Point of View on Gravity

H.B. Nielsen[2], Niels Bohr Institut, (et al?)

“Bled” , July , 2023

[2]Speaker at the Work Shop “What comes beyond the Standard Models” in Bled.

Viewing Gravity from reparametrisation Invariant Fundamental Theory

Gravity or Physics with propagation, needs a breaking of scale invarince

since the equation of motion

$$g^{\mu\nu}\partial_\mu\partial_\nu\phi = 0 \tag{1}$$

needs an upper index $g^{\mu\nu}$ for being reparametrization invariant. A non-zero $g^{\mu\nu}$ represent a spontaneous break down of a symmetry involving say scaling or reparametrizations.

The theorem: Spontaneous breaking of Reparametrization Symmetry Needed

Theorem: *If a theory with reparametrization invariance is not spontaneously broken - meaning the vacuum is totally reparametrization invarint - then propagation in this world is impossible.*

The speculative suggestion: If reparametrization transformations constitue a fundamental symmetry there would be waves going from one point to another; so only to the extend that vacuum has some breaking of the symmetry of reparametrization we can get propagation. So not much interesting physics could go on without this spontaneous breaking. Gravity-like fields - basically $g^{\mu\nu}$ - non-zero in vacuum is needed.

Side remark on Dimensional Reduction in the Spontaneous Breakdown

Even if dimension d of the fundamental space were high, we could have that the rank of the $g^{\mu\nu}$ tensorfield in vacuum were lower. In that case the world in which we could propagate would be the lower dimension.
But say spinor-fields would anyway have to have numbers of components matching the fundamental dimension, but alas: There is no spinor representation of the general linear group which the transformation group of the tangent space for the symmetries we care for in this talk! So can a model with reparametriztion unvariance as fundamental have spinors at all?

In spaces with local linear deformation like: Reparametrization invariant or Projective space, No Signature

It is the $g^{\mu\nu}(x)$ that has the signature - in physical world 3+1 -, so without $g^{\mu\nu}$ Minkowski and Euklidean spaces are the same.

Our Point of View: Start with a Scaling Containing Symmetry.

Having in mind our work with Astri Kleppe[1, 2] of deriving locality of the theory from a reparametrisation invaliant theory - though allowed not to be invariant under variations of the measure - for an extremely genral action, we suggest to assume a symmetry involving - at least locally - such a reparametrisation invariance to be assumed before e.g. getting gravity.
That is to say: We want to assume either reparametrisation invariance or something like that, end after that hopefully derive or understand gravity and locality.

Geometries with (local) scale symmetry

Examples of how you can have local scale symmetry:

- **Full Reparametrization Symmetry** This is the symmetry of coordinate shifts in the General relativity.
- **Projective geometry space** The symmetry of the projective space is a smaller group than that of general relativity. (I am personally especially attached to projective geometry, because I made my leaving from teaching it for 6 years.)

By local scale symmetry we mean that there are symmetries so that in the tangent space to any point we have symmetry under scaling up by any (real) factor this tangent space.
In the present work we really want to have locally not only scale invariance but invariance under any real linear transformation $GL(d, \mathrm{R})$.

Starting Point: Locally General Linear Transformation Symmetry

Our starting assumption - in this work - is that there are such symmetries assumed that for every point on the manifold you must have symmetry under general linear transformations in the tangent space:

$$\begin{aligned}
(dx^1, dx^2, ..., dx^d) &\rightarrow ((Adx)^1, (Adx)^2, ..., (Adx)^d) && (2)\\
&= (A^1_\mu dx^\mu, A^2_\mu dx^\mu, ..., A^d_\mu dx^\mu) && (3)\\
\text{for any real matrix} A^\mu_\nu &\in \mathrm{M_{d\,\times\,d}}(\text{in the curled indices}). && (4)
\end{aligned}$$

Having such a symmetry will be enough for guaranteeing that we for a general functionally analytic action $S[fields]$ shall get locallity[1, 2].

Propagation Requires Breaking of the Locally General Linear Symmetry

Usually the propagation of particles in say free approximation is given by a D'alembertian equation of motion

$$(\Box + m^2)\phi = 0 \quad (5)$$

but to have local general linear transformation invariance:

$$(g^{\mu\nu}\partial_\mu\partial_\nu + m^2)\phi = 0 \quad (6)$$

we need the $g^{\mu\nu}$!

If such a $g^{\mu\nu}$ is non-zero in vacuum, we have a **spontaneous break down** of the symmetry, because the $g^{\mu\nu}$ field transform non-trivially under the local genral linear tranformations.
So we only can propagate (normally) any particles, provided we break (spontaneously) this locally general linear symmetry!

Reviewing our “ Derivation of Locality”

Let us review Astri Kleppes and HBN’s [1, 2] “derivation” of locality under the assumptions of **diffeomorphism symmetry** (invariance under reparamterizations) for a very general action S not being a priori local but rather only having the diffeomorphism symmetry and being **Taylor expandable** in **local fields**:

$$S[\phi] \quad = \quad \sum_n \frac{1}{n!} \sum (\text{or integral}) \frac{\partial}{\partial\phi(x_1)} \cdots \frac{\partial}{\partial\phi(x_n)} S[0]\phi(x_1)\cdots\phi(x_n),$$

where here ϕ stands for a very general fields, possibly with many indices, and S is an a priori non-local action.

The Taylor Expnasion for Functional in Integral form

The **functional Taylor expansion** in the more functional notation (i.e. without imagining a lattice cut off say):

$$S[\phi] \quad = \quad \sum_n \frac{1}{n!} \int \frac{\delta}{\delta\phi(x_1)} \cdots \frac{\delta}{\delta\phi(x_n)} S[0]_{\mu\cdots\nu} \phi(x_1) \cdots \phi(x_n) \Pi_i dx_i^{\mu} \cdots dx_i^{\nu}$$

Here the δ means functional derivative

The Crucial Point: All Points can by Symmetry(Difeomorphism symmetry) be brought into Any Other one, Trasitivity

When there is no distance a priori in the just manifold with diffeomorphism symmetry or the projective space, you at least, if you do not go to higher order interaction with several fields multiplied, **field at one point will interact the same way with fields at any other point, except the very point itself.** Thus you either interactions between all points, or interaction of the fields at the same point, i.e. locality.

More on Transitivity

When a group G acts on a space X

$$\alpha : G \times X \rightarrow X \quad (7)$$

$$\text{denoting } \alpha(g, x) = gx \quad (8)$$

$$\text{so if } gx_1 = x_2, \quad (9)$$

it means the group element g brings the element $x_1 \in X$ into $x_2 \in X$, then we say G **acts n-transitively provided there for any two ordered sets of n different points in X,** $(x_1, x_2, ..., x_n)$ **and** $(y_1, y_2, ..., y_n)$ **exist a group element** $g \in G$ **such that**

$$gx_i = y_i \text{ for all } i. \quad (10)$$

We say it is sharply n-transitive, when this g is unique.

d-dimensional projective space has a symmetry group acting almost(!) $(d+2)$-transitively

Examples:

- Under the action of diffeomorphisms on a manifold the action is infinitely transitive.
- In d-dimensional projective space $PS(d, \mathrm{R})$ the symetry group acts essentially $(d+2)$-transitively, but not truly so, because the immage of points say on a line remains on a line. Only the projective line $PS(1, \mathrm{R})$ is truly 3-transitive.
- Euclidean spaces are only (1-)transitive under their symmetry. It is the translation group that acts transitively. When the group conserves the length of line there can be no even 2-transitivity.

We derived a form of the Diffeomorphism Invariant Taylor Expandable Action $S[\phi]$ as a Function $F(S_1[\phi], S_2[\phi], ..., S_n[\phi])$ of usual local action integral.

Indeed the terms in the functional Taylor expansion will be of the form that groups of factors are at the same point inside the groups, and that then these point are integrated over all the space(manifold). Denoting the possible integral over local field combinations

$$S_i[\phi] \quad = \quad \int \mathsf{L}_{i\ \mu\nu\rho\kappa}(x) dx^\mu \Lambda dx^\nu \Lambda \cdots \Lambda dx^\kappa \qquad (11)$$

we get the form

$$S[\phi] \quad = \quad F(S_1[\phi], ..., S_n[\phi]). \qquad (12)$$

This form was studied by my student Stillits[1]

But this was not Really Locality!

The form, which we derived from the diffeomorphism invariance

$$S[\phi] \quad = \quad F(S_1[\phi], ..., S_n[\phi]) \tag{13}$$

is not truly local; we should have had a linear combination

$$S[\phi] \quad \overset{=}{wanted} \quad a_1 S_1[\phi] + ... + a_n S_n[\phi]. \tag{14}$$

However, if we construct the equation of motion by putting the functional derivatiive of (15) we get the form (14).

Equations of Motion got Already Local, but...

The equations of motion for an action of the form - derived from the diffeoemorphism symmetry

$$S[\phi] = F(S_1[\phi], ..., S_n[\phi]) \tag{15}$$

becomes

$$\begin{aligned} 0 &= \frac{\delta S[\phi]}{\delta\phi(x)} && (16) \\ &= \sum_i \frac{dF}{dS_i}|_\phi * \frac{\delta S_i[\phi]}{\delta\phi} && (17) \\ &= \frac{\delta}{\delta\phi}\sum_i \frac{dF}{dS_i}|_\phi * S_i[\phi] \text{ considering } \tfrac{dF}{dS_i}|_\phi = a_i \text{ constants} \end{aligned}$$

The coefficients $\frac{dF}{dS_i}|_\phi$ do depend on the fields ϕ, but integrated over all time and all space.

Effectively these coefficients

$$a_i = \frac{dF}{dS_i}|_\phi \tag{18}$$

to the various possible local actions $S_i[\phi]$ **do depnd on the fields** ϕ but since they depnd via integrals over all space time, we can in pracsis take them as constants. Indeed they are the **coupling constants** which we just fit to experiments. But it means that our lack of completing the derivation of locallity means: **The coupling constants - say fine structure constant etc. - depends huge integrals over space time**, although comosed in a way which depends on the fundamental non-local action, which we do not know (yet?).

We did not get full locallity! Coupling constants depend on all space-time

We got, that **the Lagrangian density only depends on the fields in the point you write this Lagrangean density**, and that is practical locality, **but we did not get locallity for the coupling constants in the sense that they with derivation "know about" what goes on all over space and time, including even future.**
. This suggests that the question of what the initial conditions should be at least in principle needs an extra discussion.
In principle there is a backreation for any choice of initial condition because it influences the couplings depeding on the development of it also to far future.

Repeating Argument using Action

Instead of looking at the equation of motion we could ask, if we could make an action

$$S[fields] \quad = \quad \int \mathcal{L}_{\mu\nu\rho\sigma}(x) dx^\mu \Lambda dx^\nu \Lambda dx^\rho \Lambda dx^\sigma, \qquad (19)$$

which is invariant under our symmetry having locally generallinear symmetry and at the same time can describe a propagation of some fields.

If a field ϕ shall not be determined locally by theother fields, but appear in equation(s) with derivatives, there must be a derivative acting on ϕ i.e. say $\partial_\mu \phi$ occuring in the Lagrangian density $\mathcal{L}_{\mu\nu\rho\sigma}$; but with what to contract the lower index μ on $\partial_\mu \phi$? To some field with an upper curled index like a vierbein V_a^μ or a $g^{\mu\nu}$? Yes but if we work in vacuum and there were no spontaneous break down of the symmetry these fields would be zero.

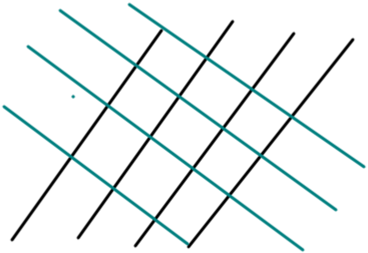
Continuing repeating Derivation of Need for Spontaneous Breaking of Locally General linear symmetry

Looking for making

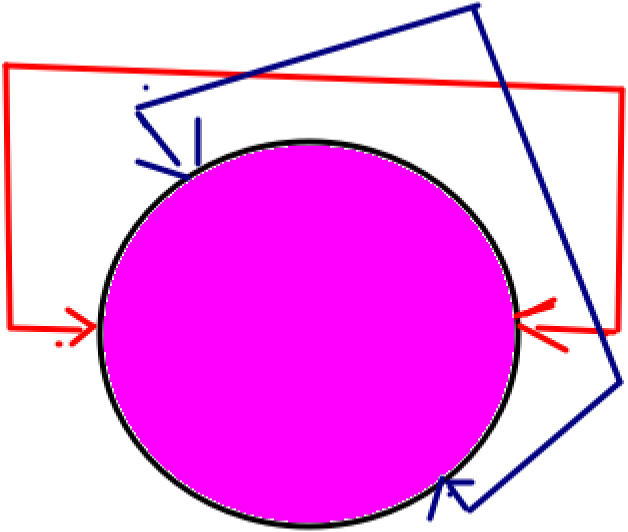
$$S[fields] \quad = \quad \int \mathcal{L}_{\mu\nu\rho\sigma}(x) dx^{\mu} \Lambda dx^{\nu} \Lambda dx^{\rho} \Lambda dx^{\sigma}, \qquad (20)$$

invariant under the symmetry, but still with fields propagating even with vacuum not breaking the symmetry. (We shall show you cannot find such an action.)
Can it help to let the $\partial_\mu \phi$ combination be contracted with a dx^{μ} to give it a chance to propagate?
Couple it directly to the dx^{μ}'s?

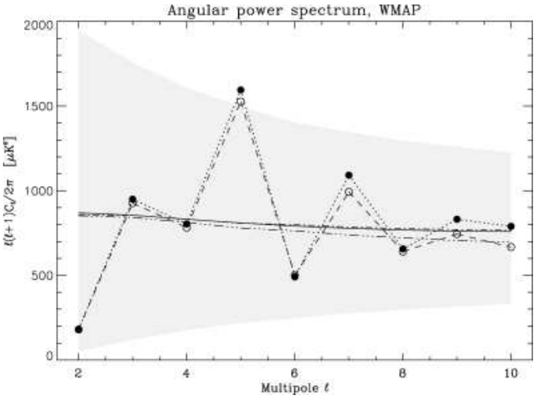
In Plane Projective Geometry All (different) Lines Cross in a Point



Bundles of parallel lines are identified with points on the line at infinity. So parallel lines cross there.



Lowest l WMAP fluctuations



Remarkable:**Even l fluctuatons are relatively high, while the odd l ones are relatively small**

If Projective Space “seen” in CMB-fluctuations, then Universe Not Much bigger than Visible Universe

The just shown:

- Y_{lm}-proportinal modes in temperature variation over sky with **even** l have **lower** fluctuation.
- Y_{lm}-proportional modes in temperature variation over sky with **odd** l have **higher** fluctuation.

if taken seriously implies that **the visible universe edge is not very far from where there is the identification of the diametrically opposite points** (on say the infinite line). So Universe would not be so huge as the very accurate flatness would indicate!

The Hugeness of the Universe ?

Dirac wondered about the huge numbers of order 10^{20}, that e.g. the age of the universe is of the order of $(10^{20})^3$ time the Planck time.

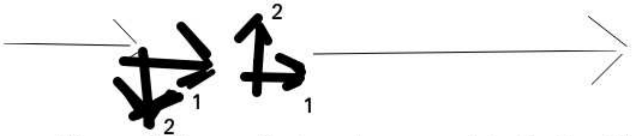
Assuming a **projective space** background for our space time could in an a priori unexpected way enforce the existence of very - infinitely - extended space time reagion(s)!

Argument goes:

- The projective space of even dimension is non-orientable.
- That enforces a hyper-surface, on which the $g^{\mu\nu}$ is is of rank one less - say for normal rank 4 it has 3 there.
- But then there $g_{\mu\nu} = \infty$.
- Appraoching this degeneracy surface the volume relative to the coordinates grow so much that an infinite universe in space and time pops out.

Non-orientability of Even Dimensional Projective space

Most easily seen in the even dimension $d = 2$.



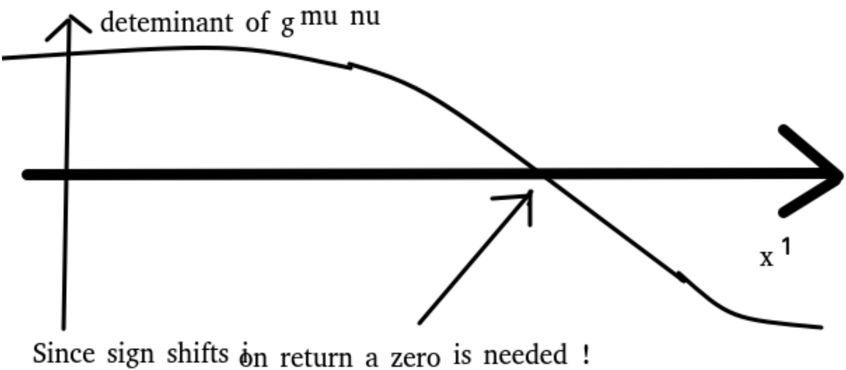
Move a little coordinate system around to the line at infinity and back from opposite side.

The Needed $g^{\mu\nu}(x)$ must be degenrate along a 3-surface

The determinant $det(g^{\mu\nu})$ cannot avoid a zero surface of dimension 3 in a 4 dimensional projective space. The sign of this determinant namely represents an orientation.
Write it the coordinates chosen locally x^1, x^2, x^3, x^4 and in a certain order say 1,2,3,4. Then

$$\begin{aligned} \text{If } det(g^{\mu\nu}) &> 0, \text{ orientation is that of ordered coordinates} \quad (21) \\ \text{If } det(g^{\mu\nu}) &< 0, \text{orientation is opposite coordinates in their order} \end{aligned}$$

Think of the determinat $det(g^{\mu\nu})$ followed around to infinite line and back the other way



We really needed upper index $g^{\mu\nu}$, so it must be "fundamentally" an effective (?) field

But the lower index ones $g_{\mu\nu}$ could just a definiton of an inverse. I.e. the $g_{\mu\nu}$ with lower indices would just be the defined as the inverse

$$g_{\mu\nu} = (-1)^{\mu+\nu} \frac{det g^{\cdot\cdot}|_{\text{left out}_{\mu\nu}}}{det(g^{\cdot\cdot})} \tag{22}$$

So when $det(g^{\mu\nu}) = 0$ **(genericly) all matrix elements of** $g_{\mu\nu}$ **go to infinity.** And so near by all distances between the point in the projective space become huge.

Characterization of Projective line as 3-transitive

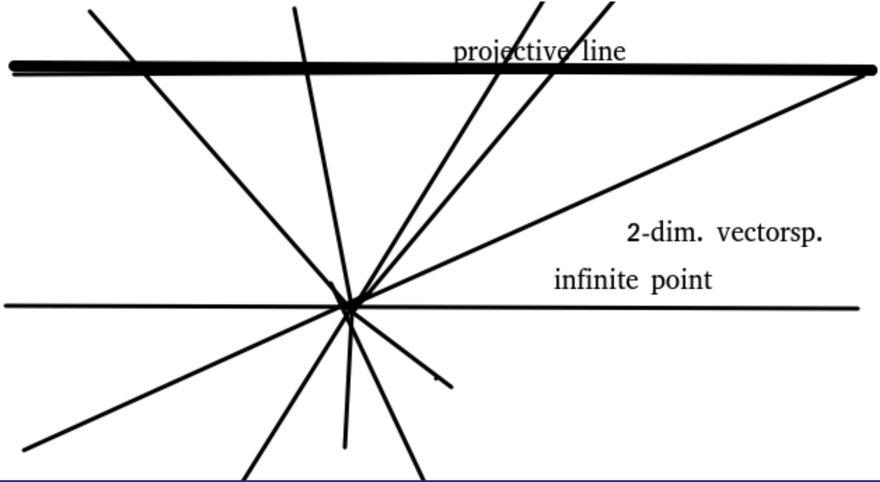
In last years Bled talk I presented a work with Masao Ninomiya[2], in which we showed that requiring for a group acting on space X in sharply 3-transitive way, essentially led you to the projective line ($=$ a one dimensional projective space.

Hope to somehow characterize projevtive spaces by some form of n-transitivity (may be next years talk?)

Projective line

The projective line is the real axis extended with one point at infinity.

Projective space of d dimension as set of Rays in Vector space of $d+1$



Conclusion

- Screwed logic: If we want to use Astris and my way to derive locallity, then space-time must be diffeomorphism invariant, or it might still go with less symmetry, such as the **Projective space time**.
- With such diffeoemorphism or projective space symmetry, there would be no propagations of signals no waves, if there does not appear by spontaneous break down a $g^{\mu\nu}$ (with upper indices) being non-zero.
- So gravity is activity due to a needed for propagation, non-zero field.

Speculative Conclusion

Fundamentally we have for some reason a projective space or a manifold with diffeomorphism invariance - in any case a space-time with symmetry group acting in a practically n-transitive way with a high n - but then either a field $g^{\mu\nu}(x)$ or some corresponding vierbein fields $V_a^\mu(x)$ (also with upper curved indices) get non-zero in the vacuum. This makes possible propagation of waves/particles along the direction of the subspace of the tangent space spanned by this $g^{\mu\nu}(x)$ or these vierbeins $V_a^\mu(x)$. So at the end the end **the Einstein general relativity four-space is imbedded into the more fundamental general manifold or projective space.**

H. B. Nielsen and A. Kleppe (The Niels Bohr Institute, Copenhagen, Denmark and SACT, Oslo, Norway) "Towards a Derivation of Space" arXiv: 1403.1410v1 [hep-th] 6 Mar 2014.

H.B. Nielsen and A. Kleppe BLED WORKSHOPS IN PHYSICS VOL. 20, NO. 2 Proceedings to the 22nd Workshop What Comes Beyond . . . (p. 135) Bled, Slovenia, July 6–14, 2019
" Deriving Locality From Diffeomorphism Symmetry in a Fiber Bundle Formalism "

P. Bielewicz, K. M. Górski, A. J. Banday "Low-order multipole maps of cosmic microwave background anisotropy derived from WMAP", Monthly Notices of the Royal Astronomical Society, Volume 355, Issue 4, December 2004, Pages 1283–1302, https://doi.org/10.1111/j.1365-2966.2004.08405.x

- Stillits, Thesis for Cand.Scient.degree at The Niels Bohr Insitute.
- H.B. Nielsen, Masao NinomiyaNovel “String Field Theory and Bound State, Projective Line, and sharply 3-transitive group” Proceedings contribution to the 24th Bled-Workshop "What comes beyond the Standard Models” Bled, July 3-11, 2021, Slovenia; arXiv:2111.05106 [physics.gen-ph] (or arXiv:2111.05106v1 [physics.gen-ph] for this version) https://doi.org/10.48550/arXiv.2111.05106