Physics of Atomic Nuclei, Vol. 57, No. 3, 1994, pp. 485 - 487. Translated from Yadernaya Fizika, Vol. 57, No. 3, 1994, pp. 514 - 516.

ELEMENTARY PARTICLES AND FIELDS
Theory

The Nonhomogeneity Problem for the Primordial Axion Field

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Received June 18, 1993

Abstract – It is shown that, in the cosmology of an invisible axion, the energy density of the axion field coherent oscillations must be distributed in the Universe in the form of archioles, i.e., nonlinear, nonhomogeneous structures reflecting large-scale distribution of the "Brownian" structure of axionic strings in the early Universe.

1. At present, an invisible axion is considered to be one of the most probable candidates for the role of a particle constituting the dark matter of the Universe (see, for example, review [1] and references cited therein). In this paper, we discuss the form of the axionic cold dark matter (CDM) density distribution in the case of a high scale of symmetry breaking $F_a \simeq 10^{10}$ GeV admissible in all axion models. This investigation is interesting for the development of methods of axionic haloscopy aimed at the search for cosmic axions and for construction of cosmological models explaining the formation of the structure of the Universe by axionic (CDM).

2. Let us now consider a simplified model of an invisible axion [2] with the Lagrangian

$$\mathcal{L} = \partial_\mu \phi^+ \partial^\mu \phi - \frac{1}{2}\lambda(\phi^+\phi - F_a^2)^2 + 2\Lambda_1^4[\cos(N\theta) - 1]. \quad (1)$$

Here, F_a is the scale of the global Peccei–Quinn $U(1)_{PQ}$-symmetry breaking [3], $\Lambda_1 \simeq 1$ GeV effectively accounts for the effects of QCD anomalies, and N is an integer characterizing a specific axion model. Let us discuss the cosmological consequences of this model [2]. After the temperature of the Universe drops below F_a, the field ϕ acquires a vacuum expectation $\langle\phi\rangle = F_a e^{i\theta}$, where the scale of θ variations is the correlation length $\xi \simeq F_a^{-1}$. Existence of non contractible closed loops, along which the phase variation is $2\pi n$ with integer n, result in the emergence of topologically stable configurations, namely, axionic strings with a linear energy density $\mu \simeq F_a^2 \ln(F_a d)$, where d is the characteristic distance between the strings ($d \geq \xi$).

At the moment of formation of axionic strings, their structure is composed of two subsystems: (a) the system of closed Brownian strings [4] with the scale-invariant length distribution and (b) the system of infinite Brownian strings [4], which comprise 80% of the total length of strings, the characteristic distance between the strings being [5]

$$L \simeq Pt. \quad (2)$$

Here, P is the probability for intersection of strings (in the simplest case, $P \simeq 1$). Loss of energy through radiation of axions [6] leads to the disappearance of closed loops at the cosmological time $\tau \leq t$ and rectification of long strings at the scale of the cosmological horizon. When the temperature of the Universe drops to $T \simeq \Lambda_1$, the term in (1) that contains θ begins to give a significant contribution, so that the energy minimum (for $N = 1$) corresponds to "vacuum" with $\theta = 0$. However, the "vacuum" value of the phase θ cannot be zero everywhere because the change in the phase around the string must be $\Delta\theta = 2\pi$. Therefore, the "vacuum" value of θ tends to zero at all points around the string except the neighborhood of the point $\theta = \pi$, where a domain wall is formed [2]. Intersection with the wall makes the phase change by 2π. The wall-like solution of the equation of motion of the axion field yields [2]

$$\theta = 4\tan^{-1}\exp(m_a x), \quad (3)$$

where the x-axis is orthogonal to the surface of the wall. The thickness of the axion wall is $\delta \simeq m_a^{-1}$, where $m_a = A_c m_\pi f_\pi / F_a$ is the mass of the axion, and the surface energy density is $\sigma = 16 m_\pi f_\pi F_a$. Here, m_π and f_π are the π-meson mass and coupling constants, respectively, and A_c depends on the axion model. Note that, in contrast to the case $N = 1$, the structure of the domain walls bounded by strings rapidly begins to dominate in the Universe in the case $N > 1$ [7].

3. After the moment $t_1 \simeq 10^{-6}\,(1\ \mathrm{GeV}/\Lambda_1)^2$ s, when the temperature of the Universe drops below $T \simeq \Lambda_1$, axions acquire mass owing to the "switch on" of the instanton effects. At the same time, axion field coherent oscillations with the energy density [8]

$$\rho_a = (39.14/2)\,(T_1^2 m_a/M_P)\,(T/T_1)^3 \theta^2 F_a^2, \quad (4)$$

where M_P is the Planck mass, "switch on" in the Universe, and massless axions produced in decay of axionic strings in the period from t_{PQ} to t_1 acquire mass.

The energy density of strings produced in the decay of axionic strings depends on the form of their spectrum $\omega(t)$:

$$\rho_a^{str} = (m_a/t^{3/2}) \int_{t_{PQ}}^{t_1} \frac{dt}{t^{3/2}} \frac{\mu}{\omega(t)}. \quad (5)$$

In [6], it was assumed that axions produced by decaying strings have the maximum wavelength, and therefore, $\omega(t) \simeq t^{-1}$, while in [9], it was supposed that the spectrum has the form $\propto k^{-}$, and hence, $\omega \simeq \ln(F_a t)t^{-1}$. Thus, the energy density of axions produced in the decay of strings is

$$\rho_a^{str} \simeq \beta_i \rho_a/\theta^2, \quad (6)$$

where $i = 1, 2$; $\beta_1 = 1$ for the case considered in [9]; and $\beta_2 = \ln(F_a t_1) \simeq 70$ for the case considered in [6]. The axions from decaying strings obey quasi-homogeneous distribution. The same is usually assumed in the cosmology of an invisible axion for the averaged energy density of coherent oscillations with the average $\theta = 1$ [8]. However, the local energy density of coherent oscillations of the axion field is governed by the local value of θ, which determines the local amplitude of the axion field coherent oscillations after "switching on" of the axion mass. The corresponding energy density distribution is strongly nonhomogeneous with a maximum density near the axion wall. In fact, near the wall, the initial value of θ is close to π, and the amplitude of coherent oscillations is determined by the difference between the initial local value of the phase $\theta(x)$ and the vacuum expectation, which is nonzero only in a narrow region inside the wall of size m_a^{-1}. The amplitude of oscillations is $\theta(x) = \pi - \varepsilon(x)$, where $0 < \varepsilon(x) \le \pi$ inside the wall $x \le m_a^{-1}$ and $\varepsilon(x) \simeq 0$ outside the wall $x > m_a^{-1}$.

In this region, the energy density of the axion field coherent oscillations is obtained from (4) and has the form

$$\rho_a^W \simeq \pi^2 \rho_a (\theta = 1). \quad (7)$$

Thus, nonhomogeneities of the axion field coherent oscillations form a strong nonlinear structure with the relative density $\delta\rho/\rho > 1$ at scales exceeding the cosmological horizon. This structure is identical to the "Brownian" distribution of axionic strings and walls formed between them. We propose that these nonhomogeneities be referred to as "archioles". Let us compare the energy density of the walls bounded by axionic strings and that of the axionic field coherent oscillations. As is known [2], the ratio of the energy density of the walls and strings to the total density in the Universe at the moment t is

$$\rho_{WS}/\rho \le (t\sigma/\mu)^{1/2} G\mu. \quad (8)$$

Taking into account the fact that $\rho \simeq (Gt^2)^{-1}$, comparing expression (8) with ρ_a/ρ at the moment $t_1 \simeq 10^{-6}$ s, and using (4), we obtain

$$\rho_{WS}/\rho_a \le 10^{-3}. \quad (9)$$

Superweak self-interaction of axions at $F_a > 10^8$ GeV splits the vacuum structure of archioles and of walls formed by axionic strings at the moment of their formation. As a result, these structures evolve independently at later stages. Because strings radiate gravitons (see [2]), evolution of strings implies that they quickly disappear at a characteristic time $\tau \simeq (G\sigma)^{-1}$. Large-scale archiole structure remains "frozen" in the radiation-dominated phase, and the small-scale evolution depends on nonlinear effects in the development of gravitational instability. Thus, the axionic CDM model of the present Universe inevitably predicts a strong nonlinear distribution of non homogeneities, i.e., of the archioles, ($\delta\rho/\rho \ge 1$) with the ratio of densities

$$\rho_a/\rho_a^{str} \simeq \begin{cases} 10^{-2}, & i = 2 \\ 1, & i = 1. \end{cases} \quad (10)$$

These nonhomogeneities have Brownian character and extend far beyond the present horizon. It seems that the existence of archioles poses a serious problem for the axionic CDM model.

In particular, such nonhomogeneities would result in much larger anisotropy of the relic radiation temperature than that compatible with experimental constraints. Clarification of reasons behind this contradiction demands a special study. Brownian character of the archiole structure and its nontrivial topology, which is a replica of the structure of axionic walls bounded by strings at the time $t \simeq 10^{-6}$ s, are interesting to study from the viewpoint of fractal theory [4].

4. In the theory of an invisible axion [1], there are two ways to avoid formation of the archiole structure at the scale of the present cosmological horizon. The first possibility is to consider a model involving a hadronic axion and an archion with the "low" energy scale $F \simeq 10^6$ GeV. In this case, archioles dissipate because of thermalization of axions at $F \le 10^8$ GeV after the QCD phase transition in the process $\pi N \to aN$ [1]. The second possibility is that of the high-energy scale $F \gg 10^{10}$ - 10^{12} GeV. In this case, the phase transition with the Peccei–Quinn symmetry breaking takes place at the inflation stage [10]. This can lead to the phase $\theta \ll 1$ ($\theta \simeq 10^{12}$ GeV$/F$) in the region of the present horizon. In this case, the archiole structure is characterized by a scale that is much larger than the present horizon, thus removing disagreement with experiments. Both possibilities correspond to the parameters of an invisible axion ($m_a \simeq 1$ eV in the first and $m_a \ll 10^{-5}$ eV in the second cases) and are beyond the bounds accessible to modern axion haloscopy. Current projects aimed at the search for axions forming CDM (see, for example, [11]) assume the values of axion parameters

($m_a \simeq 10^{-5}$ - 10^{-3} eV), for which the problem of archioles is of crucial importance.

ACKNOWLEDGMENTS

We are grateful to A.G. Doroshkevich and A.V. Vorob'ev for discussion of some of the problems considered in this paper.

This work was completed at the scientific-educational center "Kosmion" and supported by the Russian Foundation for Fundamental Research (grant no. 93-02-2951).

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