Quantum Electrodynamics: A Few Additions to a Jewel of Theoretical Physics

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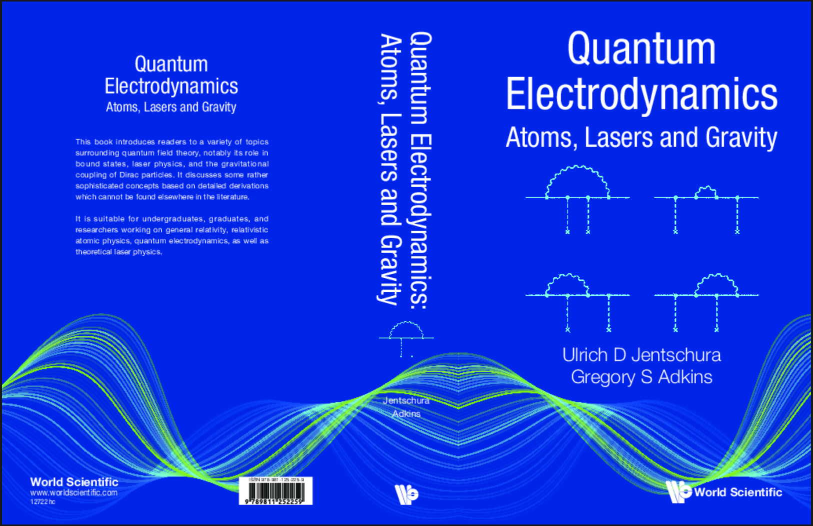
Abstract

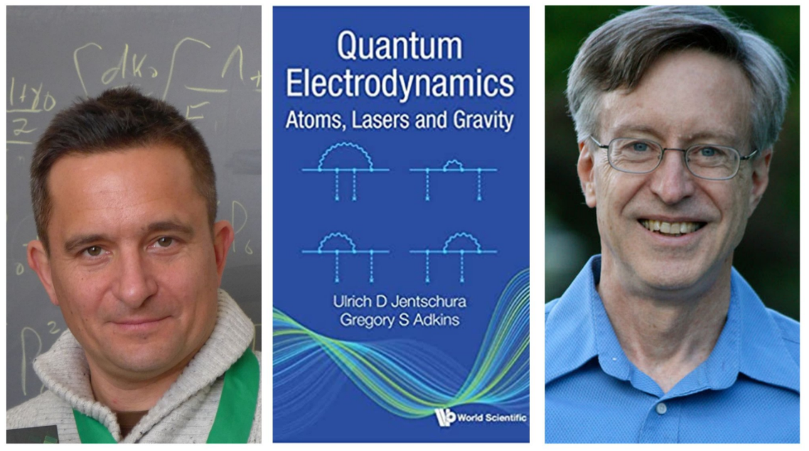
The purpose of this talk is to introduce a few new results recently presented in [U. D. Jentschura and G. S. Adkins, Quantum Electrodynamics: Atoms, Lasers and Gravity *(World Scientific, Singapore, 2022)]. The development of quantum electrodynamics started when Bethe, Feynman and Schwinger, and Tomonaga, developed the concept of renormalized quantum field theory, to deal with the infinities that arose in perturbative calculations of scattering processes. The characteristic element of the calculations was the emergence of so-called loop corrections, which describe the self-interaction of the quantum fields. The application of the formalism to bound states is marred with additional difficulties, due to the presence of two distinct energy scales, which have to be matched at the end of any precise calculation. Nevertheless, the theory has enabled theorists to calculate transition energies in simple atomic systems like hydrogen and helium to unprecendented accuracy, approaching 13 or 14 decimals. As an example of higher-order calculations which could further enhance our understanding of bound systems, the eighth-order Foldy–Wouthuysen transformation will be described. Our talk also focuses on searches for physics beyond the Standard Model (proton radius puzzle, X17 boson) which have an overlap with current precision atomic physics. In particular, prospects for a definitive resolution of the proton radius puzzle will be discussed.*

The Book: An “Advertisement”

Textbook and Monograph

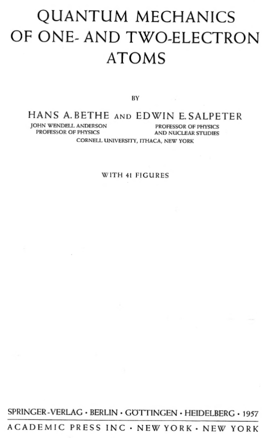


Textbook and Monograph



Background

For more than 60 years, the book of Hans Albrecht Bethe (portrait, left) and Edwin Ernest Salpeter (portrait, right) has been a cornerstone in the description of few-body atomic systems. It is a masterpiece.


QUANTUM MECHANICS
OF ONE- AND TWO-ELECTRON
ATOMS

BY

HANS A. BETHE AND EDWIN E. SALPETER
JOHN WENDELL ANDERSON PROFESSOR OF PHYSICS
PROFESSOR OF PHYSICS AND NUCLEAR STUDIES
CORNELL UNIVERSITY, ITHACA, NEW YORK

WITH 41 FIGURES

SPRINGER-VERLAG · BERLIN · GÖTTINGEN · HEIDELBERG · 1957
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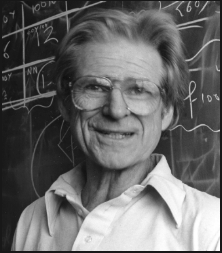


Perhaps, a Little Warning

=== Bound-State Quantum Electrodynamics ===

may belong to the more difficult, and more technically demanding subfields of theoretical physics. The field combines the intricacies of modern quantum field theory (including the concepts of regularization and renormalization) with the additional technical challenges of the bound-state formalism.

One needs a specific mindset to work on the subject.

Still, low-energy precision experiments based on quantum electrodynamics serve as a tool to look for effects beyond the Standard Model, in or from the low-energy sector.

Table of Contents (812 Pages, 19 Chapters)

1. (I) Chapters 1–6: Advanced Quantum Mechanics Toward Field Quantization
(Introduction; From Unit Systems for the Microworld to Field Quantization; Time–Ordered Perturbations; Bound–Electron Self–Energy and Bethe Logarithm; Interatomic and Atom–Surface Interactions; Racah–Wigner Algebra)
2. (II) Chapters 7–10: From Relativistic Quantum Mechanics to QED
(Free Dirac Equation; Dirac Equation for Bound States, Lasers and Gravity; Electromagnetic Field and Photon Propagators; Tree–Level and Loop Diagrams, and Renormalization)
3. (III) Chapters 11–17: QED and Bound States
(Foldy–Wouthuysen Transformation and Lamb Shift; Relativistic Interactions for Many–Particle and Compound Systems; Fully Correlated Basis Sets and Helium; Relativistic Many–Particle Calculations; Beyond Breit Hamiltonian and On–Shell Form Factors; Bethe–Salpeter Equation; NRQED: An Effective Field Theory for Atomic Physics)
4. (IV) Chapters 18+19: Concepts of Quantum Field Theory and QED
(Fermionic Determinants and Effective Lagrangians; Renormalization–Group Equation)

Examples: Some Highlights, Perhaps

Calculation of Bethe Logarithms

"Almost analytic" representation of the Bethe logarithms:

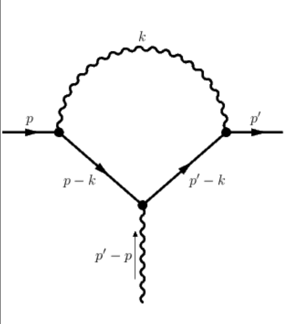
$$\begin{aligned}\ln k_0(1S) &= 10\,\ln(2) - 2\zeta(2) - 1 \\ &\quad + \sum_{k=2}^{\infty} \frac{16\,k}{(k-1)^2\,(k+1)^2}\,\Phi\left(\frac{1+k}{1-k}, 1, 2\,k\right) \\ &= 2.98412\,85557\,65497\,61075\,97770\,90013\,79796\,99751 \\ &\quad 80566\,17002\,00048\,15926\,13924\,06576\,62306\,75532 \\ &\quad 86860\,62013\,30404\,72249\,.\end{aligned}$$

- The Bethe logarithm is a sum over virtual excitations of the hydrogen atom, where the excitation energies enter the sum in terms of their logarithms. Hans Bethe came up with the concept in 1947, in order to explain the splitting between the $2S_{1/2}$ and $2P_{1/2}$ energy levels in hydrogen observed by Willis Lamb. The Bethe logarithm contributes to the *leading-order self-energy*.
- The formula given above (Φ is the Lerch transcendent or *incomplete zeta function*) is the result of an improved understanding of the mathematical structure of the Schrödinger–Coulomb Green function, which is explained in the book. It can be used in order to search for closed-form expressions, using the PSLQ algorithm.

Vertex, Loop and Electron Form Factors

We use dimensional regularization. $d = 4 - 2\varepsilon$, $D = 3 - 2\varepsilon$.
(A bit of Humor: "This was not available to Landau fifty years ago.")
Recommend $\overline{\text{MS}}$ scheme:

$$e^2 = (4\pi)^{1-\varepsilon}\,\alpha\,\mu^{2\varepsilon}\,\mathrm{e}^{\gamma_E\varepsilon}\,.$$



(Detailed Derivation!)
(Nothing Swept Under Rug!)

Vertex function:

$$\Gamma^\nu_{\mathrm{R}}(q) = F_1(q^2)\,\gamma^\nu + F_2(q^2)\,\frac{\mathrm{i}}{2m}\sigma^{\nu\mu}q_\mu\,,$$

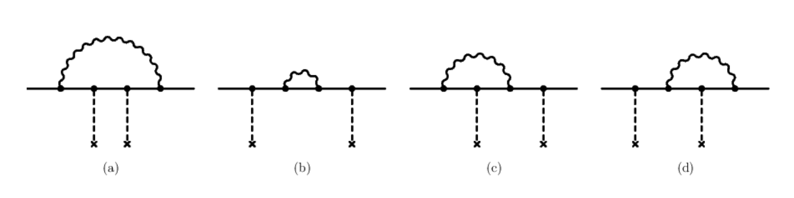
$$F_1(q^2) = 1 + \frac{\alpha}{\pi}\left[\frac{q^2}{m^2}\left(-\frac{1}{8} - \frac{1}{6\varepsilon} + \frac{1}{6}\,\ln\left(\frac{m^2}{\mu^2}\right)\right) + \mathcal{O}\left(\frac{q^4}{m^4}\right)\right],$$

$$F_2(q^2) = \frac{\alpha}{2\pi}\left[1 + \frac{1}{6}\frac{q^2}{m^2} + \frac{1}{30}\frac{q^4}{m^4} + \mathcal{O}\left(\frac{q^6}{m^6}\right)\right].$$

Surprise: Dimensional regularization is easier than Pauli–Villars.
Also: Lamb shift using dimensional regularization.

Binding Corrections to the Lamb Shift and Forward Scattering

- Diagrams:



- The original result was due to Bethe, Baranger and Feynman:

$$\Delta E = \frac{\alpha}{\pi} \frac{(Z\alpha)^5 m}{n^3} A_{50}(nL_j)\,, \qquad A_{50}(nL_j) = 4\pi\,\delta_{L0} \left(\frac{139}{128} - \frac{1}{2}\ln(2) \right) .$$

- Modern Approach: Dispersion Relation! Idea: Let the incoming Coulomb momentum, which is initially space-like, $q^2 = -\vec{q}^2 \leq 0$, continue into the time-like domain, $q^2 \to Q^2 > 4m^2$, where the expression for the energy correction develops a branch cut in the complex plane. Then, write a dispersion relation which connects the cut to real energy shift. This simplifies the calculation dramatically.

General Results for the 8th–Order Foldy–Wouthuysen Transformation

Result for the eigth-order terms
(Obtained using advanced computer algebra):

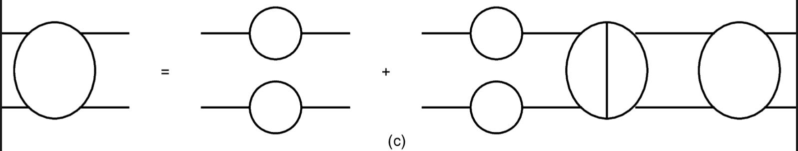
$$
\begin{aligned}
H^{[8]} = & -\frac{5}{128m^7}(\vec{\sigma}\cdot\vec{\pi})^8 - \frac{\mathrm{i}\,e^2}{32m^4}\left[\vec{\sigma}\cdot\vec{E}, \vec{\sigma}\cdot\partial_t\vec{E}\right] \\
& + \frac{7\,e^2}{192m^5}\left[\vec{\sigma}\cdot\vec{\pi}, \vec{\sigma}\cdot\vec{E}\right]\left[\vec{\sigma}\cdot\vec{\pi}, \vec{\sigma}\cdot\vec{E}\right] - \frac{3\,e^2}{64m^5}\left\{\vec{\sigma}\cdot\vec{\pi}, \vec{\sigma}\cdot\vec{E}\right\}\left\{\vec{\sigma}\cdot\vec{\pi}, \vec{\sigma}\cdot\vec{E}\right\} \\
& - \frac{e^2}{24m^5}\left[\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, (\vec{\sigma}\cdot\vec{E})^2\right]\right] + \frac{e^2}{48m^5}\left\{(\vec{\sigma}\cdot\vec{\pi})^3, \vec{\sigma}\cdot\partial_t\vec{E}\right\} \\
& - \frac{5\mathrm{i}e}{1024m^6}\left[\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, \vec{E}\right]\right]\right]\right]\right] \\
& - \frac{\mathrm{i}\,e}{32m^6}\left\{\vec{\sigma}\cdot\vec{\pi}, \left\{\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, \vec{E}\right]\right]\right]\right\}\right\} \\
& - \frac{\mathrm{i}\,e}{48m^6}\left\{\vec{\sigma}\cdot\vec{\pi}, \left\{\vec{\sigma}\cdot\vec{\pi}, \left\{\vec{\sigma}\cdot\vec{\pi}, \left\{\vec{\sigma}\cdot\vec{\pi}, \left[\vec{\sigma}\cdot\vec{\pi}, \vec{E}\right]\right\}\right\}\right\}\right\}.
\end{aligned}
$$

The general results for the eighth-order transformation could be the starting point for a series in improvements of theoretical predictions.
Computer algebra, including the somewhat sophisticated mapping of operator algebras, was used in the derivation.

Relativistic Recoil Correction (Salpeter)

- Way I (Chapter 15): Ad hoc approach, matching scattering amplitudes
- Way II (Chapter 16): Ab initio, relativistic Bethe–Salpeter equation

$$G = S + S\,K\,G$$



- Way III (Chapter 17): From an Effective Field Theory (NRQED)
  (There is a Bethe–Salpeter Equation of NRQED!)
  (It corresponds to the Schrödinger equation!)

Precision Experiments and Theory: Candidates for BSM Effects

Proton Radius Puzzle

X17 Boson

Proton Radius Puzzle

Enigma Since 2011: In a Nutshell

- Spectroscopic results can be “reverse engineered” in order to find the proton radius.
- Measurements of the Lamb shift in muonic hydrogen (2011, Paul–Scherrer Institute) yield a result of about $r_p \approx 0.84\,\text{fm}$.
- Measurements of the Lamb shift in ordinary hydrogen by the Paris group (Biraben and Nez, with notable results communicated in 1998 and 2018) yield a result of about $r_p \approx 0.88\,\text{fm}$.
- A recent measurement of a $2S$–$8D$ transition in ordinary hydrogen, completed at Colorado State University in 2022, yields a result of $r_p \approx 0.86\,\text{fm}$.
- The results of electron scattering experiments at Mainz (2010, $r_p \approx 0.88\,\text{fm}$) and PRad Brookhaven (2019, $r_p \approx 0.84\,\text{fm}$) are not in mutual agreement.
- Two PRLs from 1969 (experiments at Brookhaven) suggest that there could be a non-universality in muon-proton versus electron-proton scattering cross sections.
- The situation is unclear and the MUSE experiment at PSI (and further spectrocopic results) could shed s more light on the problem.

Forgotten Physical Review Letters (plural!) from 1969

PRL 23, 153 (1969) published on 21–JUL–1969
(coincidentally, the precise day when mankind set foot on the moon)

VOLUME 23, NUMBER 3 PHYSICAL REVIEW LETTERS 21 JULY 1969

HIGH-ENERGY MUON-PROTON SCATTERING: MUON-ELECTRON UNIVERSALITY*

L. Camilleri,† J. H. Christenson, M. Kramer,‡ and L. M. Lederman
Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

and

Y. Nagashima and T. Yamanouchi
University of Rochester, Rochester, New York 14627,
and Brookhaven National Laboratory, Upton, New York 11973
(Received 10 April 1969)

Measurements of the μ-p elastic cross section in the range $0.15 < q^2 < 0.85$ $(\mathrm{GeV}/c)^2$ are compared with similar e-p data. We find an apparent disagreement between the muon and electron experiments which can possibly be accounted for by a combination of systematic normalization errors.

Could this be a hint for new physics?
The signal seen in 1969 matches the discrepancy seen in the proton radius derived from hydrogen versus muonic hydrogen spectroscopy (both sign and magnitude).

Forgotten Physical Review Letters (plural!) from 1969

PRL 23, 149 (1969) published on 21–JUL–1969
(coincidentally, the precise day when mankind set foot on the moon)

HIGH-ENERGY MUON-PROTON SCATTERING: ONE-PHOTON EXCHANGE TESTS*

L. Camilleri,† J. H. Christenson, M. Kramer,‡ and L. M. Lederman
Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

and

Y. Nagashima and T. Yamanouchi
University of Rochester, Rochester, New York 14627
and Brookhaven National Laboratory, Upton, New York 11973
(Received 10 April 1969)

Muon-proton elastic scattering has been studied in the range $0.15 < q^2 < 0.85\ (\mathrm{GeV}/c)^2$ with μ^+ and μ^- beams of 6 and 11 GeV/c and a μ^- beam of 17 GeV/c. Cross sections have been determined with uncertainties as small as 2%. Rosenbluth straight-line plots and comparisons of the μ^+ and μ^- cross sections show no deviation from the one-photon exchange approximation.

Could this be a hint for new physics?
The signal seen in 1969 was carefully checked for the validity of the one-photon approximation (Rosenbluth approximation):

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}q^2}\right|_{\mu,e} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}q^2}\right|_{\mathrm{NS}} \frac{1}{\cot^2(\theta/2)} \left[2\tau G_M(q^2) + \frac{G_E^2(q^2) + \tau\, G_M(q^2)}{1+\tau}\, \cot^2(\theta/2)\right]$$

Non–Universality of about 4 % Seen in 1969

Plot from PRL 23, 153 (1969)
[Sachs form factor, muon versus electron scattering]:

VOLUME 23, NUMBER 3 PHYSICAL RE

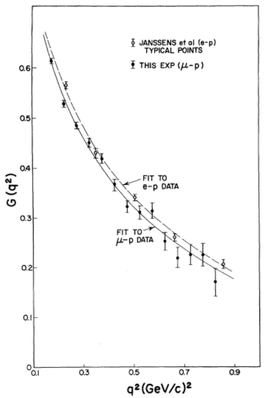


FIG. 1. Measurements of the form factor $G(q^2)$ vs q^2 for this experiment and for the e-p data of Janssens et al. Not all of the electron data are shown. The solid and dashed curves represent fits to the muon and electron data, respectively.

8% change in the cross sections
4% change in the form factors
2% change in the rms radius

$$\langle r^2 \rangle_p = r_p^2 = 6\hbar^2 \left. \frac{\partial G_E(q^2)}{\partial q^2} \right|_{q^2=0}$$

Could explain, e.g., a difference

$$\left. \sqrt{\langle r^2 \rangle_p} \right|_e \approx 0.86\,\mathrm{fm}\,,$$

$$\left. \sqrt{\langle r^2 \rangle_p} \right|_\mu \approx 0.84\,\mathrm{fm}\,,$$

[U.D.J., J.Phys.Conf.Ser. **2391**, 012017 (2022)]

Two (Perhaps, Three) Approaches to the Proton Radius Determination

Way #1: Scattering Experiments

$$\langle r^2 \rangle_p = r_p^2 = 6\hbar^2 \left. \frac{\partial G_E(q^2)}{\partial q^2} \right|_{q^2=0}$$

Way #2A: Muonic Hydrogen Spectroscopy

$$\Delta E = \frac{2}{3} \frac{(Z\alpha)^4 \mu c^2}{\pi n^3} \left(\frac{\mu c r_p}{\hbar} \right)^2$$

Way #2B: Hydrogen Spectroscopy

We have $Z = 1$. The reduced mass μ is roughly 200 times larger for muonic bound systems as compared to ordinary hydrogen. The finite-size effect is proportional to μ^3, and thus, muonic hydrogen is very sensitive probe of the proton radius.

(Not Yet Published) Reevaluation of Two–Loop Vacuum Polarization

Muonic bound systems are sensitive to vacuum-polarization corrections. For a theoretical overview: [UDJ, Ann. Phys. (N.Y.) **326**, 500 (2011)]

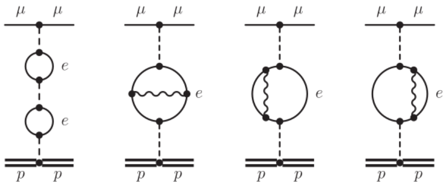


Fig. 1. Two-loop vacuum-polarization diagrams.

The two-loop vacuum-polarization correction contributes an energy shift of about 1.5081 meV to the muonic hydrogen Lamb shift and had never been reevaluated beyond the classic works of Kallen and Sabry (1955), and Barbieri and Remiddi (1973). A reevaluation using dimensional regularization and integration-by-parts techniques (S. Laporta and UDJ, in preparation) sheds additional light on the problem, in view of a comparison to a proton size puzzle of 0.3 meV in the muonic hydrogen Lamb shift.

Muonic Hydrogen Measurement [2010]: Lamb Shift

2010 measurement: $r_p = 0.84184(67)\,\text{fm} \approx 0.84\,\text{fm}$.
Nature **466**, 213 (2010):

Vol 466 | 8 July 2010 | doi:10.1038/nature09250 nature

LETTERS

The size of the proton

Randolf Pohl[1], Aldo Antognini[1], François Nez[2], Fernando D. Amaro[3], François Biraben[2], João M. R. Cardoso[3], Daniel S. Covita[3,4], Andreas Dax[5], Satish Dhawan[5], Luis M. P. Fernandes[3], Adolf Giesen[6]†, Thomas Graf[6], Theodor W. Hänsch[1], Paul Indelicato[2], Lucile Julien[2], Cheng-Yang Kao[7], Paul Knowles[8], Eric-Olivier Le Bigot[2], Yi-Wei Liu[7], José A. M. Lopes[3], Livia Ludhova[8], Cristina M. B. Monteiro[3], Françoise Mulhauser[8]†, Tobias Nebel[1], Paul Rabinowitz[9], Joaquim M. F. dos Santos[3], Lukas A. Schaller[8], Karsten Schuhmann[10], Catherine Schwob[2], David Taqqu[11], João F. C. A. Veloso[4] & Franz Kottmann[12]

present calculations[11–15] of fine and hyperfine splittings and QED terms, we find $r_p = 0.84184(67)$ fm, which differs by 5.0 standard deviations from the CODATA value[3] of 0.8768(69) fm. Our result implies that either the Rydberg constant has to be shifted by $-110\,\text{kHz}/c$ (4.9 standard deviations), or the calculations of the QED effects in atomic hydrogen or muonic hydrogen atoms are insufficient.

Recent Hydrogen Measurement [2018, French]: $1S$–$3S$

2018 measurement [Paris]: $r_p = 0.877(13)\,\mathrm{fm}$. Oops...

PHYSICAL REVIEW LETTERS **120,** 183001 (2018)

New Measurement of the $1S-3S$ Transition Frequency of Hydrogen: Contribution to the Proton Charge Radius Puzzle

Hélène Fleurbaey, Sandrine Galtier,* Simon Thomas, Marie Bonnaud,
Lucile Julien, François Biraben, and François Nez
*Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-Université PSL,
Collège de France, 4 place Jussieu, Case 74, 75252 Paris Cedex 05, France*

Michel Abgrall and Jocelyne Guéna
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(Received 8 December 2017; revised manuscript received 9 March 2018; published 4 May 2018)

We present a new measurement of the $1S-3S$ two-photon transition frequency of hydrogen, realized with a continuous-wave excitation laser at 205 nm on a room-temperature atomic beam, with a relative uncertainty of 9×10^{-13}. The proton charge radius deduced from this measurement, $r_p = 0.877(13)$ fm, is in very good agreement with the current CODATA-recommended value. This result contributes to the ongoing search to solve the proton charge radius puzzle, which arose from a discrepancy between the CODATA value and a more precise determination of r_p from muonic hydrogen spectroscopy.

[The experimental approach taken by the Paris group should be largely independent of cross-damping terms.]

Recent Hydrogen Measurement [2019, Canada]: $2S$–$2P_{1/2}$

2019 measurement [Toronto]: $r_p = 0.833(10)\,\mathrm{fm}$.
Leads to a small "Canadian proton".

ATOMIC PHYSICS

A measurement of the atomic hydrogen Lamb shift and the proton charge radius

N. Bezginov[1], T. Valdez[1], M. Horbatsch[1], A. Marsman[1], A. C. Vutha[2], E. A. Hessels[1]*

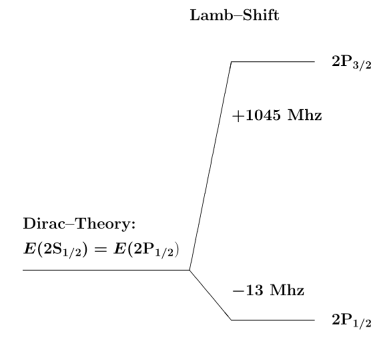
The surprising discrepancy between results from different methods for measuring the proton charge radius is referred to as the proton radius puzzle. In particular, measurements using electrons seem to lead to a different radius compared with those using muons. Here, a direct measurement of the $n = 2$ Lamb shift of atomic hydrogen is presented. Our measurement determines the proton radius to be r_p = 0.833 femtometers, with an uncertainty of ±0.010 femtometers. This electron-based measurement of r_p agrees with that obtained from the analogous muon-based Lamb shift measurement but is not consistent with the larger radius that was obtained from the averaging of previous electron-based measurements.

Bezginov *et al.*, *Science* **365**, 1007–1012 (2019)   6 September 2019

[1]Department of Physics and Astronomy, York University, Toronto, Ontario M3J 1P3, Canada. [2]Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada.
*Corresponding author. Email: hessels@yorku.ca

Lamb Shift and Fine Structure [Canada, Perhaps a Caveat]

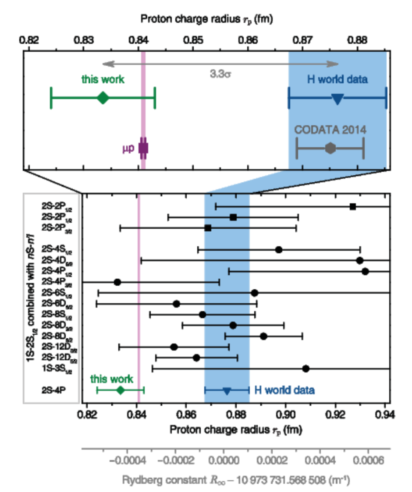
Hydrogen spectrum ($n = 2$ manifold, without hyperfine structure):



Perhaps a little caveat: The $2P_{1/2}$–$2P_{3/2}$ fine-structure is nearly independent of the proton radius and can be calculated to very high precision; its measurement would constitute an important consistency check for the smallness of the "Canadian protons".

"French versus Canadian and German Protons"

One might ask, jokingly:
"Are French protons larger than German and Canadian protons?"
Blue: *Decades* of work of the French [Paris] group
Green: Result of the 2017 measurement of the Garching group
[Science **358**, 79 (2017)] (in agreement with the Toronto measurement)



Recent Hydrogen Measurement [2022]: $2S$–$8D$

2022 measurement: $r_p = 0.8584(51)\text{fm} \approx 0.86\,\text{fm}$.
Colorado State University with support from the Russian Quantum Center.
“Size of American protons between German/Canadian and French ones.”

PHYSICAL REVIEW LETTERS **128,** 023001 (2022)

Measurement of the $2S_{1/2} - 8D_{5/2}$ Transition in Hydrogen

A. D. Brandt,[1] S. F. Cooper,[1] C. Rasor,[1] Z. Burkley,[1] A. Matveev,[2] and D. C. Yost[1,*]
[1]*Department of Physics, Colorado State University, Fort Collins, Colorado 80523, USA*
[2]*Russian Quantum Center, Skolkovo, Moscow 143025, Russia*

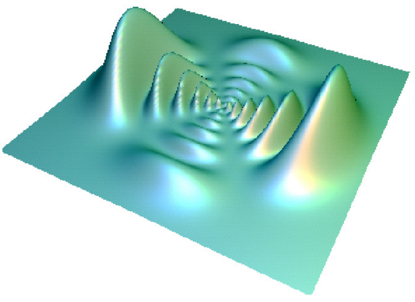
(Received 26 September 2021; revised 15 November 2021; accepted 7 December 2021; published 13 January 2022)

We present a measurement of the hydrogen $2S_{1/2} - 8D_{5/2}$ transition performed with a cryogenic atomic beam. The measured resonance frequency is $\nu = 770649561570.9(2.0)$ kHz, which corresponds to a relative uncertainty of 2.6×10^{-12}. Combining our result with the most recent measurement of the $1S - 2S$ transition, we find a proton radius of $r_p = 0.8584(51)$ fm and a Rydberg constant of $R_\infty = 10973731.568332(52)\ \text{m}^{-1}$. This result has a combined 3.1σ disagreement with the Committee on Data for Science and Technology (CODATA) 2018 recommended value.

DOI: 10.1103/PhysRevLett.128.023001

Recent Hydrogen Measurement [2022]: $2S$–$8D$

by the way... example of a higher-order coefficient:
$A_{60}(8D_{5/2}) = 0.034\,607\,492(1)$
[see U.D.J., E.-O. Le Bigot, P. J. Mohr, P. Indelicato, G. Soff, Phys. Rev. Lett. **90**, 163001 (2003)]



X17 Boson

Perhaps New Physics!?!?! (Group of Attila Krasznahorkay, ATOMKI)

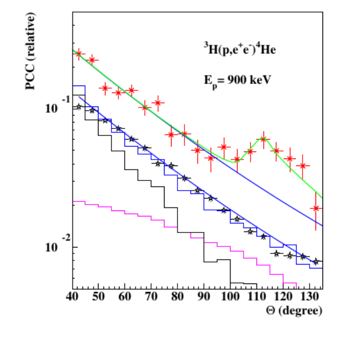
Beryllium (2016):

$$^{7}\mathrm{Li} + p \to {}^{8}\mathrm{Be}^{*} \to {}^{8}\mathrm{Be} + \gamma \to {}^{8}\mathrm{Be} + e^{+}e^{-} \qquad (1^{+} \to 0^{+})$$

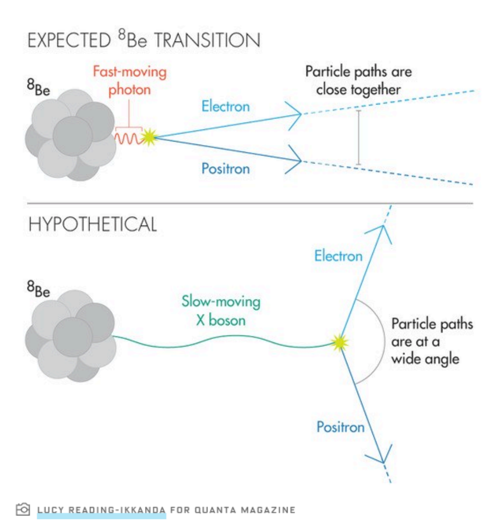
Helium (2019):

$$^{3}\mathrm{H} + p \to {}^{4}\mathrm{He}^{*} \to {}^{4}\mathrm{He} + \gamma \to {}^{4}\mathrm{He} + e^{+}e^{-} \qquad (0^{-} \to 0^{+})$$

From arXiv:1910.10459:



Caricature



Mass of new particle: about 17 MeV.

Possible Theoretical Explanations

Group of Jonathan Feng (PRL, 2016):
"The X17 might be a protophobic vector boson."

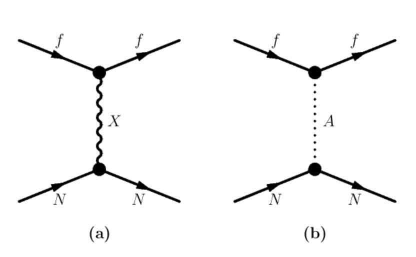
Paper of Ellwanger and Moretti (JHEP, 2016):
"The X17 might be a light pseudoscalar boson."

Let us remember that in atomic physics precision experiments, we would actually like to see deviations of experimental observations from experiments attributable to "new physics". This has been a significant motivation pushing the theoretical and experimental efforts for the last couple of decades.

Recent attempts at alternative explanations for the Hungarian observations, but noone has carried out any experiment. Room for improvement: Angular resolution, in Hungary!

Light Vector and Pseudoscalar Particles and Atomic Physics

We investigate, **irrespective of the Hungarian experimental results**, what the effect, what the effect of a light (mass in the approximate range from 10 MeV to 100 MeV) vector or pseudoscalar new particle is for atomic-physics experiments.



Here, f denotes the bound fermion (typically, an electron or a muon) and N denotes the atomic nucleus. (Inspired by the mentioned theoretical papers of Feng *et al.*, and of Ellwanger and Moretti.)
Unfortunately, the mass range of 17 MeV (give or take) is quite problematic for atomic-physics studies, because the Yukawa potentials are almost indistiguishable from a nuclear-size effect for electronic bound states. Way out: study muonic systems.

Effective Hamiltonian for Vector Boson Exchange

Vector exchange leads to the following contribution to HFS:

$$H_{\mathrm{HFS},V} = \frac{h'_f\, h'_N}{16\,\pi\, m_f\, m_N} \left[-\frac{8\pi}{3}\delta^{(3)}(\vec{r})\,\vec{\sigma}_f \cdot \vec{\sigma}_N \right.$$
$$- \frac{m_X^2\, \left(\vec{\sigma}_f \cdot \vec{r}\,\vec{\sigma}_N \cdot \vec{r} - r^2\,\vec{\sigma}_f \cdot \vec{\sigma}_N\right)}{r^3}\,\mathrm{e}^{-m_X\, r}$$
$$- (1 + m_X\, r)\,\frac{3\,\vec{\sigma}_f \cdot \vec{r}\,\vec{\sigma}_N \cdot \vec{r} - r^2\,\vec{\sigma}_f \cdot \vec{\sigma}_N}{r^5}\,\mathrm{e}^{-m_X\, r}$$
$$\left. - \left(2 + \frac{m_f}{m_N}\right)\,(1 + m_X\, r)\,\frac{\vec{\sigma}_N \cdot \vec{L}}{r^3}\mathrm{e}^{-m_X\, r} \right].$$

Derivation:
[Phys. Rev. A **101**, 062503 (2020)]

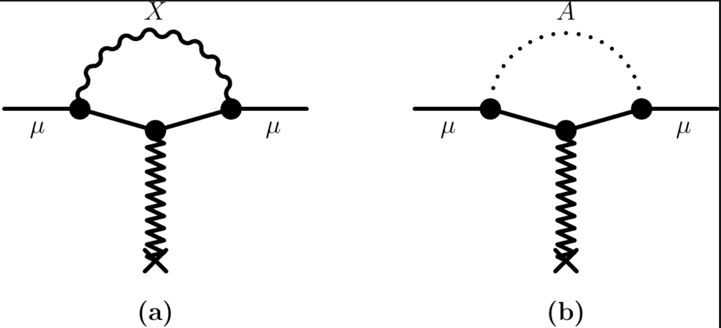
Effective Hamiltonian for Pseudoscalar Boson Exchange

Pseudoscalar exchange exclusively contributes to the HFS:

$$H_{\mathrm{HFS},A} = \frac{h_f\, h_N}{16\,\pi\, m_f\, m_N} \left[\frac{4\pi}{3} \delta^{(3)}(\vec{r})\, \vec{\sigma}_f \cdot \vec{\sigma}_N \right.$$
$$- \frac{m_X^2\, \vec{\sigma}_f \cdot \vec{r}\, \vec{\sigma}_N \cdot \vec{r}}{r^3}\, \mathrm{e}^{-m_X\, r}$$
$$\left. + (1 + m_X\, r)\, \frac{3\, \vec{\sigma}_f \cdot \vec{r}\, \vec{\sigma}_N \cdot \vec{r} - \vec{\sigma}_f \cdot \vec{\sigma}_N\, r^2}{r^5}\, \mathrm{e}^{-m_X\, r} \right] .$$

Leaves Lamb shift invariant!
[Phys. Rev. A **101**, 062503 (2020)]

Bound on the Muon Coupling Parameter



Vector model:

$$\hbar'_\mu = (\hbar'_\mu)_{\rm opt} = 5.6 \times 10^{-4}\,.$$

Pseudoscalar model:

$$\hbar_\mu = (\hbar_\mu)_{\rm max} = 3.8 \times 10^{-4}\,.$$

Enhancement of X17 Effects in Muonic Systems

Example: Relative correction to the S state splitting is

$$\frac{E_{X,V}(nS_{1/2})}{E_F(nS_{1/2})} \approx -\frac{2h'_f h'_N}{g_N \pi} \frac{Z\, m_r}{m_X},$$
$$\frac{E_{X,A}(nS_{1/2})}{E_F(nS_{1/2})} \approx \frac{h_f h_N}{g_N \pi} \frac{Z\, m_r}{m_X}.$$

Have the reduced mass m_r in the numerator after dividing by the leading-order Fermi splitting.
(Electronic systems: relative corrections to HFS of order 10^{-9}.)
(So: Concentrate on muonic systems)
Just to clarify:
The nuclear g factor g_N is used in a specific normalization [Phys. Rev. A **101**, 062503 (2020)].

Predictions for Muonic Deuterium

S states (with realistic estimates for coupling parameters):

$$\frac{E_{X,V}^{(\mu d)}(nS_{1/2})}{E_F(nS_{1/2})} \doteq 3.8 \times 10^{-6}\,,$$

$$\frac{E_{X,A}^{(\mu d)}(nS_{1/2})}{E_F(nS_{1/2})} \doteq -1.0 \times 10^{-6}\,.$$

P states:

$$\frac{E_{X,V}^{(\mu d)}(nP_{1/2})}{E_F(nP_{1/2})} \approx 2.5 \times 10^{-7} \left(1 - \frac{1}{n^2}\right)\,,$$

$$\frac{E_{X,A}^{(\mu d)}(nP_{1/2})}{E_F(nP_{1/2})} \approx 6.6 \times 10^{-8} \left(1 - \frac{1}{n^2}\right)\,.$$

This could be measurable but an enhanced understanding of nuclear polarization effects might be required for S states.
For P states, nuclear effects are strongly suppressed.

Predictions for True Muonium ($\mu^+\mu^-$)

Define

$$\chi_V(nS) = \frac{4}{7}\frac{E_{X,V}(nS)}{E_F(nS)} + \frac{3}{7}\frac{E_{\mathrm{ANN},V}(nS)}{E_{\mathrm{ANN},\gamma}(nS)}\,,$$
$$\chi_A(nS) = \frac{4}{7}\frac{E_{X,A}(nS)}{E_F(nS)} + \frac{3}{7}\frac{E_{\mathrm{ANN},A}(nS)}{E_{\mathrm{ANN},\gamma}(nS)}\,.$$

Obtain the estimates

$$\chi_V(nS) \mathrel{\dot{\approx}}\; 1.3\times 10^{-6}\,,$$
$$\chi_A(nS) \mathrel{\dot{\approx}}\; 2.1\times 10^{-6}\,.$$

This could very well be measurable; only a very moderate improvement of the accuracy of the predictions for hadronic vacuum polarization is required.

Conclusions

- Textbook/monograph is available.
  It covers aspects of atomic physics,
  advanced quantum mechanics, to quantum
  field theory and the Bethe–Salpeter equation.
- Proton radius puzzle: Requires additional
  experimental efforts for verification.
  Stimulates the evaluation of higher-order
  binding corrections to the Lamb shift.
- X17 boson:
  Possible addition to the low-energy sector
  of the Standard Model
  Could be detected in exotic atomic systems.
- Low-energy precision physics is one of the
  most promising instruments to detect conceivable
  low-energy additions to the Standard Model.

Thank You for Your Attention!

- Hope You Had Fun!
- Questions are Welcome!