The Stochastics Gravitational Waves Background.

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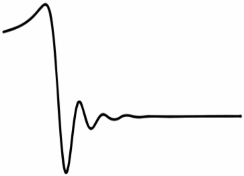
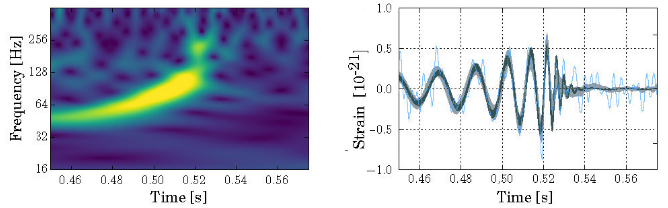


XXVI Bled Workshop 14th july, 2023.

Introduction

- Since 2015, a number of binary merger of two black holes, a black hole and a neutron star, and two neutron stars have been detected and well studied.
- The latter observation is followed by an electromagnetic spectrum observations, which gave rise to the field of gravitational wave multimessenger astronomy.



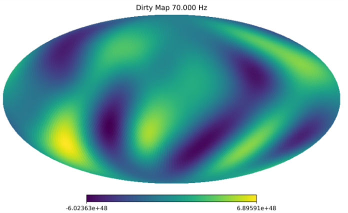
- In addition our Universe is surrounded by stochastic gravitational waves backround (SGWB).

Ligo

Motivation

To map the gravitational waves,

- *Core-collapse supernovae..*
- *Magnetars..*
- *Primordial Black holes..*



[Ain, Suresh & Mitra, PRD 98, 024001, 2018]

Sources and Detectors

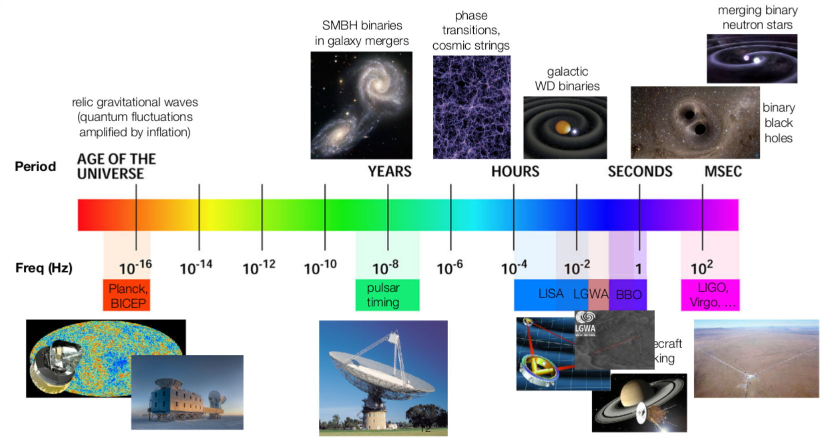


Figure: Several detectors and the corresponding sources to be detected.

[Joe's lectures;Les Houches Summer School 2018]

SGWB

The normalized GW energy spectrum is commonly used to describe the SGWB, as follows,

$$\Omega_{gw}(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d \ln f}. \quad (1)$$

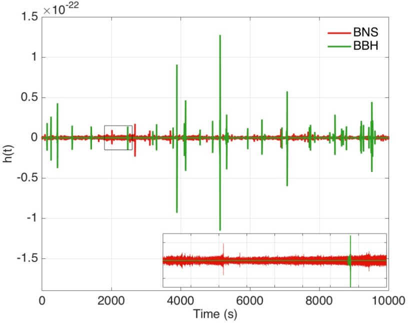
If the no of GW sources per unit comoving volume $R_V(z)$, then the above expresion becomes measured in the source frame time,

$$\Omega_{gw}(f) = \frac{f}{\rho_c H_0} \int_0^{\infty} \frac{R_V(z) dz}{(1+z) E(\Omega_m, \Omega_\lambda, z)} \frac{dE_{gw}}{df} \bigg|_{f(1+z)} \quad (2)$$

The redshift influence of the comoving volume is captured by $E(\Omega_m, \Omega_\lambda, z)$.

Many models give power law spectra in our band,

$$\Omega_{gw}(f) = \Omega_\alpha \left(\frac{f}{f_{ref}} \right)^{\alpha} \quad (3)$$



- individually undetectable (subthreshold)
- but detectable as a collectivity via their common influence on multiple detectors..
- combined signals describe statistically -stochastic gravitational wave background

Examples of Popcorn and Gaussian mixed signal. Popcorn for binary blackholes and gaussian for binary neutron stars.

[Abbott, B. P., et al., PRL 120, 091101, 2018]

Cross-Correlation

In a single gravitational wave detector, a stochastic background appears as noise.

- The signal s(t) from that detector would be the sum of the gravitational wave, h(t), and the noise, n(t).

$$s(t) = n(t) + h(t)$$

- A SGWB magnitude is much lower than the noise, n(t)$\gg$h(t).

- The only means of recognising a stochastic background is to take the correlation between two detector outputs.

$$< s_1(t)s_2(t) > \approx < h_1(t)h_2(t) >$$

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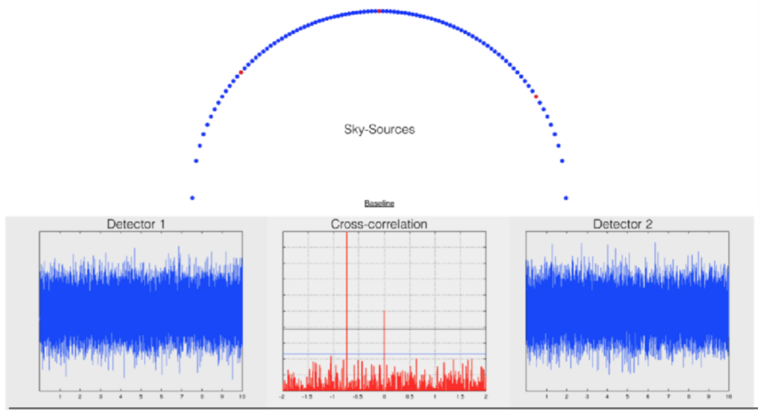
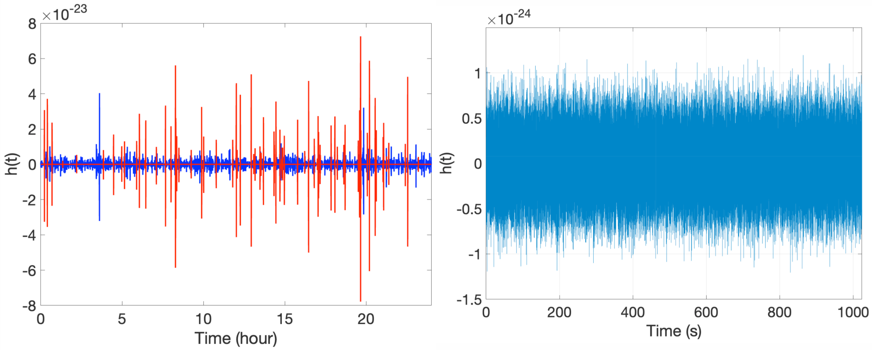


Figure: There are three sources of different strength (strong, medium and weak) marked in red.



Examples of interferometer timestreams populated by GWBs with different time-domain properties: Popcorn (left panel), and Gaussian (right panel).

[Regimbau, T., Symmetry 2022, 14(2), 270]

Types of SGWB

(i) can differ in spatial distribution

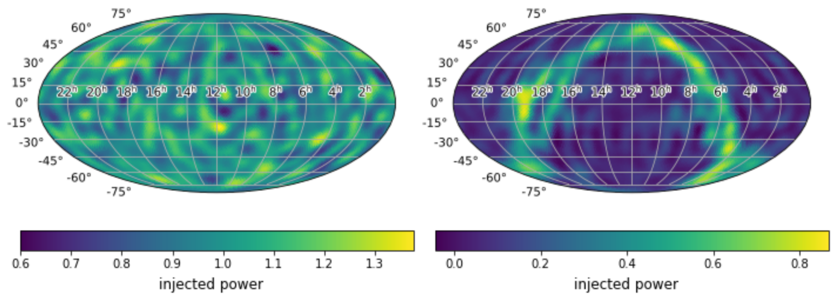
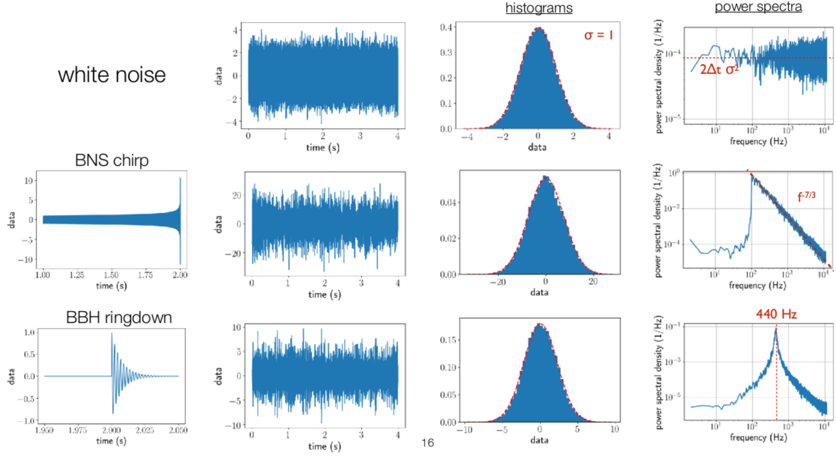


Figure: Simulated sky maps of GW power for a statistically isotropic background (left panel) and an anisotropic background (right panel).

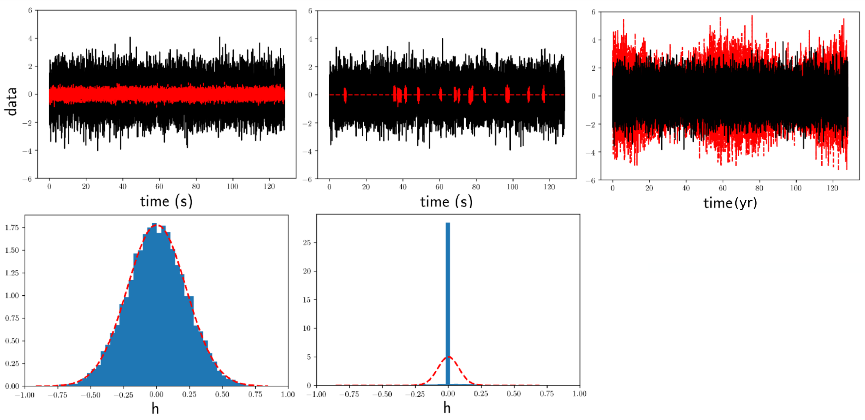
[Joe's lectures;Les Houches Summer School 2018]

(ii) can differ in their power spectra..



[Joe's lectures;Les Houches Summer School 2018]

(iii) can differ in temporal distribution and amplitude..



[Joe's lectures;Les Houches Summer School 2018]

Summary of sources

- the most probable backgrounds produced via cosmological or astrophysical phenomena..

- Cosmoloical Source
  - Cosmic String
  - Phase Transitions
  - Amplification of Vacuum Fluctuations
  - Colliding vacuum bubble
  - Inflation-produced gravitational

- Astophysical Sources
  - Binary Neutron Stars
  - Binary Black holes
  - Core-collapse supernovae
  - Magnetars
  - Primordial Blackhole mergers
  - Population III to Population II

Star formation rate

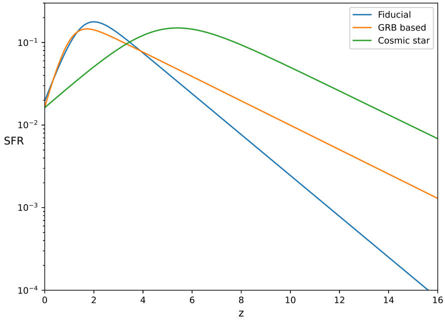


Figure: SFR as a function of redshift for different models explored here.

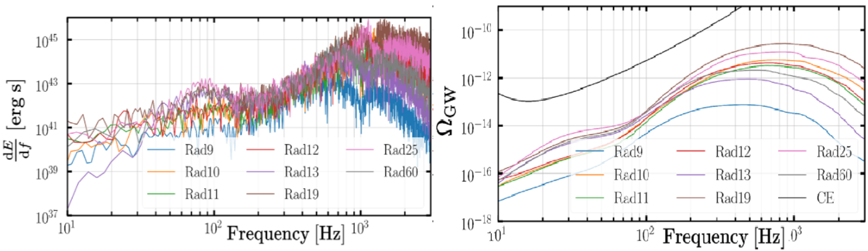
The star formation rate followed by the relation,

$$R_*(z) = \nu \frac{a \exp(b(z - z_m))}{a - b + b \exp(a(z - z_m))}$$

- Fiducial model:
$\nu = 0.178\ M_\odot yr^{-1} Mpc^{-3}, z_m = 2,$
$a = 2.37$ and $b = 1.8.$
- GRB data:
$\nu = 0.146\ M_\odot yr^{-1} Mpc^{-3}, z_m = 1.72,$
$a = 2.8$ and $b = 2.46.$
- Cosmic star:
$\nu = 0.15\ M_\odot yr^{-1} Mpc^{-3}, z_m = 5.4,$
$a = 0.933$ and $b = 0.66.$

Core-collapse supernovae

The simulation of core-collapse of seven non-rotating progenitors with ZAMS masses of 9 $M_\odot$, 10 $M_\odot$, 11 $M_\odot$, 12 $M_\odot$, 13 $M_\odot$, 19 $M_\odot$, 25 $M_\odot$, and 60 $M_\odot$.



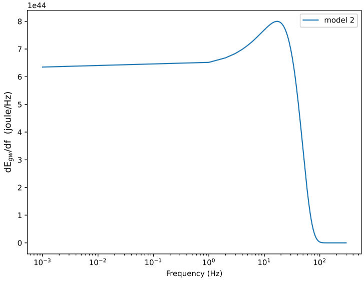
Total time-integrated GW spectra $\frac{dE}{df}$ and the corresponding Ω_{gw}.

[Radice, D., et al., Astrophys. J. Lett. 876, L9 (2019).]

The contribution of the energy spectrum from the core collapse of a single star which reflects the neutron star birth rate..

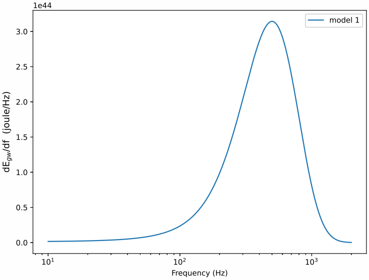
following the gaussian spectrum..

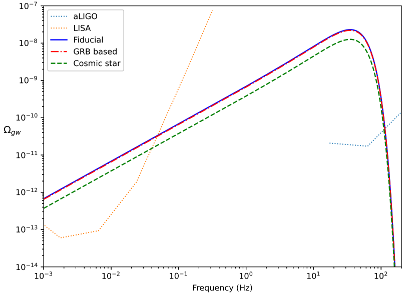
$$\frac{dE_{gw}}{df_e} = A \exp\left[-\frac{(f_e - f_0)^2}{2\Delta^2}\right],$$

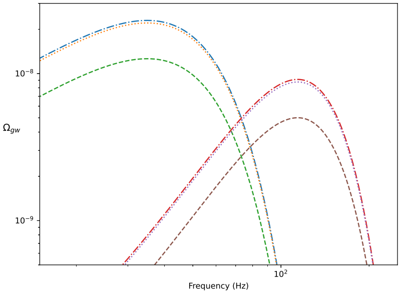


following the neutrino anisotroy spectrum..

$$\frac{dE_{gw}}{df_e} = \frac{G}{c^5} E_\nu^2 \exp(-2f_e/b)\left[1+\frac{f_e}{a}\right]^6,$$



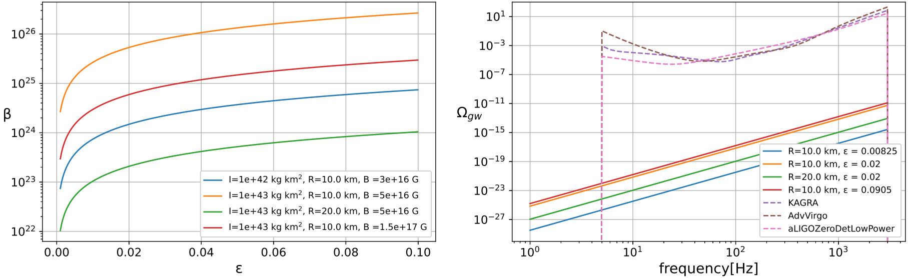




Characteristics strain of the core-collapse supernovae producing neutron stars.

- Ellipticity of the poloidal field configuration is

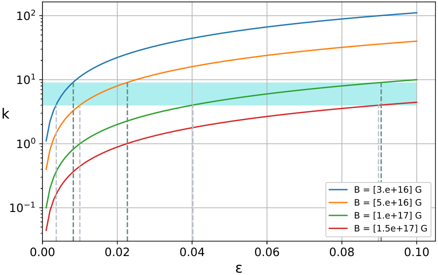
$$\epsilon = \beta \left(\frac{R^8 B^2}{4 G I^2} \right). \tag{4}$$

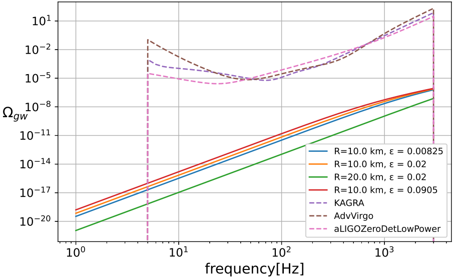


Expected evolution of the dimensionless parameter (β) and the corresponding $\Omega_{gw}(f)$.

- The ellipticity can be modeled for twisted-torus configuration with the help of a dimensionless parameter (k),

$$\epsilon = k\left(\frac{B}{10^{15}}\right)^2 \times 10^{-6}. \quad (5)$$





The dimensionless parameter (k) versus the ellipticity (e) and the corresponding $\Omega_{gw}(f)$.

Primordial Black holes

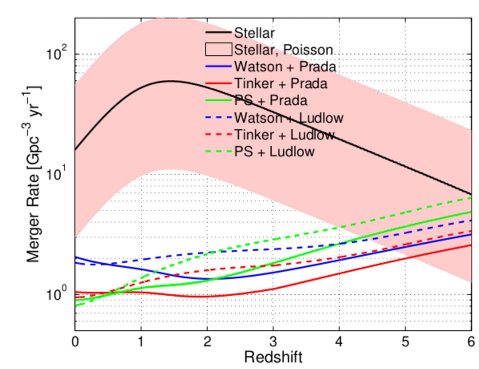


Figure: Primordial BBH merger rate per comoving volume as a function of redshift.

[Mandic, Bird, & Cholis PRL 117, 201102, 2016]

The Navarro-Frank-White (NFW) halo density model, which takes the form:

$$\rho_r = \frac{4\rho_s}{\frac{r}{R_s}(1+\frac{r}{R_s})^2}$$

Summary and Conclusion

- The astrophysical background is also promising because it would reveal details about the **physical characteristics** of compact objects and how they changed as **redshift increased**.
- If the background is dominated by local ($z \leq 2$) universe, it will be **anisotropic**
- Anisotropies exhibit a **range of variability** depending on the basic astrophysical model for star formation, distribution, and collapse.
- It is essential to model the astrophysical background as **accurately** as possible to extract data on its strength, frequency range, and statistical characteristics—anything that may help **differentiate** it from the cosmological signal or detached overlapping sources.
- A number of **ongoing or upcoming** experiments in **various frequency bands** promise thrills and surprises!

. . . and we expect many more weaker signals. . .