domination epoch in the crosscut direction is given by formula

$$l_{nl} \approx \frac{3}{\sqrt{10}} t_{eq}^{1/3} t_{pres}^{2/3} \left(\frac{\tilde{t}}{t_{eq}}\right)^{1/2} \left(\frac{\mu_A}{\delta}\tilde{t}G\right)^{1/2} \tag{16}$$

and close to 10^{14}cm at the epoch just after beginning of CDM dominance in the Universe.

The abundance of such axionic mini-clusters is easy to find by using the numerical results of U(1) string formation [3]. The point is that the probability for element of string pass through isolated cell with the Hubble scale at the moment $\tilde{t}$ is equal to 0.88. Thus it is possible to say that 88% of axionic CDM are is mini-clusters, and 66%-80% of these mini-clusters are distributed in the form of Brownian chains with far correlation exceeding the Hubble scale.

3.3 Spatial variation of axion CP- violation and spatial dependent baryon excess

In the early Universe after PQ phase transition axion plays the role of spatially varying CP-violating phase until the temperature falls down T_1 when the axion mass is switched on. So if axion induced CP violation contributes baryosynthesis, the corresponding contribution into baryon excess is spatial dependent. The distribution of inhomogeneous baryon excess reproduces in this case the distribution of axion phase in the period of baryosynthesis. Since the latter distribution follows the crosscut of axion string distribution one obtains fractal distribution of the baryon density due to effect of infinite axion strings. The maximal baryon density corresponds to $\theta(x) = \pi/2$ whereas the maximal axion CDM density follows the direction $\theta(x) = \pi$, what may provide the physical mechanism for biasing in axionic CDM models of structure formation.

The extreme case of axion induced CP violation is the axion phase dominance in baryosynthesis. In [11] phenomenological model of baryon excess being the sum of constant, B, and axion phase induced, $A\sin\theta(x)$, parts was considered. Axion dominance, taking place at A>B, should lead to antibaryon excess generation with the maximal effect along the axion phase equal to $\theta(x) = 3\pi/2$. Antimatter domains are formed initially as bents along the axion strings with the angular width in the crosscut around $3\pi/2$ determined by the ratio (A-B)/(A+B). So the model represents an interesting example of antimatter domains in baryon dominated Universe, first considered in [12].

It was shown in [11] that such a scenario can naturally lead to annihilation of the most part of antimatter before the cosmological nucleosynthesis, so that the amount of antimatter surviving in domains and annihilating on their border does not contradict the astrophysical constraints on antimatter annihilation after big bang nucleosynthesis [11-12]. The scale of domains surviving up to the modern period turns to coincide with the Jeans scale for baryon matter after recombination. So the model [11] predicts the existence of isolated antimatter objects of globular cluster type even in our Galaxy. The existence of such objects can lead to antinuclear component of cosmic rays accessible in the antimatter searches planned in AMS and PAMELA experiments.

4. CONCLUSIONS

High PQ-symmetry breaking scale makes axion elusive for direct experimental study, but the model of invisible axion can be tested in the combination of its cosmological, astrophysical and physical effects. This approach of cosmoparticle physics is even more important in the unification of axion with other extensions of the standard model such as horizontal unification [13], identifying PQ symmetry with family symmetry breaking. The development of cosmoparticle physics will fix the proper place and relevance of invisible axion as the necessary element in the future complete particle theory.

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REFERENCES

1. J.E.Kim, Phys. Rep. 150 (1987) 1
2. M.Yu.Khlopov and A.S.Sakharov, Phys. Atom. Nucl. 57 (1994) 485
3. T.Vachaspati and A.Vilenkin, Phys. Rev. 30D (1984) 2036, T.Vachaspati, Phys. Rev. 40D (1991) 3723
4. M.Yu.Khlopov, D.D.Sokoloff and A.S.Sakharov, Phys. Atom. Nucl. 59 (1996) 1005
5. E.F.Bunn, A.R.Liddle and M.White, Phys. Rev. D 54 (1996) 5917R
6. R.Davis, Phys.Lett. 180B, 277 (1986)
7. D.Harari and P.Sikivie, Phys.Lett. 195B (1987) 361
8. R.A.Battye and E.D.S.Shellard, Phys. Rev. Lett. 73, (1994) 2954
9. C.J.Hogan and M.G.Rees, Phys. Lett. (1988) 228
10. E.W. Kolb and I.I.Tkachev, Phys. Rev. 49D (1994) 5040
11. M.Yu.Khlopov, Gravit&Cosm. 4 (1998) 69
12. V.M.Chechetkin et al, Phys.Lett. 180B (1986) 277
13. M.Yu.Khlopov and A.S.Sakharov, Phys. Atom. Nucl. 57 (1994) 690