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Cosmology of spontaneously broken gauge family symmetry with axion solution of strong CP-problem

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Abstract. The $SU(3)_H$ model of spontaneously broken local family symmetry is considered as a simplest version of realistic quantum flavourdynamics, giving reasonable explanation of the mass hierarchy and mixing pattern of quarks and leptons. This scheme can naturally possess one or two additional global $U(1)$ symmetries, which can play the role of Peccei-Quinn symmetry. The model predicts: existence of the neutrino Majorana masses with definite hierarchy, existence of familon being simultaneously invisible axion (or arion) and Majoron, relationship between neutrino lifetimes relative to familon decays. Thereby, the model provides the unified physical ground for all the main types of dark matter, considered in the theory of large scale structure of the universe.

1. The problem of fermion families remains one of the central problems of particle physics. The standard $SU(3)\otimes SU(2)\otimes U(1)$ model, as well as its possible "vertical" extensions in the one family framework like $SU(5)$, $SO(10)$ etc., does not contain any deep physical grounds for the explanation of fermion mass hierarchy and their weak mixing pattern due to arbitrariness of Yukawa couplings. The identity of quark and lepton families:

$$(u, d, e, \nu_e), \qquad (c, s, \mu, \nu_\mu), \qquad (t, b, \tau, \nu_\tau) \tag{1}$$

relative to strong and electroweak interactions strongly suggests the existence of "horizontal" symmetry between them. The concept of local horizontal symmetry $SU(3)_H$ with left-handed quark and lepton components being $SU(3)_H$ triplets and the right-handed ones – antitriplets, first proposed in [1], is attractive to be considered (generalization on the case of n families, $SU(n)_H$, is trivial). In this approach the "reflection" hypothesis (RH) (being called earlier the hypothesis of horizontal hierarchy [2–4]) is reasonable, according to which the structure of fermion mass matrices reflects the pattern of horizontal symmetry breaking and the mass hierarchy between families is related to a definite hierarchy in this breaking. Indeed, the mass terms transform as $3\times 3=\bar{3}+6$ and hence may arise as a result of $SU(3)_H$ breaking only.

The simplest realization of RH was suggested in [3, 4] providing that the quark and lepton masses are induced in a "see-saw" manner due to their mixing with some additional superheavy fermions. In this approach along with the local $SU(3)_H$ the global $U(1)_H$ symmetry could be included naturally [5, 6]. Its breaking results in the existence of Goldstone boson α, which is simultaneously axion (or arion), singlet Majoron and familon*. Depending on the heavy fermion content there are two possibilities, with the quark and lepton mass hierarchy being in direct or inverted relation with the hierarchy of the $SU(3)_H\otimes U(1)_H$ symmetry breaking. It may be shown [6] that in these approaches it is possible to reproduce all the popular ansatz's for quark and lepton mass matrices.

In the present paper the whole pattern of physical and cosmological implications of the inverse hierarchy model is analysed and confronted with the possibilities of their experimental and astronomical search. Flavour nondiagonal transitions with α emission induce the decays $\mu\to e\alpha$, $\tau\to\mu\alpha$, $K\to\pi\alpha$, $B\to K\alpha$ etc., being available for experimental searches. On the other hand, the model provides the unified description of all the main types of dark matter, considered in the theory of the cosmological large scale structure: i) cold dark matter (CDM) in the form of oscillating primordial axion field, ii) hot dark matter (HDM) in the form of massive stable neutrino ν_τ and iii) unstable dark matter (UDM) in the form of unstable neutrinos decaying into lighter neutrino and archion with the dominance of relativistic or nonrelativistic decay products in the modern universe.

2. Let us consider the standard $SU(3)\otimes SU(2)\otimes U(1)$ model with gauge horizontal symmetry $SU(3)_H$ between families (1). Quarks and leptons are in following representations of $SU(2)\otimes U(1)\otimes SU(3)_H$:

$$f_{L\alpha}:\ \begin{pmatrix} u \\ d \end{pmatrix}_{L\alpha} (2,\, 1/3,\, 3), \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_{L\alpha} (2,\, -1,\, 3)$$

$$f_R^\alpha:\ u_R^\alpha(1,\, 4/3,\, \bar{3}), \qquad d_R^\alpha(1,\, -2/3,\, \bar{3}), \qquad e_R^\alpha(1,\, -2,\, \bar{3}) \tag{2}$$

* We call this particle archion

where we retain family ($SU(3)_H$) index: $\alpha=1, 2, 3$. Let us introduce additional fermions in the form [3]:

$$F_L^\alpha:\ U_L^\alpha(1, 4/3, \bar{3}),\quad D_L^\alpha(1, -2/3, \bar{3}),\quad E_L^\alpha(1, -2, \bar{3}),\quad N_L^\alpha(1, 0, \bar{3})$$
$$F_{R\alpha}:\ U_{R\alpha}(1, 4/3, 3),\quad D_{R\alpha}(1, -2/3, 3),\quad E_{R\alpha}(1, -2, 3). \tag{3}$$

Note, that these fermions cancel the $SU(3)_H$ anomalies of quarks and leptons (2). For the minimal Higgs content, appropriate for the realistic model the most general Yukawa couplings allowed by the gauge symmetry are

$$g_f \bar{f}_{L\alpha} F_{R\alpha} \phi^0 + G_{nF} \bar{F}_{R\alpha} F_L^\beta \xi_{\alpha\beta}^{(n)} + G_f \bar{F}_L^\alpha f_R^\alpha \eta + \text{h.c.};\quad n=1, 2 \ldots, \tag{4}$$

for quarks and charged leptons ($f=u, d, e$, $F=U, D, E$) and

$$g_\nu \bar{\nu}_{L\alpha} N_{R\alpha} \phi^0 + G_{nN} \bar{N}_{R\alpha} N_L^\beta \xi_{\{\alpha\beta\}}^{(n)} + \text{h.c.} \tag{5}$$

for neutrinos ($N_R \equiv C\bar{N}_L$, $\nu_R \equiv C\bar{\nu}_L$). Here ϕ^0 is the neutral component of the standard $SU(2)\otimes U(1)$ Higgs doublet ϕ (2, -1, 1) ($\langle\phi^0\rangle \equiv v = (\sqrt{8}G_F)^{-1/2} = 175$ GeV), η is the singlet scalar ($\langle\eta\rangle \equiv \mu$) and $\xi^{(n)}$, $n=1, 2, 3\ldots$, is a set of $SU(3)_H$ symmetry breaking scalars which may be either sextets $\xi_{\{\alpha\beta\}}$ or antitriplets $\xi_{[\alpha\beta]} = \varepsilon_{\alpha\beta\gamma}\xi^\gamma$. We assume the general structure for their VEVs matrix:

$$\hat{V}_H = \sum \langle \xi^{(n)} \rangle = \begin{pmatrix} r_1 & p_1 & p_2 \\ \pm p_1 & r_2 & p_3 \\ \pm p_2 & \pm p_3 & r_3 \end{pmatrix} \tag{6}$$

with the natural (about 5–10 fold) hierarchy $r_1 > p_1 > r_2$, $p_2 > p_3 > r_3$ (see for details [2, 6]). Signs + and − correspond to the cases of sextet and triplet scalars ξ_{nd} generating the nondiagonal VEVs. We shall not concretize further their $SU(3)_H$ content, mentioning only those cases, when sextet and triplet representations result in different consequences.

Yukawa couplings (4), (5) are invariant relative to global axial $U(1)_H$ transformations:

$$f_L \to e^{i\omega} f_L,\quad f_R \to e^{-i\omega} f_R,\quad F_L \to e^{-i\omega} F_L,\quad F_R \to e^{i\omega} F_R,$$
$$\phi \to \phi,\quad \eta \to \eta,\quad \xi^{(n)} \to e^{2i\omega} \xi^{(n)};\quad n=1, 2\ldots. \tag{7}$$

This $U(1)_H$ symmetry will be maintained also by the Higgs potential providing that there are no trilinear couplings between the horizontal scalars $\xi^{(n)}$. Such a couplings are not induced by any other (gauge or Yukawa) interactions, so their absence in the Lagrangian seems to be natural [6].

Inserting the scalar VEVs into Yukawa couplings (4), (5) one obtains full 6×6 fermion mass matrices:

$$\begin{array}{c|cc} & f_R & F_R \\ \hline \bar{f}_L & 0 & g_f v \\ \bar{F}_L & G_f \mu & \hat{M}_F \end{array},\qquad \begin{array}{c|cc} & \nu_R & N_R \\ \hline \bar{\nu}_L & 0 & g_\nu v \\ \bar{N}_L & g_\nu v & \hat{M}_N \end{array}. \tag{8}$$

Where $\hat{M}_F = \sum G_{nF} \langle \xi^{(n)} \rangle$, $F=U, D, E, N$ (note, that only sextet ξ scalars contribute into Majorana mass matrix $\hat{M}_N$). So, one has Dirac "see-saw" mechanism of the quark and lepton mass generation [3] and ordinary Majorana "see-saw" for neutrino masses, where N_R play the role of right-handed neutrinos [7]. The mass matrices obtained from the block-diagonalization of (8) have the form:

$$\hat{m}_f = g_f G_f v \mu \hat{M}_F^{-1} (f=u, d, e),\qquad \hat{m}_\nu = (g_\nu v)^2 \hat{M}_N^{-1}. \tag{9}$$

Therefore, the mass hierarchy between families appears to be inverted with respect to the hierarchy of symmetry breaking

$$SU(3)_H \otimes U(1)_H \xrightarrow{v_3} SU(2)_H \otimes U(1)'_H \xrightarrow{v_2} U(1)''_H \xrightarrow{v_1} I \tag{10}$$

where $v_3 = r_1$, $v_2 = (p_1^2 + 2r_2^2)^{1/2}$ and $v_1 = (p_2^2 + p_3^2 + 2r_3^2)^{1/2}$. Here the intermediate $SU(2)_H \otimes U(1)'_H$ horizontal symmetry is maintained between the second and third families and the remaining $U(1)''_H$ is appropriate to the third family only. This hierarchy could be estimated typically factor the difference in Yukawa coupling constants G_{nF}, which are supposed to be of the same order, as

$$v_1 : v_2 : v_3 \sim \sqrt{m_1} : \sqrt{m_2} : \sqrt{m_3} \sim 1 : 10 : 100 \tag{11}$$

where $m_{1,2,3}$ are the quark and lepton masses of corresponding families.

3. The breaking of global $U(1)_H$ ($U(1)''_H$) symmetry results in the existence of archion – Goldstone boson α, having both flavour diagonal and flavour non-diagonal couplings with quarks and leptons and thus being the "singlet" familon of the type [8, 9], which is different from the octet familons arising in the context of spontaneously broken global $SU(3)_H$ symmetry [10]. The flavour diagonal couplings of α are generally pseudoscalar* and the non-diagonal ones can be pure scalar, pseudoscalar or their mixture, depending on the phase structure of fermion mass matrices. The typical values for these coupling constants with e.g. charged leptons can be estimated as

$$g_{\tau\tau} \simeq m_\tau/v_1,\qquad g_{\mu\mu} \simeq (m_\mu/m_\tau)\, m_\mu/v_1,$$
$$g_{ee} \simeq (m_e/m_\tau)\, m_e/v_1,\qquad g_{\tau\mu} \simeq \sqrt{m_\mu m_\tau}/2v_1,$$
$$g_{\tau e} \simeq \sqrt{m_e m_\tau}/2v_1,\qquad g_{\mu e} \simeq (m_\mu/m_\tau)\sqrt{m_e m_\mu}/2v_1 \tag{12}$$

The tree-level couplings of α with quarks are the similar.

In our minimal $SU(2)\otimes U(1)\otimes SU(3)_H$ version with the fermion sets (2), (3) QCD and electromagnetic anomalies of global $U(1)_H$ current are cancelled in parallel with $SU(3)_H$ anomalies. So α is the arion type particle [12] with vanishingly small mass, having no couplings $\alpha G\tilde{G}$ and $\alpha F_{\text{em}} \tilde{F}_{\text{em}}$ induced by fermion triangles. Its interactions with the ordinary matter (first family quarks and

* Accounting for the pseudogoldstone nature of α being axion or arion tiny diagonal scalar couplings are also possible giving rise to long range interactions, which may generate effects of the fifth force

leptons) are highly suppressed removing the strong astrophysical restrictions on the scale v_1. The lower bound on this scale following from the analysis of ν-signal from supernova SN 1987A is about $5 \cdot 10^5$ GeV.

Somewhat stronger restrictions follow from the analysis of familon decays $\mu \to e\alpha$, $K \to \pi\alpha$, $\tau \to \mu\alpha$, $B \to K(K^*)\alpha$. For the typical values of familon coupling constants given in (12) the branching ratios of this decays can be estimated as

$$\begin{aligned} &\mathrm{Br}(\mu \to e\alpha) \simeq 2 \cdot 10^{-4} (10^6\,\mathrm{GeV}/v_1)^2, \\ &\mathrm{Br}(\tau \to \mu\alpha) \simeq 3 \times 10^{-3} (10^6\,\mathrm{GeV}/v_1)^2, \\ &\mathrm{Br}(K \to \pi\alpha) \simeq 4 \cdot 10^{-4} (10^6\,\mathrm{GeV}/v_1)^2, \\ &\mathrm{Br}(B \to K\alpha) \simeq 3 \times 10^{-2} (10^6\,\mathrm{GeV}/v_1)^2. \end{aligned} \tag{13}$$

Then the experimental upper limits $\mathrm{Br}(\mu \to e\alpha) < 2.6 \cdot 10^{-6}$ [13] and $\mathrm{Br}(K \to \pi\alpha) < 3.8 \cdot 10^{-8}$ [14], respectively, turn into lower bounds on the scale v_1: $v_1 > 8 \cdot 10^6$ GeV, $v_1 > 10^8$ GeV. It should be noted that these restrictions strongly depend on the concrete structures of the fermion mass matrices. E.g. if assume the Fritzsch ansatz [15] for the VEVs matrix $\hat{V}_H(6)$: $p_2, r_2, r_3 = 0$ (i.e. the inverted Fritzsch form [3, 6] for the quark and lepton mass matrices) the above restrictions from $\mu \to e\alpha$ and $K \to \pi\alpha$ will be lowered by factors m_μ/m_e and m_s/m_d, respectively. We would like to emphasize that in the case of pseudoscalar nondiagonal couplings $\alpha\bar{s}\gamma_5 d$ the strongest restriction from $K \to \pi\alpha$ decay is removed at all due to vanishing matrix element $\langle K|\bar{s}\gamma_5 d|\pi\rangle$ [8, 9]. As for $\tau \to \mu\alpha$ and $B \to K(K^*)\alpha$ decays for $v_1 \sim 10^6$ GeV their search could be available in near future.

In the case of α being arion $U(1)_H$ is not related to Peccei-Quinn symmetry [16], so the question of strong CP violation remains opened. However, there are two ways to solve this problem in the given model. The first, straightforward solution implies the introduction of some additional set of heavy fermions, e.g.

$$\begin{aligned} &\begin{pmatrix} U \\ D \end{pmatrix}_L^\alpha (2, 1/3, \bar{3}), \begin{pmatrix} N \\ E \end{pmatrix}_L^\alpha (2, -1, \bar{3}), \\ &\begin{pmatrix} U \\ D \end{pmatrix}_{R\alpha} (2, 1/3, 3), \begin{pmatrix} E \\ N \end{pmatrix}_{R\alpha} (2, -1, 3), \end{aligned} \tag{14}$$

which could also contribute the "see-saw" scheme of quark and lepton mass generation together with (3). Some kind of such additional set arises naturally with the extension of considered scheme to GUTs. E.g. in $SU(5) \otimes SU(3)_H$ with quarks and leptons (2) arranged in left handed multiplets $(\bar{5} + 10, 3)_L$ [1, 2] the heavy fermions (14) form together with (3) the representations $(\bar{5} + 5, \bar{3})_L$ and $(10 + \overline{10}, \bar{3})_L$ [3] (cancellation of $SU(3)_H$ anomalies requires also introduction of pure $SU(3)_H$ fermions, e.g. $(1, 15)_L$ and $(1, 3)_L$).

In this case owing to the presence of extra heavy fermions $U(1)''_H$ acquires the color, as well as electromagnetic anomalies and turns out to be the Peccei-Quinn symmetry. Thereby, α becomes the invisible axion of nearly hadronic type [17] with decay constant $f_\alpha = 2v_1$, mass $m_\alpha = A_c(10^6\,\mathrm{GeV}/v_1) \cdot 3$ eV and lifetime $T(\alpha \to 2\gamma) \simeq 2\tau(\pi^0 \to 2\gamma)(m_\pi/m_\alpha)^5$, having strongly suppressed couplings with electrons (see (12)). Here A_c is the color anomaly of $U(1)''_H$ current. In the case $A_c = 1$ (e.g. in $SU(5) \otimes SU(3)_H$ model with heavy fermions in $(10 + \overline{10}, \bar{3})_L$ only) the effective potential of axion field is nondegenerate and the Universe is free of domain walls.

The scale v_1 is then restricted from below by astrophysical evaluations of stellar energy losses due to axion emission: $v_1 > 10^6$ GeV (Sun and red giants [18]) and the data from SN 1987A excludes the range $2 \cdot 10^6\,\mathrm{GeV} < v_1 < 4 \cdot 10^9$ GeV [19], so there is a narrow allowed window near 10^6 GeV, for which the familon decays (13) are still of interest. Certainly, the wide interval between $v_1 \simeq 4 \cdot 10^9$ GeV and cosmological upper bound [20] $v_1 \simeq 10^{12}$ GeV is also possible.

The second solution is related to the singlet scalar field η. For η complex Yukawa couplings (4) acquire the additional chiral global symmetry $U(1)_\eta$:

$$\begin{aligned} &f_L \to e^{i\sigma} f_L; \quad f_R \to e^{-i\sigma} f_R; \quad F_L \to e^{i\sigma} F_L; \quad F_R \to e^{i\sigma} F_R \\ &\phi \to \phi, \quad \eta \to e^{2i\sigma}\eta, \quad \xi^{(n)} \to \xi^{(n)}, \qquad n = 1, 2 \ldots . \end{aligned} \tag{15}$$

This symmetry, playing the role of Peccei-Quinn symmetry, is broken at the scale $f_a = \sqrt{2}\mu$, which leads to the appearance of invisible axion a of the Dine-Fishler-Srednicki-Zhitnitsky (DFSZ) type [21] with $x = 1$, having only diagonal couplings with quarks and leptons – $g_{ee} = m_e/\mu$, $g_{uu} = m_u/\mu$ etc. Then the astrophysical lower bounds from red giants $\mu > 10^8$ GeV [18] and from SN 1987A $\mu > 10^{10}$ GeV [19] and cosmological upper bound $\mu < 10^{12}$ GeV [20] leave the window between 10^{10} GeV and 10^{12} GeV for μ variation.

So, in the framework of our model both popular types of axion (DFSZ and hadronic) can be included. The difference of our hadronic axion from the standard one [17] removes the cosmological problem of the latter, connected with the overabundance of superheavy stable quarks in the Universe, since such quarks (3) and (14) are unstable owing to their mixing with light ones. In general case when both $U(1)_H$ and $U(1)_\eta$ symmetries are present, one superposition of Goldstone bosons α and a, connected with the color anomaly free subgroup of $U(1)_H \otimes U(1)_\eta$ will be the arion and the other one will be the axion.

4. According to (9) the hierarchy of neutrino Majorana masses is similar to the ordinary quark and lepton mass hierarchy:

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} \sim m_e : m_\mu : m_\tau . \tag{16}$$

If the mass of τ-neutrino is larger than 1–10 eV, then accounting for existing upper limits on lepton mixing from the searches for ν-oscillations, it is naturally to expect that the relationship (16) is almost exact. According to (9) the mass of heaviest neutrino (ν_τ) is determined by the mass of the lightest from heavy neutral leptons N (N_3), so in view of hierarchy (11) we have

$$m_{\nu_\tau} = (g_\nu v)^2 / M_{N_3} \sim (g_\nu^2 / G_N)(10^{12}\,\mathrm{GeV}/\mathrm{v}_1)\,\mathrm{KeV}. \tag{17}$$

Note, that N_3 decays in light particles due to small mixing with ν_τ with lifetime $\tau_{N_3} \propto m_{\nu_\tau}^{-1}$. The analysis [6, 22] of primordial black hole (PBH) formation by these metastable particles at the stage of their dominance in the very early Universe leads to the lower bound on ν_τ mass: $m_{\nu_\tau} \geqq 0.1$ eV and, on the other hand, to upper limit on the scale $v_1 : v_1 \leqq 10^{14}$ GeV.

Archion α appears to be the singlet Majoron type particle [23] related to the appearance of the Majorana masses of right-handed neutrinos N_R. For the scalars ξ_{nd} being sextets the neutrino mass matrix $\hat{m}_\nu$ (9) is nondiagonal and the Heavier neutrino ν_H decays into lighter one $\nu_L : \nu_H \to \nu_L \alpha$ are possible with the lifetime

$$\tau(\nu_H \to \nu_L \alpha) = 16\pi / g_{HL}^2 m_H \tag{18}$$

where $g_{\nu_\mu \nu_\tau} \simeq \sqrt{m_{\nu_\mu} m_{\nu_\tau}} / 2 v_1$ etc. similar to (12). For triplet ξ_{nd} $\hat{m}_\nu$ is diagonal and neutrinos are stable.

5. The present model exhibites the simplest variant of unified physical framework for analysis of practically all the main types of dark matter, considered in the cosmological theory of large scale structure formation. The model predicts the hierarchy of neutrino masses and lifetimes relative to familon decays and the existence of axion. Relative contribution of neutrinos and axions into the cosmological density is determined by the parameters of the model and, first of all, by the scale of horizontal symmetry breaking.

Let us consider first the case without second global symmetry $U(1)_\eta$. For the total ρ_{tot} and baryon ρ_B modern densities being fixed, the relationship

$$\rho_\alpha(v_1) + \rho_{\nu_\tau}(v_1) + \rho_{\nu_\mu}(v_1) + \rho_{\nu_e}(v_1) + \rho_B = \rho_{\mathrm{tot}} \tag{19}$$

turns to be the equation for the value of v_1 giving the discrete set of cosmological models with different types of dark matter, forming the structure of the Universe. Since baryonic forms of dark matter are rather improbable one has six realistic possibilities in the framework of considered model:

1) The primordial axion density ρ_α dominance (CDM): If α is axion, its primordial field oscillations contribute the modern cosmological density as $\rho_\alpha \simeq (v_1 / 4 \cdot 10^{11}\ \mathrm{GeV})\, \rho_{cr}$ [20]. One can easily estimate that the density of primordial thermal axions is always small: $\rho_\alpha^T = m_\alpha n_\alpha^T$, where $n_\alpha^T \leqq 0.1\, n_\nu \leqq 0.03\, n_\gamma$. According to [24] the intensive axion emission by decaying axion cosmic string structure may increase the cosmological axion density, so that its modern value is equal to $\rho_\alpha = (v_1 / 2 \cdot 10^{10}\ \mathrm{GeV})\, \rho_{cr}$.

The sensitivity of ρ_α to the existence of the cosmic string network makes it possible to probe the conditions of $U(1)_H''$ phase transition in their relationship with the inflational stage. Indeed, the typical scale of $\theta = \alpha / f_\alpha$ variation on 2π, needed for axion string formation, is of the order of horizon size for the period of $U(1)_H''$ phase transition. Since this scale is usually very small averaging over it gives the estimation $\theta \sim 1$ for the mean amplitude of axion coherent oscillations, and the contribution of axion emission of strings is essential. If such transition takes place at inflational stage (or earlier) this scale extends exponentially, so that the string network is too rarefied to give any significant contribution into the axion density. In this case the axion density is determined by axion condensate with the oscillation amplitude θ, fixed by its arbitrary value at the beginning of inflation, so that very small values of this amplitude $\theta \ll 1$ are possible [25].

2) Massive stable neutrino ν_τ density ρ_{ν_τ} dominance (HDM), when its mass $m_{\nu_\tau} \approx 24$ eV and the lifetime $\tau(\nu_\tau \to \nu_\mu \alpha)$ exceeds the age of the Universe t_u: $\rho_{\nu_\tau} = m_{\nu_\tau} n_\nu$, where $n_\nu = (3/11)\, n_\gamma$ is the standard big bang neutrino number density [26]. Accounting for (17) and varying g_ν^2 / G_N from 10^{-5} to 10^{-1} the corresponding scale v_1 varies in the range $10^8 \div 10^{12}$ GeV. Note, that for the definite range of the parameter g_ν^2 / G_N it is possible to realize the combined scenario CDM + HDM with the dominance in the Universe of comparable amounts of CDM (ρ_α) and HDM (ρ_{ν_τ}).

The following possibilities correspond to UDM scenarios [27]:

3) The dominance of density of archions ρ_α and neutrinos ρ_{ν_μ}, relativistic decay products of massive unstable ν_τ with mass $m_{\nu_\tau} = 50 \div 100$ eV and lifetime $\tau(\nu_\tau \to \nu_\mu \alpha) = (2/\pi)\, t_u (24\ \mathrm{eV}/m_{\nu_\tau})^2 = 4 \cdot 10^{16} - 10^{16}$ s: $\quad \rho_\alpha = \rho_{\nu_\mu} \propto v_1^{3/2}$. From (18) with typical $g_{\nu_\mu \nu_\tau}$ we estimate that the scale $v_1 \sim 10^8$ GeV, which fixes the value $(g_\nu^2 / G_N) \sim 10^{-5}$.

4) The dominance of nonrelativistic ν_μ from ν_τ decay and primordial ν_μ for the same range of parameters, if $m_{\nu_\mu} \sim 10$ eV and $\tau(\nu_\tau \to \nu_\mu \alpha) \leqq (2/\pi)\, t_u (m_{\nu_\mu} / m_{\nu_\tau})^2$.

5) The dominance of relativistic archions ρ_α and neutrinos ρ_{ν_e} from the decay of massive unstable ν_μ with $m_{\nu_\mu} = 50$–100 eV and $\tau(\nu_\mu \to \nu_{e_\alpha}) = (2/\pi)\, t_u (12\ \mathrm{eV}/m_{\nu_\mu})^2 = 10^{16} - 2 \cdot 10^{15}$ s. For typical $g_{\nu_e \nu_\mu}$ this corresponds to the scale $v_1 \sim 10^6$ GeV and $g_\nu^2 / G_N \sim 10^{-5}$. Then the condition of fast ν_τ decays with $m_{\nu_\tau} \sim$ few keV and $\tau(\nu_\tau \to \nu_\mu \alpha) \sim 10^8$ s is authomatically satisfied.

6) The dominance of nonrelativistic (or semirelativistic) axions ρ_α (for $v_1 \sim 10^6$ GeV the axion mass $m_\alpha \sim$ few eV), being the products of early ν_τ decays and of ν_μ decays with $\tau(\nu_\mu \to \nu_e \alpha) / t_u \leqq (2 m_\alpha / m_{\nu_\mu})^2$. In this case the main contribution into the inhomogeneously distributed dark matter is from nonrelativistic axions from early ν_τ decays with the axion concentration n_α^i, being equal to the concentration of primordial ν_τ, $n_{\nu_\tau}^{\mathrm{prim}}$. Since the concentration of nonrelativistic axions from $\nu_\mu \to \nu_e \alpha$ decay, being uniformly distributed in the Universe, is $n_\alpha^u = n_{\nu_\mu}$ and n_{ν_μ} is determined by the concentration of primordial ν_μ, $n_{\nu_\mu}^{\mathrm{prim}}$, and by the concentration of ν_μ from early $\nu_\tau \to \nu_\mu \alpha$ decays, being equal to $n_{\nu_\tau}^{\mathrm{prim}}$, one obtains for $n_{\nu_\mu}^{\mathrm{prim}} = n_{\nu_\tau}^{\mathrm{prim}}$ $\Omega^u \geqq 2\Omega^i$, so that the total density being equal to the critical one the density of dark matter in the inhomogenities should be no more, than $\Omega_{dm}^i < 0.3$.

Note that the cases 5)–6) correspond to a nontrivial scenario of the cosmological evolution of inhomogenities, combining the attractive features of both the models of early neutrino decays [28] and of unstable dark matter scenarios [27] and deserving special consideration. In this scenario short period of ν_τ dominance in the Universe at $t \sim 10^6 \div 10^8$ s provides the survival and further development of short-wave ν_μ and axion density perturbations at the scales ~ 10–100 kpc and long (from 10^{11} s to 10^{16} s) period of ν_μ dominance gives birth to the formation of the cosmological large scale structure (at the scales $\sim 10 \div 100$ Mpc). Note, that the condition of the growth of initial density fluctuations into the observed structure for the observed isotropy of the microwave thermal background is satisfied for the neutrino mass and lifetime hierarchy, predicted by the presented model, if either $m_{\nu_\tau} \leqq 100$ eV (cases 1)–4)), or $(v_1/10^8\,\mathrm{GeV})^2 (100\,\mathrm{eV}/m_{\nu_\mu}) < 10^{-3}$ (cases 5)–6)) [6]. In the latter case the acount for radiative decays $\alpha \to 2\gamma$ results in the prediction of nonthermal electromagnetic background, ionizing the matter at $z > 10^3$ and reducing thus the predicted anisotropy of the thermal background [29].

The cosmological dark matter density dependence on the scale v_1 is shown on Fig. 1. The solutions of (19), corresponding to the cases 1)–6) are also shown.

It should be emphasized that the cases 3) and 4), requiring $v_1 \sim 10^8$ GeV are excluded in view of SN 1987A data, if α is axion. For α being arion these cases become allowed, but the cases 1) and 6) are absent, and the case 1) may be realized by the additional DFSZ axion a only. Since the scale μ related to a is restricted from below by SN 1987A data: $\mu > 10^{10}$ GeV, significant contribution of CDM in cosmological density is guaranteed in this situation, thus providing for appropriate choice of the scale v_1 interesting possibilities of combained HDM+CDM or UDM+CDM cosmological models. The case 6) can not be realized, since arion produced in ν-decays is practically massless particle.

6. The considered simplest model of quantum flavourdynamics based on local $SU(2) \otimes U(1) \otimes SU(3)_H$ symmetry with associated global $U(1)_H$ satisfies the requirement of RH: after the horizontal symmetry breaking at the scale $v_H > 10^6$ GeV it reduces to standard $SU(2) \otimes U(1)$ scheme. The Yukawa couplings of the single Higgs doublet ϕ are then determined by the VEV matrix $\hat{V}_H$ of horizontal scalars. As a consequence, the flavour changing neutral currents are naturally suppressed [30] at the electroweak scale v.

The model contains a rather wide set of parameters. But: i) the number of these parameters is smaller than in the standard model without horizontal symmetry and it will be reduced further with the extention to GUTs ii) the bulk of these parameters is fixed by the experimental data on quark and lepton properties and, finally, iii) the set of new nontrivial phenomena, predicted by the model, provides in principle the complete check of the model and determination of all the parameters. These phenomena arise at a high energy scale of horizontal symmetry breaking $v_H > 10^6$ GeV which can not be achieved even in the far future at accelerators. However, combination of experimental searches of their indirect effects in the processes with known particles (rare familon decays $\mu \to e\alpha$, $\tau \to \mu\alpha$ etc., ν-oscillations, $2\beta_{0\nu}$-decay and so on) together with their astrophysical effects (search of Solar axions with Helioscopic detectors or axions from primordial condensate with Haloscopic detectors [31]) makes it possible to study physics, predicted at this scale. In any case, determination in astronomical observations of the dominant form of dark matter, formed the structure of the universe, turns to be the way of precise measurement of the magnitude v_H. In particular, in the case of "low" scale horizontal symmetry ($v_1 \sim 10^6$ GeV) the set of these data makes the system of equations for unknown model parameters overdetermined.

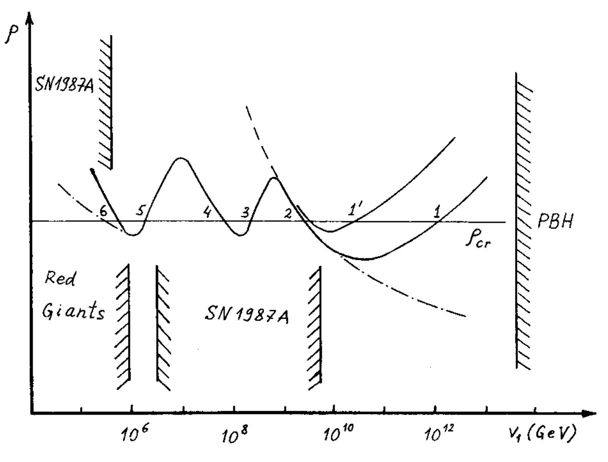

Fig. 1. Cosmological density dependence on the scale v_1. The solid line corresponds to cosmological density in the case, when ξ_{nd} are sextets and α is axion. Solutions of (19) 1–6 correspond to the dark matter scenarios 1)–6) (1′ corresponds to the dominance of ρ_α from cosmic string network). The dashed line corresponds to triplet ξ_{nd}, where there are no neutrino decays and, finally, the dotted line corresponds to the case α is arion. The astrophysical and cosmological bounds on v_1 are also shown for the cases α being axion (lower restrictions) and arion (upper restrictions)

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References

1. J.L. Chkareuli: Pis'ma ZhETF 32 (1980) 684
2. Z.G. Berezhiani, J.L. Chkareuli: Pis'ma ZhETF 35 (1982) 494; Yad. Fiz. 37 (1983) 1043
3. Z.G. Berezhiani: Phys. Lett. 129B (1983) 99
4. Z.G. Berezhiani: Phys. Lett. 150B (1985) 177
5. Z.G. Berezhiani, M.Yu. Khlopov: Proc. XXIV Rencontres de Moriond, ed. J. Tran Thanh Van, p. 179. Edition Frontieres, 1989
6. Z.G. Berezhiani, M.Yu. Khlopov. Preprint SAO-23 (1989); Yad. Fiz. 51 (1990) 1157, 1479
7. Z.G. Berezhiani, J.L. Chkareuli: Pis'ma ZhETF 37 (1983) 285
8. A.A. Anselm, N.G. Uraltsev: ZhETF 84 (1983) 1961
9. A.A. Anselm, Z.G. Berezhiani: Phys. Lett. 162B (1985) 349
10. F. Wilczek: Phys. Rev. Lett. 49 (1982) 1549; D. Reiss. Phys. Lett. 115B (1982) 217; G.B. Gelmini, S. Nussinov, T. Yanagida. Nucl. Phys. B219 (1983) 31
11. E. Fishbach et al.: Phys. Rev. Lett. 56 (1986) 3; for theoretical discussions see C.T. Hill, G.G. Ross: Nucl. Phys. B311 (1989) 253 and also [8] and [18]
12. A.A. Anselm: Pis'ma ZhETF 36 (1982) 46; A.A. Anselm, N.G. Uraltsev: Phys. Lett. 116B (1982) 161
13. A. Jodidio et al.: Phys. Rev. D34 (1986) 1967

14. Y. Asano et al.: Phys. Lett. 107B (1981) 159
15. H. Fritzch: Nucl. Phys. B155 (1979) 183
16. R.D. Peccei, H.R. Quinn: Phys. Rev. D16 (1977) 1791
17. J.E. Kim: Phys. Rev. Lett. 43 (1979) 103; M.A. Shifman, A.I. Vainshtein, V.I. Zakharov: Nucl. Phys. B166 (1980) 493
18. J. Kim: Phys. Rep. 150 (1987) 1; H.Y. Cheng. ibid. 158 (1988) 1
19. J. Ellis, K.A. Olive: Phys. Lett. 193B (1987) 525; G. Raffelt, D. Seckel: Phys. Rev. Lett. 60 (1988) 1793; M.S. Turner: ibid 1796
20. J. Preskill, M. Wise, F. Wilczek: Phys. Lett. 120B (1983) 127; L. Abbot, P. Sikivie: ibid., 133; M. Dine, W. Fishler: ibid., 137
21. A.R. Zhitnitsky: Yad. Fiz. 31 (1980) 497; M. Dine, W. Fishler, M. Srednicki: Phys. Lett. 104B (1981) 497
22. M.Yu. Khlopov, A.G. Polnarev: Phys. Lett. B97 (1980) 383
23. V. Chikashige, R.N. Mohapatra, R.D. Peccei: Phys. Rev. Lett. 45 (1980) 1926
24. R.L. Davis: Phys. Lett. 180B (1986) 225
25. A.D. Linde: Phys. Lett. 200B (1988) 272
26. A.D. Dolgov, Ya.B. Zeldovich: Rev. Mod. Phys. 53 (1981) 3; Ya.B. Zeldovich, M.Yu. Khlopov: Usp. Fiz. Nauk 135 (1981) 45
27. A.G. Doroshkevich, A.A. Klypin, M.Yu. Khlopov: Pis'ma Astron. Zh. 11 (1985) 483; Astron. Zh. 65 (1988) 248; in: Large scale structures of the universe, A. Szalay, J. Audouze (eds.), p. 293. Dordrecht: Reidel 1988
28. M. Davis, M. Lecar, C. Pryor, E. Witten: Ap. J. 250 (1981) 423
29. Z.G. Berezhiani, M.Yu. Khlopov, R.R. Khomeriki: FERMILAB-PUB-89/203A; Yad. Fiz. 52 (1990) 104
30. S.L. Glashow, S. Weinberg: Phys. Rev. D15 (1977) 1958; E.A. Paschos: ibid., 1964
31. P. Sikivie: Proc. Int. Conf. CP-violation in particle physics and astrophysics, Chateau de Blois, May 22-26 1989. J. Tran Thanh Van (ed.), p. 319. Gif-sur-Yvette: Edition Frontieres 1989